A Theory of Digital Ecosystems

Paul Heidhues

Mats Kösters

Botond Kőszegi

UNIVERSITÄT BONN

Universität zu Köln

A Theory of Digital Ecosystems[*]

Paul Heidhues
DICE

Mats Köster
CEU

Botond Kőszegi
University of Bonn

July 8, 2024

Abstract

We develop a model of digital ecosystems based on the assumption that a multi-market firm can use a sale in or data from one market to steer users toward its products in other markets. Due to this "cross-market leverage," a market leader at an "access point" (where users begin their online journeys) has a high value from offering services in connected markets (where users continue their journeys), and can thus make profitable takeovers. Indeed, because the firm has the threatening outside option of acquiring, and steering users toward, its target's competitor, it can take over the target at a discount. In contrast, other firms have no or smaller incentives for takeovers, explaining why ecosystems grow out of market leaders at access points. Conversely, cross-market leverage also implies that once an ecosystem has grown, it has an increased value of controlling access points, so it may go to great lengths to dominate these markets.

Our theory's logic suggests that ecosystems have mixed implications for consumer welfare. Under plausible assumptions, a to-be ecosystem takes over market leaders, and this consolidation of good services across markets benefits consumers in the short run. But an ecosystem's takeovers and dominance of access points lower incentives for entry and innovation, and lower the efficiency of access-point markets with superior alternatives. Hence, the long-run welfare implications of ecosystem growth are often negative.

Keywords: digital ecosystems, takeover, contestability, entry, envelopment, default effects, steering.

---

1 Introduction

In this paper, we formulate a novel theory of digital ecosystems — firms that offer a wide range of online services — by connecting two observations that are central in policy reports as well as academic research about these firms. First, ecosystems aim to keep consumers through setting defaults, manipulating attention, or employing other means of steering. Second, ecosystems tend to develop in a major part through takeovers.

Based on the first fact, we make the core assumption that an ecosystem can use a sale in or data from one market to steer consumers toward its product in another market. For instance, Google Search preferentially lists videos from Google-owned Youtube and connections from Google Flights, surely to increase traffic to these services (Schechner et al., 2020). Due to this "cross-market leverage," a market leader at an "access point" (where users begin their online journeys; Google Search in our example) has a high value from offering services users may want next. As a result, such a firm has an incentive to take over firms in other markets. Indeed, because the firm has the threatening outside option of taking over and then steering users toward a competitor, it can obtain a preferred target at a discount. We show that other firms do not have similar incentives for takeovers, explaining the second observation, and adding the prediction that ecosystems grow out of firms that control access points. Conversely, cross-market leverage means that once an ecosystem has grown, it can have an extremely high value for attracting consumers at access points. It is therefore willing to spend tremendous resources to steer consumers toward itself in these markets, for instance by buying the default position there or developing its product to be the first mover.

Our analysis indicates that ecosystems have mixed implications for consumer welfare. Focusing on services that are socially valuable, we argue that the short-run implications are often positive, while the long-run implications are often negative. Under plausible assumptions, a to-be ecosystem takes over market leaders, and this consolidation of good services across markets benefits consumers. But the fact that an ecosystem harms existing firms

through takeovers lowers the incentives for entry and innovation, which in turn often harms consumers. Furthermore, an ecosystem may stifle superior rivals at a new access point, or even newly superior rivals at an old access point, again potentially harming consumers.

We introduce our model in Section 2. Consumers use two services, a and b, starting with the access-point service a. The services are offered by single-market firms and possibly ecosystems, which all derive a fixed benefit from a service of theirs being used. We think of a service as "better" than another if it is utility-maximizing for more consumers. But boundedly rational consumers may fail to choose optimally for themselves, giving rise to steering effects. Specifically, in each market consumers' attention is drawn toward one "default" firm, which obtains an increase in its market share at the expense of other, potentially better firms. In our basic model, the default firm at the access point a is allocated exogenously; suppose (as we will endogenize below) that it is the best firm in the market. Default determination in market b, in turn, captures cross-market leverage: if the consumer bought from an ecosystem in market a, then the ecosystem becomes the consumer's default firm in market b. Otherwise, the consumer's default firm in market b is selected randomly. For predictions regarding the takeover targets of to-be ecosystems and the short-run welfare effects of ecosystem growth, we also impose quality-steering complementarity, whereby firms offering a better service benefit more from the default position.

We discuss several possible microfoundations for the default firm's benefit, such as pre-selecting the service in an app or link, including it in the consideration set of consumers with limited attention, using data from previous interactions to manipulate the consumer's attention, or setting one's product as the consumer's reference point. While different in the underlying psychology, the microfoundations have some common qualitative implications for a firm's valuation of the default position. Indeed, in Appendix A we identify plausible conditions under which the microfoundations also satisfy quality-steering complementarity.

In Section 3, we study the emergence of ecosystems from an economy with only single-market firms. Suppose that a single firm G in market a can make sequential take-it-or-leave-it

acquisition offers to firms in market b until one accepts or all reject. Because of cross-market leverage, firm G can direct its users in market a toward an acquired target, so it can make more profits than the target can by itself. But in addition to this valuation incentive for growing, firm G also has a strategic advantage in the process. Namely, if a target t rejected G's offer, then G would take over a competitor t' of t, and direct users to t'. Because this would divert users away from t, it diminishes the position of t. As a result, G can take over any preferred target t at a discount.

To sharpen our understanding of takeover incentives, we study several alternative games. Comparing the profits of all cross-market acquisitions, we establish that the takeover in which the best firm in market a acquires the best firm in market b is the most profitable. In particular, this forward integration is more profitable than backward integration, where the leader in market b acquires the leader in market a. The reason is that a firm in market b cannot steer users away from a firm in market a, so it does not enjoy a takeover discount. Furthermore, if there are multiple targets, then even the presence of multiple potential acquirers may fail to protect targets from acquisition at prices below their standalone values. We demonstrate this in the case where targets are equally valuable. Then, an increase in the number of potential acquirers can actually worsen the outcome for targets. Intuitively, the equilibrium price for takeovers is determined by the last target's value, which can be low because consumers are steered towards already acquired firms.

Taken as a whole, the above logic yields several conclusions. First and most importantly, it implies that market leaders at the access point grow into digital ecosystems, and do so primarily by taking over the best available firms. Second, the logic can explain why it is increasingly tech firms rather than investors specializing in tech — who do not benefit from takeover discounts — that make successful takeover offers (Eisfeld, 2024). Third, the takeover discount suggests that as long as there are available targets, an ecosystem may prefer to expand through takeovers rather than product development. To mention two of numerous examples, Google added both YouTube and Google Flights to its services through

takeovers. Nevertheless, if targets are unavailable, a dominant firm at the access point has an incentive to develop products for connected markets itself. Because of cross-market leverage, it still benefits more from the new service than other firms. For example, Microsoft (a leader in operating systems) spent tremendous resources to develop its browser and search engine.

In Section 4, we return to the beginning of the consumer's journey, the access-point market a. We endogenize the determination of the default firm by assuming that an outside seller auctions off this position to firms in the market. This corresponds to a situation where an original equipment manufacturer (OEM) like Apple or Samsung sets the default search engine for a fee. Other negotiation mechanisms, for instance bilateral bargaining, yield similar qualitative insights. Alternatively, the race to offer a new access product first may often be seen as an all-pay auction whose winner obtains the default position through prominence, again yielding similar insights.

As a benchmark, we show that with single-market firms, the best firm wins the auction. This justifies our assumption above that the best firm in market a becomes the default there, solidifying its market share and putting it in the best position to grow into an ecosystem. The situation is different, however, once an ecosystem has developed. An ecosystem G's cross-market leverage gives rise to a "default multiplier" in its willingness to pay for the default position in market a. The willingness to pay can be especially high when there is a second multi-market firm that may lure the consumer into its own ecosystem, making it difficult for G to win the consumer back. As a result, payments for the default position in market a can be extremely high. A striking example consistent with this prediction is the $18-20 billion annual payment that Google makes for being the default search engine on Apple devices — which constitutes 36% of its search advertising revenue from these devices.[1]

---

[1] See, for example, `https://t1p.de/r9drd` (accessed on June 11, 2024). Google also pays $400-$450 million per year for the default position on Firefox (see `https://t1p.de/1gvpq` accessed on June 11, 2024). That capturing the consumer's attention at access points is a key part of Google's strategy is clear from Google's internal communications as well. For instance: "There's tremendous power in the default OS access points but it's pay to play.... There is no substitute for the default access points: we should continue to explore broad default access across all OS (including newer and emerging access points)" (see, e.g.,

In Section 5, we identify what the above positive analysis of ecosystems implies for welfare and policy. Reflecting the core mechanism of our paper, we focus on interventions that disrupt the ability of digital ecosystems to steer consumers. These interventions include "leverage policies" aimed at weakening cross-market leverage, and "access-point policies" designed to regulate the sale of defaults. An obvious, if extreme and indirect, leverage policy is to restrict digital firms' ability to engage in takeovers. But many other policies are possible, and already in place in the European Union's Digital Market Act (DMA). For instance, Articles 6(3) and 6(4), which require that users can easily change default settings on operating systems, virtual assistants, web browsers, and app stores, can be seen as access-point policies. Similarly, Articles 6(5) and 6(6), which prohibit access-point firms from favoring their own products through rankings or impediments to switching, can be seen as leverage policies.

On the one hand, the development of a digital ecosystem tends to increase short-run welfare by enabling boundedly rational consumers to make better choices. This occurs because a digital ecosystem consolidates the best services under one umbrella, improving defaults in all affected markets. The idea that such an arrangement simplifies or improves consumers' lives explains why many consumers appear to love (steering by) digital ecosystems.[2] On the other hand, the long-run welfare effects of a digital ecosystem are more nuanced and often negative. First, potential inefficiencies arise as a digital ecosystem is willing to outbid a better rival at a new access point or a newly superior rival at an old access point. In such a case, access-point policies raise welfare in these critical markets. Second, our theory validates recent regulatory concerns about the stifling effects of digital ecosystems on innovation and entry (e.g., Crémer et al., 2019, Scott Morton et al., 2019, Furman et al., 2019). Because a digital ecosystem often outbids better rivals at the access point and cheaply acquires leaders in other markets, it lowers market "contestability" — the entry capacity of

---

https://www.justice.gov/d9/2023-09/416682.pdf).

[2] See, for example, https://t1p.de/htpki (accessed June 17, 2024).

non-dominant firms — everywhere. As a result, both access-point and leverage policies can increase contestability throughout the economy.

We conclude in Section 6 by speculating why ecosystems are less common in offline settings. While analogues do exist (e.g., tropical hotels with many services), in general offline firms appear to have less steering ability than their online counterparts.

Related literature Our paper belongs to the quickly growing theoretical literature on ecosystems and the digital economy. No previous paper, however, analyzes our main question, the role of cross-market leverage and steering in the emergence of digital ecosystems. Indeed, although there are informal accounts (e.g., Eisenmann et al., 2011, Condorelli and Padilla, 2020), to our knowledge the only existing formal theory applicable to ecosystem growth is by Chen and Rey (2023). Chen and Rey assume exogenously that consumers obtain a convenience or bundling benefit from buying from the same firm, which motivates conglomerate mergers. Our theory also features a consumer benefit from ecosystems, but this arises endogenously due to the nature of the mergers taking place. Moreover, our microfoundation leads to different main predictions. First, while our theory predicts that takeovers will be initiated by firms at (or closer to) consumers' access points, Chen and Rey's theory makes no prediction in this regard. Second, while our theory predicts that ecosystems will go to great lengths to secure access points, in Chen and Rey's model this appears unnecessary. At the same time, Chen and Rey study pricing implications of ecosystems, which we do not. In addition, a small recent literature explores the implications of default effects or inertia in single-market models of the digital economy (Chen and Schwartz, 2023, Ostrovsky, 2023, Hovenkamp, 2023, Decarolis et al., 2022, Denicolò and Polo, 2024), without analyzing ramifications for ecosystems. Most closely related, Chen and Schwartz (2023) and Hovenkamp (2023) develop theories based on what we refer to as a switching-cost model of "default effects." In line with our benchmark Proposition 4, they show that usually better firms have a greater willingness to pay for the default position. Finally, recent theoretical research

also investigates aspects of the digital economy, including pricing (e.g., Jeon et al., 2023), within-market steering (e.g., Teh and Wright, 2020, Hidir and Vellodi, 2020, Heidhues et al., 2023b), entry for buyout (e.g., Bryan and Hovenkamp, 2020) or innovation (e.g., Madsen and Vellodi, forthcoming), that are completely different from our main focus.

Our assumption that there is a default firm that consumers are more likely to choose can — consistent with our terminology — be seen as incorporating a generalized default effect into our model. A vast literature in behavioral economics documents that defaults matter (e.g., Madrian and Shea, 2001, Choi et al., 2004, Johnson and Goldstein, 2003, 2004, Jones, 2012, Chetty et al., 2014, Blumenstock et al., 2018, Jachimowicz et al., 2019, Brown and Previtero, 2020, Brot-Goldberg et al., 2021) and academics, policymakers, as well as firms recognize that defaults are equally if not more important in digital settings.[3] Furthermore, it is universally recognized that firms have some control over consumers' defaults, giving rise to the steering ability that is central to our model.

2 Basics: Firm Payoffs with Steering

We begin by specifying a model of demand when consumers are susceptible to default effects and steering. In later sections, we will use this framework to study mergers and competition for the default position. Because our main interest is therefore not in default effects per se, but in firm behavior, we specify firm payoffs with defaults in reduced form. In Appendix A, we provide microfoundations for our assumptions based on the main mechanisms for default effects described in the literature. These mechanisms presume that consumers compare and choose between products imperfectly or in a context-dependent way. First, consistent with

---

[3] Among academics, see for instance Johnson et al. (2002), Altmann et al. (2019), Fletcher and Vasas (2023), and Fletcher et al. (forthcoming). Among policy reports, see Scott Morton et al. (2019) and Furman et al. (2019). Among firms, extensive Google memos document that default settings "can be a powerful strategic weapon"; a view that is shared by its competitors, as is evident from the testimony by Neeva, DuckDuckGo, and Microsoft managers (see `https://t1p.de/sco1p`). For Google's own estimates of the value of defaults see, for example, `https://t1p.de/ceovi`. Both links accessed July 8, 2024.

a strict notion of defaults, the consumer may receive the default option unless she actively avoids it. Then, the default matters because searching for or switching to something else is costly. Second, in a generalization of default effects, the default option may attract a consumer's attention. Then, the default matters because it is more likely to enter the consumer's consideration set, become her reference point, or be taken by her as a recommendation.

2.1   Setup

Basic Assumptions   There are two markets, a and b, served by firms 1 through n. Market a is an access point, such as a browser, search engine, map, or operating system, or in the future potentially a service on wearables or self-driving cars. Market b is a follow-on service. We denote by $\mathcal{N}^s \subseteq \{1, \ldots, n\}$ the set of firms offering service $s \in \{a, b\}$, define $n^s := |\mathcal{N}^s|$, and assume $n^s \geq 2$. We call a firm serving one market a single-market firm, and a firm serving both markets an ecosystem. Prices are fixed at zero, and a firm's profit equals its total demand across the two markets.[4] We normalize the size of the potential market a to 1.

In each market s, consumers are steered toward the product of one firm, which we call the *default* firm. Firm i obtains exogenously given demand $q_{ij}^s \geq 0$ in market s when firm j is the default in market s. Throughout the paper, we maintain:

Assumption 0 (Steering). For all i and $j \neq i$, we have $q_{ii}^s > q_{ij}^s$.

Assumption 0 can be thought of as the definition of steering or default effects in our context: that having the default position is beneficial for a firm. All of our microfoundations in Appendix A imply choice behavior satisfying this assumption.

For now, we assume that the default in market a is assigned exogenously, and endogenize this selection in Section 4. At the same time, we suppose that the default in market b depends on what happened in market a. Specifically, any ecosystem becomes the default in

---

[4] This is consistent with the common situation in digital markets, where a firm benefits not from direct sales to consumers, but from collecting data and/or showing ads.

market b with probability equal to its market-a demand. For the probability that remains after accounting for the market-a demands of ecosystems, the default position is divided equally between all firms in market b (including ecosystems). These assumptions capture the cross-market leverage that results if consumers use service b after service a. If a consumer used an ecosystem in market a, then the ecosystem sets itself as the default in market b for that consumer. If, however, the consumer used a single-market firm in market a, then no firm in market b is systematically favored.

Quality-Steering Complementarity   For some of our results, we impose more structure on the effect of steering on demand. Such structure is not necessary for our main positive results, that ecosystems grow from leaders at access points through takeovers, and that they have a high value from controlling access points. The additional structure is necessary, however, for our predictions regarding the takeover targets of to-be ecosystems and the short-run welfare effects of ecosystem growth.

Within a market s, we think of a firm i as "better" or "higher-quality" than firm j if

$$\sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s > \sum_{\ell \in \mathcal{N}^s} q_{j\ell}^s,$$

i.e., i's demand when the default is assigned randomly and with equal probability is strictly higher than j's. This definition reflects a common property of all of our microfoundations: if no firm is favored on average, then demand indicates quality. In that case, even a boundedly rational consumer is more likely to choose a better option.

Assumption 1 (Quality-Steering Complementarity)**.** Let $i, i', j, j' \in \mathcal{N}^s$.

    I. If j is better than j', then $q_{ij'}^s \geq q_{ij}^s$ for all $i \neq j, j'$.

    II. If i is better than i', then $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$ for all $j \neq i, i'$.

    III. If i is better than j, then $q_{ii}^s - q_{ij}^s > q_{jj}^s - q_{ji}^s$.

Broadly, Assumption 1 imposes that better firms benefit more from holding the default position than worse firms. Three specific conditions incorporate this complementarity. A better default attracts more consumers away from a competitor, lowering the competitor's demand (Part I). In addition, a better default attracts more consumers from all options, so a better firm gains more from replacing a given rival as the default (Part II). And for the same reason, a better firm benefits more from replacing a worse firm as the default than vice versa (Part III).

In Appendix A, we identify plausible conditions under which our microfoundations lead to consumer behavior satisfying Assumption 1. Depending on the microfoundation, this is either always the case, or if the default effect is sufficiently weak. Intuitively, a better firm often benefits more from drawing a boundedly rational consumer's attention, as it induces the consumer to take into account the firm's quality in her choice.

Discussion of Setup For transparency and tractability, our model makes some unrealistic simplifying assumptions. First, while we have assumed that there is a single service b that follows the single access point a in consumers' online journeys, there may be multiple candidates for both. For example, a consumer may start her search for vacations in a browser or on a map, after which she may look up flights to or weather at a candidate location. Similarly, she may be interested in further services after market b. She may, for instance, use a browser to read news, then perform a search for videos, and then watch a video. These alternatives do not affect our qualitative prediction that ecosystems grow from access points toward connected services. But the presence of multiple candidates for access points and subsequent services does add the obvious and realistic qualification that ecosystems will consist not just of two, but of potentially many connected services.

Second, while our specification of cross-market leverage implicitly assumes that all consumers move from service a to service b, the sequence of service usage is not always obvious or uniform across consumers. For instance, some consumers first search and then send an

email, whereas others proceed in the opposite direction. Nevertheless, our points continue to hold if there are a few central services — search in this case — that a disproportionate number of consumers start with.

Third, while we assign the default in market b by dividing consumers that have not used an ecosystem in market a equally among all firms, it seems plausible that prominent firms are systematically favored. For example, unless steered by a competitor, consumers may use the leading search engine simply because its name has become a synonym for search. Our results continue to hold as long as ecosystems can steer consumers across markets.

Fourth, to focus on the implications of cross-market leverage, our model ignores two considerations that are commonly considered central for the dominance of large digital firms (see Calvano and Polo, 2021, for a review): network effects and economies of scope in data. Network effects — that more users make the service better — apply almost exclusively within market, and all of our results are driven by cross-market effects. To go further, our theory shows that economies of scope in data — that combining consumer data from different markets may increase conversions for advertisers — are unnecessary for digital ecosystems to emerge. Indeed, the data view does not explain our main results regarding the direction of ecosystem growth or the tremendous effort ecosystems expend to secure access points. Still, our findings are not only consistent with, but typically reinforced by within-market network effects or economies of scope in data, which by definition increase the value of large firms.

2.2 Preliminaries: Default Advantage and Default Externality

As a useful step for our analysis, we decompose steering effects. We define firm i's *default advantage* as its gain in demand when it rather than a randomly chosen firm is the default:

$$\alpha_i^s := q_{ii}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Relatedly, we define firm j's *default externality* on firm i as the change in firm i's demand when firm j rather than a randomly chosen firm is the default:

$$\eta_{ij}^s := q_{ij}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Using Assumption 0, it is easy to establish that:

Lemma 1 (Default Advantage and Externalities)**.**

I. For any firm $i \in \mathcal{N}^s$, we have $\alpha_i^s > 0$.

II. For any firm $i \in \mathcal{N}^s$, there exists another firm $j \neq i$, such that $\eta_{ij}^s < 0$.

III. If all firms in market s are symmetric, then $\alpha_i^s = (n^s - 1)|\eta_{ji}^s|$ for all i and $j \neq i$.

Part I simply restates the premise that the default position is valuable in terms of a firm's default advantage. More substantively, Part II says that there must exist a rival of firm i that imposes a negative default externality on i. Intuitively, if firm i would benefit from facing everyone of its competitors as the default, the default position would be undesirable, contradicting Assumption 0. Finally, Part III notes that if all firms are symmetric, firm i's default advantage is equal to the corresponding loss in market shares of all its rivals.

3 The Emergence of Ecosystems

In this section, we show that cross-market leverage leads to the emergence of ecosystems through — primarily — takeovers. Throughout this section, we assume that the default in market a is exogenously given, and thus suppress the default in a firm's market-a demand.

3.1 Takeovers in the Shadow of Cross-Market Leverage

We first identify a strategic advantage in making a takeover in the next market, and investigate which firm has the greatest incentive to make a takeover. As a start, suppose that all

firms are single-market firms. We consider a simple takeover game: one firm in the access-point market a, firm G, can make sequential take-it-or-leave-it offers to the firms in market b until one firm accepts an offer or all firms reject an offer. We look for subgame-perfect equilibrium outcomes.

As a point of comparison, notice that a target t's "standalone value," i.e., its demand in the absence of a takeover, is

$$V_t^b := \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{t\ell}^b.$$

Without cross-market leverage, this is how much G would value t, and also how much t would be willing to sell for. Hence, in that case the profit from a takeover would be zero.

Proposition 1 (Takeover Discounts and Profits).

I. In any subgame-perfect equilibrium of the takeover game, there are $t_1, t_2 \in \mathcal{N}^b$ such that (i) firm G takes over t_1 at a price $V_{t_1}^b - q_G^a|\eta_{t_1 t_2}^b|$, thereby making a profit of $q_G^a(\alpha_{t_1}^b + |\eta_{t_1 t_2}^b|)$; and (ii) t_1 and t_2 solve $\max_{t_1', t_2' \in \mathcal{N}^b} \alpha_{t_1'}^b - \eta_{t_1' t_2'}^b$.

II. Under Assumption 1, t_1 and t_2 are firms with the best and second-best service b.

The proposition states that G makes a profitable takeover. Its profit is due to two considerations that arise from cross-market leverage. First, straightforwardly, if G acquires a target t, then it can use its cross-market leverage to raise t's demand. Hence, G values any target above its standalone value. Second, less obviously, G obtains a "takeover discount" — it pays a price below the target's standalone value. Intuitively, in equilibrium t knows that if it did not sell to G, then another target t' would. In that case, firm G would use its cross-market leverage to increase the demand of t' and thereby lower the demand of t. This threat weakens the bargaining position of t. Furthermore, in equilibrium the firms t and t' maximize G's takeover profit.[5]

---

[5] Our proof establishes the last result by noting that G can first make low-ball offers to all firms but t and t', and once these offers are rejected, approach t. Then, t knows that if it rejected the offer, G would

In general, t and t' could be any two firms in market b, but under Assumption 1, they are the best firms in the market. Then, firm G buys the best firm because this firm benefits the most from the default position, and hence is the most profitable firm to steer consumers to. In addition, the second-best firm is the best threat because (by Part I of Assumption 1) making it the default attracts the most consumers away from the best firm.

Having analyzed takeovers by a single firm, and having identified the takeover discount, we now explore which firm is most likely to initiate a takeover. To do so, we calculate the expected profit each firm can make in a takeover game of the type above, where it makes sequential take-it-or-leave-it offers to the firms in the other market. Because a takeover is costly to prepare and execute, the firm with the most to gain is most likely to undertake the necessary investment. Alternatively, one may think of a "law firm" or consultancy, which organizes the acquisition and reaps part of the profits from it. Such an intermediary also has an interest in bringing about the most profitable takeover.

Proposition 2 (Profits of Forward Integration vs. Backward Integration)**.**

I. Consider any firm i in market b. The highest profit it can earn is through taking over the leader ℓ in market a. It can do so at a price V_ℓ^a, making an additional profit of $q_\ell^a \alpha_i^b$.

II. Firm ℓ can earn strictly more from a takeover than $q_\ell^a \alpha_i^b$, and it can earn weakly more than any other firm $j \neq \ell$ in market a.

Part I identifies the profits from "backward integration" — when a firm in market b expands into market a. Cross-market leverage allows the firm to direct some of the target's consumers to itself, so it makes a profit from such a takeover. In doing so, however, the firm does not obtain a takeover discount. Intuitively, taking over the target's competitor does not decrease the target's demand, so it is not threatening. Accordingly, Part II says that

---

take over t'. This logic of course relies on the assumption that G cannot approach firms that previously rejected it. If G had less commitment power regarding which firms it will approach after getting rejected by t, its profit could be lower, but the qualitative logic of Proposition 1 would still hold; indeed, often there is also a subgame-perfect equilibrium in which G approaches its preferred target first.

the profit from backward integration is lower than that from "forward integration" — when a firm in market a expands into market b. Overall, the firm with the greatest incentive to make takeovers is the leader in market a.

Propositions 1 and 2 imply that market leaders at access points, like Google or Microsoft, are prone to growing into ecosystems through profitable mergers. Both specific examples and general observations are consistent with our story. For instance, Google took over Youtube and added it to its services, Google took over Where 2 Technologies that became Google Maps, Google took over ITA Software to create Google Flights, and Microsoft took over Hotmail and turned it into Outlook.[6] More generally, "strategic mergers" are common in digital markets (Eisfeld, 2024) despite the growing importance of acquisitions by financial firms elsewhere (e.g., Vild and Zeisberger, 2014, and references therein), and most mergers between digital firms can be classified as vertical or conglomerate mergers.[7] Furthermore, as we predict, digital firms "tend to acquire companies at lower prices" than financial investors (Eisfeld, 2024, p. 57).

The logic of our model also allows us to make further observations. First, consider the possibility that for exogenous reasons firms in market b are not willing to sell. Suppose furthermore that the leader G in market a can internally develop service b at a cost c_{int}, whereas an outsider can develop a service with the same market share at a cost c_{ext}. For simplicity, we impose that the outsider makes the development decision first, and if it does not develop the service, then G can do so. The outsider develops the service if its standalone value

---

[6] See, for example, `https://t1p.de/lsvbk` (YouTube), `https://t1p.de/86x05` (Google Maps), `https://t1p.de/aia1s` (ITA Software), and `https://t1p.de/w3dhr` (Hotmail). Other examples include Google taking over Upstartle and 2Web Technologies to create Google Docs and Google Sheets (`https://t1p.de/med6m`); Apple buying the MP3 player SoundJam MP to create iTunes (`https://t1p.de/vfuyv`); Microsoft extending its range of services by taking over Skype and LinkedIn (`https://t1p.de/1dddj` and `https://t1p.de/g9bwj`); Amazon acquiring Audible, LoveFilm (now Amazon Prime Video), and Twitch (`https://t1p.de/52vaf`, `https://t1p.de/7cpmu`, and `https://t1p.de/4p5ue`); Meta taking over WhatsApp and Instagram (`https://t1p.de/g98rr`). All links accessed on June 6, 2024.

[7] The observation that digital ecosystems grow from access points into adjacent markets through takeovers is not restricted to western societies. The two dominant ecosystems in China, Alibaba and Tencent, grew out of the Alibaba's dominant marketplace (Taobao) and Tencent's dominant social media and messaging service (WeChat) into ecosystems that offer a broader range of services (Prüfer et al., 2024).

exceeds c_{ext}. Otherwise it does not, in which case G develops the service if the standalone value plus the default advantage that G can generate exceeds c_{int}. Hence, because of its ability to steer, firm G may develop the service even if $c_{int} > c_{ext}$. In this sense, G has a larger incentive than other firms to develop new services in adjacent markets. Without cross-market leverage, in contrast, G would never be the one developing the service if $c_{int} > c_{ext}$. Consistent with our prediction, for instance, Microsoft spent tremendous resources to develop and improve its browser (first Internet Explorer, now Edge), Google developed its email service Gmail in house, and Uber developed its food delivery service UberEats.[8]

Notwithstanding its incentive to self-develop service b if necessary, firm G often prefers to acquire the service through a takeover instead. In particular, suppose that G could hire a team and self-develop the service, or let the team start a company offering the service and take over the company. Firm G prefers the takeover if c_{int} is above the takeover price. Hence, G may use a takeover even if c_{int} is well below the profits it can make with the service. Again, without cross-market leverage this would never be the case. Note, however, that even if firm G makes a takeover in equilibrium, its option to self-develop the product remains relevant. Much like taking over a competitor, this can serve as a threatening outside option that lowers the preferred target's takeover price.[9]

Second, while we simplify things by assuming two markets, the logic of our results applies equally or even more strongly to the kinds of more complex situations we have mentioned previously. As one relevant extension, there may be multiple markets that consumers want to use immediately after the access point. Then, iterated application of Propositions 1 and 2 implies that the leader at the access point takes over firms in each of the other markets. Alternatively, there may be further services that consumers want to use after

---

[8] See `https://t1p.de/l6y2x` and `https://t1p.de/a5egg` (Microsoft), (`https://t1p.de/86x05`) (Google), and `https://t1p.de/2u84f` (Uber). Notably, also Apple and Google developed their own web browsers (`https://t1p.de/pqiz6` and `https://t1p.de/69unz`). All links accessed June 6, 2024.

[9] Relatedly, Katz (1987) argues that large downstream buyers can more credibly threaten to enter an input market in the presence of scale economies, and hence get offered a lower input price even absent entry.

market b. In particular, consider a situation in which the consumer uses another service c after service b, and for simplicity suppose that market c emerges after markets a and b have been established.[10] Applying Propositions 1 and 2 first to markets a/b says that the leader in market a takes over a firm in market b. Now applying the same logic to markets b/c says that if the new ecosystem is the leader in market b, it will take over a firm in market c. Indeed, the ecosystem will be the leader in market b if cross-market steering is sufficiently strong, or if it was created through the takeover of a sufficiently good firm in market b. Under Assumption 1, in particular, the ecosystem merges the best firms in markets a and b (Proposition 1, Part II), so it will always be the leader in market b. In that case, it expands further by taking over the leader in market c.

Third, our theory says that if forward integration is not possible (for a reason outside our model) but backward integration is, then backward integration will occur. Indeed, Part I of Proposition 2 says that backward integration is profitable due to cross-market leverage, even if less profitable than forward integration. We expect this type of takeover to occur when there are already ecosystems in place, and a new access point with small single-market firms emerges. For practical reasons, such as financing or know-how, it seems implausible that a small startup would take over an ecosystem like Google or Meta. Hence, in this case backward integration naturally occurs. Consistent with this prediction, Google took over Fitbit and Meta acquired Oculus VR.[11]

While we focus on the emergence of ecosystems through takeovers, as usual in simple models of vertical relations, contracting solutions can — in theory — substitute for the merger. The complexity of more realistic settings, however, provides a rationale for why firms exploit cross-market leverage through takeovers rather than contracting solutions. In practice, contracting solutions can be difficult to design and enforce, especially in highly

---

[10] The example we have mentioned previously, where the consumer first uses a browser to read news, then performs a search for videos, and then watches a video, is consistent with such a situation. Video sharing developed well after browsers and search engines.

[11] See `https://t1p.de/o9wui` (Google) and `https://t1p.de/p8kvb` (Meta) both accessed June 19, 2024.

dynamic digital markets. If steering is based on choice architecture, for example, firms would have to agree every time the service in market a adapts their customer interface. Similarly, when improving their service in market b, a firm would face a hold-up problem unless the steering price is credibly fixed. Furthermore, in the plausible scenario with further markets following market b, a consumer steered into market b can be steered once more into market c. In such an environment, finding appropriate contracting solutions will be even more difficult. And a common approach for advertising — auctioning of a desirable position instantaneously — is presumably harder to implement for most steering situations. If, for example, consumers are steered to different services at different times, then even consumers with limited attention may eventually become aware of these services, decreasing their "steerability."[12]

3.2   Multiple Acquirers and Takeover Prices

Our model in the previous subsection assumes that a single firm can make takeover bids, giving the acquirer substantial bargaining power. A natural conjecture is that the presence of multiple potential acquirers introduces competition for takeovers and thus protects targets from being bought below their standalone values. We evaluate this conjecture in a variant of the above takeover game, and find that it is not generally correct.

Consistent with Proposition 2, we focus on forward integration. A given subset $\mathcal{A} \subseteq \mathcal{N}^a$ of the firms in market a can make takeovers, and all firms in market b are potential takeover targets. For expositional purposes, we suppose that all potential targets offer an equally good service b, and firms in $\mathcal{A}$ differ in their market-a demand. We thus drop subscripts referring to the target (e.g., on standalone values), and denote by $\bar{\mathcal{A}}(x)$ the x firms in $\mathcal{A}$ with the highest market-a demand. In every round of the game, one of the remaining targets is offered for sale, and each potential acquirer that has not yet made a takeover can submit a takeover offer. The target can accept any offer. The game ends when all potential targets

---

[12] More generally steering to different targets may be less effective in case consumers learn through word of mouth.

have been up for sale.

We characterize (subgame-perfect) equilibrium outcomes following the usual convention that takeover offers are cautious; that is, no offer is above the additional profits a firm can generate when acquiring the target. Formally, we define *(dynamically) cautious offers* recursively: if a subgame following the current round of offers has a unique cautious subgame perfect equilibrium outcome, we require firms to make cautious offers (i.e., use iteratively weakly undominated strategies) in the reduced game in which these subgames are replaced by the corresponding equilibrium payoffs.[13]

Proposition 3 (Multiple Acquirers). *In any equilibrium with cautious offers, firms in $\bar{\mathcal{A}}(n^b)$, but no other firms, make a takeover. The equilibrium takeover price is $f^* < V^b$. Specifically,*

$$
f^* = \begin{cases} V^b - |\eta^b| \sum_{i \in \mathcal{A}} q_i^a & \text{if } n^b > |\mathcal{A}|, \\ V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a & \text{if } n^b = |\mathcal{A}|, \\ V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a & \text{if } n^b < |\mathcal{A}|. \end{cases} \tag{1}
$$

Proposition 3 says that even with multiple acquirers, the acquirers can always take over their targets at a price below the targets' standalone value (in the absence of *any* takeover). This means that the possibility of takeovers always harms targets. Intuitively, other acquirers not only introduce competition for an acquiring firm, but also affect the value of targets. Specifically, on the equilibrium path, the last target taken over is less valuable to an acquirer, because the ecosystems created in preceding takeovers will steer consumers away. Furthermore, in equilibrium the price of all takeovers is determined by this marginal acquisition, and hence is low.

The above logic implies that with an abundance of targets ($n^b > |\mathcal{A}|$), targets are actually made worse off with an increase in the number of acquirers. In that case, the abundance

---

[13] Requiring the use of iteratively undominated strategies amounts to ruling out that takeover offers are driven by the fear that some firm with a low demand in market a bids aggressively above its possible additional profits.

of targets means that other acquirers lower a target's value without inducing competition. Even if there are more acquirers than targets ($n^b < |\mathcal{A}|$), the decrease in value due to other takeovers outweighs the price-increasing effect of competition. If a target refuses to sell, it expects to obtain little demand due to successful acquirers steering consumers to all its rivals. While there is also competition for targets, the former effect dominates. This is because targets are identical and potential acquirers compete for the extra demand they can create through steering. Because unsuccessful acquirers have fewer consumers to steer than successful ones, they could create only less extra demand. Hence, unsuccessful acquirers are not willing to pay enough for a takeover to compensate a target for the loss in demand due to its rivals being taken over by acquirers with more consumers to steer.

4   Default Setting at the Access Point

Having established that cross-market leverage leads to ecosystems, we investigate the implications of ecosystems for steering incentives in the access point market a. Our analysis is motivated by the high-stakes practice of default setting on digital devices. As in the case of Google's astronomical payment to Apple mentioned in the introduction, such settings are often negotiated between the original equipment manufacturer (OEM) and digital firms.[14] Going further, for commonly used Android devices, Google enforces its default position through a nexus of usage contracts for the operating system (i.e., Google Android) and "must-have" apps (e.g., Google Play Store or Google Maps).[15] The obvious aim of such arrangements is to induce the owner to use Google services when she first picks up her device.

While acquiring the literal default position is our main motivation, there are other plau-

---

[14] Chris Barton (Google): "We need to incentivize carriers to ship Google by using the same approach we at Google have used for many years: 'We will pay you revenue share in return for exclusive default placement.' This contract is an exchange." (internal email, April 2011, `https://www.justice.gov/d9/2023-09/416302.pdf`).

[15] See Heidhues et al. (2023a) for a discussion of how Google locks in its default position through interlocking contracts.

sible ways of steering or getting a consumer at the access point. These will be especially pertinent in the near future, when new products with access-point services, such as self-driving cars or smart wearables (e.g., fitness tracking smart watches), become widely used. For example, a firm may spend extra resources to plant a position at a new access point by developing the product, such as the operating system of a self-driving car, even if other providers are better positioned for the innovation. A first-mover advantage may then benefit the firm for an extended period.

Formally, we assume that the default position in market a is sold through a sealed-bid second-price auction. Other negotiation mechanisms, for instance bilateral bargaining, yield similar qualitative insights. We look for cautious Nash equilibria in which all firms use a weakly undominated strategy.[16] For stating our results, we define $\Delta_{ij}^a := q_{ii}^a - q_{ij}^a \in (0,1)$, which is a single-market firm i's willingness to pay to replace j as the default in market a.

4.1   Benchmark: Single-Market Firms

As a benchmark, we suppose that all firms are single-market firms.

Proposition 4 (Superior Defaults). *If Assumption 1 holds, then a firm $i \in \mathcal{N}^a$ that offers the best service a wins the auction for the default position, and it pays at most $\max_{j \in \mathcal{N}^a \setminus \{i\}} \Delta_{ji}^a$.*

Proposition 4 says that with only single-market firms, at least under Assumption 1, a top-quality firm becomes the default. Upon winning the default position, such a top-quality firm attracts away most consumers from all other options, and it thus values the default position the most. Chen and Schwartz (2023) and Hovenkamp (2023) establish similar results in slightly different settings.

---

[16] It is well-known that the second-price sealed-bid auction has a continuum of equilibria in weakly dominated strategies both with independent private values as well as known values (Blume and Heidhues, 2004).

4.2 Ecosystems and Access Points

We turn to our main interest, how ecosystems value and bid for access points. For simplicity, we suppose that there are two firms in market a, G and M, of which G is an ecosystem, and M may be a single-market firm or an ecosystem.

Proposition 5 (Potential for Inferior Defaults)**.**

I. Suppose that M is a single-market firm. Then, if $\Delta_{GM}^a \left(1 + \alpha_G^b\right) > \Delta_{MG}^a$, firm G wins the auction for the default position with probability 1, and pays Δ_{MG}^a.

II. Suppose that M is an ecosystem, and let $\eta_{GM}^b, \eta_{MG}^b < 0$. Then, if

$$\Delta_{GM}^a \left(1 + \alpha_G^b + \eta_{MG}^b\right) > \Delta_{MG}^a \left(1 + \alpha_M^b + \eta_{GM}^b\right),$$

firm G wins the auction for the default position with probability 1, and pays

$$\Delta_{MG}^a \left(1 + \alpha_M^b\right) + \Delta_{GM}^a |\eta_{MG}^b|.$$

As in the case of single-market firms (Proposition 4), the winner and payment in the auction are determined by firms' valuations of the default position in market a. In the case of ecosystems, however, cross-market leverage means that this valuation includes considerations from market b. By Part I, when firm G competes against a single-market firm, then its willingness to pay to replace firm M as the default in market a is $\left(1 + \alpha_G^b\right)\Delta_{GM}^a$. When replacing firm M as the default in market a, firm G's demand for this service increases by Δ_{GM}^a. Because of cross-market leverage, this increase in demand for service a translates into a higher probability of becoming the default in market b, which comes with a default advantage of α_G^b. Hence, when replacing firm M as the default in market a, firm G earns an additional profit of $\Delta_{GM}^a \alpha_G^b$ in market b. In the extension of our model in which firm G offers services in many markets, it can monetize a sale at the access point in even more ways. We can capture such a situation in reduced form by assuming that firm G's default

advantage α_G^b is large, leading to a large "default multiplier" $1 + \alpha_G^b$.[17]

By Part II, when G competes against another ecosystem, its willingness to pay to obtain the default position is even higher. Intuitively, facing a multi-market competitor adds the additional concern that if the competitor wins the default position, its cross-market leverage makes it more difficult to attract the consumer in market b. More precisely, since firm M exerts a default externality of η_{GM}^b on firm G in market b, firm G's willingness to pay is $\Delta_{GM}^a \left(1 + \alpha_G^b\right) + \Delta_{MG}^a |\eta_{GM}^b|$. Firm M's valuation, which determines firm G's payment in case it wins, is determined analogously.

Proposition 5 has two economically important implications. First, even under Assumption 1, an ecosystem may obtain the default position in market a when it does not offer the better service there. Recall that under Assumption 1, $\Delta_{GM}^a > \Delta_{MG}^a$ if and only if firm G has the better product. Against a single-market firm (Part I), firm G may win with an inferior product if it has a sufficiently large default multiplier. Against an ecosystem (Part II), it may win with an inferior service a if it offers a better product in market b[18] or — considering

---

[17] More concretely, consider the extensions discussed in Section 3.1. Suppose that, in addition to the access point, there are markets $b_1, \ldots, b_S$, and the multi-market firm G is active in all of these markets. First, suppose that each consumer wants to use exactly one service b_s upon leaving the access point, but different consumers demand different services. For example, some consumers search for vacation spots (market a) and book a flight afterwards (market b_1) while others search for yesterday's game (market a) to watch its highlights (market b_2). Let $w_i \in (0,1)$ be the share of consumers interested in service b_i. Firm G's willingness to pay for the default position in market a then is

$$\left(1 + \sum_{s=1}^{S} w_s \alpha_G^{b_s}\right) \Delta_{GM}^a.$$

Second, suppose that upon leaving the access-point market all consumers want to use all other services in a (for simplicity) fixed and identical sequence. For example, a consumer might search for an interview about yesterday's game (market a) that she translates to her mother-tongue (market b_1) and upon reading it (market b_2) emails the translation to a friend (market b_3). In this case, firm G's willingness to pay for the default position in market a is

$$\left(1 + \sum_{s=1}^{S} \prod_{m=1}^{s} \alpha_G^{b_m}\right) \Delta_{GM}^a.$$

Either way, firm G's willingness to pay for the default strictly increases in the number of services it offers.

[18] Under Assumption 1 (Part III), $q_{GG}^b - q_{GM}^b = \alpha_G^b - \eta_{GM}^b > \alpha_M^b - \eta_{MG}^b = q_{MM}^b - q_{MG}^b$ if and only if G offers a better product than M. Hence, G needs to offer a better service than M for $\alpha_G^b + \eta_{MG}^b > \alpha_M^b + \eta_{GM}^b$.

the extension to many markets discussed above — is active in more markets.

Second, Proposition 5 implies that firm G may make an extremely large payment for the default position. When competing against a single-market firm, this is the case if the competitor is very good. When competing against an ecosystem, the same is the case also if cross-market steering is very effective. These results, especially the latter one, explain the great lengths to which Google goes for securing the default position in the market for search (e.g., by outbidding Microsoft for being the default search engine on iOS devices).[19] Similarly, Google appears to be spending tremendous resources to develop a self-driving car, which is considered a major new access point in the making.[20] Note that theories of conglomerates based on consumer-side complementarities, such as one-stop-shopping benefits (Chen and Rey, 2023), predict that such measures are unnecessary. If consumers had an inherent preference to use the services of a multi-market firm, then they should flock to Google without Google having to spend a lot to steer them in its direction.

Proposition 5 analyzes what happens when firm G competes either against a single-market firm or against an ecosystem. An interesting additional issue arises when firm G competes against both types of firms at the same time. Then, the fact that firm G's willingness to pay for the default position is higher when competing against another ecosystem can lead to multiple equilibria. This can happen when the single-market firm offers the best service a. In one equilibrium, the ecosystems bid high in an effort to prevent the other ecosystem from winning. Then, an ecosystem wins and pays a lot. In the other equilibrium, the single-market firm wins, and the ecosystems bid less aggressively exactly because they are not worried about the other ecosystem winning. In this case, the winner pays a lower amount.

The above logic implies that despite having no direct interest in either market, an OEM

---

[19] See, for example, `https://t1p.de/r9drd` (accessed June 7, 2024).

[20] See, for example, `https://t1p.de/uo5sm`. Other examples include Google developing — partly, through takeovers — a smart TV platform that "amalgamates various streaming services, live TV, and a plethora of apps, all accessible through a single, user-friendly interface" (`https://t1p.de/1aetd` and `https://t1p.de/khbht`). All links accessed July 3, 2024.

may want to exclude good single-market competitors from bidding for the default position at the access point.[21] By doing so, the OEM guarantees that the ecosystems compete against each other, ensuring itself the profits from the better equilibrium above.

5   Welfare and Policy Implications

Following a number of high-profile policy reports (Scott Morton et al., 2019, Furman et al., 2019, Crémer et al., 2019) in the US, UK, and EU respectively, policymakers recently have proposed a variety of laws with the aim to curb the power of "big tech." The market power of big tech raises at least two policy concerns. One is that due to their entrenched market position, entry of new competitive firms in their established markets has become infeasible to the long-run detriment of consumers. Another is that digital ecosystems can leverage their market power from some core platform into other complementary markets — including through the use of what we refer to as steering — thereby forestalling competition and making the economy less dynamic. As a response to these concerns, the European Union passed the Digital Market Act (DMA) with the aim of increasing "fairness" and "contestability" in the digital economy.[22]

For example, Articles 6(5) and 6(6) of the DMA prohibit ecosystems (referred to as gatekeepers) from giving their own complementary services preferential treatment through ranking or other means. Such "leverage policies" — weakening cross-market leverage — are meant to open up these services to "fair" competition and complement a variety of other

---

[21] See Ostrovsky (2023) for how an auction designer who does have a vested interest in the access-point market can subtly manipulate the auction to generate more desirable outcomes.

[22] Contestability is defined in Recital 32 of the DMA as "[...] ability of undertakings to effectively overcome barriers to entry and expansion and challenge the gatekeeper on the merits of their products and services." This recital and others argue that there is weak contestability in the digital sector. Recital 33 defines the lack of fairness as a situation in which a "gatekeeper obtains a disproportionate advantage" and its business users do not "capture the benefits resulting from their innovative or other efforts."

Reflecting the widespread policy concern regarding big tech's market power, similar laws were introduced but not passed in the US congress (Crémer et al., 2021). Furthermore, the UK and Germany have recently introduced new laws and regulations that also aim to limit ecosystems' market power.

laws that limit the steering of consumers in the EU.[23] In addition, Articles 6(3) and 6(4) of the DMA necessitate that users can easily change default settings on operating systems, virtual assistants, web browsers, and app stores. Such "access-point policies" — regulating the sale of default positions — limit the ability to use defaults to steer demand in access-point markets. Indeed, in response EU consumers now often see a choice screen when, for example, first using a browser on a new mobile device. Going beyond the above mentioned, and already implemented, leverage policies, there has been also some discussion of either breaking up big tech or limiting the ability of big tech firms to engage in further takeovers.[24] Motivated by these policy ideas, we discuss welfare and policy implications of our theory, focusing on interventions that interfere with steering by ecosystems.

In doing so, we restrict attention to valuable services, such as search, maps, or the operating system of a self-driving car, where it is optimal for the consumer to find and use the provider that gives her the highest utility. Our analysis does not apply to services such as social networks that a person may overconsume,[25] potentially even more so with a better default option. We suspect that in such situations regulating the harm directly is a natural policy prescription.

Going further, we assume that in our online setting with many low-stake decisions, con-

---

[23] The steering of consumers is also restricted through various other EU regulations including the EU's Digital Service Act (e.g., Art. 25), Data Act, AI Act, as well as the Unfair Commercial Practices Directive (for an overview see Busch and Fletcher, 2024), with the aim of protecting consumers from "dark patterns" or online choice architecture that misleads or otherwise induces the consumer to act in the firm's rather than their own interest. Going beyond forbidding to mislead or pressure consumers, the DMA as well as various competition cases are explicitly concerned about ecosystems' ability to distort competition through defaults, rankings, and other design choices. See, for example, the European Commission's Google Shopping decision (`https://t1p.de/e1g1a`) or the US and Plaintiff States versus Google LLC case (`https://t1p.de/65om`). Both links accessed July 4, 2024. Similarly, China's drafted guidelines that, if adopted, restrict its dominant ecoystems' ability to self-preference, thereby limiting cross-market leverage (Prüfer et al., 2024).

[24] The proposal to break up big tech companies has been, for example, a central part of Senator Warren's campaign for US President (see, e.g., `https://t1p.de/rtm8k`, accessed July 5, 2024). Scott Morton et al. (2019) argue for scrutinizing big tech acquisitions more heavily. On this topic, see also Cabaral et al. (2021).

[25] See Allcott et al. (2020) for empirical evidence that the average user spends too much time on Facebook, and Bursztyn et al. (2024) for experimental evidence suggesting that a large share of TikTok and Instagram users would be better off if these platforms did not exist.

sumers (on average) benefit from a better default. This rules out situations in which a product's default status creates "as-if switching costs" (Goldin and Reck, 2022), making it optimal for a policymaker to force active choice through selecting a bad default.[26] When forced choice is indeed desirable, policies such as choice screens that induce choice already benefit consumers in the short run, and will have the same long-term welfare benefits as those we discuss below.[27] Finally, once we impose that consumers benefit from a better default, our microfoundations also suggest quality-steering complementarity (Assumption 1), and hence we impose it to simplify our discussion.

Given our assumptions on the nature of products being sold and the properties of default effects, the short-run welfare effects of the development of a digital ecosystem are positive. Propositions 1 and 2 imply that the best firm in the access-point market takes over the best firms in successive markets; and Proposition 5 implies that if the default position in the access-point market is sold through an auction, an ecosystem that has grown in this way acquires that position too. As a result, the probability that the best firm becomes the default increases in each market, benefiting consumers. Accordingly, both leverage policies and access-point policies lower welfare.

The long-run welfare effects of digital ecosystems, however, are more complex and often negative. As an immediate caveat to the short-run effects, Proposition 5 implies potential inefficiencies at a new access point, or when another firm becomes better at an existing access point. In that situation, an ecosystem may be willing to outbid others despite having a worse product. Hence, both leverage and access-point policies can improve welfare in the access-point market. If the ecosystem still has the best products in connected markets, however, such a policy does come at the cost of lowering welfare there. Since the ecosystem cannot

---

[26] All but one — namely, defaults as reference points — of our microfoundations imply that a better default increases consumer welfare (see Appendix A for details).

[27] To work well, forced choice policies require the consumer to become informed and deliberately select from the available options. In many online settings, the effort this requires may not be desirable from a consumer's perspective.

steer the consumer to itself at the access point, its ability to do so in other markets is also reduced.

The logic of the development of digital ecosystems also implies several barriers to the contestability of markets — the ability of non-dominant firms to overcome barriers to entry. To identify entry incentives, we assume that there is a single ecosystem G, which in line with Propositions 1 and 2 offers the best existing services in both markets, and consider a single-market potential entrant e into a given market. Furthermore, to focus on contestability by socially valuable firms, we assume that the entrant e has a better product than G.

We start by studying incentives to enter the access-point market. As an immediate corollary to Proposition 4, firm G is the highest bidder absent entry. Straightforwardly, leverage policies lower firm G's bid in the auction, thereby increasing contestability of the access-point market. To illustrate this especially clearly, consider the extreme case in which G is initially a monopolist and the policy alleviates any cross-market leverage. Such a policy ensures that the entrant wins the auction and even if the entrant won anyways, it now pays less due to the policy.[28] More subtly, we argue that access-point policies raise contestability unless an entrant has a significantly better product than G. First, consider an entrant who would not win the auction upon entering the market. In this case, firm G obtains the default position, and because it offers the best service a, this reduces the entrant's demand compared to a randomly assigned default.[29] Hence, auctioning off the default position makes entry less profitable. Even if e wins the auction upon entering, it benefits from the default being auctioned off only if the default advantage α_e^a compensates for the required payment. This necessitates that the entrant offers a product that is drastically better than the previously best product in the market. The product must not only be sufficiently good for e to outbid G (and other firms), but also that the auction leaves e with sufficient surplus.

---

[28] By Proposition 5, absent the policy, G wins the auction against a superior entrant if $\Delta_{Ge}^a(1+\alpha_G^b) > \Delta_{eG}^a$. And if the entrant wins the auction absent the policy, it will have to pay $\Delta_{Ge}^a(1+\alpha_G^b)$ for the default position.

[29] The last part follows from Part II of Lemma 1 in combination with Part I of Assumption 1.

Somewhat counterbalancing the above negative effect is that the presence of G can increase entry incentives in a new access-point market in which it is not yet present. As we have discussed in Section 3.1, G has a strong incentive to acquire such firms through backward integration, so an entrant may be able to sell expensively. Once G is present at the access point, however, the above logic implies that it reduces further entry incentives.

We now move on to entry into the non-access-point market b. Consider a leverage policy that lowers the probability that G sets the default in market b. Since G offers the best existing service b, any such policy raises an entrant's profit conditional on entering market b and thus makes the market more contestable. In the extension of our model where additional markets are connected to market b, this logic has further implications. Since the ecosystem derives a default advantage $\alpha_G^b > 0$ in market b, a leverage policy also lowers the probability that it sells in a connected market. By the same argument, therefore, the policy encourages entry also in connected markets.

The above argument assumes a single potential entrant, so it ignores that a policy that makes entry in market b more likely may actually result in a firm entering this market, which in turn affects incentives to enter other markets. If the entrant offers a sufficiently good service in market b, however, the ecosystem loses demand upon entry. This in turn reduces the probability with which the ecosystem can set the default in connected markets. Thus, entry into these other markets becomes more likely. Combining these arguments, we conclude that there is a "double dividend" to encouraging good entry into markets that form a key part of digital ecosystems.

Finally, we discuss entry incentives for market b, or markets connected to it, in which the ecosystem firm G is not yet active. Consider an entrant e that decides whether or not to enter the market before G can take over one of the then existing firms. By Proposition 1, the ecosystem takes over the entrant conditional on entry, and secures a discount that is increasing in its leverage from market a. Hence, any leverage policy raises contestability.

6   Conclusion

Given our results, the obvious question arises: would the same forces not generate ecosystems in the offline world, just like they do in the online world? While we do not have a fully precise answer, several observations may be helpful for answering the question. For starters, in some settings offline ecosystems do emerge. For example, tropical hotels often feature an array of services, including airport transfers and a selection of water sports and tours, that keep travelers from looking for other options. Nevertheless, such arrangements do seem less dominant in offline settings. Unlike tropical hotels, for example, hotels in Paris or London are usually not part of an ecosystem. A simple explanation may be that offline sellers are less able to steer consumers than online firms. Even if a hotel in Paris recommends a particular restaurant to a traveler, she may be tempted by the many other restaurants she encounters. Since takeovers are costly, they may make sense only if steering by the acquirer can substantially increase a target's market share, and this is simply not the case in most offline settings. Tropical hotels are arguably in a unique position to steer because many travelers are reluctant to venture outside due to their lack of knowledge about the culture or environment. We have, however, not evaluated this hypothesis systematically, and doing so is an important topic for future research.

References

Allcott, Hunt, Luca Braghieri, Sarah Eichmeyer, and Matthew Gentzkow, "The Welfare Effects of Social Media," *American Economic Review*, 2020, *110* (3), 629–676.

Altmann, Steffen, Armin Falk, Paul Heidhues, Rajshri Jayaraman, and Marrit Teirlinck, "Defaults and Donations: Evidence from a Field Experiment," *Review of Economics and Statistics*, 2019, *101* (5), 808–826.

Blume, Andreas, "Bertrand without Fudge," *Economics Letters*, 2003, *78* (2), 167–168.

_ **and Paul Heidhues**, "All Equilibria of the Vickrey Auction," *Journal of Economic Theory*, 2004, pp. 170–177.

Blumenstock, Joshua, Michael Callen, and Tarek Ghani, "Why Do Defaults Affect Behavior? Experimental Evidence from Afghanistan," *American Economic Review*, 2018, *108* (10), 2868–2901.

Brot-Goldberg, Zarek, Timothy Layton, Boris Vabson, and Adelina Yanyue Wang, "The Behavioral Foundations of Default Effects: Theory and Evidence from Medicare Part D," 2021. Working Paper.

Brown, Jeffrey R. and Alessandro Previtero, "Saving for Retirement, Annuities, and Procrastination," 2020. Working Paper.

Bryan, Kevin A. and Erik Hovenkamp, "Antitrust Limits on Startup Acquisitions," *Review of Industrial Organization*, 2020, *56*, 615–636.

Bursztyn, Leonardo, Benjamin R Handel, Rafael Jimenez, and Christopher Roth, "When product markets become collective traps: The case of social media," 2024.

Busch, Christoph and Amelia Fletcher, "Harmful Online Choice Archtecture," Technical Report, Centre on Regulation in Europe May 2024.

Cabaral, Lúis, Justus Haucap, Geoffrey Parker, Georgios Petropoulos, Tommaso Valletti, and Marshall Van Alstyne, "The EU Digital Markets Act: A Report from a Panel of Economic Experts," Technical Report, Publications Office of the European Union, Luxembourg 2021.

Calvano, Emilio and Michele Polo, "Market Power, Competition and Innovation in Digital Markets: A Survey," *Information Economics and Policy*, 2021, *54*, 100853.

Chen, Yongmin and Marius Schwartz, "Assigning Default Position for Digital Goods: Competition, Regulation, and Welfare," 2023. Working Paper.

Chen, Zhijun and Patrick Rey, "A Theory of Conglomerate Mergers," 2023. Working Paper.

Chetty, Raj, John N. Friedman, Søren Leth-Petersen, Torben Heien Nielsen, and Tore Olsen, "Active vs. Passive Decisions and Crowd-Out in Retirement Savings Accounts: Evidence from Denmark," *Quarterly Journal of Economics*, 2014, *129* (3), 1141–1219.

Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick, "For Better or For Worse: Default Effects and 401(k) Savings Behavior," in David Wise, ed., *Perspectives in the Economics of Aging*, University of Chicago Press, 2004, pp. 81–121.

Condorelli, Daniele and Jorge Padilla, "Harnessing Platform Envelopment in the Digital World," *Journal of Competition Law & Economics*, 2020, *16* (2), 143–187.

Crémer, Jacques, David Dinielli, Amelia Fletcher, Paul Heidhues, Monika Schnitzer, Fiona M. Scott Morton, and Katja Seim, "International Coherence in Digital Platform Regulation: An Economic Perspective on the U.S. and EU Proposals," 2021. Policy Discussion Paper No. 5, Digital Regulation Project, Yale Tobin Center for Economic Policy.

__ , **Yves-Alexandre de Montjoye, and Heike Schweitzer**, "Competition Policy for the Digital Era Final Report," Technical Report, European Commission 2019. Available at `https://ec.europa.eu/competition/publications/reports/kd0419345enn.pdf`.

Decarolis, Francesco, Muxin Li, and Filippo Paternollo, "Competition and Defaults in Online Search," 2022. Working Paper.

Denicolò, Vincenzo and Michele Polo, "Acquisitions, Innovation and the Entrenchment of Monopoly," *Rand Journal of Economics*, 2024. Forthcoming.

Eisenmann, Thomas, Geoffrey Parker, and Marshall Van Alstyne, "Platform Envelopment," *Strategic Management Journal*, 2011, *32* (12), 1270–1285.

Eisfeld, Luise, "Entry and Acquisitions in Software Markets," *Working Paper*, 2024.

Fletcher, Amelia and Zita Vasas, "Implications of Behavioural Economics for the Procompetitive Regulation of Digital Platforms," 2023. Working Paper, avaliable at SSRN.

__ , **Gregory S. Crawford, Jacques Crémer, David Dinielli, Paul Heidhues, Michael Luca, Tobias Salz, Monika Schnitzer, Fiona M. Scott Morton, Katja Seim, and Michael Sinkinson**, "Consumer Protection for Online Markets and Large Digital Platforms," *Yale Journal on Regulation*, forthcoming.

Furman, Jason, Diane Coyle, Amelia Fletcher, Derek McAuley, and Philip Marsden, "Unlocking Digital Competition: Report of the Digital Competition Expert Panel," Technical Report, UK Government Publication, HM Treasury 2019. `https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/785547/unlocking_digital_competition_furman_review_web.pdf`.

Goldin, Jacob and Daniel Reck, "Optimal Defaults with Normative Ambiguity," *Review of Economics and Statistics*, 2022, *104* (1), 17–33.

Heidhues, Paul, Allesandro Bonatti, L. Elisa Celis, Gregory S. Crawford, David Dinielli, Michael Luca, Tobias Salz, Monika Schnitzer, Fiona M. Scott Morton,

Michael Sinkinson, and Jidong Zhou, "More Competitive Search Through Regulation," *Yale Journal on Regulation*, 2023, *40* (3), 915–972.

_ , **Mats Köster, and Botond Kőszegi**, "Steering Fallible Consumers," *Economic Journal*, May 2023, *133 (May)*, 1430–1465.

Hidir, Sinem and Nikhil Vellodi, "Privacy, Personalization and Price Discrimination," *Journal of the European Economic Association*, April 2020, *19* (2), 1342–1363.

Hovenkamp, Erik, "The Competitive Effects of Search Engine Defaults," 2023. Working Paper.

Jachimowicz, Jon M., Shannon Duncan, Elke U. Weber, and Eric J. Johnson, "When and Why Defaults Influence Decisions: A Meta-analysis of Default Effects," *Behavioural Public Policy*, 2019, *3* (2), 159–186.

Jeon, Doh-Shin, Yassine Lefouili, Yaxin Li, and Timothy Simcoe, "Ecosystems and Complementary Platforms," 2023. Working Paper.

Johnson, Eric J. and Daniel G. Goldstein, "Defaults and Donation Decisions," *Transplantation*, 2004, *78* (12), 1713–1716.

_ **and Daniel Goldstein**, "Do Defaults Save Lives?," *Science*, 2003, *302* (5649), 1338–1339.

_ , **Steven Bellman, and Gerald L. Lohse**, "Defaults, Framing and Privacy: Why Opting In $\neq$ Opting Out," *Marketing Letters*, 2002, *13* (1), 5–15. Kluwer Academic Publishers. Manufactured in The Netherlands.

Jones, Damon, "Inertia and Overwithholding: Explaining the Prevalence of Income Tax Refunds," *American Economic Journal: Economic Policy*, 2012, *4* (1), 158–185.

Katz, Michael L., "The Welfare Effects of Thrid-Degree Price Discrimination in Intermediate Good Markets," *American Economic Review*, 1987, *77* (1), 154–167.

Kőszegi, Botond and Matthew Rabin, "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 2006, *121* (4), 1133–1166.

Lleras, Juan Sebastián, Yusufcan Masatlioglu, Daisuke Nakajima, and Erkut Y. Ozbay, "When More Is Less: Limited Consideration," *Journal of Economic Theory*, 2017, *170*, 70–85.

Madrian, Brigitte C. and Dennis F. Shea, "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior," *Quarterly Journal of Economics*, 2001, *116* (4), 1149–1187.

Madsen, Erik and Nikhil Vellodi, "Insider Imitation," *Journal of Political Economy*, forthcoming.

Ostrovsky, Michael, "Choice Screen Auctions," *American Economic Review*, 2023, *113* (9), 2486–2505.

Prüfer, Jens, Inge Graef, and Doh-Shin Jeon, "Regulation of Digital Platforms and the State's Use of Platform Technologies in China," Technical Report DP 2024-05, TILEC Discussion Paper 2024.

Schechner, Sam, Kirsten Grind, and John West, "Searching for Video? Google Pushes Youtube over Rivals," *The Wall Street Journal*, 2020, *July 14.*

Scott Morton, Fiona M., Pascal Bouvier, Ariel Ezrachi, Bruno Jullien, Roberta Katz, Gene Kimmelman, Douglas Melamed, and Jamie Morgenstern, "Stigler Committee on Digital Platforms Final Report," Technical Report, Committee for the Study of Digital Platforms, Market Structure and Antitrust Subcommittee, Stigler Center for the Study of the Economy and the State 2019.

Teh, Tat-How and Julian Wright, "Intermediation and Steering: Competition in Prices and Commissions," 2020. Working Paper.

Vild, Jan and Claudia Zeisberger, "Strategic Buyers vs. Private Equity Buyers in an Investment Process," *Working Paper*, 2014.

A Behavioral Foundations of Default Effects

We provide a series of microfoundations for default effects. We consider a single representative consumer who values service $s \in \{a, b\}$ offered by firm $i \in \mathcal{N}^s$ at v_i^s, which is drawn from a distribution with continuous CDF F_i^s. (Alternatively, we can think of a mass of ex-ante homogeneous consumers.) The consumer's valuations are drawn independently across firms and markets. Within market s, the value distributions can be ranked in terms of (weak) first-order-stochastic dominance (FOSD) and have common support. These rankings correspond to the "better than" quality rankings that we have introduced in the text. Throughout, we assume that the consumer chooses a service s based on within-market considerations only. All results generalize, however, to a forward-looking consumer who understands, and takes into account, the steering by ecosystems.

A.1 Overview

Technically Unsavvy Consumer Consider, for example, a consumer who is assigned a default product and does not know that this default can be changed or, alternatively, believes that doing so is prohibitively costly. If the value of the default product is above some critical threshold $\underline{v} \in \mathbb{R}$, the consumer uses the default service. Otherwise, the consumer refrains from using any service in market s (i.e., she selects some outside-option activity). This model of a technically unsavvy consumer satisfies Assumption 0 if and only if $F_i^s(\underline{v}) < 1$ for all $i \in \mathcal{N}^s$, and it further satisfies Assumption 1 if and only if $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ whenever F_i^s first-order stochastically dominates F_j^s. Moreover, for valuable services, consumer welfare is higher with a better default.

Switching costs In a less extreme version of the above model, the consumer incurs physical switching costs of $\gamma > 0$ when switching away from the default product. To simplify notation, we assume that the consumer uses one of the services. This model generally sat-

isfies Assumption 0. To also establish Assumption 1, we strengthen our assumptions on the distributions of values: all distributions have full support on $\mathbb{R}$, and can be ranked in terms of their reversed hazard rates.[30] Then, if $n^s \geq 3$ and switching costs are small enough, this model also satisfies Assumption 1.[31] Under the same distributional assumptions, consumer welfare is higher with a better default.

Sequential Search   A consumer observes the value (but not the identity) of the default for free and can then search for other products at an increasing cost. For simplicity, search costs increase sufficiently fast for the consumer to search at most once. A searching consumer draws one of the non-default products at random (i.e., search is non-directed). The model implies Assumption 0, and it satisfies Assumption 1 if and only if $n^s \geq 3$ and $F_i^s(v) \neq F_j^s(v)$ for small enough v whenever F_i^s first-order stochastically dominates F_j^s. Consumer welfare is higher with a better default.

Consideration Sets   The consumer considers only $k^s \in \{1, \ldots, n^s - 1\}$ offers in market s, which comprise her consideration set. The consideration set includes the default product with certainty, and any non-default product with probability $(k^s-1)/(n^s-1)$. From her consideration set, the consumer selects the product with the highest realized value v_i^s (as in Lleras et al., 2017). This model can be thought of as a simultaneous search model in which the consumer (perhaps optimally) chooses the size of her consideration set before her attention is drawn to the default product. For example, we can think of a consumer who considers products based on her "mood," and of the default firm as the only one being able to predict the consumer's mood (e.g., using data from past interactions) and tailor its product accordingly.[32] The

---

[30] We provide a formal definition of "dominance in reversed hazard rates" in Appendix A.3. A sufficient condition for dominance in reversed hazard rates is that the two distributions satisfy the monotone likelihood ratio property.

[31] The literal cost of switching to another option are arguably small; in particular, in the digital context we consider.

[32] Specifically, the consumer starts considering products that fit her mood, and then fills her consideration set by randomly selecting among other, non-mood-congruent products.

model implies Assumption 0, and it satisfies Assumption 1 if and only if $n^s \geq 3$ and $k^s \geq 2$. Consumer welfare is higher with a better default.

Loss Aversion Suppose that the default acts as a reference point, and the consumer dislikes falling short of this reference point. Each product i is defined by a pair (v_i^s, d_i^s) with $v_i^s \sim G^s$ and $d_i^s \in \mathbb{R}_{>0}$. We think of d_i^s as an app-design dimension affecting the consumer's utility in an additively separable way (as in most applications of Kőszegi and Rabin, 2006). Let $F_i^s(u) = G^s(u - d_i^s)$ be the distribution of $v_i^s + d_i^s$. We assume that, if the consumer uses the product of firm j instead of the default product offered by firm i, she experiences a loss of λd_i (with $\lambda > 1$) because she cannot enjoy firm i's proprietary design features, and a gain of d_j from using firm j's product instead. In addition, the consumer has some reference point in the service dimension, and we think of the distribution G^s as being "gain-loss adjusted." The model satisfies Assumption 0. To establish also Assumption 1, we again assume that any F_i^s and F_j^s can be ranked in terms of their reversed hazard rates. With this, Assumption 1 holds as long as the consumer is not too loss averse. But, when gain-loss utility is not welfare relevant, which seems plausible for the digital goods we have in mind, a better default does not necessarily improve consumer welfare (see also Goldin and Reck, 2022).

Other Models of As-If Switching Costs Our loss-aversion based model of default effects is mathematically equivalent to a model of product-specific switching costs. Similarly, other mechanism give rise to "as if" switching costs, with similar positive and normative implications.[33] The consumer may, for example, (mis)interpret the default status of a product as a recommendation, indicating high quality and thereby introducing non-physical switching costs. Or a consumer may want to procrastinate on making a choice, which again introduces

---

[33] Non-physical switching costs are arguably higher than the physical costs of switching, which makes the restriction to small switching costs for some of our results stronger. At the same time, models of non-physical switching costs often impose more structure on how switching costs are tied to values, which can help with satisfying Assumption 1.

non-physical switching costs.

A.2   Technically Unsavvy Consumer

Consider market s, and let $n^s \geq 2$. The consumer is assigned a default service i, which she uses if its value v_i is above a threshold $\underline{v} \in \mathbb{R}$. Otherwise, the consumer refrains from using service s.

Lemma 2 (Technically Unsavvy Consumer).

    I. Assumption 0 holds if and only if $F_i^s(\underline{v}) < 1$ for all i.

    II. Assumption 1 holds if and only if $F_i^s(\underline{v}) < 1$ and $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ for all i and all $j \neq i$.

Proof. <u>Part I</u>. The consumer never uses a non-default product. Hence, for any i and $j \neq i$,

$$q_{ii}^s - q_{ij}^s = 1 - F_i^s(\underline{v}).$$

Hence, Assumption 0 holds if and only if $F_i^s(\underline{v}) < 1$. This proves Part I.

    <u>Part II</u>. Part I of Assumption 1 holds because for any i and $j \neq i$, we have $q_{ii}^s = 1 - F_i^s(\underline{v}) \geq 0$ and $q_{ij}^s = 0$. Next, we observe that for any i, i', and $j \neq i, i'$,

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = F_{i'}^s(\underline{v}) - F_i^s(\underline{v}) = (q_{ii}^s - q_{ii'}^s) - (q_{i'i'}^s - q_{i'i}^s)$$

which is positive if and only if $F_{i'}^s(\underline{v}) > F_i^s(\underline{v})$. Because any two distribution F_i^s and $F_{i'}^s$ can be ranked in terms of FOSD, Parts III and IV of Assumption 1 thus hold if and only if $F_i^s(\underline{v}) \neq F_{i'}^s(\underline{v})$ for all i and i' with $i \neq i'$. This completes the proof of Part II.   □

Consumer Welfare Suppose $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ for all i and all $j \neq i$, and in line with Section 5, let $\underline{v} \geq 0$; that is, the service is valuable in that the consumer "buys reasonably" (Heidhues et al., 2023b). If i is the default, consumer welfare is $\int_{\underline{v}}^{\infty} v \, dF_i^s(v)$. Since all F_ℓ^s have the same support,

$$\int_{\underline{v}}^{\infty} v \, dF_i^s(v) - \int_{\underline{v}}^{\infty} v \, dF_j^s(v) = \underline{v}\big(F_j^s(\underline{v}) - F_i^s(\underline{v})\big) + \int_{\underline{v}}^{\infty} F_j^s(v) - F_i^s(v) \, dv.$$

Because $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ and $\underline{v} \geq 0$, the above is positive if F_i^s first-order stochastically dominates F_j^s. Hence, for valuable services expected consumer welfare is higher with a better default.

A.3   Switching Costs

Consider market s, and let $n^s \geq 2$. We study a slightly more general model than the one described in Section A.1 by allowing for product-specific switching costs. Specifically, we assume that the consumer observes the value of all products, but incurs a switching cost $\gamma_i > 0$ when firm i's product is the default and the consumer chooses the product of firm $j \neq i$. Moreover, we assume that, if F_i^s first-order stochastically dominates F_j^s, then $\gamma_i \geq \gamma_j$. The consumer uses one service.

Lemma 3 (Switching Costs: Steering). *Assumption 0 holds.*

Proof. When firm i's product is the default, it is chosen if and only if $v_i + \gamma_i \geq \max_{\ell \in \mathcal{N}^s \setminus \{i\}} v_\ell$. This happens with probability

$$q_{ii}^s = \int \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v).$$

When firm j's product is the default, the consumer buys the product of firm i with probability

$$q_{ij}^s = \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v).$$

We obtain

$$q_{ii}^s - q_{ij}^s = \int F_j^s(v + \gamma_i) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v) - \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v)$$

$$> \int \left(F_j^s(v + \gamma_i) - F_j^s(v - \gamma_j) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v),$$

which is positive because $\gamma_i, \gamma_j > 0$. This completes the proof. $\qquad\square$

To also establish Assumption 1, we impose more structure on the values distributions. First, we assume that all distributions have full support on $\mathbb{R}$, which implies that any F_i^s admits a density f_i^s.[34] Second, we assume that any two distributions can be ranked in terms of their reversed hazard rates, which is stronger than imposing a ranking in terms of first-order stochastic dominance. Formally, F_i^s dominates F_j^s in the reversed hazard rate if $f_i^s(v)/F_i^s(v) > f_j^s(v)/F_j^s(v)$ for all $v \in \mathbb{R}$.

Lemma 4 (Switching Costs: Quality-Steering Complementarity). *Suppose that F_i^s dominates F_j^s in the reversed hazard rate for all $j \in \mathcal{N}^s$. Then, for any $n^s \geq 3$ and any such $\{F_\ell^s\}_{\ell \in \mathcal{N}^s}$, there exists some $\bar{\gamma} > 0$, such that for any $\gamma_i < \bar{\gamma}$, Assumption 1 holds.*

Proof. First, consider j and $j' \neq j$, with F_j^s dominating $F_{j'}^s$ in the reversed hazard rate. Then,

$$
\begin{aligned}
q_{ij'}^s - q_{ij}^s &= \int \left(F_{j'}^s(v - \gamma_{j'})F_j^s(v) - F_j^s(v - \gamma_j)F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j,j'\}} F_\ell^s(v) \, dF_i^s(v) \\
&\geq \int \left(F_{j'}^s(v - \gamma_j)F_j^s(v) - F_j^s(v - \gamma_j)F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j,j'\}} F_\ell^s(v) \, dF_i^s(v),
\end{aligned}
$$

where the inequality follows from $\gamma_j \geq \gamma_{j'}$ and $F_{j'}^s$ being increasing. Moreover, because F_j^s dominates $F_{j'}^s$ in the reversed hazard rate, we have

$$
\frac{\partial}{\partial v} \frac{F_j^s(v)}{F_{j'}^s(v)} \propto f_j^s(v)F_{j'}^s(v) - f_{j'}^s(v)F_j^s(v) > 0.
$$

This implies, in turn, that

$$
\frac{F_j^s(v)}{F_{j'}^s(v)} > \frac{F_j^s(v - \gamma_j)}{F_{j'}^s(v - \gamma_j)} \quad \text{or, equivalently,} \quad F_{j'}^s(v - \gamma_j)F_j^s(v) - F_j^s(v - \gamma_j)F_{j'}^s(v) > 0. \tag{2}
$$

Hence, $q_{ij'}^s > q_{ij}^s$, which means that Part I of Assumption 1 holds.

---

[34] If v_i^s has full support on $\mathbb{R}$, then F_i^s is strictly increasing and thus almost everywhere differentiable.

Second, consider i and $j \neq i$, with F_i^s dominating F_j^s in the reversed hazard rate. To verify Part III of Assumption 1, we define $G^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v)$ and observe

$$
\begin{aligned}
q_{ii}^s - q_{ij}^s &= \int F_j^s(v + \gamma_i) G^s(v + \gamma_i) \, dF_i^s(v) - \int F_j^s(v - \gamma_j) G^s(v) \, dF_i^s(v) \\
&= 1 - \int F_i^s(u - \gamma_i) G^s(u) \, dF_j^s(u) - \int F_j^s(u) F_i^s(u - \gamma_i) \, dG^s(u) \\
&\quad - \left(1 - \int F_i^s(u + \gamma_j) G^s(u + \gamma_j) \, dF_j^s(u) - \int F_j^s(u - \gamma_j) F_i^s(u) \, dG^s(u) \right) \\
&= \int \left(F_i^s(u + \gamma_j) G^s(u + \gamma_j) - F_i^s(u - \gamma_i) G^s(u) \right) dF_j^s(u) \\
&\quad + \int \left(F_j^s(u - \gamma_j) F_i^s(u) - F_j^s(u) F_i^s(u - \gamma_i) \right) dG^s(u) \\
&= q_{jj}^s - q_{ji}^s + \int \left(F_j^s(u - \gamma_j) F_i^s(u) - F_j^s(u) F_i^s(u - \gamma_i) \right) dG^s(u) \\
&\geq q_{jj}^s - q_{ji}^s + \int \left(F_j^s(u - \gamma_j) F_i^s(u) - F_j^s(u) F_i^s(u - \gamma_j) \right) dG^s(u),
\end{aligned}
$$

where the inequality follows from $\gamma_i \geq \gamma_j$ and F_i^s being increasing. Hence, by (2), it follows that $q_{ii}^s - q_{ij}^s > q_{jj}^s - q_{ji}^s$ and thus Part III of Assumption 1.

Third, consider i and $i' \neq i$, with F_i^s dominating $F_{i'}^s$ in the reversed hazard rate, and denote as $\widetilde{G}^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i, i', j\}} F_\ell^s(v)$. Then, since $F_i^s(v)$ and $F_{i'}^s(v)$ admit a density by assumption,

$$
\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) &= \int F_j^s(v + \gamma_i) \widetilde{G}^s(v + \gamma_i) F_{i'}^s(v + \gamma_i) f_i^s(v) \, dv \\
&\quad - \int F_j^s(v + \gamma_{i'}) \widetilde{G}^s(v + \gamma_{i'}) F_i^s(v + \gamma_{i'}) f_{i'}^s(v) \, dv \\
&\quad - \int F_j^s(v - \gamma_j) \widetilde{G}^s(v) \left[F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv \\
&\geq \int F_j^s(v + \gamma_i) \widetilde{G}^s(v + \gamma_i) \left[F_{i'}^s(v + \gamma_i) f_i^s(v) - F_i^s(v + \gamma_i) f_{i'}^s(v) \right] dv \\
&\quad - \int F_j^s(v - \gamma_j) \widetilde{G}^s(v) \left[F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv,
\end{aligned}
$$

where inequality follows from $\gamma_i \geq \gamma_{i'}$, and F_j^s, $F_{i'}^s$ and $\widetilde{G}^s$ being increasing.

41

We distinguish two cases. First, let $\gamma_j = \gamma_i + \epsilon$ for some $\epsilon > 0$. Then, at $\gamma_i = 0$, the above is

$$\int \widetilde{G}^s(v) \left[F_j^s(v) - F_j^s(v-\epsilon) \right] \left[F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv > 0,$$

where the inequality follows from $\epsilon > 0$ and F_j^s having full support, and F_i^s and $F_{i'}^s$ satisfying the reverse hazard rate. By continuity, there exists some constant $\bar{\gamma}_{ii'j} > 0$, such that for any $\gamma_i < \bar{\gamma}_{ii'j}$, we have $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$. Note that the constraint is getting tighter for smaller ϵ.

Second, let $\gamma_j = \gamma_i - \epsilon$ for some $\epsilon \in [0, \gamma_i]$. Hence, if $\gamma_i = 0$, then also $\gamma_j = 0$. It follows that $q_{ii}^s - q_{ij}^s = q_{i'i'}^s - q_{i'j}^s$ if $\gamma_i = 0$. We now take, for a fixed ϵ, the partial derivative of the lower bound on $(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s)$ derived above with respect to γ_i. Clearly, both integrands are bounded from above and from below. Hence, by Lebesgue's dominated convergence theorem, we can exchange differentiation and integration. This implies that this partial derivative is given by

$$\int \left[F_{i'}^s(v+\gamma_i) f_i^s(v) - F_i^s(v+\gamma_i) f_{i'}^s(v) \right] \frac{\partial}{\partial \gamma_i} \left(F_j^s(v+\gamma_i) \widetilde{G}^s(v+\gamma_i) \right) dv$$

$$+ \int \left[f_{i'}^s(v+\gamma_i) f_i^s(v) - f_i^s(v+\gamma_i) f_{i'}^s(v) \right] F_j^s(v+\gamma_i) \widetilde{G}^s(v+\gamma_i) \, dv$$

$$+ \int f_j^s(v-\gamma_j) \widetilde{G}^s(v) \left[F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv.$$

Evaluating this partial derivative at $\gamma_i = 0$, we obtain

$$\int \left[F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] \left(2 f_j^s(v) \widetilde{G}^s(v) + F_j^s(v) \tilde{g}(v) \right) dv,$$

which is strictly positive by the reversed hazard rate condition. Together with $q_{ii}^s - q_{ij}^s = q_{i'i'}^s - q_{i'j}^s$ at $\gamma_i = 0$, this implies that again, there exists some $\bar{\gamma}_{ii'j} > 0$, such that for any $\gamma_i < \bar{\gamma}_{ii'j}$, we have $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$. Here, the constraint is getting tighter as ϵ gets bigger. Further noticing that there are only finitely many combinations of i, i', and j, we conclude that $\bar{\gamma} := \min_{i,i',j \in \mathcal{N}^s} \bar{\gamma}_{ii'j} > 0$. Hence, if $\gamma_i < \bar{\gamma}$ for all i, also Part II of Assumption 1 holds. This completes the proof. $\qquad\square$

Consumer Welfare Suppose that switching costs are independent of the default: $\gamma_\ell \equiv \gamma$ for all $\ell \in \mathcal{N}^s$. Denote as ν_i^s the value (net of switching costs) of the product *chosen* by the consumer when product i is the default. The value ν_i^s is drawn from the cumulative distribution function

$$\mathbb{P}[\nu_i^s \leq v] = F_i^s(v) \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F_\ell^s(v + \gamma).$$

Notice that

$$\mathbb{P}[\nu_i^s \leq v] - \mathbb{P}[\nu_j^s \leq v] = \left[F_i^s(v) F_j^s(v + \gamma) - F_i^s(v + \gamma) F_j^s(v) \right] \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v + \gamma).$$

Hence, if F_i^s dominates F_j^s in the reversed hazard rate, ν_i^s first-order stochastically dominates ν_j^s. As a result, expected consumer welfare is higher with a better default.

Large Switching Costs To see that the restriction to small enough switching costs has bite, let $n^s = 3$, $\gamma_i = \gamma$, and assume that v_i^s is drawn from a normal distribution with a mean μ_i and a variance of 1. As illustrated in Figure 1 for the case of $\mu_1 = \frac{1}{2}$, $\mu_2 = 0$, and $\mu_3 = -\frac{1}{2}$, Part II of Assumption 1 does *not* hold in general. In this particular example, the upper bound on the switching cost γ for Assumption 1 to hold is given by $\bar{\gamma} \approx 2.15$.
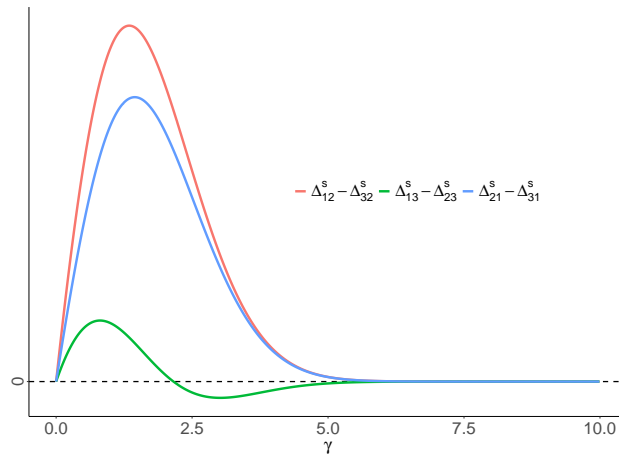

Figure 1: *Violation of Assumption 1 (Part II) with normal values. Note:* $\Delta_{ij}^s = q_{ii}^s - q_{ij}^s$.

A.4 Sequential Search

Consider market s, and let $n^s \geq 2$. We assume that the consumer observes the value, but not the identity of the default product for free and can then search at a cost. The first search costs $c_1 > 0$, the second search costs $c_2 \geq c_1$, and the kth search costs $c_k \geq c_{k-1}$. For simplicity, we set $c_2 = \infty$, thereby effectively limiting the consumer's behavior to searching at most once. If the consumer searches for another product, she randomly draws one of the non-default products.

To get started, suppose that firm i's product is the default. The consumer decides to search if and only if the realized value v_i^s lies below some cutoff $\hat{v}_i^s \in \mathbb{R}$. Moreover, because the consumer does not learn the identity of the default firm, she does not learn anything about the quality of the non-default firms and thus about the value of search. Formally, the cutoff satisfies $\hat{v}_i^s = \hat{v}^s$. Then:

Lemma 5 (Sequential Search).

I. Assumption 0 holds.

II. Assumption 1 holds if and only if $n^s \geq 3$ and for all $i, i' \in \mathcal{N}^s$, there exists some $v \leq \hat{v}^s$, such that $F_{i'}^s(v) \neq F_i^s(v)$.

Proof. <u>Preliminaries</u>. First, suppose i's product is the default. Firm i's profit in market s then is

$$
\begin{aligned}
q_{ii}^s &= \mathbb{P}\big[v_i^s > \hat{v}^s\big] \;+\; \mathbb{P}\big[v_i^s \leq \hat{v}^s\big] \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s \leq \hat{v}^s\big] \\
&= \mathbb{P}\big[v_i^s > \hat{v}^s\big] \;+\; \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \bigg\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] - \mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s > \hat{v}^s\big] \bigg\} \\
&= \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \bigg\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big] \Big(1 - \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s > \hat{v}^s\big] \Big) \bigg\} \\
&= \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{\bigg\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s\big] \bigg\}}_{=: \; \mathcal{A}_{i\ell}},
\end{aligned}
\tag{3}
$$

where the second equality holds by the law of total probability.

Second, suppose firm j's product is the default. Firm i thus makes a sale only if the consumer decides to search *and* firm i is the random firm that she finds. Firm i's profit in market s then is

$$
\begin{aligned}
q_{ij}^s &= \frac{1}{n^s - 1} \mathbb{P}\big[v_j^s \le \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_j^s | v_j^s \le \hat{v}^s\big] \\
&= \frac{1}{n^s - 1} \underbrace{\left\{ \mathbb{P}\big[v_i^s > v_j^s\big] - \mathbb{P}\big[v_j^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_j^s | v_j^s > \hat{v}^s\big] \right\}}_{=:\, \mathcal{B}_{ij}},
\end{aligned}
\tag{4}
$$

where the second equality again follows from the law of total probability.

Third, suppose that the default is determined at random. Using Eq. (3) and (4), we conclude that firm i's profit in market s is then given by

$$
\frac{1}{n^s(n^s - 1)} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \left\{ \mathcal{A}_{i\ell} + \mathcal{B}_{i\ell} \right\}.
$$

Fourth, we express the probabilities above in terms of the underlying CDFs. We observe

$$
\mathbb{P}\big[v_i^s > v_\ell^s\big] = \int_{-\infty}^{\infty} F_\ell^s(v) \, dF_i^s(v),
$$

and

$$
\mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s \le v_\ell^s | v_i^s > \hat{v}^s\big] = \mathbb{P}\big[v_i^s \le v_\ell^s, v_i^s > \hat{v}^s\big] = 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_\ell^s(v) \, dF_i^s(v).
$$

This implies that

$$
\begin{aligned}
\mathcal{A}_{i\ell} &= \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s \le v_\ell^s | v_i^s > \hat{v}^s\big] \\
&= \int_{-\infty}^{\hat{v}^s} F_\ell^s(v) \, dF_i^s(v) + 1 - F_i^s(\hat{v}^s) \\
&= 1 - F_i^s(\hat{v}^s) + F_\ell^s(\hat{v}^s) F_i^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v) \, dF_\ell^s(v),
\end{aligned}
\tag{5}
$$

45

where the last equality follows from integration by parts, as well as

$$
\begin{aligned}
\mathcal{B}_{ij} &= \mathbb{P}\big[v_i^s > v_j^s\big] - \mathbb{P}\big[v_j^s > \hat{v}^s\big]\mathbb{P}\big[v_i^s > v_j^s | v_j^s > \hat{v}^s\big] \\
&= \int_{-\infty}^{\infty} F_j^s(v)\ dF_i^s(v) - \left(1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v)\right) \\
&= 1 - \int_{-\infty}^{\infty} F_i^s(v)\ dF_j^s(v) - 1 + F_j^s(\hat{v}^s) + \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\
&= F_j^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v)\ dF_j^s(v),
\end{aligned}
\tag{6}
$$

where the second equality again follows from integration by parts.

Finally, we define

$$
\begin{aligned}
\mathcal{C}_{ij} &:= \mathcal{A}_{ij} - \mathcal{B}_{ij} \\
&= 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_j^s(v)\ dF_i^s(v) + 1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\
&= 2 - F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s) - \big(1 - F_j^s(\hat{v}^s)F_i^s(\hat{v}^s)\big) + \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\
&= 1 - F_i^s(\hat{v}^s)\big(1 - F_j^s(\hat{v}^s)\big) - F_j^s(\hat{v}^s),
\end{aligned}
\tag{7}
$$

where the second equality follows from integration by parts. This also means that $\mathcal{C}_{ij} = \mathcal{C}_{ji}$.

<u>Part I</u>. Subtracting (4) from (3) gives

$$
q_{ii}^s - q_{ij}^s = \frac{1}{n^s - 1}\left(\mathcal{C}_{ij} + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \mathcal{A}_{i\ell}\right).
$$

Because all F_ℓ^s have common support, we have $\mathcal{A}_{i\ell}, \mathcal{C}_{ij} > 0$. Hence, Assumption 0 holds.

<u>Part II</u>. Suppose F_i^s first-order stochastically dominates F_j^s. Since $\mathcal{C}_{ij} = \mathcal{C}_{ji}$,

$$
(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) = \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \big\{\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell}\big\}.
$$

Using Eq. (5), we conclude

$$
\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell} = \big(F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s)\big)\big(1 - F_\ell^s(\hat{v}^s)\big) + \int_{-\infty}^{\hat{v}^s} \big(F_j^s(v) - F_i^s(v)\big)\ dF_\ell^s(v) \geq 0,
$$

holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Moreover, if $n^s = 2$, then clearly $q_{ii}^s - q_{ij}^s = q_{jj}^s - q_{ji}^s$. Combining both, we conclude that Part III of Assumption 1 holds if and only if $n^s \geq 3$ and for all $i, i' \in \mathcal{N}^s$, there exists some $v \leq \hat{v}^s$, so that $F_{i'}^s(v) \neq F_i^s(v)$.

Next, consider three firms i, i', and $j \neq i, i'$. We have

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = \frac{1}{n^s - 1}\left(\mathcal{C}_{ij} - \mathcal{C}_{i'j} \;+\; \mathcal{A}_{ii'} - \mathcal{A}_{i'i} \;+\; \sum_{\ell \in \mathcal{N}^s \setminus \{i, i', j\}} \left\{\mathcal{A}_{i\ell} - \mathcal{A}_{i'\ell}\right\}\right).$$

To verify Part II of Asssumption 1, let F_i^s first-order stochastically dominate $F_{i'}^s$. Then, by (7),

$$\mathcal{C}_{ij} - \mathcal{C}_{i'j} = \left(F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s)\right)\left(1 - F_j^s(\hat{v}^s)\right) \geq 0,$$

holding with a strict inequality if and only if $F_{i'}^s(\hat{v}^s) > F_i^s(\hat{v}^s)$. Moreover, by (5), we obtain

$$\mathcal{A}_{ii'} - \mathcal{A}_{i'i} = F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s) + \int_{-\infty}^{\hat{v}^s} F_{i'}^s(v) \, dF_i^s(v) - \int_{-\infty}^{\hat{v}^s} F_i^s(v) \, dF_{i'}^s(v) \geq 0,$$

holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Finally, by the same argument as above, we have $\mathcal{A}_{i\ell} \geq \mathcal{A}_{i'\ell}$, holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Hence, Part II of Assumption 1 holds if and only if for all $i, i' \in \mathcal{N}^s$, there exists some $v \leq \hat{v}^s$, so that $F_{i'}^s(v) \neq F_i^s(v)$.

Finally, for any j, j', and $i \neq j, j'$, we obtain

$$q_{ij'}^s - q_{ij}^s = \frac{1}{n^s - 1}\left(\mathcal{C}_{ij} - \mathcal{C}_{ij'} + \mathcal{A}_{ij'} - \mathcal{A}_{ij}\right).$$

If F_j^s first-order stochastically dominates $F_{j'}^s$, then, by (7),

$$\mathcal{C}_{ij} - \mathcal{C}_{ij'} = \left(1 - F_i^s(\hat{v}^s)\right)\left(F_{j'}^s(\hat{v}^s) - F_j^s(\hat{v}^s)\right) \geq 0,$$

holding with a strict inequality if and only if $F_{j'}^s(\hat{v}^s) > F_j^s(\hat{v}^s)$. Similarly, by (5),

$$\mathcal{A}_{ij'} - \mathcal{A}_{ij} = \int_{-\infty}^{\hat{v}^s} \left\{F_{j'}^s(v) - F_j^s(v)\right\} \, dF_i^s(v) \geq 0,$$

holding with a strict inequality if and only if $F_{j'}^s(v) \neq F_j^s(v)$ for some $v \leq \hat{v}^s$. Hence, Part I of Assumption 1 holds if and only if $F_{j'}^s(v) \neq F_j^s(v)$ for some $v \leq \hat{v}^s$. $\qquad\square$

Consumer Welfare   If product i is the default product, then consumer welfare equals

$$\mathbb{P}\big[v_i^s \geq \hat{v}^s\big]\mathbb{E}\big[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\big] + \big(1 - \mathbb{P}\big[v_i^s \geq \hat{v}^s\big]\big)\frac{1}{n^s - 1}\sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{E}\big[\max\{v_i^s, v_\ell^s\} - c_1 | v_i^s \leq \hat{v}^s\big]$$

$$= \int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \frac{1}{n^s - 1}\bigg[\int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}}\int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v)\bigg]$$

$$= \frac{n^s - 2}{n^s - 1}\int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \frac{1}{n^s - 1}\bigg[\int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}}\int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v)\bigg],$$

where $G_{i\ell}^s$ is the CDF of $\max\{v_i^s, v_\ell^s\}$. Because v_i^s and v_ℓ^s are independent, we have

$$G_{i\ell}^s(v) = \mathbb{P}\bigg[\big\{\max\{v_i^s, v_\ell^s\} \leq v\big\} \cup \big\{v_i^s \leq \hat{v}^s\big\}\bigg] = \begin{cases} F_i^s(v)F_\ell^s(v) & \text{if } v < \hat{v}^s, \\[2mm] F_i^s(\hat{v}^s)F_\ell^s(v) & \text{if } v \geq \hat{v}^s, \end{cases}$$

and the corresponding density is given by

$$g_{ij}^s(v) = \begin{cases} f_i^s(v)F_j^s(v) + F_i^s(v)f_j^s(v) & \text{if } v < \hat{v}^s, \\[2mm] F_i^s(\hat{v}^s)f_j^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

First, we observe that, for any $\ell \neq i, j$,

$$G_{i\ell}^s(v) - G_{j\ell}^s(v) = \begin{cases} \big(F_i^s(v) - F_j^s(v)\big)F_\ell^s(v) & \text{if } v < \hat{v}^s, \\[2mm] \big(F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s)\big)F_\ell^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

Hence, if F_i^s first-order stochastically dominates F_j^s, then $G_{i\ell}^s$ first-order stochastically dominates $G_{j\ell}^s$ for all $\ell \neq i, j$, and as a consequence, also

$$\int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v) > \int_{-\infty}^{\infty} v - c_1 \; dG_{j\ell}^s(v) \quad \text{for all } \ell \neq i, j.$$

Second, we observe that

$$\int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) - \int_{-\infty}^{\infty} v - c_1 \; dG_{ji}^s(v)$$

$$= F_i^s(\hat{v}^s)\int_{\hat{v}^s}^{\infty} v - c_1 \; dF_j^s(v) - F_j^s(\hat{v}^s)\int_{\hat{v}^s}^{\infty} v - c_1 \; dF_i^s(v)$$

$$= F_i^s(\hat{v}^s)\big(1 - F_j^s(\hat{v}^s)\big)\mathbb{E}\big[v_j^s - c_1 | v_j^s \geq \hat{v}^s\big] - F_j^s(\hat{v}^s)\big(1 - F_i^s(\hat{v}^s)\big)\mathbb{E}\big[v_i^s - c_1 | v_i^s \geq \hat{v}^s\big],$$

48

and thus

$$\left(\int_{\hat{v}^s}^{\infty} v \, dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1 \, dG_{ij}^s(v) \right) - \left(\int_{\hat{v}^s}^{\infty} v \, dF_j^s(v) + \int_{-\infty}^{\infty} v - c_1 \, dG_{ji}^s(v) \right)$$

$$= \left(1 - F_i^s(\hat{v}^s)\right)\left(1 - F_j^s(\hat{v}^s)\right)\left(\mathbb{E}\left[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\right] - \mathbb{E}\left[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\right] \right) + c_1\left(F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s) \right).$$

Notice that

$$\mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] = \begin{cases} \frac{F_i^s(v)}{1 - F_i^s(\hat{v}^s)} & \text{if } v \geq \hat{v}^s \\ \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for any $v \geq \hat{v}^s$, if F_i^s first-order stochastically dominates F_j^s, then

$$\mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] - \mathbb{P}[v_j^s \leq v | v_j^s \geq \hat{v}^s]$$

$$= \frac{1}{\left(1 - F_i^s(\hat{v}^s)\right)\left(1 - F_j^s(\hat{v}^s)\right)} \left[F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_j^s(\hat{v}^s)F_i^s(v) \right]$$

$$\leq \frac{1}{\left(1 - F_i^s(\hat{v}^s)\right)\left(1 - F_j^s(\hat{v}^s)\right)} \left[F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_i^s(\hat{v}^s)F_i^s(v) \right]$$

$$= -\frac{F_j^s(v) - F_i^s(v)}{1 - F_j^s(\hat{v}^s)},$$

where the inequality follows from the fact that $F_j^s(v) \geq F_i^s(v)$. We conclude that the dominance relation is preserved when truncating the distributions of v_i^s and v_j^s at $\hat{v}^s$. This, in turn, implies that if F_i^s first-order stochastically dominates F_j^s, then $\mathbb{E}\left[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\right] \geq \mathbb{E}\left[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\right]$.

Combining both observations, if F_i^s first-order stochastically dominates F_j^s, then consumer welfare increases by more than

$$\frac{n^s - 2}{n^s - 1} \left[\mathbb{P}\left[v_i^s \geq \hat{v}^s\right]\mathbb{E}\left[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\right] - \mathbb{P}\left[v_j^s \geq \hat{v}^s\right]\mathbb{E}\left[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\right] \right] > 0$$

if product i replaces product j as the default. Hence, a better default improves consumer welfare.

A.5 Consideration Sets

Consider market s, and let $n^s \geq 2$. The consumer considers $k^s \in \{1, 2, \ldots, n^s - 1\}$ offers, and selects the one with the highest realized value v_i^s among the offers in her consideration set. The consumer considers the default product with certainty. Any non-default product enters the consideration set with the same probability of $(k^s-1)/(n^s-1)$; that is, attention to non-default products is random.

Lemma 6 (Consideration Sets).

    I. Assumption 0 holds.

    II. Assumption 1 holds if and only if $n^s \geq 3$ and $k^s \geq 2$.

Proof. <u>Preliminaries</u>. Consider some firm i offering service s, and refer to $r_{ii}^s \in [0, 1]$ as its (average) "conversion rate" conditional on being in the consideration set and facing $k^s - 1$ random competitors. This is exactly what we call in the main text the firm's demand conditional on being the default.

To see how the conversion rate depends on the value that firm i and its rivals offer, we denote as $\mathcal{N}^s(x)$ a subset of $x \geq 1$ firms that are active in market s. We introduce the CDF

$$q_{\mathcal{N}^s(x)}(v) \ := \ \prod_{\ell \in \mathcal{N}^s(x)} F_\ell^s(v);$$

that is, the probability that the products of all firms in the set $\mathcal{N}^s(x)$ have a realized value less or equal to v. Using this notation, we can write the conversion rate of firm i as

$$r_{ii}^s \ = \ \sum_{\mathcal{N}^s(k^s-1):\ i \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v). \tag{8}$$

When firm i is in the consideration set, all $n^s - 1$ rivals are equally likely to be drawn into one of the $k^s - 1$ remaining spots. Hence, there are $\binom{n^s-1}{k^s-1}$ different sets of potential competitors, each of which occurs with probability $1/\binom{n^s-1}{k^s-1}$. Fixing a set of competitors $\mathcal{N}^s(k^s - 1)$, the consumer uses the service of firm i if and only if it offers the highest value; that is, $v_i^s > v_j^s$

for all $j \in \mathcal{N}^s(k^s - 1)$. This happens to be the case with probability $\int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v)$. Because all distributions F_j^s have common support, we have $r_{ii}^s > 0$. By the same argument, we get $r_{ii}^s < 1$ if and only if $k^s \geq 2$.

Now consider firms i and j with F_i^s first-order stochastically dominating F_j^s. Then,

$$
\begin{aligned}
r_{ii}^s - r_{jj}^s \quad &\propto \sum_{\mathcal{N}^s(k^s-1): \ i,j \notin \mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_j^s(v) \right\} \\
&+ \sum_{\substack{\mathcal{N}^s(k^s-1): \ i \notin \mathcal{N}^s(k^s-1), \\ j \in \mathcal{N}^s(k^s-1)}} \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v) - \sum_{\substack{\mathcal{N}^s(k^s-1): \ j \notin \mathcal{N}^s(k^s-1), \\ i \in \mathcal{N}^s(k^s-1)}} \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_j^s(v) \\
&\geq \sum_{\mathcal{N}^s(k^s-1): \ i,j \notin \mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_j^s(v) \right\} \\
&+ \sum_{\substack{\mathcal{N}^s(k^s-1): \ i \notin \mathcal{N}^s(k^s-1), \\ j \in \mathcal{N}^s(k^s-1)}} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_j^s(v) \right\} \\
&\geq 0,
\end{aligned}
$$

where the first inequality holds because $F_j^s(v) \geq F_i^s(v)$ and the second one follows from the fact that $q_{\mathcal{N}^s(k^s-1)}(v)$ is increasing in v together with F_i^s first-order stochastically dominating F_j^s. Moreover, whenever $k^s \geq 2$, the second inequality is strict, which in turn implies $r_{ii}^s > r_{jj}^s$.

Similarly, in this subsection, we refer to $r_{ij}^s \in [0, 1]$ as the (average) conversion rate of firm i conditional on being in the consideration set and facing firm j as well as $k^s - 2$ random competitors. Notice that this conversion rate is *not* the same as the demand of firm i when firm j's product is the default. Using the same notation as above, we can write this conversation rate as

$$
r_{ij}^s \quad = \sum_{\substack{\mathcal{N}^s(k^s-1): \ i \notin \mathcal{N}^s(k^s-1), \\ j \in \mathcal{N}^s(k^s-1)}} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-1)}(v) \ dF_i^s(v). \tag{9}
$$

This implies, in particular, that

$$
r_{ij}^s - r_{ij'}^s \quad = \sum_{\mathcal{N}^s(k^s-2): \ i,j,j' \notin \mathcal{N}^s(k^s-2)} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-2)}(v) \left[F_j^s(v) - F_{j'}^s(v) \right] dF_i^s(v).
$$

Hence, if F_j^s first-order stochastically dominates $F_{j'}^s$ and $k^s \leq n^s - 1$, then $r_{ij}^s < r_{ij'}^s$.

<u>Part I</u>. For any i and $j \neq i$, we have

$$q_{ii}^s - q_{ij}^s = r_{ii}^s - \frac{k^1 - 1}{n^1 - 1} r_{ij}^s = \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v). \qquad (10)$$

Since all F_ℓ^s have the same support, the above is postive. Hence, Part Assumption 0 holds.

<u>Part II</u>. Suppose F_j^s first-order stochastically dominates $F_{j'}^s$. Then,

$$
\begin{aligned}
q_{ij'}^s - q_{ij}^s &= \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i,j' \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1),\\ j' \in \mathcal{N}^s(k^s-1)}} \int \left(F_{j'}^s(v) - F_j^s(v) \right) \prod_{\ell \in \mathcal{N}^s(k^s-1)\backslash\{j'\}} F_\ell^s(v)\ dF_i^s(v) > 0,
\end{aligned}
$$

where the second equality follows from the definition of $q_{\mathcal{N}^s(k^s-1)}(v)$ and the inequality holds (weakly) as F_j^s first-order stochastically dominates $F_{j'}^s$. The inequality is strict since all F_ℓ^s have no mass points and common support. Hence, if $k^s \geq 2$, Part I of Assumption 1 holds.

Next, consider firms, i and i', with F_i^s first-order stochastically dominating $F_{i'}^s$. Then,

$$
\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) &= \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i',j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_{i'}^s(v) \\
&\geq \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i',j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1),\\ i' \in \mathcal{N}^s(k^s-1)}} \int \left(F_{i'}^s(v) - F_i^s(v) \right) \prod_{\ell \in \mathcal{N}^s(k^s-1)\backslash\{i'\}} F_\ell^s(v)\ dF_i^s(v) \\
&\geq 0,
\end{aligned}
$$

where the first inequality follows from the fact that $q_{\mathcal{N}^s(k^s-1)}(v)$ monotonically increases in v and F_i^s first-order stochastically dominates $F_{i'}^s$ while the second one simply follows from first-order stochastic dominance. Whenever $k^s \geq 2$, the inequalities are strict because the distributions have no mass points and common support. Hence, if $k^s \geq 2$, Part II of Assumption 1 holds.

Finally, let F_i^s first-order stochastically dominate F_j^s, and observe that

$$
\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) =& \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&- \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \\
\geq& \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
&- \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\
=&\ 0,
\end{aligned}
$$

where the inequality follows from $q_{\mathcal{N}^s(k^s-1)}(v)$ being increasing and F_i^s first-order stochastically dominating F_j^s, and it is strict if $k^s \geq 2$. Hence, if $k^s \geq 2$, Part III of Assumption 1 holds. $\qquad\square$

Consumer Welfare   Denote as ν_i^s the value of the product *chosen* by the consumer when product i is the default. The value ν_i^s is drawn from the cumulative distribution function

$$
\mathbb{P}[\nu_i^s \leq v] = F_i^s(v) \sum_{\mathcal{N}^s(k^s-1):\ i \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).
$$

Notice that

$$
\mathbb{P}[\nu_i^s \leq v] - \mathbb{P}[\nu_j^s \leq v] = \left(F_i^s(v) - F_j^s(v) \right) \sum_{\mathcal{N}^s(k^s-1):\ i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).
$$

Hence, if F_i^s first-order stochastically dominates F_j^s, ν_i^s first-order stochastically dominates ν_j^s, and expected consumer welfare is higher with a better default.

A.6 Loss Aversion

Consider market s, and let $n^s \geq 2$. Suppose that each product i is defined by a pair (v_i^s, d_i^s) with $v_i^s \sim G^s$ and $d_i^s \in \mathbb{R}_{>0}$. We think of d_i^s as an app-design dimension, which affects the consumer's utility in an additively separable way. Let $F_i^s(u) = G^s(u - d_i^s)$ be the distribution of $v_i^s + d_i^s$. We assume that, if the consumer uses the product of firm j instead of the default product i, she experiences a loss of λd_i (with $\lambda > 1$) because she cannot enjoy firm i's proprietary design features, and a gain of d_j from using firm j's product instead. In addition, the consumer has some reference point in the service dimension, and we think of the distribution G^s as being "gain-loss adjusted."

Lemma 7 (Loss Aversion).

I. Assumption 0 holds.

II. For any $n^s \geq 3$ and any $\{F_\ell^s\}_{\ell \in \mathcal{N}^s}$ that can be ranked in terms of the reversed hazard rate, there exists some $\bar{\lambda} > 1$, such that for any $\lambda \in (1, \bar{\lambda})$, Assumption 1 holds.

Proof. We simply show that model is mathematically equivalent to the switching-cost model in Section A.3. Suppose that product i is the default. Because we think of v_i^s as being gain-loss adjusted, and we assume that the reference point in this dimension is independent of a product's default status, the consumer chooses the product of firm i over that of a rival j if and only if

$$v_i > v_j + d_j - \lambda d_i \quad \text{or, equivalently,} \quad v_i + d_i > v_j + d_j - \underbrace{(\lambda - 1)d_i}_{=: \, \gamma_i}.$$

The claim now follows immediately from Lemmas 3 and 4. $\square$

Consumer Welfare   In the context of digital markets, it seems plausible to assume that gain-loss utility is not welfare relevant. From a normative perspective, our model of loss-aversion based default effects is therefore different from the switching-cost model in Section A.3. In fact, with loss aversion, a better default product does not necessarily improve

54

consumer welfare (see also Goldin and Reck, 2022). To see this most clearly, suppose that there exists a product $w \in \mathcal{N}^s$ with

$$\mathbb{P}[v_w + d_w + \gamma_w < v_j + d_j] \approx 1.$$

for all $j \neq w$. Then, if w is the default, the consumer makes an active choice with almost probability 1, and thus almost certainly chooses the welfare-maximizing option. Hence, with loss-aversion based default effects it can be optimal to force choice by setting a bad default.

B Proofs

B.1 Preliminaries

Proof of Lemma 1. <u>Part I</u>. By Assumption 0, we immediately have

$$\alpha_i^s = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} (q_{ii}^s - q_{i\ell}^s) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{(q_{ii}^s - q_{i\ell}^s)}_{>0} > 0.$$

<u>Part II</u>. Let $i \in \mathcal{N}^s$, and for the sake of a contradiction, suppose $\eta_{ij}^s \geq 0$ for all $j \neq i$. Then,

$$\alpha_i^s = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (q_{ii}^s - q_{i\ell}^s) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (\alpha_i^s - \eta_{i\ell}^s) \leq \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \alpha_i^s = \frac{n^s - 1}{n^s} \alpha_i^s < \alpha_i^s,$$

where the strict inequality follows from $\alpha_i > 0$; a contradiction.

<u>Part III</u>. Suppose all firms are symmetric, and denote by $\bar{q}$ a firm's demand when being the default and by $\underline{q}$ a firm's demand when one of the rivals is default. By Assumption 0, we have $\bar{q} > \underline{q}$, and by symmetry,

$$\alpha_i^s = \frac{n^s - 1}{n^s} (\bar{q} - \underline{q}) \quad \text{and} \quad \eta_{ij}^s = -\frac{1}{n^s} (\bar{q} - \underline{q}).$$

Hence, $(n^s - 1)|\eta_{ij}^s| = \alpha_i^s$. $\qquad\qquad \square$

B.2 The Emergence of Ecosystems

Proof of Proposition 1. <u>Part I</u>. We solve the game backwards. If firm G takes over a target t, it earns an additional gross profit of $V_t^b + q_G^a \alpha_t^b$, which by Part I of Lemma 1, is greater than t's standalone value V_t^b. Standard ultimatum-bargaining game analysis, hence, implies that in any subgame in which there is a single target t left, G must offer V_t^b and t always accepts that offer.

Now consider any subgame in which a target t' receives an offer and there is one remaining target $t \neq t'$ left thereafter. By subgame perfection, t' accepts the offer only if it is greater than $V_{t'}^b + q_G^a \eta_{t't}^b$, and whenever it accepts the offer in equilibrium, by standard arguments, the offer cannot be strictly greater than $V_{t'}^b + q_G^a \eta_{t't}^b$. When an offer of $V_{t'}^b + q_G^a \eta_{t't}^b$ is accepted, G earns net profits of $q_G^a(\alpha_{t'}^b - \eta_{t't}^b)$ while otherwise it earns $q_G^a \alpha_t^b$. Hence, G earns at most $q_G^a \max\{\alpha_{t'}^b - \eta_{t't}^b, \alpha_t^b\}$. Similarly, if firm G instead approached t first, it would earn at most $q_G^a \max\{\alpha_t^b - \eta_{tt'}^b, \alpha_{t'}^b\}$. Therefore, in any subgame with just two targets left, G earns at most

$$\max_{t_1, t_2 \in \mathcal{N}^b} q_G^a \left(\alpha_{t_1}^b - \min\left\{0, \eta_{t_1 t_2}^b\right\}\right) = \max_{t_1, t_2 \in \mathcal{N}^b} q_G^a \left(\alpha_{t_1}^b - \eta_{t_1 t_2}^b\right) \equiv \pi', \qquad (11)$$

where the equality follows because by Part II of Lemma 1, for every t_1 there exists some t_2' such that $\eta_{t_1 t_2'}^b < 0$. Because we have a finite number of targets, the maximum in (11) exists.

Next, we observe that in any subgame with two or more remaining targets in which t accepts G's offer, G must offer at least $V_t^b + q_G^a \min_{t' \in \mathcal{N}^b \setminus \{t\}} \min\{0, \eta_{tt'}^b\}$; for otherwise the target is better of rejecting the offer no matter what happens in later subgames. This, however, implies that G's profits from the takeover in any subgame are at most

$$\max_{t' \in \mathcal{N}^b} q_G^a \left(\alpha_t^b - \min\left\{0, \eta_{tt'}^b\right\}\right) \leq \max_{t_1, t_2 \in \mathcal{N}^b} q_G^a \left(\alpha_{t_1}^b - \min\left\{0, \eta_{t_1 t_2}^b\right\}\right) = \pi'.$$

Finally, let t_1' and t_2' be targets for which the maximum in (11) is attained. By standard arguments, in any subgame with only t_1' and t_2' left, G's offer must be accepted by the target it first approaches. By approaching all targets other than t_1' and t_2' beforehand and offering them zero, which must be rejected, G can induce the subgame in which only t_1'

56

and t_2' remain. In the subgame-perfect equilibrium of this subgame, G earns π' from the takeover. Since G cannot earn more than π' in any other subgame, offering zero to all other targets first and playing a subgame-perfect equilibrium strategy when only t_1 and t_2 remain is part of a subgame-perfect equilibrium strategy for G. Since doing so is always a possible deviation, furthermore, G must earn π' from the takeover in any subgame-perfect equilibrium. Existence of a subgame perfect equilibrium follows from standard arguments. Since G must offer any target t at least $V_t^b + q_G^a \min_{t' \in \mathcal{N}^b \setminus \{t\}} \eta_{tt'}^b$, for G to earn π', it must take over a target t_1 from a pair t_1, t_2 that attains (11) at the price specified in the proposition.

<u>Part II</u>. We show that *under Assumption 1*, $(t_1, t_2) \in \{\arg \max_{t_1', t_2' \in \mathcal{N}^b} \alpha_{t_1'}^b - \eta_{t_1' t_2'}^b \}$ *implies* t_1 *and* t_2 *offer the best and second-best service* b, *respectively.* For any t_1 and t_2, we have

$$\alpha_{t_1}^b - \eta_{t_1 t_2}^b = q_{t_1 t_1}^b - q_{t_1 t_2}^b.$$

By Part I of Assumption 1, for any t_1 the right-hand side is larger for a better t_2. Fixing t_2, by Part II of Assumption 1, the right-hand side is greater for a better t_1. Hence, t_1 and t_2 must belong to the firms offering the two best services. Focusing on the best two firms, by Part III of Assumption 1, the right-hand side is maximized if t_1 is one of the best firms. $\quad\square$

Proof of Proposition 2. <u>Part I</u>. Suppose that firm $i \in \mathcal{N}^b$ plays the takeover game. Suppose i wants to take over a firm $j \in \mathcal{N}^a$. Because firm i can not direct consumers into market a, it has to pay firm j's standalone value for j to accept the offer. Upon taking over firm j, however, firm i can direct j's consumers in market a to its own product in market b, generating a default advantage α_i^b. Hence, firm i earns additional profits of $q_j^a \alpha_i^b > 0$ from taking over firm j. By definition, these profits are highest when taking over the market leader ℓ because $q_\ell^a \geq q_k^a$ for all $k \in \mathcal{N}^a$.

<u>Part II</u>. By Proposition 1, the market leader ℓ in market a earns additional profits of

$$q_\ell^a \left(\alpha_i^b + |\eta_{ii^*}^b| \right) > q_\ell^a \alpha_i^b$$

57

from taking over firm i in market b (where i^* is i's strongest rival). By definition, the market leader ℓ generates more profits from taking over i than any other firm $j \in \mathcal{N}^a$ does. $\qquad\square$

Proof of Proposition 3. Since all targets are identical, for any $t_1, t_2 \in \mathcal{N}^b$, we write η^b instead of $\eta^b_{t_1 t_2}$. Similarly, for any $t \in \mathcal{N}^b$, we write α^b and V^b instead of α^b_t and V^b_t. For any round of bidding, we distinguish between a *bidding subgame* in which all remaining acquirers decide what bid to submit, and an *acceptance subgame* in which the target that is currently up for sale decides whether to accept one of the bids. We will subsequently derive the equilibrium takeover price in the following three cases: (a) $n^b < |\mathcal{A}|$, (b) $n^b = |\mathcal{A}|$, and (c) $n^b > |\mathcal{A}|$.

<u>Case (a).</u> Let $n^b < |\mathcal{A}|$. We solve the game backwards and proceed by induction over the number of remaining targets.

Induction hypothesis: *Consider any bidding subgame with $k \geq 1$ remaining targets. Denote as $A(k) \subseteq \mathcal{A}$ the set of potential acquires in this subgame that have not yet taken over a target, and note that $|A(k)| \geq k+1$ because there are more potential acquirers than targets (i.e., $n^b < |\mathcal{A}|$). (In slight abuse of notation, we suppress the dependence of this subgame on the prior history, which determines the identity of potential acquirers. This is without loss of generality because these identities are fixed from this point onward.) Denote by $\ell_i \in A(k)$ the potential acquirer with the ith highest market share in market a among the firms in $A(k)$. Define as $\bar{A}(k, x) := \{\ell_1, \ldots, \ell_x\}$ the set of the x firms with the highest market shares in $A(k)$, and set $\bar{A}(k, 0) = \emptyset$. We hypothesize that all firms in $\bar{A}(k, k)$ take over one of the remaining targets at a price*

$$f^*(k) := V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right) + q_{\ell_{k+1}}^a \alpha^b. \tag{12}$$

Induction anchor: Consider any bidding subgame with a single target left (i.e., $k = 1$). We write $f^* = f^*(1)$. The induction hypothesis states that ℓ_1 *takes over the last target at a*

price of

$$f^* = V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus A(1)} q_i^a + q_{\ell_2}^a \alpha^b. \tag{13}$$

We verify the induction hypothesis in two steps.

1. Step: equilibrium construction. We construct a cautious equilibrium in which ℓ_1 takes over the last target at a price of f^*. Denote the target's adjusted standalone value by

$$\nu := V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus A(1)} q_i^a.$$

and the takeover revenue of a potential acquirer $j \in A(1)$ by

$$\pi_j := \nu + q_j^a \alpha^b.$$

We now argue that the target accepting any highest bid at or above ν (breaking ties randomly), firm ℓ_1 bidding f^*, firm ℓ_2 submitting a bid by uniformly randomizing over $[f^* - \epsilon, f^*]$, and any other firm $j \neq \ell_1, \ell_2$ bidding ν is a cautious equilibrium of this subgame for small enough ϵ.[35]

We first verify that the above is a subgame perfect equilibrium. If the target rejects the highest bid, it earns ν. Hence, it is optimal to accept any highest bid at or above ν.

Firm ℓ_1 has no incentive to bid more than f^* as then it would take over the target at a strictly higher price, and it does not want to bid less than $f^* - \epsilon$ as then it would lose and earn nothing in market b instead of $(q_{\ell_1}^a - q_{\ell_2}^a)\alpha^b > 0$. If firm ℓ_1 deviates to $f \in (f^* - \epsilon, f^*)$, it earns at most

$$\frac{f - (f^* - \epsilon)}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right). \tag{14}$$

At $f = f^*$, the profit in (14) equals $(q_{\ell_1}^a - q_{\ell_2}^a)\alpha^b$; that is, the candidate equilibrium profit. Moreover, the derivative of (14) with respect to f is

$$\frac{1}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right) - \left(1 - \frac{f^* - f}{\epsilon}\right) > \frac{1}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right) - 1 > \frac{(q_{\ell_1}^a - q_{\ell_2}^a)\alpha^b}{\epsilon} - 1 > 0,$$

---

[35] The equilibrium we construct is inspired by Blume (2003).

which holds for all $f \in (f^* - \epsilon, f^*)$ if ϵ is small enough. Hence, firm ℓ_1 has no incentive to deviate. In fact, for ϵ small enough bidding f^* is firm ℓ_1's strict best reply.

Finally, no firm $j \neq \ell_1$ has an incentive to bid above f^* — a bid the target would accept — as then they would earn negative profits from the takeover, and they are indifferent between all bids (weakly) below the takeover price because independently of the size of such a bid they will lose for certain and earn zero profits in market b. Hence, no firm $j \neq \ell_1$ has an incentive to deviate.

It remains to be shown that all firms play iteratively weakly undominated strategies in the reduced bidding game in which all acceptance subgames with a unique equilibrium payoff are replaced by the corresponding equilibrium payoffs. This follows from the lemma below.

Lemma 8 (Iteratively Weakly Undominated Strategies).

I. Bidding ν is iteratively weakly undominated for any potential acquirer.

II. For any potential acquirer $\ell_j \neq \ell_1$, bidding $f \in (\nu, \pi_{\ell_j})$ is iteratively weakly undominated, and for firm ℓ_1, bidding $f \in (\nu, \pi_{\ell_2})$ is iteratively weakly undominated.

III. For firm ℓ_2, any mixed strategy with support in $(\nu, \pi_{\ell_2}) = (\nu, f^*)$ and a continuous CDF is iteratively weakly undominated.

IV. For firm ℓ_1, bidding $\pi_{\ell_2} = f^*$ is iteratively weakly undominated.

Proof. <u>Preliminary</u>. Note that all acceptance subgames following *multiple* highest bids at or above ν do *not* have a unique equilibrium payoff, as the equilibrium payoffs depend on whom the target selects, and are thus identical in the original and the reduced game; that is, the reduced game is still an extensive-form game. In particular, for any j making a highest bid, the target accepting j's bid with probability 1 is an equilibrium of the acceptance subgame.

<u>Part I</u>. We argue that in the reduced bidding game, for any potential acquirer, it is iteratively weakly undominated to bid ν. We do so by induction over the rounds of elimination of weakly dominated strategies. Bidding ν is trivially undominated in the first round of elimination; for example, it is the strict best reply to every rival bidding strictly below

ν. Now suppose that ν is weakly undominated for all potential acquirers after k rounds of elimination, and consider round $k + 1$. For every potential acquirer j, bidding ν is a strict best response to all its rivals bidding ν and the target accepting j's offer with probability one. Hence, for every potential acquirer, bidding ν is weakly undominated in the $(k + 1)$th round of elimination. This proves the claim.

Part II. Again, we proceed by induction, starting with the potential acquirer $\ell_j \neq \ell_1$ that has the lowest demand in a. Suppose that after k rounds of elimination, bidding $f \in (\nu, \pi_{\ell_j})$ is iteratively weakly undominated for all $\ell_{j'}$ with $j' \leq j$, and note this is trivially true after zero rounds of elimination. We now argue that it then is also weakly undominated to bid $f \in (\nu, \pi_{\ell_j})$ after $k + 1$ rounds of elimination for all potential acquirers $\ell_{j'}$ with $j' \leq j$. By Part I, it is iteratively weakly undominated to bid ν for all acquirers, and from now on presume that all potential acquirers $\ell_{j''}$ with $j'' > j$ do so. For any potential acquirer $j' \leq j$, bidding f is a strict best response to all rivals with an index weakly below j bidding f and the target accepting j''s offer with probability one. Hence, for any j', bidding f is weakly undominated in the $(k + 1)$th round of elimination.

Part III. Suppose that firm ℓ_1 bids $f' \in [\nu, \pi_{\ell_2})$ and all firms $j \neq \ell_1, \ell_2$ bid ν, which is weakly undominated by Parts I and II. Now consider two (potentially) mixed strategies of firm ℓ_2 with support in (ν, π_{ℓ_2}), and denote the corresponding CDFs as G and H with $G \neq H$. Assume that G is continuous. When bidding according to G, firm ℓ_2's additional profits in market $\hat{S}$ are

$$\big(1 - G(f')\big)\big(\pi_{\ell_2} - \mathbb{E}_G[f | f \geq f']\big) = \big(1 - G(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \, dG(f).$$

Similarly, when bidding according to H, firm ℓ_2 earns additional profits of

$$\big(1 - H(f')\big)\big(\pi_{\ell_2} - \mathbb{E}_H[f | f \geq f']\big) = \big(1 - H(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \, dH(f).$$

For the sake of a contradiction, suppose that the (potentially) mixed strategy with CDF H weakly dominates the mixed strategy with CDF G. Then, it follows from the last two

equations that for any $f' \in [\nu, \pi_{\ell_2})$,

$$\big(G(f') - H(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \, dH(f) + \int_{f'}^{\pi_{\ell_2}} f \, dG(f) \geq 0. \tag{15}$$

Integration by parts implies that (15) is equivalent to

$$-\big(H(f') - G(f')\big)(\pi_{\ell_2} - f') + \int_{f'}^{\pi_{\ell_2}} H(f) - G(f) \, df \geq 0. \tag{16}$$

We derive a contradiction in three steps. First, evaluating (15) at $f' = \nu$, we conclude that

$$\mathbb{E}_H[f] \leq \mathbb{E}_G[f].$$

Hence, H cannot first-order stochastically dominate G, so that $G(f) < H(f)$ for some $f \in [\nu, \pi_{\ell_2})$. Define $\bar{f} := \sup\{\arg\sup H(f) - G(f)\}$, and notice that $H(\bar{f}) - G(\bar{f}) > 0$.

Second, because H is right-continuous and G is continuous, also $H - G$ is right-continuous. This in turn implies that the left-hand side of (16) is right-continuous. Hence, if $\bar{f} < \pi_{\ell_2}$,

$$-\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f) \, df \geq 0.$$

Third, to establish a contradiction, we distinguish the following two cases: $\bar{f} < \pi_{\ell_2}$ and $\bar{f} = \pi_{\ell_2}$. If $\bar{f} < \pi_{\ell_2}$, $H(f) - G(f)$ cannot be constant on $(\bar{f}, \pi_{\ell_2})$. Hence, there exists some $\hat{f} \in (\bar{f}, \pi_{\ell_2})$ and some $\zeta > 0$, such that $\sup_{f \in (\hat{f}, \pi_{\ell_2})} H(f) - G(f) < H(\bar{f}) - G(\bar{f}) - \zeta$. It thus follows that

$$-\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f) \, df$$
$$< -\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(\bar{f}) - G(\bar{f}) \, df - \int_{\hat{f}}^{\pi_{\ell_2}} \zeta \, df$$
$$= -\zeta(\pi_{\ell_2} - \hat{f})$$
$$< 0;$$

a contradiction. If $\bar{f} = \pi_{\ell_2}$, $H(f)$ must jump at $f = \pi_{\ell_2}$. Since H is right-continuous and does not first-order stochastically dominate G, and since G is continuous, there exists $f^s < \bar{f}$,

so that $H(f^s) = G(f^s)$ and $H(f) < G(f)$ for all $f \in (f^s, \pi_{\ell_2})$. At $f' = f^s$, the left-hand side of (15) is

$$\underbrace{(G(f^s) - H(f^s))}_{=0} \pi_{\ell_2} - \int_{f^s}^{\pi_{\ell_2}} f \, dH(f) + \int_{f^s}^{\pi_{\ell_2}} f \, dG(f) = \mathbb{E}_G[f|f \geq f^s] - \mathbb{E}_H[f|f \geq f^s] < 0,$$

where the inequality holds as H first-order stochastically dominates G on (f^s, π_{ℓ_2}); a contradiction.

<u>Part IV.</u> Suppose that all of ℓ_1's rivals bid according the candidate equilibrium strategies, which are weakly undominated by Parts I and III. Then, as we have argued above, bidding f^* is firm ℓ_1's strict best reply and thus weakly undominated. $\qquad\square$

2. Step: unique equilibrium outcome. We show that ℓ_1 takes over the last target at f^* in any cautious equilibrium. First, we argue that in any cautious equilibrium the infimum $\underline{f}$ of the highest equilibrium takeover bid is at least f^*. Suppose not, i.e. $\underline{f} < f^*$. We show that then both firms ℓ_1 and ℓ_2 bid weakly above $\underline{f}$ with probability one. If one of them does not, it must in equilibrium with positive probability submit bids below f^* that loose with probability one. This firm could then deviate and move this probability mass to a bid $(\underline{f} + f^*)/2$ and win with positive probability thereby earning positive expected profits. Furthermore, if the distribution of the highest equilibrium takeover bid has no mass point at $\underline{f}$, both ℓ_1 and ℓ_2 must bid above $\underline{f} + \epsilon'$ for some $\epsilon' > 0$ with probability one; for otherwise, the expected profits as $f \to \underline{f}$ from above approach zero while those of deviating and bidding $(\underline{f} + f^*)/2$ do not; a contradiction. This, however, contradicts that $\underline{f}$ is the infimum of the highest bid. Thus, with positive probability both ℓ_1 and ℓ_2 bid $\underline{f}$, and win when doing so. But since they earn strictly positive profits when winning at this price, either of them would be better of moving the probability mass from $\underline{f}$ to $\underline{f} + \epsilon''$ for some small enough $\epsilon'' > 0$. We conclude that the infimum of the highest equilibrium bid is at least f^*.

Second, we argue that the supremum of the highest equilibrium bid in a cautious equilibrium is f^*. Bidding at or above f^* is weakly dominated for all firms other than ℓ_1. Hence,

firms other than ℓ_1 must bid strictly below f^* with probability one. Thus, firm ℓ_1 wins with probability one when bidding f^*, and hence cannot bid strictly above f^* either. We conclude that the highest equilibrium bid must be f^* in any cautious equilibrium with probability one. Furthermore, it must be submitted by ℓ_1, and thus ℓ_1 wins with probability one in any cautious equilibrium.

Induction step: Suppose that the induction hypothesis holds for any bidding subgame with $k \geq 1$ remaining targets. Consider a bidding subgame with $k + 1$ remaining targets, and note that there are at least $k + 2$ remaining acquirers. Define $A(k + 1)$ and $\bar{A}(k + 1, x)$ analogously to the above.

The induction step follows in five steps: in steps one to four, we verify that the induction hypothesis must hold in any cautious equilibrium of a bidding subgame with $k + 1$ remaining targets; in step five, we then construct a cautious equilibrium of this bidding subgame.

1. Step: no firm $\ell_j \in A(k + 1)$ with $j \geq k + 2$ takes over the current target. Recall that we rank acquirers in terms of their market shares in market a, with a lower index j indicating a higher market share. Let ℓ_w be the firm buying target $k + 1$, and let $\pi_j(w)$ be firm j's additional subgame perfect equilibrium profit in the bidding subgame with k remaining targets as a function of w.

For the sake of a contradiction, suppose firm ℓ_j with $j \geq k + 2$ takes over the current target with positive probability. Then, by the induction hypothesis the remaining targets are acquired the firms in $\bar{A}(k + 1, k)$. Hence, ℓ_j's gross profits in market b are

$$\pi_j(j) = V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k)} q_i^a \right) + \alpha^b q_{\ell_j}^a.$$

Using the induction hypothesis, we construct a reduced game by replacing any continuation subgame following the current acceptance subgame (i.e. following the target's acceptance decision) by its unique equilibrium payoffs, and replacing all possible acceptance subgames in which the target is not indifferent (between accepting and rejecting some offer) by its unique equilibrium payoffs.

If ℓ_j does not take over the current target, it will not make a takeover; thus, its profits from the takeover market will be zero. If it takes over the current target, its profits will be $\pi_j(j)$. In the reduced game, thus, it is weakly dominated for firm ℓ_j to bid at or above $\pi_j(j)$. Hence, because ℓ_j takes over the current target with positive probability, and because it plays a weakly undominated strategy in any cautious equilibrium, the infimum $\underline{f}$ of the highest bid must be strictly below $\pi_j(j)$.

We next argue that any firm $\ell_{j'} \in \bar{A}(k+1,k)$ makes greater profits when taking over the current target at $\underline{f}$ than when not doing so, irrespective of which rival ℓ_w would take over the current target otherwise. To see why, let $f^* = f^*(k+1)$, and consider three cases. First, if $w \leq k$, by the induction hypothesis, any firm $\ell_{j'} \in \bar{A}(k+1,k)$ takes over a remaining target at a price

$$f^* = V^b - |\eta^b|\left(\sum_{i \in \mathcal{A} \backslash A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k)} q_i^a\right) + \alpha^b q_{\ell_{k+2}}^a \geq \pi_j(j),$$

and following the takeover in market b earns additional gross profits of

$$\pi_{j'}(w) = V^b - |\eta^b|\left(\sum_{i \in \mathcal{A} \backslash A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k+1) \backslash \{\ell_{j'}\}} q_i^a\right) + \alpha^b q_{\ell_{j'}}^a.$$

This implies additional net profits of $\alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > 0$. If instead firm $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} = \pi_{j'}(w) - \underline{f} > \pi_{j'}(w) - \pi_j(j) \geq \pi_{j'}(w) - f^* = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a),$$

where the first equality follows from the fact that ℓ_w would take over one of the remaining targets.

Second, if $w = k+1$, then by the induction hypothesis any firm $\ell_{j'} \in A(k+1,k)$ takes over a remaining target at a price

$$f' = V^b - |\eta^b|\left(\sum_{i \in \mathcal{A} \backslash A(k+1)} q_i^{a\hat{S}} + \sum_{i \in \bar{A}(k+1,k-1)} q_i^a + q_{\ell_{k+1}}^a\right) + \alpha^b q_{\ell_{k+2}}^a > f^*,$$

and earns additional gross profits of $\pi_{j'}(k+1) = \pi_{j'}(j')$. The additional net profits are $\pi_{j'}(j') - f'$. If $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* > \pi_{j'}(j') - f'.$$

Third, if $w \geq k+2$, then by the induction hypothesis a firm $\ell_{j'} \in A(k+1,k)$ takes over a remaining target at a price

$$f'' = V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \backslash A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k-1)} q_i^a + q_{\ell_w}^a \right) + \alpha^b q_{\ell_{k+1}}^a,$$

and earns additional gross profits of

$$\pi_{j'}(w) = V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \backslash A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k) \backslash \{\ell_{j'}\}} q_i^a + q_{\ell_w}^a \right) + \alpha^b q_{\ell_{j'}}^a.$$

and additional net profits of

$$\pi_{j'}(w) - f'' = \alpha^b \left(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a \right) + |\eta^b| \left(q_{\ell_{j'}}^a - q_{\ell_k}^a \right).$$

If $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a).$$

Hence, because

$$\alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > \alpha^b \left(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a \right) + |\eta^b| \left(q_{\ell_{j'}}^a - q_{\ell_k}^a \right),$$

firm $\ell_{j'}$ strictly prefers winning at $\underline{f}$ also in this last case.

Observe that there are at least two firms in $\bar{A}(k+1,k)$. Because any firm $\ell_{j'} \in \bar{A}(k+1,k)$ strictly prefers winning to loosing when bidding $\underline{f}$, by a similar argument than in the induction anchor (2. Step), it cannot be the case that two or more firms in $\bar{A}(k+1,k)$ have

66

a mass point at $\underline{f}$. Furthermore, if no rival has a mass point at $\underline{f}$, as the bid of a firm $\ell_{j'}$ approaches $\underline{f}$ from above its profits approach those of losing for certain. In that case, however, by a similar argument than in the induction anchor, there exists an $\epsilon' > 0$ such that firm $\ell_{j'}$ is strictly better of moving any probability mass from below $\underline{f} + \epsilon'$ to $\underline{f} + \epsilon'$. Hence, any firm in $\bar{A}(k + 1, k)$ has an incentive to bid strictly above $\underline{f}$ with probability one. In this case, however, firm $\ell_{j'}$ lowest bid is bounded below by $\underline{f} + \epsilon'$, contradicting that $\underline{f}$ is the infimum of the highest bid. Hence, a firm in $A(k + 1, k + 1)$ takes over the current target.

2. Step: the infimum $\underline{f}$ of the highest equilibrium bid for the current target is weakly above f^*. Suppose otherwise. By bidding in the interval $(\underline{f}, f^*)$, firm ℓ_{k+2} can earn positive profits. Hence, in equilibrium, firm ℓ_{k+2} must win with positive probability, contradicting the first step above.

3. Step: the supremum $\bar{f}$ of the highest equilibrium bid for the target currently on sale is weakly below f^*. Suppose otherwise. First, recall that bidding at or above f^* is weakly dominated for every firm ℓ_j with $j \geq k + 2$. Eliminating bids weakly above f^* for all $j \geq k + 2$, we now argue that it is weakly dominated to bid above f^* also for firm ℓ_{k+1}. To see why, notice that if firm ℓ_{k+1} bids above f^* it either wins or looses to a firm $j \leq k$. In either case, by the induction hypothesis, the remaining $k + 1$ targets are bought by the firms $\bar{A}(k + 1, k + 1)$. Furthermore, if ℓ_{k+1} looses, by the induction hypothesis, it pays f^*. Hence, when bidding f^* firm ℓ_{k+1} always earns $\alpha^b(q^a_{\ell_{k+1}} - q^a_{\ell_{k+2}})$, and bidding strictly above f^* is weakly dominated for firm ℓ_{k+1} in the reduced bidding game.

Now consider a firm that attains the supremum; that is, a firm that bids in $((\bar{f}+f^*)/2, \bar{f}]$ and wins with positive probability. If the firm moves the probability mass from $((\bar{f}+f^*)/2, \bar{f}]$ to $(\bar{f}+f^*)/2$ instead, it either wins at a lower price or looses to another firm in $\bar{A}(k + 1, k)$ and, by the induction hypothesis, takes over another target at f^*. In either case it earns higher profits, and hence has an incentive to deviate; a contradiction.

4. Step: firm ℓ_{k+1} cannot take over the current target with positive probability. Suppose

it does. Then, it must bid f^* with positive probability, since $\underline{f} = \bar{f} = f^*$. By the induction hypothesis, if ℓ_{k+1} takes over the current target, in the following subgame the firms in $\bar{A}(k+1, k)$ take over the remaining targets at a price of $f' > f^*$, and hence earn strictly less than when taking over the current target at f^*. But if a firm in $\bar{A}(k+1, k)$ deviates and bids $f^* + \epsilon'$ for any $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small $\epsilon' > 0$.

5. *Step: equilibrium construction.* We sketch a cautious equilibrium of the bidding subgame with $k+1$ remaining targets in which all firms in $\bar{A}(k+1, k+1)$ take over a target at f^*. Using the induction hypothesis, following the current round of bidding a cautious equilibrium with the desired properties exist for any bidding subgame with k remaining targets. We thus specify behavior only in the current round of bidding. By an argument analogous to that in the induction anchor (1. Step and in particular, Lemma 8), the current target accepting any highest bid above

$$ V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right) $$

with probability one (breaking ties randomly), every firm in $\bar{A}(k+1, k)$ bidding f^*, firm ℓ_{k+1} uniformly randomzing over $(f^* - \epsilon, f^*)$ for small enough $\epsilon > 0$, and any other firm j bidding

$$ V^b - |\eta^b| \left(\sum_{i \in \mathcal{A} \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right) $$

is a weakly undominated equilibrium of the reduced game in which all subgames with a unique equilibrium outcome are replaced by the corresponding equilibrium payoffs. Hence, it is part of a cautious equilibrium of this subgame.

Finally, we argue that $f^* < V^b$. Let $\underline{q} := \max_{j \in \mathcal{A} \setminus \bar{A}(n^b-1)} q_j^a$. By definition, for any $i \in \bar{A}(n^b - 1)$, we have $q_i^a > \underline{q}$. Thus, because all targets are symmetric, we have

$$ f^* = V^b - |\eta^b| \sum_{i \in \bar{A}(n^b-1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{A}(n^b-1)} q_j^a \; < \; V^b - \underline{q} \underbrace{\left(|\eta^b|(n^b - 1) - \alpha^b \right)}_{= \, 0 \text{ by Part III of Lemma 1}} \; = \; V^b. $$

<u>Case (b)</u>. Let $n^b = |\mathcal{A}|$. We solve the game backwards. Consider first any subgame with one potential acquirer j and one target, which implies that all other firms in $\mathcal{A}$ have taken over another target before. Since j makes a take-it-or-leave-it offer, it must offer the amount at which the target is indifferent between accepting and rejecting, and since there is a strictly positive gain of trade the target must accept in equilibrium. Hence, the takeover price is

$$V^b - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus \{j\}} q_i^a.$$

We proceed by induction over the number of remaining targets, focusing on subgames in which there are as many targets as potential acquirers. (Notice that we can use Case (a) to deal with subgames in which there are more acquirers than targets.)

Induction hypothesis: *Consider any subgame in which there are $k \geq 2$ potential acquirers and k targets left. Let ℓ_k be the potential acquirer with the lowest market share in market a among the k remaining potential acquirers. Then, in any cautious equilibrium of this subgame, a potential acquirer other than ℓ_k takes over the current target at a price of*

$$f^*(k) := V^b - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

Induction anchor: Consider any subgame with two potential acquirers ℓ_1 and ℓ_2 and two targets. Let $q_{\ell_1}^a > q_{\ell_2}^a$. We have to show that ℓ_1 takes over the current target at a price $f^* = f^*(2)$.

First, we observe that if the current target is not taken over, then by Case (a), firm ℓ_1 takes over the remaining target in the next round. Anticipating this takeover, the current target's anticipated standalone value is f^*. Hence, the current target accepts any highest bid strictly above f^*, and rejects any highest bid strictly below f^*; at f^* the current target is indifferent.

Second, we show that the infimum $\underline{f}$ of the highest equilibrium bid is at least f^*. Suppose not. Then, by our first observation, with positive probability, the current target is not taken

over. Anticipating that ℓ_1 would take over the last remaining target in the following subgame, ℓ_2 could earn net profits of $q_{\ell_2}^a \alpha^b$ from taking over the current target at f^*. Hence, for small enough $\epsilon' > 0$, shifting probability mass from $(\underline{f}, f^*)$ to $f^* + \epsilon'$ is a profitable deviation for ℓ_2; a contradiction.

Third, we argue that in any cautious equilibrium the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose otherwise. Because the target accepts all bids above f^*, both ℓ_1 and ℓ_2 must attain $\bar{f}$ for otherwise the firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. To establish a contradiction, we observe that for firm ℓ_2 it is weakly dominated to assign positive mass to bids in $((f^*+\bar{f})/2, \bar{f}]$. If ℓ_1 submits a bid weakly below $(f^*+\bar{f})/2$, then firm ℓ_2 is strictly better of bidding $(f^*+\bar{f})/2$ with probability one, as this way it pays a lower price, both when losing and taking over the last target at f^* or when winning at this price. If firm ℓ_1 submits a bid above $(f^*+\bar{f})/2$, firm ℓ_2 is (weakly) better of bidding $(f^*+\bar{f})/2$ with probability one and losing for certain, in which case it takes over the remaining target at f^* in the following subgame. Hence, assigning positive mass to bids in $((f^*+\bar{f})/2, \bar{f}]$ is weakly dominated for ℓ_2 in the reduced bidding game; a contradiction.

Fourth, we show that firm ℓ_1 takes over the current target with probability 1. Note that firm ℓ_2 cannot take over the current target with positive probability. Suppose it does. Then, it must bid f^* with positive probability. If ℓ_2 takes over the current target, in the following subgame ℓ_1 takes over the remaining target at a price of $f' > f^*$, and hence earns strictly less than when taking over the current target at f^*. But if ℓ_1 deviates and bids $f^* + \epsilon'$ for any $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small $\epsilon' > 0$. Similarly, the target must accept a bid of f^* by firm ℓ_1 with probability 1, for otherwise ℓ_1 could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. Since if ℓ_2 takes over the current target ℓ_1 pays a strictly higher price in the next round, similar arguments as in Case (a) imply that firm ℓ_1 bidding f^* and firm ℓ_2 uniformly randomizing over $(f^* - \epsilon, f^*)$

is part of a cautious equilibrium of this subgame for small enough $\epsilon > 0$.

Induction step: Suppose the induction hypothesis holds for any subgame with $k \geq 2$ remaining targets and potential acquirers. Consider a subgame with $k + 1$ remaining targets and acquirers. As before, we denote $f^* = f^*(k + 1)$. We follow the same five steps as in the induction anchor.

First, we observe that if the current target is not taken over, by Case (a), all firms except for ℓ_{k+1} take over one of the remaining targets in the following rounds. Anticipating this, the current target's anticipated standalone value is f^*. The current target thus accepts any highest bid strictly above f^*, and rejects any highest bid strictly below f^*; at f^* the current target is indifferent.

Second, we show that the infimum $\underline{f}$ of the highest equilibrium bid is at least f^*. Suppose not. Then, with positive probability, the current target is not taken over. Anticipating that it would not make a takeover otherwise, firm ℓ_{k+1} could earn net profits of $q^a_{\ell_{k+1}} \alpha^b$ from taking over the current target at f^*. Hence, for small enough $\epsilon' > 0$ shifting probability mass from $(\underline{f}, f^*)$ to $f^* + \epsilon'$ is a profitable deviation for ℓ_{k+1}; a contradiction.

Third, we argue that in any cautious equilibrium the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose not. Because the target accepts all bids above f^*, at least two firms must attain $\bar{f}$ for otherwise the only firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. Notice that, by the same argument as in the induction anchor, assigning positive probability mass to bids above $(f^* + \bar{f})/2$ is weakly dominated for ℓ_{k+1}, so it cannot be ℓ_{k+1} that attains $\bar{f}$. Now consider two firms ℓ_i and ℓ_j with $i, j \neq k + 1$ that attain $\bar{f}$. Both firms have an incentive to shift probability mass from $((f^* + \bar{f})/2, \bar{f}]$ to $(f^* + \bar{f})/2$ because they either win at a lower price or — by the induction hypothesis — take over one the remaining targets at f^* in the following rounds; a contradiction.

Fourth, we show that the current target is taken over with probability 1 by a firm other than ℓ_{k+1}. Note that ℓ_{k+1} cannot take over the current target with positive probability.

71

Suppose it does. Then, it must bid f^* with positive probability. If ℓ_{k+1} takes over the current target, in the following subgame the remaining acquirers take over the remaining targets at a price of $f' > f^*$, and hence earn strictly less than when taking over the current one at f^*. If either of these firms deviates and bids $f^* + \epsilon'$ for any $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small $\epsilon' > 0$; a contradiction. Similarly, the target must accept an equilibrium bid of f^* with probability 1, for otherwise a firm other than ℓ_{k+1} could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. By similar arguments as in Case (a), firm $\ell_1, \ldots, \ell_k$ bidding f^* and firm ℓ_{k+1} uniformly randomizing over $(f^* - \epsilon, f^*)$ is part of a cautious equilibrium of this subgame for small enough $\epsilon > 0$.

<u>Case (c)</u>. Let $n^b > |\mathcal{A}|$. We solve the game backwards, and begin by solving subgames with one more acquirer than targets. (Other relevant subgames are captured by (a) and (b).)

Induction hypothesis: *Consider a subgame with $k \geq 1$ remaining potential acquirers and $k + 1$ remaining targets. The current target is taken over at a price of*

$$f^* := V^b - |\eta^b| \sum_{i \in \mathcal{A}} q_i^a.$$

Induction anchor: Let $k = 1$, so that there are one acquirer ℓ and two targets left. If the current target is not taken over, by Case (b), ℓ takes over the last remaining target at a price

$$f' = V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{\ell\}} q_i^a.$$

The current target's profits in that case are f^*. Thus the target must reject any bid strictly less than f^*, and accept any bid strictly above f^*. Hence, firm ℓ has a strict incentive to lower any bid strictly above f^*. For the sake of a contradiction, suppose the target rejects f^*. Then, ℓ pays $f' > f^*$ for the takeover, and thus has an incentive to deviate by bidding $(f^*+f')/2$, a bid that the current target accepts. Hence, ℓ takes over the current target at f^*.

Induction step: Suppose that the induction hypothesis holds for k remaining acquirers and remaining $k + 1$ targets. Now consider a subgame with $k + 1$ remaining acquirers and $k + 2$ targets. Let ℓ_k be the remaining acquirer with the lowest market share in market a.

First, if the current target is not taken over, by Case (b), all remaining targets are bought at

$$f'' = V^b - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

The current target's profits in that case are f^*. Thus, the current target must reject any bid strictly less than f^*, and accept any bid strictly above f^*; at f^* the current target is indifferent.

Second, we argue that the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose not. Since the target accepts all bids above f^*, at least two firms must attain $\bar{f}$ for otherwise the only firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. Either firm has an incentive to shift probability mass from $({(f^* + \bar{f})}/{2}, \bar{f}]$ to ${(f^* + \bar{f})}/{2}$ since they either win at a lower price or by the induction hypothesis, take over a remaining target at f^* in the following rounds; a contradiction.

Third, we show that the target accepts a bid of f^*. Suppose otherwise. Then, by Case (b), all potential acquirers take over one of the remaining targets at a price f''. Hence, for small enough $\epsilon' > 0$, by bidding $f^* + \epsilon'$ for the current target, each potential acquirer could make a takeover at a lower price; a contradiction.

Fourth, by an argument similar to Lemma 8 (Part I), bidding f^* is weakly undominated for every potential acquirer. Hence, every potential acquirer bidding f^* is part of a cautious equilibrium.

We finally argue that any acquirer takes over a target at price f^* in any subgame with $k \geq 1$ remaining potential acquirers and $k + x$ remaining targets for any $x \geq 2$. First, we note that no target would sell strictly below f^* because even if all acquirers buy a rival it earns f^* upon rejecting an offer. Hence, no target can be bought at a price strictly below f^*. Second,

we observe that no acquirer would bid more than f^* because such an acquirer could deviate and not bid until a subgame is reached in which there are k' acquirers and $k'+1$ targets left. In such a subgame, it can take over a target at f^*. Hence, in equilibrium an acquirer takes over a target at f^*. $\qquad\square$

B.3 Access-Point Markets

Proof of Proposition 4. Let firm 1 offer the best service a. We first observe that for every firm $j \neq 1$, bidding strictly above Δ^a_{j1} is weakly dominated. By Part I of Assumption 1,

$$\Delta^a_{jj'} = q^a_{jj} - q^a_{jj'} \leq q^a_{jj} - q^a_{j1} = \Delta^a_{j1}.$$

To see that bidding $b > \Delta^a_{j1}$ is weakly dominated by bidding Δ^a_{j1}, we consider three cases. First, in case the highest rival bid is strictly above b or below Δ^a_{j1}, both bids obtain the same payoff. Second, in case the highest rival bid lies in $(\Delta^a_{j1}, b]$, bidding Δ^a_{j1} yields a strictly higher payoff because j strictly prefers to lose the auction at any price strictly above Δ^a_{j1}, no matter who obtains the default. Third, in case the highest bid is Δ^a_{j1}, bidding Δ^a_{j1} yields a strictly higher payoff if the highest rival bid is made by a firm other than 1 and the same payoff when the highest rival bid is made by firm 1. This also establishes that if firm 1 wins the auction, it pays at most $\max_{j\neq 1} \Delta^a_{j1}$.

Now suppose that, for the sake of a contradiction, a firm $j \neq 1$ that does not offer the best service in market a, wins with positive probability. Hence, there exists some interval of bids $(\underline{b}, \overline{b}]$ in which firm j bids with positive probability and conditional on doing so wins the auction. Since bidding above Δ^a_{j1} is weakly dominated for firm j, we must have

$$\overline{b} \leq \Delta^a_{j1} = q^a_{jj} - q^a_{j1} < q^a_{11} - q^a_{1j} = \Delta^a_{1j}, \tag{17}$$

where the strict inequality follows from Part III of Assumption 1. Because firm j wins with positive probability either firm 1 bids below $\overline{b}$ with positive probability or both firms have a mass point at $\overline{b}$ in their bid functions. We next argue that in either case, firm 1 can profitably

74

move the probability mass in its bid function from weakly below $\bar{b}$ to Δ_{1j}^a. In case firm 1 would have won the auction with its original bid this does not affect the allocation and hence does not affect firm 1's payoff. Similarly, if firm 1 still loses the auction when bidding Δ_{1j}^a the allocation is unaffected. If, however, firm 1 wins the auction, it must replace some other bidder j' (not necessarily equal to j). Because also j' must bid below $\Delta_{j'1}^a$, we have that the winning bid firm 1 replaces satisfies

$$b' \leq \Delta_{j'1}^a < \Delta_{1j'}^a. \tag{18}$$

Conditional on replacing the winning bid b', firm 1 thus gains at least $\Delta_{1j'}^a - \Delta_{j'1}^a > 0$. Finally, because prior to the deviation winning bids fall in the interval $(\underline{b}, \bar{b}]$ with positive probability, the latter happens with positive probability. Hence, firm 1 has a strictly profitable deviation; a contradiction. Hence, a firm offering the best service a wins with probability 1. $\quad\square$

Proof of Proposition 5. It is well-known that with two bidders $i \in \{G, M\}$, it is weakly dominant for i to bid its value (for the default). Hence, to prove the claim, we simply calculate these values.

$\underline{\text{Part I}}$. Suppose firm M is a single-market firm. Hence, firm M's willingness to pay to replace G as the default is Δ_{MG}^a. When firm G replaces M as the default in market a, it earns additional profits of Δ_{GM}^a in that market and it increase the probability of being the default in market b by Δ_{GM}^a. Hence, G values the default position in market a at $\Delta_{GM}^a(1 + \alpha_G^b)$. The claim follows.

$\underline{\text{Part II}}$. Suppose M is an ecosystem. We start by deriving G's willingness to pay to replace M as the default in market a. When firm G is the default in market a, it makes profits of

$$q_{GG}^a + q_{GG}^a q_{GG}^b + q_{MG}^a q_{GM}^b + (1 - q_{GG}^a - q_{MG}^a)\frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b = q_{GG}^a + q_{GG}^a \alpha_G^b + q_{MG}^a \eta_{GM}^b + \frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b.$$

When firm M is the default in market a, firm G earns

$$q^a_{GM}+q^a_{GM}q^b_{GG}+q^a_{MM}q^b_{GM}+(1-q^a_{GM}-q^a_{MM})\frac{1}{n^b}\sum_{\ell\in\mathcal{N}^b}q^b_{G\ell} = q^a_{GM}+q^a_{GM}\alpha^b_G+q^a_{MM}\eta^b_{GM}+\frac{1}{n^b}\sum_{\ell\in\mathcal{N}^b}q^b_{G\ell}.$$

Using $q^a_{GG}-q^a_{GM}=\Delta^a_{GM}$ and $-(q^a_{MM}-q^a_{MG})=-\Delta^a_{MG}$, we obtain

$$\text{WTP}_{GM} = \Delta^a_{GM}(1+\alpha^b_G)-\Delta^a_{MG}\eta^b_{GM} = \Delta^a_{GM}(1+\alpha^b_G)+\Delta^a_{MG}|\eta^b_{GM}|,$$

where the last equality follows from the fact that $\eta^b_{GM}<0$. Similarly, because $\eta^b_{MG}<0$,

$$\text{WTP}_{MG} = \Delta^a_{MG}(1+\alpha^b_M)+\Delta^a_{GM}|\eta^b_{MG}|.$$

The claims follows by observing that

$$\text{WTP}_{GM} > \text{WTP}_{MG} \quad \text{if and only if} \quad \Delta^a_{GM}(1+\alpha^b_G+\eta^b_{MG}) > \Delta^a_{MG}(1+\alpha^b_M+\eta^b_{GM}). \qquad \square$$