Forschung am ivwKöln
Band 7/2018

Resilience and Intergenerational Fairness in Collective Defined Contribution Pension Funds

Oskar Goecke

ivwKöln
Institut für Versicherungswesen

Fakultät für Wirtschafts- und Rechtswissenschaften

Technology
Arts Sciences
TH Köln

Forschung am **ivw**Köln, Band 7/2018

Oskar Goecke
Forschungsstelle FaRis

Resilience and Intergenerational Fairness in Collective Defined Contribution Pension Funds

Abstract

A pension system is resilient if it able to absorb external (temporal) shocks and if it is able to adapt to (longterm) shifts of the socio-economic environment. Defined benefit (DB) and defined contribution pension plans behave contrastingly with respect to capital market shocks and shifts: while DB-plan benefits are not affected by external shocks they totally lack adaptability with respect to fundamental changes; DC-plans automatically adjust to a changing environment but any external shock has a direct impact on the (expected) pensions. By adding a collective component to DC-plans one can make these collective DC (CDC)-plans shock absorbing - at least to a certain degree. In our CDC pension model we build a collective reserve of assets that serves as a buffer to capital market shocks, e.g. stock market crashes. The idea is to transfer money from the collective reserve to the individual pension accounts whenever capital markets slump and to feed the collective reserve whenever capital market are booming. This mechanism is particular valuable for age cohorts that are close to retirement. It is clear that withdrawing assets from or adding assets to the collective reserve is essentially a transfer of assets between the age cohorts. In our near reality model we investigate the effect of stock market shocks and interest rate (and mortality) shifts on a CDC- pension system. We are particularly interested in the question, to what extend a CDC-pension system is actually able to absorb shocks and whether the intergenerational transfer of assets via the collective reserve can be regarded as fair.

Resilience and Intergenerational Fairness in Collective Defined Contribution Pension Funds

Oskar Goecke

Resilience and Intergenerational Fairness in Collective Defined Contribution Pension Funds

Content

1 Introduction

All over the world defined benefit pension plans (*DB*-plans) are in retreat, meaning that young employees entering working life must accept defined contribution pension plans (*DC*-plans).[1] There are several reasons for this development, including: increased risk awareness among employers, intensified regulation and a low interest rate environment.

Employees and labour unions regard the shift from *DB* to *DC* as a massive reduction of labour rights since the investment risk is put on the weak shoulders of employees. This fact cannot be denied. However, one can also argue that the transition from *DB* to *DC* is just proof that *DB* plans are unsustainable in the sense that they lack flexibility to adjust to a changed economic environment. As a consequence, inevitable adjustments had to be made by closing old *DB* systems and in doing so putting the financial burden of the obsolete *DB* plans on the shoulders of the younger generation.[2] This generation is hit twice since at the same time the social security pension systems are under reconstruction with the obvious outcome for the young.[3]

Compared to *DB*-plans, pure (individual) *DC*-plans are "over-reactive" in the sense that pension benefits are directly linked to the time value of the pension pot. Equity market shocks, shifts of the yield curve or changing life expectancy instantaneously hit the expected pension or the pension in payment.

The idea behind *collective DC*- (*CDC*-) plans is to introduce a collective component to a *DC*-plan to buffer external shocks or shifts in order to stabilise (expected) pension payments. The collective reserve in a *CDC* system can be regarded as an unallocated fund of assets. This fund must be fed by contributions or asset returns. Payments into and withdrawals from the collective reserve constitute an intergenerational transfer of assets.

In the following we present a multi generation *CDC*-pension model including rules for when and how the intergenerational transfer is to be carried out. The main purpose of this paper is to apply the concept of *resilience* to a pension system. Resilience is the ability of a system to absorb (single) external shocks and to adapt to (permanent) shifts of the socio-economic environment. Our approach allows us to explicitly measure the intergenerational transfer.

---

[1] Cf. [OECD 2011], p. 15.

[2] We have the same effect if the benefits of a *DB* plan remain untouched but the contributions are adjusted.

[3] Cf. [House of Commons 2016] p. 15-16.

The utility increasing effect of intergenerational risk transfer has been proven by many authors using different methods. [Gordon/ Varian 1988] use a stylised overlapping generation model to prove that the government should play an active role by borrowing or saving in the capital market to improve risk allocation between generations. [Gollier 2007] addresses the intergenerational risk transfer in a pension fund with a stable number of new young workers replacing the retirees who get a lump sum payment as pension benefit. Using expected utility theory, Gollier can prove that if all generations save into a common pension fund the expected utility for every generation can be increased. [Westerhout 2011] discusses the question of how the intergenerational risk transfer in a pension system can be designed in such a way that every generation really takes advantage of the system. [Cui e.a. 2011] argue in the same spirit as [Gollier 2007], however their pension model is more realistic in the sense that their model works with current pension payments (instead of lump sum benefits) and they introduce an absorbing funding surplus, which finances the intergenerational transfer. Furthermore [Cui e.a. 2011] use option price techniques to value the intergenerational transfer.

Our contribution is to discuss the *resilience* of a *CDC* pension scheme with respect to *intergenerational fairness*. We say that a pension scheme is resilient, if it is able to absorb external (single) *shocks* (e.g. a crash of market value of equities) and it is able to adjust to (permanent) *shifts* (e.g. shift of interest rates or mortality). It is desirable that a single stock market crash does not affect pensions in payment to full extend. However, as in defined contribution system with no external sponsor any protection of the group of pensioners is implicitly financed by an intergenerational transfer from the young to the old. Young participants will regard this kind of intergenerational transfer as *fair* because they expect that sooner or later the effects of the down shock will be compensated by an up shock. However, if e.g. the risk-free interest rate shifts to a new lower level, say combined with a lower inflation rate, then the understanding of intergenerational fairness *could* be that all age cohorts have to bear the consequences. Under these circumstances a waving of pension adjustments or a cut of pensions in payment could be compelling from the perspective of intergenerational fairness.

The setup of this paper is as follows. Following this introduction, section 2 introduces our basic pension model and section 3 the asset liability management (*ALM*) rules. The resilience test in section 4 constitutes the main part of this paper. To test the resilience of the pension system we have to define a steady state position (section 4.1). Then we apply capital market shock (section 4.2) and capital market shift (section 4.3) scenarios to the system. Finally in section 4.4 we discuss the effects of a mortality shift.

2 Basic Model

2.1 Population Model

2.1.1 *CDC* Pension Fund

We consider a pension fund for active and retired employees. The active employees pay periodic contributions to build up a pension capital. At a certain *retirement age z* the individual pension capitals are converted into a life annuity. The pension fund is exclusively financed by the regular contributions; there is no external entity that could step in if the pension fund runs out of assets. Examples of such scenarios would be if assets do not perform as expected or if the retirees live longer than expected resulting in pension benefits having to be adjusted. In extreme cases pension payments may have to be cut. On the other hand, overperforming assets or declining life expectancy eventually result in higher pension benefits.

In the case of a defined contribution (*DC*) pension fund, the contributions determine the pension benefits. If observed asset returns or mortality rates deviate from the expected values the pension benefits have to be adjusted while contributions remain unchanged. In contrast, in a definded benefit (*DB*) scheme, the contributions would be adjusted but not the promised benefits. The standard design of a *DC* schemes is an individual *DC* scheme, where each participant pays contributions into a personal pension pot, at retirement the accrued capital of the pension pot determines the paid benefits.

To our understanding the characteristic feature of a *collective DC* (*CDC*) pension fund is that there is a collective reserve, i.e. part of the total assets can be used to balance unexpected losses on the asset side or actuarial losses on the liability side. The following FIGURE 1 shows the stylised balance sheet of the pension fund. We have to explain when and how the collective reserve is deployed and refilled.

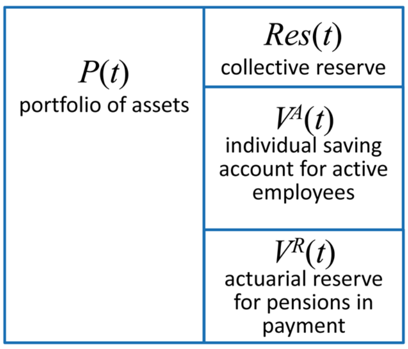


FIGURE 1: Stylised Balance Sheet

We assume that employees enter the system at a fixed *entry age* x_0 and that they remain in the population until death. If an employee dies before age z the balance of the personal account is paid out. From the retirement age of z onwards an annuity is paid until the person dies.

Here we list some basic notations with respect to the population model:

t: time index $t = 0, 1, \dots, T$

x_0: fixed entry age, if not stated otherwise we set $x_0 = 20$

z: fixed retirement age, if not stated otherwise we set $z = 65$

ω: maximal age, if not stated otherwise we set $\omega = 115$

$L(t, x)$: number of persons of the (t, x)-cohort, i.e. the number of persons who are x years old at time t. We assume that each age cohort is homogeneous, i.e. all members share the same mortality risk and have the same pension entitlements.

$\tilde{p}(t,x) = L(t+1, x+1) / L(t,x)$: survival probability for the (t, x)-cohort. This is a random variable conditioned to the avaible information at time t, observable at time t+1.

$\hat{p}(t,x)$: estimated survival probability for the (t, x)-cohort for the time interval $[t, t+1]$ based on the information up to time t

$p_a(t,x)$: actuarial survival probability for the (t, x)-cohort. These values are used to calculated the actuarial reserve for pensions due. The actuarial survival probalities could be *best* or *prudent* estimates. We do *not* model an ongoing updating of $p_a(t, x)$ to match the experienced mortality rate up a certain date. However, in the course of our discussion we will also examine the effect of a mortality shift.

By definition of ω we have $\tilde{p}(t,\omega) = \hat{p}(t,\omega) = p_a(t,\omega) = 0$ for all t.

We do not model the *idiosyncratic mortality risk*, i.e. the risk that a single person dies in a certain time period. Instead, we allow for non integer $L(t, x)$ and assume that

$$L(t+1, x+1) = L(t,x)\ \tilde{p}(t,x),$$

where the random variable $\tilde{p}(t,x)$ represents the *systematic mortality risk*.

We think of $\hat{p}(t,x)$ as any reasonable *best estimate* for $\tilde{p}(t,x)$. In practice, the phrase *best estimate* does not necessarily imply that $\hat{p}(t,x) = \mathbf{E}\left(\tilde{p}(t,x)\right)$.[4] We distinguish between $p_a(t,x)$ and $\hat{p}(t,x)$ to allow for safety margins with respect to mortality rates.

We regard the initial population $\left(L(0,x): x_0 \le x \le \omega\right)$ and the new entrants $\left(L(t,x_0): t \ge 0\right)$ as deterministic.

$L^A(t) := \sum_{x=x_0}^{z-1} L(t,x):$ total number of active employees at time *t*

$L^R(t) := \sum_{x=z}^{\omega} L(t,x):$ total number of retirees at time *t*

$L(t) := L^A(t) + L^R(t):$ total population at time *t*.

For convenience we define $L(t, x) := L(t, x_0)$ for all $x < x_0$ and $L(-1, x) := L(0, x)$ for all x, assuming that before time $t = 0$ we had a stable population. If not stated otherwise we calibrate our model population such that $L(0, 20) = 1000$.

2.1.2 Steady State Population and Population Dynamics

The *best estimate probabilities* $\hat{p}(t,x)$ are taken from the mortality tables *Richttafeln 2005G*,[5] which are the generally accepted standard tables for calculating book reserves for *DB*- plans in Germany. The entry age of the *Richttafeln 2005G* is $x_0 = 20$ and the terminal age is $\omega = 115$, i.e. $\hat{p}(t,115) = 0$ for all *t*. The *Richttafeln 2005G* are derived from social security data for male and female employees and comprise tables for all birth cohorts between 1891 and 2005. If indicated we will present separate results for a male and a female population. However, most calculations are performed on the basis of a *hybrid male/female population*. To this end we define *hybrid survival probabilities* by $\hat{p}(t,x) = \frac{1}{2}\left(\hat{p}^{(male)}(t,x) + \hat{p}^{(female)}(t,x)\right)$. One should be aware of the fact that the resulting *hybrid population* is not the population of a 50 - 50 mixed male/female population.

[4] For example, in the stochastic CDB-model (as described in the [Cairns e.a. 2006]) the "natural" best estimate is not necessarily an unbiased estimator.

[5] "Reference tables" [Heubeck et al. 2006]

Our calculations are based on an initial (male, female or hybrid) population defined by

$$L(0,20) = 1000,\ \ L(0,x+1) = \hat{p}(2018,x)\,L(0,x)\ \text{ for } x = 20,\ \ldots,\ 115,$$

i.e. all calculations start in 2018.

To illustrate the path to a steady state population we stipulate that all cohorts born after 2005 have the mortalitiy rates as the 2005- generation, i.e.

$$\hat{p}(t,x) = \hat{p}(2005+x,x)\ \text{ for all } t \geq 2005 + x.$$

This implies that from year 2120 the population is stationary.

Steady State Population

One of the main objectives of this paper is to analyse how our *CDC*-pension system responds to external capital market and or longevity shocks or shifts. We apply the external shock/ shift to a *CDC*-system which is in a *steady state* with respect to mortality and capital market returns.

To this end we define a *steady state population* $\{L(x),\ x = 20,\ldots,115\}$, where the number of members of an age cohort does not change over time:

$$L(x) := L(0,x)\ \text{ and }\ p(x) = \hat{p}(2018,x)\ \text{for all } x.$$

Population Dynamics

In addition to the shock/ shift scenario analysis we will perform simulations. For these we take into account the cohort specific mortality trends as estimated in *Richttafeln 2005G*. Furthermore, we will also consider different scenarios with respect to the number of new entrants to the pension fund at the age of $x_0 = 20$.

The evolution of the population in time will depend on the number of entering employees. We will consider four scenarios for population dynamics:

A constant number of new entrants:	$L(t, x_0) = 1000$ for all $t \geq 0$.
+1% growing number of new entrants:	$L(t, x_0) = 1000\,(1.01)^t$
-1% shrinking number of new entrants:	$L(t, x_0) = 1000\,(1.01)^{-t}$
winding up after 10 years:	$L(t, x_0) = 1000$ for $t = 0, \ldots, 10$ and $L(t, x_0) = 0$ for $t > 10$.

The following FIGURE 2 exhibits the population development for these scenarios for a male and a female population

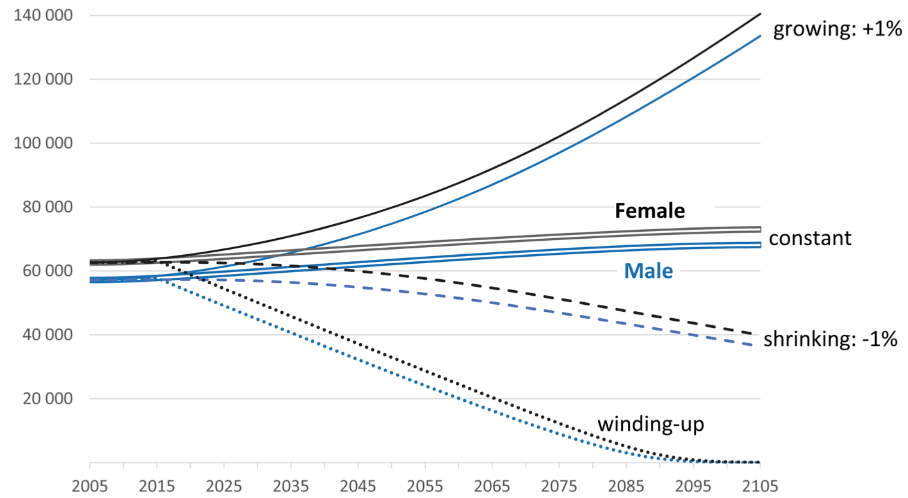


FIGURE 2: Population development for 2005-2105, for a female and a male population for various new entrant scenarios.

We observe an increasing population despite a constant number of new entrants. This is due to the fact that we use cohort mortality tables with increasing survival probabilities. Since we assume that the secular trend of improving mortality rates comes to an end for generations born 2005 or later, from the year 2120 onwards we have a steady state population.

2.2 Liabilities

We distinguish between the liabilities with respect to active employees and the pension liabilities for retirees.

Pensions liabilities for active employees

Each active employee has an individual pension account to which the regular contributions and a "*profit participation*" are credited.

For $x_0 \le x \le z$ let $v(t, x)$ denote the individual accumulated pension capital at time t. We assume that at time t (at the *beginning* of $[t, t{+}1]$) every active employee ($x < z$) pays a constant contribution c, which is credited to his or her personal account. $C(t) = L^A(t)\, c$ denotes the total of all contributions payable at time t.

We assume that all individual accounts share the same "*profit participation*" $\eta(t{+}1)$. We understand $\eta(t{+}1)$ to be the profit participation for the time period $[t, t{+}1]$, which is credited at time t+1. If $\eta(t{+}1)$ is determined at time *t, i.e. at the beginning* of $[t, t{+}1]$ then we will call this a *prospective declaration* of profit participation; if $\eta(t{+}1)$ is determined at the end $[t, t{+}1]$ we call this a *retrospective declaration*. The phrase "*profit participation*" is commonly used with life insurance contracts, where it refers to payments on top of a guaranteed interest rate. In our model, there is no guaranteed interest rate, we explicitly allow for a negative $\eta(t)$.

At time $t = 0$ we assume that each member of the $(0, x)$ – cohort (for x: $x_0 < x \leq z$) has an initial pension capital $v(0, x)$. Furthermore, we assume that all newly entering employees (at age x_0) start with zero pension capital, i.e. $v(t, x_0) = 0$ for all t.

For the individual accounts we have the following recursion:

$$v(t+1, x+1) = \big(v(t,x) + c\big)\exp\big(\eta(t+1)\big) \qquad \text{for } t \geq 0\ x_0 \leq x < z$$

Note that $v(t, x)$ is the individual pension capital "a logical second" before the contribution c is credited. The final pension capital $v(t, z)$ is not paid out but is converted into a life annuity.

If a person of the $(t\text{-}1, x\text{-}1)$ - cohort, $x_0 < x \leq z$, dies within the time interval $[t\text{-}1, t]$, then at time t the accrued capital $v(t,x) = \big(v(t-1, x-1) + c\big)\exp\big(\eta(t)\big)$ is paid out to the surviving dependents. The total death benefit payable at time $t \geq 0$ is

$$D(t) := \sum_{x = x_0+1}^{z} \big(L(t-1, x-1) - L(t,x)\big)\, v(t,x)\,.$$

Note that according to our convention $L(\text{-}1, x) = L(0, x)$.

Since the death benefit equals the accumulated pension capital, the survival probabilities do not effect the individual pension accounts. However, the mortality rates for the active employees has an effect on the population structure of the pension fund.

At time t the death benefit $D(t)$ is paid out and the accrued contributions of the (t, z)-cohort, namely $V_z(t) := L(t,z)\ v(t,z)$, is converted into an annuity.

We define

$$V^A(t) := \sum_{x = x_0+1}^{z} L(t,x)\, v(t,x) + D(t)\,.$$

$V^A(t)$ denotes the total liability with respect to the group of active employees a logical second *before* $D(t)$ is paid out and $V_z(t)$ is converted. Note that $V^A(t)$ does not comprise the new contributions $C(t)$ that are credited at the beginning of $[t, t+1]$. For notational convenience we set $V^A(t+) := V^A(t) - D(t) - V_z(t) + C(t)$, the liabilities with respect to active employees a logical second *after* the death benefit is paid, the (t, z)-cohort become retirees and the new contributions flow in.

Remark

$$V^A(t) = \sum_{x=x_0+1}^{z} L(t-1, x-1)\, v(t,x) \qquad (Eq.\ 1)$$

Liabilities for pensions due

We assume that within the (t, x) - cohort ($z < x \leq \omega$) all retirees are entitled to the same pension $b(t, x)$. The pensions are paid in advance, i.e. the pensions for the time period $[t, t+1]$ are payable a "logical second" after time t. $B(t) := \sum_{x=z}^{\omega} L(t,x)\, b(t,x)$ denotes the sum of all pensions due at time t.

To calculate the actuarial reserve for all pensions in payment, we assume a fixed (time independent) actuarial interest rate μ_a and actuarial survival probabilities $p_a(t, x)$. Implicitly we assume that all $p_a(t, x)$ are „known" at time $t = 0$.

Let $\ddot{a}(t, x)$ denote the actuarial reserve for a 1€ - pension paid in advance for members of the (t, x)-cohort. $\ddot{a}(t, x)$ can be defined by the following backward recursion:

$$\ddot{a}(t,\omega) = 1 \quad \text{and} \quad \ddot{a}(t,x) = 1 + p_a(t,x)\exp(-\mu_a)\, \ddot{a}(t+1, x+1) \quad \text{for } x = z, ..., \omega\text{-}1.$$

We define $V^R(t) := \sum_{x=z+1}^{\omega} L(t,x)\, b(t,x)\, \ddot{a}(t,x)$, the pension liabilities with respect to the cohort of retierees. Note that $V^R(t)$ does not comprise $V_z(t)$.

At time t the (t, z)-cohort members own an individual pension capital of $v(t, z)$ which is immediately after time t converted into a life annuity

$$b(t,z) := \frac{v(t,z)}{\ddot{a}(t,z)}, \text{ thus } V_z(t) = L(t, z)\, b(t, z)\, \ddot{a}(t, z).$$

$V^R(t+) := V^R(t) + V_z(t) - B(t) = \sum_{x=z}^{\omega-1} L(t,x)\, b(t,x)\left(\ddot{a}(t,x) - 1\right)$ are the liabilities for due pensions a logical second *after* $V_z(t)$ is converted and due pensions are paid.

We assume that at the end of each period $[t, t+1]$ all pensions are adjusted by the same rate $\varepsilon(t+1)$, i.e. $b(t+1, x+1) = \exp\left(\varepsilon(t+1)\right) b(t,x)$ for $x = z, ..., \omega - 1$. As with $\eta(t+1)$ the pension adjustment $\varepsilon(t+1)$ can be *declared prospectively* at time t or *retrospectively* at time $t+1$.

Remark

From the account's point of view, the death benefit $D(t)$ belongs to $[t\text{-}1, t]$ while $B(t)$ and $C(t)$ belong to $[t, t+1]$. Therefore the balance sheet liability of the pension fund should be $V_{bal}(t) = V(t) - D(t)$ rather than $V(t)$. We may justify our "accounting trick" by assuming that at the end of $[t\text{-}1, t]$ the death benefit has yet not been paid out and that $D(t)$ just represents provisions for benefits due but yet not paid.

The reason why we include $D(t)$ in the liabilities $V^A(t)$ is pure pragmatism. By this we can combine net cash outflow in a single entity and this simplifies notation considerably. For notational convenience we define (cf. Figure 3):

$CF(t) := B(t) + D(t) - C(t)$: *total outgoing cash flow* at time t

$V(t) := V^A(t) + V^R(t)$ *total liability* just *before* $CF(t)$ is paid out

$V(t+) := V(t) - CF(t) = V^A(t+) + V^R(t+)$ *total liability* just *after* $CF(t)$ is paid out.

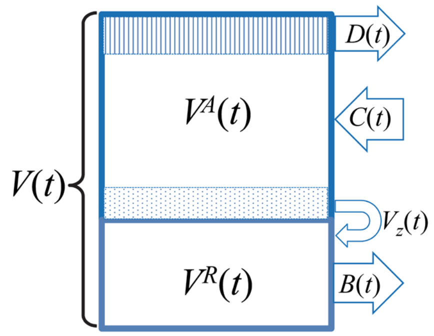


FIGURE 3: Liabilities and cash flows

Since in case of death just the existing reserve is paid out, there is actually no mortality risk for the cohort of active workers. So the question might be: Why not simplify the model by setting $L(t, x) = L(t, z)$ for $x < z$ and skipping the death benefit? However, in our model we have a *collective reserve* and as we will see, the age structure of the whole pension population does have effect on the performance.

2.3 Assets

Let $P(t)$ denote the value of assets at time t, just *before* the cash flow $CF(t) = D(t) + B(t) - C(t)$ is paid out. Denote by $\tilde{\mu}(t+1)$ the log-return on pension assets for the investment period $[t, t+1]$, then $P(t+1) = \big(P(t) - CF(t)\big)\exp\big(\tilde{\mu}(t+1)\big)$.

We assume that $\tilde{\mu}(t+1)$ is a random variable, which can be decomposed as

$$\tilde{\mu}(t+1) = \mu(\sigma_t) + \sigma_t X_{t+1}, \qquad (Eq.\ 2)$$

where $\mu(\sigma)$ is a real valued increasing function on $[0, \sigma_{max}]$, and $X_1, X_2, \ldots$ are stochastically independent identically distributed random variables with zero expectation and variance one.

The idea behind (*Eq*. 2) is that at time t the pension manager decides on the risk exposure σ_t for the following investment period $[t, t+1]$. They can choose a *risk free investment* ($\sigma_t = 0$) or, depending on the risk appetite, a more risky investment with a higher expected return. In our model the assets are controlled by only one parameter σ_t , which serves as risk indicator. The time index $t+1$ in $\tilde{\mu}(t+1)$ or X_{t+1} indicates the time, when these random variables can be observed.

Remarks

1. Consider the *continuous time Black Scholes model* with a risk free asset $dA_t = A_t\,\bar{\mu}\,dt$ and a risky asset $dS_t = S_t\,(\mu_M\,dt + \sigma_M dW_t)$. We may think of S_t as a broadly diversified portfolio of equities, representing the *market portfolio* in terms of the capital asset pricing theory. Consider a *right-continuous* nonnegative real valued *risk exposure process* $(\sigma(t))_{t\geq 0}$, where $\sigma(t)$ only depends on the information up to time t. We interpret $\sigma(t)$ to be the risk exposure we take at time t.

   Define $(S_t^\sigma)_{t\geq 0}$ by $S_0^\sigma = S_0$ and $d\,S_t^\sigma = S_t^\sigma\left(\mu(\sigma_t)\,dt + \sigma_t\,dW_t\right)$. Then this is *arbitrage free* within the Black Scholes framework if and only if

   $$\mu(\sigma) = \bar{\mu} + r_{SR}\,\sigma - \tfrac{1}{2}\sigma^2 \text{ with } r_{SR} := \frac{\mu_M + \frac{1}{2}\sigma_M^2 - \bar{\mu}}{\sigma_M} \qquad (Eq.\ 3)$$

   The economic interpretation is the following: Suppose our portfolio has value P_t at time t and we wanted to pursue a *constant mix strategy* for the time interval $[t, t+1]$. This means that at any $t' \in [t, t+1[$ the relative proportion of risky assets in our portfolio is constant at some level $\beta \in [0,1]$. So at time t we invest βP_t in S_t and $(1-\beta)\,P_t$ in A_t . Since the value processes for S_t and A_t will diverge within $[t, t+1]$ we have to rebalance our portfolio continuously.

   Then for $\sigma = \beta\,\sigma_M$ we get $P_{t+1} = P_t \exp\left(\bar{\mu} + r_{SR}\,\sigma - \tfrac{1}{2}\sigma^2 + \sigma(W_{t+1} - W_t)\right)$ and $\ln\left(\dfrac{P_{t+1}}{P_t}\right) = \mu(\sigma) + \sigma\,(W_{t+1} - W_t)$. Thus, a constant mix strategy within the Black Scholes framework statisfies (*Eq*. 2).

2. As an alternative we could pursue a *buy and hold strategy*: At time t invest βP_t in S_t and $(1-\beta)\,P_t$ in A_t and then wait until $t+1$. Then we get

   $P_{t+1} = P_t\left((1-\beta)\exp(\bar{\mu}) + \beta\exp\left(\mu_M + \sigma_M\,(W_{t+1} - W_t)\right)\right)$ and

$$\mathbf{E}(P_{t+1}) = P_t\left((1-\beta)\exp(\overline{\mu}) + \beta\exp(\mu_M + \tfrac{1}{2}\sigma_M^2)\right).$$

If we define $i_A = \exp(\overline{\mu}) - 1$ and $i_S = \exp(\mu_M + \tfrac{1}{2}\sigma_M^2) - 1$, we get

$$\mathbf{E}\left(\frac{P_{t+1}}{P_t}\right) = (1-\beta)(1+i_A) + \beta(1+i_S) = 1 + i_A + \beta(i_S - i_A).$$

However, one may convince oneself that $\tilde{\mu}(t+1) := \ln\left(P_{t+1} / P_t\right)$ cannot be decomposed as in (*Eq*. 2).

3 Asset Liability Management

3.1 Basic Relations

We define $\rho(t) := \ln\left(P(t) / V(t)\right)$ - the *log-reserve ratio* or simply the *reserve ratio*. He have $\rho(t) > 0$ iff $P(t) > V(t)$. In the following $\rho(t)$ will be the fundamental control variable for the asset liability management (*ALM*). For practioners, the cover ratio $P(t)/V(t)$ rather than $\rho(t)$ is taken as the indicator of the "wellbeing" of a pension fund. Clearly, it makes no difference whether we control $\rho(t)$ or $P(t)/V(t)$. But, as we will see, $\rho(t)$ simplifies notations. Note that for $P(t)/V(t) \approx 1$ (say $0.8 < P(t)/V(t) < 1.2$) we have $1 + \rho(t) \approx P(t)/V(t)$.

At time t (i.e. based on the information up to time t) the pension manager has to dedide on σ_t , the risk exposure for the coming time period $[t, t+1]$. If we apply a *prospective declaration*, then also $\eta(t+1)$ and $\varepsilon(t+1)$ are determined at time t. It is clear that if we want to guarantee a minimum cover ratio (or reserve ratio) then we must apply a retrospective declaration.

For the following propositions we define for $t \geq 0$:

$$w(t,x) := \frac{L(t,x)\,b(t,x)\left(\ddot{a}(t,x)-1\right)}{V^R(t) + V_z(t) - B(t)} = \frac{L(t,x)\,b(t,x)\left(\ddot{a}(t,x)-1\right)}{V^R(t+)} \quad \text{(for } x \geq z\text{)}$$

$$\tilde{\pi}(t+1) := -\ln\left(\sum_{x=z}^{\omega-1} \frac{\tilde{p}(t,x)}{p_a(t,x)}\, w(t,x)\right)$$

$$\hat{\pi}(t+1) := -\ln\left(\sum_{x=z}^{\omega-1} \frac{\hat{p}(t,x)}{p_a(t,x)}\, w(t,x)\right)$$

$$\lambda(t) := \frac{CF(t)}{V(t)}$$

$$\delta(t) := \ln\left(\frac{P(t)-CF(t)}{V(t)-CF(t)}\right) - \ln\left(\frac{P(t)}{V(t)}\right) = \ln\left(\frac{1-\lambda(t)\exp\left(-\rho(t)\right)}{1-\lambda(t)}\right)$$

$$\gamma(t) := \frac{V^R(t)+V_z(t)-B(t)}{V(t)-CF(t)} = \frac{V^R(t+)}{V(t+)}.$$

Remarks

1. Since $V^R(t+) = \sum_{x=z}^{\omega-1} L(t,x)\, b(t,x)\left(\ddot{a}(t,x)-1\right)$, $w(t,x)$ is the relative weight of the (t, x)-cohort in $V^R(t+)$. Note that $\sum_{x=z}^{\omega-1} w(t,x) = 1$.

2. $\hat{\pi}(t+1)$ is the weighted safety margin if the actuarial assumptions with respect to the survival probabilities are set so that $p_a(t,x) > \hat{p}(t,x)$. If we use best estimate survival probabilities for actuarial valuation we have $\hat{\pi}(t+1) = 0$.

3. $\tilde{\pi}(t+1)$ and $\hat{\pi}(t+1)$ only depend on the survival probabilities for the cohort of retirees. $\tilde{\pi}(t+1)$ measures to what extent the experienced and the actuarially presupposed mortality rates diverge. If the actuarial assumptions include safety margins then $\tilde{\pi}(t+1)$ is *expected* to be positive.

   We regard $\hat{\pi}(t+1)$ as *the best estimate* for $\tilde{\pi}(t+1)$ based on information up to time t. As practitioners we *do* use the phrase "best estimate" rather generously. In particular we do *not* stipulate that $\mathbf{E}\left(\tilde{\pi}(t+1)\right) = \hat{\pi}(t+1)$. One should note that $\hat{p}(t,x) = \mathbf{E}\left(\tilde{p}(t,x)\right)$ for all x and t does not imply that
   $\mathbf{E}\left(\tilde{\pi}(t+1)\right) = \hat{\pi}(t+1)$.

4. If $p_a(t,x) = \hat{p}(t,x)$ then $\hat{\pi}(t+1) = 0$ and $\tilde{\pi}(t+1) = -\ln\left(\sum_{x=z}^{\omega-1} \frac{\tilde{p}(t,x)}{\hat{p}(t,x)} w(t,x)\right)$.

   $\tilde{\pi}(t+1)$ can be interpreted as the *weighted mortality effect*.

5. $\lambda(t)$ can be interpreted as the *liquidity ratio*, the ratio of outgoing money to the total liabilities. Note that $\lambda(t) < 1$ since $CF(t) < V(t)$.

6. Since $V(t) > CF(t)$ (by definition) $\delta(t)$ is well defined provided $P(t) > CF(t)$.

7. Note that $\rho(t) = 0$ implies $\delta(t) = 0$. For $\rho(t) > 0$ $\delta(t)$ is *positive* and *increasing* in $CF(t)$ and for $\rho(t) < 0$ $\delta(t)$ is *negative* and *decreasing* in $CF(t)$. $\delta(t)$

measures the effect of the cashflow $CF(t)$ on the reserve ratio $\rho(t)$. $CF(t)$ has no effect on the absolute value of the reserve $P(t) - V(t)$, but $CF(t) \neq 0$ effects the reserve ratio. If $CF(t) < 0$, which is typical for a young population, the reserve ratio will decrease. This effect is similar to the *stock dilution effect* when additional common shares are issued. A share buy-back program has an opposite effect. So we call $\delta(t)$ the *stock effect*. As we will see below, the stock effect will be positive if the pension system is in a *steady state*. It is also positive if the pension system is unwinding.

8. $\gamma(t)$ can be interpreted as the *weighted age burden*. $\gamma(t) = 0$ means that there are no pension liabilities, and $\gamma(t) = 1$ implies that there are no liabilities for active workers.

Proposition 1

If $\eta(t+1)$ is the profit participation for the individual pension accounts and if the pensions are adjusted by $\varepsilon(t+1)$, then we have the following recursions for the liabilities:

$$V^A(t+1) = \exp\big(\eta(t+1)\big) V^A(t+) \qquad (Eq.\ 4)$$

$$V^R(t+1) = \exp\big(\varepsilon(t+1) + \mu_a - \tilde{\pi}(t+1)\big) V^R(t+) \qquad (Eq.\ 5)$$

If $\varepsilon(t+1) = \eta(t+1) - \mu_a + \tilde{\pi}(t+1)$, then

$$V(t+1) = \exp\big(\eta(t+1)\big)\big(V(t) - CF(t)\big) = \exp\big(\eta(t+1)\big) V(t+) \qquad (Eq.\ 6)$$

$$\rho(t+1) - \rho(t) = \tilde{\mu}(t+1) - \eta(t+1) + \delta(t) \ . \qquad (Eq.\ 7)$$

Proof

To prove (*Eq*. 4) we use definition (*Eq*. 1) and the fact that $v(t, x_0) = 0$:

$$V^A(t+1) = \sum_{x=x_0+1}^{z} L(t, x-1)\, v(t+1, x)$$

$$= \exp\big(\eta(t+1)\big) \sum_{x=x_0+1}^{z} L(t, x-1)\, \big(v(t, x-1) + c\big)$$

$$= \exp\big(\eta(t+1)\big) \left(C(t) + \sum_{x=x_0}^{z-1} L(t, x)\, v(t, x) \right)$$

$$= \exp\big(\eta(t+1)\big)\big(V^A(t) - D(t) - V_z(t) + C(t)\big).$$

To verify ($Eq.$ 5) we take ($Eq.$ 2) and use the definition of $w(t, x)$ and $\tilde{\pi}(t+1)$ and the recursion for $\ddot{a}(t, x)$:

$$\begin{aligned}
V^R(t+1) &= \sum_{x=z}^{\omega-1} L(t+1, x+1)\, b(t+1, x+1)\, \ddot{a}(t+1, x+1) \\
&= \exp\left(\mu_a + \varepsilon(t+1)\right) \sum_{x=z}^{\omega-1} \tilde{p}(t,x)\, L(t,x)\, b(t,x) \frac{\ddot{a}(t,x)-1}{p_a(t,x)} \\
&= \exp\left(\mu_a + \varepsilon(t+1)\right)\left(V^R(t) + V_z(t) - B(t)\right) \sum_{x=z}^{\omega-1} \frac{\tilde{p}(t,x)}{p_a(t,x)} w(t,x) \\
&= \exp\left(\mu_a + \varepsilon(t+1) - \tilde{\pi}(t+1)\right)\left(V^R(t) + V_z(t) - B(t)\right).
\end{aligned}$$

($Eq.$ 6) and ($Eq.$ 7) follow directly from ($Eq.$ 4) and ($Eq.$ 5) and the definition of $\delta(t)$.

♦

Remark

If we determine $\eta(t+1)$ and $\varepsilon(t+1)$ *retrospectively*, i.e. on the basis of information up to time t+1, then according to ($Eq.$ 7) $\eta(t+1)$ and $\varepsilon(t+1)$ can be defined such that any predetermined reserve level $\rho(t+1)$ can be reached. For example, if we define $\eta(t+1) = \tilde{\mu}(t+1) + \delta(t)$ and $\varepsilon(t+1) = \eta(t+1) - \mu_a + \tilde{\pi}(t+1)$, then $\rho(t+1) = \rho(t)$. If this was our *ALM*-strategy, we wouldn't need a collective reserve! However, in this setting capital market risks and the mortality risk would directly affect the individual accounts or pensions. The main benefit of a collective system, namely the intergenerational risk sharing, would then not be enabled.

Fixing $\eta(t+1)$ and $\varepsilon(t+1)$ at time t (and not at time $t+1$) reflects the idea of *defined ambition*.[6] This is attractive for savers and retirees because they know in advance, how their contributions are accrued and how the pensions are adjusted.

In Proposition 1 we have set $\varepsilon(t+1) = \eta(t+1) - \mu_a + \tilde{\pi}(t+1)$, which can only be determined retrospectively. Thus for a prospective declaration we have to replace $\tilde{\pi}(t+1)$ by $\hat{\pi}(t+1)$.

Proposition 2

If in the situation of Proposition 1 we define $\varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$ then

$$V(t+1) = \exp\left(\eta(t+1) + Y_{t+1}\right)\left(V(t) - CF(t)\right) \qquad (Eq.\ 8)$$

[6] c.f. [Day et al. 2014]

$$\rho(t+1)-\rho(t)=\sigma_t X_{t+1}-Y_{t+1}+\mu(\sigma_t)+\delta(t)-\eta(t+1)\,, \qquad (Eq.\ 9)$$

where $Y_{t+1} := \ln\left(1+\gamma(t)\left(\exp\left(\hat{\pi}(t+1)-\tilde{\pi}(t+1)\right)-1\right)\right)$.

Proof

The definition of $\varepsilon(t+1)$ together with Proposition 1 shows that

$$\begin{aligned}
V(t+1) &= V^A(t+1)+V^R(t+1) \\
&= \exp\left(\eta(t+1)\right)V^A(t+)+\exp\left(\eta(t+1)+\hat{\pi}(t+1)-\tilde{\pi}(t+1)\right)V^R(t+) \\
&= \exp\left(\eta(t+1)\right)V(t+)\left(\frac{V^A(t+)}{V(t+)}+\frac{V^R(t+)}{V(t+)}\exp\left(\hat{\pi}(t+1)-\tilde{\pi}(t+1)\right)\right) \\
&= \exp\left(\eta(t+1)\right)V(t+)\left(1+\gamma(t)\left[\exp\left(\hat{\pi}(t+1)-\tilde{\pi}(t+1)\right)-1\right]\right) \\
&= \exp\left(\eta(t+1)+Y_{t+1}\right)V(t+)\,.
\end{aligned}$$

(*Eq.* 9) is a direct consequence of (*Eq.* 8) and the definition von $\delta(t)$.

♦

Remark

1. (*Eq.* 9) will be the basis for the *ALM*-strategies which are presented in the next section. The change of the reserve ratio, $\rho(t+1)-\rho(t)$, can be broken down into
   - the stochastic capital market effect $\tilde{\mu}(t+1)=\mu(\sigma_t)+\sigma_t X_{t+1}$
   - the stochastic longevity effect Y_{t+1}
   - the structural stock effect $\delta(t)$
   - the profit participation $\eta(t)$.
2. Admittedly, the definition of Y_{t+1} is a little bit clumsy, but it serves perfectly to isolate the longevity risk. Y_{t+1} depends on the *weighted age burden* $\gamma(t)$ and the difference between the estimated and the observed longevity effect $\hat{\pi}(t+1)-\tilde{\pi}(t+1)$.

   If $\gamma(t)=0$ then $Y_{t+1}=0$, and if $\gamma(t)=1$ then $Y_{t+1}=\hat{\pi}(t+1)-\tilde{\pi}(t+1)$.
   Using the 2nd order Taylor approximation for the function $\Delta \mapsto \ln\left(1+\gamma(t)\left(\exp(\Delta)-1\right)\right)$, we get the following approximation:

   $$Y_{t+1}\approx \Delta\gamma(t)\left(1+\tfrac{1}{2}\Delta\left(1-\gamma(t)\right)\right) \text{ with } \Delta := \hat{\pi}(t+1)-\tilde{\pi}(t+1)\,. \qquad (Eq.\ 10)$$

3. If $\hat{\pi}(t+1)=0$, and especially if $p_a(t,x)=\hat{p}(t,x)$ for all x, then

   $$Y_{t+1}=\ln\left(1+\gamma(t)\sum_{x=z}^{\omega-1}\left(\frac{\tilde{p}(t,x)}{p_a(t,x)}-1\right)w(t,x)\right).$$

4. The pension adjustment $\varepsilon(t+1)=\eta(t+1)-\mu_a+\hat{\pi}(t+1)$ can be regarded as *fair*, since there is no systematic transfer of capital between the young and the old. If the actuarial surviving probabilities $p_a(t, x)$ are calculated with safety margins, then the initial pensions $b(t, z)$ are lower compared to a best estimate pension. Then $\hat{\pi}(t+1)$ ensures that the safety margins are (on average) refunded to the cohort of retirees. However, within the cohort of retirees high safety margins with respect to $p_a(t, x)$ do have a redistributional effect, since higher pension adjustments are unilaterally favourable for long living retirees.

3.2 ALM – Strategies

We now come to the question of how to control the *CDC*-pension fund described above. Capital market opportunities and risks, mortality rates and the number of new entrants are exogenous variables, of which only the capital market risk can be controlled to a certain extent. Our *CDC*-pension fund is *self financing* in the sense that there is no outside institution that can step in if capital markets perform extremely badly or people live much longer than expected. On the other hand, the pension member can be sure that every contribution paid into the system is exclusively used for death or pension benefits.

Since the pension fund itself does not guarantee any benefits, there must be some good arguments for employees to entrust their contribution to such a system. Actually, the only good reason to enter such a collective system is that the employees have a good chance to get a *better risk-return profile* than in an individual saving and dissaving arrangement.

Before presenting *ALM*-rules for the *CDC* pension fund, let us state some principles that the *ALM* has to comply with:

Principle 1: The benefits a person receives are calculated on the basis of their personal pension capital at retirement age. Especially all pension members within an age cohort are treated equally.
The idea behind this principle is that the sole purpose of the collective element in the *CDC* plan is to enable an intertemporal risk transfer. Thus, in the absence of risk a *CDC* plan should be nothing but a simple *DC* plan with a one-to-one correspondence between contributions and benefits on the individual level.

Our *CDC* model complies with *Principle* 1 since the pensions are calculated on the basis of accumulated contribution and furthermore, $\eta(t)$ and $\varepsilon(t)$ apply equally to active workers and retirees respectively.

Principle 2: It must be ensured that $P(t) \geq V(t)$, i.e. $\rho(t) \geq 0$. We think of a *capital funded* system, which in general means that pension benefits are prefunded by regular contributions. In contrast, in a *pay-as-you-go* provision system the currently paid benefits are covered by currently incoming contributions. Instead of $P(t) \geq V(t)$ for all t, we could require that at any time all pension liabilities can be settled even if there are no further contributions. However, in a system with no guarantees the expression "all pension liabilities" is rather vague or has to be made precise. In our model the understanding of $V^R(t)$ is that this is the actuarial reserve under the assumption that the currently paid pensions are kept constant in future. Note that $V^R(t)$ is *not* the market consistent value of the pension liabilities since we do not price the pension fund's implicit option to increase or reduce future payments if circumstances require.

In our model *CDC* pension system we can ensure $P(t) \geq V(t)$ only if we allow for a *retrospective declaration*. In case of a *prospective declaration* $P(t) \geq V(t)$ can only be ensured with a certain degree of probability. Thus, in the case of a prospective declaration we have to take a weakened version of *Principle* 2:

Principle 2': It must be ensured that $P(t) \geq V(t)$, but for a transitional period $P(t) < V(t)$ is accepted provided measures are taken to restore full funding.

Principle 3: No age cohort is systematically preferred or put at a disadvantage compared to others (*intergenerational equity*). The requirement of generational equity is fundamental for any pension system - capital funded or pay-as-you-go. This issue is widely discussed in literature.[7]

Admittedly, *Principles* 2, 2' and 3 are put in rather vague terms. They convey the idea of a "fair" pension system, but *fairness* is not an actuarial concept. At this point, it is worth to mention the fundamental concept, which *John Rawls* (1921-2002) worked out in his seminal book "*A Theory of Justice*". He addresses the problem of justice between generations from an abstract perspective so that his rules are not directly applicable to a funded pension scheme.[8] However, his idea of a social contract agreed upon behind the "*veil of ignorance*" ("*in the original position*") can be applied to the question of a *fair* pension scheme. Behind the veil of ignorance people do not know in advance whether their generation will be lucky or unlucky with respect to the individual life span and to the future development of capital markets.

[7] Cf. [European Union 2016] IORP II Directive, Article 7.

[8] Rawls explicitly addressed the issue of intergenerational fairness – cf. [Rawls 1971], Chapter 44, pp. 251-258.

Principle 2 requires that the *ALM* has to control the cover ratio $P(t)$ / $V(t)$ or - which is equivalent - the reserve ratio $\rho(t)$. If $\rho(t)$ threatens to fall below zero or some threshold $\rho_{\min}$ measures have to be taken , e.g. pension cuts and/ or the reduction of the risk exposure on the asset side. Intergenerational equity (*Principle* 3) requires that $\rho(t)$ is also capped above, since an unreasonable large reserve ratio indicates that there is a systematic transfer from the old to the young.

As pointed out, generational equity requires that $\varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$, otherwise there would be a systematic income transfer between old and young.

In our setting *a feasible retrospective ALM rule* is a rule which at time t (on the information up to time t) determines $\eta(t)$ and σ_t, such that *Principles* 1, 2 and 3 are satisfied. A *feasible prospective ALM-rule* is a rule which at time t (on the information up to time t) determines $\eta(t+1)$ and σ_t, such that *Principles* 1, 2' and 3 are satisfied.

We can think of a wide range of *ALM*-strategies that comply with the above principles. The *ALM* rules we use here are taken from [Goecke 2013]. The model presented there is time continuous and restricted to the accumulation phase. But the basic features can be transferred the discrete case. In particular we adopt the idea of a *strategic reserve rati*o $\hat{\rho}$ and a *strategic risk exposure* $\hat{\sigma}$. The pair $(\hat{\rho}, \hat{\sigma})$ represents a state of equilibrium in the sense that if we observe a reserve ratio $\rho(t) = \hat{\rho}$ then we choose $\sigma_t = \hat{\sigma}$ as the risk exposure. $\eta(t)$ is chosen such that the reserve ratio remains unchanged provided capital market returns and mortality rates are just as expected. Another feature taken from [Goecke 2013] is that whenever $\rho(t) \neq \hat{\rho}$ we adjust σ_t and $\eta(t+1)$ in dependence of the *reserve gap* $\rho(t) - \hat{\rho}$.

We now state our *basic ALM-strategy* in the prospective version. For real numbers $(\hat{\rho}, \hat{\sigma}, a,\ \theta, \sigma_{\max})$ we define

(*ALM* 1) $\quad \sigma_t = \hat{\sigma} + a\big(\rho(t) - \hat{\rho}\big)$ and $0 \le \sigma_t \le \sigma_{\max}$

(*ALM* 2) $\quad \eta(t+1) = \mu(\sigma_t) + \delta(t) + \theta\big(\rho(t) - \hat{\rho}\big)$

(*ALM* 3) $\quad \varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$.

Remarks:

1. The reserve gap $\rho(t) - \hat{\rho}$ rather than the reserve ratio $\rho(t)$ is the decisive control variable of the *CDC*-pension system. However, due to the stock effect $\delta(t)$ the absolute level of $\rho(t)$ *does* have influence on the process.

2. (ALM 1) is motivated by the following considerations. Suppose at time t we determine the risk exposure σ_t under the side constraint, that with probability 1-α the reserve ratio does not fall below $\rho_{\min}$, i.e. $\mathbf{P}\left(\rho(t+1)\le\rho_{\min}\right)\le\alpha$.

   For $\eta(t+1)=\mu(\sigma_t)+\delta(t)+\theta\left(\rho(t)-\hat{\rho}\right)$ - c.f. (ALM 2) - this is equivalent to

   $$\mathbf{P}\left(\sigma_t X_{t+1}-Y_{t+1}\le\rho_{\min}-\rho(t)+\theta\left(\rho(t)-\hat{\rho}\right)\right)\le\alpha .$$

   Let $VaR_\alpha>0$ denote the *α-value at risk* of X_{t+1}, i.e. $\mathbf{P}\left(X_{t+1}\le -VaR_\alpha\right)=\alpha$, then for $Y_{t+1}=0$ (i.e. neglecting the mortality risk) we get

   $$\sigma_t\le\frac{\hat{\rho}-\rho_{\min}+(1-\theta)\left(\rho(t)-\hat{\rho}\right)}{VaR_\alpha}.$$

   Thus, if we seek maximal risk exposure under the constraint $\mathbf{P}\left(\rho(t+1)\le\rho_{\min}\right)\le\alpha$, then we have to define

   $$\sigma_t=\hat{\sigma}+a\left(\rho(t)-\hat{\rho}\right)\text{ with }\hat{\sigma}=\frac{\hat{\rho}-\rho_{\min}}{VaR_\alpha},\ a=\frac{1-\theta}{VaR_\alpha}.$$

   If we use the Black Scholes framework for the capital market (cf. *Remark* 1 of section 2.3), then X_t is normally distributed with variance 1 and expectation 0. On the basis of the *Solvency 2* security level of 1-α = 99.5% we get VaR_α = 2.5758. Suppose that the regulator allows a temporary underfunding of 90% and a "normal" funding ratio of 115%, then $\rho_{\min}$ = ln(0.9) = -10.54%, $\hat{\rho}$ = ln(1.15) = 13.98%, and

   $$\hat{\sigma}=\frac{\hat{\rho}-\rho_{\min}}{VaR_\alpha}=0.0952\ \text{ and }\ a=\frac{1-\theta}{VaR_\alpha}.$$

   Assuming that a broadly diversified portfolio of stocks has a volatility of about 19%, $\hat{\sigma}=0.095$ corresponds to an equity ratio of about 50%.

   The question of how to calibrate θ, we will answer in view of Prop. 3, below.

3. Parameter a in (ALM 1) determines the *adjustment speed* with respect to the risk exposure. For a = 0 we have a constant mix strategy throughout the time horizon. If a > 0 then the risk appetite for the asset allocation changes in line with the positive or negative reserve gap $\rho(t)-\hat{\rho}$. The case a < 0 corresponds to a massive anti cyclic investment strategy, because we then increase the risk exposure after bad experience with the pension assets. However, this strategy massively increases the risk of encountering negative reserve ratios.

4. The side constraint $0 \leq \sigma_t \leq \sigma_{\max}$ allows us to keep the risk exposure within reasonable limits. In our setting $\sigma_t = 0$ implies a risk free investment. We should be aware that even a portfolio of AAA- government bonds is not risk free, since bond prices are driven by market interest rates. So in practice we must choose a σ_t not below some $\sigma_{\min} > 0$. We could skip the upper bound $\sigma_{\max}$ if we allowed for leverage instruments. However usually these instruments are prohibited for pension funds.
5. According to (*ALM* 1) and (*ALM* 2) the risk exposure and the profit participation are linear function of the reserve gap. Since $P(t)/V(t) \approx 1+\rho(t)$ for $0.8 \leq P(t)/V(t) \leq 1.2$, we can say that risk exposure and profit participation are approximately linearly dependent of the reserve gap. Since a low cover ratio $P(t)/V(t) << 1$ is generally regarded as more critical than a high cover ratio, the transition from $P(t)/V(t)$ to $\ln(P(t)/V(t))$ is at least plausible.
6. In (*ALM* 2) $\eta(t+1)$ has three components:
   - $\mu(\sigma_t)$ ensures a fair participation in the portfolio returns. All pension members directly share the expected asset returns.
   - $\delta(t)$ ensures that the capital returns from the collective reserve are evenly redistributed to the pension members.
   - The term $\theta\left(\rho(t)-\hat{\rho}\right)$ represents an *intergeneration risk transfer*. If the observed reserve ratio falls behind the target ratio, then all members have to put extra money aside to fill the gap. If there is a positive reserve gap then all members get an equal share. It is obvious that the generation of young employees would prefer a strong reserve because this allows a higher risk exposure and, in the long run, a higher return on investment. The pensioners would be rather reluctant to strengthen the collective reserve. In [Goecke 2013] this term (for $\theta > 0$) ensures the mean reverting property of the stochastic process $\rho(t)$. Economically, $\theta < 0$ makes no sense; it is also clear that with $\theta = 0$ we had no control over the reserve. The case $\theta > 1$ implies an overreaction – cf. Prop. 3 below.
7. As pointed out, the prospective declaration in (*ALM* 2) cannot ensure that $P(t) \geq V(t)$. In order to safeguard a minimum reserve ratio $\rho_{\min}$, we can define a *retrospective variant* of (*ALM* 2) by

$$\eta^{(retro)}(t+1) = \mu(\sigma_t) + \delta(t) + Min\left(\theta\left(\rho(t)-\hat{\rho}\right), \sigma_t X_{t+1} - Y_{t+1} + \rho(t) - \rho_{\min}\right).$$

Note that if $\sigma_t X_{t+1} - Y_{t+1} + \rho(t) - \rho_{\min} < \theta\left(\rho(t) - \hat{\rho}\right)$ then by Proposition 2 we have $\rho(t+1) = \rho_{\min}$.

8. In [Goecke 2013] a time continuous version of (*ALM* 1) and (*ALM* 2) is analysed for a pure *CDC*- saving fund. On the basis of a Black Scholes-type capital market model limit distributions for $\rho(t)$, $\eta(t)$ and $\sigma(t)$ can be derived. Furthermore, the question of optimality of the parameters $(\hat{\rho}, \hat{\sigma}, a, \theta, \sigma_{\max})$ is discussed.

The following proposition is a direct consequence of Proposition 2 and serves as motivation for (*ALM* 2) and (*ALM* 3):

Proposition 3

If $\eta(t+1) = \mu(\sigma_t) + \delta(t) + \theta\left(\rho(t) - \hat{\rho}\right)$ and $\varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$ then

$$\rho(t+1) - \rho(t) = \sigma_t X_{t+1} - Y_{t+1} - \theta\left(\rho(t) - \hat{\rho}\right) \qquad (Eq.\ 11)$$

$$\mathbf{E}\left(\rho(t+1) - \hat{\rho} \middle| \rho(t)\right) = (1-\theta)\left(\rho(t) - \hat{\rho}\right) + \sigma_t\, \mathbf{E}(X_{t+1}) - \mathbf{E}(Y_{t+1})\,. \qquad (Eq.\ 12)$$

♦

Remarks:

1. (*Eq*. 11) shows that for $0 < \theta < 1$ we have an exponential damping with respect to the reserve gap. $\theta = 1$ implies an instant refilling or depletion of the reserve gap. A low θ-value means that accrued contributions and pensions in payment are adjusted sluggishly which increases the probability that $\rho(t)$ falls below zero. In the extreme case $\theta = 0$ the total risk $\sigma_t X_{t+1} - Y_{t+1}$ is dumped on the reserve and we have a random walk reserve, i.e.

$$\rho(t+1) = \rho(0) + \sum_{s=0}^{t}\left(\sigma_s X_{s+1} - Y_{s+1}\right)$$

2. (*Eq*. 12) is directly linked to *Principle* 3 stipulating equity between generations. Let us first assume that $\mathbf{E}(Y_{t+1}) = 0$, i.e. that our assumptions with respect to future mortality are unbiased. We interpret the reserve gap $\rho(t) - \hat{\rho}$ as a burden (if negative) or as a legacy (if positive) for future generations. If $0 < \theta <$ 1 then the burden or bequest is systematically reduced over time. Thus, if our *CDC* model started with $\rho(0) = \hat{\rho}$, no generation (including unborn new entrants) is systematically favoured or disfavoured by the collective intergenera-

tional risk transfer. A disruption of the equilibrium $\rho(0) = \hat{\rho}$ resulting from exogenous capital market or mortality risks is shared between generations. One may argue that a generation entering the *CDC*-model at *t* with $\rho(t) - \hat{\rho} < 0$ is systematically put at a disadvantage. However, years before entering the *CDC*-system they have had an equal chance of entering when $\rho(t) - \hat{\rho} > 0$. If one were to start a greenfield *CDC* fund with no initial collective reserve, the first generations would clearly have a disadvantage.

3. From (*Eq.* 12) if follows that $\theta > 1$ would destabilise the system, because an underfunding would be followed by an expected overfunding and vice versa.
4. From the viewpoint of intergenerational equity we should calibrate $\hat{p}(t,x)$ such that $\mathbf{E}(Y_{t+1}) = 0$.
5. For $\mathbf{E}(X_{t+1}) = \mathbf{E}(Y_{t+1}) = 0$ we have $\mathbf{E}\big(\rho(t+1) - \hat{\rho} \big| \rho(t)\big) = (1-\theta)\big(\rho(t) - \hat{\rho}\big)$. To calibrate θ, we could require that the reserve gap $\rho(t) - \hat{\rho}$ should halve within *m* years. Then we have to set $\theta := 1 - (0.5)^{1/m}$.
6. We are a little bit negligent in writing $\mathbf{E}(Y_{t+1})$ - what we have in mind is the expectation based on the information up to time *t*. The same applies to the expression $\mathbf{E}\big(\ldots \big| \rho(t)\big)$ - using this notation we just want to stress that we observe $\rho(t)$ at time *t*.
7. In view of the definition of Y_{t+1} in Proposition 2 the condition $\mathbf{E}(Y_{t+1}) = 0$ is quite ambitious. Actually, if we have a stochastic model for $\tilde{p}(t,x)$ - even a very simple one - there is little hope to define $\hat{p}(t,x)$ such that $\mathbf{E}(Y_{t+1}) = 0$. But what we can do is, given $\hat{p}(t,x)$, estimate $\mathbf{E}(Y_{t+1})$ to get an idea how far away from 0 we are.
8. If $\theta = 1$ then $\rho(t+1) - \hat{\rho} = \sigma_t X_{t+1} - Y_{t+1}$ and $\eta(t+1) = \mu(\sigma_t) + \delta(t) + \sigma_t X_t - Y_t$ for all *t*. This means that there is a 1-year time lag between the risk occurrence and the time it would affect the individual accounts and the pensions.
9. Let us consider the following modification of our *CDC*-model: Suppose we stipulate that every contribution and every benefit paid out has to be "*reserve-neutral*" in the sense that any *in*- or *out*flow does not affect the reserve ratio $\rho(t)$. This means that if the contribution *c* is credited to the personal account an additional collective contribution of $(\exp(\rho(t)) - 1)\, c(t)$ has to be paid into the collective reserve. Conversely, if a pension $b(t)$ is paid out the pensioner

gets an additional bonus of (exp($\rho(t)$) - 1) $b(t)$ paid from the collective reserve. The same applies to death benefits.

If then, as in Proposition 2, we define $\varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$ for this *reserve-neutral CDC- variant* model we get

$$P(t+1) = \left(P(t) - \exp\left(\rho(t)\right) CF(t)\right) \exp\left(\tilde{\mu}(t+1)\right),$$

$$V(t+1) = \left(V(t) - CF(t)\right) \exp\left(\eta(t+1)\right) \text{ and}$$

$$\rho(t+1) - \rho(t) = \sigma_t X_{t+1} - Y_{t+1} + \mu(\sigma_t) - \eta(t+1). \qquad (Eq.\ 13)$$

(*Eq.* 13) is the same as (*Eq.* 9) of Proposition 2 except that the stock effect $\delta(t)$ has vanished. For $\eta(t+1) = \mu(\sigma_t) + \theta\left(\rho(t) - \hat{\rho}\right)$ we then get exactly the same equations (*Eq.* 11 and 12) as in Proposition 3.

3.3 Individual Saving and Dissaving

The main purpose of this paper is to compare collective defined contribution (*CDC*-) plans with individual defined contribution (*IDC*-) plans. In our understanding an individual arrangement is one where we do not actively utilise a collective reserve. In the following we want to show that *IDC* can be regarded as a special case of the *CDC*-framework introduced above.

In the *IDC*-case there is no reserve to puffer capital market or mortality risks. Thus $\rho(t) = \rho(0) = \hat{\rho} = 0$ and $\delta(t) = 0$. If we set $\varepsilon(t+1) = \eta(t+1) - \mu_a + \tilde{\pi}(t+1)$ then by (*Eq.* 9) of Proposition 1 we get $0 = \rho(t+1) - \rho(t) = \tilde{\mu}(t+1) - \eta(t+1)$, i.e. $\eta(t+1) = \tilde{\mu}(t+1)$. Thus, with respect to the cohort of active members, an *IDC*-system is just a *CDC*-system without reserves.

Remarks

1. If we require $\rho(t) = \rho(0) = \hat{\rho} = 0$ and set $\varepsilon(t+1) = \eta(t+1) - \mu_a + \hat{\pi}(t+1)$ as in Proposition 2 we get $\eta(t+1) = \tilde{\mu}(t+1) - Y_{t+1}$. In this case, the cohort of active members shares the mortality risk Y_{t+1} which only depends on the mortality of the retirees.
2. By setting $\varepsilon(t+1) = \tilde{\mu}(t+1) - \mu_a + \tilde{\pi}(t+1)$ the longevity risk is shared by the cohort of retirees collectively, irrespectively to what degree the particular age

cohort contributes to the longevity risk. If we wanted to eliminate the intergenerational risk sharing we would have to introduce cohort-specific pension adjustment rates by

$$\varepsilon(t+1,x+1) := \tilde{\mu}(t+1) - \mu_a + \ln\left(\frac{p_a(t,x)}{\tilde{p}(t,x)}\right). \qquad (Eq.\ 14)$$

Both versions of $\varepsilon(t+1)$ are variants of an *actuarial tontine*, where the idiosyncratic longevity risk is transferred to a cohort of retirees. In the first case we could speak of an *open tontine* allowing for new entrants.[9] One may convince oneself that if pensions were adjusted according (*Eq.* 14) then

$P(t,x) = L(t,x)\, r(t,x)\, \ddot{a}(t,x)$ for all $x > z$ and $t = t_0 + x - z$,

if at time t_0 for $x = z$ the initial cohort capital $P(t_0, z)$ satisfies

$$P(t_0,z) = L(t_0,z)\, r(t_0,z)\, \ddot{a}(t_0,z)\,.$$

3.4 Steady State Analysis

To measure the intergenerational redistribution in a *CDC*- pension system we need a reference point to compare. To this end we look at a *CDC*-system which is in a *stationary state*, when - loosely spoken - nothing changes. To make things precise we say that the pension fund is in a *steady state*, if the following conditions hold:

- we have a *stable population*, i.e. $L(t, x) = L(x)$ is independent of t and $Y_t \equiv 0$,
- we have a *stable return on assets*, i.e. $\tilde{\mu}(t) = \mu$ for all t and $X_t \equiv 0$,
- $\ddot{a}(t,x) = \ddot{a}(x)$ is independent of t with a constant actuarial interest rate μ_a and deterministic mortalities rates $\tilde{p}(t,x) = \hat{p}(t,x) = p_a(t,x) = p(x)$,
- $\rho(t) = \rho$, $\eta(t) = \eta$ and $\varepsilon(t) = \varepsilon$ and for all t.

If contributions, profit participation and pension adjustments are constant, then also $V^A(t)$, $V^R(t)$, $D(t)$, $R(t)$, $CF(t)$, $V(t)$, $P(t)$, $\lambda(t)$ and $\gamma(t)$ must be constant - so we can skip the time index "..(t)".

From $P = (P - CF)\exp(\mu)$ we get $\lambda = \dfrac{CF}{V} = \exp(\rho)\left(1 - \exp(-\mu)\right)$. We may assume $\lambda < 1$. So we can express the *stock effect* (cf. section 3.1) in terms of ρ and μ:

[9] c.f. [Milevsky, Salisbury 2015], [McKeever 2010]

$$\delta = \ln\left(\frac{1-\lambda\exp(-\rho)}{1-\lambda}\right) = -\ln\left(\exp(\rho)+\exp(\mu)-\exp(\rho+\mu)\right).$$

$\eta = \mu + \delta = -\ln\left(1+\exp(\rho-\mu)-\exp(\rho)\right)$ is thus the *steady state profit participation*. Note that $\lambda < 1 \Leftrightarrow \exp(\rho)+\exp(\mu)-\exp(\rho+\mu) > 0$.

Remark: If we set $i := \exp(\mu)-1$ and $RQ := (P-V)/V = \exp(\rho)-1$ we can write $\delta = -\ln(1-i\,RQ) \approx i\,RQ$. Thus δ is just the *extra profit participation* from the *collective reserve* (*P*-*V*).

Observe that η and ε are not uniquely determined by the steady state condition. For example, by setting $\eta = 0$ we would unilaterally favour the cohort of retirees since then all of the capital gains $\left(\exp(\mu)-1\right) P$ would be used to increase the pensions in payment. Actually, for a given η we can find a suitable ε such that we have a steady state in the above sense.

Among all (η, ε) -combinations only $\varepsilon = \eta - \mu_a$ ensures intergenerational equity in the sense that there is no permanent transfer from old to young or vice versa. However, if $\varepsilon = \eta - \mu_a$ then according to Proposition 2 we must have $\eta = \mu + \delta$.

Example

For $\rho(t) = \rho = 0.2$ and $\tilde{\mu}(t) = \mu = 0.025$ we can calculate the *steady state stock effect* by $\delta = -\ln\left(\exp(\rho)+\exp(\mu)-\exp(\rho+\mu)\right) = 0.0056206$. The *intergenerationally fair* (η, ε) – combination is given by $(\eta, \varepsilon) = (\mu + \delta, \mu + \delta - \mu_a)$.

However, if we allow for a systematic cross subsidising between the cohorts of savers and retirees there are further (η, ε) - combinations which are feasible in the sense that these combinations are steady state compatible. On the basis of the hybrid steady state population[10] and $\mu_a = 0.01$ we calculate the feasible (η, ε)- combinations – cf. Figure 4.

[10] hybrid population as in section 2.1.2.

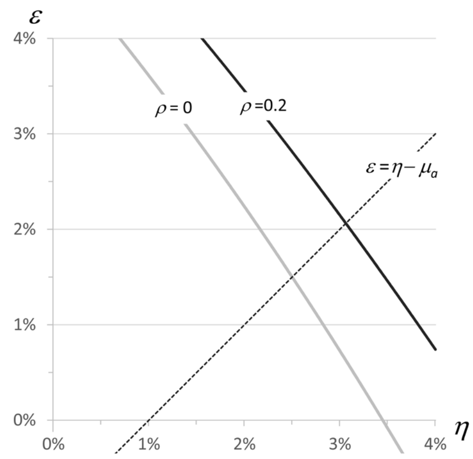


FIGURE 4: Steady state compatible (η, ε)- combinations for a hybrid steady state population for $\mu = 0.025$, $\mu_a = 0.01$ and $\rho = 0/\ 0.2$. The dotted line indicates intergenerationally fair (η, ε) - combinations.

If we regard ε as a function of η we notice a leverage effect between η and ε. At the reserve ratio of $\rho = 0.2$ the *intergenerationally fair* (η, ε)- combination would be $(\eta,\varepsilon) = (\mu+\delta, \mu+\delta-\mu_a) = (3.06\%, 2.06\%)$, i.e. young and old equally participate in the stock effect. But also $(\eta, \varepsilon) = (2.50\%, 2.81\%)$ is feasible in the sense that the reserve ratio is kept constant. However in this case only the old would benefit from the stock effect.

Remark

One can extend the above considerations to a *dynamic steady state*, where the contributions are inflation adjusted by a fixed rate w, i.e. $c(t+1) = c\exp(w)$. It is easy to verify that the *steady state stock effect* will then be

$$\delta_w = -\ln\left(\exp(\rho) + \exp(\mu - w) - \exp(\rho + \mu - w)\right).$$

We summarise the results in the following

Proposition 4

If the pension system is in a dynamic steady state with inflation adjusted contributions at the rate of w, then the steady state *intergenerationally fair profit participation η and pension adjustment ε* are given by

$$\eta = \mu + \delta_w \text{ and } \varepsilon = \eta - \mu_a, \qquad (Eq.\ 15)$$

where $\delta_w := -\ln\left(\exp(\rho) + \exp(\mu - w) - \exp(\rho + \mu - w)\right)$.

♦

The comparison of *CDC*- and *IDC*-pension arrangements must take into account that members of a *CDC*- plan receive additional returns from the collective reserve, namely the stock effect δ which is positive provided ρ>0.

To measure the effect we calculate $TV_{IDC}(x)$ and $TV_{CDC}(x)$, the *time value* of future (death and pension) benefits *minus* future contributions for members the *x*-cohort in the *IDC* and *CDC*-case. Then $TV_{CDC}(x)$ - $TV_{IDC}(x)$ measures the effect of the extra return of $\delta = -\ln(e^{\rho} + e^{\mu} - e^{\rho+\mu})$ from the collective reserve.

Figure 5 illustrates this for $\mu = 0.025$, $\mu_a = 0.01$, ρ=0.15 and $\delta \approx 0.0041$. For example, an employee, aged $x_0 = 20$, entering the *CDC* plan will receive more benefits with a time value of about 4.39 contribution rates. This is exactly the time value of the additional return of δ.

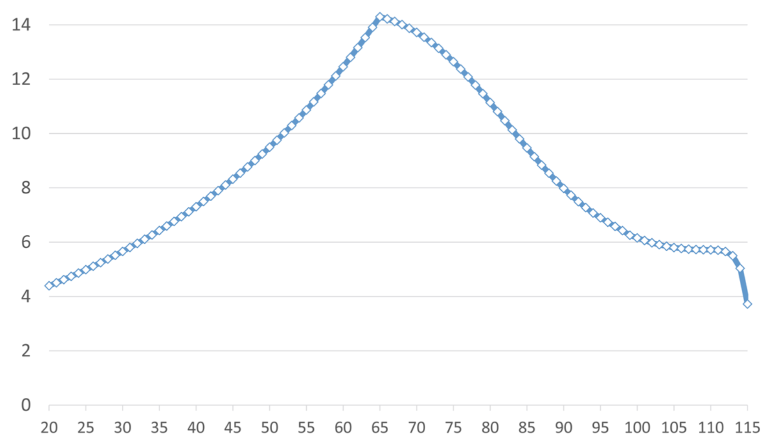


FIGURE 5: Value added per head ($TV_{CDC}(x)$ - $TV_{IDC}(x)$) in a *CDC*-pension scheme in steady state with a constant capital market return of μ=0.025, a reserve ratio of ρ= 0.15 and a contribution rate of c = 1.

4 Resilience Test

A pension system is *resilient*, if it is able to *absorb external* (*single*) *shocks* and *adapt to a* (*lasting*) *shift* of the economic environment. Our resilience test works as follows: We start from a steady state situation and then apply a shock or a shift scenario and analyze the effects on the pension benefits. In a *DC* pension system all disturbances from outside must be compensated by adjusting the pension benefits. In the *IDC*-version we do not allow for risk transfer between generations, so the *IDC*-version will serve as a reference model to evaluate different *ALM*-strategies of the *CDC*-model.

4.1 Steady State Original Position

We assume that our pension system starts from a steady state position[11] with following parameters:

- annual contributions $c = 1$ payable from age $x_0 = 20$ until age $z-1 = 64$
- constant capital market returns $\tilde{\mu}(t) = \mu = 0.025$
- a stationary population with time independent survival probabilities
  $\tilde{p}(t,x) = \hat{p}(t,x) = p_a(t,x) = p(x) = \dfrac{L(x+1)}{L(x)}$,
  where $p(x)$ are the male/ female hybrid survival probabilities as described in section 2.1.2. We then have $\tilde{\pi}(t) = \hat{\pi}(t) = 0$ for all t.
- constant number of new entrants $L(x_0) = L(20) = 1000$
- fixed actuarial interest rate of $\mu_a = 0.01$ and annuity factors for $x \geq z$
  $\ddot{a}(x) := \sum_{k=0}^{\omega-x} \dfrac{L(x+k)}{L(x)} \exp(-k\mu_a)$; $\ddot{a}(z) = 17.9249$
- pensions in payment are adjusted at the rate of $\varepsilon = \mu - \mu_a = 0.015$.

For the *IDC*-model the accrued pension capital at the age of x: $x_0 \leq x \leq z = 65$ is then

$$v(x) := \frac{\exp\big((x-x_0)\mu\big)-1}{1-\exp(-\mu)},$$

and the initial steady state pension is $b(z) = \dfrac{v(z)}{\ddot{a}(z)} = \dfrac{84.2531}{17.9249} = 4.7003$.

To make *IDC*- and *CDC*-plans comparable we assume that in the *CDC*-case we start with a *steady state* resevere ratio $\hat{\rho} = 0$. Then in the steady state situation pensions and death benefits are identical for *IDC*- and *CDC*-plans. We define

$$P_x := \begin{cases} L(x)\, v(x) & \text{for } x_0 \leq x < z \\ L(x)\, r(x)\, \ddot{a}(x) & \text{for } z \leq x \leq \omega \end{cases} \quad \text{and}$$

$$D_x := \begin{cases} \big(L(x-1) - L(x)\big)\, v(x) & \text{for } x_0 < x \leq z \\ 0 & \text{for } z < x \leq \omega \end{cases}$$

to be the *steady state* pension capital for the x-cohort and the *death benefit* for those who die between age x-1 and x. We denote by P the total *steady state* pension capital and V the total *steady state* pension liabilities. Under the assumption that $\hat{\rho} = 0$ we

[11] cf. section 3.4

calculate $P = V = \sum_{x=x_0+1}^{\omega} P_x + \sum_{x=x_0+1}^{z} D_x = 2\,530\,615 + 5\,851 = 2\,536\,466$. Note that by our convention P and V comprise the death benefits for the decedents of the foregoing year.

4.2 Capital Market Shock

A capital market shock is associated with an equity market crash or boom. Starting from a steady state situation with a constant investment return of $\mu = 0.025$ we assume that at time T_0 (i.e. at the end of $[T_0\text{-}1, T_0]$) we observe a return of $\mu + \mu_\Delta$ with $\mu_\Delta = +0.2$ ("*up-scenario*") or $\mu_\Delta = -0.2$ ("*down-scenario*"). In the following our wording always refers to the down-scenario, however the derived formulars apply to either cases.

4.2.1 Capital Market Shock Effect on *IDC*-Plans

Instantly upon observation of the capital market shock the individual pension accounts and the annuities are adjusted. Consider the (T_0, x)-cohort, i.e. the generation of persons aged x at time T_0. For $x_0 \leq x \leq z$ the personal pension capital at T_0 will be $v'(x) := \exp(\mu_\Delta)\, v(x)$ instead of $v(x)$. After T_0 the annual return is again μ, therefore the resulting annuity (z-x years later) will be cut by factor

$$1 - \frac{v(x)}{v(z)} \exp\big((z-x)\mu\big)\big(1 - \exp(\mu_\Delta)\big)$$ - cf. FIGURE 6.

For $x > z$ the due pension will be $b'(x) = \exp(\mu_\Delta)\, b(x)$ instead of $b(x)$. From time T_0+1 onwards pensions will again be adjusted by $\varepsilon = \mu - \mu_a$.

The capital market shock has the strongest effect on persons aged z or older. Their benefits would be cut by about 18% compared to the pre-shock level.

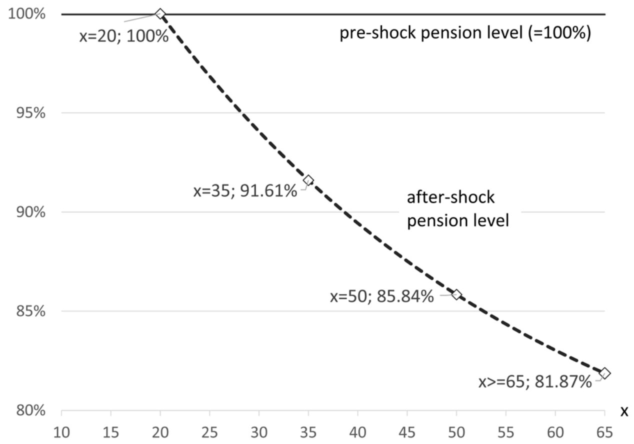


FIGURE 6: Down-shock scenario (μ_Δ= -0.2) for *IDC*-plans: Effect on the expected pension level, depending on the age x at time T_0.

4.2.2 Capital Market Shock Effect on *CDC*-Plans

In the steady state scenario we have a constant expected return $\mu(\hat{\sigma}) = \mu$ and no external disturbances, i.e. $X_t = Y_t = 0$ and $\hat{\rho} = 0$. Then $\rho(t) = \delta(t) = 0$ and $\eta(t+1) = \mu$ for all $t < T_0$. In the steady state original position for all ages x the cohort pension capital P_x and the individual pension capital $P_x / L(x)$ coincide with the time value of future benefits minus contributions.

Now consider a single interest rate shock at time T_0 (i.e. $\hat{\sigma} X_{T_0} = \mu_\Delta$). Applying rule (*ALM* 2) we have $\eta(t+1) = \mu + \delta(t) + \theta\,\rho(t)$ for all t and $\eta(T_0) = \mu$. Therefore at time T_0 neither the individual accounts $v(x)$ nor the due pensions $r(x)$ are affected. However, the total pension capital at time T_0 falls to $\exp(\mu_\Delta)\,P$ and $\rho(T_0) = \mu_\Delta$ and $\delta(T_0) = \ln(e^{\mu} + e^{-\mu_\Delta} - e^{\mu-\mu_\Delta})$. By Proposition 3 we know that $\rho(T_0 + k) = (1-\theta)^k \mu_\Delta$, so for $0 < \theta < 2$ $\rho(t)$ converges to $\hat{\rho} = 0$. In the special case θ=1 we get $\rho(T_0 + 1) = 0$ and $\eta(T_0+1) = \mu + \mu_\Delta + \delta(T_0) = \mu + \ln\left(1 + e^{\mu}(e^{\mu_\Delta} - 1)\right)$. Note that $\varepsilon(t) = \eta(t) - \mu_a$.

Due to the non-trivial stock effect $\delta(t)$ there is no simple formula for $\eta(t)$. Therefore we just illustrate $\eta(t)$ for the down-scenario (μ_Δ = -0.2) for different levels of θ - cf. FIGURE 7.

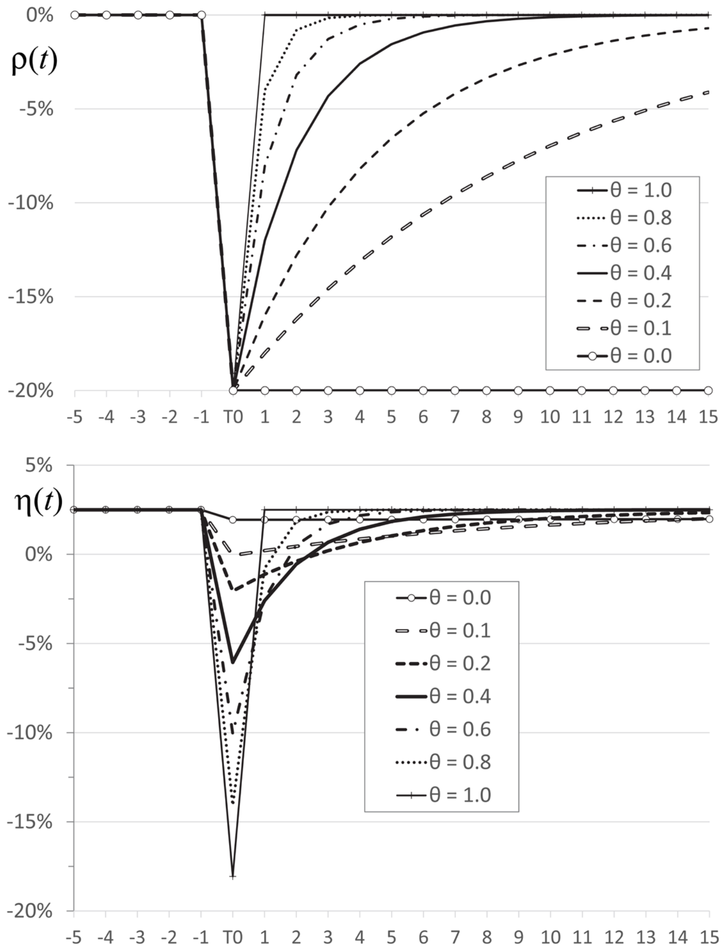


FIGURE 7: Effect of a capital market down-shock (μ_Δ= - 0.2) at time T_0 on the reserve ratio $\rho(t)$ (top chart) and the profit participation $\eta\,(t)$ (bottom chart) for alternative levels of θ.

For θ= 0 the reserve ratio will remain at the level of ρ= -0.2 forever. This means that all future generations have to pay the bill: Due to the negative stock effect we have $\eta = \mu - \delta = \mu - \ln(e^{\mu} + e^{-\mu_\Delta} - e^{\mu-\mu_\Delta})$ = 2.04% instead of μ= 2.5%.

For 0 < θ< 2 the reserve ratio returns to the steady state level. If we wanted to avoid a negative profit participation, we would have to choose $\theta \le 0.1$ with the consequence that it takes about 7 years to halve the after-shock reserve gap of 20%.

Our goal is to measure the intergenerational effects of a *CDC*-plan compared to an *IDC*-plan in a shock scenario. To this end for each (T_0, x)-cohort we calculate $TV_{CDC}(T_0, x)$, the time value of future benefits minus future contributions. Note that for *IDC*-plans the time value equals the cohort's pension capital i.e.

$$TV_{IDC}(T_0,x) = \exp(\mu_\Delta)\,P_x = \exp(\mu_\Delta)\begin{cases} L(x)\,v(x) & \text{for } x < z \\ L(x)\,r(x)\,\ddot{a}(x) & \text{for } x \ge z \end{cases}.$$

We take $\Delta TV(x) := TV_{CDC}(T_0, x) - TV_{IDC}(T_0, x)$ as a measure of the intergenerational asset transfer for the (T_0, x)-cohort, and $\Delta TV(x)/L(x)$ as the individual effect.

Fig. 8 illustrates the intergenerational redistribution in the down-shock scenario. Let us consider the *CDC*-plan with $\theta = 0.2$. Then the steady state pension capital is maximal for the (T_0, z)-cohort – we get $P_z = 75798$. In the *IDC*-case the pension capital falls to $\exp(\mu_\Delta)\, P_z = 62058$. In the *CDC*-case the pension capital of the (T_0, z)-cohort remains unchanged after the shock, but the *time value of future pensions* reduces to 64206. This means that the *CDC*-plan causes an intergenerational redistribution of $\Delta TV(z) = 64206 - 62059 = 2147$ in favour of the (T_0, z)-cohort.

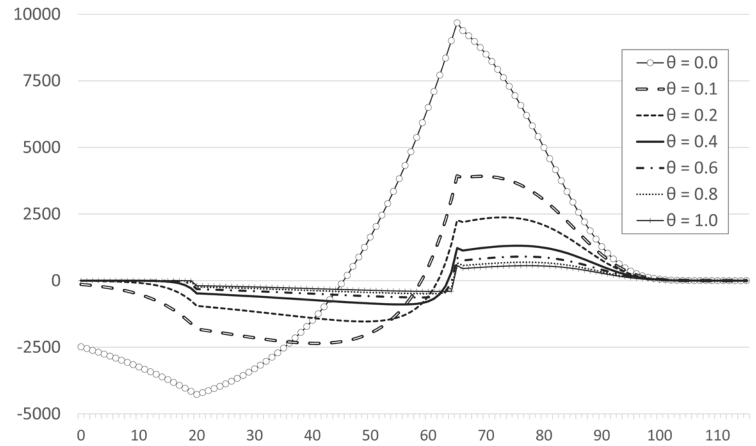


FIGURE 8: $\Delta TV(x)$ for age cohorts $0 \le x \le 115$ for different levels of θ for a down-shock scenario ($\mu_\Delta = -0.2$).

There is an additional (small) redistribution effect in favour of the death benefits payable at time T_0 after the shock. While in the *CDC*-case the total death benefit is not affected at T_0, in the *IDC*-case the death benefit is reduced by factor $\exp(\mu_\Delta)$. It is clear that the total effect over all generations (including future generations of new entrants) must be zero.

If we look at the effects per capita we see that the positive or negative redistribution effects amounts to a multiple of the regular contribution (which is 1 in our calculations) – cf. Fig. 9. For example, in the case $\theta = 0.2$ and $\mu_\Delta = -0.2$ each single member of the (T_0, z)-cohort receives a transfer of $\Delta TV(z)/\, L(65) = 2.39$. In the extreme case $\theta = 0$ the reserve will remain at the after shock level of -0.2 for ever so that all future generations will be charged. This extreme case again shows that a *CDC*-system could be misused by the generation 50+, who may have strong influence on *ALM*-decisions and who are prone to postpone unpleasant decisions.

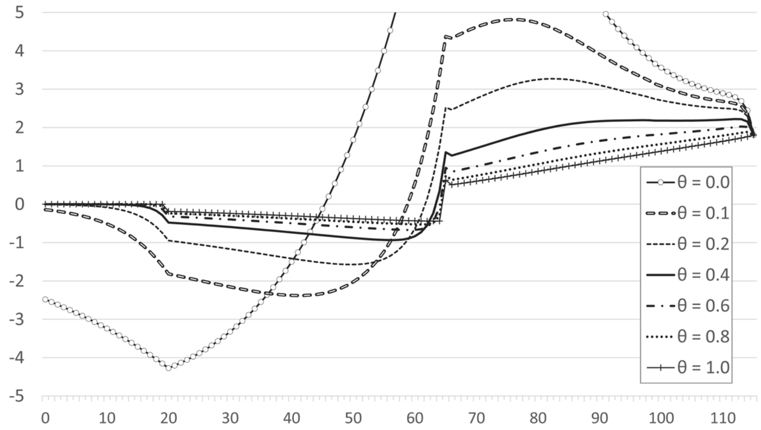


FIGURE 9: Redistribution effect per head ($\Delta TV(x)/L(x)$) for different levels of θ for a down-shock scenario (μ_Δ= - 0.2).

The following table shows the intergenerational redistribution for a single capital market down and up shock in relation to the pre-shock total pension capital. Note that for θ = 0 the reserve ratio will remain at the level at time directly after the shock. i.e $\rho(t) = \mu_\Delta$= - 0.2 for all $t \geq T_0$.

	Capital Market Down Shock (μ_Δ = -0.2)		Capital Market Up Shock (μ_Δ = +0.2)	
θ	Redistribution to the older generation	Beneficiary age cohorts	Redistribution to the younger generation	Beneficiary age cohorts
0	9.11%	≥46	11.86%	≤58
0.1	3.52%	≥59	4.67%	≤61
0.2	2.17%	≥62	2.86%	≤63
0.3	1.56%	≥64	2.06%	≤63
0.4	1.22%	≥64	1.62%	≤64
0.5	1.00%	≥65	1.33%	≤64
0.6	0.86%	≥65	1.13%	≤64
0.7	0.75%	≥65	0.99%	≤64
0.8	0.66%	≥65	0.87%	≤64
0.9	0.60%	≥65	0.79%	≤64
1.0	0.54%	≥65	0.71%	≤64

TABLE 1: Overall redistribution effect in % of total pre-shock pension capital in favour of the older generation (down-scenario) or younger generation (up-scenario) for different levels of θ.

4.3 Capital Market Shift

We now want to analyse the effect of an interest rate shift on a pure bond portfolio. We analyse a sudden but permanent *interest rate shift* from μ to $\mu' := \mu + \mu_{shift}$ from some time T_0 onwards. We associate this stylised situation with a non-expected decision of the central bank to adjust interest rates.[12] This interest rate shift has then two effects: Firstly, new fixed income investments bear an interest rate of μ' instead of μ, and secondly, there is a price effect on existing bond investments. If the interest rate shift occurs at the beginning of the time period $[T_0, T_0 + 1]$ then the market value of a bond portfolio with an average duration of D will chance by factor $f \approx \exp(-D\ \mu_{shift})$ instantly after the shift. From time T_0 onwards all assets including new investments will have a return of μ'. If $D > 0$ then the interest rate up/down shift results in a single down/up shock followed by a permanent up/down shift.

We want to check how *IDC*- and *CDC*-plans adapt to this permanent change of the capital market. In our wording we concentrate on a *down shift scenario* ($\mu_{shift} < 0$). It is quiet obvious that, cum grano salis, in an up-shift scenario the same happens in the other direction. To keep the variants of our calculations in limits we do not adjust the actuarial interest rate μ_a , so that the annuitisation factors $\ddot{a}(x)$ remain unchanged.

If not stated otherwise our numerical examples are calculated on the basis of

$$\mu = 2.5\%,\ \mu_{shift} = -1.0\%,\ \mu' = \mu + \mu_{shift} = 1.5\%,\ \mu_a = 1\%,\ \varepsilon' = \mu' - \mu_a = 0.5\%.$$

Furthermore, we consider the price effect due to the interest rate shift by assuming that the time value of assets change by factor $f_D := \exp(-D \cdot \mu_{shift})$ for $D = 0$, $D = 5$ and $D = 10$.

4.3.1 Capital Market Shift Effect on IDC-Plans

From time T_0 onwards the individual pension capital bears interest at the lower rate $\mu' = 1.5\%$, pensions in payment are adjusted by $\varepsilon' = 0.5\%$.

We illustrate the effect for the group of active members – cf. FIGURE 10 below. For example a person aged $x = 20$ or younger at time T_0 will be affected most, because they experience the lower interest rates for the whole accumulation phase. Their pension capital at age $z = 65$ will be 64.75 instead of 84.25, that is about 77% of the pre-shift level. This is independent of the duration of the underlying assets. For older

[12] Actually central banks can only determine the short term interest rates, long term interest rates can only be influenced indirectly.

members the positive duration effect for $D > 0$ can overcompensate the reduced future returns. However, after retirement the pensions are only adjusted by $\varepsilon' = 0.5\%$ instead of $\varepsilon = 1.5\%$.

Pension in payment will experience a single increase by factor f_D followed by reduced pension adjustments. For older pensioners the duration effect at time T_0 might overcompensate the reduced adjustment rate.

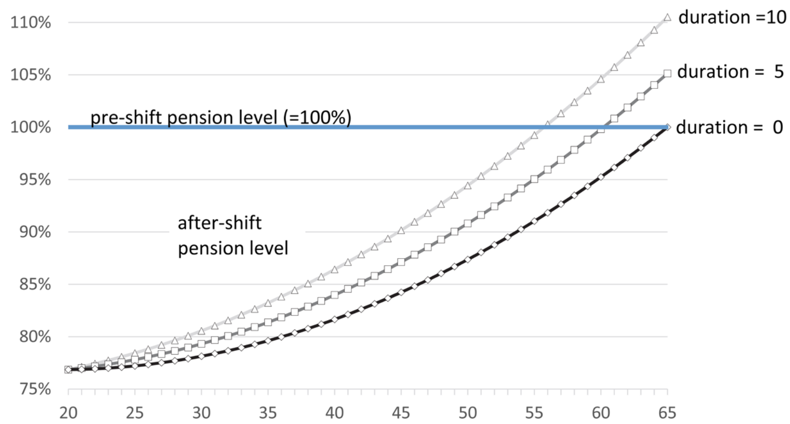


FIGURE 10: Down-shift scenario (μ_{shift} = -1%) for *IDC*-plans: Effect on the expected pension level at age z, depending on the age x at time T_0 and the duration.

Let $TV_{IDC}(x, T_0)$ denote the *time value* of future benefits (pension and death benefit) minus contributions at time T_0 immediately *after* the shift. If we want to calculate $TV_{IDC}(x, T_0)$ market consistently it must be calculated on the basis of the shifted discount rate $\mu + \mu_{\text{shift}}$. Then clearly $TV_{IDC}(T_0, x) = f_D P_x$, where P_x denotes the pre-shift pension-capital for the x-cohort.

4.3.2 Capital Market Shift Effect on CDC-Plans

We assume that the pension management instantly recognises the interest rate shift as permanent. According to (*ALM* 2) for $t \geq T_0$

$$\eta(t+1) = \mu' + \delta(t) + \theta\big(\rho(t) - \hat{\rho}\big).$$

Due to the interest rate shift, at time T_0 the assets have to be revalued. As above, we assume that $P' := f_D P$ is the value of assets immediately after revaluation. Accordingly, after revaluation we have

$$\rho(T_0) = \ln\left(\frac{P'}{V}\right) = -D\,\mu_{shift} \quad \text{and}$$

$$\delta(T_0) = \ln\left(\frac{P' - CF}{V - CF}\right) - \ln\left(\frac{P'}{V}\right) = \ln\left(1 - \exp(\mu)(1 - f_D)\right) + D\mu_{shift} .$$

Here we used the fact that in the steady state situation $CF = (1\text{-}\exp(-\mu))\, P$. Following (*ALM* 2) we get

$$\eta(T_0 + 1) \; = \mu' + \delta(T_0) + \theta\left(\rho(T_0) - \hat{\rho}\right) = \mu' + \ln\left(1 - \exp(\mu)(1 - f_D)\right) + (1 - \theta)\, D\mu_{shift}$$

and

$$\begin{aligned}\rho(T_0 + 1) - \rho(T_0) &= \ln\left(\frac{(P' - CF)\exp(\mu')}{(V - CF)\exp(\eta(T_0 + 1))}\right) - \rho(T_0) \\ &= \mu' - \eta(T_0 + 1) + \delta(T_0) = -\theta\, \rho(T_0) \;\; .\end{aligned}$$

We could have derived this directly from (*Eq*. 11) of Prop. 3. More generally we get

$$\rho(T_0 + k) = -(1 - \theta)^k\, D\, \mu_{shift} .$$

For $D = 0$ the reserve ratio is not affected at all. Due to the non-trivial stock effect for $D > 0$ there is no simple formula for $\eta(t)$ for $t > T_0+1$. So we just present numerical results – cf. FIGURE 11. For $D > 0$ we observe an increase of the reserve ratio at time T_0. After T_0 the reserve is drawn down depending on the speed parameter θ.

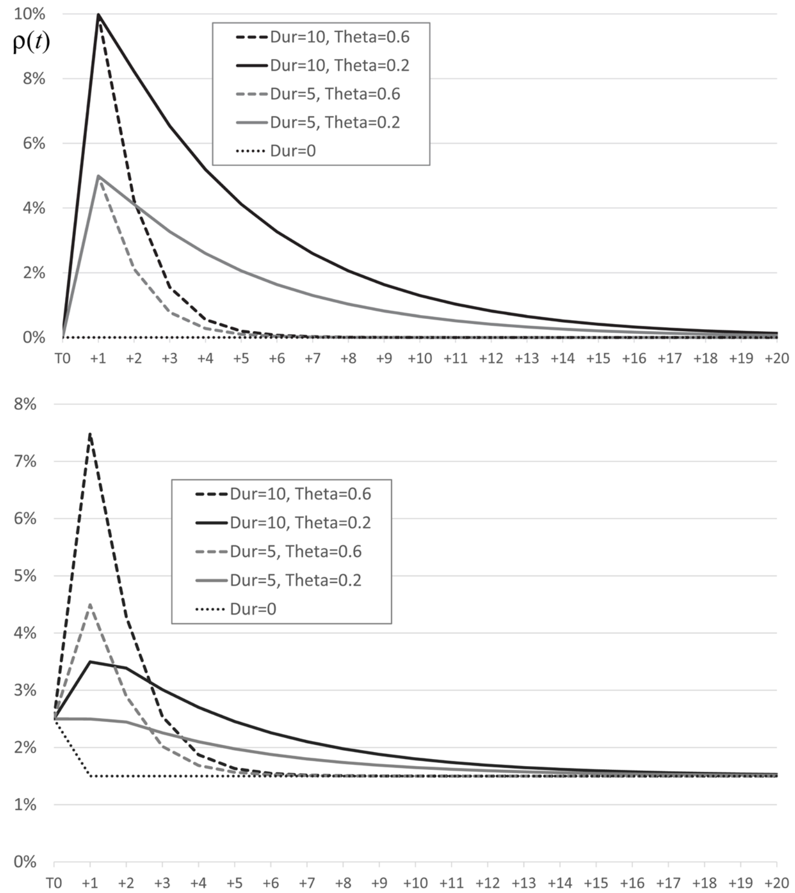


FIGURE 11: Down-shift scenario (μ_{shift}= -0.01): Effect on the reserve ratio $\rho(t)$ (TOP CHART) and on the profit participation $\eta(t)$ (BOTTOM CHART) for θ = 0.2/ 0.6 and D = 0/ 5/ 10.

The following charts illustrate the intergenerational redistribution effect $\Delta TV(x) := TV_{CDC}(T_0, x) - TV_{IDC}(T_0, x)$. It is clear that $\Delta TV(x) = 0$ if $D = 0$. For $D > 0$ in the *CDC*-system the collective reserve built at time T_0 is gradually drawn down, so that younger age cohorts profit from this. The intergenerational asset transfer is larger for higher durations and lower adjustment speed parameters θ - cf. FIGURE 12. Clearly the overall effect sums up to zero, i.e. $\sum_{x \le 115} \Delta TV(x) = 0$.

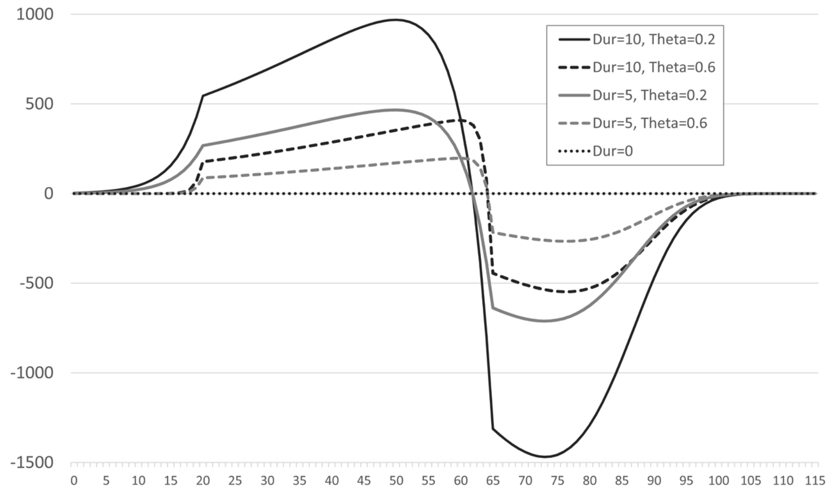


FIGURE 12: Intergenerational redistribution $\Delta TV(T_0, x)$ after a capital market down-shift (μ_{shift} = - 0.01) in a *CDC*-pension system for instant recognition for age cohorts $0 \leq x \leq 115$ and for different θ and D-values.

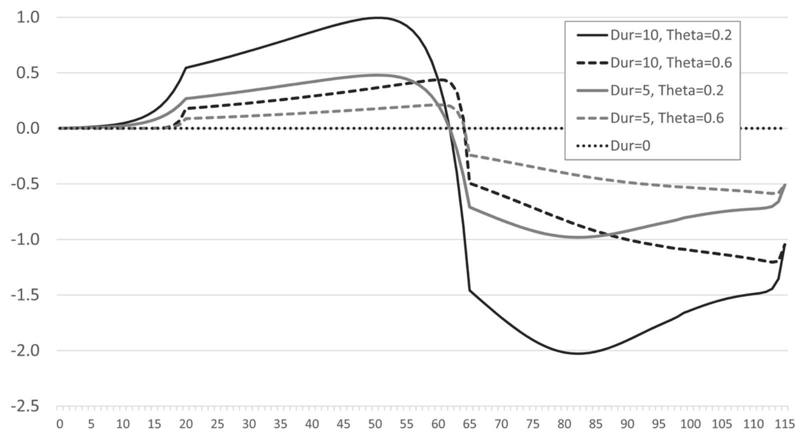


FIGURE 13: Intergenerational redistribution per head $\Delta TV(T_0, x)/ l_x$ after a capital market down-shift (μ_{shift} = - 0.01) in a *CDC*-pension system for instant recognitionfor age cohorts $0 \leq x \leq 115$ and for different θ / D-values.

It is worth noting that an interest rate down shift leads to an intergenerational transfer from old to young despite the fact that the young generations will receive lower pensions in future compared to those who already have retired. At first glance this seems to be paradoxical. However, one should note that we adjusted the discount rate in line with the interest rate μ. Due to the interest rate down shift the coupon rates of existing bonds constitute a reserve, built up by the older generation and (partially) handed down to the younger generation.

4.4 Mortality Shift

In practice, changes of mortality rates evolve gradually and it usually takes some years to recognise these changes as being systematic rather than just deviations from the mean. But eventually the actuary in charge has to adjust the mortality tables, that is from one day to the other they have to calculate on the basis of new "shifted" mortality tables. This is the situation we want to analyse with respect to a *CDC*-plan.

To make things precise, let $p(x)$ be the *steady state* survival probabilities and for a real number Δ define *shifted survival probabilities* for x by

$$p_{\Delta}(x) := \left(1 + \exp(-\Delta)\left(p(x)^{-1} - 1\right)\right)^{-1}$$

with the implicit convention, that $p_{\Delta}(x) = p(x)$ if $p(x) = 1$ or $p(x) = 0$. Note that we assumed that $p(x) = 1$ for $x < x_0$ and $p(\omega) = 0$.

One may verify that for $0 < p(x) < 1$ $\operatorname{logit}\left(1 - p_{\Delta}(x)\right) = \operatorname{logit}\left(1 - p(x)\right) - \Delta$. This notion of a mortality shift is motivated by the *CBD*- model[13] which assumes that $\operatorname{logit}\left(1 - p(x)\right) = \ln\left(\frac{1-p(x)}{p(x)}\right)$ can be approximated by a linear function - at least for ages $x \geq 60$.[14] By $\ddot{a}_{\Delta}(x)$ we denote the annuity factors based on $p_{\Delta}(.)$.

We now define $\tilde{p}(t,x) = p(x)$ for $t < T_0$-1 and $\tilde{p}(t,x) = p_{\Delta}(x)$ for $t \geq T_0$-1. Note that according to our convention $\tilde{p}(T_0 - 1, x - 1)$ refers to the survival probabilities of the (T_0-1, x-1)-cohort, observable at T_0.

The following *TABLES* 2 and 3 show the effect of a Δ = +/- 0.5 mortality shift on the further life expectancy and annuity factors for $x = 65$. *Cum grano salis*, a Δ = +0.5 shift increases the life expectancy of men to that of women. Just for comparison, a 25% reduction of mortality rates (as the Solvency 2 standard formula requires [15]) increases life expectancy of about 2.4 years for men and 2.3 years for women. So a shift of Δ = +/-0.5 in our setting stands for a rather massive mortality shift.

[13] Named after the authors Cairns, Blake, Dowd of [Cairns et al.2006]

[14] Cf. [Cairns et al.2006] p. 692

[15] Cf. [EIOPA 2014] p. 33

	Further life expectancy at age 65								
	Male members				Female members				hybrid
	period	birth cohort			period	birth cohort			period
	2018	1953	1973	1993	2018	1953	1973	1993	2018
Δ = 0	17.82	19.40	22.02	24.45	21.50	23.45	25.94	28.23	19.49
Δ = 0.5	21.47	23.40	26.00	28.39	24.92	27.21	29.67	31.92	23.07
Δ = -0.5	14.45	15.70	18.32	20.80	18.27	19.92	22.46	24.81	16.14

TABLE 2: Effect of a mortality shift on the further life expectancy for males and females for the year 2018 (period table) and for birth years 1953, 1973, 1993 (cohort table). The last column refers to the steady state hybrid population.

	Annuitising factor at age 65 for μ_a = 0.01								
	Male members				Female members				hybrid
	period	birth cohort			period	birth cohort			period
	2018	1953	1973	1993	2018	1953	1973	1993	2018
Δ = 0	16.53	17.79	19.93	21.86	19.58	21.09	23.05	24.80	17.92
Δ = 0.5	19.49	20.97	23.01	24.84	22.27	23.98	25.84	27.51	20.79
Δ = -0.5	13.71	14.74	16.94	18.97	16.96	18.28	20.34	22.20	15.16

TABLE 3: Effect of a mortality shift on the annuitising factors for males and females for the year 2018 (period table) and for birth years 1953, 1973, 1993 (cohort table). The last column refers to the steady state hybrid population.

We want to distinguish two strategies depending on whether the pension management considers the observed mortality shift as exceptional or as permanent:

Strategy 1 (delayed recognition): The observed mortality shift is not recognised as permanent but regarded as a statistical outlier. Consequently, the actuarial assumptions remain unchanged, i.e. $p_a(t,x) = \hat{p}(t,x) = p(x)$ for all *t*.

Strategy 2 (instant recognition): The observed mortality shift is regarded as permanent and the actuarial assumptions are adjusted instantly, i.e.

$$p_a(t,x) = \hat{p}(t,x) = \begin{cases} p(x) & \text{for } t < T_0 \\ p_\Delta(x) & \text{for } t \geq T_0 \end{cases} .$$

Accordingly, from time T_0 onwards the annuity factors $\ddot{a}(x)$ are replaced by $\ddot{a}_\Delta(x)$. In particular, new pensions are calculated on the basis of $\ddot{a}_\Delta(z)$. Despite the wording "*instant recognition*" there is a time lag of one year, i.e. at time T_0 we *expected* $L(T_0\text{-}1, x\text{-}1)\, p(x)$ survivors of the $(T_0\text{-}1, x\text{-}1)$-cohort, but we *observe* $L(T_0\text{-}1,$

x-1) $p_\Delta(x)$ survivors. From time $T_0 + 1$ onwards we observe exactly what we expect.

4.4.1 Mortality Shift Effect on IDC-Plans

By construction, at any time t and for any (t, x)-cohort the *time value of future benefits minus contributions* - denoted by $TV_{IDC}(t, x)$ - is exactly the pension capital for the particular age cohort. Thus there is no intergenerational redistribution. However, there is an *intra*-generational redistribution depending what strategy we pursue.

Strategy 1 (delayed recognition)

If we do not realise that the mortality shift is permanent, then year by year we have to adjust the pensions in payment to compensate the difference between expected and observed number of survivors. Then for $t \geq T_0$ and $x > z$ we get

$$\varepsilon(t,x) = \ln\left(\frac{b(t,x)}{b(t-1,x-1)}\right) = \varepsilon + \ln\left(\frac{p(x-1)}{p_\Delta(x-1)}\right) = \varepsilon - \Delta + \ln\left(1 + (e^\Delta - 1)\, p(x-1)\right), \tag{Eq. 16}$$

where $\varepsilon = \mu - \mu_a$ is the steady state adjustment rate. Note that the initial pension for the (T_0, z)-cohort is not affected by the mortality shift.

Strategy 2 (instant recognition)

In this case from time T_0 onwards pensions in payment and the new pensions are calculated on the basis of the adjusted annuity factors $\ddot{a}_\Delta(x)$. So at time T_0 all pensions in payment experience a significant adjustment: downward- (for $\Delta > 0$) or upward- (for $\Delta < 0$). For $x = z$ the pension capital is annuitised on the basis of $\ddot{a}_\Delta(z)$, which means that compared to the pre-shift situation the z-cohort faces a one-time cut/increase by factor $\ddot{a}(z)/\ddot{a}_\Delta(z)$. For $x > z$ the pensions experience a one-time adjustment of

$$\varepsilon(T_0,x) = \ln\left(\frac{r(T_0,x)}{r(T_0-1,x-1)}\right) = \varepsilon + \ln\left(\frac{p(x-1)}{p_\Delta(x-1)}\,\frac{\ddot{a}(x)}{\ddot{a}_\Delta(x)}\right). \tag{Eq. 17}$$

Since from T_0+1 onwards we observe what we expect, the pensions are adjusted by the pre-shift rate of $\varepsilon = \mu - \mu_a$.

Comparison Strategy 1/ Strategy 2

Let us compare the two strategies for the *IDC*-case for Δ = +0.5. In Figure 14 the line denoted by "*steady state*" marks the growth of the future pensions starting from the 100% level in T_0 if no mortality shifted occurred.

For *Strategy* 1 year by year the pensions have to be adjusted by $\varepsilon(t, x)$ - cf. (*Eq*. 16). $\varepsilon(t, x)$ decreases in x, so that the pension adjustments become more and more massive for the very old. Note that the initial pension for $x = z = 65$ starts at the 100% level because the accrued pension capital is not affected by the mortality shift.

For *Strategy* 2 there is a massive one-off adjustment at time T_0, but thereafter the benefits are again adjusted by the pre-shift rate $\varepsilon = \mu - \mu_a$.

Comparing both strategies we recognise that the instant recognition is much more attractive for those retirees who live longer. Consider e.g. the cohort of 80-year-old retirees at time T_0. For them the pensions according to Strategy 1 will be higher than the Strategy 2 pensions for the next 7 years. After time T_0+7 for Strategy 1 the pensions are cut progressively such that at time T_0+15 the pension is less than 60% of the pre-shift level. If *Strategy 2* is applied instead, the initial cut at T_0 by 20.6% is overcompensated from time T_0+15 onwards. It is obvious that *Strategy* 1 entails a substantial money transfer from those who live long to those who die early.

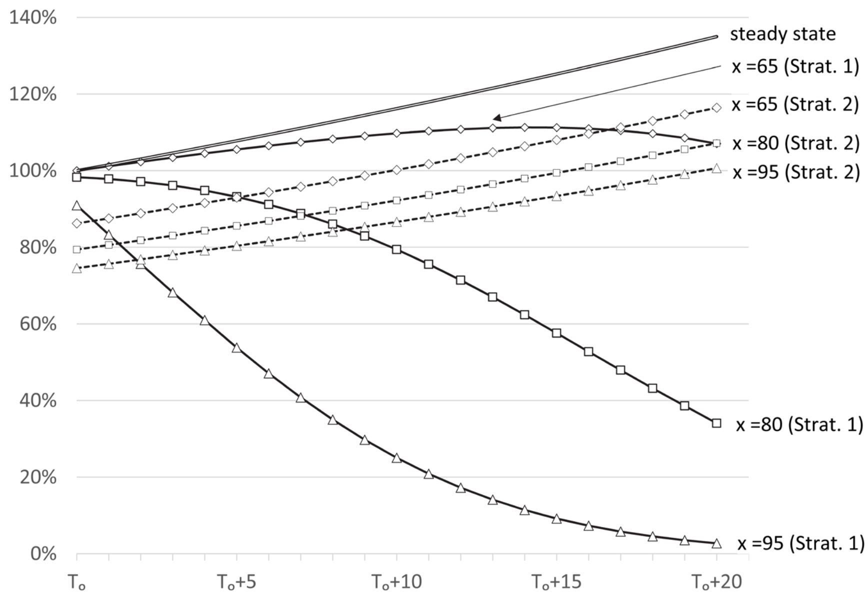


FIGURE 14: Effect on *IDC*-pensions after mortality shift for Strategy 1 and 2 for the age cohorts of retirees who are 65, 80 and 95 years old at time T_0.

4.4.2 Mortality Shift Effect on *CDC*-Plans

<u>*Strategy 1 (delayed recognition)*</u>

For $t \geq T_0$ -1 by Prop. 2 and 3 we get

$$Y_{t+1} = \ln\left(1 + \gamma(t) \sum_{x=z}^{\omega-1} \left(\frac{p_\Delta(x)}{p(x)} - 1 \right) w(t,x) \right), \qquad (Eq.\ 18)$$

$$\rho(t+1) = -Y_{t+1} + (1-\theta)\rho(t),$$

$$\eta(t+1) = \mu + \delta(t) + \theta\,\rho(t) \text{ and } \varepsilon(t+1) = \eta(t+1) - \mu_a . \qquad (Eq.\ 19)$$

In particular $\rho(T_0) = -Y_{T_0}$ and $\eta(T_0) = \mu$.

Different from the *IDC*-case, here all age cohorts are treated equally and the cohort of active workers becomes involved.

The effect on the profit participation $\eta(t)$ and the reserve ratio $\rho(t)$ is more complex than in the case of a capital market shift, since the age profile of the population changes and it takes $\omega - x_0 = 95$ years until a new steady state population is reached - see FIGURE 15.

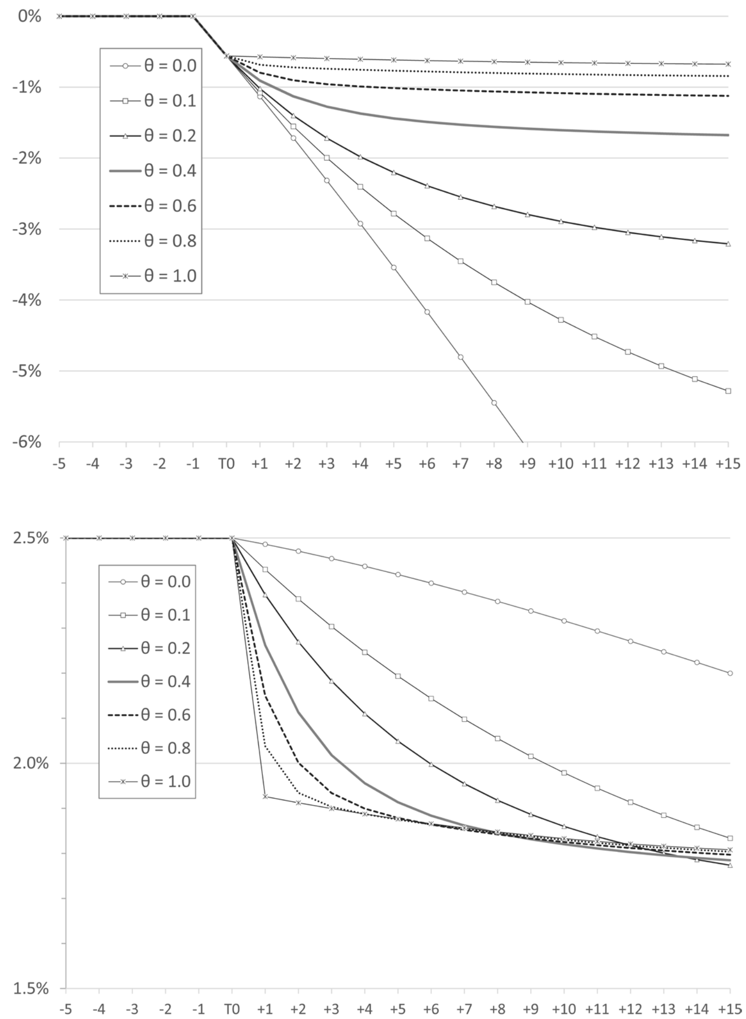


FIGURE 15: Effect of a mortality down-shift (Δ =+0.5) on the reserve ratio $\rho(t)$ (top chart) and profit participation $\eta(t)$ (bottom chart) in a *CDC*-pension system for *Strategy 1* for alternative levels of θ .

Since from time T_0 onwards, the age burden $\gamma(t)$ and the weights $w(t, x)$ deviate from the steady state values, the Y_t -process is not trivial. Clearly, for $\theta = 0$ $\rho(t)$ does not converge, neither does Y_t. One may check that $\rho(T_0 + k) = -\sum_{j=0}^{k}(1-\theta)^j \, Y_{T_0+k-j}$. So we can deduce that for $0 < \theta < 2$, $\rho(t)$ converges provided Y_t converges. The following table shows the new steady state values for ρ and η. E.g. for $\theta = 0.2$ and $\Delta = +0.5$ the *CDC*-system will converge to a new steady state with a permanent negative reserve of $\rho = -3.17\%$. The steady state profit participation ($\eta = 1.79\%$) falls behind the capital market return ($\mu = 2.50\%$) because the mortality shift has to be financed year by year and furthermore due to the negative reserve we have a negative stock effect (in this case $\delta = -0.0788\%$).

	Δ = + 0.5			Δ = -0.5		
θ	η	ρ	TV (x_0=20)	η	ρ	TV (x_0=20)
10%	1.72%	-6.25%	-3.8082	3.47%	7.66%	5.7557
20%	1.79%	-3.17%	-3.2344	3.35%	3.76%	4.3964
30%	1.81%	-2.12%	-3.0320	3.31%	2.49%	3.9811
40%	1.82%	-1.59%	-2.9286	3.29%	1.86%	3.7799
50%	1.83%	-1.28%	-2.8658	3.28%	1.49%	3.6611
60%	1.83%	-1.06%	-2.8237	3.27%	1.24%	3.5828
70%	1.84%	-0.93%	-2.7934	3.27%	1.08%	3.5272
80%	1.84%	-0.80%	-2.7706	3.26%	0.93%	3.4857
90%	1.84%	-0.71%	-2.7529	3.26%	0.82%	3.4535
100%	1.84%	-0.64%	-2.7386	3.26%	0.74%	3.4279

TABLE 4: *Strategy 1:* Profit participation (η), reserve ratio (ρ) and time value of future benefits minus contributions (*TW*) for new entrants (x_0=20) in the adjusted steady state after a mortality shift of Δ = +/- 0.5.

We now turn to the question of to what extent a mortality shift induces a transfer of wealth between the age cohorts. To this end we first calculate the time value $TV_{CDC}(T_0, x)$ of future benefits (including death benefits) minus future contributions for each (T_0, x)-cohort immediately after the shift occurred. Then the difference $TV_{CDC}(T_0, x)$ - $TV_{IDC}(T_0, x)$ is a suitable figure to measure the intergenerational wealth transfer. We also calculate $\dfrac{TV_{CDC}(T_0,x) - TV_{IDC}(T_0,x)}{L(T_0,x)}$, the individual contribution (positive or negative) to the intergenerational transfer.

Consider for example the (T_0, z)-cohort. At time T_0 we observe more survivors than expected, namely $L_\Delta(z) = \dfrac{p_\Delta(z-1)}{p(z-1)} L(z)$ instead of $L(z)$. The total pension capital for this cohort is $L_{\Delta}(x)\ v(x) = 76059.39$ for both, the *IDC* and the *CDC*-case. In the *IDC*-case all future pensions are paid from this capital stock. In the *CDC*-case the pensions are adjusted according to (*Eq.*19). For $\theta = 0.2$ the time value of all pensions paid to the (T_0, x)-cohort amounts to 83171.67. The difference 6706.07 is the intergenerational wealth transfer in favour of the (T_0, x)-cohort, which comes to an individual transfer of 7.88. In other words, each single member receives a subsidy of about eight contribution rates.

Let's now look at the (T_0, x_0)-cohort. At time T_0 the pension capital is zero. In the *IDC*-case all members of this cohort know that every Euro they pay into the system bears an interest rate of $\mu = 0.025$ and will be paid back – at least on average. In the *CDC*-regime ($\theta = 0.2$) we get $TV_{CDC}(T_0, x_0) = -3\,528.47$ and an individual transfer of -3.53. This means that a new entrant has to realise that more than 3 of the future contribution rates are transferred to the old generation.

The following FIGURE 16 illustrates the intergenerational transfer on cohort-level including cohorts of unborn. It is clear that the total sum taken over all existing and future generations must add up to zero.

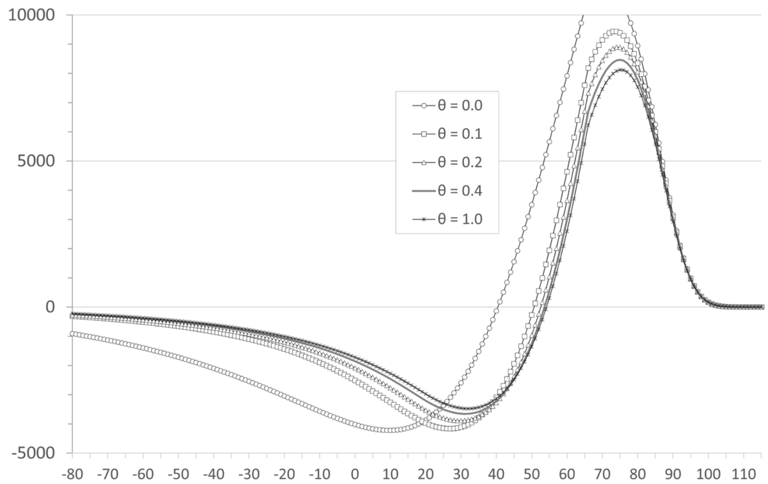


FIGURE 16: Intergenerational redistribution after a mortality down-shift ($\Delta = +0.5$) in a *CDC*-pension system for *Strategy 1* for age cohorts $x \geq -80$ and for $\theta = 0/\ 0.1/\ 0.2/\ 0.4/\ 1$.

FIGURE 17 shows the transfer on individual level for $x \geq 0$. Since the old age cohorts have fewer members the individual effect is more significant.

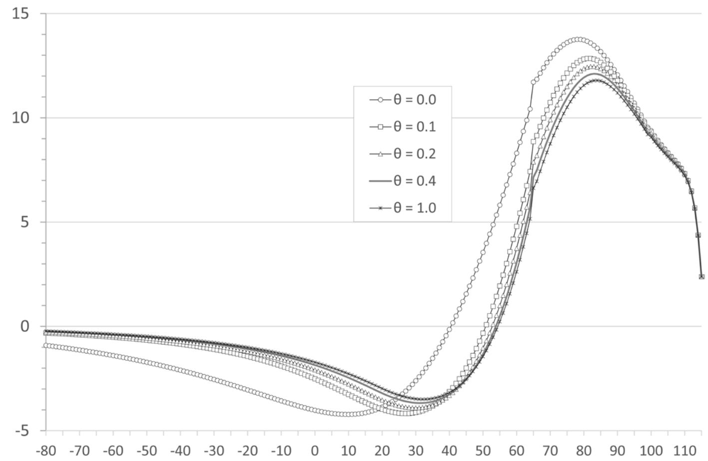


FIGURE 17: Intergenerational redistribution per head after a mortality down-shift (Δ = +0.5) in a *CDC*-pension system for *Strategy 1* for ages $x \geq 0$ and for θ=0.1/ 0.2/ 0.6/ 0.8.

Strategy 2 (Instant Recognition)

Instant recognition means that at time T_0 the liabilities in the balance sheet are adjusted to comply with the new survival probabilities. But the benefits payable at T_0 (pensions and death benefits) remain unchanged i.e. $\eta(T_0) = \mu$ and $\varepsilon(T_0) = \mu - \mu_a$. Furthermore, we assume that the new pensions for the (T_0, z)-cohort are calculated on the basis of $\ddot{a}(z)$. However from T_0+1 onwards we apply $\ddot{a}_\Delta(z)$.

Let $P(t)$ resp. $V(t)$ denote the total of assets resp. liabilities at time $t \geq T_0$. By our convention $P(t)$ and $V(t)$ include the death benefit payable in t for active workers who die in [t-1, t]. Thus we have $P(T_0) = P$, the steady state value of assets. Let us denote by $L'(x)$ the number survivors of the (T_0-1, x-1)-cohort after the mortality shift.

Proposition 5

$$V(T_0) = V + \left(L'(z) - L(z)\right)v(z) + \sum_{x=z}^{\omega}\left(\frac{p_\Delta(x-1)}{p(x-1)}\frac{\ddot{a}_\Delta(x)}{\ddot{a}(x)} - 1\right)L(x)\,\ddot{a}(x)\,r(x) \qquad (Eq.\ 20)$$

Proof

$$V(T_0) = \sum_{x=x_0}^{z-1} L(x-1)\,v(x) + \left(L'(z) - L(z-1)\right)v(z) + \sum_{x=z}^{\omega} L'(x)\,\ddot{a}_\Delta(x)\,r(x)$$

$$= \sum_{x=x_0}^{z-1} L(x-1)\,v(x) + \left(L(z) - L(z-1)\right)v(z) + \sum_{x=z}^{\omega} L(x)\,\ddot{a}(x)\,r(x)$$

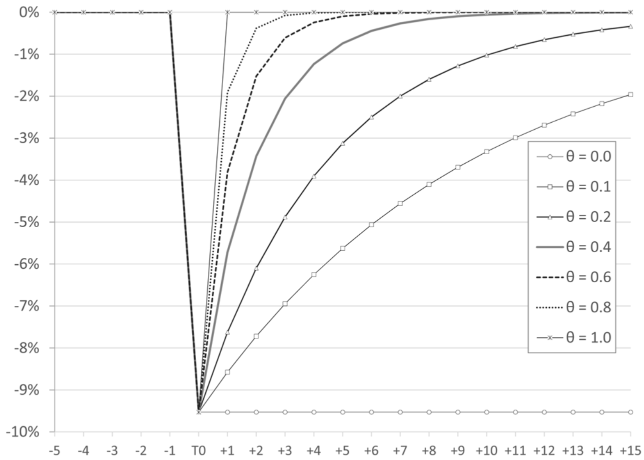
$$+ \left(L'(z) - L(z)\right)v(z) + \sum_{x=z}^{\omega}\left(\frac{p_\Delta(x-1)}{p(x-1)}\ddot{a}_\Delta(x) - \ddot{a}(x)\right)L(x)\,r(x)$$

Since $V = \sum_{x=x_0}^{z-1} L(x-1)\,v(x) + \big(L(z)-L(z-1)\big)\,v(z) + \sum_{x=z}^{\omega} L(x)\,\ddot{a}(x)\,r(x)$ we get (*Eq.* 20).

♦

Note that for $\Delta > 0$ $\big(L'(z)-L(z)\big)v(z) < 0$ and $\left(\dfrac{p_\Delta(x-1)}{p(x-1)}\dfrac{\ddot{a}_\Delta(x)}{\ddot{a}(x)} - 1\right) > 0$.

If the mortality shift is recognised instantly, the effect on the reserve ratio and the profit participation strongly resembles the situation after a capital market down shock. We illustrate the effects in FIGURE 18 below. As in Figure 7 we see that for $\theta > 0$ the reserve ratio will gradually return to the steady state level $\hat{\rho} = 0$.



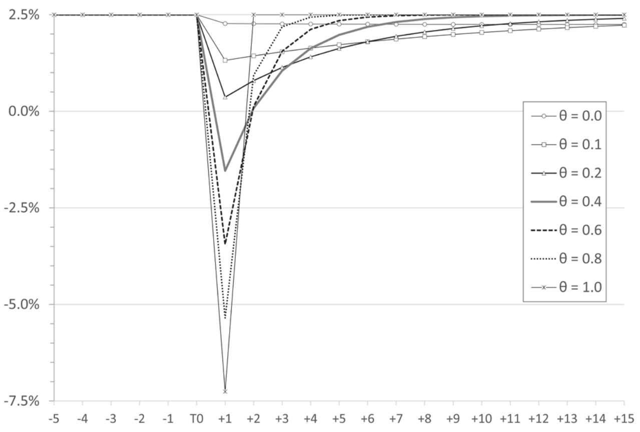


FIGURE 18: Effect of a mortality down-shift (Δ =+0.5) on the reserve ratio $\rho(t)$ (top chart) and profit participation $\eta(t)$ (bottom chart) in a *CDC*-pension system for *Strategy 2* (instant recognition) for alternative levels of θ.

As for *Strategy 1* we measure the intergenerational wealth transfer by comparing the time value of future benefits minus contributions for the (T_0, x)-cohorts. The instant recognition of the mortality shift (Δ = +0.5) has a mild effect on the (T_0, x)-cohorts for $x \geq z$ since their pensions are only indirectly affected via reduced $\varepsilon(t)$. However those who enter retirement at T_0+1 or later have to endure a double impact: firstly the profit participation and future pension increases will go down to refill the reserve and secondly, their initial pensions are calculated on the basis of the shifted mortality. This is illustrated in FIGURE 19 and 20 below. We notice a sharp cut at age $x = 65$ which is a result of the fact that due to the instant recognition of the mortality shift, from time T_0 onwards all new pensions are calculated on the basis of the shifted mortality rates. We see that the redistributional effect of a mortality shift differs clearly from that of a capital market shock- compare Figure 19/ 20 and Figure 8/ 9.

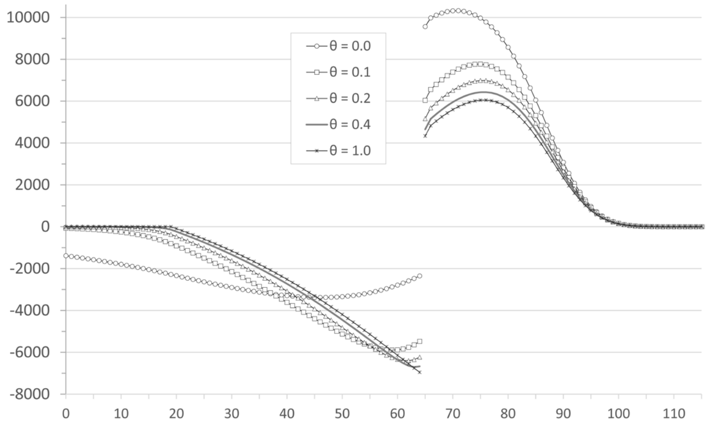


FIGURE 19: Intergenerational redistribution after a mortality down-shift ($\Delta = +0.5$) in a *CDC*-pension system for *Strategy 2* for age cohorts $x \geq 0$ and for θ=0.1/ 0.2/ 0.6/ 0.8.

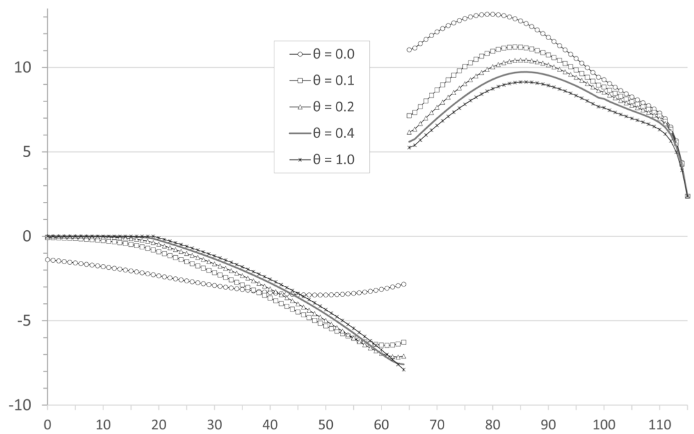


FIGURE 20: Intergenerational redistribution per head after a mortality down-shift ($\Delta = +0.5$) in a *CDC*-pension system for *Strategy 2* for ages $x \geq 0$ and for θ=0.1/ 0.2/ 0.6/ 0.8.

Comparison of Strategy 1/ 2 (delayed/ instant recognition)

The avoidance of cutting pensions in payment seems to be a touchstone of a pension plan. Accordingly, the managers of a pension plan will be very reluctant to actually cut pensions. If we look at the effect of Strategy 1 or 2 on the pensions in payment (cf. FIGURE 21) then it is clear that the "procrastination policy" (Strategy 1) is very attractive. We know from the analysis above that Strategy 1 shifts the burden of longer life expectance to future generations, who inherit an eternal loan from the old.

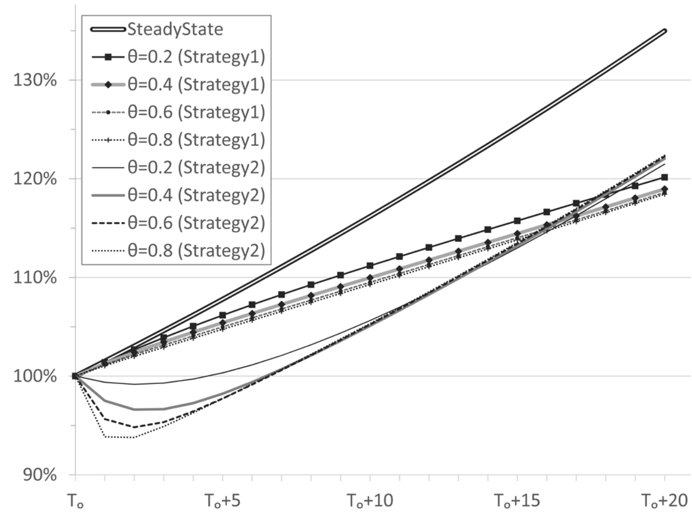


FIGURE 21: Pension level for pensions in payment after a mortality down-shift ($\Delta = +0.5$) for Strategy 1 and 2 for different levels of θ. 100% marks the pre-shift pension level.

Both strategies imply a massive wealth transfer between the generations. TABLE 5 below shows the wealth transfer in favour of the older generations as a proportion of the steady state total pension capital (= 2543840). Strategy 1 turns out to produce a stronger transfer than Strategy 2. If we compare the age cohorts that profit from the transfer we see that Strategy 1 is attractive for active employees aged 54 and over. One may guess that for many pension plans these age cohorts are dominant in the representative bodies, so one might expect that in real life there will be a strong tendency to postpone the updating of the mortality tables.

	Mortality Shift (Δ = +0.5)			
	Strategy 1 (delay recognition)		**Strategy 2** (instant recognition)	
θ	Redistribution in % of total pension assets	burdened age cohorts	Redistribution in % of total pension assets	burdened age cohorts
0	13.78%	x≤40	8.88%	x≤64
0.1	10.12%	x≤50	6.97%	x≤64
0.2	9.22%	x≤52	6.31%	x≤64
0.3	8.81%	x≤53	6.00%	x≤64
0.4	8.58%	x≤53	5.82%	x≤64
0.5	8.44%	x≤54	5.71%	x≤64
0.6	8.34%	x≤54	5.63%	x≤64
0.7	8.27%	x≤54	5.57%	x≤64
0.8	8.21%	x≤54	5.53%	x≤64
0.9	8.17%	x≤54	5.49%	x≤64
1.0	8.13%	x≤54	5.46%	x≤64

TABLE 5: Overall redistribution effect from young to old of a mortality shift (Δ = +0.5) for Strategy 1 and Strategy 2 in % of total pre-shift pension capital.

Neither Strategy 1 nor 2 should be the choice in practice! There are good arguments to apply a mixed strategy by adjusting mortality rates step by step.

5 Concluding Remarks

The primary purpose of *collective DC*-plans is smooth away the ups and downs of capital market returns, which are particularly volatile for stock markets. If there is no external institution to step in if equities slump, the smoothing can only be done by some kind of intergenerational risk transfer. Intergenerational risk transfer is going on since decades but in general unilaterally at the cost of the younger generation. The shift from *DB*- to *DC*-plans is only one example. So the challenge is to find rules that allow for a fair risk transfer between age cohorts. Our proposal for such rules is guided by the concept of *resilience*. We apply these rules to several capital market shock and shift scenarios and to mortality shift scenarios. We measure the intergenerational effects; so we have instrument to measure intergenerational equity. Our conclusion is that *collective DC*-plans are the better alternative compared to *pure DC*-plans.

References

[Cairns e.a. 2006] Cairns, A. J. G., D. Blake and K. Dowd: *A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration*, The Journal of Risk and Insurance, 73, (2006), 687-718.

[Cui e.a. 2011] Cui, J., Jong, F. de, Ponds, E.: *Intergenerational risk sharing with funded pension schemes*. Journal of Pension Economics & Finance, 10 (1): 1-9.

[Day et al. 2014] Day, N., Ohta, A., Thomas, A., 2014. *Defined Ambition: Consumer perspectives -Qualitative research among employers, individuals and employee benefit consultants*, Department for Work & Pension (UK), Research Report No 866.

[EIOPA 2014] European Insurance and Occupational Pensions Authority: *The underlying assumptions in the standard formula for the Solvency Capital Requirement calculation*, EIOPA-14 -322, 25-07-2014, https://eiopa.europa.eu/Publications/Standards/EIOPA-14-322_Underlying_Assumptions.pdf.

[European Union 2016] Directive (EU) 2016/2341 of the European Parliament and of the Council of 14 December 2016 on the activities and supervision of institutions for occupational retirement provision (IORPs), http://eur-lex.europa.eu/legal-content/EN /TXT/ PDF/?uri=CELEX:32016L2341&from=EN.

[Goecke 2013] Goecke, Oskar: *Pension saving schemes with return smoothing mechanism*; Insurance: Mathematics and Economics 53 (2013), S. 678-689.

[Gollier 2008] Gollier, Christian: *Intergenerational Risk-Sharing and Risk-Taking of a Pension Fund*; Journal of Public Economics, 92(5-6): 1463-1485.

[Gordon/ Varian 1988] Gordon, R. H., Varian, H.R.: *Intergenerational risk sharing*; Journal of Public Economics, 37, 185-202.

[Heubeck et al. 2006] Heubeck, Klaus; Herrmann, Richard; D'Souza, Gabriele: *Die Richttafeln 2005 G – Modell, Herleitung, Formeln*, Blätter der DGVFM, Band XXVII, Heft 3, April 2006, S. 473-517.

[House of Commons 2016] House of Commons -Work and Pensions Committee: *Intergenerational fairness*; Third Report of Session 2016–17. DOI: https://www.publications.parliament.uk/pa/cm201617/cmselect/cmworpen/59/59.pdf.

[McKeever 2010] McKeever, Kent: *A Short History of Tontines*, Fordham Journal of Corporate & Financial Law, Vol XV (2010), pp. 491-521.

[Milevsky, Salisbury 2015] Milevsky, M.A., Salisbury, T.S.: *Optimal Retirement Income Tontines*, Insurance: Mathematics and Economics (2015), Vol. 64(1), p. 91-105.

[OECD 2013] OECD (ed.): *Pension at a Glance 2013: OECD and G20 Indicators*, OECD Publishing, Paris. DOI: http://dx.doi.org/10.1787/pension_glance-2013-en.

[Rawls 1971] Rawls, John: A Theory of Justice, Cambridge, Massachusetts 1971, revised edition 1999.

[Westerhout 2011] Westerhout, E: *Intergenerational risk sharing in time-consistent funded pension schemes*, CPB Netherlands Bureau for Economic Policy Analysis, Discussion paper 176 (April 2011).

Figures and Tables

Impressum

Diese Veröffentlichung erscheint im Rahmen der Online-Publikationsreihe „Forschung am **ivw**Köln“. Eine vollständige Übersicht aller bisher erschienenen Publikationen findet sich am Ende dieser Publikation und kann hier abgerufen werden.

Forschung am ivwKöln, 7/2018
ISSN (online) 2192-8479

Oskar Goecke:
Resilience and Intergenerational Fairness in Collective Defined Contribution Pension Funds

Köln, Dezember 2018

Schriftleitung / editor's office:

Prof. Dr. Jürgen Strobel

Institut für Versicherungswesen /
Institute for Insurance Studies

Fakultät für Wirtschafts- und Rechtswissenschaften /
Faculty of Business, Economics and Law

Technische Hochschule Köln /
University of Applied Sciences

Gustav Heinemann-Ufer 54
50968 Köln

Tel. +49 221 8275-3270
Fax +49 221 8275-3277
Mail juergen.strobel@th-koeln.de
Web www.th-koeln.de

Kontakt Autor / Contact author:

Prof. Dr. Oskar Goecke

Institut für Versicherungswesen /
Institute for Insurance Studies

Fakultät für Wirtschafts- und Rechtswissenschaften /
Faculty of Business, Economics and Law

Technische Hochschule Köln /
University of Applied Sciences

Gustav Heinemann-Ufer 54
50968 Köln

Tel. +49 221 8275-3278
Fax +49 221 8275-3277
Mail oskar.goecke@th-koeln.de
Web www.ivw-koeln.de

Publikationsreihe „Forschung am ivwKöln“

Die Veröffentlichungen der Online-Publikationsreihe "Forschung am ivwKöln" (ISSN: 2192-8479) werden üblicherweise über Cologne Open Science (Publikationsserver der TH Köln) veröffentlicht. Die Publikationen werden hierdurch über nationale und internationale Bibliothekskataloge, Suchmaschinen sowie andere Nachweisinstrumente erschlossen.

Alle Publikationen sind auch kostenlos abrufbar unter www.ivw-koeln.de.

2018

6/2018	Miebs: Kapitalanlagestrategien für die bAV – Herausforderungen für das Asset Management durch das Betriebsrentenstärkungsgesetz. Proceedings zum 13. FaRis & DAV Symposium am 8. Dezember 2017 in Köln
5/2018	Goecke, Heep-Altiner, Knobloch, Schiegl, Schmidt (Hrsg.): FaRis at ICA 2018 – Contributions to the International Congress of Actuaries 2018 in Berlin . Beiträge von FaRis Mitgliedern zum Weltkongress der Aktuare vom 4. bis zum 8. Juni 2018 in Berlin
4/2018	Knobloch: Die Pfade einer bewerteten inhomogenen Markov-Kette - Fallbeispiele aus der betrieblichen Altersversorgung
3/2018	Völler, Müller-Peters: InsurTech Karte ivwKöln 1/2018 - Beiträge zu InsurTechs und Innovation am ivwKöln
2/2018	Schmidt, Schulz: InsurTech. Proceedings zum 12. FaRis & DAV Symposium am 9. Juni 2017 in Köln
1/2018	Institut für Versicherungswesen: Forschungsbericht für das Jahr 2017

2017

8/2017	Materne, Pütz: Alternative Capital und Basisrisiko in der Standardformel (non-life) von Solvency II
7/2017	Knobloch: Konstruktion einer unterjährlichen Markov-Kette aus einer jährlichen Markov-Kette - Eine Verallgemeinerung des linearen Ansatzes
6/2017	Goecke, Oskar (Hrsg.): Risiko und Resilienz. Proceedings zum 11. FaRis & DAV Symposium am 9. Dezember 2016 in Köln
5/2017	Grundhöfer, Dreuw, Quint, Stegemann: Bewertungsportale - eine neue Qualität der Konsumenteninformation?
4/2017	Heep-Altiner, Mehring, Rohlfs: Bewertung des verfügbaren Kapitals am Beispiel des Datenmodells der „IVW Privat AG“
3/2017	Müller-Peters, Völler: InsurTech Karte ivwKöln 1/2017 - Beiträge zu InsurTechs und Innovation am ivwKöln
2/2017	Heep-Altiner, Müller-Peters, Schimikowski, Schnur (Hrsg.): Big Data für Versicherungen. Proceedings zum 21. Kölner Versicherungssymposium am 3. 11. 2016 in Köln
1/2017	Institut für Versicherungswesen: Forschungsbericht für das Jahr 2016

2016

13/2016	Völler: Erfolgsfaktoren eines Online-Portals für Akademiker
12/2016	Müller-Peters, Gatzert: Todsicher: Die Wahrnehmung und Fehlwahrnehmung von Alltagsrisiken in der Öffentlichkeit (erscheint 2017)
11/2016	Heep-Altiner, Penzel, Rohlfs, Voßmann: Standardformel und weitere Anwendungen am Beispiel des durchgängigen Datenmodells der „IVW Leben AG“
10/2016	Heep-Altiner (Hrsg.): Big Data. Proceedings zum 10. FaRis & DAV Symposium am 10. Juni 2016 in Köln
9/2016	Materne, Pütz, Engling: Die Anforderungen an die Ereignisdefinition des Rückversicherungsvertrags: Eindeutigkeit und Konsistenz mit dem zugrundeliegenden Risiko
8/2016	Rohlfs (Hrsg.): Quantitatives Risikomanagement. Proceedings zum 9. FaRis & DAV Symposium am 4. Dezember 2015 in Köln

7/2016	Eremuk, Heep-Altiner: Internes Modell am Beispiel des durchgängigen Datenmodells der „IVW Privat AG“
6/2016	Heep-Altiner, Rohlfs, Dağoğlu, Pulido, Venter: Berichtspflichten und Prozessanforderungen nach Solvency II
5/2016	Goecke: Collective Defined Contribution Plans - Backtesting based on German capital market data 1955 - 2015
4/2016	Knobloch: Bewertete inhomogene Markov-Ketten - Spezielle unterjährliche und zeitstetige Modelle
3/2016	Völler (Hrsg.): Sozialisiert durch Google, Apple, Amazon, Facebook und Co. – Kundenerwartungen und –erfahrungen in der Assekuranz. Proceedings zum 20. Kölner Versicherungssymposium am 5. November 2015 in Köln
2/2016	Materne (Hrsg.): Jahresbericht 2015 des Forschungsschwerpunkts Rückversicherung
1/2016	Institut für Versicherungswesen: Forschungsbericht für das Jahr 2015

2015

11/2015	Goecke (Hrsg.): Kapitalanlagerisiken: Economic Scenario Generator und Liquiditätsmanagement. Proceedings zum 8. FaRis & DAV Symposium am 12. Juni 2015 in Köln
10/2015	Heep-Altiner, Rohlfs: Standardformel und weitere Anwendungen am Beispiel des durchgängigen Datenmodells der „IVW Privat AG“ – Teil 2
9/2015	Goecke: Asset Liability Management in einem selbstfinanzierenden Pensionsfonds
8/2015	Strobel (Hrsg.): Management des Langlebigkeitsrisikos. Proceedings zum 7. FaRis & DAV Symposium am 5.12.2014 in Köln
7/2015	Völler, Wunder: Enterprise 2.0: Konzeption eines Wikis im Sinne des prozessorientierten Wissensmanagements
6/2015	Heep-Altiner, Rohlfs: Standardformel und weitere Anwendungen am Beispiel des durchgängigen Datenmodells der „IVW Privat AG“
5/2015	Knobloch: Momente und charakteristische Funktion des Barwerts einer bewerteten inhomogenen Markov-Kette. Anwendung bei risikobehafteten Zahlungsströmen
4/2015	Heep-Altiner, Rohlfs, Beier: Erneuerbare Energien und ALM eines Versicherungsunternehmens
3/2015	Dolgov: Calibration of Heston's stochastic volatility model to an empirical density using a genetic algorithm
2/2015	Heep-Altiner, Berg: Mikroökonomisches Produktionsmodell für Versicherungen
1/2015	Institut für Versicherungswesen: Forschungsbericht für das Jahr 2014

2014

10/2014	Müller-Peters, Völler (beide Hrsg.): Innovation in der Versicherungswirtschaft
9/2014	Knobloch: Zahlungsströme mit zinsunabhängigem Barwert
8/2014	Heep-Altiner, Münchow, Scuzzarello: Ausgleichsrechnungen mit Gauß Markow Modellen am Beispiel eines fiktiven Stornobestandes
7/2014	Grundhöfer, Röttger, Scherer: Wozu noch Papier? Einstellungen von Studierenden zu E-Books
6/2014	Heep-Altiner, Berg (beide Hrsg.): Katastrophenmodellierung - Naturkatastrophen, Man Made Risiken, Epidemien und mehr. Proceedings zum 6. FaRis & DAV Symposium am 13.06.2014 in Köln
5/2014	Goecke (Hrsg.): Modell und Wirklichkeit. Proceedings zum 5. FaRis & DAV Symposium am 6. Dezember 2013 in Köln
4/2014	Heep-Altiner, Hoos, Krahforst: Fair Value Bewertung von zedierten Reserven
3/2014	Heep-Altiner, Hoos: Vereinfachter Nat Cat Modellierungsansatz zur Rückversicherungsoptimierung
2/2014	Zimmermann: Frauen im Versicherungsvertrieb. Was sagen die Privatkunden dazu?
1/2014	Institut für Versicherungswesen: Forschungsbericht für das Jahr 2013

2013

11/2013 Heep-Altiner: Verlustabsorbierung durch latente Steuern nach Solvency II in der Schadenversicherung, Nr. 11/2013

10/2013 Müller-Peters: Kundenverhalten im Umbruch? Neue Informations- und Abschlusswege in der Kfz-Versicherung, Nr. 10/2013

9/2013 Knobloch: Risikomanagement in der betrieblichen Altersversorgung. Proceedings zum 4. FaRis & DAV-Symposium am 14. Juni 2013

8/2013 Strobel (Hrsg.): Rechnungsgrundlagen und Prämien in der Personen- und Schadenversicherung - Aktuelle Ansätze, Möglichkeiten und Grenzen. Proceedings zum 3. FaRis & DAV Symposium am 7. Dezember 2012

7/2013 Goecke: Sparprozesse mit kollektivem Risikoausgleich - Backtesting

6/2013 Knobloch: Konstruktion einer unterjährlichen Markov-Kette aus einer jährlichen Markov-Kette

5/2013 Heep-Altiner et al. (Hrsg.): Value-Based-Management in Non-Life Insurance

4/2013 Heep-Altiner: Vereinfachtes Formelwerk für den MCEV ohne Renewals in der Schadenversicherung

3/2013 Müller-Peters: Der vernetzte Autofahrer – Akzeptanz und Akzeptanzgrenzen von eCall, Werkstattvernetzung und Mehrwertdiensten im Automobilbereich

2/2013 Maier, Schimikowski (beide Hrsg.): Proceedings zum 6. Diskussionsforum Versicherungsrecht am 25. September 2012 an der FH Köln

1/2013 Institut für Versicherungswesen (Hrsg.): Forschungsbericht für das Jahr 2012

2012

11/2012 Goecke (Hrsg.): Alternative Zinsgarantien in der Lebensversicherung. Proceedings zum 2. FaRis & DAV-Symposiums am 1. Juni 2012

10/2012 Klatt, Schiegl: Quantitative Risikoanalyse und -bewertung technischer Systeme am Beispiel eines medizinischen Gerätes

9/2012 Müller-Peters: Vergleichsportale und Verbraucherwünsche

8/2012 Füllgraf, Völler: Social Media Reifegradmodell für die deutsche Versicherungswirtschaft

7/2012 Völler: Die Social Media Matrix - Orientierung für die Versicherungsbranche

6/2012 Knobloch: Bewertung von risikobehafteten Zahlungsströmen mithilfe von Markov-Ketten bei unterjährlicher Zahlweise

5/2012 Goecke: Sparprozesse mit kollektivem Risikoausgleich - Simulationsrechnungen

4/2012 Günther (Hrsg.): Privat versus Staat - Schussfahrt zur Zwangsversicherung? Tagungsband zum 16. Kölner Versicherungssymposium am 16. Oktober 2011

3/2012 Heep-Altiner/Krause: Der Embedded Value im Vergleich zum ökonomischen Kapital in der Schadenversicherung

2/2012 Heep-Altiner (Hrsg.): Der MCEV in der Lebens- und Schadenversicherung - geeignet für die Unternehmenssteuerung oder nicht? Proceedings zum 1. FaRis & DAV-Symposium am 02.12.2011 in Köln

1/2012 Institut für Versicherungswesen (Hrsg.): Forschungsbericht für das Jahr 2011

2011

5/2011 Reimers-Rawcliffe: Eine Darstellung von Rückversicherungsprogrammen mit Anwendung auf den Kompressionseffekt

4/2011 Knobloch: Ein Konzept zur Berechnung von einfachen Barwerten in der betrieblichen Altersversorgung mithilfe einer Markov-Kette

3/2011 Knobloch: Bewertung von risikobehafteten Zahlungsströmen mithilfe von Markov-Ketten

2/2011 Heep-Altiner: Performanceoptimierung des (Brutto) Neugeschäfts in der Schadenversicherung

1/2011 Goecke: Sparprozesse mit kollektivem Risikoausgleich