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ABSTRACT

Identification and Estimation of Continuous-Time Job Search Models with Preference Shocks*

This paper applies some of the key insights of dynamic discrete choice models to continuous-time job search models. Our framework incorporates preference shocks into search models, resulting in a tight connection between value functions and conditional choice probabilities. In this environment, we establish constructive identification of the model parameters, including the wage offer distributions off- and on-the-job. Our framework makes it possible to estimate nonstationary search models in a simple and tractable way, without having to solve any differential equations. We apply our method using Hungarian administrative data. Longer unemployment durations are associated with lower offer arrival rates, resulting in accepted wages falling over time. Counterfactual simulations indicate that increasing unemployment benefits by 90 days results in a 14-day increase in expected unemployment duration.

JEL Classification: J64, C31, C41, J31

Keywords: job search, identification, dynamic discrete choice

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1 Introduction

Canonical job search models typically involve workers accepting jobs that exceed a reservation wage (Burdett, 1978). When the search environment is nonstationary, for example when benefits expire after a certain duration, estimation is substantially complicated as the reservation wage changes over time (van den Berg, 1990). In addition, these strict cutoff rules make it difficult to accommodate the wage cuts that are prevalent in the data (Jolivet et al., 2006).

Motivated by these issues, this paper applies some of the key insights from the dynamic discrete choice literature to continuous-time job search models. The main idea of our approach is to adapt conditional choice probabilities (henceforth CCP) to a job search environment. To do so, we incorporate preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities. These shocks, which can be interpreted as the stochastic component of job switching costs, affect the instantaneous utility of accepting a particular job offer. As a result and consistent with existing empirical evidence that workers tend to accept particular job offers with probabilities that are significantly different from zero or one (Krueger and Mueller, 2016), future job offers associated with particular wages will be accepted only probabilistically from the perspective of the worker.

Our approach has two key advantages. The first is related to identification. We consider a class of nonstationary job search models that incorporate on-the-job search, non-pecuniary job attributes, and involuntary wage transitions. A key contribution of our paper is to establish constructive identification of all of the model parameters, up to the discount rate. Central to our identification strategy is the existence of preference shocks that allow us to trace out the full wage offer distribution from the observed job-to-job transitions, and then express the employment and unemployment value functions as known functions of the conditional probabilities of accepting particular job offers. Building on this framework, we derive transparent closed-form expressions for most of the model parameters, where the expressions depend on the hazard rates associated with the different types of labor market transitions.

The second advantage is computational. Although the empirical labor search literature is extensive and rapidly growing, structural estimation of these models often remains a challenge. This is particularly true for models in nonstationary environments, which tend to be the norm rather than the exception in the context of job search (van den Berg, 1990, 2001, Cahuc et al., 2014). We provide in this paper a novel empirical framework that makes it possible to estimate nonstationary job search models in a simple, tractable, and transparent way.

We illustrate our method using rich longitudinal administrative data from Hungary. The dataset consists of half of the population, i.e., 4.6 million individuals, who are linked across 900 thousand firms. An important feature of the Hungarian data is that individuals are observed on a monthly basis, making it possible to follow the labor force transitions at a high frequency. In practice, we consider a flexible parametric specification that allows for unobserved heterogeneity through worker and job types, and devise a tractable sequential estimation procedure that adapts the insights of Arcidiacono and Miller (2011) and Lentz et al. (2023) to this nonstationary search environment. Estimation proceeds in three stages. We first estimate the distribution of worker and job types by adapting to our context the Classification EM algorithm implemented in Lentz et al. (2023). Given the worker type distribution and job classification, we then estimate the parameters associated with job-to-job transitions, before estimating in a third and last step the unemployed-side parameters.

Estimates of the model reveal substantial worker and firm heterogeneity, with more productive workers earning more at all firm types. More productive workers are also more likely to work at high-paying firm types as, regardless of employment status, they receive a disproportionately large share of their job offers from such firms.

The data show significant decreases over time in accepted wages for those who find a job before benefit expiration. Our estimated model allows us to disentangle the mechanisms that lead to the negative relationship between unemployment duration and accepted wages. Part of the decline in accepted wages is driven by offer arrival rates, as a disproportionate share of job offers from high-paying firms arrive early in the unemployment spell; part is driven by non-pecuniary benefits falling over time; and part is due to the anticipation of pecuniary benefit expiration. The latter two mechanisms result in workers becoming increasingly less selective over time. Anticipation of unemployment benefits expiration plays an important role: counterfactual simulations indicate that extending unemployment benefits by 90 days would increase average unemployment duration by 14 days.

This paper brings together two strands of the literature. First, we contribute to the literature on the identification and estimation of structural dynamic discrete choice models (Rust, 1994, Heckman and Navarro, 2007, survey by Blundell, 2017). Since the seminal articles of Hotz and Miller (1993) and Magnac and Thesmar (2002), CCP methods have been increasingly used as a way to identify, and estimate complex dynamic discrete choice models at a limited computational cost (see surveys by Aguirregabiria and Mira, 2010 and Arcidiacono and Ellickson, 2011). While CCP methods have been used a variety of settings, they have been mostly used in a discrete time environment. Exceptions are Arcidiacono et al. (2016), Agarwal et al. (2021) and Llull and Miller (2018), who apply these methods to estimate continuous-time dynamic equilibrium models of market competition, an equilibrium model of kidney allocations, and a stationary dynamic model of job and location choices in the context of internal migration in Spain, respectively. We contribute to this literature by establishing the usefulness of CCP methods to constructively identify, and then estimate continuous-time job search models.

Since the seminal work of Flinn and Heckman (1982), a large number of papers have structurally estimated various types of job search models (see Eckstein and van den Berg, 2007 for a survey, and French and Taber, 2011 for an overview of the identification of job search models). In this literature, structural parameters are generally estimated via maximum likelihood or indirect inference methods, where the full model needs to be solved within the estimation procedure, and typically follows a strict job acceptance rule based on whether the offer exceeds the reservation wage. Nonstationarity in job search, which arises in particular when the level of unemployment benefits varies over the unemployment spell, is an important case where the computational demands are especially high. Since the important work of van den Berg (1990) who structurally estimated a continuous-time nonstationary search model,[1] examples of structural estimates of nonstationary job search models remain scarce.[2]

We contribute to this literature by providing a new empirical framework, based on a constructive identification strategy, that makes it possible to estimate a rich class of nonstationary job search models in a simple and tractable way. Key to our identification strategy is the stochastic nature of job acceptance. This aspect of our analysis shares similarities with Sorkin (2018) as well as recent work by Lentz et al. (2023) and Lamadon et al. (2024), which also allow for random preference shocks in job search environments.[3] Related to the

[1]See also Wolpin (1987), which is, to the best of our knowledge, the first study to estimate a (discrete time) nonstationary search model.

[2]Notable exceptions include Cockx et al. (2018), Launov and Walde (2013), Robin (2011), Lollivier and Rioux (2010), Paserman (2008), and Frijters and van der Klaauw (2006).

[3]At a high level, our approach also shares similarities with Bonhomme et al. (2019) who propose a flexible stochastic job mobility framework that accommodates preferences for non-pecuniary job attributes. See also Aizawa and Fang (2020) who estimate an equilibrium search model that incorporates labor supply preference shocks.

stochastic nature of job acceptance, our paper also fits into the literature that accommodates job-to-job transitions involving wage cuts. Such wage cuts, which are prevalent in practice (Jolivet et al., 2006), are typically rationalized through measurement errors in wages, exogenous reallocation ("Godfather shocks"), non-pecuniary amenities associated with lower wages, or higher option value associated with the poaching firm as in the Bertrand competition setup of Postel-Vinay and Robin (2002). Within our framework, preference shocks are key to rationalizing the high prevalence of wage cuts observed in the Hungarian context, where more than a third of job-to-job transitions involve a wage cut of at least 5%.

Our paper also complements the work of Sullivan and To (2014) and Taber and Vejlin (2020) who consider the identification of search models that allow for non-pecuniary job attributes. In contrast to these papers, we consider a nonstationary environment and establish constructive identification of the model parameters, most of them being obtained as closed-form expressions of the underlying hazard rates. Another important difference with Taber and Vejlin (2020) is that, while they consider an equilibrium search framework, our framework is set in partial equilibrium.

Additionally, our empirical illustration fits into the vast empirical literature that investigates the impact of unemployment benefit levels and duration on labor supply (see, e.g., Le Barbanchon et al., 2017, Nekoei and Weber, 2017, and surveys by Le Barbanchon et al., 2024, Schmieder and von Wachter, 2016 and Krueger and Meyer, 2002). Consistent with many of these studies, our estimation results provide evidence that nonstationarity plays an important role in describing the search environment over the course of the unemployment spell. A central and distinctive feature of our empirical strategy is that it leverages the direct links that exist between reduced-form hazard rates from unemployment to employment, or from one job to another, and the structural parameters of the model. As such, our paper provides a bridge between reduced-form and structural approaches that have been used in the context of job search. Beyond the specific application we consider in this paper, a similar approach can be readily used to identify and estimate a wide range of search models (see Gyetvai, 2024, for a recent application to occupational mobility).

The remainder of the paper is structured as follows. In Section 2, we introduce and discuss the general setup of the nonstationary search model we consider throughout the paper. Section 3 establishes identification of the model parameters. In Section 4 we discuss the data used to estimate the model. Section 5 presents our estimation procedure, with Section 6 discussing estimation and counterfactual simulation results. Finally, Section 7 concludes. The online appendix gathers the proofs and additional details pertaining to some of our identification results, details about the data, and about the estimation procedure, as well as additional estimation results.

2 Model

2.1 The environment

Consider an economy in continuous time with infinitely lived workers, who discount the future at a rate $\rho > 0$. Both employed and unemployed workers engage in job search. Job offers are characterized by a wage, w, and a job type, s. Job types capture non-wage characteristics such as firm, occupation, industry, or any particular non-monetary job attribute. The distribution of wages and job types is assumed to be discrete with a finite number of support points, denoted by W and S respectively. The support for wages and job types is given by $\Omega_w = \{\underline{w}, \ldots, \overline{w}\}$ and $\Omega_s = \{\underline{s}, \ldots, \overline{s}\}$. Conditional on receiving an offer from a particular job type s, the wage offer distribution depends on whether one is currently employed. The probability mass functions (pmf) of the wage offer distributions evaluated at wage w are given by f_w^s for the employed, and g_w^s for the unemployed.

We model job offer arrivals from the different job types as Poisson processes, and allow employed and unemployed workers to sample job offers at different frequencies. While working at a job of type s, the offer arrival rate for jobs of type s' is given by $\lambda^{ss'}$. The offer arrival rate for the unemployed for type-s jobs may vary with the duration of the unemployment spell, which we denote by t, and is given by $\lambda^s(t)$. Unemployed workers also receive benefits that are allowed to depend on the duration of the spell.[4] The unemployed offer arrival rates ($\lambda^s(t)$) and the flow payoff of unemployment ($b(t)$)—which may include both pecuniary and non-pecuniary components—are the two sources of nonstationarity in this setup.

While this model shares many of the features of the job search models that have been estimated in the literature, a central distinction is that it incorporates preference shocks into the search framework. This feature is instrumental to our approach as it allows us to connect the value functions of unemployment and employment to the conditional choice probabilities of accepting particular job offers. Specifically, in addition to a wage and a job type, any given job offer is associated with a preference shock, ε, which is assumed to be drawn independently from a standard logistic distribution whenever a new job offer arrives.

[4]In practice, following much of the empirical search literature, we treat unemployment and non-participation as a single state.

The ε shock affects the instantaneous utility of accepting a particular job offer. Our model also incorporates job switching costs, which in our application play an important role in fitting the observed job mobility flows. ε can be interpreted as the stochastic component of job switching costs.

2.2 Value of employment

As is standard in job search models, we work with the value function which maps the current state (i.e., the wage and the job type for the employed) into the expected present lifetime utility, under the assumption that the worker behaves optimally given their information at the time of each future decision. The value function for the employed can be expressed as depending on three components. The first one is the *flow payoff* for being employed in a job that pays w and is of type s. We express this flow payoff as the sum of two parts: the utility of the wage paid, u_w, and the non-pecuniary payoff of working in a job of type s, ϕ^s. Without loss of generality, we normalize $\phi^1 = 0$. The second component relates to *exogenous transitions* of which there can be two types. First, workers may be laid off and become unemployed, which happens at a rate δ_0^s.[5] Second, within the same firm, they may exogenously transition to a different wage w' and job type s'. These involuntary within-firm changes occur at a rate $\delta_{ww'}^{ss'}$, with the normalization that $\delta_{ww}^{ss} = 0$.

The third and final component of the value function captures *endogenous transitions*. Namely, workers may receive an offer from another firm for a job of type s' at a rate $\lambda^{ss'}$ and then decide whether to accept it or stay with their current job. These voluntary transitions are associated with an instantaneous cost of switching jobs, $c^{ss'}$, where we assume that the switching costs are symmetric (i.e. $c^{ss'} = c^{s's}$ for all s, s'). These transitions may or may not involve a wage change, and may occur both between or within job types. Combining these three components, the Bellman equation for the value of employment V_w^s associated with a job (w, s) can then be written as:

$$\begin{aligned} \left(\rho + \delta_0^s + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} + \sum_{s'} \lambda^{ss'}\right) V_w^s \quad &= \quad u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'} V_{w'}^{s'} \\ &\quad + \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \mathbb{E}_\varepsilon \max\left\{V_{w'}^{s'} - c^{ss'} + \varepsilon, V_w^s\right\} \end{aligned} \tag{2.1}$$

where $V_0(0)$ is the value of unemployment immediately upon entering an unemployment

[5] Our identification strategy would readily apply to a more general setup where the utility of work is not additively separable in the wage and non-pecuniary component, or where transitions to unemployment are allowed to be wage-specific. We do not consider this more general model for ease of exposition.

spell.

Given the logistic assumption on ε and following McFadden (1978) and Arcidiacono and Miller (2011), we can express Equation (2.1) as:[6]

$$\left(\rho + \delta_0^s + \sum_{w'} \sum_{s'} \delta_{ww'}^{ss'}\right) V_w^s = u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'} \sum_{s'} \delta_{ww'}^{ss'} V_{w'}^{s'} - \sum_{w'} \sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln(1 - p_{ww'}^{ss'}) \quad (2.2)$$

Because of the uncertainty associated with the preference shocks, even after conditioning on the wage and job type associated with the job offer, as well as the current wage and job type, job offers are accepted probabilistically. Namely, the probability of a job of type s' paying w' being accepted given current job type s paying w is:

$$p_{ww'}^{ss'} = \frac{\exp\left(V_{w'}^{s'} - c^{ss'}\right)}{\exp\left(V_w^s\right) + \exp\left(V_{w'}^{s'} - c^{ss'}\right)} \quad (2.3)$$

2.3 Value of unemployment

We now turn to the problem of the unemployed individuals. Indexing by t time spent unemployed, it is useful in this nonstationary environment to first write the Bellman equation for the unemployment value function $V_0(t)$ in discrete time:[7]

$$\begin{aligned} V_0(t) &= b(t)\Delta t + \frac{\Delta t}{1 + \rho\Delta t} \sum_w \sum_s \lambda^s(t) g_w^s \mathbb{E}_\varepsilon \max\left\{V_w^s + \varepsilon, V_0(t + \Delta t)\right\} \\ &\quad + \frac{1 - \sum_s \lambda^s(t)\Delta t}{1 + \rho\Delta t} V_0(t + \Delta t) \end{aligned}$$

where Δt denotes the discrete time unit. The first term corresponds to the flow utility of unemployment, the second term captures the probability of receiving an offer in the time interval $(t, t + \Delta t]$ for each possible wage multiplied by the corresponding ex ante value function, and the last term is the probability of not receiving an offer multiplied by the value of remaining unemployed at time $t + \Delta t$. Rewriting this equation and letting $\Delta t \to 0$ yields the following differential equation:

$$\rho V_0(t) = b(t) + \sum_w \sum_s \lambda^s(t) g_w^s \mathbb{E}_\varepsilon \max\left\{V_w^s - V_0(t) + \varepsilon, 0\right\} + \dot{V}_0(t) \quad (2.4)$$

[6] The units of the value function are then relative to the (normalized) scale of the preference shocks.

[7] Note that we implicitly normalize to zero the switching cost from unemployment to employment, which in our setup is not separately identified from the value of unemployment.

where $\dot{V}_0(t)$ is the derivative of $V_0(t)$ with respect to unemployment duration. This term represents the change in the option value of job search due to variation over time in the value of unemployment. In the particular case where nonstationarity arises as a result of over-time changes in the level of unemployment benefits, the option value of searching for a job will decrease as job seekers get closer to the unemployment benefit expiration date.

Making again a logistic assumption on the preference shocks in a similar fashion to the employed side, we can rewrite Equation (2.4) as follows:

$$\rho V_0(t) = b(t) - \sum_w \sum_s \lambda^s(t) g_w^s \ln\left(1 - p_w^s(t)\right) + \dot{V}_0(t) \tag{2.5}$$

where $p_w^s(t)$ the probability of accepting a job offer of type s and wage w at time t and is given by:

$$p_w^s(t) = \frac{\exp(V_w^s)}{\exp(V_w^s) + \exp(V_0(t))} \tag{2.6}$$

Equation (2.5) is a simple linear first-order differential equation in $V_0(\cdot)$. In the absence of preference shocks, $V_0(t)$ would satisfy instead the following nonlinear differential equation:

$$\rho V_0(t) = b(t) + \sum_s \sum_w \lambda^s(t) g_w^s \max\left\{V_w^s - V_0(t), 0\right\} + \dot{V}_0(t)$$

This type of nonlinear differential equation would need to be solved numerically, similar to van den Berg (1990) in a simpler context without on-the-job search.

3 Identification

We provide in the following a simple and constructive identification strategy for the parameters of the job search model introduced in Section 2. We first focus on a setup without unobserved heterogeneity. In particular, we start by assuming that job types s are observed by the econometrician. We then discuss in Subsection 3.5 how identification proceeds in the presence of worker- as well as job-level unobserved heterogeneity. Our identification results hold in an empirical setting where one has access to longitudinal data on (i) across-firm job-to-job transitions, (ii) within-firm transitions, (iii) transitions from unemployment to employment, and (iv) transitions from employment to unemployment.

Recall that we assume that wages are drawn from a discrete distribution with finite support. This distribution can be thought of as a discrete approximation to an underlying continuous wage distribution. We maintain this assumption throughout our analysis for simplicity, but our identification strategy readily applies to the case of continuous wage distributions.[8]

3.1 Assumptions

We first introduce four assumptions that relate to the types of transitions that are observed in the data. We denote by A1, A2, A3 and A4, respectively, the assumptions that the following hazard rates are identified from the data:

A1 $h_{ww'}^{ss'}$, the hazard rate of moving from a job with wage w and type s to a job with wage w' and type s' (in a different firm);

A2 $h_w^s(t)$, the hazard rate out of unemployment at time t to a job that pays w and is of type s;

A3 $\delta_{ww'}^{ss'}$, the hazard rate of within-firm wage (w to w') and type (s to s') changes;

A4 δ_0^s, the hazard rate from a type-s job to unemployment.

As is standard for this class of models, we also maintain the assumption that the discount rate ρ is known.

In the following, we show that these hazard rates can be used to recover closed-form expressions for the employed and unemployed wage offer distributions (f_w^s and g_w^s); the pecuniary and non-pecuniary payoffs of the job (u_w and ϕ^s), each up to a constant; the cost of switching jobs ($c^{ss'}$); the job offer arrival rates for those who are employed and unemployed ($\lambda^{ss'}$ and $\lambda^s(t)$); and the flow payoff of unemployment ($b(t)$).

3.2 Hazard rates

Our starting point is the expression of the job-to-job hazard rates and of the hazard rates out of unemployment as a function of the model parameters. For the employed individuals, note that the hazard of moving from a job of type s that pays w to a job of type s' that pays w' is, by definition, the product of three terms: (i) the job arrival rate ($\lambda^{ss'}$), (ii) the

[8]Specifically, the key observation here is that, for any given pair of wages (w, w'), the hazard rates associated with the transitions to wage w' conditional on current wage w are directly identified from the data. Such hazard rates are also known in the statistical literature as the conditional mark-specific hazard function (see Sun et al., 2009, Equation (1) p.395).

probability that the offered wage is w' ($f^{s'}_{w'}$), and (iii) the probability of accepting the job ($p^{ss'}_{ww'}$):

$$h^{ss'}_{ww'} = \lambda^{ss'} f^{s'}_{w'} p^{ss'}_{ww'} \tag{3.1}$$

There are S^2W^2 such hazard rates that, by Assumption A1, are directly identified from the data. These are used to recover S^2 offer arrival rates, $S(W-1)$ wage offer probabilities $(f^{s'}_{w'})_{w',s'}$, and S^2W^2 job acceptance probabilities. As will become clear from the discussion below, the structure that our search model imposes on these acceptance probabilities is key to solving this identification problem.

On the unemployed side, the hazard rates take a similar form. Specifically, the hazard out of unemployment to job type s and wage w at time t, $h^s_w(t)$, is the product of the offer arrival rate of job type s, $\lambda^s(t)$, the probability that a job type s will offer w, g^s_w, and the probability that such an offer would be accepted, $p^s_w(t)$:

$$h^s_w(t) = \lambda^s(t) g^s_w p^s_w(t) \tag{3.2}$$

In the following, we show how the hazard rates out of unemployment, combined with the parameters previously identified from the employed side, allow us to separately identify the offer arrival rates, the wage offer distributions, and the acceptance probabilities.

3.3 Employed-side parameters

We first establish identification of the employed wage offer distributions for each job type. Identification comes from analyzing the hazard rates of transitions to jobs that are of the same type and pay the same amount as the current jobs (h^{ss}_{ww}, at different wage levels. The key to identifying the wage offer distribution is that, from Equation (2.3), the probability of accepting a job in this case is invariant to the wage level w: $p^{ss}_{ww} = p^{ss}_{w'w'} = \frac{\exp(-c^{ss})}{1+\exp(-c^{ss})}$ for all $(w, w') \in \Omega^2_w$.

It follows from this invariance property, combined with the expression of the job-to-job hazard rates in Equation (3.1), that when the transitions are to same-type and same-pay jobs the ratio of the hazards for two different initial wages coincides with the ratio of the pmfs for these two wages:

$$\frac{f^s_w}{f^s_{w'}} = \frac{h^{ss}_{ww}}{h^{ss}_{w'w'}}$$

Our first identification result directly follows:

Lemma 1 *Assume that Assumption A1 holds. Then f^s_w is identified and can be written as follows:*

$$f^s_w = \frac{h^{ss}_{ww}}{\sum_{w'} h^{ss}_{w'w'}} \tag{3.3}$$

Inspection of the proof indicates that this result does not rely on the distributional assumption that the preference shocks ε are drawn from a logistic distribution. However, as is clear from the rest of our analysis, this assumption does play a central role in obtaining closed-form expressions for the other model parameters.[9] This result shows that one can identify the full wage offer distribution without having to make a recoverability assumption on the underlying distribution (Flinn and Heckman, 1982). By introducing preference shocks to the search environment, we are able to trace out the entire wage offer distribution from the observed job-to-job transitions.

Having recovered the wage offer distributions, we next show identification of the on-the-job offer arrival rates, $\lambda^{ss'}$. Given the employed wage offer distribution and conditional on job type, combining pairs of hazards that involve transitions from wages w to w' and from w' to w allows us to recover the offer arrival rate. This is because the value functions embedded in the two hazards are the same, so that combining them in an appropriate way results in these value functions differencing out.

Consider first the offer arrival rates for jobs of the same type, λ^{ss}. The basis for our identification strategy comes from two alternative ways of expressing $p^{ss}_{ww'}$. The first one is based on Equation (2.3), which expresses $p^{ss}_{ww'}$ as a function of the value functions for the two jobs, and the cost of switching. The second one builds on the mapping between $p^{ss}_{ww'}$ and $h^{ss}_{ww'}$ given by Equation (3.1). The two together imply:

$$p^{ss}_{ww'} = \frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'}} = \frac{\exp(V^s_{w'} - c^{ss})}{\exp(V^s_{w'} - c^{ss}) + \exp(V^s_w)} \tag{3.4}$$

Rearranging the second equality results in the associated log-odds ratio:

$$\ln\left(\frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'} - h^{ss}_{ww'}}\right) = V^s_{w'} - V^s_w - c^{ss} \tag{3.5}$$

[9] We conjecture that one could rely on a similar identification strategy with alternative distributional assumptions (such as, e.g., normal distribution) on the preference shocks, as long as one maintains additive separability of the preference shocks. However, this would come at the cost of losing the closed-form nature of our results, resulting in a less transparent identification strategy.

Adding the log-odds ratio for the reverse job-to-job transition, from a job paying w' to a job paying w, results in an expression that does not depend on wages:

$$\ln\left(\frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'} - h^{ss}_{ww'}}\right) + \ln\left(\frac{h^{ss}_{w'w}}{\lambda^{ss} f^s_{w} - h^{ss}_{w'w}}\right) = -2c^{ss} \tag{3.6}$$

Evaluating this expression for another set of wage transitions, between w and $\tilde{w}$, and setting the two equal then yields the following closed-form expression for λ^{ss}:

$$\lambda^{ss} = \frac{\left(f^s_w h^{ss}_{w\tilde{w}} + f^s_{\tilde{w}} h^{ss}_{\tilde{w}w}\right) h^{ss}_{ww'} h^{ss}_{w'w} - \left(f^s_w h^{ss}_{ww'} + f^s_{w'} h^{ss}_{w'w}\right) h^{ss}_{w\tilde{w}} h^{ss}_{\tilde{w}w}}{f^s_w f^s_{\tilde{w}} h^{ss}_{ww'} h^{ss}_{w'w} - f^s_w f^s_{w'} h^{ss}_{w\tilde{w}} h^{ss}_{\tilde{w}w}} \tag{3.7}$$

Having identified λ^{ss}, the switching cost c^{ss}, differenced value functions $V^s_{w'} - V^s_w$ and conditional choice probabilities $p^{ss}_{ww'}$ are then directly identified from Equations (3.6), (3.5) and (3.4), respectively.

Lemma 2, proved in Appendix A.1.1, extends the same logic to transitions across jobs of different types.

Lemma 2 *(i) Assume that Assumption A1 holds and that there exists a triplet* $(w, w', \tilde{w}) \in \Omega^3_w$ *such that* $f^s_{\tilde{w}} h^{ss}_{ww'} h^{ss}_{w'w} \neq f^s_{w'} h^{ss}_{\tilde{w}w} h^{ss}_{w\tilde{w}}$*. Then* λ^{ss}*,* $p^{ss}_{ww'}$ *and* c^{ss} *are identified.*

(ii) For $x \in \{w', \tilde{w}\}$ *and* $s \neq s'$*, let* $A_x = f^{s'}_x f^s_x h^{ss'}_{ww} h^{s's}_{ww} - f^{s'}_w f^s_w h^{ss'}_{xx} h^{s's}_{xx}$*,* $B_x = f^{s'}_x h^{s's}_{xx} h^{ss'}_{ww} h^{s's}_{ww} - f^{s'}_w h^{s's}_{ww} h^{ss'}_{xx} h^{s's}_{xx}$*, and* $C_x = f^s_w h^{ss'}_{xx} h^{ss'}_{ww} h^{s's}_{ww} - f^s_w h^{ss'}_{ww} h^{ss'}_{xx} h^{s's}_{xx}$*. Assume that Assumption A1 holds and that there exists a triplet* $(w, w', \tilde{w}) \in \Omega^3_w$ *such that the following conditions hold:*

(a) $A_{w'} \neq 0$

(b) $B_{w'} A_{\tilde{w}} - B_{\tilde{w}} A_{w'} \neq 0$

(c) $A_{w'} C_{\tilde{w}} - A_{\tilde{w}} C_{w'} \neq 0$

then $\lambda^{ss'}$*,* $p^{ss'}_{ww'}$*,* $c^{ss'}$ *and* $V^{s'}_{w'} - V^s_w$ *are identified.*

Further, when the conditions stated in (i) and (ii) are met, there are closed-form expressions for $\lambda^{ss'}$*,* $c^{ss'}$*,* $p^{ss'}_{ww'}$ *and* $V^{s'}_{w'} - V^s_w$*, for all* $(w, w', s, s') \in \Omega^2_w \times \Omega^2_s$*, as a function of the underlying hazard rates.*

Note that each of the sufficient conditions listed above are testable using the observed job-to-job transition rates.

Finally, we turn to unbundling the differenced employed value functions, $V_{w'}^{s'} - V_w^s$, in order to recover the pecuniary (u_w) and non-pecuniary (ϕ^s) components of a job. We illustrate identification of u_w in a simpler setting where there are no within-firm involuntary wage changes ($\delta_{ww'}^{ss'} = 0$), before turning to Lemma 3 for the general case.

Recall that from Lemma 2 we have already recovered $V_{w'}^{s'} - V_w^s$. Identification of $u_{w'} - u_w$ follows immediately from the expression of the Bellman equation for the value of employment given in Equation (2.2), considering the within-job-type differences $V_{w'}^s - V_w^s$:

$$(\rho + \delta_0^s)(V_{w'}^s - V_w^s) = u_{w'} - u_w - \sum_{\tilde{w}} \sum_{\tilde{s}} \lambda^{s\tilde{s}} f_{\tilde{w}}^{\tilde{s}} \left[\ln(1 - p_{w'\tilde{w}}^{s\tilde{s}}) - \ln(1 - p_{w\tilde{w}}^{s\tilde{s}}) \right] \tag{3.8}$$

The only unknown is $u_{w'} - u_w$. It directly follows that u_w is known up to a constant, a standard feature for discrete choice models which carries over to this search environment.

Lemma 3, proved in Appendix A.1.2, extends this result to accommodate within-firm involuntary wage changes ($\delta_{ww'}^{ss'} \neq 0$) and further states that u_w is fully identified if one assumes CRRA preferences over wages. It also establishes that non-pecuniary payoffs ϕ^s are not identified from the employment transitions alone, and provides their degree of under-identification in this setup.

Lemma 3 *Given Assumptions A1, A3, and A4:*

(i) u_w *is identified up to a constant and has a closed-form expression as a function of the employed hazard rates.*

(ii) When workers have CRRA preferences so that $u_w = \frac{\alpha w^{1-\theta}}{1-\theta}$, *both* α *and the risk aversion parameter* θ *are identified.*

(iii) Given u_w *and the normalization* $\phi^1 = 0$, *the non-pecuniary payoffs* ϕ^s *are a known linear function of* $V_0(0)$, *given by:*

$$\phi^s = (\rho + \delta_0^s) \left(\tilde{\kappa^s} - \left(\frac{\delta_0^s}{\rho + \delta_0^s} - \frac{\delta_0^1}{\rho + \delta_0^1} \right) V_0(0) \right)$$

where $\tilde{\kappa^s}$ *is known.*[10]

Key to Part *(ii)* of Lemma 3 is that, using a monotonicity argument detailed in Appendix A.1.2, the curvature parameter θ, and then the scale parameter α, can be recovered

[10]The location normalization $\phi^1 = 0$ reflects the fact that the decision to accept an offer of a job of type s', given current job type $s \neq s'$, depends on the associated non-pecuniary payoffs only through the difference $\phi^{s'} - \phi^s$.

from the utility differences identified in Part (i). In the next section we establish identification of $V_0(0)$, allowing in turn to identify ϕ^s (up to a reference job type), by combining the employment hazards with the hazards out of unemployment. As is clear from the expression of ϕ^s in (iii), a notable special case is one where the destruction rates do not vary across job types, in which case the non-pecuniary payoffs are directly identified from the employed-side parameters.

3.4 Unemployed-side parameters and main identification result

We now turn to the identification of the parameters that govern the transitions out of unemployment. We first introduce some mild regularity conditions regarding the model parameters that are allowed to vary over the course of the unemployment spell.

A5 The function $t \mapsto \lambda^s(t)$ is continuous on the positive real line. $t \mapsto b(t)$ admits a finite number of discontinuity points, is right-continuous and admits a left-hand limit at any such points.

We use Assumption A5 when differentiating, in the last part of the identification argument, the unemployed value function $V_0(t)$ with respect to time.[11] As with the employed-side parameters, we begin by recovering the wage offer distributions, g^s_w. Key to the identification argument is leveraging what we have already recovered on the employed side. Namely, for a given job type s and unemployment duration t, the difference in the log odds from accepting, out of unemployment, a job that pays w and accepting a job that pays w' can be written as the difference in the employment value functions associated with these two jobs:

$$\ln\left(\frac{p^s_w(t)}{1-p^s_w(t)}\right) - \ln\left(\frac{p^s_{w'}(t)}{1-p^s_{w'}(t)}\right) = V^s_w - V^s_{w'} \tag{3.9}$$

where it follows from Lemma 2 that the right-hand side, which does not vary over the course of unemployment, is identified from the employed side alone. A remarkable implication is that, for any given job offer type and duration of unemployment, the job acceptance probabilities out of unemployment are identified, up to a constant, without exploiting any information from the transitions out of unemployment.

[11] This type of regularity condition is standard in continuous-time dynamic optimization models. See also van den Berg (1990), who imposes similar regularity conditions in the context of a continuous-time non-stationary search model. His empirical illustration features a discrete change in benefits, consistent with allowing for finitely many discontinuity points in $b(t)$.

We next use the hazards out of unemployment to express the left-hand side of Equation (3.9) as a function of the model primitives, using the relationship $p_w^s(t) = \frac{h_w^s(t)}{\lambda^s(t) g_w^s}$.[12] Denoting the (identified) differenced value function $V_w^s - V_{w'}^s$ as $\kappa_{ww'}^{ss}$, this results in the following relationship between the job offer arrival rates $\lambda^s(t)$, for any given unemployment duration t and job type s, and the probabilities of receiving a wage offer w and w' (g_w^s and $g_{w'}^s$):

$$\begin{aligned} \frac{1}{\lambda^s(t)} &= \frac{g_w^s h_{w'}^s(t) \exp(\kappa_{ww'}^s) - g_{w'}^s h_w^s(t)}{h_w^s(t) h_{w'}^s(t) \left(\exp(\kappa_{ww'}^s) - 1\right)} && (3.10) \\ &\equiv A_{ww'}^s(t) g_w^s - B_{ww'}^s(t) g_{w'}^s && (3.11) \end{aligned}$$

where $A_{ww'}^s(t)$ and $B_{ww'}^s(t)$ are known quantities that involve the job-to-job and unemployment hazard rates. Evaluating this expression for an alternative pair of wages, say $\{\tilde{w}, \tilde{w}'\}$, allows us to difference out the term that depends on the offer arrival rate:

$$0 = A_{ww'}^s(t) g_w^s - B_{ww'}^s(t) g_{w'}^s - A_{\tilde{w}\tilde{w}'}^s(t) g_{\tilde{w}}^s + B_{\tilde{w}'\tilde{w}}^s(t) g_{\tilde{w}'}^s \quad (3.12)$$

This yields, for any given unemployment duration t, a linear system involving $W-1$ unknown parameters $(g_w^s)_w$, and $W-2$ generically non-redundant equations. Evaluating Equation (3.12) for different unemployment durations then results in a generally overdetermined system.

We provide in Lemma 4 below conditions under which this yields identification of the wage offer distribution out of unemployment. To do so, we first rewrite the system of restrictions obtained by pooling Equations (3.12) for a finite set of durations $\mathbf{t}_k = (t_1, t_2, \dots, t_k) \in (\mathbb{R}_+^*)^k$ as $M^s(\mathbf{t}_k) g^s = b^s(\mathbf{t}_k)$, where $g^s = (g_1^s, g_2^s, \dots, g_{W-1}^s)'$, $M^s(\mathbf{t}_k)$ is a matrix of dimension $k(W-2) \times (W-1)$, and $b^s(\mathbf{t}_k)$ is a vector of dimension $k(W-2)$. This corresponds to the reduced system after excluding the redundant equations, and evaluating Equation (3.12) at the wage tuples $\{(1, 2, 1, \tilde{w}) : W \geq \tilde{w} \geq 3\}$.

Lemma 4 *Given Assumptions A1 through A4, $W \geq 3$, and assuming that there exists a set of durations $\mathbf{t}_k^*$ such that $M^s(\mathbf{t}_k^*)$ is full rank with $M^s(\mathbf{t}_k^*) g^s = b^s(\mathbf{t}_k^*)$, the unemployed wage offer distribution for job type s, $(g_w^s)_w$, is identified as the least squares solution to this system.*

We provide in Appendix A.3 the expression of the rank condition and of the unemployment wage offer distribution when $W = 3$, using hazard rates out of unemployment evaluated at

[12] This substitution follows directly from Equation (3.2).

two distinct unemployment durations ($k = 2$).[13] As is clear from this special case, key to the identification of the wage offer distribution out of unemployment is the nonstationarity of the search environment, specifically the hazard rates out of unemployment $h^s_w(t)$ exhibiting some variation over the course of unemployment. Variation over two distinct unemployment durations, under the rank condition that is derived in Appendix A.3 for $k = 2$ and $W = 3$, is sufficient for identification. In practice, the unemployment wage offer distribution is generally heavily over-identified.

Having recovered the wage offer distribution, identification of the arrival rate $\lambda^s(t)$ follows directly from Equation (3.10) as all the terms on the right-hand side are either directly identified from the data ($h^s_w(t)$), or already identified from the employed side ($\kappa^{ss}_{ww'}$ and g^s_w). The job acceptance probabilities are then immediately identified from Equation (3.2).

The last key model component that remains to be identified is the flow utility of unemployment, $b(t)$. Our identification strategy involves recovering in a preliminary step the unemployment value function $V_0(t)$ and its time derivative $\dot{V}_0(t)$. To recover the unemployment value function, we express the following log odds by normalizing the future value of working relative to staying at the same job:

$$\begin{aligned} \ln\left(\frac{p^s_w(t)}{1-p^s_w(t)}\right) &= V^s_w - V_0(t) \\ &= \Big(u_w + \phi^s + \delta^s_0 V_0(0) + \sum_{w'}\sum_{s'} \delta^{ss'}_{ww'}\left[c^{ss'} + \ln\left(p^{ss'}_{ww'}\right) - \ln\left(1-p^{ss'}_{ww'}\right)\right] \\ &\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f^{s'}_{w'} \ln\left(1-p^{ss'}_{ww'}\right)\Big)\Big/(\rho+\delta^s_0) - V_0(t) \end{aligned} \tag{3.13}$$

where the second equality follows from Equation (2.2).

Note that Equation (3.13) is linear in the two unknowns, $V_0(t)$ and $V_0(0)$. Identification proceeds in two steps. First, by evaluating Equation (3.13) at $t = 0$ and for the reference job type ($s = 1$), we immediately recover $V_0(0)$, the value of unemployment at the start of the unemployment spell. We can then recover $V_0(t)$ for any $t \geq 0$ using Equation (3.13), taking $V_0(0)$ as given. Finally, differentiating the unemployment value function with respect to unemployment duration t yields $\dot{V}_0(t)$.

Given these earlier steps, identification of the flow utility of unemployment is straightforward. For any given unemployment duration t, this directly follows by solving for $b(t)$ in the Bellman equation (2.5). Namely:

[13] Similar results can be derived for cases with $W > 3$ and $k > 2$, at the cost of more tedious expressions.

$$b(t) = \rho V_0(t) + \sum_w \sum_s \lambda^s(t) g_w^s \ln\left(1 - p_w^s(t)\right) - \dot{V}_0(t) \tag{3.14}$$

The following lemma summarizes the identification results for the unemployment parameters:[14]

Lemma 5 *Given Assumptions A1-A5, the offer arrival rates $\lambda^s(t)$, the conditional choice probabilities $p_w^s(t)$, the flow payoff of unemployment $b(t)$, the value function of unemployment and its derivative, $V_0(t)$ and $\dot{V}_0(t)$, are identified.*

An important consequence of Lemma 5 is that the non-pecuniary payoffs ϕ^s, which from Lemma 3 were only known up to $V_0(0)$, are now also identified (up to the normalization $\phi^1 = 0$). Taking stock, a key implication of these results is that, by exploiting the tight connection between value functions and conditional choice probabilities, we are able to recover the structural parameters of this nonstationary job search model without solving any differential equation.

Finally, our main identification result follows from Lemmas 1 through 5:

Theorem 1 *Given Assumptions A1-A5, all of the employed and unemployed-side parameters are identified subject to a normalization of one u_w and one ϕ^s, and subject to the rank conditions from Lemmas 2 and 4.*

3.5 Worker- and job-level unobserved heterogeneity

The identification strategy provided above can be adapted to accommodate worker-level unobserved heterogeneity.[15] Namely, assume that workers belong to one of a finite number of unobserved heterogeneity types, where the model parameters are allowed to vary across types. The previous constructive strategy still identifies the structural parameters, from knowledge of the type-specific hazard functions and the distribution of heterogeneity types. The distribution of types can be identified from the observed transitions from unemployment to employment, by using the identification results from Heckman and Singer (1984) for duration models with unobserved heterogeneity but without covariates.[16] Alternatively,

[14] Note that, for any potential discontinuity point t^* of the flow utility of unemployment, $\dot{V}_0(t^*)$ denotes the right-hand derivative of $V_0(.)$ at that point.

[15] Our identification strategy trivially applies to a setup with worker-level observed heterogeneity, starting from the hazard rates which are in that case conditional on workers' observed heterogeneity.

[16] These results apply to a class of duration models that are characterized by a Box-Cox baseline hazard. We thank Jim Heckman for useful discussions on this point.

a similar reasoning as in the dynamic model of Bonhomme et al. (2019) identifies the distribution of worker-level unobserved heterogeneity from the observed job-to-job transitions. One can then identify in a second step, taking as given the distribution of heterogeneity types, the type-specific hazards associated with the job-to-job, job-to-unemployment and unemployment-to-job transitions.

Our identification strategy also extends to a framework where job types s are unobserved to the econometrician. Namely, following the approach initially proposed by Bonhomme et al. (2019) and recently extended by Lentz et al. (2023), one can classify firms into a finite number of firm classes using k-means clustering. Our constructive identification strategy then still applies in a second step, setting s equal to the firm class associated with a particular job, and taking, as in Lentz et al. (2023), the partition of firms into classes as given.

In practice, the longitudinal dimension of the data, along with the dynamics of the model, are central to the identification of the worker- and job-type distributions. In particular, job-to-job transition rates to and from particular groups of firms, along with the associated wage changes, are informative about job-type specific wage offer distributions and non-pecuniary job attributes. For example, firms that are consistently associated with higher wages for their incumbent workers may be classified as having some combination of a better wage offer distribution or poorer job amenities. Serial correlation in wages and durations of employment spells are informative about worker-level unobserved heterogeneity types. If a particular worker is seen at multiple jobs with high wages, this suggests he may be a more productive type. Overall, the type classifications result from correlations over time for workers or firms that cannot be rationalized solely by the independent shocks in the model.

3.6 Extensions

Aggregate shocks Our identification strategy can be extended to allow for aggregate shocks to the economy. Namely, we assume that the economy is in one of K states, $k \in \{1, \ldots, K\}$, with the transition rate from state k to k' denoted by $q_{kk'}$. The aggregate state of the economy then affects the job destruction rates, the within-employer type and wage transitions, the job offer arrival rates, the wage offer distributions and the flow utility of unemployment. Our constructive identification holds in this case as well, under the assumption that the econometrician observes the market state (implying $q_{kk'}$ is identified) and where the hazard rates in A1 through A4 are observed conditional on the market state (see Appendix A.2).

Scale of preference shocks It is also possible to relax the assumption that the distributions of preference shocks share the same scale parameter across the employed and unemployed parts of the model. Letting σ denote the scale parameter of the preference shocks affecting unemployed workers, the key difference is that Equation (3.9) is now given by:

$$\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right) - \ln\left(\frac{p_{w'}^s(t)}{1-p_{w'}^s(t)}\right) = \frac{V_w^s - V_{w'}^s}{\sigma} \tag{3.15}$$

As a result, the relationship between the job offer arrival rate and the wage offer distribution out of unemployment also depends on the unknown scale parameter σ, with:

$$\frac{1}{\lambda^s(t)} = \frac{g_w^s h_{w'}^s(t)\exp(\kappa_{ww'}^s/\sigma) - g_{w'}^s h_w^s(t)}{h_w^s(t)h_{w'}^s(t)\left(\exp(\kappa_{ww'}^s/\sigma) - 1\right)} \tag{3.16}$$

Evaluating this expression for an alternative pair of wages and taking the difference yields a system of identifying restrictions that are linear in the g_w^s's and nonlinear in σ. The Bellman equation for the unemployment value also needs to be adjusted accordingly, with the flow utility of unemployment then being identified from:

$$b(t) = \rho V_0(t) + \sigma \sum_w \sum_s \lambda^s(t) g_w^s \ln\left(1 - p_w^s(t)\right) - \dot{V}_0(t) \tag{3.17}$$

Nonstationary unemployment wage offer distribution A third possible extension consists in relaxing the assumption that the wage offer distribution is constant over the course of unemployment. Doing so requires imposing some structure on the evolution of the wage offer distribution over the course of unemployment. Denoting by $G_w(t)$ the cdf of the wage offer distribution out of unemployment at time t evaluated at wage w, one such restriction is given by assuming that, for all $t \geq 0$, $G_w(t) = G_w(0)^{\alpha(t)}$ (with $\alpha(t) > 0$ and $\alpha(0) = 1$). Assuming further a flexible parametric specification for $\alpha(t) \equiv \alpha(t, \theta_\alpha)$, and following a similar reasoning as with the baseline specification with time-invariant wage offer distribution, Equation (3.10) yields a generally overdetermined (nonlinear) system in $(G_w(0))_w$ and θ_α.

4 Application to job search in Hungary: background and data

We now turn to the data used in our empirical application, describing the institutional background, the data available for employed and unemployed workers, and the corresponding descriptive patterns. The descriptive analysis reveals two motivating facts for the application. On the employed-side, the data reveal a substantial share of job-to-job changes that entail wage cuts. Among unemployed workers, accepted wages decline sharply as unemployment duration increases.

4.1 Institutional background

Hungary had a two-tier unemployment insurance system during the observational period for our unemployed sample (January 2004 to October 2005). Only those with a sufficiently long work history were eligible for the second-tier benefits, and benefit payments in the second tier were lower than in the first. Those who exhausted benefits in both tiers were eligible for social assistance. Tier 1 benefits expired in 270 days and Tier 2 benefits expired in an additional 90 days. We focus on unemployed workers leaving unemployment in Tier 1, because Tier 2 benefits were low ($114 per month on average over our period of interest) and very similar to the amount of social assistance that anyone is eligible for, regardless of prior work history. As such, Tier 2 benefits likely did not provide significant further incentive to remain in unemployment.[17]

4.2 Data

We estimate the model using linked employer-employee data from Hungarian administrative records, provided by the Center for Economic and Regional Studies at the Hungarian Research Network (HUN-REN CERS).[18]

The sample consists of half of the population, i.e., 4.6 million individuals, linked across 900 thousand firms. On the individual side, a *de facto* 50% random sample of the Hungarian

[17]In practice, we choose to censor durations at 269 days as a disproportionally large number of workers are recorded as claiming Tier 1 benefits up until exactly 270 days. This suggests that some of these workers might actually have started working before that point.

[18]The linked administrative data collection is the property of the data owners of the National Health Insurance Fund, Central Administration of Pension Insurance, Educational Authority, National Tax and Customs Administration, National Labor Office, and the Hungarian State Treasury and their legal successors.

population is observed; every Hungarian citizen born on January 1, 1927 and every second day thereafter is included. A distinctive feature of the Hungarian data is their frequency: job spells are observed on a monthly basis, and unemployment spells are observed at a daily frequency. When working, one individual can be present in at most two work arrangements: labor market measures, such as wages and days worked, are observed separately for each one of them. We also have information on demographics, total earnings and days worked, and, for job seekers, unemployment benefit payments. On the firm side, all firms are included at which any sampled individual is observed to have worked for at least one month. From these data, we can infer the length of the employment spells, as well as employment-to-employment transitions from changes in firm identifiers.

We define our sample by restricting the full dataset in three main ways. First, we use employment spells over the full sample period of January 2003 to December 2007 and unemployment spells from January 2004 to October 2005.[19] Second, we focus on males who were older than 25 in the beginning of our sample and younger than 50 at the end: we drop females from our sample to abstract from differential labor market flows resulting in part from childbearing decisions; and we drop older males to abstract from differential search behavior as retirement nears, with a retirement age of 62 for males during this time period. Third, we exclude self-employed workers and incorporated firms with only one employee: we single out small firms with less than 25 employees and will estimate heterogeneity types among larger firms later in the paper.[20]

Because of some recoding of jobs around the first day of the year, we treat employment spells that go past December 31st of a particular year as right-censored. Given that the employed dataset tracks where individuals are employed on the 15th of the month, there can be issues with distinguishing whether there was an employment-to-employment transition versus a short employment spell between two jobs. These issues are most heightened when observations in two consecutive months are affected. As a result, we further right censor jobs at October 31st in each year to ensure consistent coding of employment-to-employment transitions within a month. Appendix B describes our data cleaning process.

Table 1 shows summary statistics for the employment spells. In a given year, one in every ten workers have two or more employment spells. Most employment spells are right-censored. Among those spells that are not right-censored, one in every four ends in a transition to

[19] Unemployment data are only available from January 2004 onward. We cut our unemployed sample in November 2005 because the UI system went through (presumably unanticipated) changes at that point in time.

[20] Our final sample consists of 2,752,895 employment and 21,308 unemployment spells.

Table 1: Summary statistics, employment spells

	Number of spells						
	1	*2*	*3*	*4*	*5*	*6*	*7*
In whole history (%)	15.4	13.3	11.8	11.3	31.8	10.9	5.4
In a given year (%)	90.3	9.0	0.6	0.0	0.0	0.0	0.0

	Destination			**Firm size**	
	EE	*EU*	*RC*	*Small firms*	*25+ firms*
Share (%)	5.0	15.7	79.3	32.2	67.8

Current wage	*Overall*	*Small firms*	*25+ firms*
Mean current wage (HUF)	3,695	2,395	4,311
Std. current wage (HUF)	2,876	1,711	3,102
Share $\underline{w}$ (%)	27.6	55.1	14.5

Notes: The top panel shows the share of individuals with a given number of employment spells in their history, as well as the share of individual-years with a given number of employment spells. Durations are right-censored at October 31st each year. The middle left panel shows the fraction of employment spells that end in an employment-to-employment transition (EE), employment-to-unemployment transition (EU), or are right-censored (RC). The middle right panel shows the fraction of employment spells by firm size: small refers to firms with 2–24 employees, 25+ refers to firms with 25 or more employees. Even though 89.1 percent of firms in our sample are small, they make up only 32.2 percent of employment spells. The bottom panel shows summary statistics of the current daily wage of employment spells, first overall then conditional on firm size. Wages are recoded as $w = \max(w, w_{\min})$. The last row shows the share of spells with wages in the lowest bin (75 to 107% of the minimum wage). 1,000 HUF ≈ 5 USD in 2004.
Source: HUN-REN CERS, authors' own calculations.

Table 2: Employment-to-employment transition counts by wage bins

Current wage	*Accepted wage* *1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	23,582	5,009	3,116	2,602	2,481	1,737	1,345	1,108	834	605
2	5,059	3,628	2,004	1,370	1,067	863	592	461	333	208
3	2,665	1,736	2,951	1,883	1,234	871	575	437	256	186
4	1,943	999	1,370	2,559	1,682	1,223	725	540	307	179
5	1,647	655	777	1,290	2,651	1,591	984	697	383	261
6	1,115	493	528	713	1,222	2,470	1,713	1,038	510	284
7	911	390	377	456	653	1,072	2,343	1,564	883	381
8	726	296	261	358	473	501	888	2,145	1,771	665
9	500	196	211	233	355	350	492	939	2,694	1,965
10	457	160	163	219	309	336	373	526	1,150	6,102

Notes: For exposition's sake, the table uses 10 wage bins instead of 25 as in our empirical illustration. The first bin contains wages between 75 and 107% of the effective minimum wage. Subsequent bins are equally sized percentiles of the distribution of current wages.
Source: HUN-REN CERS, authors' own calculations.

another job, with the remaining spells entailing transitions to unemployment. A third of employment spells take place at small firms, and the average wage at small firms is about half of what larger firms pay.

For the purposes of estimation, we discretize wages into $W = 25$ bins. The first bin contains wages around the minimum wage (between 75 and 107% of the effective minimum wage in a given year), with the remaining bins set to be evenly distributed based on the distribution of current wages in each calendar year.[21] Whenever we use wage levels in a given bin (e.g., for the utility of wages), we take the mean wage in each bin of the distribution of current wages in 2003, except for the first bin where we use the 2003 minimum wage. For the purposes of describing the data below, we follow a similar procedure but discretize wages into ten bins.

Table 2 shows the number of employment-to-employment transitions to particular wage bins given the current wage bin. Excluding transitions to the first bin, the most populous cells are those that involve within-bin transitions, the second most populous cells are ones involving a transition to one bin higher, and the third most populous cells are ones involving a transition to one bin lower, suggesting heterogeneity in worker productivity may be important to account for in estimation. There are also a number of transitions involving substantial wages changes in both directions.

Table 3 takes this analysis one step further by examining how often employment-to-employment transitions result in wage increases or decreases. Overall, wages increase by 17.2% as a result of an employment-to-employment transition, but this average masks a large amount of heterogeneity: a third of transitions involve a wage decrease over 5% while over 40% of them involve a wage increase over 5%. Large wage increases are more common for transitions out of small firms, while large wage decreases occur more frequently out of larger firms.

Taken together, the descriptives reported in Tables 2 and 3 provide support for the model described in Section 2. There is clear evidence that individuals are moving to jobs that involve significant wage cuts. This empirical fact is consistent with a search model where individuals value more than just the wage. Furthermore, wage patterns differ across small and larger firms, which highlights the importance of modeling firm type-specific amenities.

Turning to the unemployment side, 45% of unemployment spells end in employment.[22] Panel (a) of Figure 1 shows the distribution of unemployment durations for those who exited unemployment during our observation window; the mean duration is 111 days. Panel (b)

[21] See Appendix B.3 for additional details on the wage discretization process.

[22] 37% of the remaining unemployment spells—those that do not end in employment—end in non-employment, while 63% are right-censored.

Table 3: Summary statistics, employment-to-employment transitions

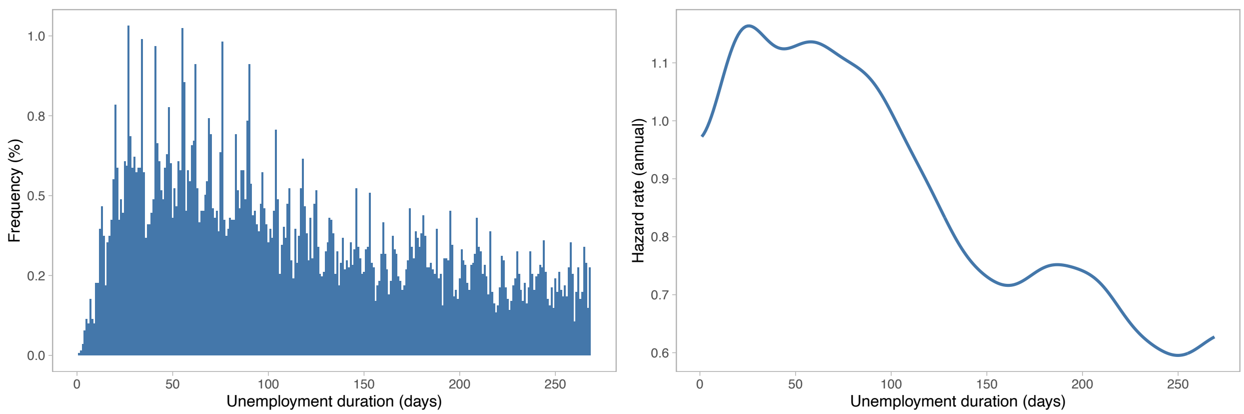
	Mean wage change (%)	**Share of transitions by wage change (%)**		
		Less than -5%	-5 *to* 5%	*More than* 5%
All EE	17.2	34.1	24.1	41.8
EE out of small firms	31.7	24.6	32.1	43.3
EE out of 25+ firms	9.8	39.0	20.0	41.0

Notes: The first column shows the mean wage change from the current to the accepted job, for all employment-to-employment (EE) transitions then by firm size. The second to fourth columns show the distribution of spells that end in an employment-to-employment transition, broken down by wage change categories. Current and accepted wages are recoded as $w = \max(w, w_{\min})$.
Source: HUN-REN CERS, authors' own calculations.

Figure 1: Unemployment durations

(a) Histogram

(b) Hazard of exiting to employment



Notes: Panel (a) shows the distribution of unemployment spells. Spells are right-censored at 269 days. Panel (b) shows the unconditional hazard rate of exiting unemployment. We calculate the hazard as the kernel-smoothed density of exiting unemployment to a job, divided by the kernel-smoothed survivor function. We use Gaussian kernels with optimal bandwidth selection and reflection for boundary correction.
Source: HUN-REN CERS, authors' own calculations.

of Figure 1 shows that, after the first month, the hazard rate of exiting unemployment to employment is generally downward-sloping, consistent with the existence of negative duration dependence. Next, we divide those who exit unemployment to a job into five categories based on their unemployment duration. Summary statistics of accepted wages for those who exit unemployment to firms of a given size in each of these durations are presented in Table 4. Consistent with unemployed workers being willing to accept lower wage offers over time, longer durations are associated with lower accepted wages, and higher probabilities of accepting a job at the minimum wage. In particular, those whose unemployment durations were less than 30 days were a little over half as likely to exit to a job paying the minimum wage as those whose durations were in the top category.

Table 4: Summary statistics, unemployment-to-employment transitions

(a) All unemployment-to-employment transitions

	Overall	*By unemployment duration (days)*				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Mean U duration (days)	110.7	20.5	46.0	75.7	130.5	221.0
Mean acc. wage (HUF)	2,743	3,079	2,951	2,769	2,656	2,474
Share $\underline{w}$ (%)	30.6	20.8	22.9	29.1	34.5	38.3
Share small firms (%)	34.1	23.8	27.7	33.5	37.2	41.9

(b) Unemployment-to-employment transitions to small firms

	Overall	*By unemployment duration (days)*				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Mean U duration (days)	122.5	20.8	46.4	76.6	129.9	222.7
Mean acc. wage (HUF)	2,101	2,323	2,299	2,109	2,045	1,981
Share $\underline{w}$ (%)	55.9	44.8	45.5	53.5	60.3	61.6

(c) Unemployment-to-employment transitions to firms with 25 or more employees

	Overall	*By unemployment duration (days)*				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Mean U duration (days)	104.7	20.5	45.8	75.2	130.8	219.8
Mean acc. wage (HUF)	3,076	3,315	3,202	3,101	3,017	2,830
Share $\underline{w}$ (%)	17.4	13.3	14.2	16.8	19.3	21.6

Notes: The table shows summary statistics of spells that end in an unemployment-to-employment transition. Accepted wages are recoded as $w = \max(w, w_{\min})$. The last row of each panel shows the share of unemployment-to-employment transitions to the lowest wage bin (75 to 107% of the minimum wage). Wage rates are daily; 1,000 HUF $\approx$ 5 USD in 2004.
Source: HUN-REN CERS, authors' own calculations.

5 Estimation

5.1 Overview

Consider a workforce populated by N individuals, indexed by i. Workers may face different wage offer distributions, job offer arrival and destruction rates, as well as different flow payoffs of unemployment in ways that are unobserved to the econometrician. In addition, each of these may be affected by the type of job with which the worker is matched. On the worker side, each individual belongs to one of R worker types, which we set in our application to $R = 4$. We allow jobs to belong to one of $F = 3$ firm types. The first of these types is assumed to be observed, and consists of firms that have fewer than 25 employees. Within the class of firms with 25 or more employees, we allow for two unobserved firm types. We collect firm types in a firm classification vector (denoted by C). Finally, recall that in our application, we discretize wages into $W = 25$ bins.

Each individual i experiences K_i employment spells, indexed by k, and $\tilde{K}_i$ unemployment spells, indexed by $\tilde{k}$. Given a job classification C, the corresponding likelihoods for these spells k and $\tilde{k}$ for individual i of type r are given by $\mathcal{L}^E_{ikr}(\theta^E; C)$ and $\mathcal{L}^U_{i\tilde{k}r}(\theta^E, \theta^U; C)$, respectively, where θ^E denote the employed-side parameters and θ^U the parameters that are unique to the unemployed likelihood. Note that the employed-side parameters enter the likelihood for the unemployment spells but the reverse is not true. We will exploit this sequential likelihood property later in our estimation procedure.

In addition to the likelihood components associated with each employed and unemployed spell, we also account for the initial wage and initial firm type, which are allowed to depend on the worker type. Denote the likelihood of the initial condition for worker i given type r as $\mathcal{L}^I_{ir}(\theta^I; C)$. After integrating over worker types where the probability of type r is given by π_r, with $\pi_R = 1 - \sum_{r<R} \pi_r$, the full maximization problem is given by:

$$\max_{C,\pi_1,\ldots,\pi_{R-1},\theta^I,\theta^E,\theta^U} \sum_i \ln\left(\sum_r \pi_r \mathcal{L}^I_{ir}(\theta^I; C) \prod_k \mathcal{L}^E_{ikr}(\theta^E; C) \prod_{\tilde{k}} \mathcal{L}^U_{i\tilde{k}r}(\theta^E, \theta^U; C)\right) \tag{5.1}$$

We estimate our model of job-to-job transitions and unemployment-to-job transitions allowing for both worker- and job-level unobserved heterogeneity. Estimation proceeds in three stages. First, building on the insights of Arcidiacono and Miller (2011), Bonhomme et al. (2022), and Lentz et al. (2023), we recover the distribution of unobserved worker types and classify firm into types. Second, conditional on these types, we estimate the parameters governing job-to-job transitions. Finally, given the parameters from the previous two stages, we estimate the parameters governing the unemployment-to-job transitions and recover the flow utility of unemployment.

Note that, after obtaining the first stage results, one could in principle compute the hazard rates given in A1–A4 by using the posterior type probabilities as weights, and conditioning on the estimated firm classification. It would then be straightforward to estimate the model parameters relying on the constructive identification results in Sections 3.3 and 3.4. We do not pursue this approach for two main reasons. First, in practice, the model is heavily overidentified despite placing little structure on the wage distribution and offer arrival rates. Second, absent any restrictions, and given $R = 4$ worker types, $F = 3$ firm types, and

$W = 25$ wage support points, we would need to estimate as many as 452 parameters.[23] As we describe in Sections 5.3 and 5.4, we impose more structure on the model to conserve on parameters, which in turn affects our estimation strategy.

5.2 Step 1: worker type distribution and job classification

In the first estimation stage, we pre-classify workers and jobs into discrete types following Lentz et al. (2023). Their algorithm, which builds upon Bonhomme et al. (2022), posits an initial classification of jobs. Given this initial job classification, the algorithm iterates between an inner Expectation-Maximization (EM) loop and an outer classification loop. The inner EM loop updates the distributions of worker types. Given the conditional probabilities of each individual belonging to any worker type, an outer classification loop updates the job-type classification where, as in the EM algorithm, the likelihood is guaranteed to weakly improve at each step.[24]

Following Arcidiacono and Miller (2011), instead of estimating the structural parameters in the first step, we substitute in reduced-form counterparts for $\mathcal{L}^E_{ikr}(\theta^E; C)$ and $\mathcal{L}^U_{i\tilde{k}r}(\theta^E, \theta^U; C)$, the details of which can be found in Appendix C.1. We denote these reduced-form counterparts by $\tilde{\mathcal{L}}^E_{ikr}(\tilde{\theta}^E; C)$ and $\tilde{\mathcal{L}}^U_{i\tilde{k}r}(\tilde{\theta}^U; C)$, respectively. This allows us to substantially speed up the estimation.[25]

5.2.1 Inner EM loop

The inner EM loop takes the job classification C as given and iterates over an Expectation and Maximization step. At the $(m + 1)$-th iteration, we first calculate the posterior probabilities that each individual belongs to each of the R worker types, denoted by $q^{(m+1)}_{ir}$, and update the population probabilities of each worker type, $\pi^{(m+1)}_r$, given the parameters $\{\theta^{I(m)}, \tilde{\theta}^{E(m)}, \tilde{\theta}^{U(m)}\}$. The calculation of $q^{(m+1)}_{ir}$ follows from Bayes' rule, with the updating

[23] For $W = 25$, $R = 4$ and $F = 3$, this is equal to the sum of the total number of wage parameters ($(W - 1)RF = 288$), offer arrival rates ($RF^2 = 36$), (symmetric) switching costs ($RF(F - 1)/2 = 12$), wage utility parameters ($R(W - 1) = 96$), utility parameters associated with compensating differentials ($R(F - 1) = 8$) and job destruction rates ($RF = 12$).

[24] Appendix C.1.4 discusses how starting values are chosen and, given the possibility of local optima, our strategy for mitigating this concern.

[25] As a check on this reduced-form specification, we compared the posterior type probabilities to their structural counterparts, obtained by updating the q's using the likelihood components from the later estimation stages. The correlation between the two sets of probabilities exceeds 99 percent for all worker types.

of the population probabilities given by the empirical average of the posterior probabilities:

$$q_{ir}^{(m+1)} = \frac{\pi_r^{(m)} \mathcal{L}_{ir}^I(\theta^{I(m)}; C) \prod_k \tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^{E(m)}; C) \prod_{\tilde{k}} \tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^{U(m)}; C)}{\sum_r \pi_r^{(m)} \mathcal{L}_{ir}^I(\theta^{I(m)}; C) \prod_k \tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^{E(m)}; C) \prod_{\tilde{k}} \tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^{U(m)}; C)} \tag{5.2}$$

$$\pi_r^{(m+1)} = \left(\sum_i q_{ir}^{(m+1)} \right) / N \tag{5.3}$$

The second step then consists of maximizing the expected complete log-likelihood function, taking the posterior worker type probabilities as given, to obtain the updated parameters $\{\theta^{I(m+1)}, \tilde{\theta}^{E(m+1)}, \tilde{\theta}^{U(m+1)}\}$. Namely:

$$\max_{\theta^I, \tilde{\theta}^E, \tilde{\theta}^U} \sum_{i,r,k,\tilde{k}} q_{ir}^{(m+1)} \left(\ln \mathcal{L}_{ir}^I(\theta^I; C) + \ln \tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^E; C) + \ln \tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^U; C) \right) \tag{5.4}$$

5.2.2 Outer classification loop

In the outer classification loop, we update the classification of firms to firm types. Namely, at the $(n+1)$-th classification step, the conditional worker type probabilities $q_{ir}^{(n)}$ and parameters $\{\theta^{I(n)}, \tilde{\theta}^{E(n)}, \tilde{\theta}^{U(n)}\}$ are taken as given and set equal to the converged values from the inner EM loop associated with the n-th classification step. The optimization problem boils down to choosing the job classification $C^{(n+1)}$ according to:

$$\max_C \sum_{i,r,k,\tilde{k}} q_{ir}^{(n)} \left(\ln \mathcal{L}_{ir}^I(\theta^{I(n)}; C) + \ln \tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^{E(n)}; C) + \ln \tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^{U(n)}; C) \right) \tag{5.5}$$

A challenge that arises when choosing a new classification is that jobs are interconnected: changing the classification for one job may affect the optimal classification of another job, which results in a difficult maximization problem. We address this issue by following Lentz et al. (2023) and seek to improve, at any given step, the likelihood rather than maximize it, relying on subsequent iterations to reach the maximum. Following their approach, we first order firms according to their size. We then choose a classification for the largest firm that results in the highest likelihood for the full problem, taking the other firm classifications as given. Then, with this updated classification for the largest firm, we proceed to classify the second largest firm, and so on. By classifying firms one by one and taking any updates to a classification of a larger firm as given, one is guaranteed to (weakly) increase the likelihood with the classification of each firm.

After completing the outer loop, the inner EM algorithm is run again. We iterate between the inner EM loop and the outer classification loop, and the full algorithm stops when the outer loop results in no changes to the job classification.

5.3 Step 2: employed-side parameters

With the classification of firms and conditional probabilities of worker types from Step 1 in hand, we now proceed to the estimation of the employed-side parameters. The general structure of the model we estimate matches that of Section 3.3 with one exception: we do not allow for exogenous wage and firm type changes ($\delta^{ss'}_{ww'} = 0$). As previously mentioned, the large number of parameters in practice necessitates imposing additional structure on the model, which naturally motivates a maximum likelihood framework. Namely, we specify the flow payoff of wages as $u_w = \alpha \ln(w)$. We also assume that there is a common switching cost c, and that job destruction rates vary only at the worker-type level: $\delta^s_r = \delta^{s'}_r$ for all $\{s, s'\}$. The discount rate, ρ, is set to 0.05. The structure placed on the wage distributions and the offer arrival rates is given in Appendix C.2.

Estimation of the employed-side parameters consists of estimating the job destruction rates, along with the parameters that govern the job-to-job hazard rates. For each employment spell k associated with worker i, we observe its duration, t_{ik}, and the wage, w_{ik}. Let $\mathbb{1}\{EU_{ik+1} = 1\}$ denote an indicator variable for individual i transitioning to unemployment during their k^{th} employment spell. Estimation of the type-r job destruction rate δ_r directly follows as the weighted number of transitions to unemployment, divided by the weighted time spent in employment:

$$\hat{\delta}_r = \frac{\sum_{i=1}^{N} q_{ir} \sum_{k=1}^{K_i} \mathbb{1}\{EU_{ik+1} = 1\}}{\sum_{i=1}^{N} q_{ir} \sum_{k=1}^{K_i} t_{ik}} \tag{5.6}$$

The job-to-job hazard rates are then used to recover the remaining parameters, with the rate of moving from a $\{w, s\}$ job to a $\{w', s'\}$ job for a type r worker given by:

$$h^{ss'}_{ww'r} = \lambda^{ss'}_r f^{s'}_{w'r} p^{ss'}_{ww'r} \tag{5.7}$$

Note that the conditional job acceptance probability, $p^{ss'}_{ww'r}$, directly follows from Equation (3.4). Embedded in Equation (3.4) are the worker type-specific value functions associated with jobs $\{w, s\}$ and $\{w', s'\}$. We solve for these employment value functions with a fixed point algorithm embedded in the maximum likelihood routine.

We collect the employed-side parameters that remain to be estimated in $\theta^E_2 \equiv (f, \lambda, \alpha, \phi, c)'$.

It follows that the likelihood contribution of a job spell k for a type-r worker i is given by:

$$\mathcal{L}^E_{ikr}(\delta_r, \theta^E_2) = \prod_{w,w',s,s'} \Big[(h^{ss'}_{ww'r})^{\mathbb{1}\{w_{ik}=w, w_{ik+1}=w', s_{ik}=s, s_{ik+1}=s'\}} \exp(-h^{ss'}_{ww'r} t_{ik})\Big]^{\mathbb{1}\{w_{ik}=w, s_{ik}=s\}}$$

Taking as given the job destruction rate estimated in a preliminary step by $\hat{\delta}_r$, we then estimate these parameters by maximizing the expected complete log-likelihood with respect to θ^E_2:

$$\max_{\theta^E_2} \sum_{i=1}^{N} \sum_{r=1}^{R} \sum_{k=1}^{K_i} q_{ir} \ln \Big(\mathcal{L}^E_{ikr}(\hat{\delta}_r, \theta^E_2)\Big) \tag{5.8}$$

5.4 Step 3: unemployed-side parameters

In the third and last estimation step, we estimate the wage offer distribution out of unemployment, g^s_{wr}, and the offer arrival rates, $\lambda^s_r(t)$, via maximum likelihood. We then estimate the flow payoff of unemployment, $b_r(t)$, based on our constructive identification strategy.

Note that the type-r hazard of leaving unemployment at duration t to wage w and firm type s is given by:

$$h^s_{wr}(t) = \lambda^s_r(t) g^s_{wr} p^s_{wr}(t) \tag{5.9}$$

The probability of accepting a job, $p^s_{wr}(t)$, depends on the value of the job, V^s_{wr}, and the value of remaining unemployed, $V_{0r}(t)$, and takes the logit form:

$$p^s_{wr}(t) = \frac{\exp(V^s_{wr})}{\exp(V^s_{wr}) + \exp(V_{0r}(t))} \tag{5.10}$$

We recover V^s_{wr} up to a constant using the estimates of the employed-side parameters. We denote this normalized value function as $\tilde{V}^s_{wr} \equiv V^s_{wr} - \delta_r V_{0r}(0)/(\rho + \delta_r)$. As a result, the unemployment value function we estimate, $\tilde{V}_{0r}(t)$, is also normalized, with $\tilde{V}_{0r}(t) \equiv V_{0r}(t) - \delta_r V_{0r}(0)/(\rho + \delta_r)$. In practice, we specify $\tilde{V}_{0r}(t)$ as a flexible function of t, the details of which, along with the parameterizations of $\lambda^s_r(t)$ and g^s_{wr}, are given in Appendix C.3.

Denote by θ^U the vector of parameters indexing the wage offer distributions, the offer arrival rates and the value of unemployment. The likelihood contribution of type-r individual i's unemployment spell $\tilde{k}$ is then given by

$$\mathcal{L}^U_{i\tilde{k}r}(\theta^E, \theta^U) = \prod_{w,s} \left\{ [h^s_{wr}(t_{i\tilde{k}})]^{\mathbb{1}\{w_{i\tilde{k}}=w, s_{i\tilde{k}}=s\}} \exp\left(-\int_0^{t_{i\tilde{k}}} h^s_{wr}(u)\, du \right) \right\} \tag{5.11}$$

where we then maximize, as in Step 2 for the employed-side parameters, the expected complete log-likelihood to obtain $\hat{\theta}^U$ (taking as given $\hat{\theta}^E$ from Step 2):

$$\max_{\theta^U} \sum_{i,r,\tilde{k}} q_{ir} \ln\left(\mathcal{L}^U_{i\tilde{k}r}(\hat{\theta}^E, \theta^U) \right) \tag{5.12}$$

We conclude this section with the estimation of the last remaining parameters, the unemployment value function $V_{0r}(t)$ and the flow utility of unemployment $b_r(t)$. We first need to calculate the (unnormalized) unemployment value function and its time derivative. Given the estimates of the employed and unemployed parameters, we compute $V_{0r}(t)$ pointwise at each duration t as follows. We first evaluate the relationship $\tilde{V}_{0r}(t) = V_{0r}(t) - \delta_r V_{0r}(0)/(\rho + \delta_r)$ at $t = 0$, which directly yields the initial value of unemployment, $V_{0r}(0)$. The unemployment value function at any given time $t > 0$ is then given by $V_{0r}(t) = \tilde{V}_{0r}(t) + \delta_r V_{0r}(0)/(\rho + \delta_r)$ and its time derivative is given by $\dot{V}_{0r}(t) = \dot{\tilde{V}}_{0r}(t)$, where the normalized value function $\tilde{V}_{0r}(t)$ is specified as a differentiable function of t.

We finally calculate the flow payoff of unemployment using the expression

$$b_r(t) = \rho V_{0r}(t) + \sum_{w,s} \lambda^s_r(t) g^s_{wr} \ln(1 - p^s_{wr}(t)) - \dot{V}_{0r}(t) \tag{5.13}$$

where all of the right-hand side parameters have been estimated in previous steps.

6 Estimation results

We begin with the employed-side parameters, showing substantial permanent heterogeneity across both worker and firm types. We then turn to the unemployed side, where the non-stationarities lie. Job offer arrival rates fall with unemployment duration, as does the flow utility of unemployment. Finally, we rely on our estimated model to perform a counterfactual simulation analysis, focusing on the effect of extending unemployment benefits on the duration of unemployment.

Table 5: Structural parameter estimates, employed side

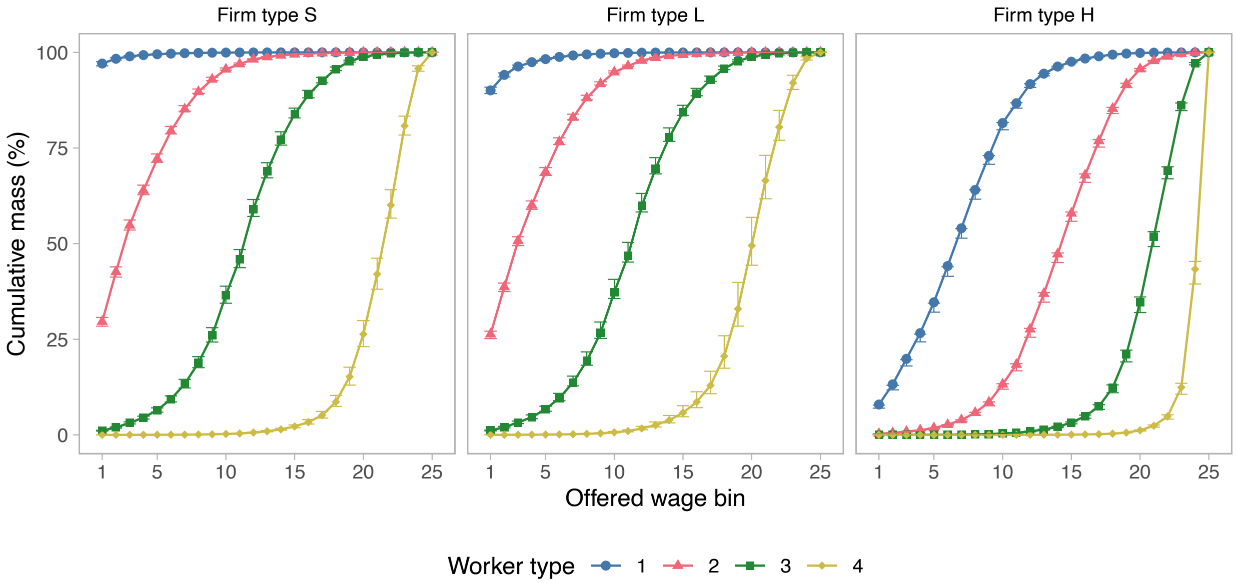
	Parameter	Estimate			
		Worker type			
		1	*2*	*3*	*4*
λ_r^{ss}	Offer arrival rate from current firm type				
	Firm type S	0.098	0.109	0.135	0.124
		(0.003)	(0.003)	(0.005)	(0.004)
	Firm type L	0.132	0.078	0.045	0.017
		(0.004)	(0.003)	(0.002)	(0.002)
	Firm type H	0.073	0.098	0.128	0.150
		(0.002)	(0.003)	(0.004)	(0.005)
$\sum_{s'} \lambda_r^{ss'}$	Total offer arrival rate conditional on current firm type				
	Firm type S	0.256	0.222	0.224	0.193
		(0.007)	(0.006)	(0.006)	(0.005)
	Firm type L	0.200	0.159	0.147	0.122
		(0.005)	(0.004)	(0.004)	(0.003)
	Firm type H	0.248	0.223	0.230	0.220
		(0.006)	(0.006)	(0.006)	(0.005)
δ_r	Job destruction rate	0.321	0.228	0.129	0.095
		(0.001)	(0.001)	(0.001)	(0.001)
α	Flow utility of log wages	0.479			
		(0.013)			
ϕ_r^s	Flow utility of firm types				
	Firm type L	-0.720	-0.246	0.013	0.163
		(0.013)	(0.011)	(0.016)	(0.032)
	Firm type H	0.133	0.007	-0.082	-0.137
		(0.012)	(0.009)	(0.011)	(0.012)
c	Job switching cost	0.164			
		(0.049)			
π_r	Type probability	0.419	0.397	0.138	0.045
		(0.002)	(0.002)	(0.001)	(0.001)

Notes: Offer arrival rates λ and job destruction rates δ are annual. The flow utility of log wages α and the job switching cost c are fixed across heterogeneity types. Bootstrap standard errors in parentheses (500 replications).
Source: HUN-REN CERS, authors' own calculations.

6.1 Employed-side results

Table 5 shows the estimates of the employed-side parameters with the exception of the initial conditions and the wage offer distribution, as well as average wages by worker type. Worker types are ordered according to their population shares, ranging from 4.5% (for type 4) to

Figure 2: Wage offer distribution, employed side



Notes: Error bars represent 95% bootstrap confidence intervals (500 replications).
Source: HUN-REN CERS, authors' own calculations.

41.9% (for type 1). The ordering also aligns with measures of productivity: average wages are more than four times higher for the most productive type than for the least (type 4 vs. type 1, see Table D.2 in Appendix D). And, in line with these large differences across types, type 1 workers also have job destruction rates that are more than three times as high as those of type 4 workers.

Recall that we consider a specification with three firm types, which we denote by S, L and H. Each of these types accounts for 32.2% (type S), 20.7% (type L) and 47.1% (type H) of the employment spells (Table D.2). More productive worker types receive a higher share of their offers from type H firms which, as we will show, tend to offer higher wages. Although there are clear patterns on the share of offers from each firm type, with offer rates increasing (decreasing) in productivity for H (L) type firms, there is less heterogeneity across worker types in terms of the total offer arrival rate. The total offer arrival rate ranges from 0.122 to 0.256, depending on worker and firm type. The differences in these total offer arrival rates are mainly driven by where the worker is currently employed, with slower offer arrival rates for workers in type L firms across the board.

The estimated parameter associated with the flow utility of log wages is 0.479, which is almost three times the magnitude of the average cost of switching jobs (0.164). The flow utility parameter is sufficiently large as to produce substantial heterogeneity in the probability of accepting a job given the current and offered wage. For example, consider a type 4 worker in a type H job who receives another type H job offer. If the worker is in the highest wage bin and the offer is from the lowest wage bin, the acceptance probability, under our estimates, would be 2%; if the worker is in the lowest wage bin and the offer is from the highest wage bin, the acceptance probability would be as large as 97%.

Figure 2 next shows the cdf of the employed wage offer distribution, by worker and firm type. For every firm type, the same productivity ordering emerges and aligns with the differences in average wages discussed above: the wage offer distribution for type 4 workers stochastically dominates that of type 3 workers, and so on down to type 1 workers. As shown in Table D.1 in Appendix D, initial wages also show the same ordering regardless of firm type.

Beyond worker types, Figure 2 shows that heterogeneity across firm types also plays an important role. Notably, type H jobs pay substantially more regardless of worker type. Wage offers from type S and L firms are similar for all worker types, except the most productive one. For type 4 workers, wages are indeed higher in small firms (type S) than in type L firms, likely reflecting that at least some of the jobs in small firms are high productivity jobs.[26]

6.2 Unemployed-side results

We now turn to the unemployed-side results. Crucial to obtaining these results are the estimates of the value functions associated with each firm type and wage. Firm types associated with higher employment-side offer arrival rates are, all else equal, more attractive because of their option value. This in turn affects the estimated unemployed offer arrival rates and wage offer distributions.

We first show the offered wage distribution for unemployed workers in Figure 3. The overall patterns are similar to that of employed workers. The same ordering of worker types holds for offered wages regardless of firm type, with type 4 workers seeing higher offered wages than type 3 workers, and so on. And, as with on-the-job wage offers, type H firms pay workers of all types more than type S and L firms, with type S and L firms offering similar wages to all worker types with the exception of the most productive one (type 4).[27]

[26]The share of job-to-job changes that involve a wage cut is around 40%, both in the data and in our estimated model. Decreasing the variance scale parameter to a quarter (tenth) of its original value lowers this share to 10% (2%), pointing to the important role that preference shocks also play in this setup.

[27]In Appendix E, we provide estimation results from a model that allows the offered wage distribution out of unemployment to vary with unemployment duration. While there is some evidence of the offered wage distribution becoming worse as unemployment duration increases, the effects are small, particularly so

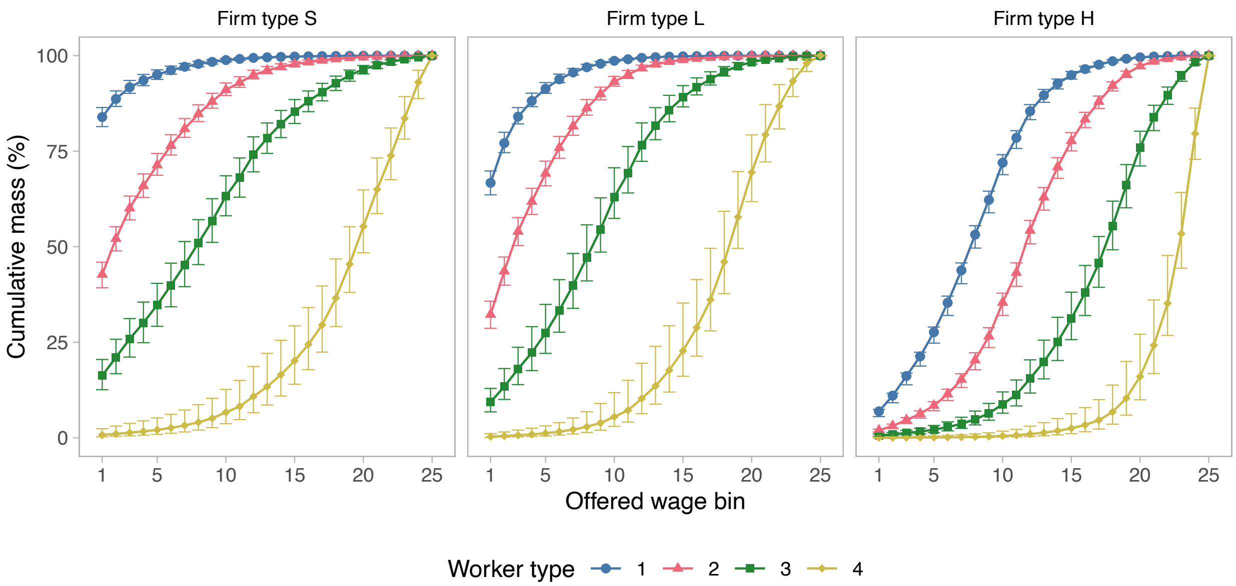
Figure 4 shows how offer arrival rates evolve over the course of unemployment. Offers from type L and H firms show negative duration dependence for all worker types.[28] For example, the least productive worker type receives offers at a rate of a little over one per year at the beginning of unemployment from type L jobs. By day 200, however, this rate has fallen by half. Interestingly, the composition of job offers vary substantially across worker types and over time. Less productive worker types (types 1 and 2) receive more offers from firm type L (lower paying large firms) than from firm type H (higher paying large firms); the reverse is true for more productive worker types (types 3 and 4). Over time, the share of offers coming from small firms (firm type S) increases. As type S jobs tend to pay significantly less than type H jobs, overall wage offers are getting worse over the course of unemployment, reflecting over time changes in the firm type composition of the job offers.

A second source of nonstationarity is the flow payoff of unemployment. The evolution of these is displayed in Figure D.1 in Appendix D. After a sharp initial increase, the flow payoff gradually decays until benefit expiration. As government-paid unemployment benefits are fixed over this time period, the nonstationarity arises from other sources, including psychic cost of unemployment which may increase with time spent unemployed. The initial increase in the flow payoff for being unemployed may be driven by startup costs associated with searching for jobs, including filling out applications. As shown in Figure D.2, job acceptance probabilities increase sharply over the course of unemployment: the probability of accepting a minimum wage job rises from 60 percent in the beginning to 82 percent at the end of the unemployment duration on average, with the largest increase being 55 percentage points for the most productive worker type at all firm types.

when compared to the heterogeneity in the offer distribution across worker and firm types.

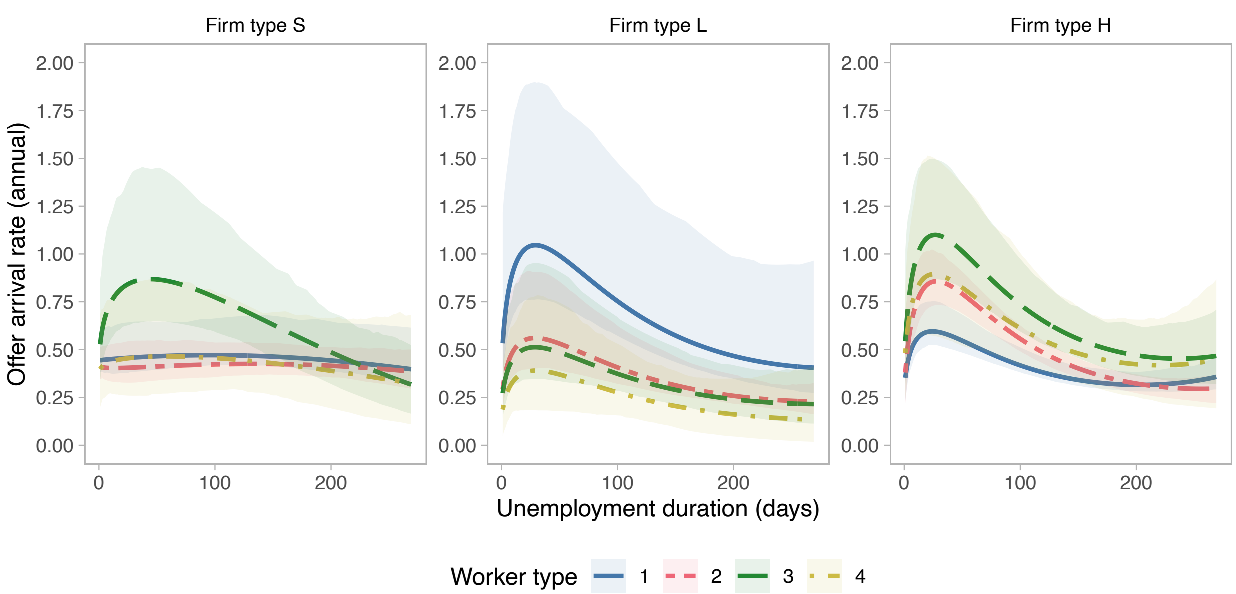
[28] The p-value of a joint significance test of the parameters that govern the evolution of $\lambda_r^s(t)$ over time is 0.037.

Figure 3: Wage offer distribution, unemployed side



Notes: Error bars represent 95% bootstrap confidence intervals (500 replications).
Source: HUN-REN CERS, authors' own calculations.

Figure 4: Offer arrival rates out of unemployment



Notes: Annualized rates. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: HUN-REN CERS, authors' own calculations.

Table 6: Dynamic composition of heterogeneity types and accepted wages

(a) Worker types

	Share (%)	*Share by unemployment duration (%)*					*Mean unemployment duration (days)*	
		1–30	*31–60*	*61–90*	*91–180*	*181–269*	*Model*	*q-weighted data*
Worker type 1	41.1	34.2	33.4	40.9	46.9	46.4	111.2	110.6
Worker type 2	45.5	51.5	50.6	45.4	40.1	42.8	97.5	96.7
Worker type 3	11.6	12.8	13.8	11.7	11.0	9.1	93.4	92.8
Worker type 4	1.9	1.5	2.2	1.9	2.0	1.8	97.1	101.6

(b) Accepted wages and firm types

	Model						**q-weighted data**					
	Overall	*By unemployment duration (days)*					*Overall*	*By unemployment duration (days)*				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*		*1–30*	*31–60*	*61–90*	*91–180*	*181–269*
Worker type 1												
Mean acc. wage (HUF)	2,184	2,268	2,247	2,205	2,146	2,127	2,183	2,365	2,273	2,150	2,141	2,072
Share firm type S (%)	35.9	30.2	29.6	33.0	39.0	41.3	35.6	27.0	27.5	34.8	39.7	41.6
Share firm type H (%)	39.8	46.9	45.0	41.3	36.3	35.3	40.0	51.7	45.2	39.4	36.1	34.2
Worker type 2												
Mean acc. wage (HUF)	2,743	2,868	2,862	2,798	2,686	2,597	2,767	2,974	2,881	2,808	2,674	2,480
Share firm type S (%)	31.9	25.2	23.5	27.1	35.1	42.4	30.6	21.3	24.9	31.0	34.4	42.5
Share firm type H (%)	42.2	49.7	50.0	45.9	38.7	32.9	43.5	52.9	47.0	43.9	40.5	31.8
Worker type 3												
Mean acc. wage (HUF)	4,169	4,402	4,302	4,179	4,032	4,022	4,177	4,896	4,151	3,928	4,031	3,696
Share firm type S (%)	36.5	31.6	33.4	36.9	40.7	37.4	36.2	28.7	38.9	38.4	36.6	40.2
Share firm type H (%)	44.9	50.6	48.4	44.8	40.8	42.7	45.3	59.5	43.1	40.4	41.6	39.9
Worker type 4												
Mean acc. wage (HUF)	8,104	8,394	8,276	8,135	7,952	7,912	8,178	8,256	8,288	8,796	7,904	7,887
Share firm type S (%)	32.3	26.4	26.2	29.8	36.0	38.4	31.3	17.6	25.5	34.2	33.2	43.9
Share firm type H (%)	48.1	52.6	51.7	48.8	44.9	45.6	48.6	56.0	50.1	49.0	47.5	42.2

Notes: The left side of Panel (a) shows the distribution of worker types exiting unemployment to employment. The right side of Panel (a) shows the average days of unemployment duration in simulated spells (model) vs. observed spells using the estimated posterior worker type probabilities as weights (q-weighted data). Panel (b) shows the mean accepted wage and the share of type S/H jobs in simulated spells (model) vs. observed spells using the estimated posterior worker type probabilities as weights (q-weighted data). Jobs are classified into 3 types as discussed in Section 5. Both panels show summary statistics across all spells as well as conditional on duration ranges. Accepted wages are daily wage levels reported in Hungarian forints (1,000 HUF $\approx$ 5 USD in 2004).
Source: HUN-REN CERS, authors' own calculations.

We next report in Table 6 how the distribution of worker and firm types, as well as accepted wages, vary over the course of unemployment. Panel (a) of Table 6 shows the distribution of worker types in the unemployment sample, both overall and conditional on leaving unemployment at different durations. The least productive worker type accounts for 46.2% of unemployed spells, but only 41.1% of those who leave unemployment. This disparity becomes even more apparent when considering the distribution of worker types conditional on leaving unemployment within 30 days, with the share that belong to the least productive type being equal to 34.2%. In other words, higher productivity is associated with being more likely to leave unemployment and, conditional on leaving within our time window, doing so at earlier durations.

The left-most columns of Panel (b) of Table 6 show wages and firm types conditional on leaving unemployment, both overall and by unemployment duration. Exiting in the first 30 days is associated with wages that are between 6% and 11% higher than exiting after 180 days. The decline in wages is driven in part by shifts over time in firm type. Those exiting at earlier times are indeed more likely to go to high-paying large firms (firm type H), with those exiting later transitioning to small firms.

Finally, the right-most columns of Panel (b) of Table 6 allow us to investigate the model fit. Here we compare the model predictions to those from the data where we use the conditional probabilities of being of each worker type, q_{ir}, as weights to get the corresponding type-specific empirical moments. The main takeaway is that the predictions from the model generally match the weighted data well, though the model slightly underpredicts the share of workers going to H type firms conditional on leaving in the first 30 days.

6.3 Counterfactual simulation

We next use the estimation results of the model to examine how extending unemployment benefits would affect unemployment duration. Note that due to data limitations our sample effectively stops right at the point of benefit expiration. To examine the effect of extending unemployment benefits, we have to make assumptions regarding offer arrival rates and the total flow utility of unemployment (including both pecuniary and non-pecuniary components) for durations longer than 269 days.

We first assume that, once benefits expire, offer arrival rates are constant at their estimated value at $t = 269$ days. We also assume that the flow payoff of unemployment is fixed beyond this point, so that both its pecuniary and non-pecuniary portions no longer vary with time.

It follows from these assumptions that the value function of unemployment is also constant.

In this continuous-time environment, any anticipated discrete change in flow unemployment benefits does not generate a discontinuity in the unemployment value function. Letting $b(T^+) = \lim_{t\to T^+} b(t)$ and $b(T^-) = \lim_{t\to T^-} b(t)$ denote, respectively, the right-limit and left-limit of $b(t)$ at benefits expiration T, and similarly for the limits of the time derivative of the unemployment value function, $\dot{V}_0(T^+)$ and $\dot{V}_0(T^-)$, it follows from Equation (5.13) that the flow payoff of unemployment after benefit expiration can be computed using:

$$b(T^+) = b(T^-) + \dot{V}_0(T^-) - \dot{V}_0(T^+) \tag{6.1}$$

where the first two terms on the right-hand side are known from the estimation results, and the last term is zero given the assumption of stationarity after benefit expiration. Given any extension of unemployment benefits, we then solve by backwards recursion at the daily level the corresponding probabilities and hazards out of unemployment, at any point in time. It is then straightforward to calculate the expected durations of unemployment under different extensions of unemployment benefits.[29]

The top panel of Table 7 shows expected durations for each worker type under the status quo, and under extensions of unemployment benefits by 90, 120, and 270 days. Increasing the duration of benefits by 90 (270) days increases unemployment duration by between 8 (26) and 19 (62) days, depending on worker type.

The bottom panel of Table 7 reports the ratios between the changes in unemployment durations and the corresponding changes in unemployment benefit duration. The results lie between 0.088 and 0.229, which fits in the range of the estimates obtained in the empirical literature on UI benefits extension (Schmieder and von Wachter, 2016, Le Barbanchon et al., 2024).

Note that these are partial equilibrium results. In practice, changing unemployment benefits may also affect, in particular, offer arrival rates. Nevertheless, this simulation exercise does illustrate how this continuous-time nonstationary search model, combined with the empirical strategy that we develop in this paper, can be easily leveraged for counterfactual analysis.

[29] Conditional on the unemployment spell lasting past benefit expiration, and under the steady state assumption, the expected remaining duration is equal to one over the sum of the hazards out of unemployment.

Table 7: Unemployment duration under counterfactual UI policies

UI policy and outcome	*Overall*	*By worker type*			
		1	*2*	*3*	*4*
Expected unemployment duration					
Baseline	381	424	351	302	364
90-day extension	394	443	359	317	375
120-day extension	399	450	362	322	379
270-day extension	426	486	377	349	402
Δ unemployment duration / Δ UI benefit duration					
90-day extension	0.155	0.214	0.088	0.160	0.123
120-day extension	0.157	0.217	0.090	0.163	0.126
270-day extension	0.167	0.229	0.097	0.175	0.139

Notes: The top half of the table shows the expected unemployment duration overall as well as by worker type from our baseline results as well as under three counterfactual UI policies that extend benefits by 90, 120, and 270 days. We assume that the offer arrival rates and flow payoffs are fixed at their level at $t = 269$ for the extensions; see the text for details. The bottom half of the table shows the change in unemployment duration projected onto the change in the duration of benefits under the three counterfactual scenarios.
Source: HUN-REN CERS, authors' own calculations.

7 Conclusion

In this paper, we extend the canonical continuous-time job search model with on-the-job search to incorporate preference shocks, thereby bringing together dynamic discrete choice methods with the empirical job search literature. We show that this approach is fruitful in several ways. First, using the insights from conditional choice probability methods, we establish constructive identification of the model parameters, even in rich nonstationary settings. A second key advantage is computational. Unlike standard nonstationary search models, our approach does not require solving a nonlinear differential equation within the maximization routine, which makes estimation fast for the class of models we consider.

We illustrate our method using Hungarian administrative data. Nonstationarities when unemployed operate through offer arrival rates, the nonpecuniary flow payoff of unemployment, and through the anticipation of benefit expiration. Our model estimates show that offer arrivals decline substantially with unemployment duration, especially from high-paying firms. Job seekers then become less selective in the jobs they are willing to take over the course of unemployment. Between offer arrival rates from high-paying firms falling faster over time and workers becoming less selective, the share transitioning to high-paying firms substantially decreases with unemployment duration. Counterfactual simulations further highlight the important role of anticipation of unemployment benefits expiration.

Beyond this application, our framework accommodates a broad class of job search models, including setups with involuntary wage changes and aggregate shocks. It can also be extended to more general forms of nonstationarity—for instance, allowing both the values of unemployment and employment to vary over time to more flexibly capture aggregate fluctuations. We leave these extensions for future research.

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Online Appendix

A Mathematical appendix

A.1 Proof of Theorem 1

A.1.1 Proof of Lemma 2 (ii)

Akin to Equation (3.6), for any triplet $(w, w', \tilde{w}) \in \Omega_w^3$:

$$\begin{pmatrix} \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \\ \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \end{pmatrix} = \begin{pmatrix} \ln\left(\frac{h_{w'w'}^{ss'}}{\lambda^{ss'} f_{w'}^{s'} - h_{w'w'}^{ss'}}\right) + \ln\left(\frac{h_{w'w'}^{s's}}{\lambda^{s's} f_{w'}^{s} - h_{w'w'}^{s's}}\right) \\ \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{ss'}}{\lambda^{ss'} f_{\tilde{w}}^{s'} - h_{\tilde{w}\tilde{w}}^{ss'}}\right) + \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{s's}}{\lambda^{s's} f_{\tilde{w}}^{s} - h_{\tilde{w}\tilde{w}}^{s's}}\right) \end{pmatrix} \tag{A.1}$$

Note that now we exploit transitions across job types s and s', thus we are able to use the same wage in the old and new jobs. This nonlinear system of two equations and two unknowns—$\lambda^{ss'}$ and $\lambda^{s's}$—can be rewritten as follows:

$$\begin{pmatrix} B_{w'}\lambda^{ss'} + C_{w'}\lambda^{s's} - A_{w'}\lambda^{ss'}\lambda^{s's} \\ B_{\tilde{w}}\lambda^{ss'} + C_{\tilde{w}}\lambda^{s's} - A_{\tilde{w}}\lambda^{ss'}\lambda^{s's} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{A.2}$$

where the A, B, C coefficients are defined in Lemma 2 (ii). Assuming $A_{w'} \neq 0$ (Condition (a) from Lemma 2 (ii)) and replacing $\lambda^{ss'}\lambda^{s's}$ in the second equation by its expression from the first equation identifies the ratio of the arrival rates, with:

$$\lambda^{s's} = \left(\frac{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}}\right)\lambda^{ss'}$$

where $A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'} \neq 0$ from Condition (c). Finally, substituting for $\lambda^{s's}$ in the first equation identifies, under Condition (b), $\lambda^{ss'}$ and then $\lambda^{s's}$, which admit the following closed-form expressions:

$$\lambda^{ss'} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}} \quad \text{and} \quad \lambda^{s's} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}} \tag{A.3}$$

Having identified the arrival rates $\lambda^{ss'}$ and the wage offer distribution f_w^s, identification of the CCPs $p_{ww'}^{ss'}$ follows. Then, we can identify $c^{ss'} + c^{s's}$, and together with the assumption that switching costs are symmetric (i.e., $c^{ss'} = c^{s's}$), $c^{ss'}$ is identified.

A.1.2 Proof of Lemma 3 (ii)–(iii)

(ii) Identification of CRRA preferences. We assume that workers are endowed with CRRA preferences, such that:

$$u(w) = \alpha \frac{w^{1-\theta}}{1-\theta}$$

From the prior identification result in Lemma 3 such that u_w is identified up to a constant, it follows that for $\tilde{w} > w' > w$, the following ratio is identified:

$$\frac{u_{w'} - u_w}{u_{\tilde{w}} - u_w} = \frac{w'^{1-\theta} - w^{1-\theta}}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.4}$$

In order to establish identification of the risk aversion parameter θ, we show that the function $\theta \mapsto \frac{y^{1-\theta}-x^{1-\theta}}{z^{1-\theta}-y^{1-\theta}}$, where $z > y > x > 0$ and extended continuously at $\theta = 1$, is monotonically increasing on $(0, \infty)$. For $\theta \in (0,1) \cup (1, \infty)$, we consider:

$$f(\theta) := \frac{y^{1-\theta} - x^{1-\theta}}{z^{1-\theta} - y^{1-\theta}} \tag{A.5}$$

$$f'(\theta) = \left(z^{1-\theta} - y^{1-\theta}\right)^{-2} \cdot \Big[\left(x^{1-\theta}\ln x - y^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) - \left(y^{1-\theta} - x^{1-\theta}\right)\left(y^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\Big] \tag{A.6}$$

$$f'(\theta) > 0 \tag{A.7}$$

$$\Leftrightarrow \left(x^{1-\theta}\ln x - x^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) + \left(x^{1-\theta}\ln y - y^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) > \left(z^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\left(y^{1-\theta} - x^{1-\theta}\right) + \left(y^{1-\theta}\ln y - z^{1-\theta}\ln y\right)\left(y^{1-\theta} - x^{1-\theta}\right) \tag{A.8}$$

$$\Leftrightarrow \left[x^{1-\theta}\ln(x/y)\right]\left(z^{1-\theta} - y^{1-\theta}\right) > \left[z^{1-\theta}\ln(y/z)\right]\left(y^{1-\theta} - x^{1-\theta}\right) \tag{A.9}$$

$$\Leftrightarrow \ln(y/x)\left[1 - (y/z)^{1-\theta}\right] < \ln(z/y)\left[(y/x)^{1-\theta} - 1\right] \tag{A.10}$$

$$\Leftrightarrow \quad (y/x)^{1-\theta}\ln(z/y) + \ln(y/x)(y/z)^{1-\theta} > \ln(y/x) + \ln(z/y) \tag{A.11}$$

The above condition holds if and only if $g(\theta) > g(1)$, where $g(\theta) \equiv (y/x)^{1-\theta}\ln(z/y) + (y/z)^{1-\theta}\ln(y/x)$. The derivative of $g(\cdot)$ is given by:

$$g'(\theta) = \ln(y/x)\ln(z/y)[(y/z)^{1-\theta} - (y/x)^{1-\theta}]$$

It follows that $g'(\theta) < 0$ on $(0, 1)$ and $g'(\theta) > 0$ on $(1, \infty)$. As a consequence, $f'(\theta) > 0$ on

$(0,1) \cup (1,\infty)$, which implies that $f(.)$ (extended continuously at $\theta = 1$) is monotonically increasing on $(0,\infty)$. Identification of θ follows.

Having identified θ, it follows that the utility coefficient α is identified and given by the following closed-form expression:

$$\alpha = \frac{u_{\tilde{w}} - u_w}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.12}$$

which yields full identification of the flow utility of wages.

(iii) Identification of ϕ^s up to $V_0(0)$. We can express the log odds ratio in terms of the structural parameters using Equation (2.2):

$$\begin{aligned}
\ln\left(\frac{p^{s\tilde{s}}_{w\tilde{w}}}{1-p^{s\tilde{s}}_{w\tilde{w}}}\right) &= V^{\tilde{s}}_{\tilde{w}} - c^{s\tilde{s}} - V^s_w \\
&= \Big(u_{\tilde{w}} + \phi^{\tilde{s}} + \delta^{\tilde{s}}_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{\tilde{s}s'}_{\tilde{w}w'} \left[c^{\tilde{s}s'} + \ln\left(p^{\tilde{s}s'}_{\tilde{w}w'}\right) - \ln\left(1-p^{\tilde{s}s'}_{\tilde{w}w'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{\tilde{s}s'} f^{s'}_{w'} \ln\left(1-p^{\tilde{s}s'}_{\tilde{w}w'}\right)\Big)\Big/(\rho+\delta^{\tilde{s}}_0) \\
&- \Big(u_w + \phi^s + \delta^s_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{ss'}_{ww'} \left[c^{ss'} + \ln\left(p^{ss'}_{ww'}\right) - \ln\left(1-p^{ss'}_{ww'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f^{s'}_{w'} \ln\left(1-p^{ss'}_{ww'}\right)\Big)\Big/(\rho+\delta^s_0) - c^{s\tilde{s}}
\end{aligned} \tag{A.13}$$

Collecting all known terms on the left-hand side, the equation can be rearranged as:

$$\tilde{\kappa}^{s\tilde{s}}_{w\tilde{w}} = \frac{1}{\rho+\delta^{\tilde{s}}_0}\,\phi^{\tilde{s}} - \frac{1}{\rho+\delta^s_0}\,\phi^s + \left(\frac{\delta^{\tilde{s}}_0}{\rho+\delta^{\tilde{s}}_0} - \frac{\delta^s_0}{\rho+\delta^s_0}\right) V_0(0) \tag{A.14}$$

where

$$\begin{aligned}
\tilde{\kappa}^{s\tilde{s}}_{w\tilde{w}} &\equiv \ln\left(\frac{p^{s\tilde{s}}_{w\tilde{w}}}{1-p^{s\tilde{s}}_{w\tilde{w}}}\right) + c^{s\tilde{s}} \\
&- \frac{u_{\tilde{w}} - \sum_{w'}\sum_{s'} \delta^{\tilde{s}s'}_{\tilde{w}w'} \left[c^{\tilde{s}s'} + \ln\left(p^{\tilde{s}s'}_{\tilde{w}w'}\right) - \ln\left(1-p^{\tilde{s}s'}_{\tilde{w}w'}\right)\right] - \sum_{w'}\sum_{s'} \lambda^{\tilde{s}s'} f^{s'}_{w'} \ln\left(1-p^{\tilde{s}s'}_{\tilde{w}w'}\right)}{\rho+\delta^{\tilde{s}}_0} \\
&+ \frac{u_w - \sum_{w'}\sum_{s'} \delta^{ss'}_{ww'} \left[c^{ss'} + \ln\left(p^{ss'}_{ww'}\right) - \ln\left(1-p^{ss'}_{ww'}\right)\right] - \sum_{w'}\sum_{s'} \lambda^{ss'} f^{s'}_{w'} \ln\left(1-p^{ss'}_{ww'}\right)}{\rho+\delta^s_0}
\end{aligned} \tag{A.15}$$

Now, since $\phi^1 = 0$, writing Equation (A.14) for $s = 1$ yields:

$$\tilde{\kappa}_{w\tilde{w}}^{1\tilde{s}} = \frac{1}{\rho + \delta_0^{\tilde{s}}} \phi^{\tilde{s}} + \left(\frac{\delta_0^{\tilde{s}}}{\rho + \delta_0^{\tilde{s}}} - \frac{\delta_0^1}{\rho + \delta_0^1} \right) V_0(0) \tag{A.16}$$

Thus, we can write $\phi^{\tilde{s}}$ as a known linear function of $V_0(0)$. Furthermore, note that when the job destruction rates are not specific to job types, i.e., $\delta_0^s = \delta_0$ for all s, the non-pecuniary payoffs ϕ^s are directly identified from Equation (A.16).

A.2 Extension: aggregate shocks

One can extend our identification strategy to accommodate aggregate shocks. Specifically, consider the case where the market economy can be in one of K different states, where the job offer arrival rates, the job destruction rates, the rates of involuntary wage mobility, the offered wage distributions, and the flow payoff of unemployment are allowed to depend on the state of the economy. We further assume that the econometrician perfectly observes the state of the economy. We denote the rate at which the economy transitions from state k to k' by $q_{kk'}$, which is identified from the observed transition rates across market states.

On the employment side, identification of the state-specific offer arrival rates, destruction and involuntary wage mobility rates, offered wage distribution and conditional choice probabilities, along with the switching cost all follow directly from the baseline case, leaving the flow payoff of employment as the only unknown parameters. The value function of employment V_{wk}^s is given by:

$$\begin{aligned} \Big(\rho + \sum_{k'} q_{kk'} + \delta_{0k}^s + \sum_{s'} \lambda_k^{ss'}\Big) V_{wk}^s &= u_w + \phi^s + \delta_{0k}^s V_{0k}(0) + \sum_{k'} q_{kk'} V_{wk'}^s \\ &+ \sum_{w'} \sum_{s'} \delta_{ww'k}^{ss'} \left[V_{w'k}^{s'} - V_{wk}^s \right] + \sum_{s'} \lambda_k^{ss'} \sum_{w'} f_{w'k}^{s'} \ln\left(1 - p_{ww'k}^{ss'}\right) \end{aligned} \tag{A.17}$$

where $V_{w'k}^{s'} - V_{wk}^s = \ln\left(p_{ww'k}^{ss'}\right) - \ln\left(1 - p_{ww'k}^{ss'}\right) + c^{ss'}$.

Subtracting off the corresponding expression for $V^s_{\tilde{w}k}$ (with $\tilde{w} \neq w$) yields:

$$\begin{aligned}
\Big(\rho + \sum_{k'} q_{kk'} + \delta^s_{0k} + \sum_{s'} \lambda^{ss'}_k\Big)\left[V^s_{wk} - V^s_{\tilde{w}k}\right] &= u_w - u_{\tilde{w}} + \sum_{k'} q_{kk'}\left[V^s_{wk'} - V^s_{\tilde{w}k'}\right] \\
&+ \sum_{w'}\sum_{s'}\Big(\delta^{ss'}_{ww'k}\left[V^{s'}_{w'k} - V^s_{wk}\right] - \delta^{ss'}_{\tilde{w}w'k}\left[V^{s'}_{w'k} - V^s_{\tilde{w}k}\right]\Big) \\
&+ \sum_{s'} \lambda^{ss'}_k \sum_{w'} f^{s'}_{w'k}\Big(\ln\Big(1 - p^{ss'}_{ww'k}\Big) - \ln\Big(1 - p^{ss'}_{\tilde{w}w'k}\Big)\Big) \qquad \text{(A.18)}
\end{aligned}$$

where the difference in value functions on the left and right-hand sides are given by the sum of the log odds ratio and the switching cost. This identifies the wage component of the flow payoff up to a constant. Identification of the non-pecuniary components ϕ^s then proceeds in a similar fashion, using instead the job-to-job transitions across job types.

Identification of the unemployment-side parameters follows from similar arguments as in Section 3.4. The same strategy applies to a context with aggregate shocks, after conditioning the hazard rates out of unemployment on the states of the economy.

A.3 Rank condition for $W = 3$ with two time periods

For simplicity we focus on the case with one job type. The system, evaluated at time periods (t_1, t_2), is written as:

$$\begin{aligned}
0 &= g_1\left(A_{12}(t_1) - B_{23}(t_1)\right) + g_2\left(-B_{12}(t_1) - A_{23}(t_1) - B_{23}(t_1)\right) + B_{23}(t_1) \\
0 &= g_1\left(A_{12}(t_2) - B_{23}(t_2)\right) + g_2\left(-B_{12}(t_2) - A_{23}(t_2) - B_{23}(t_2)\right) + B_{23}(t_2)
\end{aligned}$$

This linear system is full rank if and only if the following condition holds:

$$\frac{g^*_1(t_2)}{g^*_1(t_1)}\left(\frac{e^{\kappa_{12}} g^*_3(t_1)\left(e^{\kappa_{23}} - 1\right) - g^*_1(t_1)\left(e^{\kappa_{12}} - 1\right)}{e^{\kappa_{12}} g^*_3(t_2)\left(e^{\kappa_{23}} - 1\right) - g^*_1(t_2)\left(e^{\kappa_{12}} - 1\right)}\right) \neq \frac{g^*_2(t_2)}{g^*_2(t_1)}\left(\frac{\left(e^{\kappa_{12}} - 1\right)\left(e^{\kappa_{23}} g^*_3(t_1) + g^*_2(t_1)\right) + \left(e^{\kappa_{23}} - 1\right) g^*_3(t_1)}{\left(e^{\kappa_{12}} - 1\right)\left(e^{\kappa_{23}} g^*_3(t_2) + g^*_2(t_2)\right) + \left(e^{\kappa_{23}} - 1\right) g^*_3(t_2)}\right)$$

where $g^*_w(t)$ denotes the probability of accepting a job that pays w conditional on leaving unemployment at time t, which is given by:

$$g^*_w(t) = \frac{h_w(t)}{h(t)}$$

with $h(t)$ denoting the hazard out of unemployment at time t.

Under this condition, this yields a closed-form solution for the wage offer distribution which is given by:

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = -\frac{1}{|A|} \begin{pmatrix} -(B_{12}(t_2) + A_{23}(t_2) + B_{23}(t_2)) & B_{12}(t_1) + A_{23}(t_1) + B_{23}(t_1) \\ B_{23}(t_2) - A_{12}(t_2) & A_{12}(t_1) - B_{23}(t_1) \end{pmatrix} \begin{pmatrix} B_{23}(t_1) \\ B_{23}(t_2) \end{pmatrix}$$

where

$$|A| = (A_{12}(t_2) - B_{23}(t_2))(B_{12}(t_1) + A_{23}(t_1) + B_{23}(t_1)) - (A_{12}(t_1) - B_{23}(t_1))(B_{12}(t_2) + A_{23}(t_2) + B_{23}(t_2))$$

and g_3 is then given by $1 - g_1 - g_2$.

B Data appendix

B.1 Sample creation

We define our analysis sample as follows:

1. Flip primary and secondary work arrangements (PWAs, SWAs)

    - In the raw data, PWA is defined as the arrangement with the highest earnings in the month. This setup may result in PWAs and SWAs flipping in the raw data, e.g., when a worker works only a few days in their PWA.
    - We address this issue by looping through all worker-months, flipping variables related to PWAs and SWAs as follows:

`month`	`firmid1`	`var1`	`firmid2`	`var2`		`month`	`firmid1`	`var1`	`firmid2`	`var2`
$t-1$	A	x_{t-1}	B	y_{t-1}	$\rightsquigarrow$	$t-1$	A	x_{t-1}	B	y_{t-1}
t	B	x_t	A	y_t		t	A	y_t	B	x_t

2. Calculate durations:

    (a) Employed: we calculate or infer spell-year durations in PWA. See Appendix B.2 for details.

    (b) Unemployed: we observe daily unemployment durations in the raw data. For spells that end after October 2005 (the end date of our sample), we flag spells as right-censored and shorten their durations by the out-of-sample portion. Therefore, our analysis sample includes unemployment spells that are censored earlier than 269 days.

3. Calculate wages:

    (a) Calculate counterfactual minimum wage earnings: how much the worker would have earned in a day working full time in a minimum-wage job.

    (b) Calculate daily wages as total earnings in a spell-year, divided by spell-year durations.

    (c) Deflate wage levels by annual mean wage growth.

    (d) Discretize wages into wage bins: see Appendix B.3 for details.

    (e) Calculate accepted wage bins.

4. Define transitions between employment (E), unemployment (U), and out of the labor force (N): possible transitions are EE, EU, UE, EN, UN, and NE.

5. Calculate firm size used for job heterogeneity types.

B.2 Correcting employment spell durations

The raw data on employment spells are recorded at a monthly frequency. In each month, the total number of days worked (`days`) and total earnings are known. Furthermore, days worked and earnings at PWAs and SWAs (`days_1`, `days_2`) are known if the arrangement was ongoing on the 15th of the month. We focus on PWAs only.

Table B.1 shows all the possible ways in which EE transitions show up in the raw data when observations on PWAs are not missing. When `days` equals `days_1`, we know with certainty that the transition happened on the boundary of the month: we label this as a clean EE transition (Panel a). When `days` does not equal `days_1`, we need to make some assumptions about the uncovered days: Panels b–d illustrate these cases that we label fuzzy. The bottom tables summarize our assumptions on the number of days worked in each PWA.

Table B.1: EE scenarios in raw data, no missing PWAs

(a) Clean EE

days	days_1	firmid1
31	31	A
30	30	A
31	31	B

⇓

no assumption needed

(b) Fuzzy EE 1

days	days_1	firmid1
31	31	A
30	16	A
31	31	B

⇓

31	A
16	A
14	B
31	B

(c) Fuzzy EE 2

days	days_1	firmid1
31	31	A
30	16	B
31	31	B

⇓

31	A
14	A
16	B
31	B

(d) Fuzzy EE 3

days	days_1	firmid1
31	31	A
30	16	B
31	31	C

⇓

31	A
$a < 14$	A
16	B
$30 - 16 - a$	C
31	C

Table B.2 summarizes our assumptions when PWA data are missing.

Table B.2: EE scenarios in raw data, missing PWAs

(a)

days	days_1	firmid1
31	31	A
25	.	.
31	31	A
	⇓	
	31	A
	25	A
	31	A

(b)

days	days_1	firmid1
31	31	A
25	.	.
31	31	B
	⇓	
	31	A
	$d < 15$	A
	$25 - d$	B
	31	B

(c)

days	days_1	firmid1
31	31	A
10	.	.
7	.	.
30	30	B
	⇓	
	31	A
	10	A
	7	B
	31	B

(d)

days	days_1	firmid1
31	31	A
20	.	.
25	.	.
31	31	B
	⇓	
	31	A
	$a < 15$	A
	$20 - a + 25 - b$	X
	$b < 15$	B
	31	B

Furthermore, we censor spells that spill over calendar years. We do so in order to track yearly wage changes observed in the raw data. Additionally, we censor spells at October 31st due to data limitations, as mentioned in the main text. As an example, a continuous E spell from March 2003 until May 2005 that pays wage w and is followed by a EE transition to a job paying w' is represented as a right-censored spell of 8 months in w, a right-censored spell of 10 months in w, and a spell of 5 months with a EE transition from w to w'.

B.3 Discretizing wages

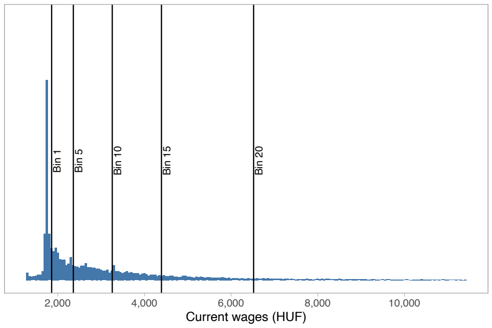
We discretize the continuously observed wages in the data into W bins, with $W = 25$ for our main results. First, we calculate the average daily wage for each worker in a given year across all months spent in employment. Then we categorize these continuous wages into discrete bins. The first bin contains wages between 75 and 107 percent of the effective minimum wage.[30] We drop wage observations below 75 percent of the effective minimum wage because we cannot distinguish between full-time and part-time earners in the data. Furthermore, we add a 7 percent padding to the right cutoff of the first bin to ensure that we include all minimum wage earners in the first bin. We then split the other wage observations, censored at the 99th percentile, evenly across the remaining $W - 1$ bins. We repeat the same discretization procedure for each calendar year: Figure B.1 demonstrates our discretization method for current and accepted wages in the year 2004.

Figure B.2 plots the resulting discrete distribution of current wages. Current wages for employment spells that lead to an employment-to-employment transition, on Panel (a),

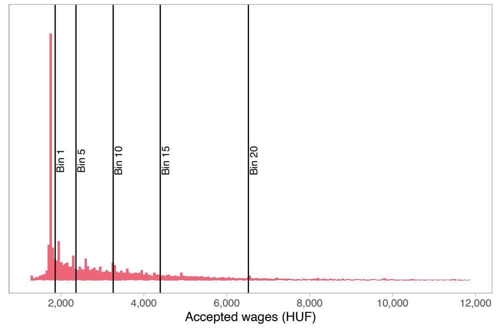
[30] During our sampling period, Hungary had a simple minimum wage policy: 50,000 HUF in 2003, 53,000 HUF in 2004, and 57,000 HUF in 2005 (1,000 HUF ≈ 5 USD in 2004).

Figure B.1: Discretizing observed wages

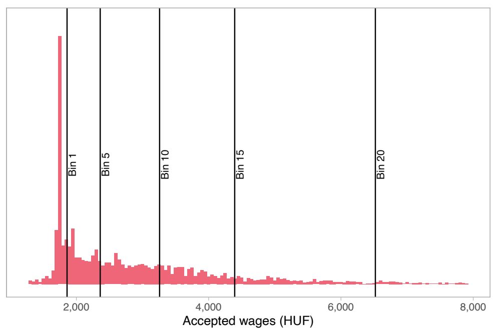
(a) Current wages



(b) Accepted wages (EE)



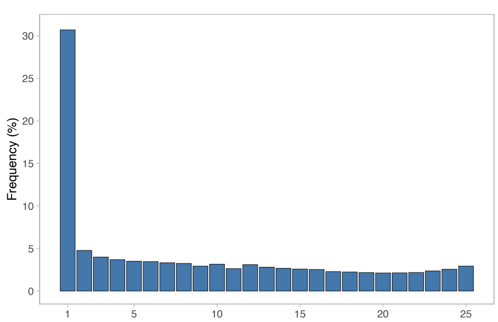
(c) Accepted wages (UE)



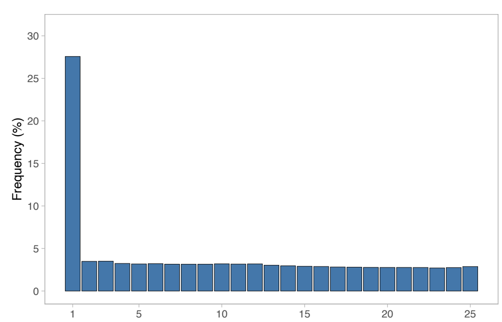
Notes: Histograms of daily wage rates in 2004 with 50 HUF bin width, truncated at the 95th percentile (1,000 HUF $\approx$ 5 USD in 2004). Vertical lines denote selected wage bin cutoffs. Panel (a): current daily wages for employment spells that lead to an EE transition. Panel (b): accepted daily wages for employment spells after an EE transition. Panel (c): accepted daily wages for employment spells after a UE transition.
Source: HUN-REN CERS, authors' own calculations.

Figure B.2: Discrete distribution of current wages

(a) Current wages (EE)



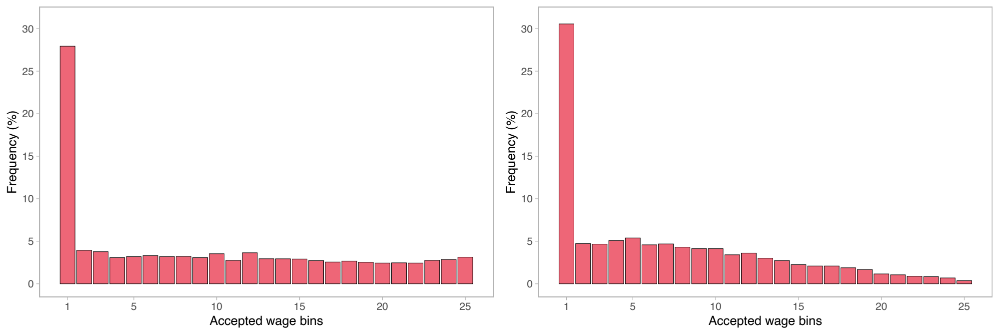
(b) Current wages (all spells)



Notes: Panel (a): discrete distribution of current wages for employment spells that lead to an EE transition. Panel (b): discrete distribution of current wages for all employment spells.
Source: HUN-REN CERS, authors' own calculations.

Figure B.3: Discrete distribution of accepted wages

(a) Accepted wages (EE) (b) Accepted wages (UE)



Notes: Panel (a): discrete distribution of accepted wages for employment spells that lead to an EE transition. Panel (b): discrete distribution of accepted wages for unemployment spells that lead to an employment spell. *Source:* HUN-REN CERS, authors' own calculations.

have a mean of 3,477 HUF (percentiles: 25th 1,721 HUF; 50th 2,398 HUF; 75th 3,871 HUF). Current wages for all employment spells, on Panel (b), have a mean of 3,677 HUF (percentiles: 25th 1,766 HUF; 50th 2,684 HUF; 75th 4,345 HUF). Similarly, Figure B.3 plots the discrete distribution of accepted wages for job-to-job and unemployment-to-employment transitions. Accepted wages for employment-to-employment transitions have a mean of 3,652 HUF (percentiles: 25th 1,769 HUF; 50th 2,623 HUF; 75th 4,198 HUF). Accepted wages out of unemployment are generally lower than those for job-to-job transitions, with a mean of 2,743 HUF (percentiles: 25th 1,663 HUF; 50th 2,208 HUF; 75th 3,177 HUF), in line with the notion that the unemployed tend to move to lower-paying jobs.

C Estimation appendix

This appendix details our estimation procedure, outlined in Section 5.

C.1 Stage 1 details: firm and worker types

In this section we specify the initial conditions likelihood, $\mathcal{L}^I_{ir}(\theta^I; C)$, the reduced form likelihood for employment spells, $\tilde{\mathcal{L}}^E_{ikr}(\tilde{\theta}^E; C)$ and the reduced form likelihood for unemployment spells, $\tilde{\mathcal{L}}^U_{i\tilde{k}r}(\tilde{\theta}^U; C)$.

C.1.1 Initial conditions

We allow for different initial wage distributions conditional on firm type and the source of the initial condition. There are three possible initial conditions: (i) the first observation in an individual's history, (ii) the first observation after a gap in one's history, and (iii) the first observation after an employment-to-employment transition across years. For (iii), recall that we censor jobs at October 31st in each year (see Section 4.2).

Estimation of the wage parameters follows an ordered logit with common cutpoints:

$$\phi^*_{w'} = \begin{cases} \theta^{\phi}_1 & \text{if } w' = 1 \\ \phi^*_{w'-1} + \exp(\theta^{\phi}_{w'}) & \text{if } 1 < w' < W \end{cases} \tag{C.1}$$

We then define the initial wage distribution as a logit cdf by allowing the intercepts to vary by firm (s'), worker (r), and initial condition (z) type, with the pmf given by:

$$f^{s'}_{w'rz} = \begin{cases} \Lambda(\phi^*_{w'} + \theta^{f1,s'} + \theta^{f2}_r + \theta^{f3}_z) & \text{if } w' = 1 \\ \Lambda(\phi^*_{w'} + \theta^{f1,s'} + \theta^{f2}_r + \theta^{f3}_z) - \Lambda(\phi^*_{w'-1} + \theta^{f1,s'} + \theta^{f2}_r + \theta^{f3}_z) & \text{if } 1 < w' < W \\ 1 - \Lambda(\phi^*_{w'-1} + \theta^{f1,s'} + \theta^{f2}_r + \theta^{f3}_z) & \text{if } w' = W \end{cases} \tag{C.2}$$

where $\Lambda(\cdot)$ denotes the logistic function and $\theta^{f2}_1 = \theta^{f3}_1 = 0$.

The likelihood of the initial wage, $\mathcal{L}^I_{ir}(\theta^I; C)$, is then given by:

$$\mathcal{L}^I_{ir}(\theta^I; C) = \prod_{k=1}^{K_i} \prod_{w',s'} \left(f^{s'}_{w'rz}\right)^{\mathbb{1}\{z_{ik}=z, w_{ik}=w', s_{ik}=s'\}} \tag{C.3}$$

where z_{ik} indicates whether spell k of individual i is an initial spell, and if so, which one of the three possibilities.

C.1.2 Employment-to-employment hazards

The reduced-form likelihood for employment spells is formed from the employment-side hazards. These employment-side hazards feature three sets of parameters: offer arrival rates, wage offer parameters, and reduced form acceptance probabilities: $h^{ss'}_{ww'r} = \tilde{\lambda}^{ss'}_r \tilde{f}^{s'}_{w'r} \tilde{p}^{ss'}_{ww'r}$. We are completely flexible on how worker types affect each of these sets of parameters, parallelizing the code so the estimation is run separately by worker type, implying that every parameter in this section varies by r.

Offer arrival rates vary across destination firm types s' and are allowed to be higher when the offer comes from the individual's current firm type:

$$\tilde{\lambda}_r^{ss'} = \exp\left(\tilde{\theta}_r^{\lambda 1,s'} + \mathbb{1}(s = s')\tilde{\theta}_r^{\lambda 2,s}\right) \tag{C.4}$$

The wage offer parameters follow an ordered logit with common cutpoints conditional on worker type, similar to initial wages (Equation C.1). Firm types then move the intercept of the logit cdf, with the pmf given by:

$$\tilde{f}_{w'r}^{s'} = \begin{cases} \Lambda(\tilde{\phi}_{w'r} + \tilde{\theta}_r^{f,s'}) & \text{if } w' = 1 \\ \Lambda(\tilde{\phi}_{w'r} + \tilde{\theta}_r^{f,s'}) - \Lambda(\tilde{\phi}_{w'-1,r} + \tilde{\theta}_r^{f,s'}) & \text{if } 1 < w' < W \\ 1 - \Lambda(\tilde{\phi}_{w'-1,r} + \tilde{\theta}_r^{f,s'}) & \text{if } w' = W \end{cases} \tag{C.5}$$

where $\Lambda(\cdot)$ denotes the logistic function and $\tilde{\theta}_1^f = 0$.

For the reduced-form job acceptance probabilities, we specify the reduced-form value function, $\tilde{V}_{wr}^s$, as a flexible function of wages, allowing it to depend on $\left[\ln(w) \quad \ln(w)^2 \quad \sqrt{\ln(w)}\right]$. The switching cost is assumed to be constant conditional on worker type. With logistic preference shocks for the new job, the probability of accepting a job of type s' and wage w' given that a type r worker is currently in a type s job paying w is:

$$\tilde{p}_{ww'r}^{ss'} = \frac{\exp(\tilde{V}_{w'r}^{s'} + \tilde{\theta}_r^{p,s'} - c_r)}{\exp(\tilde{V}_{w'r}^{s'} + \tilde{\theta}_r^{p,s'} - c_r) + \exp(\tilde{V}_{wr}^{s} + \tilde{\theta}_r^{p,s})} \tag{C.6}$$

where $\tilde{\theta}_1^{p,s'} = 0$ for all s' and $c_r > 0$ for all r.

The reduced-form likelihood of a particular employment spell of duration t_{ik}, denoted by $\tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^E; C)$, is then given by:

$$\begin{aligned} \tilde{\mathcal{L}}_{ikr}^E(\tilde{\theta}^E; C) = \prod_{w,w',s,s'} & \Big[(\tilde{\lambda}_r^{ss'} \tilde{f}_{w'r}^{s'} \tilde{p}_{ww'r}^{ss'})^{\mathbb{1}\{w_{ik}=w, w_{ik+1}=w', s_{ik}=s, s_{ik+1}=s'\}} \times \\ & \exp(-\tilde{\lambda}_r^{ss'} \tilde{f}_{w'r}^{s'} \tilde{p}_{ww'r}^{ss'} t_{ik})\Big]^{\mathbb{1}\{w_{ik}=w, s_{ik}=s\}} \end{aligned} \tag{C.7}$$

C.1.3 Unemployment-to-employment hazards

The reduced-form likelihood of unemployment spells has three components: (i) the unconditional hazard of exiting unemployment, (ii) the probability of the accepted firm type conditional on leaving unemployment but not conditional on the wage, and (iii) the probability of the accepted wage conditional on accepting a job at a particular firm type.

First, the unconditional exit hazard $h_r(t)$ is a flexible function of unemployment duration t, depending on $\begin{bmatrix}1 & t & \ln(t) & t^2\end{bmatrix}$. Second, the probability of accepting a type s' job is specified as logit depending on unemployment duration flexibly across worker and firm types:

$$\tilde{p}_r^{s'}(t) = \frac{\exp(\tilde{\theta}_r^{p1,s'} + \tilde{\theta}_r^{p2,s'} t + \tilde{\theta}_r^{p3,s'} \ln(t))}{\sum_{s'} \exp(\tilde{\theta}_r^{p1,s'} + \tilde{\theta}_r^{p2,s'} t + \tilde{\theta}_r^{p3,s'} \ln(t))} \tag{C.8}$$

where $\tilde{\theta}_r^{p1,1} = \tilde{\theta}_r^{p2,1} = \tilde{\theta}_r^{p3,1} = 0$ for all r. Third, the accepted wage distribution follows a similar logit structure as for the initial conditions and the employed side:

$$\tilde{g}_{w'r}^{s'}(t) = \begin{cases} \Lambda(\tilde{\phi}_{w'} + \tilde{\theta}_r^{g1,s'} + \tilde{\theta}_r^{g2,s'} \ln(t) & \text{if } w' = 1 \\ \Lambda(\tilde{\phi}_{w'} + \tilde{\theta}_r^{g1,s'} + \tilde{\theta}_r^{g2,s'} \ln(t)) - \Lambda(\tilde{\phi}_{w'-1} + \tilde{\theta}_r^{g1,s'} + \tilde{\theta}_r^{g2,s'} \ln(t)) & \text{if } 1 < w' < W \\ 1 - \Lambda(\tilde{\phi}_{w'-1} + \tilde{\theta}_r^{g1,s'} + \tilde{\theta}_r^{g2,s'} \ln(t)) & \text{if } w' = W \end{cases} \tag{C.9}$$

where $\Lambda(\cdot)$ denotes the logistic function and $\tilde{\theta}_1^{g1,s'} = 0$ for all s'.

The reduced-form likelihood of a particular unemployment spell of duration $t_{i\tilde{k}}$, denoted by $\tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^U; C)$, is then given by:

$$\tilde{\mathcal{L}}_{i\tilde{k}r}^U(\tilde{\theta}^U; C) = \prod_{w',s'} \left(\tilde{g}_{w'r}^{s'}(t_{i\tilde{k}})\tilde{p}_r^{s'}(t_{i\tilde{k}})\right)^{\mathbb{1}\left\{w_{i\tilde{k}}=w', s_{i\tilde{k}}=s'\right\}} \times \exp\left(-\int_0^{t_{i\tilde{k}}} h_r(u)\,du\right) \tag{C.10}$$

C.1.4 Starting values and local optima

Having both unobserved firm and worker types leads to concerns about local optima which are in turn affected by starting values. For firm types, we first classify type S as firms with less than 25 employees: this type is not updated further. For the remaining (larger) firms, we initially parse type L and H firms as those with low vs. high compensating differentials: we calculate the average distance between current and accepted wages at each firm and assign types based on below vs. above the median wage difference. Then we update the classification of L and H firm types by estimating the parameters following the strategy given in Section 5.2, given a set of starting values. Once the outer loop is complete and there is no further updating of the firm classifications, we take these firm classifications and perform twenty estimation runs where each time we draw new starting values in the neighborhood of the converged parameters and re-estimate the model. Of these twenty, we take the parameters and firm classifications that resulted in the highest likelihood. We then perform twenty additional estimation runs where each time we draw starting values in the neighborhood of the new converged parameters and re-estimate the model. We use the firm classifications and parameters that resulted in the highest likelihood as our final first-stage estimates.

C.2 Stage 2 details: employed-side functional forms

Having recovered the firm classification and for each worker their conditional probabilities of each worker type, we estimate the employed-side parameters in one step. Section 5.3 shows the likelihood function: here we detail the functional forms.

The wage offer distribution follows a similar logit cdf structure as in Stage 1, where we specify the pmf of the wage offer distribution as:

$$f^{s'}_{w'r} = \begin{cases} \Lambda(\phi_{w'} + \theta^{f,s'}_r) & \text{if } w' = 1 \\ \Lambda(\phi_{w'} + \theta^{f,s'}_r) - \Lambda(\phi_{w'-1} + \theta^{f,s'}_r) & \text{if } 1 < w' < W \\ 1 - \Lambda(\phi_{w'-1} + \theta^{f,s'}_r) & \text{if } w' = W \end{cases} \tag{C.11}$$

where $\Lambda(\cdot)$ denotes the logistic function and $\theta^{f,1}_1 = 0$.

Next, we allow the offer arrival rates to vary by firm and worker type, and we also allow them to be scaled up when the offer comes from the same firm type as an individual's current one. That is, we write offer arrival rates as:

$$\lambda^{ss'}_r = \exp(\theta^{\lambda 1,s'}_r (1 - \mathbb{1}(s \neq s')\theta^{\lambda 2,s})) \tag{C.12}$$

where $\theta^{\lambda 2,s} \in (0,1)$ for all s. Finally, there is a switching cost, c, that is common across firm types. The probability of accepting a job of a particular type and wage conditional on one's current firm type is then:

$$p^{ss'}_{ww'r} = \frac{\exp(V^{s'}_{w'r} - V^s_{wr} - c)}{1 + \exp(V^{s'}_{w'r} - V^s_{wr} - c)} \tag{C.13}$$

The value functions are obtained by iterating to a fixed point at each step of the optimization routine.

C.3 Stage 3 details: unemployed-side functional forms

We estimate the unemployed-side parameters in the final stage. Section 5.4 shows our procedure, here we detail our chosen functional forms.

We write the offered wage distribution as a logit cdf yet again, but taking the cutpoints as given from the employed side, $\hat{\phi}$. Specifically, we write the cdf as:

$$G^{s'}_{w'r} = \begin{cases} \Lambda(\theta^{G1,s'}\theta^{G2}_r(\hat{\phi}_{w'} + \hat{\theta}^{f,s'}_r + \theta^{G3,s'} + \theta^{G4}_r)) & \text{for } w' < W \\ 1 & \text{for } w' = W \end{cases} \tag{C.14}$$

where $\Lambda(\cdot)$ denotes the logistic function, $\theta^{G1,s'} > 0$ for all s', $\theta^{G2}_1 = 1$ and $\theta^{G2}_r > 0$ for $r > 1$, $\theta^{G4}_1 = 0$, and $\hat{\phi}_{w'}$ and $\hat{\theta}^{f,s'}_r$ are employed-side estimates. From here, we write the pmf as:

$$g^{s'}_{w'r} = \begin{cases} G^{s'}_{w'r} & \text{for } w' = 1 \\ G^{s'}_{w'r} - G^{s'}_{w'-1,r} & \text{for } w' > 1 \end{cases} \tag{C.15}$$

Note that, despite taking the cutpoints from the employed side, the wage offer distribution is very flexible as both the level and variance of the wage offer distribution are allowed to vary by firm and worker type.

Next we express the nonstationary offer arrival rates as a flexible function of unemployment duration:

$$\lambda^{s'}_r(t) = \exp(\theta^{\lambda t1,s'}_r + \theta^{\lambda t2,s'}_r(\theta^{\lambda t3}t + \theta^{\lambda t4}\ln(t+1)) + \theta^{\lambda t5,s'}t^2) \tag{C.16}$$

where $\theta^{\lambda t2,1}_1 = 1$. Finally, we also specify the (normalized) value function of unemployment as a flexible function of unemployment duration:

$$\tilde{V}_{0r}(t) = \begin{bmatrix} 1 & t & \ln(t+1) & t^2 \end{bmatrix} \theta^V_r \tag{C.17}$$

C.3.1 Constraints

In practice, we impose a handful of constraints on the maximum likelihood estimation of the unemployed-side parameters to keep the optimization routine away from parameter regions where the likelihood is difficult to evaluate and/or where the optimizer may get stuck at a local optimum. Specifically:

1. The total offer arrival rate across all firm types and time periods is less than 7,

2. The probability of accepting a job in the highest wage bin at $t = T$ is below .9999 for type L firms,

3. We discipline the flow payoff of unemployment by restricting it to be always less than $\alpha \ln(w_r^{90})$. Here w_r^{90} denotes the 90^{th} percentile of wages for r-type workers employed at H-type firms, and $\alpha \ln(w_r^{90})$ is the corresponding flow payoff,[31]

4. The flow payoff of unemployment is monotonically decreasing in t after 60 days of unemployment (allowing for non-pecuniary startup costs in finding a job at the beginning of an employment spell), and

5. The value function of unemployment is monotonically decreasing in t.

Note that constraints 2 and 5 are assumptions based on behavior rather than on primitives. However, neither of these constraints bind in practice. The one constraint that binds is constraint 3, at one point in time. The likelihood only slightly changes when this constraint is relaxed.

[31] Recall that H-type firms pay substantially more than S-type and L-type firms.

D Additional results

This appendix contains estimation results that are referenced, but not presented, in the main text.

Table D.1 shows the average initial wage across and within heterogeneity types. Initial wages for the most productive worker type (type 4) are 4.7 times higher than for the least productive type (type 1). Initial wages at highly productive firms (type H) are twice as high as at small firms (type S).

Table D.1: Initial wages across heterogeneity types

	Overall	*By worker type*			
		1	*2*	*3*	*4*
Mean initial wage (HUF)					
Across firm types	3,289	2,201	3,051	5,355	10,443
Firm type S	2,262	1,734	2,073	3,314	7,647
Firm type L	2,517	1,861	2,411	3,943	7,872
Firm type H	4,481	2,798	4,147	7,177	12,795

Notes: The table shows the mean initial wage across and within firm types, first for all worker types (Overall) then separately by worker types. Wage rates are daily; 1,000 HUF $\approx$ 5 USD in 2004.
Source: HUN-REN CERS, authors' own calculations.

Table D.2 shows the distribution of employment spells across and within heterogeneity types, as well as the mean current wage. 32.2% of employment spells take place at small firms (type S); two thirds of the remaining employment spells at large firms are at highly productive ones (type H). There is substantial sorting of workers and firms on productivity: more productive worker types tend to work at more productive firms. Current wages show similar patterns to initial wages: more productive workers earn more and more productive firms pay more.

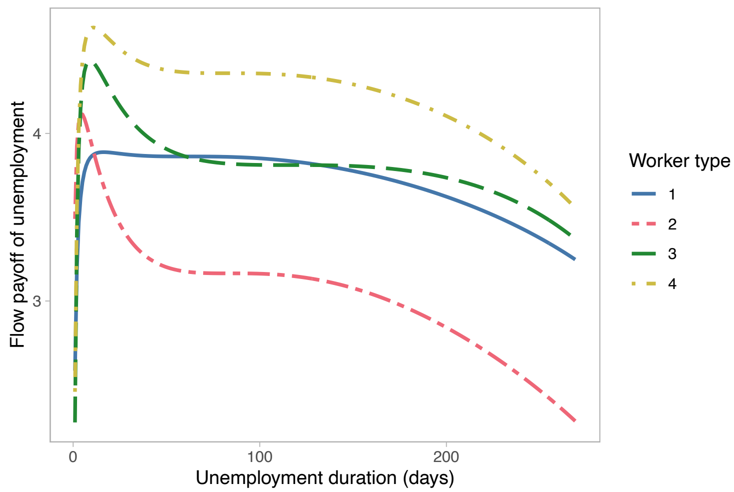
Table D.2: Share of employment spells and current wages across heterogeneity types

	Overall	*By worker type*			
		1	*2*	*3*	*4*
Share of employment spells (%)					
Firm type S	32.2	35.7	30.8	27.0	29.6
Firm type L	20.7	21.9	21.1	18.4	14.7
Firm type H	47.1	42.4	48.1	54.6	55.7
Mean current wage (HUF)					
Across firm types	3,677	2,436	3,377	5,680	10,355
Firm type S	2,366	1,800	2,126	3,317	7,455
Firm type L	2,380	1,836	2,262	3,399	6,596
Firm type H	5,142	3,282	4,668	7,617	12,887

Notes: The top half of the table shows the share of employment spells within firm types, first for all worker types (Overall) then separately by worker types. The bottom half shows the mean current wage in these spells across and within firm types, first for all worker types (Overall) then separately by worker types. Wage rates are daily; 1,000 HUF $\approx$ 5 USD in 2004.
Source: HUN-REN CERS, authors' own calculations.

Turning to the estimation results for the unemployed side, Figure D.1 shows the flow payoff of unemployment. After an initial jump, flow payoffs decrease then flatten out, before tapering off as UI benefits expire. This pattern highlights the importance of a nonstationary specification for the flow unemployment payoff.

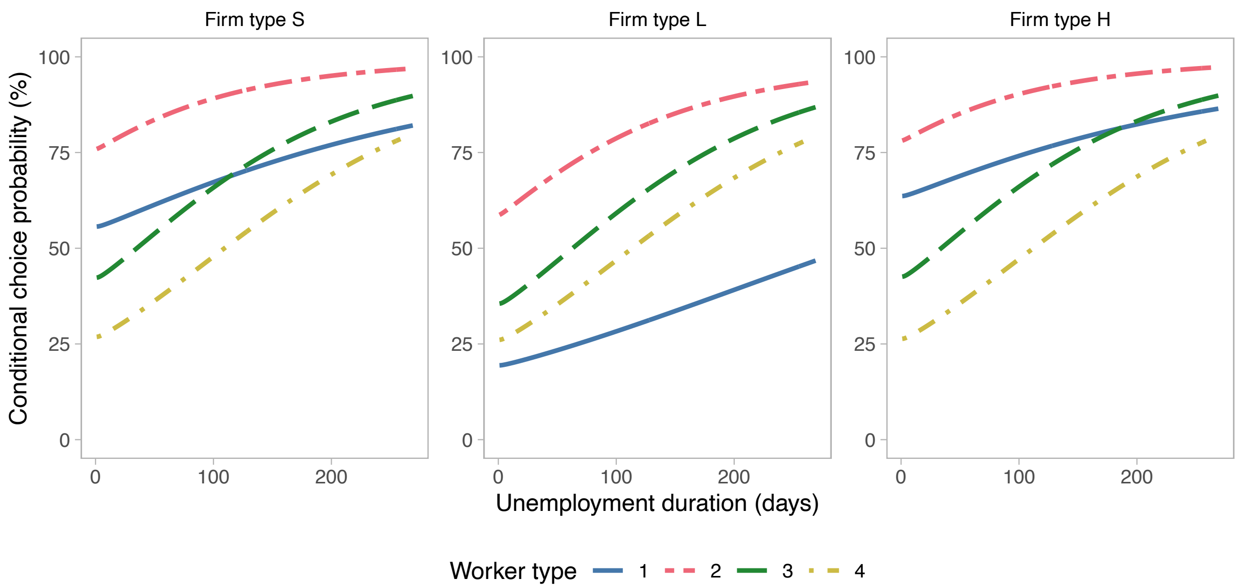
Figure D.1: Flow payoff of unemployment



Source: HUN-REN CERS, authors' own calculations.

Figure D.2 shows the CCPs of exiting unemployment over the benefit horizon to the lowest wage bin. As durations increase, the rate at which minimum wage offers also increases and especially so for the more productive types. Note that, from Table 5, the two least productive worker types have a lower amenity value for L type firms and also have lower offer arrival rates when employed conditional on working for an L type firm. These two features translate into lower probabilities of accepting minimum wage jobs from L type firms for these worker types.

Figure D.2: CCPs of exiting unemployment to wage bin 1



Source: HUN-REN CERS, authors' own calculations.

Finally, Table D.3 displays our estimates of the parameters for the offer arrival rates and the value function in unemployment, as shown in Equations C.16 and C.17.

Table D.3: Parameter estimates for unemployed offer arrival rates and value function

(a) $\theta_r^{\lambda 1,s'}$

		Worker type			
		1	*2*	*3*	*4*
Firm type	S	-0.7357	-0.8154	-0.9050	-0.7586
		(0.2962)	(0.1443)	(0.4485)	(0.1930)
	L	-1.7640	-1.6948	-2.1792	-1.1073
		(0.4743)	(0.3771)	(0.6910)	(0.4263)
	H	-1.2134	-1.6075	-1.3042	-1.5834
		(0.4027)	(0.4208)	(0.4429)	(0.4080)

(b) $\theta_r^{\lambda 2,s'}\theta^{\lambda 3}$

		Worker type			
		1	*2*	*3*	*4*
Firm type	S	-0.0056	0.0005	-0.0014	-0.0001
		(0.0023)	(0.0014)	(0.0039)	(0.0012)
	L	-0.0092	-0.0094	-0.0105	-0.0097
		(0.0031)	(0.0020)	(0.0061)	(0.0023)
	H	-0.0107	-0.0120	-0.0096	-0.0084
		(0.0020)	(0.0018)	(0.0034)	(0.0019)

(c) $\theta_r^{\lambda 2,s'}\theta^{\lambda 4}$

		Worker type			
		1	*2*	*3*	*4*
Firm type	S	0.2260	-0.0215	0.0563	0.0027
		(0.1022)	(0.0576)	(0.1664)	(0.0567)
	L	0.3672	0.3749	0.4204	0.3883
		(0.1518)	(0.1087)	(0.2631)	(0.1247)
	H	0.4271	0.4816	0.3834	0.3348
		(0.1197)	(0.1271)	(0.1646)	(0.1244)

(d) $\theta^{\lambda 5,s'}$

Firm type	S	-0.0563
		(0.0559)
	L	0.2227
		(0.0947)
	H	0.3243
		(0.0771)

(e) θ_r^V

		Worker type			
		1	*2*	*3*	*4*
Polynomial term	1	21.8108	12.1628	28.9051	9.6021
		(0.9053)	(1.2466)	(1.1945)	(0.3750)
	t	-0.0101	-0.0093	-0.0096	-0.0052
		(0.0020)	(0.0017)	(0.0025)	(0.0008)
	$\ln(t+1)$	0.0224	0.0108	0.0211	0.0113
		(0.0053)	(0.0088)	(0.0075)	(0.0014)
	t^2	0.0573	0.0738	0.0472	0.0232
		(0.0653)	(0.0661)	(0.0923)	(0.0295)

Notes: Panels (a) to (d) show the structural parameter estimates associated with offer arrival rates for the unemployed (Equation C.16). Panel (e) shows the structural parameter estimates associated with the value function for the unemployed (Equation C.17). Bootstrap standard errors in parentheses (500 replications).
Source: HUN-REN CERS, authors' own calculations.

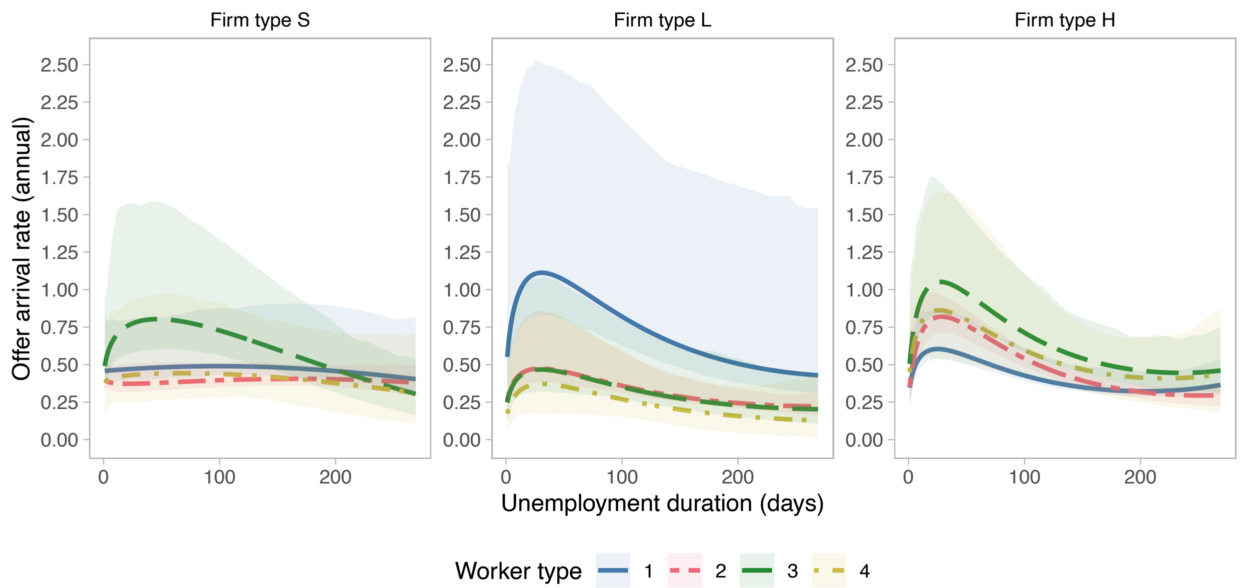
E Unemployed-side results from specification with time-varying wage offers

We estimate an alternative specification of the unemployed-side model where wage offers are nonstationary. We write the pmf as a function of unemployment duration as:

$$g_{w'r}^{s'}(t) = \begin{cases} \left[G_{w'r}^{s'}\right]^{1+\theta^{g1}\ln(t+1)+\theta^{g2}t} & \text{for } w' = 1 \\ \left[G_{w'r}^{s'}\right]^{1+\theta^{g1}\ln(t+1)+\theta^{g2}t} - \left[G_{w'-1,r}^{s'}\right]^{1+\theta^{g1}\ln(t+1)+\theta^{g2}t} & \text{for } w' > 1 \end{cases} \tag{E.1}$$

where we restrict $(\theta^{g1}, \theta^{g2})$ to ensure that $\alpha(t) = 1 + \theta^{g1}\ln(t+1) + \theta^{g2}t$ is non-negative, and its derivative is non-positive over the estimation time window.[32] Figures E.1 and E.2 show the offer arrival rate and minimum wage offer estimates, respectively. The patterns are qualitatively similar to our baseline estimates.

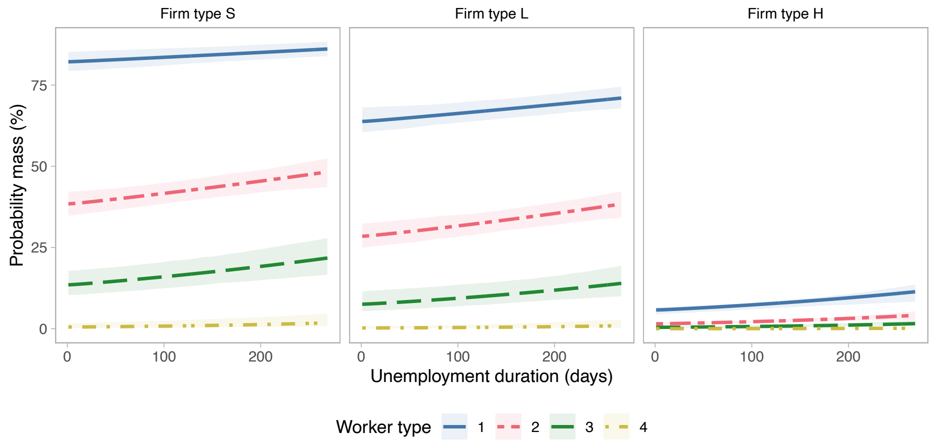
Figure E.1: Offer arrival rates out of unemployment



Notes: Annualized rates. Shaded regions represent 95% bootstrap confidence band (500 replications).
Source: HUN-REN CERS, authors' own calculations.

[32] Our estimates of these structural parameters are $\hat{\theta}^{g1} = 0.0018$ and $\hat{\theta}^{g2} = -0.0921$. The p-value of the test of their joint statistical significance is 1.77-05.

Figure E.2: Minimum wage offers over time, unemployed side



Notes: Shaded regions represent 95% bootstrap confidence bounds (500 replications).
Source: HUN-REN CERS, authors' own calculations.

Table E.1 shows the composition of heterogeneity types and accepted wages out of unemployment over time, analogously to Table 6. The results are qualitatively similar to our main results.

Table E.1: Dynamic composition of heterogeneity types and accepted wages

(a) Worker types

	Share (%)	Share by unemployment duration (%)					Mean unemployment duration (days)	
		1–30	*31–60*	*61–90*	*91–180*	*181–269*	*Model*	*q-weighted data*
Worker type 1	41.1	34.2	33.4	40.9	46.9	46.4	111.3	110.6
Worker type 2	45.5	51.5	50.6	45.4	40.1	42.8	97.2	96.7
Worker type 3	11.6	12.8	13.8	11.7	11.0	9.1	93.1	92.8
Worker type 4	1.9	1.5	2.2	1.9	2.0	1.8	96.7	101.6

(b) Accepted wages and job types

	Model						**q-weighted data**					
	Overall	*By unemployment duration (days)*					*Overall*	*By unemployment duration (days)*				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*		*1–30*	*31–60*	*61–90*	*91–180*	*181–269*
Worker type 1												
Mean acc. wage (HUF)	2,179	2,292	2,263	2,211	2,134	2,091	2,183	2,365	2,273	2,150	2,141	2,072
Share job type S (%)	35.9	30.4	29.6	33.0	39.0	41.3	35.6	27.0	27.5	34.8	39.7	41.6
Share job type H (%)	39.8	46.9	45.1	41.3	36.2	35.4	40.0	51.7	45.2	39.4	36.1	34.2
Worker type 2												
Mean acc. wage (HUF)	2,733	2,891	2,885	2,809	2,669	2,533	2,767	2,974	2,881	2,808	2,674	2,480
Share job type S (%)	31.9	25.5	23.5	27.1	35.0	42.3	30.6	21.3	24.9	31.0	34.4	42.5
Share job type H (%)	42.2	48.9	50.0	46.2	38.9	33.0	43.5	52.9	47.0	43.9	40.5	31.8
Worker type 3												
Mean acc. wage (HUF)	4,144	4,460	4,340	4,188	3,984	3,876	4,177	4,896	4,151	3,928	4,031	3,696
Share job type S (%)	36.5	31.8	33.4	36.9	40.6	37.3	36.2	28.7	38.9	38.4	36.6	40.2
Share job type H (%)	44.9	50.4	48.4	44.8	40.8	42.9	45.3	59.5	43.1	40.4	41.6	39.9
Worker type 4												
Mean acc. wage (HUF)	8,056	8,529	8,365	8,165	7,859	7,608	8,178	8,256	8,288	8,796	7,904	7,887
Share job type S (%)	32.3	26.7	26.2	29.7	35.9	38.4	31.3	17.6	25.5	34.2	33.2	43.9
Share job type H (%)	48.1	52.5	51.7	48.9	44.9	45.7	48.6	56.0	50.1	49.0	47.5	42.2

Notes: The left side of Panel (a) shows the distribution of worker types exiting unemployment to employment. The right side of Panel (a) shows the average days of unemployment duration in simulated spells (model) vs. observed spells using the estimated posterior worker type probabilities as weights (q-weighted data). Panel (b) shows the mean accepted wage and the share of type S/H jobs in simulated spells (model) vs. observed spells using the estimated posterior worker type probabilities as weights (q-weighted data). Jobs are classified into 3 types as discussed in Section 5. Both panels show summary statistics across all spells as well as conditional on duration ranges. Accepted wages are daily wage levels reported in Hungarian forints (1,000 HUF $\approx$ 5 USD in 2004).
Source: HUN-REN CERS, authors' own calculations.