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ABSTRACT

Assortative Matching and Wages: The Role of Selection*

We develop a random search model with two-sided heterogeneity and match-specific productivity shocks to explain why high-productivity workers tend to work at high-productivity firms despite low-productivity workers gaining about as much from such matches. Our model has two key predictions: i) the average log wage that a worker receives is increasing in the worker's and employer's productivity, with low-productivity workers gaining proportionally more at high-productivity firms and ii) there is assortative matching between a worker's productivity and that of her employer. Selective job acceptance drives these patterns. All workers are equally likely to meet all firms, but workers have higher surplus from meeting firms of similar productivity. The high surplus meetings result in matches more frequently, generating assortative matching. Only the subset of meetings that result in matches are observed in administrative wage data, shaping wages. We show that our findings are quantitatively consistent with recent empirical results. Moreover, we prove this selection is not detected using standard empirical approaches, highlighting the importance of theory-guided empirical work. Our results imply that encouraging high-wage firms to hire low-wage workers may be less effective at reducing wage inequality than wage patterns suggest.

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1 Introduction

Why do high-wage workers tend to work at high-wage firms, even though low-wage workers seem to enjoy similar gains in log wages from such jobs? Following the path-breaking study by Abowd, Kramarz and Margolis (1999) using administrative wage data, these empirical regularities have been extensively documented, yet the economic forces generating them remain unknown. Bonhomme, Lamadon and Manresa (2019, p. 701) write that "the presence of strong sorting, together with the absence of strong complementarities in wages, is difficult to reconcile with models where sorting is driven by complementarities in production, as in Becker (1973)." This paper proposes a natural reconciliation.

We prove that a random search model with productive complementarities between heterogeneous workers and firms, match-specific productivity shocks, and Nash-bargained wages offers a unified explanation for these patterns. Under simple conditions, average log wages increase with the permanent component of both worker and firm productivity, while being submodular. This means that low-productivity workers' log wages are more sensitive to firm productivity than are high-productivity workers' log wages. When the production technology is suitably complementary, we prove that there is assortative matching between high-productivity workers and high-productivity firms.[1]

Our theoretical results arise from a novel mechanism: selective hiring. While all workers are equally likely to meet all firms, the likelihood that a match forms depends on the worker's and firm's productivity. High-productivity firms rarely hire low-productivity job seekers, but when match-specific productivity is high enough, these workers are hired at a wage above the average wage earned by other low-productivity workers. Similarly, low-productivity firms seldom hire high-productivity workers unless match-specific productivity is unusually high. This selective hiring creates a systematic difference between the matches we observe in administrative data and all meetings between workers and firms, driving both sorting patterns and the behavior of wages.

We enrich our baseline model with on-the-job search to match the quantitative evidence. This means that employed workers can continue to search for better job opportunities, creating selection in both hiring and worker turnover. We calibrate this extended model to match results in two influential studies: Bonhomme, Lamadon and Manresa (2019) using Swedish data, and Kline, Saggio and Sølvsten (2020) using data from the Veneto region of Italy.[2]

[1] A corollary of these results is that high-productivity workers are high-wage workers while high-productivity firms are high-wage firms, which is important for interpretation. In our model productivity drives behavior, but administrative data sets often record wages rather than productivity. We maintain this distinction in our terminology throughout the paper.

[2] These papers tackle the incidental parameters problem (limited mobility bias) in ordinary least squares estimates of the Abowd, Kramarz and Margolis (1999) model.

Our analytical results from the simpler model without on-the-job search carry over to this more realistic environment: log wages are increasing and slightly submodular, though approximately additively separable, in worker and firm productivity; and there is positively assortative matching when the production technology is suitably complementary. In addition, we show that despite its sparse parameterization, our model exactly replicates key empirical findings from these two studies, namely the magnitudes of the variance of worker effects, the variance of firm effects, their covariance, and the variance of the residual. That selective hiring and turnover can explain these patterns suggests that they may be a central force behind worker-firm sorting and observed wages.

The match-specific component of wages is fundamental to our analysis and represents a significant departure from previous research. While earlier studies acknowledged significant residual wage variation, they typically treated these residuals as measurement error or time-varying worker heterogeneity that does not affect job choice. In our framework these residuals are evidence of the match-specific shocks that drive selective hiring.

Selective hiring in turn reshapes our understanding of how firms influence wages. As Card, Cardoso, Heining and Kline (2018, p. S16) note: "in the firm-switching literature, a key question is whether conventionally estimated firm-specific pay premiums predict the wage changes associated with exogenously induced job accessions and separations." The validity of such predictions rests on an "exogenous mobility" assumption, that wage residuals average to zero for each worker regardless of employer. Selective hiring in our model leads to a violation of this assumption. As a result, conventional estimates of firm-specific pay premiums do not accurately predict a worker's potential wages at alternative employers. Firm fixed effects are not pure wage premiums.

A natural question is whether this violation of exogenous mobility can be detected in the data. We prove that constructing a definitive test of exogenous mobility using only standard administrative data is impossible without imposing additional assumptions like random search. This result, which builds on insights about lack-of-identification from Flinn and Heckman (1982), highlights the potential limitations of reduced-form approaches for understanding wage determination and worker-firm sorting. In particular, when we apply the widely-used Card, Heining and Kline (2013) "event study" test to data generated from our calibrated model, it fails to detect evidence of endogenous mobility, instead producing patterns that have previously been interpreted as supporting exogenous mobility. This is despite the fact that we know exogenous mobility is violated in our model, with selection driving the shape of measured wages both with and without on-the-job search.

Our final contribution is to distinguish between the surplus derived from meetings and the surplus derived from matches. We define the surplus from meetings as the value that an unemployed worker puts on meeting a particular type of firm before knowing the realization of match quality. The surplus from matches is the expected value conditional on the unemployed worker accepting the job. We find that while the surplus from matches is increasing in the firm's productivity, the surplus from meetings may not be, particularly for the least productive workers. Moreover, the surplus from meetings is log supermodular in our quantitative analysis, so the most productive workers gain proportionately more from meeting the most productive firms. This cautions against policies designed to mitigate wage inequality by encouraging high productivity jobs to search for low productivity workers.

Our paper contributes to several strands of literature. First, we build on empirical work using matched employer-employee data to study wage determination and sorting (Abowd, Kramarz and Margolis, 1999; Andrews, Gill, Schank and Upward, 2008; Card, Heining and Kline, 2013; Card, Cardoso and Kline, 2016; Alvarez, Benguria, Engbom and Moser, 2018; Song, Price, Guvenen, Bloom and Von Wachter, 2019; Bonhomme, Lamadon and Manresa, 2019; Kline, Saggio and Sølvsten, 2020; Bonhomme, Holzheu, Lamadon, Manresa, Mogstad and Setzler, 2023). We provide a simple and novel theoretical mechanism that can explain the empirical regularities documented in these papers.

Second, Card, Cardoso, Heining and Kline (2018), Lamadon, Mogstad and Setzler (2022), and Lamadon, Lise, Meghir and Robin (2024) develop theoretical models with idiosyncratic and systematic differences in amenity valuations that are consistent with the same set of empirical findings. These papers use a static discrete choice framework while we use a dynamic search model. We thus address a key previously unresolved question raised by Card, Cardoso, Heining and Kline (2018, p. 18): "to what extent do insights from simple static wage-setting models of workplace differentiation carry over to dynamic labor market settings with search or mobility frictions?" A more fundamental difference is that the models in these earlier papers satisfy the exogenous mobility assumption, so selection does not affect measured wages. Future research should investigate the data needed to separately identify the mechanisms proposed by these distinct approaches.

Third, we build on a large literature that studies how selection affects measured wages, dating back to early econometric analyses of the Roy (1951) model (Borjas, 1987; Heckman and Honore, 1990). Noe (2020) offers a recent theoretical analysis of this question in an environment where a worker is choosing between a finite number of alternative jobs at a point in time. The previous research does not study how selection shapes regressions of wages on worker and firm fixed effects, nor does it look at the connection between selection and assortative matching.

Fourth, we contribute to the theoretical literature on conditions for assortative matching in labor markets with random search (Shimer and Smith, 2000; Hagedorn, Law and

Manovskii, 2017; Lopes de Melo, 2018; Bagger and Lentz, 2019). Our model extends this literature by incorporating match-specific shocks, as in Goussé, Jacquemet and Robin (2017)'s model of the marriage market, and highlighting the role of selection in job acceptance decisions. Our characterization of conditions for assortative matching recalls similar conditions in competitive search models (Shi, 2001, 2005; Shimer, 2005; Eeckhout and Kircher, 2010) and models with non-transferable utility (Smith, 2006; Bonneton and Sandmann, 2023). The details of the mechanisms are quite different, and in particular selection does not play a role in those papers. Those earlier papers also do not try to fit the empirical findings on how wages depend on worker and firm productivity. Our model bridges this gap by providing a framework that not only explains sorting patterns but also shows how the same selection force that generates sorting drives the wage patterns in administrative data sets.

The rest of the paper is structured as follows. Section 2 presents our model. In Section 3, we derive conditions for the average log wage to be an increasing and submodular function of worker and firm productivity. Section 4 derives conditions for assortative matching both of high-productivity workers to high-productivity jobs and of high-wage workers to high-wage jobs. Section 5 extends our model to allow for on-the-job search, whereby employed workers can continue to search for better jobs. In Section 6, we show that our model can quantitatively match the wage variance decompositions in Bonhomme, Lamadon and Manresa (2019) and Kline, Saggio and Sølvsten (2020). Section 7 discusses the difficulty of testing for exogenous mobility and shows that the "event study" proposed by Card, Heining and Kline (2013) fails to detect the endogenous mobility that is central to our quantitative model. Section 8 compares the surplus from meetings with the surplus from matches and discusses the implications for policies designed to alleviate wage inequality. Section 9 concludes by discussing how our results affect the economic interpretation of patterns in administrative wage data.

2 Model

We formulate a search model with two-sided heterogeneity (Shimer and Smith, 2000) and match-specific heterogeneity (Goussé, Jacquemet and Robin, 2017). The model is formulated in continuous time and we focus on steady states and so drop time arguments in what follows.

2.1 Assumptions

There is measure M of risk-neutral workers and measure N of risk-neutral firms. Everyone discounts the future at rate $r > 0$. There are X worker types indexed by $x = 1, \dots, X$. The population measure of type-x workers is $m_x > 0$, with $\sum_{x=1}^{X} m_x = M$. There are Y firm types indexed by $y = 1, \ldots, Y$, with population measure $n_y > 0$ and $\sum_{y=1}^{Y} n_y = N$. Workers can be either unemployed or matched to one firm; likewise, firms can be either vacant or matched to one worker. Thus, in this model, a firm and a job are identical.

Search is random and for now only unemployed workers and vacant jobs can search. Let u_x be the population measure of unemployed type-x workers, so that $\frac{u_x}{m_x}$ is the unemployment rate for type x. Similarly, let v_y be the population measure of type-y vacancies, with $\frac{v_y}{n_y}$ the vacancy rate of y. All unemployed workers contact vacant type-y firms according to a Poisson process with arrival rate ρv_y for $y = 1, \ldots, Y$, where $\rho > 0$. Likewise, all vacant firms contact unemployed type-x workers at rate ρu_x for $x = 1, \ldots, X$.[3]

When a worker and firm meet, they learn the value of a match-specific productivity shock z drawn from a distribution function with density $s(z)$ and survival function $S(z)$. The shock is independent across all worker and firm pairs. By definition, the survival function $S : \mathbb{R}_+ \to \mathbb{R}_+$ is non-increasing. For expositional simplicity, we also assume that S is continuous and strictly positive for all $z > 0$. Finally, we assume the mean productivity draw, $z^* \equiv \int_0^\infty z s(z) dz$, is finite, which is necessary for existence of an equilibrium.

After an unemployed type-x worker meets a vacant type-y firm and draws a match-specific productivity shock z, they decide whether to match. If they match, they stop searching and produce flow output $z f_{x,y}$. A matched worker and firm split the surplus according to Nash bargaining, with worker's bargaining power equal to $\gamma \in (0, 1)$. We assume $f_{x,y} > 0$ for all x and y. Matches end at rate $\delta > 0$, leaving the worker unemployed and the job vacant.

2.2 Value Functions

In the remainder of this section, we develop the formal model. We emphasize in advance that our analytical results in Sections 3 and 4 rely on only three expressions: the selection equation (6), the structural wage equation (9), and the steady state equation (12).

We start by formulating the value functions of workers and firms. For a type-x unemployed worker, let the value be V_x^u:

$$rV_x^u = \rho \sum_{y=1}^{Y} v_y \int_0^\infty \max \left\{ V_{x,y}^e(z, W_{x,y}(z)) - V_x^u, 0 \right\} s(z) dz. \tag{1}$$

[3]This is a quadratic matching technology (Diamond and Maskin, 1979), so the total number of matches is a homogeneous of degree two in unemployment $\{u_x\}_{x=1}^{X}$ and vacancies $\{v_y\}_{y=1}^{Y}$. It is straightforward to prove Propositions 2–5 with a linear matching technology (homogeneous of degree one), at the cost only of additional notational complexity. Similarly, our numerical results carry over exactly to an environment with a linear matching technology. This is a consequence of some normalizations that we discuss in Appendix C.2.

At rate ρv_y, the worker meets a vacant type y firm. They then draw match-specific productivity z from a distribution with density $s(z)$. After that, they decide whether to match. If they do, the worker's value jumps to $V^e_{x,y}(z, W_{x,y}(z))$, the value of a type x worker matched to a type y firm in a match with productivity z and earning the equilibrium wage $W_{x,y}(z)$. Nash bargaining implies that both parties agree on whether to match, so the worker matches whenever the value of being in the match, $V^e_{x,y}(z, W_{x,y}(z))$, exceeds the value of being unmatched, V^u_x.

Once the worker is in the match, we have the corresponding Bellman equation

$$rV^e_{x,y}(z, W) = W + \delta(V^u_x - V^e_{x,y}(z, W)). \tag{2}$$

This equation describes a type x worker at a type y firm with match-specific shock z earning an arbitrary wage W. The worker earns the wage until the match ends exogenously. This implies that the worker will accept the match ($V^e_{x,y}(z, W) \geq V^u_x$) if and only if $W \geq rV^u_x \equiv \bar{w}_x$, the worker's reservation wage.

The Bellman equations for firms are symmetric:

$$rV^v_y = \rho \sum_{x=1}^{X} \int_0^\infty u_x \max\left\{V^f_{y,x}(z, W_{x,y}(z)) - V^v_y, 0\right\} s(z)dz \tag{3}$$

$$rV^f_{y,x}(z, W) = zf_{x,y} - W + \delta(V^v_y - V^f_{y,x}(z, W)). \tag{4}$$

Notably, a type y firm earns flow profit $zf_{x,y} - W$ when employing a type x worker in a match with productivity z and paying a wage W. The firm will accept the match ($V^f_{y,x}(z, W) \geq V^v_y$) if and only if $zf_{x,y} - W \geq rV^v_y \equiv \bar{\pi}_y$, the firm's reservation profit.

We define the match surplus as

$$V^s_{x,y}(z) \equiv V^e_{x,y}(z, W) + V^f_{y,x}(z, W) - V^u_x - V^v_y = \frac{\max\{zf_{x,y} - \bar{w}_x - \bar{\pi}_y, 0\}}{r + \delta}, \tag{5}$$

where the second equation follows from equations (2) and (4) and the definitions of the reservation wage $\bar{w}_x$ and reservation profit $\bar{\pi}_y$. The match surplus is positive, so there exists a wage W which is acceptable both to the worker ($V^e_{x,y}(z, W) \geq V^u_x$) and firm ($V^f_{y,x}(z, W) \geq V^v_y$), if and only if the match-specific productivity shock z exceeds the reservation level $\bar{z}_{x,y}$, where

$$\bar{z}_{x,y} \equiv \frac{\bar{w}_x + \bar{\pi}_y}{f_{x,y}}. \tag{6}$$

This *selection equation* is central to our analysis. As we discuss in Sections 3 and 4, it shapes which wages we observe in the data and how often we observe different types of matches. For now, an implication of the reservation productivity level is that we can rewrite the match surplus as

$$V^s_{x,y}(z) = \frac{f_{x,y}\max\{z - \bar{z}_{x,y}, 0\}}{r+\delta}. \tag{7}$$

Finally, when the match surplus is positive, we use Nash bargaining to pin down the equilibrium wage $W_{x,y}(z)$:

$$W_{x,y}(z) = \arg\max_W (V^e_{x,y}(z,W) - V^u_x)^\gamma (V^f_{y,x}(z,W) - V^v_y)^{1-\gamma}. \tag{8}$$

Using equations (2) and (4), as well as the definitions of the reservation wage and reservation profit, it is straightforward to show that this implies the *structural wage equation*

$$W_{x,y}(z) = \bar{w}_x + \gamma(zf_{x,y} - \bar{w}_x - \bar{\pi}_y) \tag{9}$$

for $z \geq \bar{z}_{x,y}$. Putting this together with the fact that a worker and firm match whenever $z \geq \bar{z}_{x,y}$, we get that type x workers match with type y firms at rate $\rho u_x v_y S(\bar{z}_{x,y})$, with matches formed whenever $W_{x,y}(z) \geq \bar{w}_x$, a type-$x$ worker's reservation wage. Symmetrically, a type-y firm agrees to match with a type-x worker whenever $zf_{x,y} - W_{x,y}(z) \geq \bar{\pi}_y$, a type-$y$ firm's reservation profit.

We can now combine these equations to get a simpler expression for a worker's reservation wage and a firm's reservation profit. Eliminate the wage from equation (2) using equation (9) and the definition of reservation productivity in equation (6) to get $V^e_{x,y}(z, W_{x,y}(z)) - V^u_x = \frac{\gamma f_{x,y}(z-\bar{z}_{x,y})}{r+\delta}$. Substitute that into equation (1) to get

$$\bar{w}_x = \frac{\gamma\rho}{r+\delta}\sum_{y=1}^{Y} v_y f_{x,y}\int_{\bar{z}_{x,y}}^{\infty}(z-\bar{z}_{x,y})s(z)dz. \tag{10}$$

Analogous steps lead to the equation for a firm's reservation profit:

$$\bar{\pi}_y = \frac{(1-\gamma)\rho}{r+\delta}\sum_{x=1}^{X} u_x f_{x,y}\int_{\bar{z}_{x,y}}^{\infty}(z-\bar{z}_{x,y})s(z)dz. \tag{11}$$

2.3 State Variables and Equilibrium

To close the model, we need to find the steady state values of u_x and v_y. To do this, we first define the steady state measure of (x, y) matches, $\phi_{x,y}$. This satisfies the *steady state equation*

$$\delta\phi_{x,y} = \rho u_x v_y S(\bar{z}_{x,y}). \tag{12}$$

The left hand side is the rate that these matches end, while the right hand side is the rate that unemployed type-x workers (measure u_x) meet vacant type-y jobs (ρv_y) in a match with an acceptable z (share $S(\bar{z}_{x,y})$ of such meetings).

By adding partner types, we can then recover the unemployment and vacancy measures:

$$u_x = m_x - \sum_{y=1}^{Y} \phi_{x,y} \tag{13}$$

$$v_y = n_y - \sum_{x=1}^{X} \phi_{x,y}. \tag{14}$$

A steady state equilibrium is given by $(\bar{w}, \bar{\pi}, \bar{z}, \phi, u, v)$ satisfying equations (6), (10), (11), (12), (13), and (14). We can prove

Proposition 1 *An equilibrium exists. In any equilibrium, the reservation wage $\bar{w}_x$ and reservation profit $\bar{\pi}_y$ are strictly positive for all x and y.*

All the proofs are in Appendix A.

We note that a model like this may have multiple equilibria (Burdett and Coles, 1997). All of our claims apply to any steady state equilibrium.

2.4 Monotonicity

We first prove a useful preliminary result, that the reservation wage and reservation profit are increasing if the production function is increasing:

Lemma 1 *Assume $f_{x,y}$ is strictly increasing in x and y. Then the reservation wage $\bar{w}_x$ and reservation profit $\bar{\pi}_y$ are strictly increasing.*

The proof is in Appendix A.

We note that since $\bar{w}_x$ and $\bar{\pi}_y$ are strictly positive, equation (6) implies $\bar{z}_{x,y}$ is strictly positive as well. And since there are a finite number of types of workers and firms, $\underline{z} \equiv \min_{x,y} \bar{z}_{x,y}$ is strictly positive as well. It follows from the definition of equilibrium that in any steady state equilibrium, the behavior of $S(z)$ at $z < \underline{z}$ does not affect the equilibrium $(\bar{w}, \bar{\pi}, \bar{z}, u, v)$. We build on this observation in Section 3.1 below.

3 Average Log Wage

The goal of this section is to characterize the *average log wage* in a match between a type-x worker and a type-y firm, $w^*_{x,y}$. To do this, we use two equations from our model. The first is the structural wage equation (9), which defines wages as a function of the worker type x, the firm type y, and the match-specific shock z. The second is the selection equation (6), which specifies that matches form if and only if the idiosyncratic shock z exceeds the threshold $\bar{z}_{x,y}$. Using these two pieces of notation, the average log wage in an (x, y) match is

$$w^*_{x,y} \equiv \frac{\int_{\bar{z}_{x,y}}^{\infty} \log(W_{x,y}(z))s(z)dz}{S(\bar{z}_{x,y})}. \tag{15}$$

We think of this as a reduced-form wage equation, capturing what we observe in the data. In this section we illustrate how selection affects the average log wage under different distributional assumptions for the match-specific shock z.

The average log wage is of considerable empirical interest. Bonhomme, Lamadon and Manresa (2019) prove that under reasonable conditions,[4] the average log wage is identified in administrative wage data. They also estimate it using Swedish data and find that the average log wage is increasing in worker and firm type and perhaps slightly submodular, meaning that the lowest type workers get a slightly larger increase in their log wage by moving to more productive jobs.

The average log wage is related to the more familiar log-linear wage equation proposed by Abowd, Kramarz and Margolis (1999), as we discuss in Appendix B. In short, that paper proposed that the log wage $w_{i,t}$ of worker i at firm $j = \mathcal{J}_{i,t}$ in period t can be expressed as

$$w_{i,t} = \alpha_i + \psi_{\mathcal{J}_{i,t}} + \varepsilon_{i,t}, \tag{16}$$

where $\varepsilon_{i,t}$ is a random variable with mean zero for all i, t, and potential employers j. Under these conditions, $w^*_{x_i,y_j} = \alpha_i + \psi_j$, where x_i is the type of worker i and y_j is the type of firm j. Our formulation allows for non-separabilities in w^*.

3.1 Pareto-Distributed Match Quality

We first characterize wage behavior under the assumption that z has a Pareto distribution with scale parameter $z_0 > 0$ and tail parameter $\theta > 1$, so $S(z) = (z/z_0)^{-\theta}$ for $z \geq z_0$ and $S(z) = 1$ otherwise. We also parameterize the contact rate as $\rho = \bar{\rho} z_0^{-\theta}$, so the rate that an unemployed worker contacts a vacant type-y firm and has productivity at least z is $\bar{\rho} z^{-\theta} v_y$ for $z > z_0$. Notably, this is independent of z_0. We focus throughout on cases where there is an interior solution for the threshold $\bar{z}_{x,y}$ for all pairs (x, y), $z_0 < \underaccent{\bar}{z} = \min_{x,y} \bar{z}_{x,y}$, as will be

[4] Our baseline model does not satisfy the sufficient conditions for identification in Bonhomme, Lamadon and Manresa (2019) due to the failure of a rank condition. Our extension with on-the-job search in Section 5 satisfies these conditions for their dynamic model.

the case if z_0 is sufficiently small.[5] Under these conditions, we have the following result:

Proposition 2 *Assume z has a Pareto distribution with scale parameter z_0, $0 < z_0 < \min_{x,y} \bar{z}_{x,y}$, and shape parameter $\theta > 1$. Then*

$$w^*_{x,y} \equiv \int_0^\infty \log\big(\bar{w}_x + \gamma(\bar{w}_x + \bar{\pi}_y)q\big)\theta(1+q)^{-\theta-1}dq. \tag{17}$$

Moreover,

1. *for all x_1 and x_2 with $\bar{w}_{x_1} < \bar{w}_{x_2}$, $w^*_{x_1,y} < w^*_{x_2,y}$ for all y;*
2. *for all y_1 and y_2 with $\bar{\pi}_{y_1} < \bar{\pi}_{y_2}$, $w^*_{x,y_1} < w^*_{x,y_2}$ for all x;*
3. *for all x_1, x_2, y_1, and y_2 with $\bar{w}_{x_1} < \bar{w}_{x_2}$ and $\bar{\pi}_{y_1} < \bar{\pi}_{y_2}$, $w^*_{x_1,y_2} + w^*_{x_2,y_1} > w^*_{x_1,y_1} + w^*_{x_2,y_2}$.*

In particular, if $f_{x,y}$ is strictly increasing, the average log wage is strictly increasing and strictly submodular.

Our proof in Appendix A allows for any transformation of the wage, not just the log. We focus in the text on the average log wage because this is the focus of the empirical literature.

There are many pieces to unpack from Proposition 2. First, the average log wage depends only on four numbers: the worker's reservation wage $\bar{w}_x$, the firm's reservation profit $\bar{\pi}_y$, the worker's bargaining power γ, and the Pareto tail parameter θ. The production technology $f_{x,y}$ does not explicitly enter this expression. We view this as both good news and bad news for empirical research. The bad news is that average log wages are not useful for learning about the production technology f. The good news is that the model makes strong and testable predictions for how the average log wage behaves across different types of matches.

Second, workers who have a higher reservation wage earn more at any type of employer, and similarly firms that have a higher reservation profit pay more to any type of worker. When the production function is strictly increasing, this implies that more productive firms pay higher wages to all worker types, in line with a variety of empirical evidence, e.g. Alvarez, Benguria, Engbom and Moser (2018). But to understand the power of Proposition 2, it is useful to consider a non-monotonic production function f. That is, suppose that there are worker types x_1 and x_2 and firm types y_1 and y_2 such that $f_{x_1,y_1} > f_{x_1,y_2}$ and $f_{x_2,y_1} < f_{x_2,y_2}$. One might conjecture that with Nash bargaining, this non-monotonicity would imply a similar ordering of average log wages, $w^*_{x_1,y_1} > w^*_{x_1,y_2}$ and $w^*_{x_2,y_1} < w^*_{x_2,y_2}$. Proposition 2

[5] For given $\bar{\rho}$, a change in z_0 does not affect the equilibrium allocation as long as $z_0 \leq \underline{z}$. Thus focusing on small values of z_0 does not change the arrival rate of "good" matches. We could sidestep any discussion of z_0 by assuming that meetings with match quality at least equal to z occur at rate $\bar{\rho}z^{-\theta}$ times the relevant vacancy or unemployment rate for any z, as in Oberfield (2018) and Buera and Oberfield (2020).

establishes that this cannot happen: $\bar{\pi}_{y_2} \gtreqless \bar{\pi}_{y_1} \Rightarrow w^*_{x,y_2} \gtreqless w^*_{x,y_1}$ for all x.

Proposition 2 also implies that all workers have the same ranking of average log wages across firm types y, determined by the reservation profit $\bar{\pi}_y$. This result stands in contrast to the findings in Shimer and Smith (2000), a similar model except that z has a degenerate distribution. In that case, an unproductive worker's wage may be maximized at low-type firm, while a more productive worker's wage is maximized at a high-type firm. See Eeckhout and Kircher (2011) and Lopes de Melo (2018) for an elaboration of this observation. The common ranking that our model predicts is in line with a large body of empirical evidence.

Finally, we prove that the average log wage is submodular in the worker's reservation wage $\bar{w}_x$ and the firm's reservation profit $\bar{\pi}_y$. That means that the average increase in the log wage of a given worker who moves from a low $\bar{\pi}_y$ firm to a high $\bar{\pi}_y$ firm is decreasing in the worker's reservation wage $\bar{w}_x$. Again, this places strong restrictions for how wages vary across different types of matches.

3.2 Exponentially-Distributed Match Quality

Next we characterize the average log wage under the assumption that z has an exponential distribution with parameter $\theta > 0$, so $S(z) = e^{-\theta z}$ for all $z \geq 0$.

Proposition 3 *Assume z has an exponential distribution with parameter $\theta > 0$. Then*

$$w^*_{x,y} = \theta \int_0^\infty \log(\bar{w}_x + \gamma q f_{x,y}) e^{-\theta q} dq. \tag{18}$$

Assume that for all $x_1 < x_2$ and y, $f_{x_1,y} < f_{x_2,y}$. Then $w^*_{x_1,y} < w^*_{x_2,y}$. Additionally, take any x, y_1, and y_2. Then $w^*_{x,y_1} \gtreqless w^*_{x,y_2}$ if and only if $f_{x,y_1} \gtreqless f_{x,y_2}$.

If $f_{x,y} = f^1_x f^2_y$ for strictly increasing functions f^1 and f^2, then $w^*_{x,y}$ is strictly increasing and strictly submodular.

Our proof of the first part of the Proposition in Appendix A allows for any transformation of the wage, not just the log. The submodularity result for a multiplicatively-separable production function relies on the logarithmic assumption.

Some of these results parallel the earlier results for a Pareto distribution. In particular, if f is strictly increasing in both x and y, then so is the average log wage. Thus the model predicts that more productive workers are paid higher wages at any firm, and that more productive firms pay higher wages to any worker.

On the other hand, when S is exponential, the average log wage depends on the entire production function f, not just on the reservation wage and profit, as was the case with a Pareto. This allows for some interesting possibilities. For example, a type-x worker's average log wage at a type-y firm is increasing in $f_{x,y}$, and so different types of workers may have a different ordering of average log wages across firm types.

When $f_{x,y}$ is strictly increasing and multiplicatively separable. Proposition 3 establishes that the average log wage is increasing and submodular, exactly as we established in Proposition 2 with the Pareto distribution. Thus once again we see that complementarity in the production function does not necessarily carry over to the average log wage.

3.3 Other Distributions

A natural question is whether monotonicity of the production function f guarantees monotonicity of the average log wage for other distributions of the match-specific shock z. In general, the answer is no. First, if the distribution of match-specific productivity shocks is degenerate, wages are generally a hump-shaped function of a firm's type for a given worker (Shimer and Smith, 2000; Eeckhout and Kircher, 2011; Lopes de Melo, 2018). Second, we are able to construct an economy where the production function is strictly increasing and z has a continuous distribution on the positive real line, yet the average log wage is not monotonic in the firm type. Still, it is easy to construct other examples where the wage is monotone in both the worker and firm types, and for this reason we believe that the special cases we highlight here are useful for understanding more general properties of the model.

3.4 Selection in the Wage Equation

The previous sections established conditions under which the average log wage $w^*_{x,y}$ is increasing and submodular. While this characterizes what we measure in administrative wage data, systematic selection in which matches form means that it may not be the most useful object for understanding how firms affect wages. To explain why, we first need to be precise about which wages we observe in administrative data and which ones we do not observe.

For every worker i of type x_i and every firm j of type y_j, including those who never meet each other, let $z_{i,j}$ denote the realization of their match-specific productivity shock. If the match would be acceptable to the worker and firm, $z_{i,j} \geq \bar{z}_{x_i,y_j}$, their potential wage satisfies equation (9):

$$W^p_{i,j} = W_{x_i,y_j}(z_{i,j}),$$

with $W^p_{i,j} > \bar{w}_{x_i}$ when $z_{i,j} > \bar{z}_{x_i,y_j}$. For realizations of the match-specific shock below $\bar{z}_{x_i,y_j}$, Nash bargaining does not uniquely determine the potential wage, but we can bound it above by the worker's reservation wage, $W^p_{i,j} \leq \bar{w}_{x_i}$. These potential wages may not even be offered to worker i if they contact firm j; instead, the firm may simply decline to offer i a job.

Now let $\mathcal{J}_{i,t}$ denote worker i's employer at time t. We only observe the wage of worker i at firm j when $\mathcal{J}_{i,t} = j$ for some t, which requires both that $z_{i,j} \geq \bar{z}_{x_i,y_j}$ and that worker i and firm j meet. This distinction between potential and realized wages helps us interpret wage differences across firms.

A fundamental question in the empirical literature (Card, Cardoso, Heining and Kline, 2018, p. S16) is whether "conventionally estimated firm-specific pay premiums predict the wage changes associated with exogenously induced job accessions and separations."[6] The average log wage of a type-x worker at a type-y firm in period t, a conventional worker- and firm-specific pay premium, conditions on realized matches:

$$w^*_{x,y} = \mathbb{E}\big(\log W^p_{i,j} | x = x_i,\ y = y_j,\ \text{and } j = \mathcal{J}_{i,t} \big).$$

In contrast, the expected log wage following an exogenously-induced accession averages over all potential pairs, including those that are mutually unacceptable:

$$w^p_{x,y} \equiv \mathbb{E}\big(\log W^p_{i,j} | x = x_i \text{ and } y = y_j \big).$$

These coincide if and only if $\bar{z}_{x,y} \leq z_0$, so all (x, y) meetings result in matches. Otherwise firm-specific pay premia overstate the wage increase from an exogenously-induced job accession.

There are two special cases where selection may not be a huge problem. First, when match-specific productivity is degenerate, $z_{i,j} = z^*$ for all pairs (i, j) as in Shimer and Smith (2000), then $w^*_{x,y}$ is the log wage in all potential (x, y) matches. However, this model cannot explain within-match wage residuals, predicts wages are generally hump-shaped in firm type (Eeckhout and Kircher, 2011; Lopes de Melo, 2018), and provides no guidance about the potential wage in an (x, y) match if we never observe such a match, as will be the case when $z^* < \bar{z}_{x,y}$. Second, if all worker-firm pairs have the same acceptance threshold, $\bar{z}_{x,y} = \bar{z}$, selection would truncate the distribution of match-specific shocks identically for all pairs, so $w^*_{x,y}$ still measures the value of randomly assigning a type x worker to a type y firm, up to a missing constant reflecting the probability the match is acceptable, $S(\bar{z})$.

In our model, however, $\bar{z}_{x,y}$ varies systematically with both worker and firm types. We argue in the next section that this systematic selection is empirically reasonable and evident in the data: it generates realistic assortative matching between workers and firms. High productivity workers are more likely to match with high productivity firms than low pro-

[6] Card, Heining and Kline (2013), Abowd, McKinney and Schmutte (2019), and others call the assumptions needed for an affirmative answer to this question "exogenous mobility." Section 3 in Kline (2024) carefully lays out these conditions. Our model violates strict exogeneity because worker i's error term in period t depends on the firm they work at. For example, if worker i would have been exogenously induced to work at a firm j with $z_{i,j} < \bar{z}_{x_i,y_j}$ in period t, the error term would have been negative.

ductivity workers are, even though low productivity workers gain proportionately more from such matches when they occur. This assortative matching pattern arises because selection differs systematically across worker-firm pairs. Observed sorting patterns thus provide independent empirical validation of our model's key mechanism, systematic selection, which simultaneously explains both assortative matching and the shape of average log wages.

4 Assortative Matching

This section shows that our model can generate (positively) assortative matching between workers and firms. As in Section 3, we only use two equations from our model. The first in the steady state equation (12), which explains how the measure of matches $\phi_{x,y}$ depends on both meeting rates and the selection threshold $\bar{z}_{x,y}$. The second is again the selection equation (6).

We find conditions under which ϕ satisfies a strong notion of assortative matching, the *monotone likelihood ratio order*. We say that ϕ has the monotone likelihood ratio order if for all $x_1 < x_2$ and $y_1 < y_2$,

$$\frac{\phi_{x_2,y_2}}{\phi_{x_2,y_1}} > \frac{\phi_{x_1,y_2}}{\phi_{x_1,y_1}}.$$

This means that high type workers are relatively more likely to match with high type firms. Manipulating the fractions, we find that this is equivalent to stating that high type firms are relatively more likely to match with high type workers. A well-known implication of the monotone likelihood ratio order is that higher type workers and firms have a better distribution of match partners in the sense of first order stochastic dominance and that the correlation between the types of matched workers and firms is strictly positive.

4.1 Assortative Matching in Productivity

We first find conditions for assortative matching for the Pareto case:

Proposition 4 *Assume z has a Pareto distribution with scale parameter z_0, $0 < z_0 < \min_{x,y} \bar{z}_{x,y}$, and shape parameter $\theta > 1$. Also assume $f_{x,y}$ is strictly increasing and weakly log-supermodular. Then the measure of matches ϕ has the monotone likelihood ratio order.*

Weak log-supermodularity of f is equivalent to $f_{x_1,y_1} f_{x_2,y_2} \geq f_{x_1,y_2} f_{x_2,y_1}$ for all $x_1 < x_2$ and $y_1 < y_2$. Since f is strictly positive, it is a statement that more productive workers have a weak comparative advantage working at more productive firms ($f_{x,y_2}/f_{x,y_1}$ is nondecreasing in x when $y_2 > y_1$) and that more productive firms have a weak comparative advantage hiring more productive workers ($f_{x_2,y}/f_{x_1,y}$ is nondecreasing in y when $x_2 > x_1$).

The results for the exponential case impose somewhat stronger assumptions.

Proposition 5 *Assume z has an exponential distribution with parameter $\theta > 0$. Also assume $f_{x,y}$ is strictly increasing and either $-1/f_{x,y}$ is weakly supermodular or $f_{x,y}$ is multiplicatively separable. Then the measure of matches ϕ has the monotone likelihood ratio order.*

For a strictly increasing function f, the assumption that $-1/f_{x,y}$ is weakly supermodular implies that $f_{x,y}$ is strictly log supermodular, while multiplicative separability is the borderline case for weak log supermodularity.

The combination of submodular average log wages (Propositions 2 and 3) and assortative matching (Propositions 4 and 5) may seem surprising. After all, if low productivity workers gain proportionately more from moving to high productivity firms, why do they work there less frequently? Once again, our model's answer is selection. We use wage data as measured in a typical administrative data set, the wage paid by a firm to its employee. Such data sets do not have information about meetings that do not result in matches, i.e. about wage offers that are below the worker's reservation wage. Low wage workers rarely get acceptable wage offers from high productivity firms, and so rarely match there; but when such wage offers do materialize, our model predicts that the average log wage is higher than at a low productivity firm.

4.2 Assortative Matching in Wages

Define the average log wage paid to a type x worker and the average log wage paid by a type y firm:

$$\lambda_x \equiv \frac{\sum_{y=1}^{Y} w^*_{x,y}\phi_{x,y}}{\sum_{y=1}^{Y} \phi_{x,y}}, \tag{19}$$

$$\mu_y \equiv \frac{\sum_{x=1}^{X} w^*_{x,y}\phi_{x,y}}{\sum_{x=1}^{X} \phi_{x,y}} \tag{20}$$

We combine our earlier results to obtain conditions for λ_x and μ_y to be strictly increasing:

Corollary 1

1. *Assume $S(z) = (z/z_0)^{-\theta}$ with $z_0 > 0$ and $\theta > 1$. Also assume $f_{x,y}$ is strictly increasing and weakly log-supermodular. Then λ_x and μ_y are strictly increasing.*
2. *Assume $S(z) = e^{-\theta z}$ with $\theta > 0$. Also assume $f_{x,y}$ is strictly increasing and either $-1/f_{x,y}$ is weakly supermodular or $f_{x,y}$ is multiplicatively separable. Then λ_x and μ_y are strictly increasing.*

The proof follows immediately from monotonicity of $w^*_{x,y}$ in both arguments (Propositions 2 and 3) and first order stochastic dominance of the match density ϕ (Propositions 4 and 5).[7]

This result implies that if f is strictly increasing and satisfies the appropriate supermodularity condition, there is an increasing mapping between a worker's type x and their average log wage λ_x, and an increasing mapping between a firm's type y and its average log wage μ_y. Using the monotonicity of λ and μ, Propositions 4 and 5 then imply that high wage workers (high λ_x) match more frequently with high wage firms (high μ_y) in the sense of the monotone likelihood ratio order. That is, we have found conditions not only for assortative matching of more productive workers to more productive firms, but also of high wage workers to high wage firms.

In Borovičková and Shimer (2020), we develop unbiased estimates of λ for each worker and μ for each firm. We also show how to obtain consistent estimates of the variance of λ_x across employed workers, the variance of μ_y across filled jobs, and the covariance between λ_x and μ_y across matched workers and firms using a short panel with many workers and firms. Using administrative data from Austria, we verify that in fact the correlation between λ and μ is strictly positive.

5 On-the-Job Search

In this section, we extend our model to allow for on-the-job search, so employed workers can search for better jobs. This implies that vacant jobs meet both employed and unemployed workers, and that filled jobs sometimes end when the employee finds a better job opportunity.

We introduce on-the-job search for two reasons. First, the assumption is realistic. Employed workers often climb a job ladder, attaining higher wages as they move from job-to-job (Topel and Ward, 1992). It is important to see whether our findings on wages and assortative matching carry over to such an environment. Second, models with on-the-job search can generate considerably more within-worker-type wage dispersion than models where only unemployed workers search (Hornstein, Krusell and Violante, 2011). This is important for the quantitative results in Section 6.

The model is the same as the baseline model but we now assume that workers can meet vacancies also when employed. An unemployed worker contacts a type-y vacancy according to a Poisson process with arrival rate $\rho_0 v_y$, while an employed worker contacts a type-y vacancy according to a Poisson process with arrival rate $\rho_1 v_y$, with $\rho_1 < \rho_0$. Following

[7] Perhaps surprisingly, monotonicity of $w^*_{x,y}$ and ϕ having the monotone likelihood ratio order do not imply monotonicity of the Abowd, Kramarz and Margolis (1999) fixed effects $\bar{\alpha}_x$ and $\bar{\psi}_y$, defined in Appendix B and equations (39) and (40).

Bonhomme, Lamadon and Manresa (2019), we assume that a type-x worker matched with a type-y firm with a match-specific productivity shock z gets a fraction γ of the joint match surplus. The resulting wage $W_{x,y}(z)$ does not depend on whether the worker has been hired from employment or unemployment, and is constant during the duration of the match, so there is no renegotiation.[8] The rest of the model is the same as our baseline model.

Selection shapes which matches we observe through two distinct channels in this model. First, as in our baseline model without on-the-job search, unemployed workers do not accept all meetings with firms, only forming matches when the match-specific productivity realization is sufficiently high. Second, employed workers provide an additional source of selection, moving to new jobs that offer them higher value than their current job. Both channels affect which wages we observe in equilibrium and how often we observe different types of matches. To understand how, we write out the model equations.

For a type-x worker, the Bellman equation for the value of being unemployed is

$$rV_x^u = \rho_0 \sum_{y=1}^{Y} v_y \int_0^\infty \max\left\{V_{x,y}^e(z) - V_x^u, 0\right\} s(z)dz, \tag{21}$$

the same as equation (1), except that we suppress the dependence of the value of employment on the wage. The value of a type x worker employed at a type y firm with a match-specific shock z is

$$\begin{aligned} rV_{x,y}^e(z) = W_{x,y}(z) + \delta(V_x^u - V_{x,y}^e(z)) \\ + \rho_1 \sum_{y'=1}^{Y} v_{y'} \int_0^\infty \max\left\{V_{x,y'}^e(z') - V_{x,y}^e(z), 0\right\} s(z')dz'. \end{aligned} \tag{22}$$

The worker earns the wage $W_{x,y}(z)$ until the match ends. This happens either exogenously at rate δ or endogenously when the worker finds a better job. At the rate $\rho_1 v_{y'}$, the worker meets a type-y' firm with a vacancy and draws a match-specific shock z' according to distribution s. If the value of being employed at y' with z' exceeds the value of current employment, $V_{x,y'}^e(z') > V_{x,y}^e(z)$, the worker accepts the new job offer.

We next formulate Bellman equations for a firm. The Bellman equation for a vacant

[8] Our model is thus different than the competing offers framework in Cahuc, Postel-Vinay and Robin (2006). We use this wage setting protocol because it is consistent with the assumptions in the dynamic model in Bonhomme, Lamadon and Manresa (2019), while the competing offers framework is not.

type-y job is

$$
\begin{aligned}
rV_y^v = \rho_0 \sum_{x=1}^{X} \int_0^\infty & u_x \max \left\{ V_{y,x}^f(z)) - V_y^v, 0 \right\} s(z) dz \\
& + \rho_1 \sum_{x=1}^{X} \sum_{y'=1}^{Y} \int_0^\infty \phi_{x,y'}(z') \left(\int_0^\infty (V_{y,x}^f(z) - V_y^v) \mathbb{1}_{V_{x,y}^e(z) > V_{x,y'}^e(z')} s(z) dz \right) dz', \quad (23)
\end{aligned}
$$

where $\phi_{x,y}(z)$ is the density of (x, y, z) matches, and $\mathbb{1}$ is an indicator function. A vacant job meets an unemployed type-x worker at the rate $\rho_0 u_x$ and decides to form a match if firm surplus is positive, $V_{y,x}^f(z) - V_y^v > 0$. At the rate $\rho_1 \phi_{x,y'}(z')$, the vacancy meets a type-x worker employed in a type-y' firm with match-specific shock z'. They draw a new match-specific shock z. If the worker prefers the offer from the poaching firm, i.e. $V_{x,y}^e(z) > V_{x,y'}^e(z')$, the worker accepts the new offer. Note that whenever the worker prefers the poaching offer, the poaching firm is willing to hire the worker.

Finally, we write the value of a type-y firm employing a type-x worker with match-specific shock z:

$$
\begin{aligned}
rV_{y,x}^f(z) = z f_{x,y} - & W_{x,y}(z) \\
& + \left(\delta + \rho_1 \sum_{y'=1}^{Y} v_{y'} \int_0^\infty \mathbb{1}_{V_{x,y'}^e(z') > V_{x,y}^e(z)} s(z') dz' \right) \left(V_y^v - V_{y,x}^f(z)) \right). \quad (24)
\end{aligned}
$$

The firm receives the profit flow $z f_{x,y} - W_{x,y}(z)$ during the duration of the match. The match can dissolve for one of two reasons: exogenously at the rate δ or endogenously due to a worker accepting an outside offer. This happens when the worker meets a type-y' firm with a vacancy and draws a shock z' such that the value of being employed there exceed worker's value of being in the current match, $V_{x,y'}^e(z') > V_{x,y}^e(z)$.

Define the match surplus $V_{x,y}^s(z) = V_{x,y}^e(z) + V_{y,x}^f(z) - V_x^u - V_y^v$. We assume that for all (x, y, z), the worker keeps a share γ of the match surplus, $V_{x,y}^e(z) - V_x^u = \gamma V_{x,y}^s(z)$, and the firm keeps the remainder, $V_{y,x}^f(z) - V_y^v = (1 - \gamma) V_{x,y}^s(z)$. Using these as well as equations (21)–(24), we find the Bellman equation for the match surplus

$$
\begin{aligned}
(r + \delta) V_{x,y}^s(z) = z f_{x,y} - & rV_x^u - rV_y^v \\
& + \rho_1 \sum_{y'=1}^{Y} v_{y'} \int_0^\infty \mathbb{1}_{V_{x,y'}^s(z') \geq V_{x,y}^s(z)} \left(\gamma V_{x,y'}^s(z') - V_{x,y}^s(z)) \right) s(z') dz'. \quad (25)
\end{aligned}
$$

Here we use the fact that $V_{x,y}^e(z) > V_{x,y'}^e(z')$ if and only if $V_{x,y}^s(z) > V_{x,y'}^s(z')$.

We can also rewrite the values of unemployed workers and vacant firms using equations (21) and (23) as well as the surplus sharing rule:

$$rV_x^u = \gamma\rho_0 \sum_{y=1}^{Y} v_y \int_0^\infty \max\{V_{x,y}^s(z), 0\} s(z) dz, \tag{26}$$

$$\begin{aligned} rV_y^v = {} & (1-\gamma)\rho_0 \sum_{x=1}^{X} \int_0^\infty u_x \max\{V_{x,y}^s(z)), 0\} s(z) dz \\ & + (1-\gamma)\rho_1 \sum_{x=1}^{X}\sum_{y'=1}^{Y} \int_0^\infty \phi_{x,y'}(z') \left(\int_0^\infty V_{x,y}^s(z) \mathbb{1}_{V_{x,y}^s(z) > V_{x,y'}^s(z')} s(z) dz \right) dz'. \end{aligned} \tag{27}$$

To close the model, we characterize the distribution of unemployed workers u_x, vacant jobs v_y and the density of matches $\phi_{x,y}(z)$. The steady-state measure of (x, y, z) matches satisfies

$$\begin{aligned} \phi_{x,y}(z) & \left(\delta + \rho_1 \sum_{y'=1}^{Y} v_{y'} \int_0^\infty \mathbb{1}_{V_{x,y'}^s(z') \geq V_{x,y}^s(z)} s(z') dz' \right) \\ & = v_y s(z) \left(\rho_0 u_x \mathbb{1}_{V_{x,y}^s(z) \geq 0} + \rho_1 \sum_{y'=1}^{Y} \int_0^\infty \mathbb{1}_{V_{x,y}^s(z) \geq V_{x,y'}^s(z')} \phi_{x,y'}(z') dz' \right). \end{aligned} \tag{28}$$

The left hand side is the measure of matches that are destroyed through exogenous separations and through endogenous on-the-job search. The right hand side counts matches which are created through a vacancy meeting an unemployed worker (first term) or an employed worker who prefers the new offer.

The measures of unemployed and vacant jobs satisfy

$$u_x = m_x - \sum_{y=1}^{Y} \int_0^\infty \phi_{x,y}(z) dz \tag{29}$$

$$v_y = n_y - \sum_{x=1}^{X} \int_0^\infty \phi_{x,y}(z) dz. \tag{30}$$

A steady state equilibrium is values V_x^u, V_y^v, and $V_{x,y}^s$, as well as matched and unmatched rates $\phi_{x,y}(z)$, u_x, and v_y solving equations (25)–(30).

Finally, using the surplus sharing rule and equation (22), we find an expression for the wage

$$W_{x,y}(z) = rV^u_x + \gamma(r+\delta)V^s_{x,y}(z) - \gamma\rho_1 \sum_{y'=1}^{Y} v_{y'} \int_0^\infty \max\left\{V^s_{x,y'}(z') - V^s_{x,y}(z), 0\right\} s(z')dz'. \quad (31)$$

The wage equals worker's flow value of unemployment plus a γ-share of the flow match surplus, adjusted for the option value of on-the-job search.

In Appendix C.1, we reformulate these equations by expressing the match-specific productivity shock as a function of the match surplus. This allows us to reduce the model to a system of ordinary differential equations, making it amenable to a numerical solution.

6 Quantitative Evaluation

In this section, we show that our on-the-job search model can generate numerical predictions that are in line with state-of-the-art empirical findings. We propose two calibrations of our model. The first, BLM, is designed to match the variance decomposition of log wages in Bonhomme, Lamadon and Manresa (2019). The second, KSS, matches the same variance decomposition in Kline, Saggio and Sølvsten (2020).

The calibrations serve three purposes. First, our analytical results showed it is possible to match the qualitative wage and sorting patterns in the empirical literature. Here we show it is possible to match their quantitative behavior. Second, we show that the analytical characterizations of average log wages and assortative matching carry over to reasonably-calibrated versions of the on-the-job search model. Third, in the remainder of our paper we use our calibrated model as a laboratory for evaluating several strands of existing research.

6.1 Calibration Strategy

In both calibrations, we set the discount rate to $r = 0.05$, and so think of a time unit as a year. We also set the match dissolution rate to $\delta = 0.25$, which implies that the expected duration of employment is 4 years. We set the number of types at $X = Y = 10$ and assume equal measures of each type, so $m_x = \frac{1}{X}$ and $n_y = \frac{1}{Y}$. This implies in particular that the total measure of workers and firms are the same.

The production function is CES with elasticity of substitution ξ,

$$f_{x,y} = \left(\tfrac{1}{2}\left((1+\Delta_w)^{x-1}\right)^{\frac{\xi-1}{\xi}} + \tfrac{1}{2}\left((1+\Delta_f)^{y-1}\right)^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi}{\xi-1}},$$

where Δ_w and Δ_f are numbers that govern the amount of heterogeneity across workers and firms, respectively. We assume that the distribution of match quality is Pareto, $S(z) = (z/z_0)^{-\theta}$, with a sufficiently small lower bound z_0 such that all (x, y) pairs reject some matches. We set the meeting rate ρ_0 so the average log wage in $(1, 1)$ match is zero, $w^*_{1,1} = 0$.[9] Finally, we assume bargaining power is equal for workers and firms, $\gamma = 0.5$.

This leaves five parameters, the dispersion in worker types Δ_w, the dispersion in firm types Δ_f, the elasticity of substitution ξ, the Pareto tail parameter θ, and the efficiency of on-the-job search ρ_1. In the text, we set $\rho_1 = 0.2\rho_0$, but show results with higher and lower values in Appendix C.3.[10] We set the remaining four parameters to match results from Bonhomme, Lamadon and Manresa (2019) and Kline, Saggio and Sølvsten (2020). We briefly discuss those papers before calibrating those four parameters.

Using administrative data from Sweden, Bonhomme, Lamadon and Manresa (2019) estimate the average log wage $w^*_{x,y}$ in an (x, y) match, as well as the share of such matches $\phi_{x,y}$. Their main findings are depicted in Figure 2 of their paper, as well as in Figure S7 in the supplemental appendix. We summarize those figures here. First, there is strong sorting between workers and firms, with low-type firms mostly employing low-type workers and high-type firms mostly employing high-type workers. Second, average log earnings are increasing in the worker type and firm type. Third, the average log earnings of the lowest worker type is the most responsive to the firm type, consistent with submodular average log wages.[11]

Bonhomme, Lamadon and Manresa (2019) also use their estimates to decompose the variance of log wages into four components: the variance of worker effects $\text{var}(\alpha)$, the variance of firm effects $\text{var}(\psi)$, twice the covariance between worker and firm effects $2\text{cov}(\alpha, \psi)$, and

[9]We prove in Appendix C.2 that an equal proportional change in ρ_0 and ρ_1 causes a proportional change in wages without affect matching patterns.

[10]In our model, ρ_1/ρ_0 determines the number of jobs that a worker holds per employment spell, or equivalently the share of new hires that come from unemployment. In the BLM calibration, workers have between 1.9 and 2.3 jobs per employment spell, so 44 to 52 percent of new hires come from unemployment. In the KSS calibration, workers have 2.7 to 3.3 jobs per employment spell, so only 30 to 37 percent of hires come from unemployment. Using French administrative data, Postel-Vinay and Robin (2002) estimate ρ_1/ρ_0 lies between 0.31 and 0.48, depending on worker occupation and skill category. Jolivet, Postel-Vinay and Robin (2006) estimate this ratio to be 0.03 and 0.19 for different European countries.

[11]Other papers present evidence consistent with submodular average log wages. Card, Heining and Kline (2013) regress log wages on worker and establishment (firm) fixed effects, as we discuss in Section B. They then sort workers and firms into deciles based on their estimated person and establishment effects and compute the average residual for each of the one hundred combinations of deciles. Figure VI in their paper shows that these residuals are on average positive when high types match with low types and negative when low types match with low types or high types match with high types. This is consistent with submodular average log wages, so low worker-types increase their average log wage by more than the typical worker when they increase the firm type. On the other hand, Card, Cardoso and Kline (2016) show analogous calculations using Portuguese data in Figures B5 and B6 of their online appendix. They find that log wage residuals are positive for low-type workers working in low-type firms and negative for low-type workers in high-type firms, which is consistent with average log wages being supermodular.

	$\text{var}(\log W)$	$\text{var}(\alpha)$	$\text{var}(\psi)$	$2\text{cov}(\alpha,\psi)$	$\text{var}(\varepsilon)$	$\text{corr}(\alpha,\psi)$
BLM	0.124	0.0747	0.0053	0.0166	0.0274	0.4190
KSS	0.1843	0.1119	0.0240	0.0294	0.0190	0.2830

Table 1: Decomposition of the variance of log wages into four components: variance of worker types, firm types, twice its covariance and the variance of the error term. The first row shows the decomposition reported in the dynamic model in Bonhomme, Lamadon and Manresa (2019). Their Table I shows the variance of wages, while the relative variance of the other components are in their Table V. The second row shows the decomposition in the Leave-Out estimator in Kline, Saggio and Sølvsten (2020). Their Table I shows the variance of the log daily wage, while their Table II shows the remaining components of the variance decomposition.

the variance of the residual $\text{var}(\varepsilon)$. They show their results in their Tables I and V.[12] We show them in the first row of our Table 1 for convenience.

Kline, Saggio and Sølvsten (2020) use a different econometric methodology to perform a similar variance decomposition using administrative data from the Veneto region of Italy. They do not estimate the average log wage for each worker-firm pair, but instead impose that the log wage is additive in the worker's and firm's unobserved type, as in Abowd, Kramarz and Margolis (1999). They address biases in ordinary least squares estimates of the log wage variance decomposition though their Leave-Out estimator. We use this decomposition, shown in their Tables I and II and again repeated in the second row our Table 1 for convenience.

The difference in the variance decomposition between BLM and KSS comes from differences in data sets (Sweden vs. Veneto, monthly earnings vs. daily wages, etc.), as well as differences in the econometric methodologies.[13] Notably, the overall variance of wages is higher in the KSS data than in the BLM data, as are all components of the decomposition except the wage residual. Firm effects are almost five times larger in the KSS data and the correlation between worker and firm types is noticeably smaller. We take no stand on which estimates are preferable, instead showing here that our model is able to match both.

For each set of estimates, we have four moments to target with the four parameters.[14]

[12]Bonhomme, Lamadon and Manresa (2019) propose two estimators, a static model and a dynamic model. We report estimates from their dynamic model because our on-the-job search model satisfies the assumptions needed for consistent estimation of their dynamic model but not of their static model. The estimated variance of the firm effects in their static model is smaller, and indeed smaller than any other estimates we are aware of in the literature.

[13]Our model makes no distinction between wages and hours for employed workers, since hours are fixed. See Bonhomme, Holzheu, Lamadon, Manresa, Mogstad and Setzler (2023) for a comparison of different econometric approaches to estimating the variance decomposition. We believe both estimators are credible.

[14]By construction, $\text{var}(\log W) = \text{var}(\alpha) + \text{var}(\psi) + 2\text{cov}(\alpha,\psi) + \text{var}(\varepsilon)$ and $\text{corr}(\alpha,\psi) = \text{cov}(\alpha,\psi)/\sqrt{\text{var}(\alpha)\text{var}(\psi)}$. Since the four targets are a non-linear function of the model parameters, there is no guarantee that we can match these moments.

	Δ_w	Δ_f	ξ	θ
BLM	0.05169	0.2698	0.964	22.15
KSS	0.07421	0.5853	4.103	81.43

Table 2: Calibrated Parameters to fit the BLM Dynamic Model and KSS Leave-Out variance decomposition of log wages. In both cases, we assume $\rho_1/\rho_0 = 0.2$, $\gamma = 0.5$, $r = 0.05$, and $\delta = 0.25$.

In calculating the model-generated moments, we assume we have infinitely much data. This means we know each worker's type x and each firm's type y. It also means that all distributions are deterministic functions of model parameters. Appendix B.2 details how we compute the worker and firm fixed effects in the model, as well as the variance decomposition of the log wage.

6.2 Calibration Results

Table 2 shows the calibrated parameters. Both calibrations hit the targets in Table 1. In both calibrations, there is considerably more dispersion in firm types than in worker types, $\Delta_f > \Delta_w$, even though the variance of worker fixed effects α is considerably larger than the variance in firm fixed effects ψ.[15] The slight difference in the correlation between α and ψ between the two sets of moments masks large differences in the elasticity of substitution ξ, which is just below one for the BLM calibration but above 4 for the KSS calibration. And finally, the smaller residual wage variance in the BLM calibration implies a much smaller tail parameter for the match-specific shock distribution.

Figure 1 is a direct analogue of Figures 2 and S7 in Bonhomme, Lamadon and Manresa (2019). The top row shows results from the BLM calibration, the bottom row from the KSS calibration. The results are qualitatively very similar to those reported in Bonhomme, Lamadon and Manresa (2019), though naturally results from our model are smoother than those from real-world data. The left panels of Figure 1 shows the average log wage as a function of firm type y. Each line corresponds to a different worker type x. For the model without on-the-job search, we proved in Proposition 2 that the average log wage is increasing in x and y and submodular. The same results hold in our calibrated model with on-the-job search, though the fact that the lines are nearly parallel implies that the average log wage

[15] In the BLM calibration, the most productive worker produces between 22.6 and 23.5 log points more than the least productive worker at a given value of z, depending on firm type. The most productive firm produces between 105 and 106 log points more that the least productive firm at a given level of z. In the KSS calibration, these differences are bigger. The most productive worker produces 3.4 to 36.1 log points more than the least productive at a given firm and match-specific shock realization. The most productive firm produces 296 to 329 log points more than the least productive one.

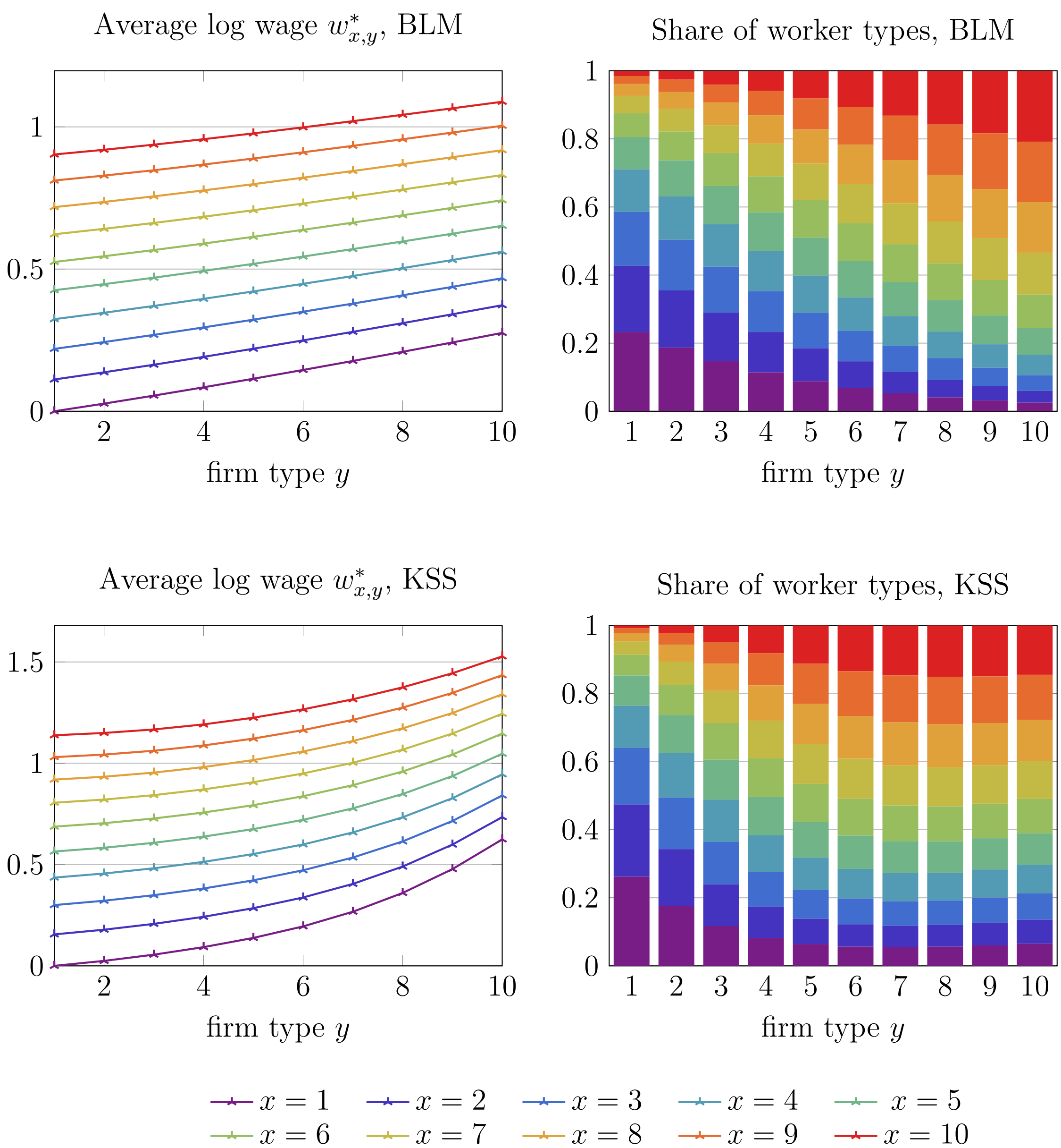

Figure 1: Average log wages and distribution of worker types conditional on firm type in the model with on-the-job search for BLM and KSS calibration. The left panels show the average log wage $w^*_{x,y}$ paid by different firms for different worker types. Each line represents one worker type. The right panels show the distribution of worker types x in firms with different firm types y. The top row shows results for BLM calibration, bottom row for KSS calibration. See Table 2 for parameter values.

is almost additively separable in worker and firm types.

The right panels of Figure 1 shows the distribution of worker types in the different firm types, $\phi_{x,y}/\sum_{x'=1}^{X}\phi_{x',y}$. The BLM calibration satisfies the monotone likelihood ratio order, with high-type firms systematically employing relatively more high-type workers. This is again consistent with the theoretical prediction without on-the-job search (Proposition 4). The KSS calibration has a positive correlation between matched worker and firm types, but the monotone likelihood ratio order is violated for the highest types of firms. That is, while there is a lot of sorting by workers across the lowest five firm types, the distribution of workers in the highest five firm types is fairly similar. This may be because the high elasticity of substitution, $\xi = 4.103$, takes us away from the assumptions in Proposition 4.

The results from both calibrations are qualitatively the same, though there are differences in magnitude resulting from different calibration targets. For example, the average log wage $w^*_{x,y}$ is more convex with respect to firm type y in the KSS calibration than BLM calibration, a consequence of the higher variance of firm fixed effects.

Figures 5–8 in Appendix C.3 show results for both a lower and a higher value of employed search intensity, $\rho_1/\rho_0 = 0.1$ and $\rho_1/\rho_0 = 0.3$. We recalibrate the four parameters Δ_w, Δ_f, ξ, and θ to match the same variance decomposition. We find the same qualitative patterns for average log wages and sorting. We conclude that our model provides a plausible mechanism for interpreting the findings in papers like Bonhomme, Lamadon and Manresa (2019) and Kline, Saggio and Sølvsten (2020).

7 Testing for Selection

Following Abowd, Kramarz and Margolis (1999), a large literature regresses the log wage $w_{i,t}$ of worker i employed by firm $\mathcal{J}_{i,t}$ at time t on fixed effects α_i for worker i, fixed effects $\psi_{\mathcal{J}_{i,t}}$ for the employer $\mathcal{J}_{i,t}$, and an error term, as in equation (16). As we discussed in Section 3.4, a key question is whether the firm effects capture the impact of an exogenously-induced job change. In our model this is not the case, because matches only form when the match-specific productivity shock z is sufficiently high. This selection violates the exogenous mobility assumption (Card, Heining and Kline, 2013; Abowd, McKinney and Schmutte, 2019; Kline, 2024). This section shows why testing for such selection using administrative wage data is impossible unless the researcher takes a stand on the data generating process. We also use our model as a laboratory for evaluating the power of one popular test, the event study proposed by Card, Heining and Kline (2013).

7.1 The Difficulty of Testing

Using administrative wage data, we cannot test whether there is selection without imposing auxiliary assumptions on the structure of the economy. This fundamental identification problem arises because we can construct alternative models that generate identical administrative wage data but have different underlying selection mechanisms. Building on an insight from Flinn and Heckman (1982), we demonstrate this by constructing a model without selection that exactly replicates the observable outcomes of our baseline model with selection. For expositional simplicity, we assume there is no on-the-job search.

In our alternative model, we modify two key assumptions: random search and the distribution of match-specific shocks. Fix $\bar{z}_{x,y}$ from our model with selective job acceptance. We assume that a type-x worker meets a type-y firm at rate $\rho S(\bar{z}_{x,y})v_y$ (rather than ρv_y). In that event, they draw a match-specific productivity shock with survival function $S(z)/S(\bar{z}_{x,y})$ for $z \geq \bar{z}_{x,y}$ (rather than $S(z)$ for $z \geq z_0$), and the wage is given by $W_{x,y}(z)$ defined in equation (9).

By construction, the acceptance thresholds are identical across the two models, but in the alternative model all meetings result in matches, eliminating selection. The two models therefore generate identical observable outcomes in administrative data sets, particularly the joint distribution of wages and match partners. This observational equivalence implies that researchers working with such data cannot test whether observed wages represent a random or selected sample from the broader distribution of potential wages.

The observational equivalence of models with and without selection poses a fundamental empirical challenge. Recall from Section 3.4 that if mobility is exogenous, we can infer the potential log wages from exogenously induced job accessions ($w^p_{x,y}$) using observed log wages ($w^*_{x,y}$) without the need for a structural model. In our baseline model, mobility is endogenous because matches only form when the match-specific productivity shock z exceeds the threshold $\bar{z}_{x,y}$, causing $w^p_{x,y}$ to be systematically smaller than $w^*_{x,y}$. The alternative model modifies meeting rates and the distribution of productivity shocks to ensure that all meetings result in matches. This implies $w^p_{x,y} = w^*_{x,y}$. It follows from the observational equivalence between these two models that administrative data alone cannot tell us the potential wages from exogenously induced accessions. Economic theory combined with alternative sources of variation, such as quasi-experimental policy changes that affect meeting rates between specific worker and firm types, could help identify whether there is selection and what are the wages from exogenously induced job accessions.

7.2 Event Studies

To circumvent the difficulty of testing for exogenous mobility directly, the literature has proposed proxies. Card, Heining and Kline (2013) introduced the best-known test. They looked at whether the wage gains of workers who move from one establishment to another are equal in magnitude and opposite in sign to the wage gains of workers who move in the opposite direction. They argue that this would not be the case in models where workers select which jobs to take based on the wage. Examples include the Burdett and Mortensen (1998) model of on-the-job search as well as our model. In these environments, workers always enjoy a wage increase when voluntarily switching jobs. Card, Heining and Kline (2013) and subsequent research by these authors find remarkable support for the "equal in magnitude and opposite in sign" prediction.

We replicate their test on data generated from our calibrated model. We describe the procedure for generating artificial data in detail in Appendix D. We stress that we apply the same sample selection criteria as Card, Heining and Kline (2013). We use our model to generate work histories for 500,000 workers at 5000 firms. We focus on workers' main job, defined as the job with the highest earnings in the given year. We classify firms into quartiles based on co-workers' average log wages. And finally, we select job switchers between years t and $t+1$ who have the same main job in years $t-1$ and t as well as years $t+1$ and $t+2$. Importantly, we do not condition on whether the move involves an intervening spell of unemployment, but we do often drop such spells because workers fail to spend two years in the same main job immediately after an unemployment spell.

Figure 2 shows the mean log wages of movers in the model-generated data for both calibrations.[16] In the figure, the move happens between years -1 and 0. We comment on a few features of the figure and compare them to the data.

First, a worker going from quartile q_1 to quartile q_2 generally earns more in both jobs if q_1 or q_2 is bigger. Card, Heining and Kline (2013) observe this in the data, writing on page 985 that "different mobility groups have different wage levels before and after a move." The same forces that lead to a firm effect in wages explains the impact of q_1 on the first wage and q_2 on the second wage. To understand the impact of q_2 on the first wage or q_1 on the second wage, we note that this event study does not condition on the worker's type. In the model, the worker's wage depends on their type and their employer's type, but not on future or past employers. However, future and past employers' types are correlated with the workers' type because of assortative matching, and thus help to predict the workers' wage.

[16] We generated 100 samples from the model to compute the standard errors. Standard errors are very small: the 95% confidence intervals for different combinations of quartiles have length between 0.01 and 0.03, and so we do not show them in the figure.

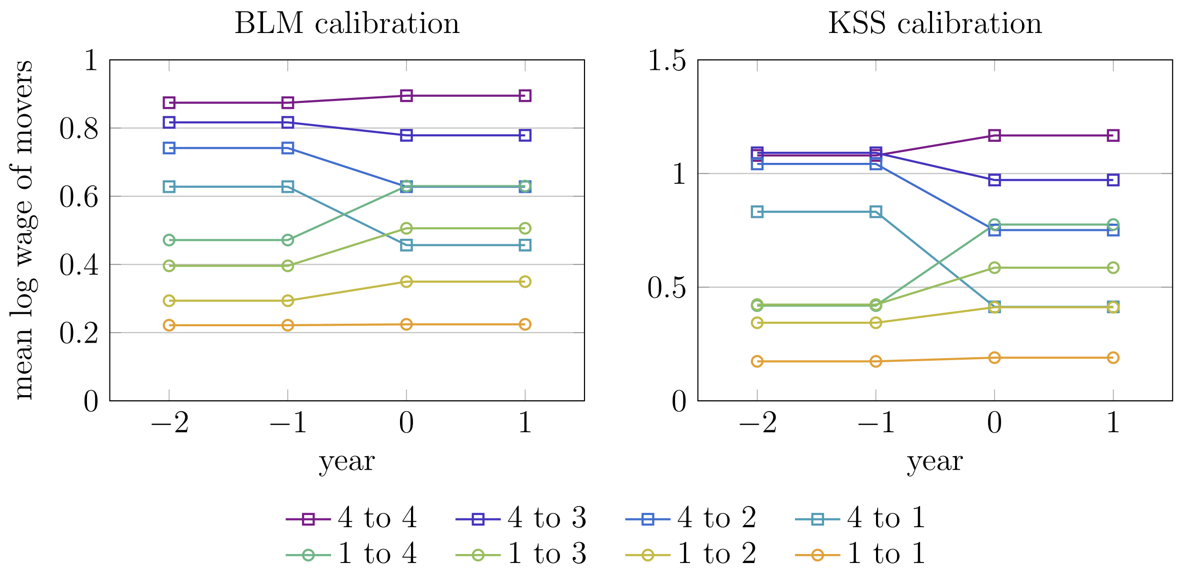

Figure 2: Mean log wages of movers in the artifical data generated from our model with on-the-job search. We select movers who change jobs between times -1 and 0, and have at least two years of tenure in both origin and destination firm. The left panel shows the results from the model calibrated to match the BLM wage decomposition, the right panel is calibrated to match the KSS wage decomposition. See Table 2 for parameter values.

This is particularly so in the BLM calibration, since matching patterns satisfy the monotone likelihood ratio order.

Second, wages are stable in the year before and after the move. In our model this happens by construction since wages do not change during the match.

Third, the mean log changes for upward and downward moves are nearly symmetric. This is easier to see in Figure 3. The figure shows pairs of quartiles q_1 and q_2, with $q_1 \neq q_2$, and depicts the mean log wage change associated with the upward and downward move. If the gains were exactly symmetric, they would be aligned along the minus 45 degree line, shown in red. We see that the data points lie very close to this line. Card, Heining and Kline (2013) argue that "This symmetry suggests that a simple model with additive worker and establishment effects may provide a reasonable characterization of the mean wages resulting from different pairings of workers to establishments (p. 985)." We agree with this statement, but our model illustrates why that does not imply the exogenous mobility restriction.

To understand why our model generates these patterns, first consider the model without on-the-job search, where all job transitions occur through unemployment. In this case, the symmetry of wage changes has a simple two-part explanation. First, the distribution of worker types moving between quartiles q_1 and q_2 is identical regardless of the direction of the

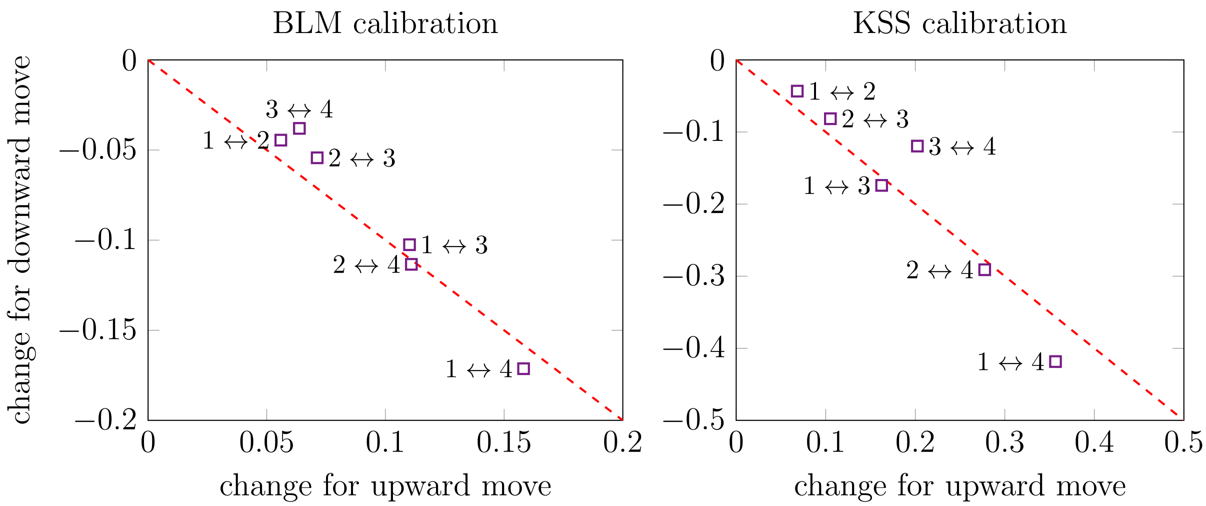

Figure 3: Mean log wage change of upward and downward movers from our model with on-the-job search. Each square labeled $q_1 \leftrightarrow q_2$ represents a pair of firm quartiles (q_1, q_2), with $q_1 < q_2$. The horizontal axis shows the mean log wage gain of moving from q_1 to q_2, while the vertical axis shows the mean log wage gain of moving from q_2 to q_1. The red dashed line represents symmetric changes for upward and downward movers. The left panel shows the results from the model calibrated to match the BLM wage decomposition, the right panel is calibrated to match the KSS wage decomposition. See Table 2 for parameter values.

move, since each worker has the same probability of moving from q_1 before an unemployment spell to q_2 after one, or having these jobs in the reverse order. Second, at each firm type, the average log wage depends only on the worker's and firm's types, not on their employment history. These two properties together guarantee exact symmetry in wage changes for upward and downward moves. Bonhomme, Lamadon and Manresa (2019) first made this observation in a model without on-the-job search and without idiosyncratic shocks.

With on-the-job search, this exact symmetry breaks down because employed workers also accept new jobs that offer higher wages. Still, the steady state assumption imposes strong restrictions on mobility patterns. Consider a simple example with just two groups of firms (high-wage and low-wage) and no sample restrictions on job tenure. In steady state, if there were no average wage change for workers who stay within the same group of firms, then the wage gains of workers moving from low-wage to high-wage firms must exactly equal the wage losses of workers moving in the opposite direction. This exact symmetry breaks down because we have four quartiles of firms, modest wage changes for workers who switch firms but stay in the same quartile, and a job tenure restriction on the sample. Nevertheless, our quantitative analysis shows that the calibrated model generates the near-symmetric patterns observed in empirical work.

We note that the magnitude of wage changes differs between our two calibrations. For example, the magnitude of wage gains for movers between quartile 1 and quartile 4 is twice as large in the KSS as the BLM calibration. This is directly related to the role of firm fixed effects. Since BLM attributes relatively little of the overall wage variation to firm effects, the wage gains and losses from interquartile moves are relatively small.

There are two takeaways from this exercise. First, we show that our model can quantitatively replicate additional patterns that have been documented for several countries. Second, it shows that the tests presented in Figure 2 and Figure 3 do not necessarily detect the presence of selection. Indeed, based on these figures one would be tempted to conclude that the exogenous mobility assumption is not violated in our model. We know that it is.

8 The Surplus of Matches and Meetings

A fundamental challenge in labor economics is evaluating policies that aim to encourage the most efficient worker-firm matches in the labor market. Place-based policies and similar interventions often try to encourage meetings between specific types of workers and firms, assuming these meetings will lead to productive matches (Bilal, 2023; Hong, 2024). However, policymakers typically must rely on observed wage patterns to guide these interventions, which our analysis reveals may be misleading. The expected surplus created by encouraging a meeting between a worker and firm can differ substantially from the average surplus observed in successful matches. This distinction matters because selective hiring means that only the most productive meetings result in matches, potentially creating a deceptive picture of which worker-firm meetings policymakers should encourage.

To rigorously evaluate these policies, we measure their marginal effects, specifically, how much value is created by generating one additional meeting between any given type of worker and firm. This measurement is challenging because we typically only observe outcomes for meetings that result in successful matches. Our model allows us to distinguish between these concepts by precisely defining and measuring two different quantities: the expected surplus from a random meeting between a worker and firm (before they learn their match quality), and the average surplus observed in realized matches.

Formally, the value of a meeting between an unemployed worker and a vacancy is captured by the match surplus $V^s_{x,y}(z)$ in equation (25). We define the average surplus in meetings as[17]

$$\bar{V}^{s,m}_{x,y} \equiv \int_{z_0}^{\infty} V^s_{x,y}(z)s(z)dz. \tag{32}$$

We also define the average surplus of a new match between an unemployed worker and a vacancy,

$$\bar{V}^{s}_{x,y} \equiv \frac{1}{S(\bar{z}_{x,y})} \int_{\bar{z}_{x,y}}^{\infty} V^s_{x,y}(z)s(z)dz. \tag{33}$$

The average surplus in meetings, $\bar{V}^{s,m}_{x,y}$, captures the expected value that an unemployed type-x worker and a vacant type-y firm would get from creating a single meeting between them, before learning their match-specific productivity z. This is distinct from the average surplus in matches, $\bar{V}^{s}_{x,y}$, which only considers meetings that actually result in employment relationships. The surplus of matches is analogous to the wage in administrative data sets, capturing only the value of meetings that result in matches. The surplus of meetings is more relevant for thinking about the marginal value of place-based policies, which alter the number of (x, y) matches by creating more meetings between type-x workers and type-y firms, many of which may not result in matches.

In Figure 4, we plot the average surplus in a new (x, y) match, $\bar{V}^{s}_{x,y}$, in the left panel, and the average surplus in any (x, y) meeting, $\bar{V}^{s,m}_{x,y}$, in the right panel. The top row shows the model calibrated to match the BLM wage decomposition, the bottom row the KSS wage decomposition.

The left panels are reminiscent of the average log wage results in left panels of Figure 1: the average surplus in matches is increasing in firm type for each worker type. Looking at this panel, one might be tempted to conclude that all workers benefit more-or-less equally from meeting higher-type firms.

The right panels of the same figure, where we show average surplus in meetings, challenge this conclusion. In the model calibrated to BLM, the average surplus in a meeting is decreasing in the firm type for the lowest-type worker. The lowest worker type gains the largest surplus from meeting with a type-1 firm and gets around 75 percent less surplus from meeting a type-10 firm. In the KSS calibration, the average surplus in a meeting is u-shaped for the lowest worker type. This worker prefers meeting a type-1 firm to meeting a firm with a median type.

The difference between the left and right panels is selection: while the left panels show the average surplus in meetings that result in matches, the right panels show the average surplus in all meetings, keeping the support of the z distribution the same for every pair. In

[17] For this definition alone, we set the lower bound on the match-specific shock distribution, z_0, equal to the minimum acceptance threshold across (x, y) pairs, $z_0 = \min_{x,y} \bar{z}_{x,y}$, where the acceptance threshold $\bar{z}_{x,y}$ is implicitly defined by $V^s_{x,y}(\bar{z}_{x,y}) = 0$.

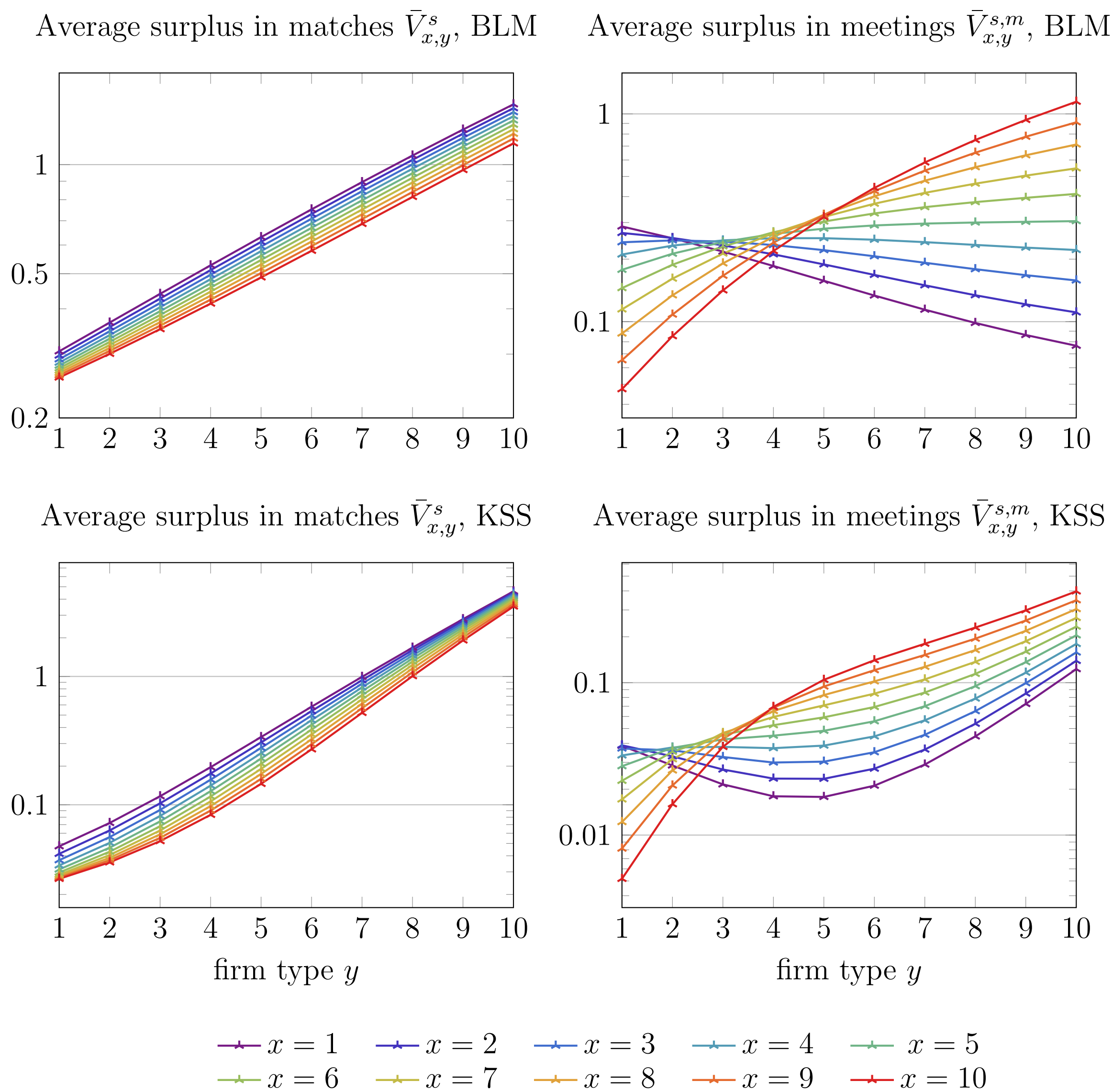

Figure 4: Average surplus in matches (left) and meetings (right) when z_0 is equal to the minimum threshold across (x, y) pairs, with $\rho_1 = 0.2\rho_0$. Top row shows the BLM calibration, bottom row KSS calibration. Each line represents one worker type and each point one firm type. Vertical scales are logarithmic. The top panels show the results from the model calibrated to match the BLM wage decomposition, the bottom panels are calibrated to match the KSS wage decomposition. See Table 2 for parameter values.

other words, the right panel is not affected by selection of worker types into firms based on the realized value of z. In light of the fact that high-type firms rarely hire low-type workers, as illustrated in the right panels of Figure 1, it should not be surprising that there is little surplus in such matches, as shown in the right panels of Figure 4.

Our structural model demonstrates that the relationship between match surplus and meeting surplus is far from straightforward. Despite monotonically increasing match surpluses in both calibrations for all worker types as firm quality rises, the surplus of meetings exhibits markedly different patterns across calibrations. Notably, in the BLM calibration, the lowest-type workers prefer meeting the lowest-type firms, while in the KSS calibration, they prefer a mix of low and very high-type firms. Crucially, in both scenarios, high-type workers stand to gain the most from meetings with higher-type firms, an insight entirely masked by the submodular nature of average log wages.

These findings underscore the importance of structural modeling in labor economics for predicting the efficacy of policies aimed at reducing wage inequality. For instance, job search assistance programs might be redesigned to target specific worker-firm pairings that maximize the surplus of meetings, rather than supporting high wage matches that may be difficult to create. Similarly, education and training policies could be tailored to enhance the gains from meetings between workers and firms, rather than simply aiming to increase average wages. Policies promoting labor mobility and firm relocation might also be refined to encourage moves that increase the likelihood of valuable meetings, not just high-wage matches. Without such structural insights, policymakers risk misinterpreting wage data and implementing counterproductive measures. By revealing these aspects of the labor market, our approach provides a more nuanced and accurate foundation for crafting effective interventions to address issues of job matching and wage inequality.

9 Conclusion

We have developed a random search model of the labor market with ex ante heterogeneous workers and firms and ex post match-specific productivity shocks. A single selection equation determines which meetings result in matches and the resulting model can explain multiple empirical patterns. When the distribution of match-specific shocks is Pareto or exponential, this selection mechanism delivers three results. First, we show that on average, more productive workers earn higher wages at any type of firm and more productive firms pay higher wages to any type of worker. Second, we show that with reasonable restrictions on the production function, there is assortative matching. And finally, we show that more productive workers earn more on average and more productive firms pay more on average, so high productivity workers and firms are the same as high wage workers and firms.

We also argue that our model is consistent with a variety of facts in the empirical labor economics literature built around the Abowd, Kramarz and Margolis (1999) wage equation and the non-separable specification in Bonhomme, Lamadon and Manresa (2019). In particular, we show how this selection mechanism shapes the decomposition of the cross-sectional variance of log wages into its four components, the variance of worker effects, the variance of firm effects, their covariance, and the variance of the residual.

Additionally, our model predicts that average log wages may be submodular—low-wage workers gain proportionately more than high-wage workers when they move from low-wage to high-wage firms—while simultaneously low-wage workers may be disproportionately employed at low-wage firms. In our model, this reflects the same selection mechanism. Low-wage workers rarely accept high-wage jobs, but when they do, productivity is so high that their average log wage is also very high.

Our theory cautions against prevailing interpretations of empirical patterns in labor economics. For instance, the "establishment-specific wage premiums" identified by Card, Heining and Kline (2013) and many other papers may not solely reflect rent-sharing, efficiency wages, or strategic wage posting. Instead, our model suggests that selection significantly shapes these premiums. It follows that these wage premia may not be informative about the wage gains from exogenously-induced accessions, as we illustrate through our quantitative analysis of the surplus from meetings and the surplus from matches. This distinction may have profound implications for policies aimed at altering sorting in the labor market to address wage inequality.

Our analysis also reveals limitations in using administrative wage data to infer properties of production functions. The relationship between worker and firm types in wage data may not directly reflect complementarities in production, as selection processes can mask these underlying relationships. This finding underscores the need for structural models to disentangle the complex interplay between production technologies, worker-firm sorting, and observed wage patterns.

The importance of selection extends to recent studies examining time-varying firm types, such as Engbom, Moser and Sauermann (2023) and Lachowska, Mas, Saggio and Woodbury (2023). Our model implies that changes in estimated firm fixed effects over time may capture shifts in selection patterns rather than actual changes in firm pay policies. This nuance is crucial for correctly interpreting empirical findings and their policy implications.

Our findings speak to work by Card, Cardoso, Heining and Kline (2018), Lamadon, Mogstad and Setzler (2022), and Lamadon, Lise, Meghir and Robin (2024), who posit that high-wage workers systematically value amenities at high-wage firms more than low-wage workers do. While this systematic difference in amenity valuations can explain the empirical patterns, it requires a particular correlation between worker productivity and amenity preferences. Our model demonstrates that selection alone—a simpler and more fundamental mechanism—can generate the same key empirical regularities. This does not rule out an important role for amenities. Indeed, we believe that idiosyncratic variation in how workers value different workplace characteristics influences individual job choices. But our parsimonious explanation suggests that selection may be the primary driver of aggregate sorting patterns. Future research should seek to understand the data needed to identify the role of selection versus both systematic and idiosyncratic amenity valuations in shaping observed wages and sorting patterns.

In summary, our approach offers a new perspective on empirical wage and worker-firm sorting patterns. By highlighting the potential role of selection in shaping observed patterns, our framework encourages a more nuanced interpretation of these findings and suggests new avenues for research. We hope that our framework will contribute to ongoing research about wage determination, sorting, and the design and limitations of effective labor market policies.

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Appendix

A Proofs

A.1 Existence and Monotonicity

Proof of Proposition 1. Consider the following $2(X+Y)$ functions of $(\bar{w}_x, u_x)_{x=1}^X$ and $(\bar{\pi}_y, v_y)_{y=1}^Y$:

$$T_{1,x}(\bar{w},\bar{\pi},u,v) = \frac{\gamma\rho}{r+\delta}\sum_{y=1}^{Y} v_y f_{x,y}\int_{\bar{z}_{x,y}}^{\infty}(z-\bar{z}_{x,y})s(z)dz,$$
$$T_{2,y}(\bar{w},\bar{\pi},u,v) = \frac{(1-\gamma\rho)}{r+\delta}\sum_{x=1}^{X} u_x f_{x,y}\int_{\bar{z}_{x,y}}^{\infty}(z-\bar{z}_{x,y})s(z)dz,$$
$$T_{3,x}(\bar{w},\bar{\pi},u,v) = m_x - \sum_{y=1}^{Y}\frac{\rho u_x v_y S(\bar{z}_{x,y})}{\delta},$$
$$T_{4,y}(\bar{w},\bar{\pi},u,v) = n_y - \sum_{x=1}^{X}\frac{\rho u_x v_y S(\bar{z}_{x,y})}{\delta},$$

where $\bar{z}_{x,y} = (\bar{w}_x + \bar{v}_y)/f_{x,y}$, as in equation (6). We refer to this mapping collectively as $T = (T_1, T_2, T_3, T_4)$. It is immediate that any vector $(\bar{w},\bar{\pi},u,v)$, together with $\bar{z}$ solving equation (6) and ϕ solving equation (12), is an equilibrium if and only if it is a fixed point of T, i.e. $T(\bar{w},\bar{\pi},u,v) = (\bar{w},\bar{\pi},u,v)$.

Now define the mapping $\tilde{T}$:

$$\tilde{T}_{1,x}(\bar{w},\bar{\pi},u,v) = \min\left\{T_{1,x}(\bar{w},\bar{\pi},u,v), \bar{\bar{w}}_x\right\},$$
$$\tilde{T}_{2,y}(\bar{w},\bar{\pi},u,v) = \min\left\{T_{2,y}(\bar{w},\bar{\pi},u,v), \bar{\bar{\pi}}_y\right\},$$
$$\tilde{T}_{3,x}(\bar{w},\bar{\pi},u,v) = \max\left\{T_{3,x}(\bar{w},\bar{\pi},u,v), 0\right\},$$
$$\tilde{T}_{4,y}(\bar{w},\bar{\pi},u,v) = \max\left\{T_{4,y}(\bar{w},\bar{\pi},u,v), 0\right\},$$

where

$$\bar{\bar{w}}_x = \frac{\gamma\rho}{r+\delta}\sum_{y=1}^{Y} n_y f_{x,y}\int_{\bar{\bar{w}}_x/f_{x,y}}^{\infty}(z-\bar{\bar{w}}_x/f_{x,y})s(z)dz,$$
$$\bar{\bar{\pi}}_y = \frac{(1-\gamma)\rho}{r+\delta}\sum_{x=1}^{X} m_x f_{x,y}\int_{\bar{\bar{\pi}}_y/f_{x,y}}^{\infty}(z-\bar{\bar{\pi}}_y/f_{x,y})s(z)dz.$$

The mapping $\tilde{T}$ is continuous. Moreover, it maps points satisfying $\bar{w}_x \in [0, \bar{\bar{w}}_x]$, $\bar{\pi}_y \in [0, \bar{\bar{\pi}}_y]$, $u_x \in [0, m_x]$, and $v_y \in [0, n_y]$ for all $x \in \{1, \ldots, X\}$ and $y \in \{1, \ldots, Y\}$ into itself. Therefore $\tilde{T}$ has a fixed point by Brouwer's fixed point theorem.

We prove that at any $(\bar{w}, \bar{\pi}, u, v)$ which is a fixed point of $\tilde{T}$, $T(\bar{w}, \bar{\pi}, u, v) = \tilde{T}(\bar{w}, \bar{\pi}, u, v)$, and thus $(\bar{w}, \bar{\pi}, u, v)$ is also a fixed point of T. In the first step we prove that $u_x > 0$ for all x. This is because if $u_x = 0$, $\tilde{T}_{3,x}(\bar{w}, \bar{\pi}, u, v) = m_x$, contradicting $(\bar{w}, \bar{\pi}, u, v)$ being a fixed point. This implies $\tilde{T}_{3,x}(\bar{w}, \bar{\pi}, u, v) = T_{3,x}(\bar{w}, \bar{\pi}, u, v)$ for all x at any fixed point. Similarly $v_y > 0$ at any fixed point of $\tilde{T}$ and so $\tilde{T}_{4,y}(\bar{w}, \bar{\pi}, u, v) = T_{4,y}(\bar{w}, \bar{\pi}, u, v)$ for all y at any fixed point.

Next, any fixed point has $\bar{w}_x > 0$ for all x: $\tilde{T}_{1,x}(\bar{w}, \bar{\pi}, u, v)$ is continuous and decreasing in $\bar{w}_x$ and is strictly positive at $\bar{w}_x = 0$ since $v_y > 0$ for all y. Similarly any fixed point has $\bar{\pi}_y > 0$ for all y.

Finally, in any fixed point of $\tilde{T}$, any solution to $\tilde{T}_{1,x}(\bar{w}, \bar{\pi}, u, v) = \bar{w}_x$ has $\bar{w}_x < \bar{\bar{w}}_x$ because $\bar{\pi}_y > 0$. This implies that the fixed point of $\tilde{T}$ also solves $\tilde{T}_{1,x}(\bar{w}, \bar{\pi}, u, v) = T_{1,x}(\bar{w}, \bar{\pi}, u, v)$ for all x. The same logic implies $\tilde{T}_{2,y}(\bar{w}, \bar{\pi}, u, v) = T_{2,y}(\bar{w}, \bar{\pi}, u, v)$ for all y in any fixed point.

In summary, we have proved that there exists a vector $(\bar{w}, \bar{\pi}, u, v)$ with $\tilde{T}(\bar{w}, \bar{\pi}, u, v) = (\bar{w}, \bar{\pi}, u, v)$, that at any such vector $T(\bar{w}, \bar{\pi}, u, v) = \tilde{T}(\bar{w}, \bar{\pi}, u, v)$ and so $(\bar{w}, \bar{\pi}, u, v)$ is a fixed point of T, and that any fixed point of T is an equilibrium. This proves an equilibrium exists.

Along the way we also proved that $\bar{w}_x > 0$ for all x and $\bar{\pi}_y > 0$ for all y at any fixed point of $\tilde{T}$, and hence in any equilibrium. ■

Proof of Lemma 1. We prove that $\bar{w}_x$ is strictly increasing. The proof that $\bar{\pi}_y$ is strictly increasing is analogous.

To find a contradiction, suppose there exists an $x_1 < x_2$ with $\bar{w}_{x_1} \geq \bar{w}_{x_2}$. Since f is monotonic, $f_{x_2,y} > f_{x_1,y}$ for all y. Then using equation (6), we have $\bar{z}_{x_1,y} > \bar{z}_{x_2,y}$ for all y. Additionally, observe that $\int_{\bar{z}}^{\infty}(z - \bar{z})s(z)dz$ is strictly positive for all $\bar{z}$, since $S(z) > 0$ for all z. Additionally, the integral is decreasing in $\bar{z}$, as can be confirmed directly. This means that $\int_{\bar{z}_{x_2,y}}^{\infty}(z - \bar{z}_{x_2,y})s(z)dz > \int_{\bar{z}_{x_1,y}}^{\infty}(z - \bar{z}_{x_1,y})s(z)dz > 0$.

Putting this together, if there exists an $x_1 < x_2$ with $\bar{w}_{x_1} \geq \bar{w}_{x_2}$,

$$\begin{aligned}\bar{w}_{x_2} &= \frac{\gamma\rho}{r+\delta}\sum_{y=1}^{Y} v_y f_{x_2,y}\int_{\bar{z}_{x_2,y}}^{\infty}(z - \bar{z}_{x_2,y})s(z)dz \\ &> \frac{\gamma\rho}{r+\delta}\sum_{y=1}^{Y} v_y f_{x_1,y}\int_{\bar{z}_{x_1,y}}^{\infty}(z - \bar{z}_{x_1,y})s(z)dz = \bar{w}_{x_1},\end{aligned}$$

where the two equations use the value function (10) and the inequality uses $f_{x_2,y} > f_{x_1,y}$ and $\int_{\bar{z}_{x_2,y}}^{\infty}(z-\bar{z}_{x_2,y})s(z)dz > \int_{\bar{z}_{x_1,y}}^{\infty}(z-\bar{z}_{x_1,y})s(z)dz > 0$, together with strict positivity of the remaining terms. But this is a contradiction, proving $\bar{w}_{x_1} < \bar{w}_{x_2}$. ■

A.2 Average Log Wage

Proof of Proposition 2. We prove a more general version of this proposition. For any strictly increasing function $G : \mathbb{R}_+ \to \mathbb{R}$, define

$$w_{x,y}^G = \frac{\int_{\bar{z}_{x,y}}^{\infty} G(W_{x,y}(z))s(z)dz}{S(\bar{z}_{x,y})}. \tag{34}$$

From equations (6) and (9), we have

$$W_{x,y}(z) = \bar{w}_x + \gamma(\bar{w}_x + \bar{\pi}_y)\left(\frac{z}{\bar{z}_{x,y}} - 1\right).$$

Using equation (34) and the functional form of the Pareto distribution, we obtain

$$w_{x,y}^G = \frac{\int_{\bar{z}_{x,y}}^{\infty} G\Big(\bar{w}_x + \gamma(\bar{w}_x + \bar{\pi}_y)\big(z/\bar{z}_{x,y} - 1\big)\Big)\theta z^{-\theta-1}dz}{\bar{z}_{x,y}^{-\theta}}.$$

Now let $q = z/\bar{z}_{x,y} - 1$ and perform a change in the variable of integration to obtain

$$w_{x,y}^G \equiv \int_0^{\infty} G\big(\bar{w}_x + \gamma(\bar{w}_x + \bar{\pi}_y)q\big)\theta(1+q)^{-\theta-1}dq.$$

Setting $G(W) = \log(W)$ for all W gives us equation (17). This equation implies that $w_{x,y}^G$ is simply a weighted average of $G\big(\bar{w}_x + \gamma(\bar{w}_x + \bar{\pi}_y)q\big)$, with the same weights for all (x, y). This means that x and y only affect $w_{x,y}^G$ through $\bar{w}_x$ and $\bar{\pi}_y$.

Now since G is strictly increasing, it is straightforward to verify that increasing either $\bar{w}_x$ or $\bar{\pi}_y$ raises the integrand in equation (17) at all $q > 0$, and hence raises $w_{x,y}^G$. This establishes the first two enumerated points.

We next prove the third point if G is strictly concave. If G is strictly convex, we prove $w_{x_1,y_2}^G + w_{x_2,y_1}^G < w_{x_1,y_1}^G + w_{x_2,y_2}^G$. If G is linear, this is an equality. Take x_1 and x_2 with $\bar{w}_{x_1} < \bar{w}_{x_2}$; and y_1 and y_2 with $\bar{\pi}_{y_1} < \bar{\pi}_{y_2}$. Let

$$\lambda \equiv \frac{(1+\gamma q)(\bar{w}_{x_2} - \bar{w}_{x_1})}{(1+\gamma q)(\bar{w}_{x_2} - \bar{w}_{x_1}) + \gamma q(\bar{\pi}_{y_2} - \bar{\pi}_{y_1})}.$$

The assumptions on x_1, x_2, y_1, and y_2 ensure that $\lambda \in (0,1)$ for all $q > 0$. Then verify algebraically that

$$\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_2})q = \lambda(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_1})q) + (1-\lambda)(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_2})q), \text{ and}$$
$$\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_1})q = (1-\lambda)(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_1})q) + \lambda(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_2})q).$$

Thus G concave (convex) implies

$$G(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_2})q) > (<) \lambda G(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_1})q) + (1-\lambda)G(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_2})q), \text{ and}$$
$$G(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_1})q) > (<) (1-\lambda)G(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_1})q) + \lambda G(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_2})q).$$

Summing these gives

$$G\big(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_2})q\big) + G\big(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_1})q\big)$$
$$> (<) G\big(\bar{w}_{x_1} + \gamma(\bar{w}_{x_1} + \bar{\pi}_{y_1})q\big) + G\big(\bar{w}_{x_2} + \gamma(\bar{w}_{x_2} + \bar{\pi}_{y_2})q\big)$$

when G is concave (convex). Integrating over the density $\theta(1+q)^{-\theta-1}$ for $q > 0$ delivers the third bullet point.

Finally, we note that if $f_{x,y}$ is strictly increasing in x and y, Lemma 1 implies $\bar{w}_x$ and $\bar{\pi}_y$ are strictly increasing. Then the three numbered conditions in the statement of the proposition imply that $w^G_{x,y}$ is strictly increasing and strictly submodular (supermodular) in x and y when G is strictly concave (convex). ■

Before proving Proposition 3, we establish a preliminary result:

Lemma 2 *Assume $f_{x,y} = f^1_x f^2_y$ for strictly positive and strictly increasing functions f^1 and f^2. Then $\bar{w}_x / f^1_x$ and $\bar{\pi}_y / f^2_y$ are strictly increasing.*

Proof. We prove that $\bar{w}_x / f^1_x$ is strictly increasing. To find a contradiction, assume there is an $x_1 < x_2$ with $\bar{w}_{x_1}/f^1_{x_1} \geq \bar{w}_{x_2}/f^1_{x_2}$. Note $x_1 < x_2$ implies $1/f^1_{x_1} > 1/f^1_{x_2}$. Equation (6) states

$$\bar{z}_{x,y} = \frac{\frac{\bar{w}_x}{f^1_x} + \frac{\bar{\pi}_y}{f^1_x}}{f^2_y},$$

and so $\bar{z}_{x_1,y} > \bar{z}_{x_2,y}$ for all y. Moreover, equation (10) implies

$$\frac{\bar{w}_x}{f^1_x} = \frac{\gamma\rho}{r+\delta} \sum_{y=1}^{Y} v_y f^2_y \int_{\bar{z}_{x,y}}^{\infty} (z - \bar{z}_{x,y}) s(z) dz.$$

Since the integral is decreasing in $\bar{z}_{x,y}$, we have $\frac{\bar{w}_{x_1}}{f^1_{x_1}} < \frac{\bar{w}_{x_2}}{f^1_{x_2}}$, a contradiction.

The proof that $\bar{\pi}_y/f^2_y$ is strictly increasing is identical, manipulating equation (11) instead of equation (10). ■

Proof of Proposition 3. For the first part of the Proposition, we again work with the more general version, using w^G defined in equation (34) for an arbitrary increasing function G. From the wage equation (9) and the exponential distribution, we have

$$w^G_{x,y} = \theta \int_{\bar{z}_{x,y}}^{\infty} G(\bar{w}_x + \gamma(z f_{x,y} - \bar{w}_x - \bar{\pi}_y))e^{-\theta(z-\bar{z}_{x,y})}dz.$$

Let $q = z - \bar{z}_{x,y}$ to get

$$w^G_{x,y} = \theta \int_0^{\infty} G(\bar{w}_x + \gamma((q + \bar{z}_{x,y})f_{x,y} - \bar{w}_x - \bar{\pi}_y))e^{-\theta q}dq.$$

From equation (6), we can reduce this to

$$w^G_{x,y} = \theta \int_0^{\infty} G(\bar{w}_x + \gamma q f_{x,y})e^{-\theta q}dq,$$

which is equivalent to equation (18) when $G(W) = \log W$ for all W. If f is strictly increasing, so is $\bar{w}_x$ (Lemma 1). And since G is strictly increasing, then the integrand in equation (18) is strictly increasing in both x and y for all q. Thus w^G is strictly increasing.

Finally we turn to the multiplicatively separable case, $f_{x,y} = f^1_x f^2_y$. Here we assume $G(W) = \log W$. We use equation (18). w^* is strictly submodular if

$$\log(\bar{w}_x + \gamma q f^1_x f^2_y)$$

is strictly submodular for all $q > 0$. So take $x_1 < x_2$ and $y_1 < y_2$. Strict submodularity requires

$$\log(\bar{w}_{x_1} + \gamma q f^1_{x_1} f^2_{y_1}) + \log(\bar{w}_{x_2} + \gamma q f^1_{x_2} f^2_{y_2}) < \log(\bar{w}_{x_1} + \gamma q f^1_{x_1} f^2_{y_2}) + \log(\bar{w}_{x_2} + \gamma q f^1_{x_2} f^2_{y_1}),$$

or equivalently

$$(\bar{w}_{x_1} + \gamma q f^1_{x_1} f^2_{y_1})(\bar{w}_{x_2} + \gamma q f^1_{x_2} f^2_{y_2}) < (\bar{w}_{x_1} + \gamma q f^1_{x_1} f^2_{y_2})(\bar{w}_{x_2} + \gamma q f^1_{x_2} f^2_{y_1}).$$

Expand the products and cancel common terms to get

$$\bar{w}_{x_1} f^1_{x_2} f^2_{y_2} + \bar{w}_{x_2} f^1_{x_1} f^2_{y_1} < \bar{w}_{x_1} f^1_{x_2} f^2_{y_1} + \bar{w}_{x_2} f^1_{x_1} f^2_{y_2}$$

or

$$\left(\frac{\bar{w}_{x_2}}{f^1_{x_2}} - \frac{\bar{w}_{x_1}}{f^1_{x_1}}\right)(f^2_{y_2} - f^2_{y_1}) > 0.$$

Lemma 2 implies the first term is positive for all $x_1 < x_2$ and the second term is positive by assumption. ■

A.3 Assortative Matching

Proof of Proposition 4. We first prove that $\bar{z}_{x,y}$ is strictly log-submodular. Substituting equation (6) for $\bar{z}$, we must prove that for $x_1 < x_2$ and $y_1 < y_2$,

$$\left(\frac{\bar{w}_{x_1} + \bar{\pi}_{y_1}}{f_{x_1,y_1}}\right)\left(\frac{\bar{w}_{x_2} + \bar{\pi}_{y_2}}{f_{x_2,y_2}}\right) < \left(\frac{\bar{w}_{x_1} + \bar{\pi}_{y_2}}{f_{x_1,y_2}}\right)\left(\frac{\bar{w}_{x_2} + \bar{\pi}_{y_1}}{f_{x_2,y_1}}\right).$$

Weak log supermodularity of f implies $f_{x_1,y_1} f_{x_2,y_2} \geq f_{x_1,y_2} f_{x_2,y_1}$. And we see that the product of the numerators on the left hand side is smaller than the product of the numerators on the right hand side if and only if

$$\bar{w}_{x_1}\bar{\pi}_{y_2} + \bar{w}_{x_2}\bar{\pi}_{y_1} < \bar{w}_{x_1}\bar{\pi}_{y_1} + \bar{w}_{x_2}\bar{\pi}_{y_2} \Leftrightarrow (\bar{w}_{x_2} - \bar{w}_{x_1})(\bar{\pi}_{y_2} - \bar{\pi}_{y_1}) > 0.$$

This is immediate because $\bar{w}$ and $\bar{\pi}$ are strictly increasing (Lemma 1).

We now use the assumption that $S(z) = (z/z_0)^{-\theta}$. Then since $\log \bar{z}_{x,y}$ is strictly submodular, $\log S(\bar{z}_{x,y}) = \theta \log z_0 - \theta \log \bar{z}_{x,y}$ is strictly supermodular. Finally, equation (12) implies that $\log \phi_{x,y}$ inherits the strict supermodularity of $\log S(\bar{z}_{x,y})$; and strict log-supermodularity of ϕ is equivalent to ϕ having the monotone likelihood ratio order. ■

Proof of Proposition 5. We split the proof into two pieces.

$-1/f_{x,y}$ **weakly supermodular.** Take any $x_1 < x_2$ and $y_1 < y_2$. Then

$$\left(\frac{1}{f_{x_2,y_2}} - \frac{1}{f_{x_2,y_1}}\right)(\bar{w}_{x_2} - \bar{w}_{x_1}) < 0,$$
$$\left(\frac{1}{f_{x_2,y_2}} - \frac{1}{f_{x_1,y_2}}\right)(\bar{\pi}_{y_2} - \bar{\pi}_{y_1}) < 0,$$

since in both cases the first term is negative (since f is positive and strictly increasing) and the second term is positive by Lemma 1. Additionally, $1/f$ submodular implies

$$\frac{1}{f_{x_1,y_1}} + \frac{1}{f_{x_2,y_2}} \leq \frac{1}{f_{x_1,y_2}} + \frac{1}{f_{x_2,y_1}}.$$

Multiply each term in the last inequality by $\bar{w}_{x_1} + \bar{\pi}_{y_1}$ and add to the preceding inequalities to get

$$\frac{\bar{w}_{x_1} + \bar{\pi}_{y_1}}{f_{x_1,y_1}} + \frac{\bar{w}_{x_2} + \bar{\pi}_{y_2}}{f_{x_2,y_2}} < \frac{\bar{w}_{x_1} + \bar{\pi}_{y_2}}{f_{x_1,y_2}} + \frac{\bar{w}_{x_2} + \bar{\pi}_{y_1}}{f_{x_2,y_1}},$$

From equation (6), this implies that $\bar{z}_{x,y}$ is strictly submodular.

Next, in steady state with an exponential distribution, we have from equation (12) that

$$\log \phi_{x,y} = \log(\rho/\delta) + \log u_x + \log v_y - \theta \bar{z}_{x,y}.$$

Since $\bar{z}_{x,y}$ is strictly submodular and the other terms are amodular, this proves that $\log \phi_{x,y}$ is strictly supermodular, i.e. that ϕ has the monotone likelihood ratio order.

$f_{x,y}$ **multiplicatively separable.** Assume $f_{x,y} = f^1_x f^2_y$ for some strictly positive and strictly increasing functions f^1 and f^2. As in the other case, we must prove that for all $x_2 > x_1$ and $y_2 > y_1$, $\bar{z}_{x,y}$ is strictly submodular:

$$\frac{\bar{w}_{x_1} + \bar{\pi}_{y_1}}{f^1_{x_1} f^2_{y_1}} + \frac{\bar{w}_{x_2} + \bar{\pi}_{y_2}}{f^1_{x_2} f^2_{y_2}} < \frac{\bar{w}_{x_1} + \bar{\pi}_{y_2}}{f^1_{x_1} f^2_{y_2}} + \frac{\bar{w}_{x_2} + \bar{\pi}_{y_1}}{f^1_{x_2} f^2_{y_1}}.$$

Regroup terms to write this as

$$\left(\frac{\bar{w}_{x_1}}{f^1_{x_1}} - \frac{\bar{w}_{x_2}}{f^1_{x_2}}\right)\left(\frac{1}{f^2_{y_1}} - \frac{1}{f^2_{y_2}}\right) + \left(\frac{\bar{\pi}_{y_1}}{f^2_{y_1}} - \frac{\bar{\pi}_{y_2}}{f^2_{y_2}}\right)\left(\frac{1}{f^1_{x_1}} - \frac{1}{f^1_{x_2}}\right) < 0.$$

Lemma 2 establishes that of $\bar{w}_x/f^1_x$ is increasing. A parallel (omitted) proof establishes monotonicity of $\bar{\pi}_y/f^2_y$. Thus both of the terms on the left hand side are negative, proving the result. ■

B Log-Linear Wage Equation

Most of the empirical literature does not aim to estimate the average log wage, $w^*_{x,y}$. Instead, following Abowd, Kramarz and Margolis (1999), authors impose a log-linear wage structure to estimate worker and firm fixed effects. In this section we analyze what that procedure recovers if our model is data-generating process, and in particular the relationship between a log-linear wage equation and the average log wage $w^*_{x,y}$.

B.1 Econometric Framework

Consider a panel data set containing the wage $W_{i,t}$ of worker $i \in \{1, \ldots, I\}$ at time $t \in \{1, 2, \ldots, T\}$ as well as the employer identifier $\mathcal{J}_{i,t} \in \{1, \ldots, J\}$. Since the worker may not always be employed, we let $\mathcal{T}_i \subseteq \{1, 2, \ldots, T\}$ denote the periods when the worker earns a wage. We assume, in line with the literature, that we observe neither a wage nor a wage offer for i when they are not employed, at $t \in \mathcal{T}_i^c \equiv \{1, 2, \ldots, T\} \backslash \mathcal{T}_i$. For notational simplicity and following the literature, we impose that each worker only works for one firm at each point in time, for example by focusing on their main job in each period that they are employed.

Following Abowd, Kramarz and Margolis (1999), we could regress the log wage on a full set of worker and firm fixed effects and an error term: For all $i \in \{1, \ldots, I\}$ and $t \in \mathcal{T}_i$,

$$\log W_{i,t} = \alpha_i + \psi_{\mathcal{J}_{i,t}} + \varepsilon_{i,t}. \tag{35}$$

We are interested in the coefficient estimates $\hat{\alpha}_i$ and $\hat{\psi}_j$ when estimating equation (35) using ordinary least squares (OLS). Regardless of the economic model and the data set, OLS is a statistical procedure which minimizes the sum of squared errors:

$$\{\hat{\alpha}_i, \hat{\psi}_j\} = \arg \min_{\{\alpha_i, \psi_j\}} \sum_{i=1}^{I} \sum_{t \in \mathcal{T}_i} (\log W_{i,t} - \alpha_i - \psi_{\mathcal{J}_{i,t}})^2. \tag{36}$$

The first order condition for α_i from equation (36) is

$$\sum_{t \in \mathcal{T}_i} (\log W_{i,t} - \hat{\alpha}_i - \hat{\psi}_{\mathcal{J}_{i,t}}) = 0. \tag{37}$$

Symmetrically, define $\mathcal{I}_{j,t}$ as the set of workers whom j employs at t, so $j = \mathcal{J}_{i,t}$ if and only if $i \in \mathcal{I}_{j,t}$.[18] Then the first order condition for ψ_j is

$$\sum_{t=1}^{T} \sum_{i \in \mathcal{I}_{j,t}} (\log W_{i,t} - \hat{\alpha}_i - \hat{\psi}_j) = 0. \tag{38}$$

Under the assumption that all workers and firms are connected though the matching graph,[19] Abowd, Creecy and Kramarz (2002) establish that equations (37) and (38) pin down $\hat{\alpha}$ and $\hat{\psi}$ up to an additive constant. That is, we can increase all the worker fixed effects by k and

[18] In our model, $\mathcal{I}_{j,t}$ has either zero or one element, depending on whether the job is filled or vacant. In real world data, firms can employ multiple workers and so $\mathcal{I}_{j,t}$ typically has multiple elements.

[19] Formally, we require that any worker i_0 can be linked to any firm j through a finite sequence of steps $t_0, t_1, \ldots, t_n$: $j_t = \mathcal{J}_{i_{t-1},t}$ for t odd and $i_t \in \mathcal{I}_{j_{t-1},t}$ for t even, with $j = \mathcal{J}_{t_n}$.

decrease all the firm fixed effects by k without changing the fit of equation (36).

B.2 Estimates in Model-Generated Data

Next consider estimating equation (35) using an ideal data set generated by our model. We assume that there is a large number of workers I and a large number of firms J. Each worker i has an unobserved type x_i, and similarly each firm j has an unobserved type y_j. We assume i and j behave according to the decision rules in our model, for simplicity described here without on-the-job search. That is, when i is unemployed, they meet a type-y vacant job in a match with productivity at least z at rate $\rho v_y S(z)$, and they accept the job and earn a wage $W_{x_i,y}(z)$ if and only if $z \geq \bar{z}_{x_i,y}$. Symmetrically, when j has a vacant job, it meets a type-x unemployed worker in a match with productivity at least z at rate $\rho u_x S(z)$, and it hires the worker and earns profits $z f_{x,y_j} - W_{x,y_j}(z)$ if and only if $z \geq \bar{z}_{x,y_j}$.

We are interested in an environment where there is a large but finite number of workers and jobs and where we observe each worker and job for a very long time, $T \to \infty$.[20] In this case, worker i with type x_i will spend a fraction u_{x_i}/m_{x_i} of their time unemployed and fraction $\phi_{x_i,y}/m_{x_i}$ of their time matched to a type-y firm. In such matches, the density of match productivity will be $s(z)/S(\bar{z}_{x_i,y})$ for $z \geq \bar{z}_{x_i,y}$ and the wage will be $W_{x_i,y}(z)$. Similarly, the relative likelihood of firm j with type y_j matching with a type-x worker is proportional to ϕ_{x,y_j}. Again, in such matches, the density of match productivity will be $s(z)/S(\bar{z}_{x,y_j})$ for $z \geq \bar{z}_{x,y_j}$ and the wage will be $W_{x,y_j}(z)$. Since there is no uncertainty about these long-run distributions, $\hat{\alpha}_i$ and $\hat{\psi}_j$ have well-behaved limits in the limit as $T \to \infty$. These limits depend only on the worker's and firm's type, since the distribution of partners and wages only depend on types. We let $\bar{\alpha}_{x_i}$ denote the limiting value of $\hat{\alpha}_i$ and $\bar{\psi}_{y_j}$ denote the limiting value of $\hat{\psi}_j$ when $T \to \infty$.[21] We are interested in characterizing and interpreting those values.

[20]In the real world, T is finite, so the OLS estimates $\hat{\alpha}_i$ and $\hat{\psi}_j$ are unbiased but noisy estimates of $\bar{\alpha}_{x_i}$ and $\bar{\psi}_{y_j}$, respectively. This creates econometric issues which we sidestep in this paper through our idealized "large T" assumption.

[21]In practice, the literature estimating equation (35) looks only at one job per worker per year. In Abowd, Kramarz and Margolis (1999), this is the job where the individual works the most days. In Card, Cardoso and Kline (2016), this is the job where the worker has the most hours during a reference week. In Bonhomme, Lamadon and Manresa (2019), a worker is only included in the sample if they are employed by a single firm in all twelve months. Since the duration distribution of all jobs is the same in our model without on-the-job search, none of these selection criterion affect the asymptotic values $\bar{\alpha}_x$ and $\bar{\psi}_y$.

Using equations (37) and (38) and the model structure, we obtain

$$\bar{\alpha}_x = \frac{\sum_{y=1}^{Y}(w^*_{x,y} - \bar{\psi}_y)\phi_{x,y}}{\sum_{y=1}^{Y}\phi_{x,y}} = \lambda_x - \frac{\sum_{y=1}^{Y}\bar{\psi}_y\phi_{x,y}}{\sum_{y=1}^{Y}\phi_{x,y}}, \tag{39}$$

$$\bar{\psi}_y = \frac{\sum_{x=1}^{X}(w^*_{x,y} - \bar{\alpha}_x)\phi_{x,y}}{\sum_{x=1}^{X}\phi_{x,y}} = \mu_y - \frac{\sum_{x=1}^{X}\bar{\alpha}_x\phi_{x,y}}{\sum_{x=1}^{X}\phi_{x,y}}, \tag{40}$$

where $w^*_{x,y}$ is the average log wage in an (x, y) match, defined in equation (15), $\phi_{x,y}$ is the measure of (x, y) matches, defined in equation (12), and λ_x and μ_y are defined in equations (19) and (20). Our model ensures that all workers and firms are connected when $T \to \infty$, since $S(\bar{z}_{x,y}) > 0$ for all (x, y). That means that we can solve equations (39) and (40) for $(\bar{\alpha}_x, \bar{\psi}_y)$ up to the additive constant k mentioned before.

If the average log wage were an additively separable function of the worker's type and the firm's type, $w^*_{x,y} = a_x + b_y$, then equations (39) and (40) would imply $\bar{\alpha}_x = a_x + k$ and $\bar{\psi}_y = b_y - k$, where k is the irrelevant additive constant discussed before. In this case, $\bar{\psi}_{y_2} - \bar{\psi}_{y_1}$ is the average difference in the log wage that any worker earns at a type-y_2 firm compared to a type-y_1 firm when following the equilibrium decision rules. Our model with a Pareto distribution of match-specific shocks implies that $w^*_{x,y}$ is increasing in x and y but is submodular rather than linear.

Finally, we follow Abowd, Kramarz and Margolis (1999), Andrews, Gill, Schank and Upward (2008), Card, Heining and Kline (2013), and others in focusing on the cross-sectional variance of $\bar{\alpha}$ and $\bar{\psi}$ as well as the covariance between α and ψ in matched pairs. Define

$$\mathbb{E}(g_{x,y}) \equiv \frac{\sum_{x=1}^{X}\sum_{y=1}^{Y} g_{x,y}\phi_{x,y}}{\sum_{x=1}^{X}\sum_{y=1}^{Y}\phi_{x,y}}$$

for any g. We let $\sigma^2_{\bar{\alpha}} \equiv \mathbb{E}\big((\bar{\alpha}_x - \mathbb{E}(\bar{\alpha}_x))^2\big)$ and $\sigma^2_{\bar{\psi}} \equiv \mathbb{E}\big((\bar{\psi}_y - \mathbb{E}(\bar{\psi}_y)))^2\big)$ denote the variance of α_{x_i} and ψ_{y_j} across employed workers i and filled jobs j. Also let $\text{cov}_{\bar{\alpha},\bar{\psi}} \equiv \mathbb{E}\big((\bar{\alpha}_x - \mathbb{E}(\bar{\alpha}_x))(\bar{\psi}_y - \mathbb{E}(\bar{\psi}_y))\big)$ denote the covariance between $\bar{\alpha}_{x_i}$ and $\bar{\psi}_{y_j}$ across matched pairs (i, j) with $j = J_{i,t}$. The unidentified additive constant k does not affect any of these moments.

If we estimate $\hat{\alpha}_i$ and $\hat{\psi}_j$ using OLS, the well-known issue of limited mobility (Andrews, Gill, Schank and Upward, 2008) biases estimates of the variances and covariances when T is finite. Throughout this paper, we assume an idealized environment, either one where $T \to \infty$, or alternatively a statistical procedure that gives unbiased estimates of the variance and covariance, as proposed by Andrews, Gill, Schank and Upward (2008), Bonhomme, Lamadon and Manresa (2019), or Kline, Saggio and Sølvsten (2020).

We find that the parameters in Table 2 deliver the variance decomposition targets in

Table 1, in line with BLM and KSS.

C On-the-Job Search Model

We show how to reformulate the model so that it is easier to solve, and then report some additional quantitative results.

C.1 Reformulation of the Model

Define $Z_{x,y}(\nu)$ to be the value of z such that the match surplus equals ν: $V^s_{x,y}(Z_{x,y}(\nu)) = \nu$. Then $z = Z_{x,y}(V^s_{x,y}(z))$ so $dz = Z'_{x,y}(\nu)d\nu$. We change variables in (25):

$$(r+\delta)\nu = Z_{x,y}(\nu)f_{x,y} - rV^u_x - rV^v_y + \rho_1 \sum_{y'=1}^{Y} v_{y'} \int_{\nu}^{\infty} (\gamma\nu' - \nu) s(Z_{x,y'}(\nu')) Z'_{x,y'}(\nu') d\nu'. \quad (41)$$

To solve this equation, we need to find the value of unemployed workers V^u_x, the value of vacancies V^v_y, and the vacancy measures v_y.

Next, we find the steady state unemployment rate by equating inflows and outflows from unemployment:

$$\delta(m_x - u_x) = \rho_0 u_x \sum_{y=1}^{Y} v_y S(Z_{x,y}(0)). \quad (42)$$

Employed workers become unemployed at rate δ, while unemployed type x workers find a job at rate $\rho_0 u_x \sum_{y=1}^{Y} v_y S(Z_{x,y}(0))$. There is no such simple equation for vacancies, however, because filled jobs end at an endogenous rate that depends on worker's quitting decision.

To find the unemployment and vacancy rates, we first let $\tilde{\phi}_{x,y}(\nu)$ be the density of the type x workers matched with type y firms with the match surplus ν. Let $\tilde{\Phi}_x(\nu)$ be the measure of worker type x who are employed in a match with surplus less than ν, $\tilde{\Phi}_x(\nu) = \sum_{y=1}^{Y} \int_0^{\nu} \tilde{\phi}_{x,y}(\nu')d\nu'$. Then it holds,

$$\left(\delta + \rho_1 \sum_{y=1}^{Y} v_y S(Z_{x,y}(\nu))\right) \tilde{\Phi}_x(\nu) = \rho_0 u_x \sum_{y=1}^{Y} v_y \left(S(Z_{x,y}(0)) - S(Z_{x,y}(\nu))\right). \quad (43)$$

The outflow from $\tilde{\Phi}_x(\nu)$, the left-hand side of (43), is either due to exogenous separations or workers accepting offers with surplus higher than ν. The inflow into $\tilde{\Phi}_X(\nu)$ is only from unemployment, and it is given by the rate at which workers receive offers with surplus between 0 and ν.

It is now straightforward to characterize density $\phi_{x,y}(\nu)$:

$$\left(\delta + \rho_1 \sum_{y'=1}^{Y} v_{y'} S(Z_{x,y'}(\nu))\right) \tilde{\phi}_{x,y}(\nu) = (\rho_0 u_x + \rho_1 \tilde{\Phi}_x(\nu)) v_y s(Z_{x,y}(\nu)) Z'_{x,y}(\nu). \tag{44}$$

The left-hand side shows the outflow from $\tilde{\phi}_{x,y}(\nu)$, which is either due to exogenous separations or workers moving to higher-surplus matches. The right-hand side shows the inflow. Workers come from unemployment by receiving an offer at the rate $\rho_0 u_x$, or from employment at the rate $\rho_1 \tilde{\Phi}_x(\nu)$ since $\tilde{\Phi}_x(\nu)$ is the measure of workers with surplus below ν. We can also use equation (43) to write

$$\begin{aligned} \rho_0 u_x + \rho_1 \tilde{\Phi}_x(\nu) &= \rho_0 u_x \left(\frac{\delta + \rho_1 \sum_{y=1}^{Y} v_y S(Z_{x,y}(0))}{\delta + \rho_1 \sum_{y=1}^{Y} v_y S(Z_{x,y}(\nu))} \right) \\ &= \frac{\delta\big(\rho_0 u_x + \rho_1 (m_x - u_x)\big)}{\delta + \rho_1 \sum_{y=1}^{Y} v_y S(Z_{x,y}(\nu))}, \end{aligned} \tag{45}$$

where the second equation eliminates $\sum_{y=1}^{Y} v_y S(Z_{x,y}(0))$ using equation (42). Thus we obtain from equation (44)

$$\tilde{\phi}_{x,y}(\nu) = \frac{\delta\big(\rho_0 u_x + \rho_1 (m_x - u_x)\big) v_y s(Z_{x,y}(\nu)) Z'_{x,y}(\nu)}{\big(\delta + \rho_1 \sum_{y'=1}^{Y} v_{y'} S(Z_{x,y'}(\nu))\big)^2}. \tag{46}$$

The measures of type-x unemployed and type-y vacancies are

$$u_x = m_x - \sum_{y=1}^{Y} \int_0^\infty \tilde{\phi}_{x,y}(\nu) d\nu, \tag{47}$$

$$v_y = n_y - \sum_{x=1}^{X} \int_0^\infty \tilde{\phi}_{x,y}(\nu) d\nu. \tag{48}$$

Next, we write equations (26) and (27) in terms of $Z_{x,y}(\nu)$:

$$rV_x^u = \gamma \rho_0 \sum_{y=1}^{Y} v_y \int_0^\infty \nu s(Z_{x,y}(\nu)) Z'_{x,y}(\nu) d\nu \tag{49}$$

$$\begin{aligned} rV_y^v = {}& (1-\gamma)\rho_0 \sum_{x=1}^{X} u_x \int_0^\infty \nu s(Z_{x,y}(\nu)) Z'_{x,y}(\nu) d\nu \\ &+ (1-\gamma)\rho_1 \sum_{x=1}^{X} \int_0^\infty \nu \tilde{\Phi}_x(\nu) s(Z_{x,y}(\nu)) Z'_{x,y}(\nu) d\nu. \end{aligned} \tag{50}$$

We can simplify equation (50) using equation (45):

$$rV_y^v = (1-\gamma)\delta \sum_{x=1}^{X} \big(\rho_0 u_x + \rho_1(m_x - u_x)\big) \int_0^\infty \frac{\nu s(Z_{x,y}(\nu)) Z'_{x,y}(\nu)}{\delta + \rho_1 \sum_{y'=1}^{Y} v_{y'} S(Z_{x,y'}(\nu))} d\nu. \tag{51}$$

Finally, rather than trying to solve the integral equation (41) directly, we differentiate it with respect to ν to get a differential equation for $Z_{x,y}(\nu)$:

$$r + \delta = Z'_{x,y}(\nu) f_{x,y} - \rho_1 \sum_{y'=1}^{Y} v_{y'} S(Z_{x,y'}(\nu)) + \rho_1(1-\gamma) \sum_{y'=1}^{Y} v_{y'} \nu s(Z_{x,y'}(\nu)) Z'_{x,y'}(\nu). \tag{52}$$

Additionally, plugging $\nu = 0$ into equation (41) and using equation (49) to simplify, we obtain the initial condition $Z_{x,y}(0) f_{x,y} = (1 - \rho_1/\rho_0) r V_x^u + r V_y^v$. For each x, this is system of N^y differential equations, easily solved numerically if we know V_x^u, V_y^v, and v_y.

To recover wages, we rewrite (31) in terms of $Z_{x,y}(\nu)$,

$$\tilde{W}_{x,y}(\nu) = rV_x^u + \gamma(r+\delta)\nu - \rho_1 \gamma \sum_{y'=1}^{Y} v_{y'} \int_\nu^\infty (\nu' - \nu) s(Z_{x,y'}(\nu')) Z'_{x,y'}(\nu') d\nu'. \tag{53}$$

To solve the model, we proceed as follows:

1. Given V_x^u, V_y^v, and v_y, we solve ODE (52) with the initial condition $Z_{x,y}(0) f_{x,y} = (1 - \rho_1/\rho_0) r V_x^u + r V_y^v$.
2. Given $Z_{x,y}(\nu)$, use equations (43)—(48) to update u_x and v_y and equations (49) and (50) to update V_x^u and V_y^v.
3. Repeat until the changes in u_x, v_y, V_x^u, and V_y^v are small.
4. Lastly recover the wage using equation (53).

C.2 Normalizations

A steady state equilibrium is described by a solution to equations (41) and (43)–(53). Here we show how to normalize the wage in both the Pareto and Exponential cases.

Pareto Case

Assume $S(z) = z^{-\theta}$. Consider an equilibrium $(V_x^u, V_y^v, Z_{x,y}(\nu), \tilde{\Phi}_x(\nu), \tilde{\phi}_{x,y}(\nu), u_x, v_y, \tilde{W}_{x,y}(\nu))$ given one value of the contact rates ρ_0 and ρ_1. Let $\hat{\rho}_0 = \lambda \rho_0$ and $\hat{\rho}_1 = \lambda \rho_1$ for some $\lambda > 0$.

Then we claim that with the contact rate $\hat{\rho}$, there is an equilibrium as follows:

$$\hat{V}_x^u = \lambda^{1/\theta} V_x^u, \quad \hat{V}_y^v = \lambda^{1/\theta} V_y^v$$
$$\hat{Z}_{x,y}(\lambda^{1/\theta}\nu) = \lambda^{1/\theta} Z_{x,y}(\nu), \quad \hat{\tilde{W}}_{x,y}(\lambda^{1/\theta}\nu) = \lambda^{1/\theta}\tilde{W}_{x,y}(\nu)$$
$$\hat{\tilde{\Phi}}_x(\lambda^{1/\theta}\nu) = \tilde{\Phi}_x(\nu)$$
$$\lambda^{1/\theta}\hat{\tilde{\phi}}_{x,y}(\lambda^{1/\theta}\nu) = \tilde{\phi}_{x,y}(\nu)$$
$$\hat{u}_x = u_x, \hat{v}_y = v_y.$$

The proof involves writing the eight equations in terms of the hatted variables evaluated at $\lambda^{1/\theta}\nu$ and simplifying. For, equation (41) becomes

$$(r+\delta)\lambda^{1/\theta}\nu = \hat{Z}_{x,y}(\lambda^{1/\theta}\nu) f_{x,y} - r\hat{V}_x^u - r\hat{V}_y^v + \hat{\rho}_1 \sum_{y_1=1}^{Y} v_{y_1} \int_{\lambda^{1/\theta}\nu}^{\infty} (\gamma\nu' - \lambda^{1/\theta}\nu)\theta \hat{Z}_{x,y_1}(\nu')^{-\theta-1}\hat{Z}'_{x,y_1}(\nu') d\nu'.$$

In the integral, do a change of variables to $\nu'' \equiv \lambda^{-1/\theta}\nu'$, so $d\nu' = \lambda^{1/\theta} d\nu''$. Also change the "hatted" variables to their unhatted counterparts using our conjectured functional forms

$$(r+\delta)\lambda^{1/\theta}\nu = \lambda^{1/\theta} Z_{x,y}(\nu) f_{x,y} - \lambda^{1/\theta} r V_x^u - \lambda^{1/\theta} r V_y^v + \lambda\rho_1 \sum_{y_1=1}^{Y} v_{y_1} \int_{\nu}^{\infty} (\gamma\lambda^{1/\theta}\nu'' - \lambda^{1/\theta}\nu)\theta \big(\lambda^{1/\theta} Z_{x,y_1}(\nu'')\big)^{-\theta-1} Z'_{x,y_1}(\nu'')\lambda^{1/\theta} d\nu''.$$

All the terms involving λ cancel, yielding equation (41). We omit the other steps in the argument, which use an identical logic.

We now show that the measures of type-x workers employed by type-y firms is unaffected by the scale parameter in the meeting rate, λ:

$$\hat{\phi}_{x,y} \equiv \int_0^\infty \hat{\tilde{\phi}}_{x,y}(\nu) d\nu = \int_0^\infty \hat{\tilde{\phi}}_{x,y}(\lambda^{1/\theta}\nu')\lambda^{1/\theta} d\nu' = \int_0^\infty \tilde{\phi}_{x,y}(\nu') d\nu' \equiv \phi_{x,y}.$$

We first define $\hat{\phi}_{x,y}$. We then use a change of variables with $\lambda^{1/\theta}\nu' = \nu$. The third equation then replaces $\hat{\tilde{\phi}}$ with our expression for $\tilde{\phi}$. Thus the right hand panels in Figure 1 are unaffected by the scale of the meeting rate.

We also calculate the average log wage that a type-x worker receives when employed by a type-y firm:

$$\hat{w}^*_{x,y} \equiv \frac{\int_0^\infty \log \hat{\tilde{W}}_{x,y}(\nu)\hat{\tilde{\phi}}_{x,y}(\nu)d\nu}{\int_0^\infty \hat{\tilde{\phi}}_{x,y}(\nu)d\nu} = \frac{\int_0^\infty \log \hat{\tilde{W}}_{x,y}(\lambda^{1/\theta}\nu')\hat{\tilde{\phi}}_{x,y}(\lambda^{1/\theta}\nu')\lambda^{1/\theta}d\nu'}{\int_0^\infty \hat{\tilde{\phi}}_{x,y}(\lambda^{1/\theta}\nu')\lambda^{1/\theta}d\nu'}$$

$$= \frac{\int_0^\infty \log\left(\lambda^{1/\theta}\tilde{W}_{x,y}(\nu')\right)\tilde{\phi}_{x,y}(\nu')d\nu'}{\int_0^\infty \tilde{\phi}_{x,y}(\nu')d\nu'} = \frac{1}{\theta}\log\lambda + w^*_{x,y}.$$

The steps are the same: define $\hat{w}^*$, do the same change of variables, replace 'hatted' variables by the 'unhatted' counterparts, and simplify. This shows how we choose λ to target an average log wage of 0 in $(1,1)$ match, and that this simply scales all average log wages by the same additive constant.

Exponential Case

Let $(V^u_x, V^v_y, Z_{x,y}(\nu), \tilde{\Phi}_x(\nu), \tilde{\phi}_{x,y}(\nu), u_x, v_y, \tilde{W}_{x,y}(\nu))$ denote an equilibrium when the distribution of match-specific shocks is exponential distribution with mean 1. We construct an equilibrium with an exponential distribution with mean θ, i.e. $S(z) = e^{-z/\theta}$, as follows:

$$\hat{V}^u_x = \theta V^u_x, \quad \hat{V}^v_y = \theta V^v_y$$
$$\hat{Z}_{x,y}(\theta\nu) = \theta Z_{x,y}(\nu), \quad \hat{\tilde{W}}_{x,y}(\theta\nu) = \theta\tilde{W}_{x,y}(\nu)$$
$$\hat{\tilde{\Phi}}_x(\theta\nu) = \tilde{\Phi}_x(\nu)$$
$$\theta\hat{\tilde{\phi}}_{x,y}(\theta\nu) = \tilde{\phi}_{x,y}(\nu)$$
$$\hat{u}_x = u_x, \hat{v}_y = v_y.$$

Again, we prove this by writing the eight equations (41) and (43)–(53) in terms of the hatted variables evaluated at $\theta\nu$ and simplifying. Start with equation (41):

$$(r+\delta)\theta\nu = \hat{Z}_{x,y}(\theta\nu)f_{x,y} - r\hat{V}^u_x - r\hat{V}^v_y + \rho_1\sum_{y_1=1}^{Y} v_{y_1}\int_{\theta\nu}^{\infty}(\gamma\nu' - \theta\nu)\frac{1}{\theta}e^{-\hat{Z}_{x,y_1}(\nu')/\theta}\hat{Z}'_{x,y_1}(\nu')d\nu'.$$

In the integral, do a change of variables to $\nu'' = \nu'/\theta$, so $d\nu' = \theta d\nu''$. Also change the hatted variables to their unhatted counterparts:

$$(r+\delta)\theta\nu = \hat{Z}_{x,y}(\theta\nu)f_{x,y} - \theta rV^u_x - \theta rV^v_y + \rho_1\sum_{y_1=1}^{Y} v_{y_1}\int_{\nu}^{\infty}(\gamma\theta\nu'' - \theta\nu)\frac{1}{\theta}e^{-Z_{x,y_1}(\nu'')}Z'_{x,y_1}(\nu'')\theta d\nu''.$$

	ρ_1/ρ_0	θ	Δ_w	Δ_f	ξ
BLM	0.1	10.809	0.084	0.361	0.967
BLM	0.2	22.150	0.053	0.270	0.964
BLM	0.3	60.810	0.024	0.221	1.026
KSS	0.1	61.311	0.193	0.855	5.414
KSS	0.2	81.430	0.074	0.585	4.103
KSS	0.3	368.43	0.016	0.513	4.099

Table 3: Calibrated parameters for different values of ρ_1. For different values of ρ_1/ρ_0, we fix r, δ, γ and calibrate the remaining four parameters to match the wage decomposition in BLM (top 3 rows) or KSS (bottom three rows). The last column shows the implied unemployment rate.

All the terms involving θ cancel, yielding equation (41). We again omit the remaining steps in the argument, which use the same logic.

The proof that the measure of type-x workers employed by type-y firms is unaffected by the mean of the exponential is the same as in the Pareto case. For the average log wage, we also have follow similar steps as we did with the Pareto:

$$\begin{aligned}\hat{w}^*_{x,y} \equiv \frac{\int_0^\infty \log \hat{\tilde{W}}_{x,y}(\nu)\hat{\tilde{\phi}}_{x,y}(\nu)d\nu}{\int_0^\infty \hat{\tilde{\phi}}_{x,y}(\nu)d\nu} &= \frac{\int_0^\infty \log \hat{\tilde{W}}_{x,y}(\theta\nu')\hat{\tilde{\phi}}_{x,y}(\theta\nu')\theta d\nu'}{\int_0^\infty \hat{\tilde{\phi}}_{x,y}(\theta\nu')\theta d\nu'} \\ = \frac{\int_0^\infty \log\left(\theta\tilde{W}_{x,y}(\nu')\right)\tilde{\phi}_{x,y}(\nu')d\nu'}{\int_0^\infty \tilde{\phi}_{x,y}(\nu')d\nu'} &= \log\theta + w^*_{x,y}.\end{aligned}$$

This shows how we choose θ to target an average log wage of 0 in $(1,1)$ match, and that this simply scales all average log wages by the same additive constant.

C.3 Quantitative Results with Different ρ_1

For comparison, we present results for the model with Pareto distributed match quality and $\rho_1 = 0.1\rho_0$ and $\rho_1 = 0.3\rho_0$. We re-calibrate the values of $\Delta_w, \Delta_f, \theta, \xi$ to match the BLM or KSS wage decompositions, and keep the other parameters the same as in the main text. Table 3 summarizes the parameter values. As search on the job becomes more effective, the amount of firm heterogeneity required to match the variance of firm effects becomes smaller, with the parameter Δ_f decreasing significantly. More search while employed also generates more wage dispersion, and so in order to keep the variance of log wages and the variance of the error term equal to the targeted values, the parameter θ has to increase to decrease the variance of the match-specific productivity shock.

Figures 5 and 6 show results with $\rho_1 = 0.1\rho_0$ and $\rho_1 = 0.3\rho_0$ for BLM calibration. The results are qualitatively the same as for BLM calibration with $\rho_1 = 0.2\rho_1$, shown in Figure 1 and Figure 4.

We now turn to Figure 7, the KSS calibration with $\rho_1 = 0.1\rho_0$. The top panel is qualitatively the same is $\rho_1 = 0.2\rho_0$: average log wages are monotone in worker and firm types, and there is sorting. The average surplus in matches is increasing in the firm type for each worker type, again same as Figure 4. The bottom right panel shows the average surplus in meetings. Every worker type has the lowest value from meeting type-1 firm, and the highest value of meeting a type-10 firm. This is different from Figure 4 where several worker types have a higher value of meeting type-1 firm than type-5 firm. An important driver of this difference is firm heterogeneity, which is calibrated to be larger in KSS $\rho_1 = 0.1\rho_0$ than in $\rho_1 = 0.2\rho_0$ (see the value of Δ_f in Table 3). The firm productivity differences are strong enough to overweight the fact that for low-type worker, a meeting with a high-type firm rarely turns into a match. This observation underlines our earlier discussion that we would not be able to conclude this without a calibrated structural model.

Figure 8 which shows KSS calibration $\rho_1 = 0.3\rho_0$ is qualitatively the same as $\rho_1 = 0.2\rho_0$ discussed in the main text.

D Event Study Methodology

We generate data from our model for 500,000 workers and 5,000 firms.

At time t=0, the economy is in the steady state, with the unemployment given by u_x and the joint distribution of matches by the ergodic distribution $\tilde{\phi}_{x,y}(\nu)$ in equation (46). Note that worker and firm types, x and y, and the value of the match, ν determine the distribution of workers' next jobs. In particular, the identities of workers and firms, and wages, are not needed at this stage.

We distinguish between employed and unemployed workers when determining worker's next job.

Consider a type-x unemployed worker. This worker meets a type-y firm with a positive match surplus at the rate $\rho_0 v_y S(Z_{x,y}(0))$. Hence, the probability that the worker's first job out of unemployment is in a y-type firm is

$$\frac{v_y S(Z_{x,y}(0))}{\sum_{y'=1}^{Y} v_{y'} S(Z_{x,y'}(0))}.$$

Conditional on that, the distribution of the match specific shock z is Pareto, with the scale parameter $Z_{x,y}(0)$. Once we draw y and z according to these distributions, we use the function $Z_{x,y}(\nu)$ to invert the value of z into the value of the match ν. Finally, we record the date

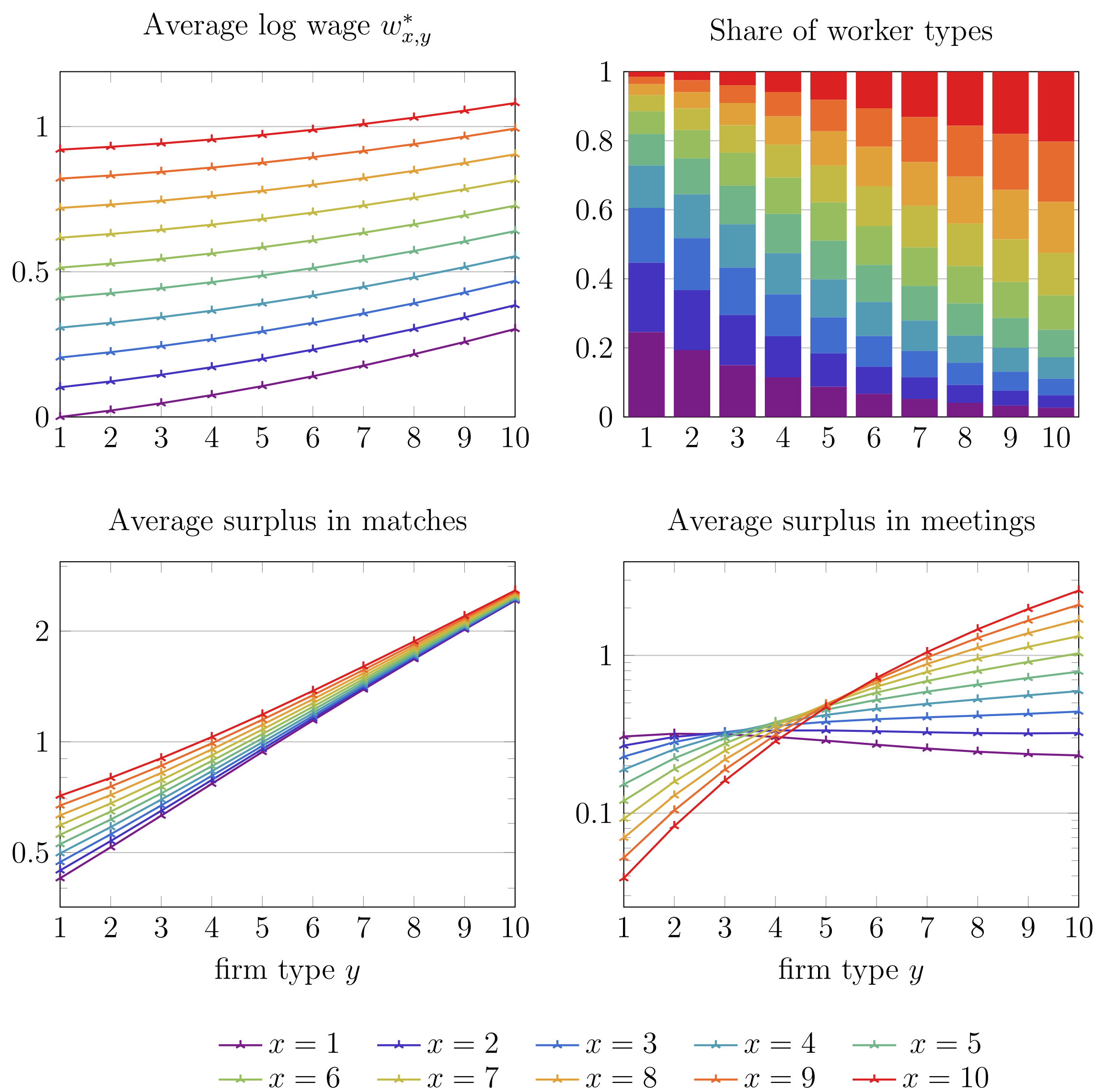

Figure 5: Results for model with on-the-job search, $\rho_1 = 0.1\rho_0$, BLM calibration. Bottom row is on a logarithmic scale.

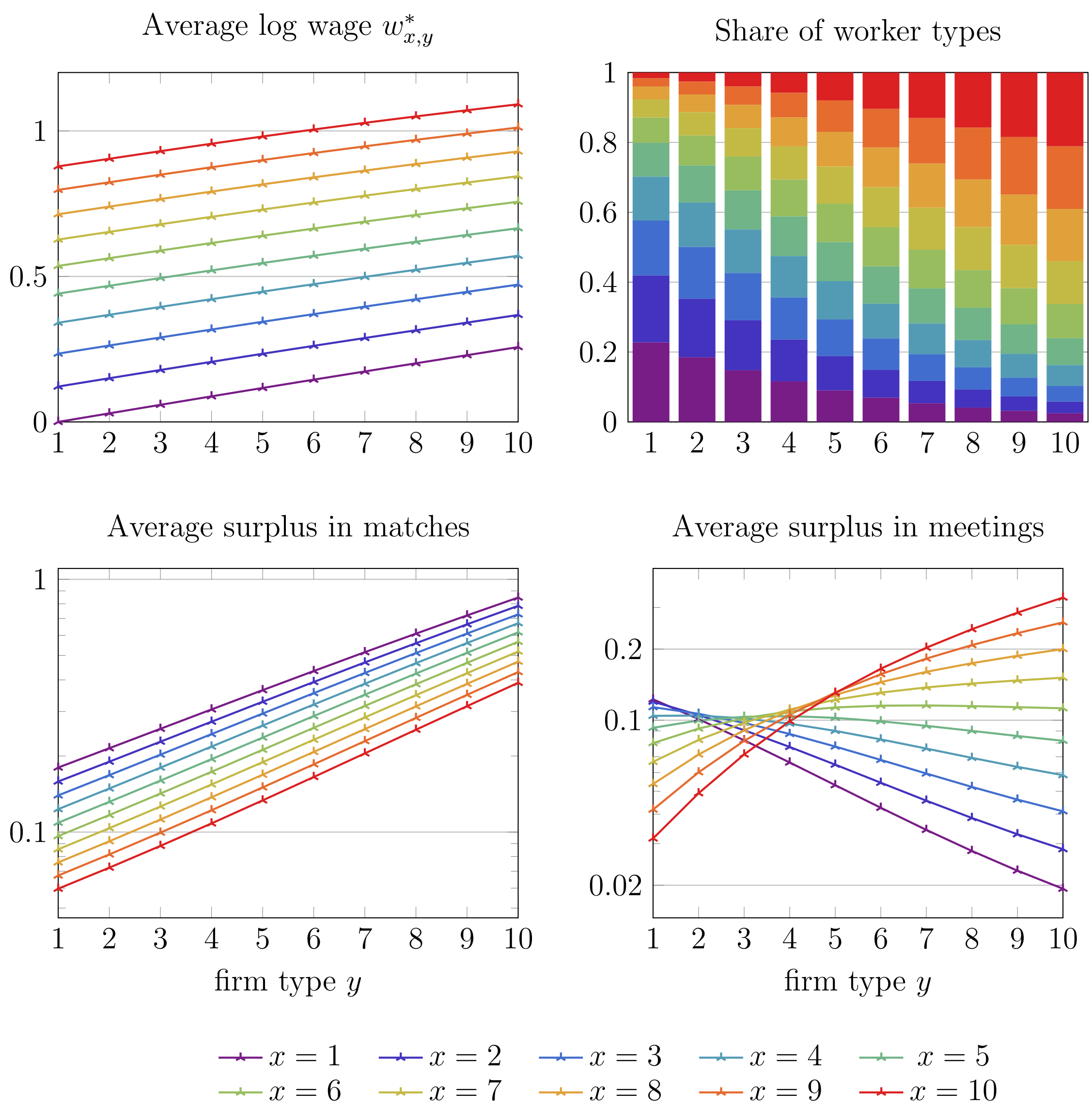

Figure 6: Results for model with on-the-job search, $\rho_1 = 0.3\rho_0$, BLM calibration. Bottom row is on a logarithmic scale.

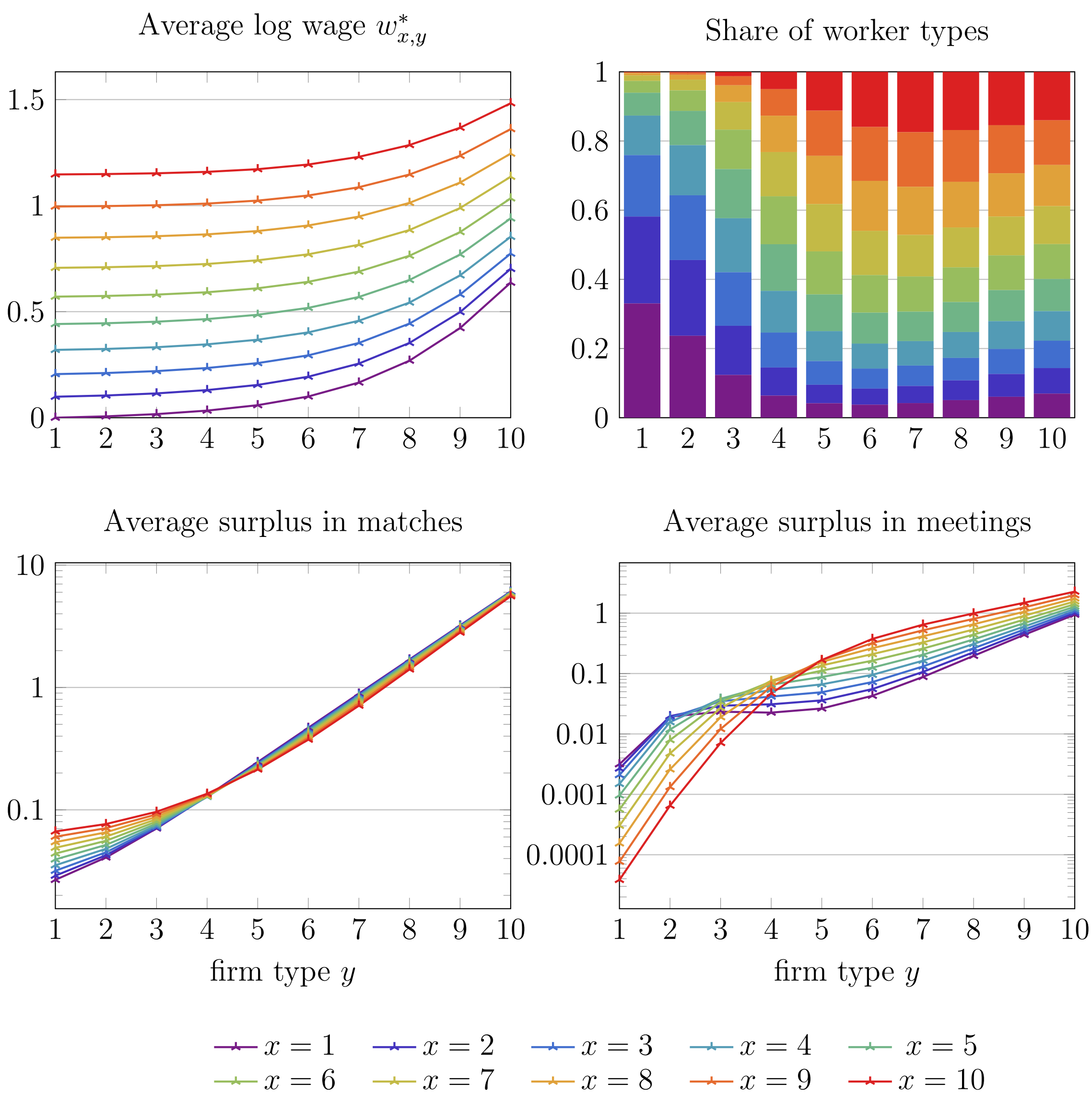

Figure 7: Results for model with on-the-job search, $\rho_1 = 0.1\rho_0$, KSS calibration. Bottom row is on a logarithmic scale.

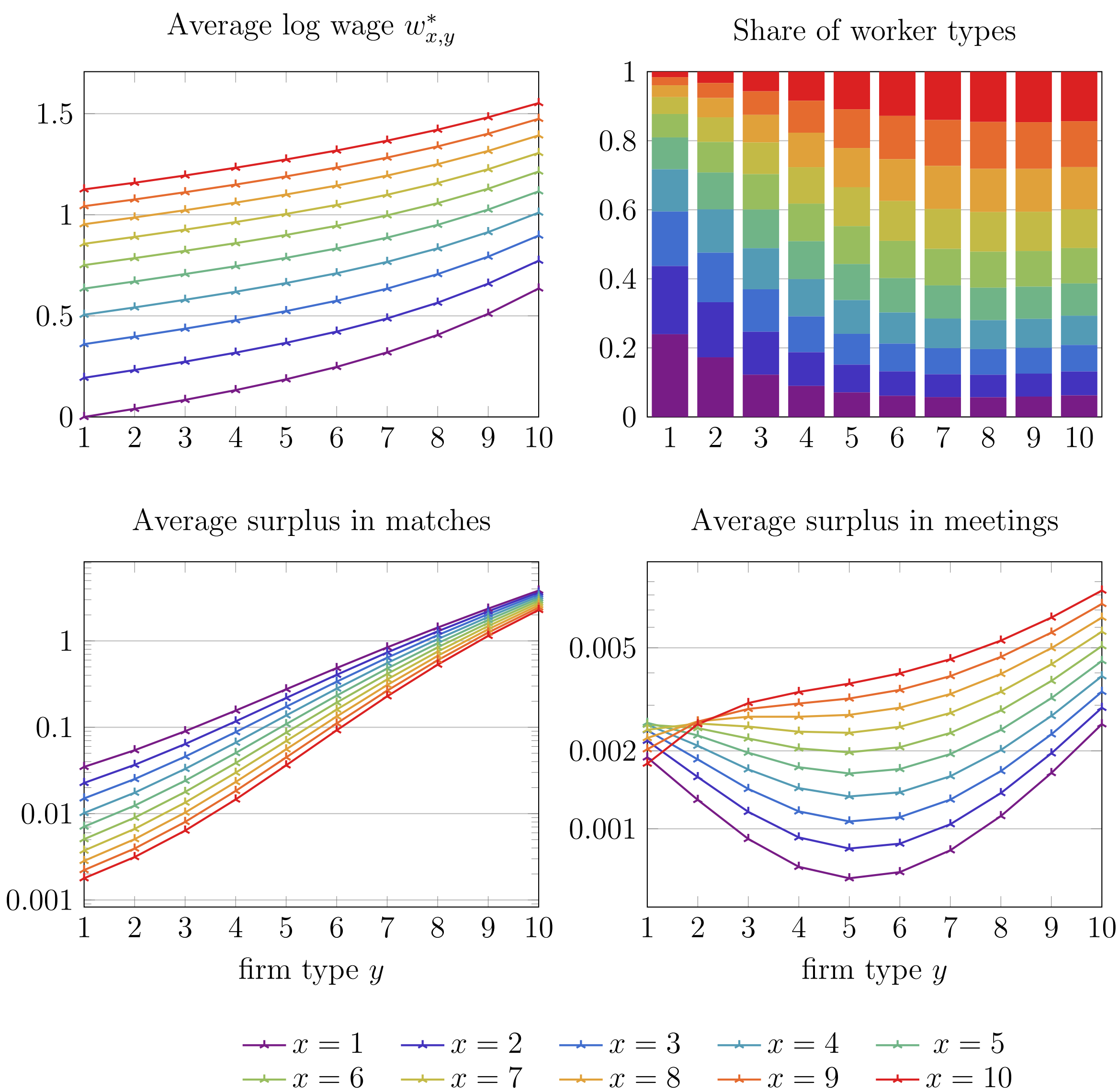

Figure 8: Results for model with on-the-job search, $\rho_1 = 0.3\rho_0$, KSS calibration. Bottom row is on a logarithmic scale.

of the end of the unemployment spell and the start of this job. The duration of an unemployment spell is exponentially distributed with the rate λ_x^u where $\lambda_x^u = \rho_0 \sum_{y_1} \upsilon_{y_1} S(Z_{x,y_1}(0))$.

Now consider an x-type worker employed at a y-type firm with value of the match ν. The worker receives a separation shock at the rate δ, and an acceptable offer from a y-type firm at the rate $\rho_1 \upsilon_y S(Z_{xy}(\nu))$. Define $\lambda_x^e(\nu)$ as

$$\lambda_x^e(\nu) = \rho_1 \sum_{y=1}^{Y} \upsilon_y S(Z_{x,y}(\nu)).$$

The probability that the worker's next transition is to unemployment is $\frac{\delta}{\delta + \lambda_x^e(\nu)}$, and the probability that the worker makes a direct transition to a y-type firm is $\frac{\rho_1 \upsilon_y S(Z_{x,y}(\nu))}{\delta + \lambda_x^e(\nu)}$. The distribution of the duration of an employment spell is exponential with the rate $\lambda_x^e(\nu) + \delta$. For the worker with the next job at a y-type firm, the distribution of match-specific shock z is Pareto with the scale parameter $Z_{x,y}(\nu)$. Again, we use $Z_{x,y}(\cdot)$ to invert the value of z into the value of the new match ν'. If the worker becomes unemployed, we use the same procedure as described above to draw the duration of the unemployment spell and the type of the next job.

We continue this procedure until we have four years of employment history for every worker.

We draw firm identities randomly from the set of firms with the given type y. If a worker makes a job-to-job transition from a y-type firm to another y-type firm, we exclude identity of the original firm from the set used to draw the new firm identity. Finally, we compute the wage associated with the match using equation (53).

We note that the first employment spell might be left-censored, and the last one right-censored, but this does not play a role in this event study.

We apply the same sample selection criteria as Card, Heining and Kline (2013). In every year, we define worker's main job as the match with the highest earnings within that year. For every worker-firm pair in the sample, we compute the average log wage of co-workers and then sort these pairs into four quartiles. Next, we select long-tenure movers. These are workers who changed main jobs between year 2 and 3, and had the same main job in years 1 and 2 as well as 3 and 4. We then sort these movers into 16 bins based on the quartile of the origin and destination firm, and compute the mean log wage of each group in years 1, 2, 3 and 4.