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ABSTRACT

Informal Incentives and Labor Markets*

This paper investigates how labor-market tightness affects market outcomes if firms use informal, self-enforcing, agreements to motivate workers. We characterize profit-maximizing equilibria and show that an increase in the supply of homogenous workers can increase wages. Moreover, even though all workers are identical in terms of skills or productivity, profit-maximizing discrimination equilibria exist. There, a group of majority workers are paid higher wages than a group of minority workers, who may even be completely excluded. Minimum wages can reduce such discrimination and increase employment. We discuss how these results relate to empirical evidence on downward wage rigidity, immigration, the gender pay gap, and credentialism.

JEL Classification: D21, D86, J21, J38, J61, J71

Keywords: informal incentives, labor supply, immigration, wage discrimination, minimum wage

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1 Introduction

Incentivizing workers is an important determinant of a firm's success. As it can be difficult to objectively assess workers' contribution to firm value, informal and self-enforcing agreements are often used to motivate them (Ahammer et al., 2023; Fahn et al., 2023; Macchiavello and Morjaria, 2023). Then, firms also need incentives to comply with such an agreement, which is particularly challenging if replacing a worker is not difficult. Therefore, by determining available alternatives, the tightness of labor markets affects not only workers' but also firms' incentives. Whereas the efficiency wage literature has improved our understanding of workers' incentives to exert effort, the role of labor market tightness for firms' incentives to compensate workers as promised is less well understood.

This paper studies how labor-market tightness shapes the design of informal incentive systems and the optimal wage setting of firms. In particular, we take into account that a higher labor supply increases a firm's chances to fill a vacancy, and consequently its temptation to replace a worker instead of compensating him for his effort. Then, firms may find it optimal to endogenously increase the cost of turnover by paying newly-hired workers a rent. The resulting higher commitment allows firms to provide stronger incentives and thus can raise profits. Investigating the interaction between labor market tightness and turnover costs, we explore conditions under which a higher supply of homogenous workers can *increase* compensation, a result that differs from both the standard competitive model of a labor market as well as efficiency wage models. We also demonstrate that discrimination against or the exclusion of one group of workers can be consistent with profit maximization, even though all workers are identical in terms of skills or productivity. In such an equilibrium, an appropriate minimum wage can reduce discrimination on labor markets *and* generate positive employment effects.

These results are based on the channel that firms "voluntarily" pay a rent to workers if a vacancy can readily be filled. We argue that this equilibrium outcome is preserved by a *social norm* determining which group of market participants — for example, firms or male/native workers — benefits most from informal incentives. Thereby, we follow Greif (1994), MacLeod and Malcomson (1998), or Ghosh and Ray (2023), who suggest that social norms can serve as a selection device in settings where multiple equilibria exist. Indeed, Lemieux et al. (2012) and Breza et al. (2021) report that social norms may sustain high wages which are above the market-clearing level.

Setup: Our analysis is based on an infinite-horizon model of an industry with many workers and firms. We build upon MacLeod and Malcomson (1998) and extend their model by introducing a general matching friction on the labor market and allowing for continuous (instead of binary) effort. In every period, each firm can employ exactly one worker. We assume that a firm with a vacancy is randomly matched with an unemployed worker with probability α^F, which increases in the extent of unemployed workers and decreases in the extent of open vacancies. With probability $1 - \alpha^F$, the vacancy remains open until the next period. If a firm is matched with an unemployed worker, the firm makes a take-it-or-leave-it offer which contains an upfront wage and a discretionary bonus potentially paid after a worker's effort choice.

Informal Incentives: Effort increases the firm's revenues but is costly for workers. Although formal (i.e., court-enforceable) incentive contracts are not feasible, a worker's effort is observable to his employer. Given this, a firm must use a relational contract to motivate a worker, in which not only the worker has to be incentivized to exert effort, but also the firm has to compensate the worker as promised (i.e., a contract must be self-enforcing).

Labor Market Tightness and Turnover Costs: A firm which reneges on a promised payment is punished by the employed worker who subsequently does not exert effort anymore. Still, a firm can replace a worker after reneging and start a new employment relationship.[1] In this case, a firm can make a credible promise only if such turnover is sufficiently costly. Because a vacancy causes a production

[1] We exclude multilateral punishments as in Levin (2002) by considering a setting in which deviations cannot be observed by non-involved parties.

loss, a lower probability of filling a vacancy α^F increases the cost of turnover. Conversely, when α^F is high, the temptation to start a new relationship is large and (ceteris paribus) reduces the willingness to compensate for effort. Then, it can be optimal for a firm to pay a rent to newly employed workers – to *endogenously* increase turnover costs and thereby its own incentives to reward a worker. Inspired by Kandori (1992), Greif (1994), and MacLeod and Malcomson (1998), we suggest that such an equilibrium is selected and sustained by a social norm that lets new workers who are offered a lower wage exert zero effort even if it exceeds their reservation wage. The resulting increased commitment of firms allows for higher equilibrium effort and lets employed workers' payoffs be strictly positive. Therefore, different from approaches with one principal and one agent such as MacLeod and Malcomson (1989) or Levin (2003) where each player's reservation payoff is exogenously given, equilibrium transfers can affect the relationship surplus. Nevertheless, the increased commitment comes with a cost because a firm needs to pay a rent even to a worker whose predecessor has left for exogenous reasons. The optimal level of total turnover costs balances a firm's commitment with such equilibrium costs, in which case equilibrium effort is below the first best.

The Effect of Higher Labor Supply on Wages: A higher labor supply increases α^F and makes it easier for firms to fill a vacancy. This generates two countervailing effects if α^F is high. On the one hand, because of the self-enforcing nature of contracts, each firm has an incentive to increase compensation to keep the total turnover cost (and consequently equilibrium effort) constant. On the other hand, a higher labor supply reduces the chances of unemployed workers to find a job, which in turn lowers a worker's outside option. The latter "efficiency-wage" mechanism puts *downward* pressure on workers' compensation, so the total effect of an increased labor supply is ambiguous. We show that it is positive if α^F is not too high and the amount of firms is fixed. Furthermore, if the amount of firms is endogenously determined by a zero-profit condition (subject to an entry cost), the resulting entry or exit of firms keeps workers' outside options constant. Then, the effect of a higher labor supply on the compensation and utility of employed workers is unambiguously positive (neutral) if the matching function exhibits increasing (constant) returns to scale. As most evidence points towards either increasing or constant returns to scale matching functions (Petrongolo and Pissarides, 2001), our results imply non-negative wage effects of a higher labor supply if firm entry is endogenous.

For lower values of α^F, workers are not paid a rent because the commitment provided by the low chances of filling a vacancy is sufficient. Effort is below the first best for intermediate values of α^F, and at the first best for low values of α^F. In the former case and with a fixed amount of firms, a higher labor supply increases each firm's profits but reduces equilibrium levels of effort and compensation. Allowing for firm entry, however, effort and compensation are pushed up to their original levels. In the latter case, effort and compensation are unaffected by α^F, but higher profits due to a lower turnover cost yield firm entry.

We argue that our result that an increase in the labor supply may not reduce (and even increase) compensation is consistent with empirical observations that we discuss in Section 5. For example, wages have been found to be downward rigid even if a recession increases the supply of labor, and a widespread rationale is a negative effect on worker morale (Campbell and Kamlani, 1997; Bewley 1998; 1999). While previous theoretical explanations for this link directly assume that wage reductions hurt worker morale, we show that it can be the consequence of a relational contract as analyzed in this paper. Our argument is further strengthened by the negative effect on worker morale only being observed for non-verifiable effort dimensions and in long-term employment relationships, as well as for new hires.

Moreover, an abundance of evidence beginning with Card (1990) has found that immigration does not necessarily worsen the labor market conditions for native workers. Although recent studies mostly focus on heterogeneity in worker skills, there is evidence that immigration can benefit native workers even when they work on the same kinds of, mostly low-skilled, tasks. Different from these studies, our mechanism also builds upon firms having wage-setting power, which is in line with recent evidence (Manning, 2003; Dube et al., 2020; Manning, 2021), as well as the discussion by Card (2022) who emphasizes the importance of studying optimal wage setting of firms in labor markets. On top, we present evidence for non-negative wage effects of immigration in the informal sector in which relational contracts seem to be particularly relevant.

Discrimination Equilibrium and Minimum Wages: In Section 4, we show that labor-market discrimination can be sustained in profit-maximizing equilibria. We derive a discrimination equilibrium in which a majority group of workers (called "insiders") is treated better than a minority group of workers (called "outsiders"), although both groups of workers only differ in a payoff- and productivity-irrelevant label. The discrimination equilibrium exists because only the *expected* rent paid to a new worker determines a firm's commitment, and it is not important how this rent is allocated among different identities. Therefore, it is possible that the rent only benefits insiders (provided the share of outsiders is not too large). We argue that such a social outcome — favoring one dominant group (e.g., male or native workers) over another, smaller, group (e.g., female or immigrant workers) — is potentially sustained by a discriminatory social norm (Onuchic, 2023).

We then show that a carefully chosen minimum wage can reduce discrimination. If a minimum wage is set between the original wage levels of insiders and outsiders, the latter benefit whereas the former are worse off. Interestingly, such a minimum wage also increases employment if we allow for firm entry/exit, because it reduces insiders' outside options and thus has a positive effect on firm profits.

We also briefly discuss a different kind of discrimination, namely that one group of workers is completely excluded. Thereby, firms can collectively increase their profits by effectively reducing α^F. This may be optimal because positive wage effects of a higher labor supply are equivalent to higher worker outside options and therefore smaller industry-wide profits (on the level of an individual firm, a reduction of α^F would not increase profits because workers' outside options are taken as given). A potential way to implement such an outcome is to require higher formal degrees for jobs than warranted by the desired skill set. We present evidence for such "credentialism," which, in particular, has been identified for jobs where relational contracts are relevant.

Evidence and Discussion: In Section 5, we provide a more detailed discussion of evidence on informal incentives, downward wage rigidity, non-negative wage effects of a higher labor supply, and discrimination in labor markets. In Section 6, we conclude with discussing the robustness of our model when allowing for positive worker bargaining power as well as other forms of endogenous turnover costs. The proofs of main results are in the Appendix. All other proofs and further numerical examples are in the Online Appendix.

Related Theoretical Literature

The standard model of the competitive labor market involves homogeneous-skill workers and no incentive problems between firms and workers; as labor supply goes up, the equilibrium wage goes down (or stays constant after capital has been adjusted). Efficiency-wage models of the labor market acknowledge the need to incentivize workers and assume that this is obtained by a combination of wages above the market-clearing level and a firing threat (Shapiro and Stiglitz, 1984; Yellen, 1984; MacLeod et al., 1994). There, a higher labor supply reduces workers' payoffs once they become unemployed, thus motivates them to work harder, and firms decrease wages in response. Hence, this "efficiency-wage effect" predicts that a higher labor supply *reduces* equilibrium wages. Incorporating a general and smooth labor-market friction α^F, we highlight that a higher labor supply can reduce a firm's credibility when making promises, to which they might optimally respond by *increasing* workers' compensation.

MacLeod and Malcomson (1998) take into account that incentives to workers are often informal and performance pay (such as bonuses) might be used to provide incentives. If firms are on the short side of the market, standard performance pay is not possible because firms would fire and replace workers when supposed to pay a bonus. In this case, firms pay workers a rent to motivate them, which is costly because such a rent has to be paid to new workers as well. Their mechanism involving endogenous turnover costs also appears in our model. Because the labor market in MacLeod and Malcomson (1998) is frictionless (firms can fill a vacancy with probability one if there is unemployment), however, a higher labor supply either reduces or has no effect on wages.

Yang (2008) extends the setting of MacLeod and Malcomson (1998) by assuming that turnover is costly. He demonstrates that higher (exogenous) turnover costs reduce total wage payments and unemployment. Fahn (2017) assumes that firms and workers bargain about the terms of the employment relationship. Workers' incentives increase in their bargaining power, thus a minimum wage can increase effort and consequently the efficiency of employment relationships.

We also contribute to the theoretical literature on labor market discrimination, i.e., pay differences that are not fully accounted for by productivity differences. Notably, different from taste-based or belief-based discrimination, discrimination in our model arises as a profit-maximizing equilibrium. This result can be categorized as a "discriminatory social norm" (see Onuchic, 2023, who provides an excellent survey covering recent theoretical contributions to the discrimination literature). Starting with the seminal work by Kandori (1992), a number of studies such as Eeckhout (2006) or Peski and Szentes (2013) have investigated discrimination or segregation among homogeneous players (except for observable, non payoff-relevant, characteristics) in repeated interactions. To the best of our knowledge, our mechanism – where endogenous (expected) turnover costs take the form of rents from which some workers get more than others – is new to the literature. By this mechanism, minority members may be hired at worse terms than the majority group in a profit-maximizing equilibrium. Moreover, several features of our equilibrium can be tested, e.g., outsiders work harder than insiders and the equilibrium social norm cannot be sustained as the amount of outsiders becomes large.

2 Model

Setup There are a mass $F > 0$ of firms and a mass $N > 0$ of workers. We first assume that F is exogenously given; we endogenize it in Section 3.3. All workers and firms are risk neutral. There are infinitely many periods $t = 1, 2, ...$, and all players have a common discount factor $\delta \in (0, 1)$.

Workers and firms either are part of a match or not, and each firm can employ exactly one worker. At the beginning of every period, unmatched players enter the labor market.

To describe the matching process, we use a general matching function $m(f_t, n_t)$ that indicates the mass of new jobs created in period t, where f_t is the mass of firm vacancies and n_t is the mass of unemployed workers in period t. Assume that $m(f_t, n_t)$ is strictly increasing in each argument and its second partial derivatives are strictly negative. We also assume that $m(0, n) = m(f, 0) = 0$, $\lim_{n\to\infty} m(f, n) = f$, and $\lim_{f\to\infty} m(f, n) = n$; these assumptions imply $m(f, n) < \min\{f, n\}$, thus indicate labor market frictions. In specific examples of the matching function we analyze below, $m(f, n)$ exhibits either constant or increasing returns to scale, for which there is substantial evidence as summarized by Petrongolo and Pissarides (2001).

In period t, each unmatched firm can fill a vacant job with probability $\alpha_t^F \equiv m(f_t, n_t)/f_t$, and an unemployed worker finds a job with probability $\alpha_t^N \equiv m(f_t, n_t)/n_t$. Once matched, each firm i can make a take-it-or-leave-it (TIOLI) offer to its worker.[2] Formally, the offer made by firm i consists of an upfront wage $w_t^i \in \mathbb{R}$ and the promise to pay a discretionary bonus $b_t^i \in \mathbb{R}$. If a worker rejects the offer, he receives his (exogenous) outside option of zero, the match separates, and the firm and worker can re-enter the matching market in the subsequent period. If a worker accepts the offer, he receives w_t^i. Then, the worker exerts effort $e_t^i \in \mathbb{R}_+$ incurring effort costs $c(e_t^i) = (e_t^i)^2/2$. After observing the worker's effort, firm i decides whether to pay a discretionary bonus b_t^i. Then, workers and firms simultaneously decide whether to

[2]This is motivated by the evidence that firms may have considerable wage-setting power even in thick labor markets (Manning, 2021, Card, 2022). We discuss general levels of bargaining power between firms and workers in Section 6.

leave the current match or not, and the match is separated if one (or both) of them chooses to leave. All workers and firms who are not part of a match re-enter the labor market. At the end of a period, each worker (whether part of a match or not) leaves the market with exogenous probability $(1-\gamma)$, after which his utility is zero. To keep the size of the labor force constant, we assume that $(1-\gamma)N$ new workers enter the labor market at the beginning of every period. The timing within a period is summarized in the following graph:

The effort of firm i's worker, e_t^i, generates firm i's revenue $e_t^i\theta$, where $\theta > 0$. Note that if a firm and a matched worker acted as a single entity, they would maximize

$$e_t^i\theta - c(e_t^i).$$

Let e^{FB} denote the resulting effort the first best, which equals $e^{FB} = c'(e^{FB}) = \theta$.

Contracts, Strategies, and Equilibrium Concept We consider situations in which effort as well as per-worker output can be observed by both the firm and the worker, but not by anyone outside the respective match. Hence, no verifiable measure of the agent's performance exists, and incentives can only be provided informally, i.e., with relational contracts.

We assume strategies are *contract specific* in the sense of Board and Meyer-Ter-Vehn (2014): actions of firms and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship.[3] Contract-specific strategies imply that firms' and workers' strategies cannot condition on any outcomes of other matches, i.e., multilateral relational contracts as in Levin (2002) are not feasible. We focus on pure strategies.

[3] In Section 4, we analyze asymmetric equilibria based on a worker's group identity.

The equilibrium concept we apply is *social equilibrium*. This concept describes a subgame-perfect equilibrium, which is restricted by the assumptions that strategies are contract-specific.[4] We derive social equilibria that maximize the profits of an individual unmatched firm, taking the behavior of other firms as given. Among these equilibria, we consider those that are (constrained) Pareto optimal.[5] We focus on the *stationary steady state*, which allows us to omit time subscripts. In the proof of Proposition 1, we show that this stationarity assumption is without loss of generality for all periods other than the very first period $t = 1$.

Three remarks are in order. First, our setup is based on the model of MacLeod and Malcomson (1998) and extends it by the introduction of labor market frictions and continuous (in contrast to binary) effort. Second, although our equilibrium-selection assumption might look strong, in Section 6 we argue that (constrained) Pareto optimality can be generated in a bargaining setting akin to Miller and Watson (2013), where individual choices are made non-cooperatively but bargaining follows the cooperative Nash-bargaining regime. In such a model, our results survive as long as a firms have considerable bargaining power. Third, the compensation structure — an upfront wage and a bonus paid at the end of a period — is assumed for simplicity and does not have to be taken literally. For example, the bonus could also be paid in the form of a salary at the beginning of the next period or correspond to future promotion opportunities, without changing expected payoffs and any of the constraints derived below. The only important feature is that the bonus is contingent on the worker exerting equilibrium effort, i.e., that it is tied to the worker keeping his job.

[4]The equilibrium concept is called *social* because a player's optimal strategy will depend on the strategies of all market participants including the ones a player never interacts with, as the possibility of a re-match determines everyone's effective outside option.

[5]In Section 6, we discuss equilibria that may involve money burning.

3 Analysis of Symmetric Equilibrium

We start with analyzing a *symmetric* profit-maximizing equilibrium (unless an equilibrium has negative profits, in which case a fraction of randomly determined firms are not active as analyzed below); see Section 4 for an asymmetric equilibrium. There, any deviation from equilibrium behavior would lead to the static Nash equilibrium with zero effort and zero payments in the respective match; thus, such a match is separated at the end of a period. This is optimal by Abreu (1988) who shows that any observable deviation should trigger the highest feasible punishment for the defector.

Equilibrium Payoffs The discounted utility stream of an employed worker in the stationary steady state equals

$$V = v + \gamma\delta V,$$

where $v = w + b - e^2/2$ is an employed worker's per-period utility. Note that discounted continuation utilities are multiplied by γ because workers might leave the market for exogenous reasons with probability $1-\gamma$, then having a utility of zero.

The utility of an unemployed worker is denoted by V^u and equals $V^u = \alpha^N V + \delta\gamma(1-\alpha^N)V^u$, where $\alpha^N = m(f,n)/n$ is the probability with which an unemployed worker finds a job. Rearranging yields

$$V^u = \frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}V.$$

A matched firm's expected profits are denoted by Π and an unmatched firm's expected profits are denoted by Π^u:

$$\begin{aligned}\Pi &= e\theta - b - w + \delta\left[\gamma\Pi + (1-\gamma)\Pi^u\right],\\ \Pi^u &= \alpha^F\Pi + \delta\left(1-\alpha^F\right)\Pi^u,\end{aligned}$$

where $\alpha^F = m(f,n)/f$ is the probability with which an unmatched firm can fill a vacancy. Rearranging yields

$$\Pi = \frac{\left[1-\delta\left(1-\alpha^F\right)\right](e\theta-b-w)}{(1-\delta)\left[1-\delta\gamma\left(1-\alpha^F\right)\right]},$$
$$\Pi^u = \frac{\alpha^F(e\theta-b-w)}{(1-\delta)\left[1-\delta\gamma\left(1-\alpha^F\right)\right]}.$$

3.1 Benchmark: Formal Contracts

We first analyze a benchmark case in which formal short-term contracts on an agent's effort are feasible. Let e^C denote the equilibrium effort under formal contracts. Then, paying $b = c(e^C)$ and $w = (1-\delta\gamma)V^u$ maximizes profits. Furthermore, $V^u = 0$ in the symmetric equilibrium. A firm keeps the full social surplus $e^C\theta - c(e^C)$ and specifies e^C to maximizes the surplus, so $e^C = e^{FB}$ is implemented. This holds irrespective of the values of α^F and α^N, therefore a change in N has no effect on a worker's compensation. Because firms can find a replacement with a larger probability if a worker leaves for exogenous reasons, their profits increase in N.

3.2 Characterization of Equilibrium with Constant F

This subsection investigates optimal contracts when the mass of firms F is exogeously given. We first pin down the two sets of conditions that must hold in equilibrium, for the market to be in the steady state as well as for the worker and firm of an individual match to play best responses.

Market Conditions Denote $f^* > 0$ as the equilibrium mass of unmatched firms and $n^* > 0$ as the equilibrium mass of unemployed workers in the stationary steady state of the labor market. Then, the mass of unmatched firms as well as unemployed workers at the beginning of a period must be the same as at the end of a period, i.e., $f^* = (1-\alpha^F)f^* + (1-\gamma)(F-f^*)$ and $n^* = (1-\alpha^N)n^* + (1-\gamma)(N-n^*)$. Equivalently,

$$f^* = \frac{1-\gamma}{1-\gamma+\alpha^F}F \qquad \text{and} \qquad n^* = \frac{1-\gamma}{1-\gamma+\alpha^N}N. \tag{1}$$

By using $\alpha^F = m(f^*, n^*)/f^*$ and $\alpha^N = m(f^*, n^*)/n^*$ and hence $\alpha^F f^* = \alpha^N n^*$, we obtain

$$\alpha^N = \frac{(1-\gamma)\,\alpha^F F}{N\,(1-\gamma+\alpha^F) - \alpha^F F} \qquad \text{and} \qquad \alpha^F = \frac{(1-\gamma)\,\alpha^N N}{F\,(1-\gamma+\alpha^N) - \alpha^N N}. \tag{2}$$

We first demonstrate comparative statics with respect to N and F:

Lemma 1

$$\frac{\partial \alpha^F}{\partial F}, \frac{\partial \alpha^N}{\partial N} < 0, \qquad\qquad \frac{\partial \alpha^F}{\partial N}, \frac{\partial \alpha^N}{\partial F} > 0.$$

These results follow basic intuition: An increase in the total labor force (holding F constant) raises the number of unemployment workers n and makes it easier for firms to fill a vacancy, whereas the increased competition makes it more difficult for an unemployed worker to find a job.

Individual Match Conditions Next, we specify the conditions that must hold for an individual match. First, it must be in the worker's interest to exert the agreed-upon effort level. Consider a deviation in which the worker chooses zero effort (which naturally is the optimal deviation). In this case, the worker does not receive the bonus and the respective match splits up, so the worker's continuation utility equals $\delta\gamma V^u$. It follows that equilibrium effort e^* must satisfy the agent's incentive compatibility (IC) constraint,

$$-(e^*)^2/2 + b + \delta\gamma V \geq \delta\gamma V^u. \tag{IC}$$

Second, as an employed worker's utility must be at least as high as his outside option V^u, the following individual rationality (IR) constraint must hold:[6]

$$V \geq V^u. \tag{IR}$$

Note that even though $V^u = \frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}V$ holds in equilibrium and $\alpha^N < 1$, (IR) cannot be omitted. This is because V^u is regarded as exogenous by each firm.

Third, a firm must pay a bonus as promised. If the firm reneges and refuses to pay the equilibrium bonus at the end of period t, the match splits up and both parties re-enter the matching market. Therefore, the maximum enforceable bonus payment is given by a dynamic enforcement (DE) constraint,

$$-b + \delta\gamma\Pi + \delta(1-\gamma)\Pi^u \geq \delta\Pi^u. \tag{DE}$$

There, we also have to take into account that even if a firm pays the bonus, its worker might leave for exogenous reasons with probability $1-\gamma$. Because

$$\Pi - \Pi^u = \frac{\left(1-\alpha^F\right)(e\theta - b - w)}{\left(1-\delta\gamma\left(1-\alpha^F\right)\right)},$$

(DE) becomes

$$b \leq \delta\gamma\left(1-\alpha^F\right)(e\theta - w). \tag{DE'}$$

(DE') describes the maximum bonus the firm can credibly promise in a relational contract. Intuitively, a high bonus may not be self-enforceable because a firm has an incentive to renege and go for a potential new match. Holding other parameters constant, (DE') is more difficult to sustain as α^F is larger. Also, sticking to its current match has to be optimal for a firm on the equilibrium path, requiring $\delta\gamma\Pi + (1-\gamma)\delta\Pi^u \geq \delta\Pi^u$ and hence $\Pi \geq \Pi^u$. Given $b \geq 0$, this condition is implied by

[6] (IR) captures that, at the end of period $t-1$, the worker stays only if he expects to receive (at least) V^u in the following period. Note that at the beginning of period t, the firm could also deviate by offering a contract with $V \in [\delta\gamma V^u, V)$ (here, $\delta\gamma V^u$ constitutes the worker's outside option from the perspective of period t, because he would have to wait until the next period before potentially finding a new match). As both parties choose to leave the match upon any deviation, however, such a deviation is not profitable for the firm; then the worker would collect the wage, choose $e_{t+1} = 0$, and leave the match.

(DE) and hence can be omitted.

Our objective is to characterize an equilibrium in which firms maximize Π^u, subject to the above constraints.[7] Propositions 1 and 2 state how varying N affects equilibrium effort and the utility of workers if we hold F constant. By Condition (1), under a fixed F, there is a one-to-one relationship between N and the probability of finding a new worker α^F in the steady state (and α^F increases in N by Lemma 1).

Proposition 1 (Optimal Informal Incentives) *There exists a profit-maximizing equilibrium with $\overline{\alpha} = 1/2$ such that*

a) For $\alpha^F > \overline{\alpha}$, equilibrium effort is $e^* = \delta\gamma\theta$, and a worker's utility is positive, i.e., $V > V^u > 0$.

- *If $N > F$ at $\alpha^F = 1/2$, then all firms are active for $\alpha^F > \overline{\alpha}$.*
- *Otherwise, there exists $\hat{\alpha} \in (1/2, 1)$ such that a fraction of firms are inactive for $\alpha^F \in (\overline{\alpha}, \hat{\alpha}]$ and that all firms are active for $\alpha^F > \hat{\alpha}$.*

b) For $\alpha^F \leq \overline{\alpha}$, equilibrium effort is $e^* = min\left\{2\left(1-\alpha^F\right)\delta\gamma\theta,\, e^{FB}\right\}$. Each worker's utility is zero (i.e., $V = V^u = 0$), and all firms are active.

Because formal contracts are not feasible, a firm's promise to reward a worker for his effort must be credible. As explained above, a worker who does not receive a promised payment responds by not exerting effort anymore. Different from "standard" relational-contracting models with one principal and one agent, a reneging firm can replace a worker and start over. Therefore, in our setting the future relationship surplus which determines enforceable effort in a relational contract also depends on the cost of turnover. This link between relationship surplus and turnover costs manifests in effort increasing in δ and γ.[8]

[7]Note that our main results would qualitatively hold if the objective was to maximize Π, the profits of a *matched* firm. However, maximizing Π^u allows us to (without loss of generality) focus on stationary contracts. If we maximized Π instead, it would be optimal to treat workers in the first period of their employment spell differently than in later ones. Thus, maximizing Π^u simplifies our exposition without affecting the core of our results.

[8]For $\alpha^F > \overline{\alpha}$, effort is $e^* = \delta\gamma\theta$; for $\alpha^F \leq \overline{\alpha}$ but sufficiently high for effort being below e^{FB}, effort is $e^* = 2\left(1-\alpha^F\right)\delta\gamma\theta$. Thus, $de^*/d\delta$, $de^*/d\gamma > 0$ in both cases.

One form of turnover cost stems from labor-market frictions which reduce the chances of finding a replacement. If frictions are small so that $\alpha^F \geq \overline{\alpha}$ (case a)), firms use another, endogenous, mechanism to make turnover costly by granting new workers a rent as in MacLeod and Malcomson (1998) or Fahn (2017); thus, firms do not utilize their wage-setting power to fully extract the relationship surplus.[9] If $N > F$ at $\alpha^F = 1/2$, then all firms are active, pay such a rent, and make positive profits for any $\alpha^F > \overline{\alpha} = 1/2$. If $N < F$ and all firms were active on the market, then they would make negative profits for α^F slightly above 1/2. Hence, there does not exist a profit-maximizing equilibrium in which all firms are active; instead, some firms must become inactive to increase the effective α^F and secure non-negative profits for the rest, in which case all firms make zero (expected) profits.[10]

Such an equilibrium with endogenous turnover costs particularly makes use of the term "social" in social equilibrium. The productivity of a firm's *current* relationship depends on the costs of starting a new relationship in the *future* — although potential new workers can't observe anything about the firm's past. The social equilibrium specifies that workers regard an offer with a lower rent as a deviation, thus firms have an incentive to compensate their workers as promised. Put differently, this social equilibrium involves a norm that high wages are paid independent of a worker's tenure.[11] In a development context where employment contracts are mostly informal, such norms have been identified by Breza et al. (2021), who find that unemployed workers do not accept offers below the prevailing wage.

If α^F is sufficiently small, the presence of the labor-market friction alone is sufficient for firms to honor their promises (case b)). Then, firms make use of their wage-setting power and leave no rents to workers. Furthermore, equilibrium effort is

[9]In the equilibrium, workers receive a positive upfront wage in every period. It is costly for firms because — unlike payments made later on — the wage paid in the first period of a worker's employment cannot be used to provide incentives. This is different from Levin (2003), where outside options are exogenously given and the relationship surplus is orthogonal to transfers.

[10]In the proof, we show that a profit-maximizing equilibrium for $N < F$ and $\alpha^F > \overline{\alpha}$ where all firms do not pay a rent also does not exist. This is because an individual firm would find it optimal to deviate and pay a rent to induce its worker to exert higher effort.

[11]See Kandori (1992), Ghosh and Ray (1996), Kranton (1996), MacLeod and Malcomson (1998), or Ghosh and Ray (2023), for more detailed discussions about the role of norms in related settings.

equal to e^{FB}. For intermediate α^F, the bonus is as high as feasible given α^F, effort is below e^{FB} and determined by a binding (DE') constraint.

Therefore, the structure of the optimal informal incentive system depends on the extent of labor market frictions which determine the size of the (future) relationship surplus. With small α^F (by small N), which implies high frictions and a large relationship surplus, incentives are strong and solely provided by the bonus. At intermediate levels of α^F, effort is reduced, but still only the bonus is used which keeps all rents with firms. For high α^F, effort stays constant and a rent is paid in the form of a positive upfront wage w (see Proposition 2 for more detailed comparative statics). This structure resembles that of Li and Matouschek (2013) or Li and Matouschek (2023). The similarity arises because both consider how different instruments can be used to sustain the optimal relational contract: when the (future) relationship surplus becomes smaller, the relationship first adjusts via lowering effort, but later on, it adjusts by giving rents.

Generally, the *optimal* level of turnover costs for firms would balance higher incentives that can be provided in a current relationship with the costs of starting new relationships later on. For $\alpha^F \leq \overline{\alpha}$, equilibrium turnover costs are "too large" because firms cannot reduce them if workers' rents are zero. For $\alpha^F > \overline{\alpha}$, equilibrium turnover costs are at the optimal level for an individual firm and equilibrium effort is below e^{FB}. This is because, at e^{FB}, having marginally smaller endogenous turnover costs (with a lower effort level) would increase the firm's profits. Moreover, by the response of its competitors, endogenous turnover costs are more expensive for a given firm than the exogenous costs stemming from labor-market frictions — because the former also increases an employed worker's outside option. Therefore, at $\alpha^F = \overline{\alpha}$, turnover costs are at that optimal level and the profits are maximized; for higher α^F, effort is the same as at $\overline{\alpha}$, but the rents for workers reduce the profits.[12]

[12]Even though it is optimal for each individual firm to pay a rent to its workers if $\alpha^F > \overline{\alpha}$, it still takes the behavior of its competitors as given. If our objective was to maximize industry-wide profits, we would have to take into account the negative externality a firm imposes on others by paying workers a rent. Then, the threshold of paying a rent would be higher than $\overline{\alpha}$.

Examples Here, we present two examples of matching functions, exhibiting either constant (Example 1) or increasing (Examples 2) returns to scale. See the Online Appendix for more details and further examples.

Example 1. Suppose $m(f,n) = (1 - e^{-\frac{n}{kf}})f$, which is discussed in a review article by Petrongolo and Pissarides (2001). It exhibits constant returns to scale and implies $\alpha^F = (1 - e^{-\frac{n}{kf}})$. For example, when $k = 2$ and $\gamma = 0.8$, $N/F = 1.11$ at $\overline{\alpha}$. Hence, workers are paid a rent once the total workforce exceeds the available jobs by 11%.

Example 2. Suppose $m(f,n) = \left(1 - e^{-k\frac{n}{F}}\right) f$, which is also discussed in a review article by Petrongolo and Pissarides (2001). It exhibits increasing returns to scale and implies $\alpha^F = \left(1 - e^{-k\frac{n}{F}}\right)$. For example, when $k = 2$ and $\gamma = 0.8$, $N/F = 1.06$ at $\overline{\alpha}$. Hence, workers are paid a rent once the total workforce exceeds the available jobs by 6%.

3.3 The Consequences of a Higher Labor Supply

We now conduct comparative statics with respect to the mass of workers N. We first analyze the case in which the mass of firms F is exogenous. We then study an extended model in which F is endogenously determined by free entry of firms.

3.3.1 Comparative Statics with a Fixed Mass of Firms

Proposition 2 presents comparative statics with respect to N, which depend on the threshold $\overline{\alpha} = 1/2$ derived in Proposition 1:

Proposition 2 (Comparative Statics with Constant F) *Suppose that F is exogenously given and that all firms are active.*

a) For $\alpha^F > 1/2$, the effort level e^* is unaffected by N, whereas total compensation $w^* + b^*$ and an employed worker's utility V may increase or decrease. Both of them increase in N if α^F is sufficiently close to $1/2$.

b) For $\alpha^F \leq 1/2$, V is unaffected by N. For intermediate values of α^F such that $e^* < e^{FB}$, $w^* + b^*$ decreases in N; for small values of α^F such that $e^* = e^{FB}$, $w^* + b^*$ is unaffected by N.

For $\alpha^F > \overline{\alpha} = 1/2$, an employed worker's compensation and utility can increase in the supply of labor (case a)): A higher N directly increases α^F, which raises an employed worker's compensation and utility. This is because total turnover costs are at their optimal level for an individual firm if $\alpha^F > \overline{\alpha}$, thus a higher α^F lets firms increase a worker's rent to the same extent. There is also a negative indirect consequence, however: Akin to the well-known efficiency wage effect (Shapiro and Stiglitz, 1984), the probability of finding an alternative job (i.e., α^N) decreases. The positive direct effect dominates the negative indirect effect if α^F is sufficiently close to $\overline{\alpha}$, because then the worker's outside option is relatively small.

When $\alpha^F \leq \overline{\alpha}$, the labor market friction is larger than optimal from an individual firm's perspective (case b)). Therefore, if N goes up, firms fully "utilize" the decreased friction and request lower effort in response to their reduced commitment. Because $V^u = 0$, there is also no indirect effect on the outside option. As effort goes down, the worker's compensation goes down as well. If frictions are so high that e^{FB} is implemented, effort is independent of N.

Examples We now present conditions for $w + b$ to increase in N for the examples introduced above.

Example 1 (continued). Suppose $m(f, n) = (1 - e^{-\frac{n}{kf}})f$. When $k = 2$, $\gamma = 0.8$, and $\delta = 0.9$, $w^* + b^*$ and V increase in N from $\alpha^F = 0.5$ to $\alpha^F = 0.8$, which is equivalent to $N/F \in [1.11, 1.43]$, and decrease in N for $\alpha^F > 0.8$. Hence, a worker's compensation increases in the labor supply when the total workforce exceeds all available jobs by 11%-43%Otherwise, the compensation decreases in N.

Example 2 (continued). Suppose $m(f, n) = \left(1 - e^{-k\frac{n}{F}}\right) f$. When $k = 2$, $\gamma = 0.8$, and $\delta = 0.9$, $w^* + b^*$ increase in N from $\alpha^F = 0.5$ to $\alpha^F = 0.75$, which is equivalent to $N/F \in [1.06, 1.075]$. Hence, a worker's compensation increases in the labor supply when the total workforce exceeds all available jobs by 6%-7.5%.

Figure 1 displays wage, bonus, and total compensation as functions of α^F (which itself is increasing in N), for Example 1 with $k = 2$, $\theta = 1$, $\delta = 0.9$, and $\gamma = 0.8$:

Figure 1 – Effort, total compensation, wage, and bonus:

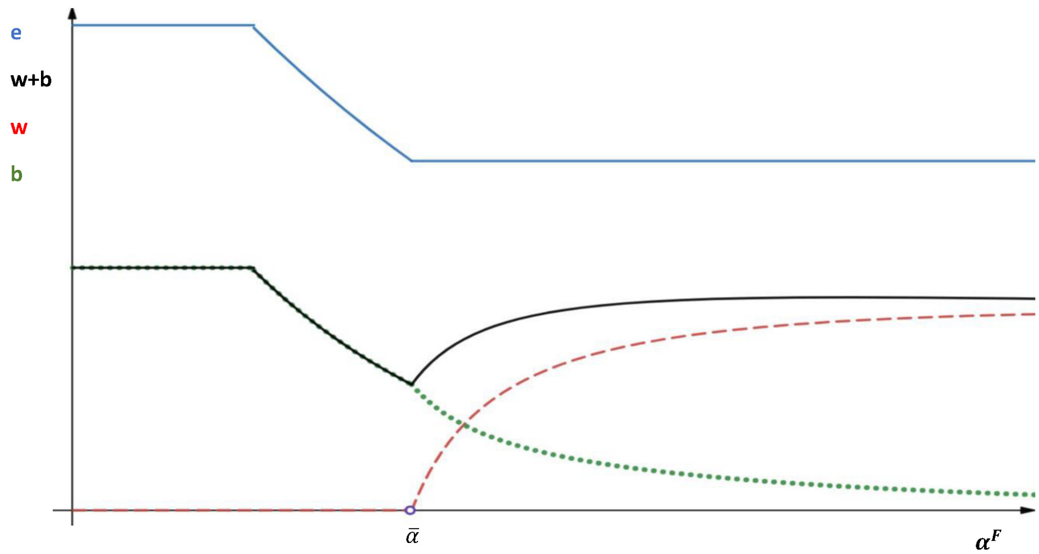

Figure 1 not only illustrates that compensation can increase in α^F and thus N, but also the structure of the optimal informal incentive system: With low α^F, incentives are strong and only the bonus is used. As α^F goes up, the bonus and effort are reduced. As soon as $\overline{\alpha}$ is reached, effort stays constant and a further increase in α^F (i.e., a reduction in the relationship surplus) makes it optimal to gradually replace the bonus with a fixed wage. Hence, different from many relational contracting models, there is no monotonic relationship between effort and future relationship surplus.

These results are based on the assumption that F is fixed. We next demonstrate that a higher labor supply never has a negative effect on a worker's compensation if F is endogenously determined by free entry, and if the matching function exhibits non-decreasing returns to scale.

Comparative Statics with an Endogenous Mass of Firms

Suppose that there exists a sufficient pool of potential entrant firms, and each of them can enter the industry by paying a one-time entry cost $K > 0$ at the beginning of the game. The zero-profit condition requires $-K+\Pi^u = 0$; moreover, $\partial\Pi^u/\partial F \leq 0$ must hold for an equilibrium to exist because otherwise, additional firms would enter.

Before discussing comparative statics, we first explore potential equilibria for each K. Let $\tilde{K} > 0$ denote a maximum value for which firms will enter the market. $\tilde{K}$ is determined by the highest profits that can be achieved, which occurs at $\alpha^F = 1/2$. The reason is that $\alpha^F = 1/2$ is associated with the optimal turnover cost and does not grant workers a rent. For all $K \in \left[0,\, \tilde{K}\right)$, there are two potential equilibria, one with $\alpha^F < 1/2$ and one with $\alpha^F > 1/2$; in the proof of Proposition 3, we show that an equilibrium with $\alpha^F < 1/2$ always exists, and an equilibrium with $\alpha^F > 1/2$ exists for all $K \in \left[\underline{K},\, \tilde{K}\right)$, where $\underline{K}$ depends on the matching function and $\underline{K} \geq 0$. Thus, $\underline{K} = 0$ is possible, in which case the latter equilibrium exists for all $K \leq \tilde{K}$.

To conduct comparative statics with respect to N, we take into account that the free-entry condition implies that any change in N must be balanced by a change in F to keep Π^u constant:

$$d\Pi^u = \frac{\partial\Pi^u}{\partial N}dN + \frac{\partial\Pi^u}{\partial F}dF = 0 \qquad \Longrightarrow \qquad \frac{dF}{dN} = -\frac{\partial\Pi^u/\partial N}{\partial\Pi^u/\partial F},$$

yielding the following results:

Proposition 3 (Comparative Statics with Endogenous F) *Suppose that F is endogenously determined by free entry subject to the entry cost $K > 0$. Then, an equilibrium exists if and only if $K \leq \tilde{K}$.*

a) There is $\underline{K} \in \left[0, \tilde{K}\right)$ such that an equilibrium with $\alpha^F > 1/2$ exists for all $K \in \left[\underline{K},\, \tilde{K}\right)$. In this equilibrium,

i) If $m(f,n)$ exhibits increasing returns to scale, then the effect of a higher N on total compensation $w^* + b^*$ and an employed worker's utility V is positive: $d(w^* + b^*)/dN > 0$ and $dV/dN > 0$.

ii) If $m(f,n)$ exhibits constant returns to scale, then $d(w^* + b^*)/dN = 0$ and $dV/dN = 0$.

iii) If $m(f,n)$ exhibits decreasing returns to scale, then $d(w^* + b^*)/dN < 0$ and $dV/dN < 0$.

b) In addition, an equilibrium with $\alpha^F < 1/2$ exists for all $K \leq \tilde{K}$. In this equilibrium, $w^* + b^*$ and V are unaffected by N.

In the equilibrium with $\alpha^F > 1/2$ (case a)), $\Pi^u = [e\theta - c(e)/\delta\gamma - (1-\delta\gamma)\,V^u]\,/\,(1-\delta)$ by Proposition 1. Therefore, free entry keeps an unemployed worker's utility V^u constant, and the effect of a higher N on an employed worker's compensation $w^* + b^*$ and utility V is determined by the sign of

$$\frac{d\alpha^F}{dN} = \frac{\partial \alpha^F}{\partial N} + \frac{\partial \alpha^F}{\partial F}\frac{dF}{dN},$$

i.e., by the total effect of an increase in labor supply on a firm's probability of filling a vacancy. In the proof of Proposition 3, we show that $d\alpha^F/dN$ is positive (resp. negative) if the matching function exhibits positive (resp. negative) returns to scale in m and f.

In the equilibrium with $\alpha^F \leq 1/2$ (case b)), the direct effects of a higher N on effort and compensation are offset by free entry. Thus, a higher N does not affect effort and utilities.

Proposition 3 shows that, under endogenous F, the consequences of a higher N on an employed worker's compensation $w^* + b^*$ and utility V are always (weakly) positive if the matching function exhibits non-decreasing returns to scale. Indeed, a review article by Petrongolo and Pissarides (2001) states that most empirical studies support either constant or increasing returns to scale.

To conclude this section, let us briefly discuss when we should expect an equilibrium with α^F above $\overline{\alpha} = 1/2$ under free entry. Proposition 3 shows that an equilibrium with $\alpha^F > \overline{\alpha}$ exists if the entry cost K is sufficiently high, which may capture a less competitive labor market for firms. Besides, while we do not take a general stand as to whether an equilibrium with high or low α^F is more likely, we would argue that high levels of α^F would be observed particularly in markets which experience a discrete increase in labor supply (for example, by immigration). This is based on the interpretation in which comparative statics with constant F would describe the short-term, endogenous F the long-term consequences of a higher labor supply. In addition, firms' adjustments would typically not be immediate, thus a labor supply increase pushing α^F above $\overline{\alpha}$ followed by gradual responses of firms would likely yield a new steady state level of $\alpha^F > \overline{\alpha}$.

Examples

We now present conditions for $w + b$ to increase in N for the examples introduced above.

Example 1 (continued). Suppose $m(f, n) = (1 - e^{-\frac{n}{kf}})f$ where $k \geq 1$. As the matching function exhibits constant returns to scale, a higher labor supply does not affect an employed worker's compensation and utility under all parameters. If $k = 1$, an equilibrium with $\alpha^F > 1/2$ exists for all $K \in \left[0, \tilde{K}\right]$ and $\gamma \in (0, 1)$. If $k = 2$ and $\gamma = 0.8$, an equilibrium with $\alpha^F > \overline{\alpha}$ exists for $K \in \left[0.72\tilde{K}, \tilde{K}\right]$. There, the total workforce must exceed all available jobs by 33%, which implies $\alpha^F \geq 0.71$.

Example 2 (continued). Suppose $m(f, n) = \left(1 - e^{-k\frac{n}{F}}\right) f$. As it exhibits increasing returns to scale, a higher labor supply increases an employed worker's compensation $w^* + b^*$ and utility V in an equilibrium with $\alpha^F > 1/2$. When $k = 2$ and $\gamma = 0.8$, an equilibrium with $\alpha^F > \overline{\alpha}$ exists for $K \in \left[0.76\tilde{K}, \tilde{K}\right]$. There, the total workforce must exceed all available jobs by 26%, which implies $\alpha^F \geq 0.63$.

4 Labor Market Discrimination and Minimum Wage

So far, our model has delivered a norm-based explanation for positive wage effects of a higher labor supply, with outcomes for all workers being the same. Now, we show that a profit-maximizing "discrimination equilibrium" also exists in which one (majority) group receives a persistent wage premium over another (minority) group, although both groups are identical, i.e., are equally productive and have the same (exogenous) outside option. Even if a firm wanted to deviate from such discrimination, doing so unilaterally would be costly. Again, we argue that social norms, potentially shaped by historical developments, may determine whether a symmetric, non-discriminatory, equilibrium is played, or instead an equilibrium in which one large group (such as male or native workers) is treated better than another small group (such as female or immigrant workers). In the discrimination equilibrium, we also show how a minimum wage can mitigate such labor market discrimination.

Suppose that there are two kinds of workers, "insiders" (the majority group) and "outsiders" (the minority group). These identities can be distinguished by firms but all workers are otherwise identical. We assume that a firm with a vacancy is randomly matched with a worker, hence targeted search is not possible. We adopt a matching function for heterogeneous agents as presented by Petrongolo and Pissarides (2001).

Only firms with an open vacancy can be matched with workers, so it is not possible for a matched firm to look for another type of worker unless it breaks up the current match. Then, presuming that insiders are not aware of the jobs outsiders apply for (Petrongolo and Pissarides, 2001), and that n^O is the mass of unemployed outsiders and n^I the mass of unemployed insiders, the matching function for outsiders is $m(n^O, f)$ and the aggregate matching function is $m(n^O + n^I, f)$.[13] Denoting a vacant firm's probability of being matched with an outsider by α^{FO} and of being matched with an insider by α^{FI}, and an unemployed outsider's (insider's) matching probability by α^{NO} (α^{NI}), we have

$$\alpha^{NO} = \frac{m(n^O, f)}{n^O}, \quad \alpha^{NI} = \frac{m(n^O + n^I, f) - m^O(n^O, f)}{n^I},$$
$$\alpha^{FO} = \frac{m(n^O, f)}{f}, \quad \alpha^{FI} = \frac{m(n^O + n^I, f) - m(n^O, f)}{f},$$

[13] If a firm may come across multiple workers within a period (by slightly extending our model), then the firm can pick whom it decides to hire. In this case, a firm will choose a worker randomly if workers are in the same group but prefers outsiders over insiders. Therefore, the above matching function already anticipates our later results of an equilibrium in which insiders are better off but generate (weakly) lower profits than outsiders.

and

$$\alpha^F = \frac{m(n^O + n^I, f)}{f} = \alpha^{FO} + \alpha^{FI}.$$

Now, a firm's profits when hiring an insider are Π^I, and Π^O when hiring an outsider. Therefore, an unmatched firm's expected profits are

$$\begin{aligned}\Pi^u =& \alpha^{FI}\Pi^I + \alpha^{FO}\Pi^O + \delta\left(1-\alpha^F\right)\Pi^u \\ =& \frac{\alpha^{FI}\Pi^I + \alpha^{FO}\Pi^O}{1-\delta\left(1-\alpha^F\right)}.\end{aligned}$$

4.1 Discrimination Equilibrium

We study characteristics of a discrimination equilibrium and focus on an equilibrium that, among equilibria that maximize individual firms' profits, is best for employed insiders. As a starting point, consider a situation with no outsiders (and in which all insiders have the same arrangement), and with α^{FI} above the threshold $\overline{\alpha}$ derived in Proposition 1. In this situation, insiders are paid a rent, and profit-maximizing effort is below the first best.

Compare this to a situation in which outsiders are also present. Importantly, a firm's optimal behavior is not uniquely determined, because only *expected* turnover costs when starting a new relationship matter. Thus, besides a symmetric equilibrium with identical outcomes for insiders and outsiders, profit-maximizing equilibria in which outsiders' payoffs are lower can also exist. There, lower payments to outsiders ceteris paribus increase a firm's profits with outsiders and hence must be accompanied by higher rents for insiders to keep expected profits for an unmatched firm constant. The best feasible arrangement for insiders involves firms' profits with outsiders to be as high as possible. As long as the mass of outsiders N^O is small, such an equilibrium pushes outsiders' payoffs to their outside option of zero and implements an effort level either at e^{FB} or determined by a firm's binding dynamic enforcement constraint. In either case, an outsider's effort e^O is strictly larger than an insider's effort e^I (which is still characterized by $e^I = \delta\gamma\theta$). Thus, outsiders might *work harder but earn less* than insiders.

Proposition 4 (Discrimination Equilibrium) *Suppose that $\alpha^{FI} > 1/2$ and all firms are active at $N^O = 0$. For sufficiently small $N^O > 0$, there exists a profit-maximizing discrimination equilibrium which favors insiders:*

a) Insiders' effort is $e^I = \delta\gamma\theta$ and $V^I > V^{uI} > 0$,

b) Outsiders' effort is $e^O > e^I$, $w^O = 0$, $b^O = c(e^O)$, and $V^O = V^{uO} = 0$.

c) Less insiders are employed than when $N^O = 0$.

If N^O is sufficiently large, there does not exist an equilibrium in which the payoff of insiders is strictly higher than the payoff of outsiders.

Proposition 4 characterizes an equilibrium in which the presence of a sufficiently small share of outsiders increases unemployment among insiders, but those who have a job are better off than without outsiders. Moreover, a higher outside option and seemingly lower productivity of insiders can emerge *endogenously* in a discrimination equilibrium. Note that insiders' effort in the discrimination equilibrium is the same as in the above symmetric equilibrium. Thus, the higher effort of outsiders implies that average productivity is highest in the discrimination equilibrium which maximizes insiders' payoffs. Finally, if the share of outsiders is sufficiently large, there is no profit-maximizing equilibrium in which insiders are treated better than outsiders. In this sense, our result speaks to discrimination against a minority group.[14]

4.2 Discrimination as a Collusion Device

So far, we have described how a norm that discriminates against outsiders benefits insiders. Such discrimination effectively manifests in a redistribution from one group to another, leaving individual firms indifferent. Now, we discuss a different form of

[14]Note that our mechanism is different from the insider-outsider theory of labor markets proposed by Lindbeck and Snower (1988) or Lindbeck and Snower (1989); there, "insiders" (employed workers) utilize extra bargaining power to push their wages above the market-clearing wage; this bargaining power is due to turnover costs which prevent replacement with cheaper "outsiders" (unemployed workers) who consequently face involuntary unemployment.

discrimination which relies on the exclusion of a group of outsiders and benefits firms. In the following, we discuss such a case without formally modelling it.

Recall that, if $\alpha^F > \overline{\alpha}$ and compensation increases in N, then profits decrease in N. Therefore, while a change in its α^F would not affect the profits of an individual firm (holding the behavior of all other firms and therefore workers' outside options constant), a reduction of the effective labor supply could increase *industry-wide profits* which are maximized at $\alpha^F = \overline{\alpha}$. Thus, assume that collectively excluding outsiders would increase industry-wide profits. Such a discrimination equilibrium can be sustained if a firm that is matched with an outsider has no incentive to hire him (hiring an outsider would be profitable if the firm could employ him at the same conditions as insiders). For example, the outsider could regard any offer as a norm violation and not exert effort, making it optimal for the firm not to grant an offer in the first place. Alternatively, if the outsider expected to be compensated in future periods, he may be willing to exert effort. Then, however, he should assume that the firm keeps hiring outsiders also in the future and consequently faces a higher α^F — which would require a higher compensation to keep total turnover cost constant. Therefore, an exclusion equilibrium in which the effective α^F is reduced because some workers are not considered certainly exists. Moreover, using exclusion as a collusion device may even be easier to sustain than collusion over consumer prices because of the potentially smaller benefits of a deviation.

4.3 Discrimination and Minimum Wage

Even though there is discrimination on labor markets, many societies have the objective to achieve more equal outcomes. Given the discrimination equilibrium described above, however, fighting discrimination would be costly for an individual firm. Reducing w^I would cause insiders to not exert effort and leave the firm, and increasing w^O would increase the firms' cost. As such society-wide norms are difficult to change, a question is whether labor-market policies can reduce discrimination or not. We demonstrate that minimum wages can be an effective tool to achieve this objective.

Assume a minimum wage $\overline{w}$ which serves as a lower bound on a worker's total compensation, $w + b$. Because b is discretionary, the upfront-wage w must exceed $\overline{w}$.

Proposition 5 (Minimum Wage) *Consider the discrimination equilibrium in Proposition 4. There exists $\hat{w} > 0$ such that the following holds for $\overline{w} \in [0, \hat{w})$:*

a) The minimum wage binds for outsiders but is slack for insiders: $w^O = \overline{w} < w^I$.

b) An increase in $\overline{w}$

- *reduces w^I as well as V^I, but increases V^O and w^O;*
- *(weakly) reduces e^O;*
- *increases Π^u and consequently employment if F is endogenous.*

In the best profit-maximizing equilibrium for insiders, $w^I > w^O$. A minimum wage in between them reduces the gap between w^I and w^O. This is not only caused by an increase in w^O, but also by a reduction of w^I, thus outsiders are better off and insiders are worse off. Moreover, the minimum wage increases profits because insiders' outside options are reduced. Therefore, if F is endogenously determined by a zero-profit condition, employment effects of such a minimum wage are positive.[15]

5 Evidence and Applications

This section presents empirical research and other evidence related to our results, as well as potential applications of our model.

5.1 Prevalence of Informal Incentives and High Wages

Informal and self-enforcing agreements are prevalent in employment and business relationships in developing (Macchiavello, 2021; Macchiavello and Morjaria, 2023) as

[15] If the minimum wage is sufficiently high for $w^I = w^O$ to hold, a further increase lets both wages (and utilities) go up. A related model in this symmetric case is investigated by Fahn (2017), who shows that a higher minimum wage increases equilibrium effort and surplus.

well as developed (Gibbons and Henderson (2012); Ahammer et al., 2023) countries. As a specific example, in a survey 20.7% of freelancers in Japan answered that they did not receive any formal contract from firms (i.e., terms and conditions of work — including payments — were agreed only informally), as many of the relations are (anticipated to be) renewed.[16] In a different survey conducted by the Cabinet Secretariat in Japan, the amount of people performing full-time freelance work was estimated at 2.1 million in February 2020 and expected to further rise. Moreover, consistent with the equilibrium outcome of our model, even though firms have substantial bargaining power, the average annual income of the full-time freelancers is higher than the national average.[17]

5.2 Downward Wage Rigidity

There is plenty of evidence that wages are downward rigid even if a recession increases unemployment and thus the supply of labor (Campbell and Kamlani, 1997; Bewley 1998; 1999). A main cause for firms' reluctance to cut the wages of (new and existing) employees even with excess labor is a negative effect on their morale (Bewley, 1995), which managers fear will increase turnover and reduce employees' effort (Campbell and Kamlani, 1997; Bewley, 2007). Although plenty of theoretical explanations exist for this link, those are based on worker preferences directly restricting a firm's wage setting and flexibility by *assuming* that wage reductions lead to a decline in morale and effort (Solow, 1979; Akerlof, 1982; Akerlof and Yellen 1988; 1990).

Our framework can provide an alternative explanation for downward wage rigidity that is also due to firms fearing a negative effect on effort (and seemingly morale). However, we do not have to assume such a link, instead it is the consequence of a norm that sustains a profit-maximizing equilibrium in a market with abundant labor

[16] https://blog.freelance-jp.org/wp-content/uploads/2020/06/2020_0612_hakusho.pdf (accessed on February 22, 2024).

[17] https://www.cas.go.jp/jp/seisaku/atarashii_sihonsyugi/freelance/dai1/siryou9.pdf (accessed on February 22, 2024).

(i.e., if N is sufficiently large for $\alpha^F > \bar{\alpha}$). For example, if F is endogenous and the matching function characterized by constant returns to scale, the compensation of new and existing workers does not respond to changes in N. Further strengthening our point, the literature is based on cases in which relational contracts seem relevant: The effort reduction caused by low morale does not affect the performance of routine tasks; instead, it is about doing the "extra thing", helping each other, making good suggestions Bewley (2007), or the quality with which services are provided (Campbell and Kamlani, 1997). Moreover, downward wage rigidity, in particular for new hires, was only observed in long-term employment relationships with low turnover (Bewley 1995; 1998; 2003) which are prerequisites for relational contracts.[18]

5.3 Non-Negative Wage Effects of a Higher Labor Supply by Immigration

The immigration literature has extensively analyzed the effects of a higher labor supply on wages. While a number of empirical studies lends support to the canonical model of the labor market and finds negative wage effects of immigration (Borjas, 2003; Borjas, 2017), many papers identify either no or even positive wage effects (Card, 1990; Winter-Ebmer and Zweimüller, 1996; Peri, 2007; Ottaviano and Peri, 2012; Peri and Yasenov, 2019)

To explain these findings, the literature has mostly focused on heterogeneity in worker skills — in particular between immigrants and native workers — and that native workers are able to switch to jobs with different skill demands (see Peri and Sparber, 2009, Ottaviano and Peri, 2012, or Peri, 2016). Then, immigration generally has positive effects on high-skill and negative effects on low-skill native workers. While such an approach explains the effects of immigration on some wages, recent evidence points towards non-negative wage effect even among low-skill native

[18]Note that a "standard" relational contracting model with exogenous outside options could also generate downward wage rigidity by specifying that any change of compensation is deemed a violation of the relational contract. However, making such a relational contract independent of the external environment would clearly not be optimal. Moreover, it would be much more difficult to explain downward wage rigidity for new hires.

workers who do not switch tasks in response to immigration (Foged and Peri, 2016; Clemens and Hunt, 2019; Tabellini, 2019).

We argue that labor-market norms might cause non-negative wage effects in settings with homogeneous workers who are incentivized by relational contracts. Although the relevance of relational contracts is often difficult to assess, those should be relevant if the legal system firms operate in is weak. Indeed, evidence points towards non-negative wage effects in these cases. For example, Fallah et al. (2019) investigate the impact of the Syrian refugee influx on labor market outcomes in Jordan. They find that employment and unemployment were unaffected, whereas hourly wages went up. Altindag et al. (2020) analyze the inflow of Syrian refugees into Turkey. These refugees mostly work in the informal sector, where they caused job losses among natives. Those natives who kept their jobs, however, experienced wage increases. Both observations are in line with our discrimination equilibrium (for which we present additional evidence below), which is further supported by Syrian refugees receiving lower wages and working more hours relative to Turkish workers.

5.4 Discrimination in Labor Markets

5.4.1 Gender Pay Gap

A huge literature has explored the existence and potential causes of a gender pay gap. While a big part of the gap can be explained by fundamental differences such as experience, human capital, or personality traits, a substantial unexplained gender difference remains (Blau and Kahn, 2017). Regarding potential explanations for the unexplained part of the gap, Blau and Kahn (2017) argue that the "formation of norms [...] is a potentially fruitful area of research that has received relatively little attention by economists". In our model, a norm that selects an equilibrium which benefits a majority group of workers or excludes a minority group can lead to discrimination on labor markets. Note that the norm arises as a profit-maximizing equilibrium, thus a unilateral deviation is not profitable. Hence, even when a norm might be the consequence of direct discrimination in the past, it can survive after other causes of discrimination have vanished.

In line with our result that discrimination in favor of insiders is only possible if their share is large, there is evidence that occupations with a high share of female workers have smaller gender pay gaps. Analyzing data from the British Office for National Statistics, the HR software provider Ciphr found that the gender pay gap is almost absent for full-time care workers where 73% of the workforce are women. Some female-dominated occupations even pay women more than men, including occupational therapists, credit controllers, special educational needs (SEN) teachers, HR officers, and publicans. To the contrary, the occupation with the largest pay gap is financial managers and directors, where 64% are men. Large gaps also exist in male-dominated occupations such as programmers/software development professionals.[19] As a further example, the Australian Workplace Gender Equality Agency (WGEA) regularly analyzes gender pay gaps, using remuneration information supplied by employers in the Census to calculate employer gender pay gaps. For 2023, the WGEA stated that the Australian gender pay gap was significantly larger in male- than in female-dominated industries (76% of employers in male-dominated industries have gender pay gaps in favour of men, compared to 41% in female-dominated industries; WGEA, 2024).

We also show that a minimum wage can reduce discrimination on labor markets. Indeed, Blau and Kahn (2000) emphasize the role of the minimum wage as an instrument that helped reduce the gender pay gap. Additional evidence is provided by Blau and Kahn (2003) who explore the role of minimum wages for reducing the gender gap in 22 countries. They identify a negative correlation between a gap and the extent to which a minimum wage binds. Importantly, the reduction of the pay gap is not only driven be an increase in female-dominated sectors, but can also be observed within sectors. For example, Majchrowska and Strawinski (2018) observe that minimum wage increases in Poland between 2006 and 2010 substantially reduced the

[19] https://www.ciphr.com/infographics/gender-pay-gap-statistics-2023, accessed on March 2, 2024.

gender wage gap, with the reduction being especially pronounced for young workers. Their results are obtained controlling for firm size, education level, industry as well as occupational group. Battisti et al. (2018) find that the introduction of a national minimum wage in Ireland in 2000 decreased the gender wage gap at the bottom of the wage distribution, and this result is not affected when including occupation and industry dummies.

In our setting, the minimum wage not only increases outsiders' wages, but also reduces insiders' wages. Although positive spillover effects have been observed in and explained by firms wanting to preserve their wage distribution, there also is evidence that a minimum wage can cause negative spillover effects.[20] Analyzing the British minimum wage, Stewart (2012) finds that the minimum wage reduced wage growth for levels slightly above. Neumark et al. (2004) observe that non-immediate spillover effects are negative. Hirsch et al. (2015) argue that wages above the minimum wage increase less than they would without such a lower bound.

Note also that our model predicts positive employment effects of a minimum wage. Excellent surveys such as Belman and Wolfson (2014) and Schmitt (2015), as well as recent work by Cengiz et al. (2019), state that there is no evidence for a systematic negative employment effect of a (moderate) binding minimum wage. In our model, positive employment effects are a side effect of reduced discrimination, in that it increases the costs of employing one group and decreases the costs of employing another group of workers, with the latter dominating.

5.4.2 Discrimination against Immigrant Workers

There is evidence that immigrant workers face discrimination. For example, Kerr and Kerr (2011) find that immigrants are paid less than natives even when conducting the same kinds of tasks, and this wage gap — although declining over time — persists in the long run. Furthermore, Battisti et al. (2018) analyze the consequences of immigration in 20 OECD countries and observe that, for each country

[20] An alternative explanation for negative spillover effects of a minimum wage is provided by Fahn and Seibel (2022).

and skill level, native workers are paid higher wages than immigrants. They discuss that natives' wage premia can be driven by either productivity differences or higher outside options. They rationalize their observations with a search-and-matching models of the labor market, where the reservation wages of immigrants are smaller by assumption. Moreover, positive effects of immigration on native workers' wages rely on both, task complementarity between native and immigrant workers and the creation of new jobs.

Our paper *endogenously* generates a higher outside option of native workers, and furthermore higher effort of immigrants. The latter is consistent with Dustmann et al. (2012), who argue that the extent of positive wage effects of immigration for some skill levels can be explained by productivity differences only if immigrant workers are more productive than natives and face a wage discount.

5.4.3 Discrimination and Credentialism

We have discussed two kinds of discrimination equilibria, one in which insiders receive preferential treatment and one in which outsiders are completely excluded. One could argue that, in many countries, such discrimination based on observable, non-productivity related, characteristics is in breach of anti-discrimination legislation. Even if this is the case, though, governments or firms can use indirect means to support such discrimination. One way is to require certificates or qualifications that are not warranted by the job requirements alone. For example, some countries impose high barriers on the recognition of foreign certificates. Because pay is often related to official qualifications, immigrants may receive less pay even when doing the same tasks as natives.

Moreover, an exclusion equilibrium can be sustained by such barriers which may even be built by firms themselves. One form is "credentialism" (or credential inflation), i.e., employers require higher formal degrees for jobs than warranted by the desired skill set. Such credentialism restricts the relevant labor pool, makes it more difficult to fill vacancies and thereby easier to sustain a relational contract. If it reduces wage payments and the outside options of workers, industry-wide profits will go up. Indeed, evidence points towards credential inflation among service jobs where relational contracts seem relevant. For example, a study by the Burning Glass Institute (BurningGlass, 2014) compared the education levels of U.S. workers with the education levels currently demanded for the same occupation, and found significant evidence for credential inflation. While these observations can partially be explained by an increased complexity of certain jobs, the skill requirements in other occupations that have experienced credential inflation — in particular, those in which some dimensions of performance (e.g., service quality) are hard to verify — have not changed. Those include jobs such as IT help-desk jobs, insurance claims clerks, executive secretaries, many HR roles, or supervisors of mechanics and installers. Related to our endogenous turnover costs, BurningGlass (2014) state that increased formal standards not related to the job duties substantially increase the time it takes to fill a position. Similar observations have been made by a Harvard Business School study (Fuller and Raman, 2017), for which the authors conducted a survey among 600 business and human resource leaders. They find substantial evidence for credential inflation among jobs with hard-to-measure performance such as "supervisors of production" "secretaries and administrative assistants," or "supervisors of blue-collar workers." Moreover, with more stringent formal requirements (even if unrelated to necessary skills), employers take substantially longer to fill a vacancy.

6 Discussion and Conclusion

This paper has investigated how labor-market tightness affects market outcomes under informal incentives. To conclude, we discuss the robustness of our model with a focus on two dimensions: general bargaining power between a firm and a worker, and alternative forms of turnover costs.

First, we discuss the consequences of workers having positive bargaining power in wage negotiations. In this case, outcomes would rely on the exact specification of the bargaining process, whether disagreement payoffs are determined by separation or only by non-production (as in Hall and Milgrom, 2008), and to what extent renegotiation would happen. Here, we discuss one particular setting which is motivated by dynamic bargaining approaches such as Ramey and Watson (1997) or Fahn (2017).[21] Assume that, at the beginning of a period, a firm and a worker bargain over how the relationship surplus is shared. The relationship surplus contains the expected discounted sum of payoffs generated in this relationship (i.e., $(e\theta - c(e)) / (1 - \delta\gamma))$ minus disagreement payoffs. Disagreement would cause a termination of the match and let both players enter the matching market in the subsequent period. The bargaining outcome would determine a worker's *minimum payoff*, which however could unilaterally be increased by a firm; thus, in equilibrium utility levels of workers would be higher than their bargaining outcomes if this also increased firms' profits. Suppose also that any deviation from equilibrium behavior leads to a termination of the employment relationship.

Given this setting, endogenous turnover costs still increase a firm's commitment and induce workers to exert higher effort. However, a positive bargaining power also provides incentives for workers to exert higher effort – because they want to remain employed to secure the associated rent in the future. If this rent is sufficiently high, a "voluntary" increase by firms is not profitable. But if the bargaining outcome is not sufficient to implement firms' desired effort, it remains optimal to increase the costs of turnover by paying workers an additional rent. The latter case is more likely if workers' bargaining power is low or if α^F is high, because a high α^F increases a firm's disagreement payoff and thus reduces the relationship surplus. Then, a higher labor supply will continue to increase an employed worker's compensation, making his bargaining power effectively irrelevant in determining his payoff. To the contrary, if bargaining outcomes would determine equilibrium payoffs (i.e., if worker bargaining power was large or α^F small), an increase in N would reduce a worker's compensation via the negative effect on his disagreement payoffs. Therefore,

[21] Those are hybrid models where individual choices are made non-cooperatively but bargaining follows the cooperative Nash-bargaining regime and deviations cause a termination of the relationship. See Miller and Watson, 2013 for an axiomatic foundation.

we would predict negative wage effects of a higher labor supply in markets where workers are paid their marginal productivities, and non-negative or even positive effects if firms have the power to set the terms of employment.

Second, we discuss the specific form of endogenous turnover costs. Individual firms would be indifferent between increasing a worker's compensation (as in our setting) or using different measures, for example letting workers temporarily reduce their effort or conduct inefficient trainings, or "money burning" that destroys surplus.[22] To assess a firm's credibility, though, it is necessary for workers to observe the realization of turnover costs. Thus, we would argue that the safest way for firms to ensure this is using options such as wages or effort reductions. It remains to discuss why, to make turnover more costly, firms do not use effort reductions in early periods instead of higher wages. Effort reductions would actually increase industry-wide profits, because workers' outside options would be zero throughout. However, if we extended the model slightly by introducing a product market where prices decrease in total output and a single firm has an impact on market outcomes (for example by making F discrete), paying higher wages would dominate effort reductions for individual firms. The reason is that a rent paid by one firm increases workers' outside options in other firms. By doing so, production becomes more expensive for other firms, which would consequently reduce their employment and output. Thus, this adverse effect on other firms increases the firm's profits.

[22]See Carmichael and MacLeod (1997) and McAdams (2011) who demonstrate that inefficiencies in the early periods of repeated interactions with anonymous re-matching might be needed to sustain cooperation later on.

Appendix

Proof of Lemma 1:

By Condition (1), $\alpha^F = m(f^*, n^*)/f^*$, and $\alpha^N = m(f^*, n^*)/n^*$, f^* and n^* are characterized by

$$F - f^* - \frac{m(f,n)}{(1-\gamma)} = 0, \quad N - n^* - \frac{m(f,n)}{(1-\gamma)} = 0.$$

The implicit function theorem yields

$$\frac{df}{dF} = \frac{\begin{vmatrix} -1 & -\frac{m_n(f,n)}{(1-\gamma)} \\ 0 & -1 - \frac{m_n(f,n)}{(1-\gamma)} \end{vmatrix}}{\begin{vmatrix} -1 - \frac{m_f(f,n)}{(1-\gamma)} & -\frac{m_n(f,n)}{(1-\gamma)} \\ -\frac{m_f(f,n)}{(1-\gamma)} & -1 - \frac{m_n(f,n)}{(1-\gamma)} \end{vmatrix}} = \frac{\left(1 + \frac{m_n(f,n)}{(1-\gamma)}\right)}{1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}} > 0,$$

$$\frac{df}{dN} = \frac{\begin{vmatrix} 0 & -\frac{m_n(f,n)}{(1-\gamma)} \\ -1 & -1 - \frac{m_n(f,n)}{(1-\gamma)} \end{vmatrix}}{\begin{vmatrix} -1 - \frac{m_f(f,n)}{(1-\gamma)} & -\frac{m_n(f,n)}{(1-\gamma)} \\ -\frac{m_f(f,n)}{(1-\gamma)} & -1 - \frac{m_n(f,n)}{(1-\gamma)} \end{vmatrix}} = -\frac{\frac{m_n(f,n)}{(1-\gamma)}}{1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}} < 0,$$

$$\frac{dn}{dF} = -\frac{\frac{m_f(f,n)}{(1-\gamma)}}{1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}} < 0, \qquad \frac{dn}{dN} = \frac{\left(1 + \frac{m_f(f,n)}{(1-\gamma)}\right)}{1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}} > 0.$$

Hence,

$$\begin{aligned}\frac{d\alpha^F}{dF} &= \frac{m_f(f,n)f - m(f,n)}{f^2}\frac{df}{dF} + \frac{m_n(f,n)}{f}\frac{dn}{dF} \\ &= \frac{m_f(f,n)f - m(f,n)\left(1 + \frac{m_n(f,n)}{(1-\gamma)}\right)}{f^2\left(1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}\right)} < 0,\end{aligned}$$

$$\begin{aligned}\frac{d\alpha^F}{dN} &= \frac{m_f(f,n)f - m(f,n)}{f^2}\frac{df}{dN} + \frac{m_n(f,n)}{f}\frac{dn}{dN} > 0, \\ \frac{d\alpha^N}{dF} &= \frac{m_f(f,n)}{n}\frac{df}{dF} + \frac{m_n(f,n)n - m(f,n)}{n^2}\frac{dn}{dF} > 0, \\ \frac{d\alpha^N}{dN} &= \frac{m_f(f,n)}{n}\frac{df}{dN} + \frac{m_n(f,n)n - m(f,n)}{n^2}\frac{dn}{dN} \\ &= \frac{m_n(f,n)n - m(f,n)\left(1 + \frac{m_f(f,n)}{(1-\gamma)}\right)}{n^2\left(1 + \frac{m_f(f,n)}{(1-\gamma)} + \frac{m_n(f,n)}{(1-\gamma)}\right)} < 0,\end{aligned}$$

where all inequalities hold by the assumption that the second partial derivatives of $m(f,n)$ are negative, which implies $m_j(f,n)j - m(f,n) \leq 0$ for $j \in \{f,n\}$. ■

Proof of Proposition 1:

Here, we first show that our stationarity assumption is without loss of generality. By standard arguments, stationary arrangements are optimal from the second period of an employment relationship. In the first period, however, wages might be different (if first-period effort or bonus were different than later values, the problem could be transformed into one that is payoff equivalent but in which only wages differ). Let w_1 denote the wage paid in the first period of an employment relationship and w denote the wage paid in all later periods. Then, the optimization problem is

$$\max \Pi^u = \frac{\alpha^F\left[e\theta - b - w_1 + \delta\gamma\left(w_1 - w\right)\right]}{\left(1-\delta\right)\left(1 - \delta\gamma\left(1-\alpha^F\right)\right)},$$

subject to

$$-e^2/2 + b + \delta\gamma w - \delta\gamma(1-\delta\gamma)V^u \geq 0 \tag{IC}$$

$$-b + \delta\gamma\left[\left(1-\alpha^F\right)e\theta + \alpha^F w_1 - w\right] \geq 0 \tag{DE}$$

$$w_1(1-\delta\gamma) + w\delta\gamma + b - e^2/2 - (1-\delta\gamma)V^u \geq 0 \tag{IR1}$$

$$w + b - e^2/2 - (1-\delta\gamma)V^u \geq 0 \tag{IR}$$

To show that it is (weakly) optimal to set $w_1 = w$, assume to the contrary that there is a profit-maximizing equilibrium with $w_1 > w$. Then, we can reduce b by $\delta\gamma\varepsilon$ and increase w by ε. This operation leaves Π^u, (IC), (DE), and (IR1) unaffected, but relaxes (IR). The opposite operation, increasing b by ε and reducing w by $\delta\gamma\varepsilon$, can be applied if $w_1 < w$, thus it is weakly optimal to set $w_1 = w$. Note that this holds for all periods except the very beginning of the game.

Therefore, (IR) is implied by (IR1) and can be omitted. The Lagrange function becomes

$$\begin{aligned}\mathcal{L} = &\frac{\alpha^F[e\theta - b - w]}{(1-\delta)(1-\delta\gamma(1-\alpha^F))}\\ &+ \lambda_{IC}\left[-e^2/2 + b + \delta\gamma w - \delta\gamma(1-\delta\gamma)V^u\right]\\ &+ \lambda_{DE}\left[-b + \delta\gamma\left(1-\alpha^F\right)(e\theta - w)\right]\\ &+ \lambda_{IR}\left[w + b - e^2/2 - (1-\delta\gamma)V^u\right],\end{aligned}$$

with first-order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial e} &= \frac{\alpha^F \theta}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} - \lambda_{IC} e + \lambda_{DE} \delta\gamma \left(1-\alpha^F\right)\theta - \lambda_{IR} e = 0, \\
\frac{\partial \mathcal{L}}{\partial b} &= -\frac{\alpha^F}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} + \lambda_{IC} - \lambda_{DE} + \lambda_{IR} = 0 \\
&\Rightarrow \lambda_{IR1} = \frac{\alpha^F}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} - \lambda_{IC} + \lambda_{DE}, \\
\frac{\partial \mathcal{L}}{\partial w} &= -\frac{\alpha^F}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} + \lambda_{IC}\delta\gamma - \lambda_{DE}\delta\gamma\left(1-\alpha^F\right) + \lambda_{IR} = 0, \\
&\Rightarrow \lambda_{DE} = \lambda_{IC}\frac{(1-\delta\gamma)}{(1-\delta\gamma(1-\alpha^F))}, \quad \lambda_{IR} = \frac{\alpha^F - (1-\delta)\delta\gamma\alpha^F\lambda_{IC}}{(1-\delta)(1-\delta\gamma(1-\alpha^F))}.
\end{aligned}
$$

Note also that, if $\lambda_{IC} = \lambda_{DE} = 0$, then $\lambda_{IR} > 0$. Therefore, we have the following three cases: **1)** $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$, **2)** $\lambda_{IC}, \lambda_{DE}, \lambda_{IR} > 0$, and **3)** $\lambda_{IC} = \lambda_{DE} = 0$ and $\lambda_{IR} > 0$. We analyze each of them in turn.

Case 1: $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$.

Now,

$$
\begin{aligned}
\lambda_{IR} =& \frac{\alpha^F - (1-\delta)\delta\gamma\alpha^F\lambda_{IC}}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} = 0 \\
\Rightarrow& \lambda_{IC} = \frac{1}{(1-\delta)\delta\gamma} \\
\Rightarrow& \lambda_{DE} = \frac{(1-\delta\gamma)}{(1-\delta\gamma(1-\alpha^F))(1-\delta)\delta\gamma},
\end{aligned}
$$

and effort is characterized by $e = \delta\gamma\theta$. Moreover, binding (IC) and (DE) constraints yield

$$
\begin{aligned}
w =& \frac{\delta\gamma\left(1-\delta\gamma\right)V^{u}+e^{2}/2-\delta\gamma\left(1-\alpha^{F}\right)e\theta}{\delta\gamma\alpha^{F}} \\
b =& \left(1-\alpha^{F}\right)\frac{\delta\gamma e\theta-e^{2}/2-\delta\gamma\left(1-\delta\gamma\right)V^{u}}{\alpha^{F}} \\
\Rightarrow w+b =& \frac{\delta\gamma\left(1-\delta\gamma\right)\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)V^{u}}{\delta\gamma\alpha^{F}} \\
&+\frac{\left(1-\delta\gamma\left(1-\alpha^{F}\right)\right)e^{2}/2-\delta\gamma\left(1-\alpha^{F}\right)e\theta\left(1-\delta\gamma\right)}{\delta\gamma\alpha^{F}} \\
=& \frac{\delta\gamma\left(1-\delta\gamma\right)\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)V^{u}}{\delta\gamma\alpha^{F}}+\left(\delta\gamma\theta\right)^{2}\frac{2\alpha^{F}-1+\delta\gamma\left(1-\alpha^{F}\right)}{2\delta\gamma\alpha^{F}},
\end{aligned}
$$

as well as

$$
\begin{aligned}
\Pi^{u} =& \frac{\frac{\delta\gamma\theta^{2}}{2}-\left(1-\delta\gamma\right)V^{u}}{\left(1-\delta\right)}, \\
V =& \left[\frac{2\alpha^{F}-1}{2}\left(\theta\delta\gamma\right)^{2}+\delta\gamma\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)V^{u}\right]/\delta\gamma\alpha^{F}.
\end{aligned}
$$

Using $V^{u}=\alpha^{N}\left[\frac{\left(2\alpha^{F}-1\right)}{2}\left(\theta\delta\gamma\right)^{2}\right]/\left[\delta\gamma\left(1-\delta\gamma\right)\left(\alpha^{F}-\alpha^{N}\right)\right]$ yields

$$
\begin{aligned}
V =& \frac{\left[1-\delta\gamma\left(1-\alpha^{N}\right)\right]\frac{\left(2\alpha^{F}-1\right)}{2}\left(\theta\delta\gamma\right)^{2}}{\delta\gamma\left(1-\delta\gamma\right)\left(\alpha^{F}-\alpha^{N}\right)}, \qquad (3)\\
w+b =& \frac{\left[1-\delta\gamma\left(1-\alpha^{N}\right)\right]\frac{\left(2\alpha^{F}-1\right)}{2}\left(\theta\delta\gamma\right)^{2}}{\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)}+\frac{\left(\theta\delta\gamma\right)^{2}}{2} \\
=& \frac{\left[1-\delta\gamma\left(1-\alpha^{N}\right)\right]\left(\delta\gamma\theta\right)^{2}\left(2\alpha^{F}-1\right)}{2\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)}, \\
\Pi =& \frac{\left(1-\delta\left(1-\alpha^{F}\right)\right)\frac{\left(1-2\alpha^{N}\right)\left(\theta\delta\gamma\right)^{2}}{2\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)}}{\left(1-\delta\right)}, \\
\Pi^{u} =& \frac{\alpha^{F}}{1-\delta\left(1-\alpha^{F}\right)}\Pi=\alpha^{F}\frac{\left(1-2\alpha^{N}\right)\left(\theta\delta\gamma\right)^{2}}{2\delta\gamma\left(1-\delta\right)\left(\alpha^{F}-\alpha^{N}\right)}. \qquad (4)
\end{aligned}
$$

Moreover, $w=\left[\frac{\left(2\alpha^{F}-1\right)}{2}\left(\theta\delta\gamma\right)^{2}\right]/\left[\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)\right]$ and
$b=\left(1-\alpha^{F}\right)\left[\frac{1-2\alpha^{N}}{2}\left(\theta\delta\gamma\right)^{2}\right]/\left(\alpha^{F}-\alpha^{N}\right)$.

The consistency requirements are $V \geq V^u$ and $V^u, \Pi^u \geq 0$. Regarding the former, let us first assume that the latter holds. Then,

$$V \geq V^u \quad \Leftrightarrow \quad \alpha^F \geq \frac{\delta\gamma\left(e\theta - (1-\delta\gamma)\,V^u\right) - c(e)}{\delta\gamma\left(e\theta - (1-\delta\gamma)\,V^u\right)}.$$

Due to symmetry, $V^u = 0$ at the threshold. Hence, this case is optimal for

$$\alpha^F > \overline{\alpha} \equiv 1 - \frac{c(e^*)}{\delta\gamma e^*\theta} = 1 - \frac{e^*}{2\delta\gamma\theta} = \frac{1}{2}.$$

Regarding $V^u, \Pi^u \geq 0$, note that (2) leads to $\alpha^F > \alpha^N \Leftrightarrow N > F$. By (3) and (4), $N > F$ is required for V (and V^u) to be positive. It remains to show that, if $N > F$, Π^u is also positive, which requires $1 - 2\alpha^N > 0$. Because $\alpha^F = \frac{(1-\gamma)\alpha^N N}{F(1-\gamma+\alpha^N)-\alpha^N N}$ by (2), $F/N = \frac{3-2\gamma}{1-\gamma+\alpha^N}\alpha^N$ holds at $\alpha^F = 1/2$. Note also that

$$\begin{aligned}
& 1 - 2\alpha^N > 0 \\
\Leftrightarrow & 2\alpha^N < 1 \\
\Leftrightarrow & (2-2\gamma)\alpha^N < 1 - \gamma \\
\Leftrightarrow & (3-2\gamma)\alpha^N < 1 - \gamma + \alpha^N \\
\Leftrightarrow & \frac{3-2\gamma}{1-\gamma+\alpha^N}\alpha^N < 1.
\end{aligned}$$

Hence, $\alpha^N < 1/2$ if and only if $N > F$ at $\alpha^F = 1/2$. Because $\frac{\partial\alpha^F}{\partial N} > 0$ and $\frac{\partial\alpha^N}{\partial N} < 0$ by Lemma 1, if $N > F$ at $\alpha^F = 1/2$, then all conditions are satisfied for all $\alpha^F > 1/2$.

If $N < F$ at $\alpha^F = 1/2$, then for the α^F above $1/2$ which do not satisfy the consistency conditions $V \geq V^u$ and $V^u, \Pi^u \geq 0$, some firms become inactive to secure non-negative profits for each active firm. To see this, given N and F, let $\tilde{F} > 0$ denote the mass of active firms in the market. Because $\alpha^F > \alpha^N \Leftrightarrow N > \tilde{F}$ is required for Π and Π^u to be non-negative, $\tilde{F} < F$ must hold: some firms must be inactive in the market. Because such an inactive firm would choose to be active if active firms earned strictly positive profits, in the steady state $\tilde{F}$ is determined by $\Pi^u = 0$ and hence $1 - 2\alpha^N = 0$. By Lemma 1 and the assumptions on the matching function, there exists such $\tilde{F} \in (0, F)$. Finally, let $\hat{\alpha} \in (1/2, 1)$ denote the value of α^F when $1 - 2\alpha^N = 0$. It is straightforward to confirm that the consistency conditions are satisfied for $\alpha^F > \hat{\alpha}$, and hence, all firms are active for $\alpha^F > \hat{\alpha}$.

Case 2: $\lambda_{IC}, \lambda_{DE} > 0$ **and** $\lambda_{IR} > 0$

Binding (IC) and (DE) constraints yield

$$
\begin{aligned}
w =& \frac{\delta\gamma (1-\delta\gamma) V^u + e^2/2 - \delta\gamma \left(1-\alpha^F\right) e\theta}{\delta\gamma\alpha^F} \\
b =& \left(1-\alpha^F\right) \frac{\delta\gamma e\theta - e^2/2 - \delta\gamma (1-\delta\gamma) V^u}{\alpha^F} \\
\Rightarrow w + b =& \frac{\delta\gamma (1-\delta\gamma) V^u \left[1 - \left(1-\alpha^F\right)\delta\gamma\right]}{\delta\gamma\alpha^F} \\
&+ \frac{\left[1 - \delta\gamma\left(1-\alpha^F\right)\right] e^2/2 - \delta\gamma\left(1-\alpha^F\right) e\theta (1-\delta\gamma)}{\delta\gamma\alpha^F},
\end{aligned}
$$

The binding (IR) constraint delivers $V = V^u = 0$ and equilibrium effort which is characterized by $\delta\gamma\left(1-\alpha^F\right) e^*\theta - e^2/2 = 0$, thus

$$
e^* = 2\left(1-\alpha^F\right)\delta\gamma\theta. \tag{5}
$$

Finally, incorporating (5) yields

$$
\begin{aligned}
w =& \frac{(e^*)^2/2 - \delta\gamma\left(1-\alpha^F\right) e^*\theta}{\delta\gamma\alpha^F} = 0, \\
b =& \frac{(e^*)^2}{2} = 2\left(1-\alpha^F\right)^2 (\delta\gamma\theta)^2, \\
\Pi^u =& \frac{\alpha^F \left(e^*\theta - c(e^*)\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} = 2\delta\gamma\theta^2 \frac{\alpha^F\left(1-\alpha^F\right)}{(1-\delta)}.
\end{aligned}
$$

Case 3: $\lambda_{IC} = \lambda_{DE} = 0$**,** $\lambda_{IR} > 0$

Now, $e^* = e^{FB} = \theta$. This case holds if

$$
\alpha^F < 1 - \frac{c(e^{FB})}{\delta\gamma e^{FB}\theta} = 1 - \frac{1}{2\delta\gamma}\left(< \frac{1}{2}\right),
$$

then

$$
\begin{aligned}
w =& 0, \\
b =& c(e^{FB}) = \frac{\theta^2}{2}, \\
\Pi^u =& \frac{\theta^2 \alpha^F}{2\left(1-\delta\right)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}.
\end{aligned}
$$

Finally, we show that in Case 1 with $N < F$ at $\alpha^F = 1/2$, a profit-maximizing equilibrium in which all firms are active and workers are not paid a rent (derived in Cases 2 or 3) does not exist. In Case 1, we have already shown that an equilibrium with $N < F$ does not exist if all employed workers receive a rent. Suppose, toward a contradiction, that all firms do not pay a rent. Then, an individual firm would find it optimal to increase wages to secure $e = \delta\gamma\theta$. This is because the profits are $\Pi^u = \frac{\frac{\delta\gamma\theta^2}{2} - (1-\delta\gamma)V^u}{(1-\delta)}$ in Case 1, whereas $\Pi^u = 2\delta\gamma\theta^2 \frac{\alpha^F\left(1-\alpha^F\right)}{(1-\delta)}$ in Case 2. These two profits are the same at $\alpha^F = 1/2$, and the former is strictly larger than the latter for any $\alpha^F > 1/2$. Given that all other firms do not pay a rent, a worker's outside option is $V^u = 0$, thus an individual firm would profitably deviate by paying a rent and generating profits in Case 1 — a contradiction. Therefore, a profit-maximizing equilibrium requires some firms to be inactive in such a case. ■

Proof of Proposition 2:

We compute comparative statics separately for the cases $\alpha^F > \overline{\alpha}$ (workers are paid a rent) and $\alpha^F \leq \overline{\alpha}$ (agents receive no rent)

1. $\alpha^F > \overline{\alpha}$

Note that $e^* = \delta\gamma\theta$ if $\alpha^F > \overline{\alpha}$ by Proposition 1. Hence, $de^*/d\alpha^F = 0$, and

$$
\begin{aligned}
w^* + b^* = & \frac{(\theta\delta\gamma)^2}{2} + \delta\gamma\theta^2 \frac{\left(2\alpha^F - 1\right)\left[(1-\delta\gamma)\left(1-\gamma+\alpha^F\right)N - \alpha^F\left(1-\gamma\delta(2-\gamma)\right)F\right]}{2\left(1-\gamma+\alpha^F\right)(N-F)\alpha^F} \\
\Rightarrow \frac{d\left(w^* + b^*\right)}{dN} = & -\delta\gamma\theta^2\left(2\alpha^F - 1\right)(1-\gamma)\frac{1-\delta\gamma\left(1-\alpha^F\right)}{2\left(1-\gamma+\alpha^F\right)(N-F)^2\alpha^F}F \\
& + \delta\gamma\theta^2\left[\frac{(1-\delta\gamma)N}{2\left(\alpha^F\right)^2(N-F)} - \frac{\left(1-\gamma\delta(2-\gamma)\right)F(3-2\gamma)}{2\left(1-\gamma+\alpha^F\right)^2(N-F)}\right]\frac{d\alpha^F}{dN}.
\end{aligned}
$$

This term can either be positive or negative. However, at $\alpha^F = \overline{\alpha} = 1/2$,

$$
\frac{d\left(w^* + b^*\right)}{dN} = 2\delta\gamma\theta^2 \frac{(1-\gamma)\left[2(1-\delta\gamma)N + \gamma\delta F\right] + (1-\delta\gamma)(N-F)}{(N-F)(3-2\gamma)}\frac{d\alpha^F}{dN},
$$

which is positive for $N > F$ because $d\alpha^F/dN > 0$ by Lemma 1. Finally, as $de^*/d\alpha^F = 0$, an employed worker's utility $V = \left(w + b - e^2/2\right)/(1-\delta\gamma)$ moves into the same direction as $w^* + b^*$.

2. $\alpha^F \leq \overline{\alpha}$

By Proposition 1, an employed worker's utility is $V = 0$ in this case, so it is unaffected by N. Moreover, $w^* + b^* = b^* = c(e^*)$.

Suppose first that α^F is sufficiently close to $\overline{\alpha}$ so that $e^* = 2\left(1-\alpha^F\right)\delta\gamma\theta < e^{FB}$. Then,

$$
\frac{de^*}{dN} = -2\delta\gamma\theta\frac{d\alpha^F}{dN} < 0, \qquad \frac{d\left(w^* + b^*\right)}{dN} = c'(e^*)\frac{de^*}{dN} < 0.
$$

Suppose second that α^F is so small that $e^* = e^{FB} = \theta$. Then,

$$
\frac{de^*}{dN} = \frac{d\left(w^* + b^*\right)}{dN} = 0.
$$

■

Proof of Proposition 3:

Existence of equilibrium under free entry We first establish for which levels of α^F (and consequently N and F) an equilibrium exists. There, we take into account that $\Pi^u = K$ and $\partial\Pi^u/\partial F \leq 0$ must hold under free entry. If the latter was violated, more firms would increase the profits of an individual firm, causing additional entry and a reduction of α^F.

First, note that the highest individual-firm profits can be achieved at $\overline{\alpha} = 1/2$: For $\alpha^F \leq 1/2$, profits are $\Pi^u = \frac{2\alpha^F\left(1-\alpha^F\right)\delta\gamma\theta^2}{(1-\delta)}$ and maximized for $\alpha^F = 1/2$, in which case they equal $\delta\gamma\theta^2/2\,(1-\delta)$. For $\alpha^F > 1/2$ (and provided $N > F$, i.e., as shown in the proof to Proposition 1, all firms are active – if there was a range with some inactive firms, Π^u would have to be zero anyway which is clearly smaller than $\delta\gamma\theta^2/2\,(1-\delta)$), profits are $\Pi^u = \left[\frac{\delta\gamma\theta^2}{2} - (1-\delta\gamma)\,V^u\right]/\,(1-\delta)$. This are maximized for $V^u = 0$, in which case its value is $\delta\gamma\theta^2/2\,(1-\delta)$ as well. Therefore, the maximum level $\tilde{K}$ that is associated with an equilibrium is

$$\tilde{K} = \frac{\delta\gamma\theta^2}{2\,(1-\delta)}.$$

For all entry costs smaller than $\tilde{K}$, there may be an equilibrium with $\alpha^F < 1/2$ and another equilibrium with $\alpha^F > 1/2$. In the following, we explore whether the condition $\partial\Pi^u/\partial F \leq 0$ is satisfied for each candidate equilibrium.

In a candidate equilibrium with $\alpha^F < 1/2$, the profits are

$$\Pi^u = \frac{2\alpha^F\left(1-\alpha^F\right)\delta\gamma\theta^2}{(1-\delta)},$$

thus

$$\frac{\partial\Pi^u}{\partial F} = \frac{2\delta\gamma\theta^2\left(1-2\alpha^F\right)}{(1-\delta)}\frac{\partial\alpha^F}{\partial F}.$$

Because $\partial\alpha^F/\partial F < 0$, $\partial\Pi^u/\partial F$ is non-positive for all $\alpha^F \leq 1/2$, thus this equilibrium always exists.

In a candidate equilibrium with $\alpha^F > 1/2$, $N > F$ must hold; otherwise, as in the proof of Proposition 1, some firm must be inactive (and $\Pi^u = 0$). Assuming

$\alpha^F > 1/2$ and $N > F$,

$$\Pi^u = \frac{\frac{\delta\gamma\theta^2}{2} - (1-\delta\gamma)\, V^u}{(1-\delta)},$$

which implies $\frac{\partial \Pi^u}{\partial F} < 0 \Leftrightarrow \frac{\partial V^u}{\partial F} > 0$. Since

$$V^u = \delta\gamma\theta^2 \frac{\left(2\alpha^F - 1\right)(1-\gamma)F}{2\,(1-\delta\gamma)\,(1-\gamma+\alpha^F)\,(N-F)},$$

$$\frac{\partial V^u}{\partial F} = \frac{\delta\gamma\theta^2(1-\gamma)}{2\,(1-\delta\gamma)} \left[\frac{\left(2\alpha^F - 1\right) N}{(1-\gamma+\alpha^F)\,(N-F)^2} + \frac{(3-2\gamma)\, F}{(1-\gamma+\alpha^F)^2\,(N-F)} \frac{\partial \alpha^F}{\partial F} \right]$$

must be positive, i.e., the first term in the squared bracket must exceed the second term (in absolute values). Note that $\partial V^u/\partial F < 0$ when $\alpha^F = 1/2$, whereas $\partial V^u/\partial F > 0$ when $N/F \to \infty$ and hence $\alpha^F \to 1$. Thus, given $N > F$, there exists an equilibrium for α^F sufficiently close to 1. If α^F is slightly above 1/2, $\partial \Pi^u/\partial F > 0$, and hence additional firm entry would reduce α^F until α^F becomes lower than 1/2, thus an equilibrium with α^F close to but above 1/2 does not exist. In terms of values of K that are consistent with an equilibrium with $\alpha^F > 1/2$, note that, for $N/F \to \infty$, $\Pi^u \to \frac{\delta\gamma\theta^2}{2(1-\delta)} = \tilde{K}$.

Lastly, we derive the condition in which an equilibrium with $\alpha^F > 1/2$ exists. At $\tilde{K} = 0$ or equivalently $\Pi^u = 0$, $\delta\gamma\theta^2/2 - (1-\delta\gamma)\, V^u = 0$, thus

$$\left(1-\gamma+\alpha^F\right) N - F\alpha^F(3-2\gamma) = 0.$$

There,

$$\frac{\partial V^u}{\partial F} = \frac{\delta\gamma\theta^2}{2\,(1-\delta\gamma)} \frac{(3-2\gamma)}{(2\alpha^F - 1)} \left[\frac{\alpha^F}{F\,(1-\gamma)} + \frac{1}{(1-\gamma+\alpha^F)} \frac{\partial \alpha^F}{\partial F} \right],$$

which can be positive or negative. Therefore, either an equilibrium with $\alpha^F > 1/2$

exists for all $K \in \left[0, \tilde{K}\right]$, or there is $\underline{K} < \tilde{K}$ such that such an equilibrium only exists for $K \in \left[\underline{K}, \tilde{K}\right]$.

Comparative Statics with respect to N We again distinguish between $\alpha^F \leq 1/2$ and $\alpha^F > 1/2$, where in the latter case we presume that the above equilibrium is played, and conduct comparative statics for each case separately.

1. $\alpha^F > 1/2$

Recall that, in the proof of Proposition 2, we have shown that

$$\begin{aligned}
\Pi^u =& \frac{\frac{\delta\gamma\theta^2}{2} - (1-\delta\gamma)\,V^u}{(1-\delta)} \\
=& \delta\gamma\theta^2 \frac{\left(1-\gamma+\alpha^F\right)N - F\alpha^F\,(3-2\gamma)}{2\,(1-\delta)\,(1-\gamma+\alpha^F)\,(N-F)}, \\
V^u =& \delta\gamma\theta^2 \frac{\left(2\alpha^F - 1\right)(1-\gamma)F}{2\,(1-\delta\gamma)\,(1-\gamma+\alpha^F)\,(N-F)}.
\end{aligned}$$

Moreover,

$$w + b = \frac{\delta\gamma\,(1-\delta\gamma)\,V^u\left(1-\left(1-\alpha^F\right)\delta\gamma\right)}{\delta\gamma\alpha^F} + (\delta\gamma\theta)^2\,\frac{2\alpha^F - 1 + \delta\gamma\left(1-\alpha^F\right)}{2\delta\gamma\alpha^F}.$$

Thus

$$
\begin{aligned}
\frac{d\left(w+b\right)}{dN} = & -\frac{\left(1-\delta\gamma\right)^{2}}{\left(\alpha^{F}\right)^{2}}V^{u}\left(\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN}\right) \\
& +\frac{\left(1-\delta\gamma\right)\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)}{\alpha^{F}}\frac{dV^{u}}{dN} \\
& +\delta\gamma\theta^{2}\frac{\left(1-\delta\gamma\right)}{2\left(\alpha^{F}\right)^{2}}\left(\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN}\right) \\
= & \frac{\left(1-\delta\gamma\right)\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)}{\alpha^{F}}\frac{dV^{u}}{dN} \\
& +\frac{\left(1-\delta\gamma\right)}{\left(\alpha^{F}\right)^{2}}\left(\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN}\right)\left[\frac{\delta\gamma\theta^{2}}{2}-\left(1-\delta\gamma\right)V^{u}\right] \\
= & \frac{\left(1-\delta\gamma\right)\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)}{\alpha^{F}}\frac{dV^{u}}{dN} \\
& +\frac{\left(1-\delta\gamma\right)}{\left(\alpha^{F}\right)^{2}}\left(\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN}\right)\left(1-\delta\right)\Pi^{u}.
\end{aligned}
$$

The free-entry condition indicates that $\Pi^{u}=\left[\frac{\delta\gamma\theta^{2}}{2}-\left(1-\delta\gamma\right)V^{u}\right]/\left(1-\delta\right)=K$ must always hold, hence $d\Pi^{u}/dN=0$. It follows that $dV^{u}/dN=0$ as well, and

$$
\frac{d\left(w+b\right)}{dN}=\left(1-\delta\right)\frac{\left(1-\delta\gamma\right)}{\left(\alpha^{F}\right)^{2}}\left(\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN}\right)\Pi^{u}.
$$

Therefore, the sign of $d\left(w+b\right)/dN$ is identical to the sign of

$$
\frac{d\alpha^{F}}{dN}=\frac{\partial\alpha^{F}}{\partial N}+\frac{\partial\alpha^{F}}{\partial F}\frac{dF}{dN},
$$

with

$$
\begin{aligned}
\frac{dF}{dN} & =-\frac{\partial\Pi^{u}/\partial N}{\partial\Pi^{u}/\partial F} \\
& =-\frac{\partial V^{u}/\partial N}{\partial V^{u}/\partial F},
\end{aligned}
$$

where $\partial\Pi^{u}/\partial F<0$ and thus $\partial V^{u}/\partial F>0$ in equilibrium.

Moreover,

$$\frac{\partial V^u}{\partial F} = \frac{\delta\gamma\theta^2(1-\gamma)}{2(1-\delta\gamma)}\left[\frac{\left(2\alpha^F-1\right)N}{(1-\gamma+\alpha^F)(N-F)^2} + \frac{(3-2\gamma)F}{(1-\gamma+\alpha^F)^2(N-F)}\frac{\partial\alpha^F}{\partial F}\right],$$

$$\frac{\partial V^u}{\partial N} = \frac{\delta\gamma\theta^2(1-\gamma)}{2(1-\delta\gamma)}F\left[-\frac{\left(2\alpha^F-1\right)}{(1-\gamma+\alpha^F)(N-F)^2} + \frac{(3-2\gamma)}{(1-\gamma+\alpha^F)^2(N-F)}\frac{\partial\alpha^F}{\partial N}\right],$$

$$\Rightarrow \frac{\partial V^u}{\partial N} = -\frac{\delta\gamma\theta^2(1-\gamma)}{2(1-\delta\gamma)}\frac{\left(2\alpha^F-1\right)}{(1-\gamma+\alpha^F)(N-F)^2}\left(F + N\frac{\frac{\partial\alpha^F}{\partial N}}{\frac{\partial\alpha^F}{\partial F}}\right) + \frac{\partial V^u}{\partial F}\frac{\frac{\partial\alpha^F}{\partial N}}{\frac{\partial\alpha^F}{\partial F}}.$$

Thus

$$\frac{dF}{dN} = \frac{\delta\gamma\theta^2\frac{(1-\gamma)\left(2\alpha^F-1\right)}{2(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)^2}\left(F + N\frac{\frac{\partial\alpha^F}{\partial N}}{\frac{\partial\alpha^F}{\partial F}}\right) - \frac{\partial V^u}{\partial F}\frac{\frac{\partial\alpha^F}{\partial N}}{\frac{\partial\alpha^F}{\partial F}}}{\frac{\partial V^u}{\partial F}}$$

and

$$\frac{d\alpha^F}{dN} = \frac{\delta\gamma\theta^2\frac{(1-\gamma)\left(2\alpha^F-1\right)}{2(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)^2}\left(F\frac{\partial\alpha^F}{\partial F} + N\frac{\partial\alpha^F}{\partial N}\right)}{\frac{\partial V^u}{\partial F}}.$$

Therefore, the sign of $d\alpha^F/dN$ – and consequently the sign of $d(w+b)/dN$ — is identical to the sign of

$$F\frac{\partial\alpha^F}{\partial F} + N\frac{\partial\alpha^F}{\partial N}.$$

From the proof of Lemma 1 and using the steady-state conditions

$$f = F - \frac{m(f,n)}{(1-\gamma)}, \qquad n = N - \frac{m(f,n)}{(1-\gamma)},$$

it follows that

$$
\begin{aligned}
&F\frac{\partial\alpha^F}{\partial F}+N\frac{\partial\alpha^F}{\partial N}\\
=&F\frac{m_f(f,n)f-m(f,n)\left(1+\frac{m_n(f,n)}{(1-\gamma)}\right)}{f^2\left[1+\frac{m_f(f,n)}{(1-\gamma)}+\frac{m_n(f,n)}{(1-\gamma)}\right]}+N\frac{m_n(f,n)\frac{f(1-\gamma)+m(f,n)}{(1-\gamma)}}{f^2\left[1+\frac{m_f(f,n)}{(1-\gamma)}+\frac{m_n(f,n)}{(1-\gamma)}\right]}\\
=&\frac{\left(\frac{m(f,n)}{(1-\gamma)}+f\right)(m_f(f,n)f+m_n(f,n)n-m(f,n))}{f^2\left[1+\frac{m_f(f,n)}{(1-\gamma)}+\frac{m_n(f,n)}{(1-\gamma)}\right]}.
\end{aligned}
$$

All this implies that the sign of $F\frac{\partial\alpha^F}{\partial F}+N\frac{\partial\alpha^F}{\partial N}$ — and consequently the sign of $d\,(w+b)\,/dN$ — is the same as the sign of

$$m_f(f,n)f+m_n(f,n)n-m(f,n),$$

which is positive (negative) if $m(f,n)$ has increasing (decreasing) returns to scale, and zero with constant returns to scale.[23]

Finally, we look at the sign of

$$\frac{dF}{dN}=-\frac{\partial V^u/\partial N}{\partial V^u/\partial F},$$

which is equivalent to the sign of $-\partial V^u/\partial N$, which can be positive or negative; see the proof of Proposition 2.

2. $\alpha^F\leq 1/2$

Now, equilibrium effort is given by

$$e^*=2\left(1-\alpha^F\right)\delta\gamma\theta \tag{6}$$

[23]To see this, note that the definition of the constant returns to scale is $m(f,n)r=m(rf,rn)$ for any $r>0$. Examining the change at $r=1$ leads to the result. The result for increasing or decreasing returns to scale can be derived in the same way.

or $e^* = e^{FB} = \theta$, whichever is smaller. If $e^* = e^{FB}$, $de^*/dN = 0$. If e^* is determined by (6), then

$$\frac{\partial e^*}{\partial \alpha^F} = -2\delta\gamma\theta < 0$$

and

$$\Pi^u = \frac{2\alpha^F \left(1 - \alpha^F\right) \delta\gamma\theta^2}{(1-\delta)}.$$

Thus,

$$\begin{aligned} \frac{\partial \Pi^u}{\partial F} &= \frac{2\delta\gamma\theta^2}{(1-\delta)} \left(1 - 2\alpha^F\right) \frac{\partial \alpha^F}{\partial F}, \\ \frac{\partial \Pi^u}{\partial N} &= \frac{2\delta\gamma\theta^2}{(1-\delta)} \left(1 - 2\alpha^F\right) \frac{\partial \alpha^F}{\partial N}, \end{aligned}$$

where the first equation implies that $\partial \Pi^u / \partial F < 0 \Leftrightarrow \alpha^F \leq 1/2$, and

$$\frac{dF}{dN} = -\frac{\frac{\partial \alpha^F}{\partial N}}{\frac{\partial \alpha^F}{\partial F}} > 0,$$

i.e., employment effects are positive, and

$$\frac{de^*}{dN} = \frac{\partial e^*}{\partial \alpha^F} \left(\frac{\partial \alpha^F}{\partial N} + \frac{\partial \alpha^F}{\partial F} \frac{dF}{dN} \right) = \frac{\partial e^*}{\partial \alpha^F} \left(\frac{\partial \alpha^F}{\partial N} - \frac{\partial \alpha^F}{\partial F} \frac{\frac{\partial \alpha^F}{\partial N}}{\frac{\partial \alpha^F}{\partial F}} \right) = 0.$$

It follows that $d(w + b)/dN = 0$.

■

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Online Appendix for "Informal Incentives and Labor Markets" by Fahn and Murooka

A Remaining Proofs

Proof of Proposition 4:

We first set up the general problem and then derive conditions for a discrimination equilibrium as described in Proposition 4 to exist. Our objective is to maximize Π^u, subject to the following set of constraints:

$$-b^I + \delta\gamma\left(\Pi^I - \Pi^u\right) \geq 0 \tag{DEI}$$

$$-b^O + \delta\gamma\left(\Pi^O - \Pi^u\right) \geq 0 \tag{DEO}$$

$$V^I - V^{uI} \geq 0 \tag{IRI}$$

$$V^O - V^{uO} \geq 0 \tag{IRO}$$

$$-c(e^I) + b^I + \delta\gamma\left[V^I - V^{uI}\right] \geq 0 \tag{ICI}$$

$$-c(e^O) + b^O + \delta\gamma\left[V^O - V^{uO}\right] \geq 0 \tag{ICO}$$

Now, $\Pi^I = \pi^I + \delta\left[\gamma\Pi^I + (1-\gamma)\Pi^u\right]$ and $\Pi^O = \pi^O + \delta\left[\gamma\Pi^O + (1-\gamma)\Pi^u\right]$, where $\pi^I = e^I\theta - w^I - b^I$ and $\pi^O = e^O\theta - w^O - b^O$. Thus,

$$
\begin{aligned}
\Pi^I =& \frac{\pi^I\left(1-\delta+\delta\alpha^{FI}\right)(1-\delta\gamma)+\delta\alpha^{FO}\left[(1-\delta)\gamma\pi^I+(1-\gamma)\pi^O\right]}{(1-\delta)(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \\
\Pi^O =& \frac{\pi^O\left(1-\delta+\delta\alpha^{FO}\right)(1-\delta\gamma)+\delta\alpha^{FI}\left[(1-\delta)\gamma\pi^O+(1-\gamma)\pi^I\right]}{(1-\delta)(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \\
\Pi^u =& \frac{\alpha^{FI}\pi^I+\alpha^{FO}\pi^O}{(1-\delta)(1-\delta\gamma+\alpha^F\delta\gamma)} \\
\Pi^I-\Pi^u =& \frac{\left(1-\alpha^{FI}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^I-\alpha^{FO}\pi^O}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \\
\Pi^O-\Pi^u =& \frac{\left(1-\alpha^{FO}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^O-\alpha^{FI}\pi^I}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)}
\end{aligned}
$$

This allows us to rewrite the optimization problem, which becomes to maximize $\alpha^{FI}\pi^I+\alpha^{FO}\pi^O$, subject to

$$-b^I+\delta\gamma\frac{\left(1-\alpha^{FI}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^I-\alpha^{FO}\pi^O}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)}\geq 0 \tag{DEI}$$

$$-b^O+\delta\gamma\frac{\left(1-\alpha^{FO}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^O-\alpha^{FI}\pi^I}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)}\geq 0 \tag{DEO}$$

$$-(e^I)^2/2+b^I+\delta\gamma\left(w^I-(1-\delta\gamma)V^{uI}\right)\geq 0 \tag{ICI}$$

$$w^I+b^I-c(e^I)-(1-\delta\gamma)V^{uI}\geq 0 \tag{IRI}$$

$$-(e^O)^2/2+b^O+\delta\gamma\left(w^O-(1-\delta\gamma)V^{uO}\right)\geq 0 \tag{ICO}$$

$$w^O+b^O-c(e^O)-(1-\delta\gamma)V^{uO}\geq 0 \tag{IRO}$$

There, $V^{uI}=\alpha^N V^I+(1-\alpha^N)\delta\gamma V^{uI}$ and $V^{uO}=\alpha^N V^O+(1-\alpha^N)\delta\gamma V^{uO}$ are taken as given by firms.

Setting up the Lagrange function and obtaining first-order conditions yields

$$
\begin{aligned}
\frac{\partial L}{\partial e^I} =& \alpha^{FI}\theta + \lambda_{DEI}\delta\gamma\frac{\theta\left(1-\alpha^{FI}\right)(1-\delta\gamma)+\alpha^{FO}\delta\gamma\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
&- \lambda_{DEO}\delta\gamma\frac{\alpha^{FI}\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - e^I\left(\lambda_{ICI}+\lambda_{IRI}\right)=0, \\
\frac{\partial L}{\partial e^O} =& \alpha^{FO}\theta - \lambda_{DEI}\delta\gamma\frac{\alpha^{FO}\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
&+ \lambda_{DEO}\delta\gamma\frac{\theta\left(1-\alpha^{FO}\right)(1-\delta\gamma)+\alpha^{FI}\delta\gamma\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - e^O\left(\lambda_{ICO}+\lambda_{IRO}\right)=0, \\
\frac{\partial L}{\partial b^I} =& -\alpha^{FI} - \lambda_{DEI}\frac{1-\delta\gamma+\alpha^{FO}\delta\gamma}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
&+ \lambda_{DEO}\frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} + \lambda_{ICI} + \lambda_{IRI} = 0 \\
\Rightarrow& \lambda_{ICI} = \alpha^{FI} + \frac{\lambda_{DEI}\left(1-\delta\gamma+\alpha^{FO}\delta\gamma\right)-\lambda_{DEO}\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - \lambda_{IRI}, \\
\frac{\partial L}{\partial w^I} =& -\alpha^{FI} - \lambda_{DEI}\delta\gamma\frac{\left(1-\alpha^{FI}\right)(1-\delta\gamma)+\alpha^{FO}\delta\gamma}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
&+ \lambda_{DEO}\frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} + \lambda_{ICI}\delta\gamma + \lambda_{IRI} = 0 \\
\Rightarrow& \lambda_{IRI} = \alpha^{FI}\left(1-\frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)}\right) \\
\Rightarrow& \lambda_{ICI} = \frac{\lambda_{DEI}}{(1-\delta\gamma)},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial b^O} =& -\alpha^{FO} + \lambda_{DEI}\frac{\delta\gamma\alpha^{FO}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
& -\lambda_{DEO}\frac{1-\delta\gamma+\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} + \lambda_{ICO} + \lambda_{IRO} = 0 \\
\Rightarrow & \lambda_{ICO} = \alpha^{FO} - \frac{\lambda_{DEI}\delta\gamma\alpha^{FO} - \lambda_{DEO}\left(1-\delta\gamma+\delta\gamma\alpha^{FI}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} - \lambda_{IRO}, \\
\frac{\partial L}{\partial w^O} =& -\alpha^{FO} + \delta\gamma\frac{\lambda_{DEI}\alpha^{FO} - \lambda_{DEO}\left[\left(1-\alpha^{FO}\right)(1-\delta\gamma) + \alpha^{FI}\delta\gamma\right]}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
& + \lambda_{ICO}\delta\gamma + \lambda_{IRO} = 0 \\
\Rightarrow & \lambda_{IRO} = \alpha^{FO}\left(1 - \frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) \\
\Rightarrow & \lambda_{ICO} = \frac{\lambda_{DEO}}{(1-\delta\gamma)}.
\end{aligned}
$$

Thus, effort levels are characterized by

$$
\begin{aligned}
\alpha^{FI}\left(\theta - e^I\right)\left(1 - \frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) + \frac{\delta\gamma\theta - e^I}{(1-\delta\gamma)}\lambda_{DEI} = 0, \\
\alpha^{FO}\left(\theta - e^O\right)\left(1 - \frac{\delta\gamma\left(\lambda_{DEO}+\lambda_{DEI}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) + \frac{\delta\gamma\theta - e^O}{(1-\delta\gamma)}\lambda_{DEO} = 0.
\end{aligned}
$$

Results

The previous analysis yields the following outcomes: Either, $\lambda_{IRI} = \lambda_{IRO} = 0$ or $\lambda_{IRI}, \lambda_{IRO} > 0$. Moreover, either $\lambda_{DEI}, \lambda_{ICI} > 0$ or $\lambda_{DEI} = \lambda_{ICI} = 0$, and equivalently for λ_{DEO} and λ_{ICO}.

For the proposition where our objective is to prove existence of a discrimination equilibrium in which insiders are better off than outsiders, we restrict our attention to $\lambda_{IRI} = \lambda_{IRO} = 0$. This is because insiders are not paid a rent in the first place when $\lambda_{IRI}, \lambda_{IRO} > 0$.

If $\lambda_{IRI} = \lambda_{IRO} = 0$, then $\lambda_{DEI} + \lambda_{DEO} = \frac{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}{\delta\gamma}$, thus effort levels are characterized by

$$\left(\delta\gamma\theta - e^I\right)\frac{\lambda_{DEI}}{(1-\delta\gamma)} = 0,$$
$$\left(\delta\gamma\theta - e^O\right)\frac{\lambda_{DEO}}{(1-\delta\gamma)} = 0.$$

Discrimination equilibrium

To show that a profit-maximizing equilibrium exists which favors insiders at the expense of outsiders, we focus on the case

$$\lambda_{ICO}, \lambda_{DEO} = 0, \quad \lambda_{ICI} = \lambda_{DEI} > 0.$$

We do so and set $\lambda_{ICO}, \lambda_{DEO} = 0$ because π^O has no direct effect on profits in such an equilbrium (see below). Then, while $e^I = \delta\gamma\theta$, e^O is not uniquely identified.

Binding (ICI) and (DEI) constraints yield

$$b^I = c(e^I) - \delta\gamma\left(w^I - (1-\delta\gamma)V^{uI}\right),$$
$$w^I = \frac{\delta\gamma e^I\theta(1-\delta\gamma)\alpha^{FI} + \delta\gamma\alpha^{FO}\pi^O - \left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)V^{uI}\right]\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right)}{\delta\gamma(1-\delta\gamma)\alpha^{FI}},$$
$$w^I + b^I = \frac{\delta\gamma\alpha^{FO}\pi^O + \alpha^{FI}\delta\gamma e^I\theta - \left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)V^{uI}\right]\left(1-\delta\gamma+\delta\gamma\alpha^{F}\right)}{\delta\gamma\alpha^{FI}},$$

hence an insider's compensation increases in π^O. Because both $V^I = \frac{w^I+b^I-(e^I)^2/2}{1-\delta\gamma}$ and $e^I = \delta\gamma\theta$, (employed and unemployed) insiders benefit from a (ceteris paribus) higher π^O. Note also that the expected profits are

$$\Pi^u = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)V^{uI}\right]}{\delta\gamma(1-\delta\gamma)},$$

thus π^O has no direct effect on an individual firm's expected profits (it affects V^{uI} which, however, cannot be affected by an individual firm).

Auxiliary problem

By the above considerations, in a potential equilibrium where outsiders are discriminated against, π^O increases insiders' payoffs without affecting Π^u. Therefore, our objective in the following is to maximize π^O. For that, a number of consistency conditions must hold. First, (DEO), (IRO) and (ICO), must be satisfied, which are

$$b^O \leq \delta\gamma\left(e^O\theta - w^O\right) - \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)V^{uI}\right) \quad \text{(DEO)}$$

$$b^O \geq c(e^O) - \delta\gamma\left(w^O - \left(1-\delta\gamma\right)V^{uO}\right) \quad \text{(ICO)}$$

$$b^O \geq c(e^O) - \left(w^O - \left(1-\delta\gamma\right)V^{uO}\right). \quad \text{(IRO)}$$

There, note that it is without loss to have $w^O = (1-\delta\gamma)V^{uO}$ and $b^O = c(e^O)$ (thus, $\pi^O = e^O\theta - c(e^O) - (1-\delta\gamma)V^{uO}$), in which case (ICO) and (IRO) are automatically satisfied and only (DEO) remains, which is

$$\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)V^{uI} \leq \delta\gamma e^O\theta - c(e^O) - \delta\gamma\left(1-\delta\gamma\right)V^{uO}. \quad \text{(DEO)}$$

Moreover, it must be optimal for firms to employ insiders instead of keeping vacancies open in the hope of attracting outsiders, i.e.,

$$\Pi^I \geq \Pi^u. \quad (7)$$

Finally, we must be in the range where insiders are paid a rent, which implies

$$V^I \geq V^{uI}. \quad (8)$$

We next show that, for sufficiently small N^O, a discrimination equilibrium with $e^O > e^I = \delta\gamma\theta$ exists.

Existence of Discrimination Equilibrium: Small N^O

First, note that because $e^I = \delta\gamma\theta$ maximizes $\delta\gamma\theta e - c(e)$, (DEO) requires $V^{uI} \geq V^{uO}$. There, we focus on $V^{uI} > V^{uO} = 0$ because a smaller V^{uO} increases π^O.

Second, note that steady-state market conditions are

$$
\begin{aligned}
f &= \frac{(1-\gamma)\,F - m(n^O + n^I, f)}{(1-\gamma)}, \\
n^I &= \frac{(1-\gamma)\,N^I + m(n^O, f) - m(n^O + n^I, f)}{(1-\gamma)}, \\
n^O &= \frac{(1-\gamma)\,N^O - m(n^O, f)}{(1-\gamma)},
\end{aligned}
$$

which allow us to rewrite

$$
\begin{aligned}
\alpha^{NO} &= \frac{(1-\gamma)\,\alpha^{FO} f}{(1-\gamma)\,N^O - \alpha^{FO} f} \\
&= \frac{(1-\gamma)\,\alpha^{FO} F}{N^O\,(1-\gamma+\alpha^F) - \alpha^{FO} F}, \\
\alpha^{NI} &= \frac{(1-\gamma)\,\alpha^{FI} f}{(1-\gamma)\,N^I - \alpha^{FI} f} \\
&= \frac{(1-\gamma)\,\alpha^{FI} F}{N^I\,(1-\gamma+\alpha^F) - \alpha^{FI} F},
\end{aligned}
$$

and hence

$$
\begin{aligned}
V^{UI} &= \frac{\alpha^{NI}}{(1-(1-\alpha^{NI})\delta\gamma)} V^I \\
&= \alpha^{NI} \frac{\alpha^{FO}\pi^O + \frac{\delta\gamma\theta^2}{2}\left[(1-\delta\gamma)\left(2\alpha^F - 1\right) - \alpha^{FO}\,(2-\delta\gamma)\right]}{(1-\delta\gamma)\left[\alpha^{FI}\,(1-\delta\gamma) - \alpha^{NI}\,(1-\delta\gamma+\delta\gamma\alpha^{FO})\right]} \\
&= \frac{(1-\gamma)}{(1-\delta\gamma)} F \frac{\alpha^{FO}\pi^O + \frac{\delta\gamma\theta^2}{2}\left[(1-\delta\gamma)\left(2\alpha^F - 1\right) - \alpha^{FO}\,(2-\delta\gamma)\right]}{\left[(1-\delta\gamma)\,(1-\gamma+\alpha^F)\,(N^I - F) + F\alpha^{FO}\,(1-2\delta\gamma+\delta\gamma^2)\right]}.
\end{aligned}
$$

Finally, we show that an asymmetric discrimination equilibrium exists given the assumptions stated in the proposition. Assume $N^O = \alpha^{FO} = 0$ and that α^{FI} either exceeds 1/2 (if $N^I > F$ at $\alpha^{FI} = 1/2$) or is above the threshold at which insiders are paid a rent (if $N^I > F$ at $\alpha^{FI} = 1/2$). Because this case describes a symmetric equilibrium among insiders in which they are paid a rent, conditions (7) and (8) are satisfied by construction, and it remains to show that (DEO) holds.

For $\alpha^{FO} \to 0$ (in which case $\alpha^{FI} = \alpha^F$), this condition becomes

$$\begin{aligned}
&\frac{(\delta\gamma\theta)^2}{2} \frac{\left(1-\gamma+\alpha^F\right) N^I - \alpha^F F\left(3-2\gamma\right)}{\left(1-\gamma+\alpha^F\right)\left(N^I-F\right)} \leq \delta\gamma e^O\theta - c(e^O) \\
\Rightarrow\ &\frac{(\delta\gamma\theta)^2}{2}\left(1 - \frac{(1-\gamma)(2\alpha^F-1)}{\left(1-\gamma+\alpha^F\right)\left(N^I-F\right)}\right) \leq \frac{(2\delta\gamma\theta - e^O)e^O}{2}.
\end{aligned}$$

Note that the right hand side of this inequality, which is maximized at $e^O = e^I$, approaches $\frac{(\delta\gamma\theta)^2}{2}$ as $e^O \to e^I$. Also, the left hand side is strictly smaller than $\frac{(\delta\gamma\theta)^2}{2}$ when $N^I > F$ and $\alpha^F > 1/2$. Hence, a discrimination equilibrium in which $e^O > e^I$ exists.

Large N^O

To prove that a discrimination equilibrium favoring insiders does not exist if N^O is sufficiently large, assume $N^O \to \infty$. Then, $\alpha^{FO} \to \alpha^F$ and $\alpha^F \to 1$, thus $\alpha^{FI} \to 0$, and (DEO) becomes

$$(1-\delta\gamma)^2\left(2-\gamma\right)\frac{N^I}{F} \leq \left(1-\gamma\right).$$

Condition (7) becomes

$$\frac{\delta\gamma\theta^2}{2} \geq \pi^O,$$

and Condition (8) becomes

$$\delta\gamma\left(\pi^O - \frac{\delta\gamma\theta^2}{2}\right)\left[1 + \frac{\left(1-\delta\gamma\right)\frac{(1-\gamma)}{(1-\delta\gamma)}F}{\left[\left(1-\delta\gamma\right)\left(2-\gamma\right)\left(N^I-F\right) + F\left(1-2\delta\gamma+\delta\gamma^2\right)\right]}\right] \geq 0,$$

i.e.,

$$\pi^O \geq \frac{\delta\gamma\theta^2}{2}.$$

Thus, both conditions can only hold simultaneously for $\pi^O = \delta\gamma\theta^2/2$. However, recall that

$$V^{uI} = \frac{(1-\gamma)}{(1-\delta\gamma)} F \frac{\alpha^{FO}\pi^O + \frac{\delta\gamma\theta^2}{2}\left[(1-\delta\gamma)\left(2\alpha^F - 1\right) - \alpha^{FO}(2-\delta\gamma)\right]}{\left[(1-\delta\gamma)\left(1-\gamma+\alpha^F\right)\left(N^I - F\right) + F\alpha^{FO}\left(1-2\delta\gamma+\delta\gamma^2\right)\right]}.$$

For $N^O \to \infty$ and $\pi^O = \delta\gamma\theta^2/2$,

$$\begin{aligned}
&\alpha^{FO}\pi^O + \frac{\delta\gamma\theta^2}{2}\left[(1-\delta\gamma)\left(2\alpha^F - 1\right) - \alpha^{FO}(2-\delta\gamma)\right] \\
=&\alpha^F\frac{\delta\gamma\theta^2}{2} + \frac{\delta\gamma\theta^2}{2}\left[(1-\delta\gamma)\left(2\alpha^F - 1\right) - \alpha^F(2-\delta\gamma)\right] \\
=&\frac{\delta\gamma\theta^2}{2}(1-\delta\gamma)\left(\alpha^F - 1\right) < 0,
\end{aligned}$$

thus V^{uI} would be negative, a contradiction.

Job Losses of Insiders

Finally, we confirm that a higher N^O increases n^I, thus unemployment among insiders. To do that, we rewrite and tag the steady-state market conditions:

$$f(1-\gamma) - (1-\gamma)F + m(n^O + n^I, f) = 0 \qquad \text{(I)}$$

$$n^I(1-\gamma) - (1-\gamma)N^I - m(n^O, f) + m(n^O + n^I, f) = 0 \qquad \text{(II)}$$

$$n^O(1-\gamma) - (1-\gamma)N^O + m(n^O, f) = 0 \qquad \text{(III)}$$

For comparative statics, we use the implicit function theorem, for which we need partial derivatives of the left-hand sides of equations I-III which, with a slight abuse of notation, are

$$\frac{\partial I}{\partial f} = (1-\gamma) + m_f(n^O + n^I, f)$$
$$\frac{\partial I}{\partial n^I} = m_n(n^O + n^I, f)$$
$$\frac{\partial I}{\partial n^O} = m_n(n^O + n^I, f)$$
$$\frac{\partial II}{\partial f} = -m_f(n^O, f) + m_f(n^O + n^I, f) > 0$$
$$\frac{\partial II}{\partial n^I} = (1-\gamma) + m_n(n^O + n^I, f)$$
$$\frac{\partial II}{\partial n^O} = -m_{n^O}(n^O, f) + m_n(n^O + n^I, f) < 0$$
$$\frac{\partial III}{\partial f} = m_f(n^O, f)$$
$$\frac{\partial III}{\partial n^I} = 0$$
$$\frac{\partial III}{\partial n^O} = (1-\gamma) + m_{n^O}(n^O, f)$$

For the following, we define

$$D \equiv \begin{vmatrix} \frac{\partial I}{\partial f} & \frac{\partial I}{\partial n^I} & \frac{\partial I}{\partial n^O} \\ \frac{\partial II}{\partial f} & \frac{\partial II}{\partial n^I} & \frac{\partial II}{\partial n^O} \\ \frac{\partial III}{\partial f} & \frac{\partial III}{\partial n^I} & \frac{\partial III}{\partial n^O} \end{vmatrix}$$
$$= \frac{\partial I}{\partial f}\frac{\partial II}{\partial n^I}\frac{\partial III}{\partial n^O} + \frac{\partial I}{\partial n^I}\frac{\partial II}{\partial n^O}\frac{\partial III}{\partial f} + \frac{\partial I}{\partial n^O}\frac{\partial II}{\partial f}\frac{\partial III}{\partial n^I}$$
$$- \frac{\partial III}{\partial f}\frac{\partial II}{\partial n^I}\frac{\partial I}{\partial n^O} - \frac{\partial III}{\partial n^I}\frac{\partial II}{\partial n^O}\frac{\partial I}{\partial f} - \frac{\partial III}{\partial n^O}\frac{\partial II}{\partial f}\frac{\partial I}{\partial n^I}$$
$$= (1-\gamma)\left[(1-\gamma) + m_{n^O}(n^O, f)\right]\left[(1-\gamma) + m_f(n^O + n^I, f) + m_n(n^O + n^I, f)\right]$$
$$> 0.$$

Therefore,

$$\frac{dn^I}{dN^O} = \frac{\begin{vmatrix} \frac{\partial I}{\partial f} & -\frac{\partial I}{\partial N^O} & \frac{\partial I}{\partial n^O} \\ \frac{\partial II}{\partial f} & -\frac{\partial II}{\partial N^O} & \frac{\partial II}{\partial n^O} \\ \frac{\partial III}{\partial f} & -\frac{\partial III}{\partial N^O} & \frac{\partial III}{\partial n^O} \end{vmatrix}}{|D|} = \frac{\begin{vmatrix} \frac{\partial I}{\partial f} & 0 & \frac{\partial I}{\partial n^O} \\ \frac{\partial II}{\partial f} & 0 & \frac{\partial II}{\partial n^O} \\ \frac{\partial III}{\partial f} & (1-\gamma) & \frac{\partial III}{\partial n^O} \end{vmatrix}}{|D|}$$

$$= (1-\gamma)\,\frac{\frac{\partial I}{\partial n^O}\frac{\partial II}{\partial f} - \frac{\partial II}{\partial n^O}\frac{\partial I}{\partial f}}{|D|}$$

$$= (1-\gamma)\,\frac{m_{n^O}(n^O,f)m_f(n^O+n^I,f) - m_n(n^O+n^I,f)m_f(n^O,f)}{|D|}$$

$$+ (1-\gamma)\,\frac{(1-\gamma)\left(m_{n^O}(n^O,f) - m_n(n^O+n^I,f)\right)}{|D|} > 0$$

$$\frac{dn^I}{dF} = \frac{\begin{vmatrix} \frac{\partial I}{\partial f} & -\frac{\partial I}{\partial F} & \frac{\partial I}{\partial n^O} \\ \frac{\partial II}{\partial f} & -\frac{\partial II}{\partial F} & \frac{\partial II}{\partial n^O} \\ \frac{\partial III}{\partial f} & -\frac{\partial III}{\partial F} & \frac{\partial III}{\partial n^O} \end{vmatrix}}{|D|} = \frac{\begin{vmatrix} \frac{\partial I}{\partial f} & (1-\gamma) & \frac{\partial I}{\partial n^O} \\ \frac{\partial II}{\partial f} & 0 & \frac{\partial II}{\partial n^O} \\ \frac{\partial III}{\partial f} & 0 & \frac{\partial III}{\partial n^O} \end{vmatrix}}{|D|}$$

$$= (1-\gamma)\,\frac{\frac{\partial II}{\partial n^O}\frac{\partial III}{\partial f} - \frac{\partial II}{\partial f}\frac{\partial III}{\partial n^O}}{|D|}$$

$$= (1-\gamma)\,\frac{m_n(n^O+n^I,f)m_f(n^O,f) - m_{n^O}(n^O,f)m_f(n^O+n^I,f)}{|D|}$$

$$+ (1-\gamma)^2\,\frac{m_f(n^O,f) - m_f(n^O+n^I,f)}{|D|} < 0.$$

$dn^I/dN^O > 0$ implies that the inflow of outsiders causes job losses of insiders if we hold F constant. If we take into account potential firm entry/exit, recall that

$$\Pi^u = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,V^{uI}\right]}{\delta\gamma\,(1-\delta\gamma)},$$

therefore Π^u goes down as long as an inflow of outsiders increases compensation and therefore V^{uI}. This reduces F which additional causes job losses of insiders.

■

Proof of Proposition 5:

In the proof of Proposition 1, we have shown that rents are paid through w. Therefore, a discrimination equilibrium with $V^I > V^O$ has $w^I > w^O$, and an exogenously increased w^O raises V^O. Moreover, w^O only enters the objective function and constraints via $\pi^O = e^O\theta - w^O - b^O$, with the exception of (DEO),

$$\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,V^{uI} \leq \delta\gamma\left(e^O\theta - w^O\right) - b^O,$$

where we have presumed that a potential minimum wage is below w^I. Finally, we have shown that V^I, V^{uI} and w^I increase in π^O, which is why maximizing V^I in a discrimination equilibrium is equivalent to maximizing π^O.

Now, assume that $w^O = \overline{w} < w^I$. An increase in $\overline{w}$ tightens (DEO), thus weakly decreases e^O. Moreover, it reduces π^O: If a higher $\overline{w}$ increased π^O, firms would have paid the higher wage in the first place given the objective is to maximize π^O. Therefore, a higher $\overline{w}$ reduces w^I and V^I, and consequently V^{uI}. Finally, as $\Pi^u = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)V^{uI}\right]}{\delta\gamma(1-\delta\gamma)}$, the higher minimum wage increases Π^u. ■

B Examples in the Main Text

Here, we analyze two specific matching functions, one exhibiting constant and one increasing returns to scale.

Example 1 (Constant Returns to Scale)

Assume the constant returns to scale matching function $m(f,n) = (1-e^{-\frac{n}{kf}})f$, with $k \geq 1$. Then, $\alpha^F = (1-e^{-\frac{n}{kf}})$ and $\alpha^F \geq 1/2 \Leftrightarrow n/f \geq k\ln 2$. Moreover, n/f is characterized by

$$\frac{f}{n}\left[(2-\gamma)\frac{N}{F} - 1 - e^{-\frac{n}{kf}}\left(\frac{N}{F} - 1\right)\right] - (1-\gamma) = 0$$

Let denote $x \equiv \frac{n}{f}$. Note that $\alpha^F = 1 - e^{-\frac{x}{k}}$ with $dx/dN > 0$.

Then, we can rewrite the equilibrium matching condition

$$\frac{1}{x}\left[(2-\gamma)\frac{N}{F} - 1 - e^{-\frac{x}{k}}\left(\frac{N}{F} - 1\right)\right] - (1-\gamma) = 0$$

to

$$\frac{N}{F} = \frac{(1-\gamma)x + 1 - e^{-\frac{x}{k}}}{\left(2-\gamma-e^{-\frac{x}{k}}\right)}.$$

The profits for the case in which workers are paid a rent are

$$\Pi^u = \delta\gamma\theta^2 \frac{\left(x - 2(1-e^{-\frac{x}{k}})\right)}{2\left(1-\delta\right)\left(x-1\right)}.$$

For $k = 1$, $x = 0.69$ at $\alpha^F = 1/2$; there, $n < f$ and consequently $N < F$ which is not consistent with an equilibrium. Then, $\Pi^u \geq 0 \Leftrightarrow x \geq 1.59 \Leftrightarrow \alpha^F \geq 0.80$. For $\gamma = 0.8$, this requires $N/F \geq 1.12$, i.e., the total workforce must exceed the available jobs by at least 12%.

For $k = 2$, $x = 1.38$ at $\alpha^F = 1/2$, with $n > f$ and $N > F$. There, for $\gamma = 0.8$, $N/F = 1.11$, hence workers are paid a rent once the total workforce exceeds the available jobs by at least 11%.

We next analyze comparative statics with respect to N under fixed F. Note that

$$w + b = \delta\gamma\theta^2 \frac{\left(1-2e^{-\frac{x}{k}}\right)\left[(1-\delta\gamma)x + \left(1-e^{-\frac{x}{k}}\right)\delta\gamma\right]}{2\left(x-1\right)\left(1-e^{-\frac{x}{k}}\right)} + \frac{\left(\delta\gamma\theta\right)^2}{2}.$$

Assume $\gamma = 0.8$ and $\delta = 0.9$. Then, for $k = 2$, $w + b$ increases in N for $x \in [1.38, 3.17]$, and decreases for $x > 3.17$ ($x = 1.38$ is the cutoff above which agents are paid a rent). This implies that compensation increases from $\alpha^F = 0.5$ to $\alpha^F = 0.80$, which is equivalent to $N/F \in [1.11, 1.43]$.

Finally, we analyze for which N and for which K (as a proportion of $\tilde{K}$) a high-α^F equilibrium exists under firms' free entry. Profits when workers are paid a rent are

$$\Pi^u = \delta\gamma\theta^2 \frac{\left(x - 2(1-e^{-\frac{x}{k}})\right)}{2\left(1-\delta\right)\left(x-1\right)}, \text{ with}$$

$$\frac{\partial \Pi^u}{\partial x} = \delta\gamma\theta^2 \frac{-2e^{-\frac{x}{k}}\left(1+\frac{(x-1)}{k}\right)+1}{2\left(1-\delta\right)\left(x-1\right)^2},$$

where the latter must be positive because $\partial x / \partial F < 0$.

For $k = 1$, $\partial x / \partial F < 0 \Leftrightarrow 1 - 2e^{-x} x > 0$. There, note that $\partial \left(1 - 2e^{-x} x\right) / \partial x = 2e^{-x}\left(x - 1\right) > 0$ for $x > 1$, which is a prerequisite for case 1. Moreover, $1 - 2e^{-x} x > 0$ at $x = 1$, thus $\partial \Pi^u / \partial x > 0$ for $x > 1$. Recall that, finally, $\alpha^F \geq 0.80 \Leftrightarrow N/F \geq 1.12$ must hold for Π^u to be positive (for the parameter value $\gamma = 0.8$), hence — as with exogenous F — an equilibrium in which workers are paid a rent exists when the total workforce exceeds total available jobs by at least 12%. It follows that, for this example, an equilibrium with $\alpha^F > 1/2$ exists for all $K \in \left[0, \tilde{K}\right]$.

For $k = 2$, $\partial \Pi^u / \partial F$ decreases for $x > 2.51$, which implies $\alpha^F \geq 0.71$ and, for $\gamma = 0.8$, $N/F \geq 1.33$. At $x = 2.51$,

$$\Pi^u \frac{(1-\delta)}{\delta \gamma \theta^2} = 0.36 = 0.72\tilde{K},$$

thus an equilibrium with $\alpha^F > 1/2$ exists for $K \in \left[0.72\tilde{K}, \tilde{K}\right]$.

Example 2 (Increasing Returns to Scale)

Assume the matching function $m = f\left(1 - e^{-k\frac{n}{F}}\right)$, which has increasing returns to scale in n and f (but constant in n, f and F), thus $\alpha^F = \left(1 - e^{-k\frac{n}{F}}\right)$ and $\alpha^F \geq 1/2 \Leftrightarrow n/F = \ln 2/k$. Then, the equilibrium matching conditions become

$$\begin{aligned} f =& F\frac{(1-\gamma)}{\left[2 - \gamma - e^{-k\frac{n}{F}}\right]}, \\ N =& F + n - f \\ \Leftrightarrow \frac{N}{F} =& \frac{n}{F} + \frac{\left(1 - e^{-k\frac{n}{F}}\right)}{\left[2 - \gamma - e^{-k\frac{n}{F}}\right]}, \end{aligned}$$

and

$$\begin{aligned} \Pi^u =& \delta\gamma\theta^2 \frac{\left(1 - \gamma + \alpha^F\right) N - F\alpha^F (3 - 2\gamma)}{2(1-\delta)\left(1 - \gamma + \alpha^F\right)(N - F)} \\ \Rightarrow \Pi^u =& \delta\gamma\theta^2 \frac{\left(2 - \gamma - e^{-k\frac{n}{F}}\right)\frac{n}{F} - 2\left(1 - e^{-k\frac{n}{F}}\right)(1-\gamma)}{2(1-\delta)\left[\frac{n}{F}\left(2 - \gamma - e^{-k\frac{n}{F}}\right) - (1-\gamma)\right]}. \end{aligned}$$

For $\gamma = 0.8$ and $k = 1$, at $\alpha^F = 1/2$ where $n/F = \ln 2$, $N/F = 1.40$. Then, workers are paid a rent once the total workforce exceeds the available jobs by at least 40%.

For $\gamma = 0.8$ and $k = 2$, at $\alpha^F = 1/2$ where $n/F = \ln 2/2$, $N/F = 1.06$. Then, workers are paid a rent once the total workforce exceeds the available jobs by at least 6%.

We next analyze comparative statics with respect to N under fixed F. By

$$\frac{N}{F} = \frac{n}{F} + \frac{\left(1 - e^{-k\frac{n}{F}}\right)}{\left[2 - \gamma - e^{-k\frac{n}{F}}\right]},$$

$$w + b = \delta\gamma\theta^2 \frac{\left(1 - 2e^{-k\frac{n}{F}}\right)\left[(1-\delta\gamma)\frac{n}{F}\left(2 - \gamma - e^{-k\frac{n}{F}}\right) + \gamma\delta(1-\gamma)\left(1 - e^{-k\frac{n}{F}}\right)\right]}{2\left(1 - e^{-k\frac{n}{F}}\right)\left[\frac{n}{F}\left(2 - \gamma - e^{-k\frac{n}{F}}\right) - (1-\gamma)\right]} + \frac{(\theta\delta\gamma)^2}{2}.$$

Assume $\delta = 0.9$ and $\gamma = 0.8$. Then, for $k = 1$, $w + b$ increases in N for $n/F \in [0.69, 2.82]$, and decreases for $n/F > 2.82 \Leftrightarrow \alpha^F > 0.94$ ($n/F = 0.69$ is the cutoff is the cutoff above which agents are paid a rent). This implies that compensation increases from $\alpha^F = 0.5$ to $\alpha^F = 0.94$, which is equivalent to $N/F \in [1.40, 3.26]$. For $k = 2$, $w + b$ increases in N for $n/F \in [0.35, 0.69]$, and decreases for $n/F > 0.69 \Leftrightarrow \alpha^F > 0.75$ ($n/F = 0.35$ is the cutoff is the cutoff above which agents are paid a rent). This implies that compensation increases from $\alpha^F = 0.5$ to $\alpha^F = 0.75$, which is equivalent to $N/F \in [1.06, 1.075]$.

Finally, we analyze for which N and for which K (as a proportion of $\tilde{K}$) a high-α^F equilibrium exists under firms' free entry. Profits when workers are paid a rent are

$$\begin{aligned}\Pi^u =& \delta\gamma\theta^2 \frac{\left(1 - \gamma + \alpha^F\right)N - F\alpha^F(3 - 2\gamma)}{2(1-\delta)(1 - \gamma + \alpha^F)(N - F)} \\ =& \delta\gamma\theta^2 \frac{\left(2 - \gamma - e^{-k\frac{n}{F}}\right)\frac{n}{F} - 2\left(1 - e^{-k\frac{n}{F}}\right)(1-\gamma)}{2(1-\delta)\left[\frac{n}{F}\left(2 - \gamma - e^{-k\frac{n}{F}}\right) - (1-\gamma)\right]}.\end{aligned}$$

Because $dn/dF < 0$, $d(n/F)/dF < 0$,

$$\frac{\partial \Pi_1^u}{\partial F} \leq 0 \Leftrightarrow \frac{\partial \Pi_1^u}{\partial (n/F)} \geq 0.$$

Now assume $\gamma = 0.8$. Then, for $k = 1$, $\partial \Pi_1^u / \partial (n/F) \geq 0$ if $n/F \geq 1.25 \Leftrightarrow \alpha^F \geq 0.71 \Leftrightarrow N/F \geq 2.03$. There,

$$\Pi^u \frac{(1-\delta)}{\delta \gamma \theta^2} = 0.455 = 0.91\tilde{K},$$

thus an equilibrium with $\alpha^F > 1/2$ exists for $K \in \left[0.91\tilde{K}, \tilde{K}\right]$. For $k = 2$, $\partial \Pi_1^u / \partial (n/F) > 0$ if $n/F \geq 0.5 \Leftrightarrow \alpha^F \geq 0.63 \Leftrightarrow N/F \geq 1.26$. There,

$$\Pi^u \frac{(1-\delta)}{\delta \gamma \theta^2} = 0.38 = 0.76\tilde{K},$$

thus an equilibrium with $\alpha^F > 1/2$ exists for $K \in \left[0.76\tilde{K}, \tilde{K}\right]$.