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Informal Elections with Dispersed Information: Protests, Petitions, and Nonbinding Voting*

Mehmet Ekmekci† and Stephan Lauermann‡

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Abstract

We study information transmission through informal elections. Our leading example is that of protests in which there may be positive costs or benefits of participation. The aggregate turnout provides information to a policy maker. However, the presence of activists adds noise to the turnout. The interplay between noise and participation costs leads to strategic substitution and complementarity effects in citizens' participation choices, and we characterize the implications for the informativeness of protests. In particular, we show that rather than being a friction, costs may facilitate information transmission by lending credibility to protest participation.

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1 Introduction

We present and study a model of certain political processes that we call *informal elections*. Examples are protests, petitions, polls, surveys, and nonbinding shareholder voting. These scenarios share certain qualitative features with elections. First, the citizens (or experts, shareholders, etc.) take a coarse action that is often binary: participate in a protest or not, sign a petition or not, etc. Second, what matters is the aggregate outcome: how many citizens participate in a protest, how many people sign a petition, etc., with the protest or petition being more effective at convincing an audience the more citizens participate. However, there is a significant difference: in a formal election, a prespecified rule determines the policy as a function of the vote count. In informal elections this is not the case; instead, the main effect of the citizens' participation is the audience's inference from the turnout, where the audience could be a particular policy maker or, for example, the general electorate. Thus, informal elections are primarily about communication.

The Condorcet jury theorem and its modern versions have shown that formal elections effectively aggregate the dispersed opinions of citizens under quite general conditions (Feddersen and Pesendorfer, 1997; Myerson, 1998).[1] We explore whether informal elections share these information aggregation properties. A variety of scenarios fit our framework:

1. **Nonbinding elections.** The board of a firm holds a nonbinding vote among the shareholders to decide whether to approve an executive compensation package or a shareholder-submitted proposal (Levit and Malenko, 2011).

2. **Petitions and protests.** A policy maker decides whether to change a policy based on a petition signed by citizens, or the general public makes an inference from turnout in a protest. For example, the Gezi Park movement informed citizens living in rural Turkey about the government's plans to replace a park with a shopping mall; other examples are provided in Battaglini (2017).

3. **Polls.** A manager holds a survey among the employees to learn about the prospects of a new product. A king asks his generals for advice in war (Wolinsky, 2002). A policy maker organizes polls to elicit the citizens' opinions on current policies (Morgan and Stocken, 2008).

Existing work has studied informal elections as "cheap talk" between multiple senders and a biased receiver. Starting with Wolinsky (2002), a central insight is that having many senders may not help with information transmission, given the receiver's bias. What is critical for this finding is that a sender's report only matters

[1] However, see Razin (2003), Bhattacharya (2013), Ekmekci and Lauermann (2020), and Ali et al. (2017) for related models where information aggregation fails.

when it is pivotal for the receiver's action, and the sender makes an inference about the state of the world conditional on his report being pivotal.

However, the cheap-talk approach omits two features of informal elections that we believe to be inherent in these settings: there are costs of participation, and there can be significant noise. Costs can arise, for example, from the time commitment or possible repercussions of participating in a protest or petition, and benefits (negative costs) may arise from its entertainment value. Noise may stem from the policy maker's inability to distinguish the motives of various protesters, from counting error, or from the presence of citizens who answer poll questions randomly. As we will see, once these two features are accounted for, in contrast to the existing cheap-talk approach, the citizens' inference from being pivotal and the potential bias of the policy maker are no longer of central importance; instead, the public good nature of political participation takes center stage.

For concreteness, we discuss protests as our leading example of informal elections, returning to polls and nonbinding voting later. In general, the meaning of turnout in a protest—and therefore its ability to influence policy or public opinion—depends on the protesters' motivations. If protesters are participating based on their information, then the turnout is informative. This is doubly true if participation is costly, because the decision to participate is then an even stronger signal.

On the other hand, if participation is primarily random and unrelated to citizens' information, then the turnout is uninformative. Indeed, politicians frequently attempt to undermine protests by arguing that the protesters' motivations are orthogonal to their information, e.g. because they are being paid or have ulterior motives.[2] Similarly, senators are worried about "bot-calling" when making inferences from calls to their offices. There are, of course, many other sources of noise, including the possibility that turnout itself is only imperfectly observable.

In this paper, we consider the following model, based on Battaglini (2017), to which we add costly participation and noise: a policy maker needs to choose one of two policies, A or B. He prefers the policy to match the unknown state of the world, α or β. There is a pool of citizens who are privately and imperfectly informed about the state of the world. The citizens also prefer the policy to match the unknown state, but their exact payoffs differ from the policy maker's. There is a protest movement in place to influence the policy maker's decision. Each citizen draws a participation cost, observes a private signal, and then chooses whether to participate in the protest. A citizen's cost c is drawn from a distribution F with 0 in the interior

[2] US President Donald Trump claimed that the protesters against Brett Kavanaugh's Supreme Court nomination had been paid to participate in the protests. The Hungarian Prime Minister Viktor Orbán, and many other politicians, often claim that left-wing protests or petitions across the world are Soros-funded. Turkey's President Recep Tayyip Erdoğan attempted to undermine the Gezi Park protests by calling the protesters "çapulcu" (looters with nothing else to do).

of its support, and her binary signal, a or b, is indicative of the state α or β, respectively. In addition, there is an exogenous, normally distributed number of activists, and the variance of the number of activists is a measure of the noise. Finally, the policy maker observes the total turnout (citizens plus noise) and chooses the policy.

In this model, citizens communicate their information to the policy maker via their participation decision. A priori, the protest may be a signal in favor of A or B. The meaning of the protest is determined in equilibrium, and we look at equilibria in which the protest is in favor of policy A. In such equilibria, the policy maker chooses A if the turnout is large and B if the turnout is small. In particular, there is a tipping point for the turnout, above which the policy maker chooses A. A citizen's participation is critical exactly when the turnout is at the tipping point. Therefore, the likelihood that the turnout is at the tipping point determines how likely a given citizen's participation is to change the policy maker's choice (i.e., how *effective* her participation will be); this anticipated effectiveness determines the citizens' participation incentives.

Citizens decide whether to participate depending on their costs, their private information, and the anticipated effectiveness of their participation. The costs c capture motives that are unrelated to the effect of the protest on the policy maker's choice, with negative costs capturing direct benefits from participation in itself. For any given signal, a or b, a citizen participates if her cost is sufficiently low, which implies signal-dependent cost cutoffs c_a and c_b, respectively. Since the protest is in favor of policy A, a citizen with signal a, who wishes to "tip" the policy from B to A, is more eager to participate than a citizen with signal b; hence $c_a > c_b$. So, citizens with high costs—larger than c_a—never participate, those with intermediate costs—between c_b and c_a—participate based on their signal (we call these *informative citizens*), and those with low costs—smaller than c_b—always participate. Because the informative citizens' participation is signal-dependent, the distribution of the turnout differs across the two states, with more participation in state α than in state β.

The policy maker faces an inference problem in which he learns about the state from the realized turnout. The larger the expected number of informative citizens, the larger the difference is in turnout across states. The informativeness of the turnout as a signal is the difference in the expected turnouts relative to the noise induced by the variance in the number of activists.

At one extreme, if there are many informative citizens relative to the noise from the activists, the turnout is very informative, and the correct policy is likely to be chosen. In this case, it is very unlikely that the turnout is at the tipping point, so participation incentives are weak. Moreover, we show that if the number of informative citizens increases further, the probability that turnout is at the tipping point decreases, which further decreases participation incentives. This *strategic substitution effect* limits how much information can be transmitted in equilibrium.

At the other extreme, if there are few informative citizens relative to the noise, the turnout distributions are close to each other, and turnout contains little information. This means the policy maker is unlikely to react to turnout; hence, the turnout is unlikely to be at the tipping point. In this case, we show that an increase in the number of informative citizens will increase the probability that turnout is at the tipping point, which in turn increases individual participation incentives, potentially inducing even more participation by informative citizens. This is a *strategic complementarity effect*. We show that this positive feedback can lead to a multiplicity of equilibria and render information transmission fragile, so that small changes in costs can unravel all participation by informative citizens.

In our main results, we characterize the *maximal* equilibrium informativeness of protests. For this, we first assume that costs follow a uniform distribution around 0 and show that equilibrium can be characterized as the fixed points of a simple mapping with a closed-form solution. Of course, there is always an uninformative babbling equilibrium. Using our characterization, we show that an informative equilibrium exists if and only if the policy maker's "burden of proof"—the minimal amount of evidence needed to change his decision from the prior optimal one—is sufficiently small. Moreover, we show that informative equilibria are easier to sustain if signals are more informative and noise is smaller. Second, we show that when the noise is large, babbling is the unique outcome, but as the noise decreases past a certain threshold, informativeness discontinuously jumps up. Similarly, for the signal precision, babbling is the unique outcome for low informativeness, and there may be an upward jump as informativeness increases. These discontinuities at the thresholds are due to the strategic complementarity effects discussed before; they indicate that the informativeness of protests can be fragile.

We then consider a general cost distribution. First, we show that our results for uniform cost distributions extend qualitatively. To investigate the role costs play in facilitating information transmission, we then study a scenario where the cost distribution is concentrated around 0, corresponding to a protest model without participation costs. In this case, we give a tight condition on the precision of citizens' information, and on the policy maker's bias relative to the citizens' preferences, that determines whether information can be transmitted. With low precision or large bias, no information is transmitted; otherwise, all available information can be transmitted in equilibrium. From this we conclude that costs improve information transmission when the policy maker's relative bias is large. This highlights an important difference between our setting and that of formal elections, in which costs impede information transmission (Krishna and Morgan, 2012). Moreover, while the possibility of information transmission in the absence of costs is determined by the policy maker's relative bias (as in the cheap-talk approach), with costs, the critical determinant is the policy maker's burden of proof (which is unrelated to his bias).

Whether protest participation exhibits strategic substitutes or complements has been debated in political economy and empirically investigated. For example, in a field experiment on protest participation decisions, Cantoni et al. (2019) find evidence of strategic substitution effects, with a citizen's probability of participation decreasing in the level to which she believes other citizens are participating; in particular, this indicates that participation decisions have interactive elements, in support of our game-theoretic protest model.[3]

We conduct our analysis in an appropriately defined "limit version" of the setting with a discrete but large number of citizens. In the limit model, there is a continuum of citizens who are (partially) altruistic, and their participation incentives are determined by the marginal effect of an additional (mass of) turnout. In Section 7, we show that these incentives are the limit of the participation incentives in the discrete setting as the number of citizens increases without bound. Our continuum model is more tractable than the discrete setting, enables the exact characterization of equilibria, and yields clean comparative statics. Unlike in prior work on protests with a continuum of citizens, the effectiveness of participation here is endogenous, and it determines the citizens' participation incentives. Our model and solution follow ideas from the ethical voting model of Feddersen and Sandroni (2006).

In Section 6, we study a variation with multiple messages/protests, which may better capture informal political processes such as polls or nonbinding voting. We identify a force towards protests being endogenously one-sided, so that, for certain parameters, only one of the two potential protests is active. Section 7 provides a microfoundation for our continuum model as the limit of models with large but finite populations. The related literature is surveyed in Section 8. We discuss the distribution and sources of noise, among other extensions, in Section 9. We sketch most proofs in the main text, relegating details to the online appendix.

2 Model of a Large Protest

The setup. The following model of a large protest builds on Battaglini (2017). There is a policy maker and a continuum of citizens. The policy maker faces a choice between two policy options, A and B. The effectiveness of each policy depends on an unknown state of the world, α or β, which the policy maker wishes to match. Each citizen has some information about the state and partially reveals her information via her decision of whether to participate in a protest. Protest

[3] For a similar field experiment and further references on strategic participation, see Hager et al. (2021).

participation is costly. The observed protest turnout includes noise in the form of a random number of activists, who participate regardless of their information.

The ex-ante probability that the state is α is $q \in (0, 1)$. The policy maker prefers A when he believes that the probability that the state is α is greater than some $\mu \in (0, 1)$, and he prefers B when he believes it is less than μ. Specifically, if the state is α, his payoff is $1 - \mu$ if the outcome is A and 0 if it is B; if the state is β, his payoff is μ if the outcome is B and 0 if it is A.

Citizens have common preferences: they prefer A when they believe that the state is α with probability greater than 1/2, and they prefer B when they believe that the state is α with probability less than 1/2. In particular, if the state is α, a citizen's payoff is u (for some $u > 0$) if the outcome is A and 0 if it is B; if the state is β, her payoff is u if the outcome is B and 0 if it is A.

The preferences of the policy maker and the citizens are aligned when the state is known, or when $\mu = 1/2$; however, they are misaligned when $\mu \neq 1/2$ and the state is uncertain. The difference $|\mu - 1/2|$ measures the conflict of interest between the citizens and the policy maker.

Each citizen receives a binary signal, $\theta \in \{a, b\}$. Conditional on the state ω, signals are identically and independently distributed across the population according to a probability distribution function $\mathbb{P}(\theta|\omega)$ that denotes the conditional probability that a citizen's signal is θ, with $\mathbb{P}(a|\alpha) > \mathbb{P}(a|\beta)$; in particular, perfectly revealing signals are a special case.

There is a protest movement in place, and each citizen decides whether to participate in the protest. Participation is costly; some citizens have positive costs while others have negative costs. Each citizen's cost c is drawn independently from her signal and from the other citizens' costs, according to a cumulative distribution function (c.d.f.) F with a density function f.

Each citizen, after observing her signal and cost, chooses whether to participate in the protest or to abstain. A strategy for the citizens is a cutoff for each signal, (c_a, c_b), such that a citizen with signal θ and cost c participates if $c < c_\theta$. We normalize the total mass of citizens to 1. Therefore, the number of citizens who participate in state ω is

$$\lambda(\omega) = \mathbb{P}(a|\omega)F(c_a) + \mathbb{P}(b|\omega)F(c_b). \tag{1}$$

The policy maker observes the citizens' turnout with noise: there is a normally distributed mass of activists with mean 0 and standard deviation σ. This noise can be interpreted simply as "counting error" reflecting the difficulty in correctly interpreting the size of a protest. It can also be thought of as a reduced-form model of aggregate uncertainty about F (the participation incentives); we discuss these interpretations and the role of the normal distribution in Section 9.2.

Including the noise, the total turnout t observed by the policy maker is normally distributed with mean $\lambda(\omega)$ and standard deviation σ; that is,

$$t \sim \mathcal{N}\left(\lambda(\omega), \sigma^2\right).$$

The policy maker forms his posterior based on his prior and the observed turnout. To avoid case distinctions, we consider $c_a \geq c_b$, and therefore, $\lambda(\alpha) \geq \lambda(\beta)$. Note that only citizens with costs between c_b and c_a base their participation on their signals and hence provide information (these are *informative citizens*). If $c_a = c_b$, that is, $\lambda(\alpha) = \lambda(\beta)$, then the protest is *uninformative*. If $c_a > c_b$, that is, $\lambda(\alpha) > \lambda(\beta)$, then the protest is *informative* and the policy maker's posterior belief that the state is α is increasing in the realized turnout, making the protest in favor of policy A.

Hence, a strategy for the policy maker is a turnout threshold (tipping point) T, such that he chooses policy A if and only if the turnout is larger than T. At turnout exactly T, the policy maker is indifferent, and so his posterior belief that the state is α is equal to μ. The inference problem is illustrated in Figure 1, which shows the densities of the turnout distributions in the two states. The density in state ω is given by $\frac{1}{\sigma}\phi(\frac{t-\lambda(\omega)}{\sigma})$, where ϕ is the density of the standard normal distribution. Hence, the posterior likelihood ratio upon observing a turnout realization t is

$$\frac{\mathbb{P}(\alpha|t)}{\mathbb{P}(\beta|t)} = \frac{q}{1-q}\frac{\phi\left(\frac{t-\lambda(\alpha)}{\sigma}\right)}{\phi\left(\frac{t-\lambda(\beta)}{\sigma}\right)}.$$

The posterior likelihood ratio is increasing in t when $\lambda(\alpha) > \lambda(\beta)$. Thus, the policy maker's best response is given by the threshold T that is the unique solution to[4]

$$\frac{\mu}{1-\mu} = \frac{q}{1-q}\frac{\phi\left(\frac{T-\lambda(\alpha)}{\sigma}\right)}{\phi\left(\frac{T-\lambda(\beta)}{\sigma}\right)}. \tag{2}$$

Given the expected turnouts $\lambda(\alpha), \lambda(\beta)$ and the policy maker's strategy T, the policy in state ω is A with probability $1 - \Phi\left(\frac{T-\lambda(\omega)}{\sigma}\right)$. Hence, the (interim) payoff of a citizen with signal θ, excluding her cost, is

$$u\mathbb{P}(\alpha|\theta)\left(1 - \Phi\left(\frac{T-\lambda(\alpha)}{\sigma}\right)\right) + u\mathbb{P}(\beta|\theta)\Phi\left(\frac{T-\lambda(\beta)}{\sigma}\right). \tag{3}$$

Additional citizen participation has the same effect on the probability of policy A

[4]If $\lambda(\alpha) = \lambda(\beta)$, then the optimal threshold is $T = \infty$ if $\mu > q$, and $T = -\infty$ if $\mu < q$; that is, the policy maker is unresponsive to turnout.

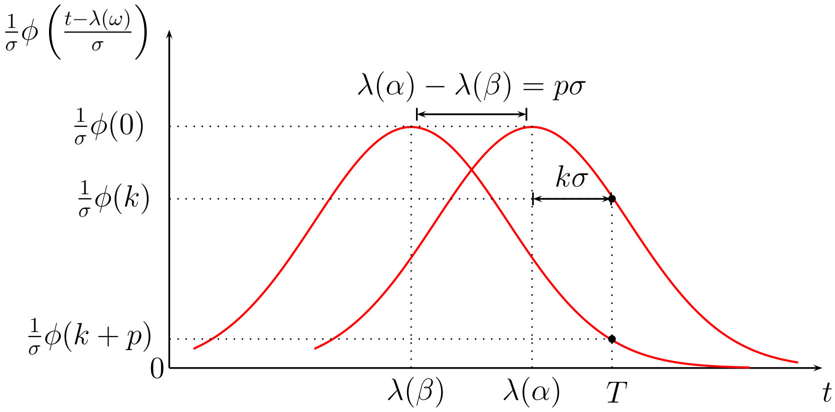


Figure 1: This figure illustrates the policy maker's inference problem when he observes a turnout t that is k standard deviations away from the mean of the turnout in state α.

as a decrease in the threshold T. Hence, the marginal effect of participation in (3) is

$$ME_\theta := u\mathbb{P}(\alpha|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\alpha)}{\sigma}\right) - u\mathbb{P}(\beta|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\beta)}{\sigma}\right). \tag{4}$$

We assume that a citizen with signal θ participates if her cost is smaller than ME_θ:

$$c_\theta = u\mathbb{P}(\alpha|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\alpha)}{\sigma}\right) - u\mathbb{P}(\beta|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\beta)}{\sigma}\right). \tag{5}$$

This assumption reflects the altruism of our citizens. Roughly speaking, if an infinitesimal mass ε of citizens with signal θ and cost c participates, then their total cost is $\varepsilon \times c$, while they evaluate the impact of their participation on the total payoff of all citizens as $\varepsilon \times ME_\theta$.

The citizens' participation incentive given by (4) is analogous to the considerations of pivotal voting in a finite population (e.g., Palfrey and Rosenthal (1983)), with the density of the turnout at T taking the place of the probability of being pivotal. It is also related to the marginal participation incentives in ethical voting (Feddersen and Sandroni (2006)). We will make these connections precise in Section 7.

Definition 1. An *ethical protest equilibrium* is a strategy profile (T, c_a, c_b) such that for the implied turnouts $\lambda(\alpha), \lambda(\beta)$ given by (1), the following hold:

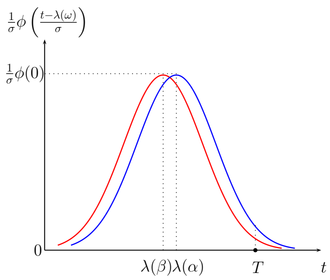
1. The strategy T satisfies (2) (the policy maker's optimality condition).

2. The strategy (c_a, c_b) satisfies (5) (the citizens' optimality condition).

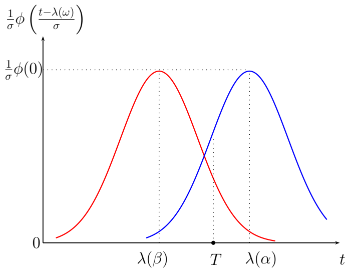
From here on, we call an ethical protest equilibrium simply an "equilibrium".

3 Protest Informativeness and Participation Incentives

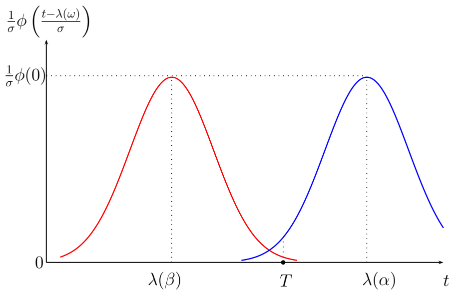
There is always an equilibrium with no information transmission (babbling), in which $c_a = c_b = 0$, and the policy maker is unresponsive ($T = \pm\infty$). We now



(a) Small p



(b) Intermediate p



(c) Large p

Figure 2: This figure illustrates how the informativeness p determines the relative location of the pivotal event T, its probability in each state, and the probability of each policy choice. Critically, the probability (density) of T is nonmonotone in p.

explore the structural properties of informative equilibria, provided they exist.

As noted, the policy maker observes the turnout, which is drawn from a normal distribution with state-dependent mean and known variance, to make an inference about the state. It is well understood that such a signal structure is more informative (in the Blackwell order) the larger the difference in the means relative to the standard deviation; hence, we define the *informativeness* of the protest to be

$$p = \frac{\lambda(\alpha) - \lambda(\beta)}{\sigma}. \tag{6}$$

Figure 2 illustrates the inference problem for the case $\mu > q$.

Figure 2 reveals a great deal about the structure of informative equilibria: the anticipated informativeness p determines the location of the policy maker's optimal threshold relative to $\lambda(\alpha)$. When p is small, T is much larger than $\lambda(\alpha)$; as p increases, T moves towards $\lambda(\alpha)$, eventually becoming smaller than $\lambda(\alpha)$. The relative location of T determines the citizens' participation incentive, given by (5), implying a new pattern of expected turnouts. Thus, starting from one anticipated level of informativeness, we have arrived at a new level of informativeness. The informativeness of an equilibrium must correspond to a fixed point of this mapping.

For the special case where F is a uniform distribution, this mapping has a closed-form expression. Specifically, we assume that F is a uniform distribution with support $[-1/2, 1/2]$, and to ensure that the cost cutoffs given by (5) are in the interior of the support, we assume $u \leq \frac{\sigma}{2\phi(0)}$. Figure 3 illustrates this mapping, and the following lemma gives the closed-form expression.

Lemma 1. *Suppose that F is a uniform distribution with support $[-1/2, 1/2]$, and $u \leq \frac{\sigma}{2\phi(0)}$. There is an equilibrium with informativeness $p^* > 0$ if and only if p^* is a fixed point of*

$$\hat{p}(p) = \frac{u}{\sigma^2} Z \phi(\kappa(p)), \tag{7}$$

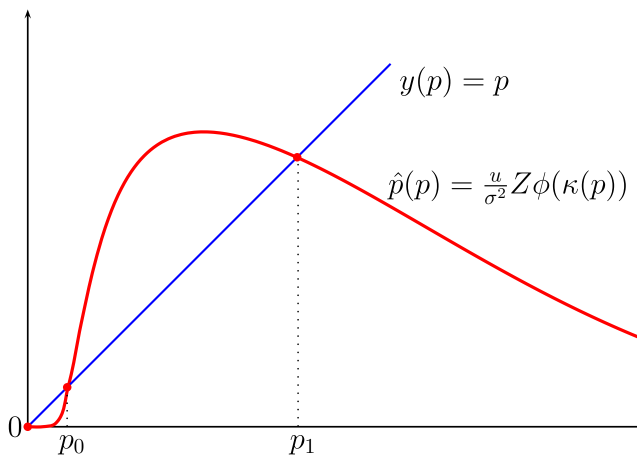


Figure 3: This figure illustrates the function $\hat{p}$. The equilibrium levels of informativeness p are given by the three intersections of $\hat{p}$ with the 45-degree line.

where $\kappa(p) = -\frac{p}{2} + \frac{1}{p}\ln\left(\frac{1-q}{q}\frac{\mu}{1-\mu}\right)$, *and*

$$Z = \left(1 + \frac{q}{1-q}\frac{1-\mu}{\mu}\right)\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right)\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right).$$

Proof. The proof is in Section A.1 of the online appendix. □

This lemma is the key to our analysis: an equilibrium is a triple (c_a, c_b, T) that is a fixed point of Equations (1), (2), and (5), which would be hard to work with. The lemma shows that this can be reduced to finding the fixed point in terms of a single variable, the informativeness p.

Let us sketch the idea. Given an equilibrium informativeness p^*, as illustrated in Figure 2, the policy maker's optimality condition determines the location of T relative to $\lambda(\alpha)$ as

$$\frac{T - \lambda(\alpha)}{\sigma} = \kappa(p^*). \tag{8}$$

The formula for κ given in the lemma is obtained using (2), noting that $\frac{T-\lambda(\beta)}{\sigma} = \kappa(p^*) + p^*$. The relative location of T, in turn, determines the citizens' participation incentives, which are given by (5):

$$c_\theta(p^*) = \frac{u}{\sigma}\mathbb{P}(\alpha|\theta)\phi\left(\kappa(p^*)\right) - \frac{u}{\sigma}\mathbb{P}(\beta|\theta)\phi\left(\kappa(p^*) + p^*\right). \tag{9}$$

Rearranging (9)[5] shows that the expected number of informative citizens is proportional to the *protest effectiveness*, measured by the density $\phi\left(\kappa(p^*)\right)$:

$$c_a(p^*) - c_b(p^*) = \frac{u}{\sigma}\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right)\left(1 + \frac{q}{1-q}\frac{1-\mu}{\mu}\right)\phi\left(\kappa(p^*)\right). \tag{10}$$

[5]For this we use the equality $\phi\left(\kappa(p^*) + p^*\right) = \phi\left(\kappa(p^*)\right)\frac{q}{1-q}\frac{1-\mu}{\mu}$ from (2).

Finally, the cost cutoffs give rise to a new pattern of expected turnouts $(\lambda(\alpha), \lambda(\beta))$ via (1). For p^* to be an equilibrium, the implied informativeness must be p^* itself. Thus, in summary, the construction of $\hat{p}$ in the proof of Lemma 1 is as follows: given any p, the relative location of the policy maker's threshold $\kappa(p)$ implies citizen cutoffs c_θ, and hence turnout patterns $\lambda(\omega)$, which then imply a new informativeness, $\hat{p}(p)$; for p^* to be an equilibrium, it must be that $\hat{p}(p^*) = p^*$.

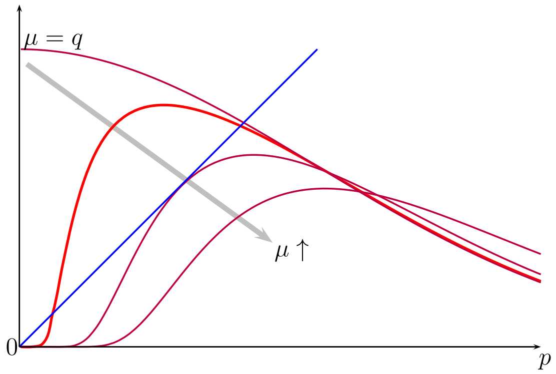
Finally, if p^* is a fixed point, then it is easy to see that it defines a unique equilibrium strategy profile. The citizens' cutoffs are determined by (9), and the policy maker's T is determined by (8); this strategy profile satisfies the equilibrium conditions by construction.

Complementarity and Substitution. We now discuss the implications of an increase in the anticipated informativeness p. Figure 3 shows that when $\mu > q$, $\hat{p}$ is first increasing and then decreasing. Put differently, for a small anticipated p, increasing p will increase the citizens' incentives to participate and hence will increase the number of informative citizens participating. We call this a *strategic complementarity effect*: if the policy maker anticipates a larger p—as implied by the participation of more informative citizens—and reacts optimally, then the citizens' best response will imply that even more informative citizens participate. However, as p increases further, the participation incentives decrease, a *strategic substitution effect.*

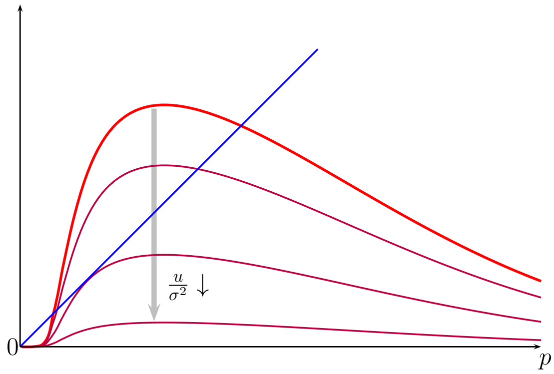
To understand this better, recall that the number of informative citizens is proportional to the protest effectiveness $\phi(\kappa(p))$, by (10). In addition, as illustrated by Figures 1 and 2, the protest effectiveness is first increasing and then decreasing in p; that is, $\phi(\kappa(p))$ is hump-shaped in p. Intuitively, for small p, the policy maker is initially unlikely to be swayed by the protest, because turnout is not very informative. However, as p increases, the turnout becomes more informative, and the policy maker is more likely to be swayed. In this region, the participation decisions of informative citizens are complementary. Finally, as p becomes very large, the participation of the "other" protesters is almost certain to ensure the desired outcome, so additional participation has little effect; eventually the participation decisions of informative citizens become substitutes.

4 Complementarity and Communication Breakdown

We now investigate under what conditions informative equilibria exist, and, if they exist, how the model's parameters affect information transmission. The results in this section are all for the case of a uniform cost distribution. In this case, equilibrium is characterized by the function $\hat{p}$. Figure 4a illustrates $\hat{p}$ for different values of the policy maker's preference, μ. When μ is large relative to q, $\hat{p}$ fails to intersect the 45-degree line at a positive p; hence, by Lemma 1, babbling is the unique equilibrium. Conversely, when μ is close to q, such intersections exist;



(a) Increasing μ moves the peak of $\hat{p}$ down and to the right.



(b) Decreasing $\frac{u}{\sigma^2} Z$ scales down $\hat{p}$.

Figure 4: This figure illustrates how the parameters affect the shape of $\hat{p} = \frac{u}{\sigma^2}\, Z\, \phi\,(\kappa(p))$.

hence, there are informative equilibria. The proposition below shows that this holds in general: whether information can be transmitted or communication breaks down depends on the distance of μ from q. Intuitively, this distance measures the policy maker's *burden of proof*, that is, the amount of evidence needed to change his decision from the prior optimal one.

Moreover, information transmission becomes easier to sustain when the noise is smaller relative to the citizens' stakes, that is, when $\frac{u}{\sigma^2}$ is higher, and when signals are (Blackwell) more informative.[6]

Proposition 1. *When the other parameters $(q, \mathbb{P}(\cdot|\cdot), \sigma, u)$ are fixed and F is uniform, there are cutoffs $d_1 < 0 < d_2$ such that an informative equilibrium exists if and only if*

$$d_1 \leq \mu - q \leq d_2.$$

An increase in $\frac{u}{\sigma^2}$ or signal informativeness (in Blackwell order) increases the cutoffs $-d_1$ and d_2.

Proof. The proof is in Section A.2 of the online appendix. □

The proposition follows from how the shape of $\hat{p}$ changes with the parameters. This is easiest to see for the second claim: an increase in $\frac{u}{\sigma^2}$ simply scales $\hat{p}$ up, preserving the existence of an informative equilibrium; see Figure 4b. A more informative signal also scales $\hat{p}$ up, because it can be shown to imply a larger Z by increasing $(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta))\,(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b))$. Conversely, as $\frac{u}{\sigma^2}$ decreases, there will be a threshold below which babbling is the unique outcome. Similarly, when the signal precision becomes too low, information transmission ceases.

The key to proving the first claim is that when $\mu \geq q$ and μ increases, the peak of $\hat{p}$ decreases, and the new peak is attained at a larger p (see Figure 4a). From

[6] The signal distribution $\mathbb{P}(\cdot|\cdot)$ becomes more informative if $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}$ increases and $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}$ decreases.

this we show that if there is no informative equilibrium for some μ—that is, if $\hat{p}$ lies entirely below the 45-degree line for that value of μ—then for any $\mu' > \mu$, the corresponding $\hat{p}$ also lies entirely below the 45-degree line. This gives us the general cutoff structure for the existence of an informative equilibrium.

To see why an informative equilibrium is harder to sustain with a larger burden of proof, note that the participation incentives are largest when $T = \lambda(\alpha)$, because then $\kappa(p) = 0$; the incentives decrease as T moves farther away from $\lambda(\alpha)$. Now consider some $\mu > q$, and fix the informativeness p such that $T = \lambda(\alpha)$. Then T moves away from $\lambda(\alpha)$ when μ increases. Hence, the effectiveness decreases with μ—that is, $\hat{p}(p)$ gets smaller—making it harder to sustain an equilibrium.

Next, consider the case $\mu = q$. In this case, T is close to $\lambda(\alpha)$, especially when p is small.[7] Technically, $\kappa(p) = -p/2$; that is, T is halfway between $\lambda(\beta)$ and $\lambda(\alpha)$. Therefore, the effectiveness is strictly positive even for small p, that is, $\hat{p}(0) > 0$, and an informative equilibrium always exists. This also explains why $\hat{p}$ is not hump-shaped in this case.

Intuitively, the breakdown of communication for a large burden of proof can be understood in terms of the nonconcavity of the value of information. When $\mu - q$ is large, for protest participation to tip the policy maker's decision, p needs to be large. That is, the peak of $\hat{p}$ is at a large p. However, providing such high informativeness is prohibitively costly since the value of convincing the policy maker has not changed. By contrast, when $\mu = q$, participation is effective even for small p because of the policy maker's indifference at the prior.

The proof provides further insights into the relationship between d_1 and d_2. An inspection of $\hat{p}$ shows that μ enters only via the functional form $\gamma := \frac{1-q}{q}\frac{\mu}{1-\mu}$, which is the quantity we call the burden of proof. The proof shows that there exists some $\hat{\gamma} > 1$, depending on the other parameters $(q, \mathbb{P}(\theta|\omega), \sigma, u)$, such that an informative equilibrium exists if and only if $\frac{1}{\hat{\gamma}} \leq \gamma \leq \hat{\gamma}$. This also reflects the symmetry between the cases $\mu > q$ and $\mu < q$.[8]

Proposition 2 states how the parameters affect the equilibrium informativeness.

Proposition 2. *When the other parameters are fixed and F is uniform, an increase in $\frac{u}{\sigma^2}$ or signal informativeness (in Blackwell order) increases the maximal informativeness, and strictly so if an informative equilibrium exists.*

For $\mu \geq q$, the maximal informativeness is decreasing in μ if

$$\frac{u}{\sigma^2}\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right)\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right) < \frac{1}{\phi(1)},$$

[7] This is in contrast to $\mu \neq q$, where T is far from $\lambda(\alpha)$ if p is small.

[8] Formally, including γ as an argument of $\hat{p}$, we have $\hat{p}(\cdot;\gamma) = \hat{p}(\cdot;\frac{1}{\gamma})$. So, for every $\mu > q$, there is some $\mu' < q$ corresponding to the same $\hat{p}$.

and is first increasing and then decreasing if

$$\frac{u}{\sigma^2}\left(\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)\right)\left(\mathbb{P}(\alpha|a)-\mathbb{P}(\alpha|b)\right)>\frac{1}{\phi(1)};$$

analogous statements hold for $\mu \le q$.

Proof. The first claim is proven in Section A.2 of the online appendix. The proof of the second claim requires a lengthy derivation, so we omit it here; it appears in Ekmekci and Lauermann (2019, Theorem 5, p. 21). □

As noted earlier and illustrated in Figure 4b, an increase in $\frac{u}{\sigma^2}$ scales $\hat{p}$ up, so the largest intersection of $\hat{p}$ with the 45-degree line increases. If $\frac{u}{\sigma^2}$ increases without bound,[9] so does the maximal informativeness of the protest, leading toward full information aggregation. On the other hand, if $\frac{u}{\sigma^2}$ decreases, the maximal informativeness decreases continuously until $\frac{u}{\sigma^2}$ crosses a threshold $\tau > 0$. At τ, $\hat{p}$ intersects the 45-degree line at a tangency point, and informativeness is strictly positive. If $\frac{u}{\sigma^2}$ falls below τ, however, then $\hat{p}$ lies entirely below the 45-degree line. Hence, there is no longer any informative equilibrium, and communication breaks down altogether; see Figure 4b. Thus, informativeness is discontinuous at τ.

An increase in noise inhibits information transmission for two separate reasons. First, it mechanically decreases the informativeness of the protest for a given number of informative citizens. Second, it decreases the effectiveness of participation for a given anticipated informativeness p, by increasing the distance between T and $\lambda(\alpha)$. A decrease in signal informativeness affects equilibrium informativeness analogously.

We have seen in Proposition 1 that an increase in the burden of proof, $|\mu - q|$, makes it harder for an informative equilibrium to exist. Nevertheless, Proposition 2 shows that, provided an informative equilibrium exists, the maximal informativeness may actually increase with the burden of proof. This happens because $\hat{p}$ is increasing in μ for large values of p, as illustrated in Figure 4a. It follows that, for sufficiently large $\frac{u}{\sigma^2}$, the maximal informativeness increases locally with μ.

We interpret the discontinuities in the maximal informativeness as indicating a fragility of the protest: if $\mu - q = d_1$, then a small change in $\frac{u}{\sigma^2}$, signal informativeness, or μ can disrupt all communication.

5 The Role of Costs in Information Transmission

How do costs affect the equilibrium informativeness? To answer this question, we allow for a general cost distribution given by some c.d.f. F that is strictly increasing on $\mathbb{R}$ and admits a strictly positive, continuous density. With a full-support distribution, we can now dispense with the assumption $u \le \frac{\sigma}{2\phi(0)}$, which was previously

[9]This is compatible with our assumption that $\frac{u}{\sigma} < \frac{1}{2\phi(0)}$ if u decreases proportionally.

used to ensure interior cost cutoffs.

The following result shows that the qualitative insights from Proposition 1 extend to general cost distributions, as do most comparative statics for informative equilibria. Moreover, for all cost distributions F, as σ becomes small, p becomes arbitrarily large.

Proposition 3. *For a fixed cost distribution F, and with the other parameters $(q, \mathbb{P}(\cdot|\cdot), \sigma, u)$ fixed, the following hold:*

1. *An informative equilibrium exists if μ is close enough to q, and babbling is the unique equilibrium if μ is close enough to 0 or 1.*

2. *If the parameters are such that an informative equilibrium exists, then an informative equilibrium also exists if signals are more informative, u is higher, or σ is smaller.*

3. *If signals become more informative, then the maximal equilibrium informativeness increases, and strictly so if an informative equilibrium exists.*

4. *For every $m > 0$, there is some $\epsilon > 0$ such that, for every $\sigma \leq \epsilon$, there is an equilibrium with informativeness $p \geq m$.*

We now ask how changes in the participation costs affect information transmission. A naive intuition may be that these costs are just a "friction", and so removing them would make the protest maximally informative. The maximal feasible informativeness is achieved if all citizens participate with signal a but not with signal b; hence, it is given by $p^{max} = \frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}$.

To test this intuition about the role of costs, the next proposition considers the benchmark where costs vanish, with F becoming concentrated around 0. Perhaps surprisingly, it turns out that costs can actually improve information transmission, and, as costs vanish, the equilibrium informativeness may also vanish.

Specifically, the proposition shows that the informativeness in the benchmark case is sharply determined by the directions of the participation incentives for citizens with different signals. We say that tipping *separates* the participation incentives if, conditional on the turnout being at the tipping point ($t = T$), citizens with signal a strictly prefer to tip the policy maker's decision from B to A (i.e. to participate), and those with signal b strictly prefer not to tip it. This holds if

$$\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)} \frac{\mu}{1-\mu} < 1 < \frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu}. \tag{11}$$

Conversely, we say that tipping *pools* the participation incentives if, conditional on $t = T$, all citizens, whatever their signals, either prefer to tip or prefer not to tip the decision. This holds if either $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)} \frac{\mu}{1-\mu} > 1$ or $1 < \frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu}$.

To understand this, consider a citizen with signal θ. The condition $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu} \gtrless 1$ answers the following question: conditional on being "pivotal", would a citizen with signal θ prefer to tip the decision from B to A? When turnout is equal to the policy maker's tipping point T, the posterior likelihood ratio of α is $\frac{\mu}{1-\mu}$, by (2). Given the citizen's own signal, the posterior likelihood ratio of the states is therefore $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu}$. If $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu} > 1$, then conditional on being pivotal and on her own signal, the citizen believes state α is strictly more likely; therefore, she strictly prefers to tip the decision. Now, if (11) holds, then citizens with different signals have different preferences about tipping the decision. However, if either $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu} > 1$ or $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu} < 1$ holds for both $\theta \in \{a, b\}$, then all citizens have the same preference. Tipping pools the incentives, that is, (11) fails, if there is a conflict of interest between the citizens and the policy maker (i.e., $\mu \neq \frac{1}{2}$) and signals are relatively uninformative (i.e., $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}$ and $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}$ are close to 1).

Proposition 4. *For every $\epsilon > 0$, if F is sufficiently concentrated around 0, then the most informative equilibrium p satisfies the following:*

1. *If tipping separates the participation incentives, then $p > p^{max} - \epsilon$.*

2. *If tipping pools the participation incentives, then $p < \epsilon$.*

Thus, if tipping separates incentives, then the protest achieves the maximal informativeness when there are no costs; that is, costs can only hurt information transmission. However, when tipping pools incentives, costs improve information transmission: no matter the other parameters, if costs are concentrated around 0, the protest contains almost no information. This effect is most prominent when the noise is small; in this case, by Proposition 3, information aggregates when there are costs, even if participation incentives are pooled. This is one of the main insights of the present paper: costs screen citizens' information and turn their participation into a credible signal.

The idea of the proof is simple: when F is concentrated around 0 and tipping separates incentives, most citizens participate if and only if their signal is a. Hence, the mass of informative citizens is equal to the citizens' total mass 1, yielding the maximal feasible informativeness p^{max}. Conversely, if tipping pools incentives, and, say, $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} > 1$, then most citizens would participate independently of their signals, implying low informativeness.

To see why costs help in this case, note that even when tipping pools the participation incentives, citizens with signal a still have stronger incentives; in other words, $0 < c_b < c_a$. Therefore, when the cost distribution is not degenerate, there is a substantial mass of citizens with costs between c_b and c_a, who base their participation decisions on their signals.

Proposition 4 reflects an insight about costless participation that was previously identified in the cheap-talk approach: Battaglini (2017) notes that when information is too noisy (signals are imprecise) or the conflict of interest is too large, even a large number of experts cannot guarantee information transmission.[10] When costs and noise are added, the economics of information transmission changes. We have already observed that costs may facilitate information transmission; here, we note some further implications.

Remark 1. *(Eagerness may hurt credibility.)* Note that participation decisions depend only on c/u, so decreasing costs is equivalent to increasing u. Hence, by similar reasoning, informativeness may be low when citizens are overly motivated, in the sense that u is large. In particular, for any $\epsilon > 0$, if u is large enough, the most informative equilibrium p satisfies the following: if tipping separates incentives, then $p > p^{max} - \epsilon$. Conversely, if tipping pools incentives, then $p < \epsilon$.[11] Thus, in the latter case, information transmission will often be nonmonotone in u, with protests having little or no informativeness for small and large u but significant informativeness for intermediate levels of u.

Remark 2. *(Burden of proof and conflict of interest.)* When there are costs, the burden of proof, $|\mu - q|$, is central to the analysis, while the conflict of interest, $\left|\frac{1}{2} - \mu\right|$, plays no particular role. By Proposition 3, for any fixed μ, there exists an informative equilibrium if q is close enough to μ, but not otherwise. When there are no costs, however, the conflict of interest is central—as in the previous literature—while the burden of proof plays no particular role, by Proposition 4.

Remark 3. *(Pivotal inference.)* The citizens' inference about the state from being at the tipping point plays no special role when there are costs. To illustrate this, consider the case in which citizens are perfectly informed about the state, that is, their signals already reveal the state, so that the inference from being pivotal is absent. In this case, the shape of $\hat{p}$ in (7) does not change, and exactly the same forces continue to determine the (im-)possibility of information transmission. Conversely, without costs, the citizens' incentives conditional on being at the tipping point depend critically on their inference about the state and on the precision of the signals.

6 Multiple Messages (Polls and Nonbinding Elections)

In many settings, citizens can express support for more than one outcome; that is, multiple (competing) protests may occur. Let us consider a variation of our model

[10] Ekmekci and Lauermann (2019) discusses the conditions in the literature.

[11] Increasing u is equivalent to making F more concentrated around 0: given some $u_1 < u_2$, inspection of (24) and (25) from the proof of Proposition 3 shows that $\hat{p}_F(p, u_2) = \hat{p}_{\tilde{F}}(p, u_1)$ for $\tilde{F}(c) = F\left(c\frac{u_2}{u_1}\right)$; that is, $\tilde{F}$ is more concentrated around 0 than F.

in which citizens have three options: participate in protest $\tilde{A}$ (for policy A), participate in a competing protest $\tilde{B}$ (for policy B), or abstain. When can both protests attract participation? Perhaps surprisingly, in this section we identify a force towards protests being endogenously one-sided; that is, for certain parameters, only one of the two potential protests can be active. As we will see, if we start with two active protests, best-response dynamics will often imply that one of the two unravels.

Moreover, whether it is protest $\tilde{A}$ or $\tilde{B}$ that survives is determined by the fundamental parameters (especially the policy maker's bias and the relative noise). This is in contrast to our main model with exogenously one-sided protests, where the equilibrium meaning of the protest is essentially arbitrary; that is, given an equilibrium with a protest in favor of A, there is a corresponding, equally informative equilibrium with a protest in favor of B.[12] Indeed, protest movements are often asymmetric; for example, there are often large protests against construction projects such as highways or airports, but rarely protests in their favor. Our model identifies one potential explanation for this asymmetry.

Formally, we add participation in $\tilde{B}$ as a third option to our basic model. In addition to the citizens, there is again normally distributed noise in each protest, with mean 0 and standard deviation $\sigma_{\tilde{A}}$ for $\tilde{A}$ and $\sigma_{\tilde{B}}$ for $\tilde{B}$. We consider ethical equilibria in which citizens follow a strategy that maximizes their total payoff, taking as given the participation of the others. To avoid case distinctions, we consider monotone equilibria in which turnout in $\tilde{A}$ is indicative of α, and turnout in $\tilde{B}$ is indicative of β; that is, the means of the turnouts satisfy $\lambda_{\tilde{A}}(\alpha) \geq \lambda_{\tilde{A}}(\beta)$ and $\lambda_{\tilde{B}}(\alpha) \leq \lambda_{\tilde{B}}(\beta)$. We say an equilibrium is *two-sided* if both inequalities are strict and *one-sided* if one inequality is strict (if neither is strict, the equilibrium is babbling). For the analysis, we consider a uniform cost distribution with support $[\underline{c}, \overline{c}]$ for some $\underline{c} \leq 0 < \overline{c}$.[13] We say that tipping pools incentives on A if $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)} \frac{\mu}{1-\mu} > 1$, and on B if $\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)} \frac{\mu}{1-\mu} < 1$, for $\theta \in \{a, b\}$.

Proposition 5. *If tipping pools incentives on A, then $\lambda_{\tilde{B}}(\alpha) = \lambda_{\tilde{B}}(\beta) = 0$; if tipping pools incentives on B, then $\lambda_{\tilde{A}}(\alpha) = \lambda_{\tilde{A}}(\beta) = 0$. In either case, informative equilibria are one-sided.*

Thus, if the policy maker's bias, $\left|\mu - \frac{1}{2}\right|$, is large relative to the signal precision, then every informative protest will be one-sided. Moreover, the proposition identifies which side is active: if μ is large (the policy maker is biased towards B), then

[12]With a uniform cost distribution, it is without loss of generality to assume the protest is in favor of A. With a general cost distribution, this is not the case, and the maximal informativeness may differ; see Section A.5. Nevertheless, all results for general costs remain true for protests in favor of B. This is because our assumptions do not otherwise treat states asymmetrically, so we can relabel them.

[13]As before, we assume that $\frac{u}{\sigma_j} < \frac{\overline{c}}{\phi(0)}$ for $j = \tilde{A}, \tilde{B}$.

every informative protest is in favor of A, and vice versa if μ is small. The proof in the appendix shows that the proposition also holds for general cost distributions, not just for the uniform distribution.

For intuition, recall from the discussion of Proposition 4 that if tipping pools incentives on A, that is, if $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} > 1$, then conditional on being at the tipping point, citizens with either signal strictly prefer policy A to policy B; thus, if the choice of protest matters, all citizens strictly prefer to join protest $\tilde{A}$ rather than $\tilde{B}$. So, combined with Proposition 4, the result shows that for those parameters for which *costless* protests are uninformative, costly protests are endogenously one-sided.

Moreover, when participation is costly, meaning $0 = \underline{c}$, there is also a force toward protests being one-sided if the conditions from Proposition 5 do not apply.

Proposition 6. *Suppose $\underline{c} = 0$. Then the protest is one-sided for generic parameters $(q, u, \mu, \mathbb{P}(\cdot|\cdot), \sigma_{\tilde{A}}, \sigma_{\tilde{B}})$.*

The proof is instructive. The arguments use the ratio

$$R = -\frac{\sigma_{\tilde{B}}^2}{\sigma_{\tilde{A}}^2}\frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\mathbb{P}(b|\beta) - \mathbb{P}(b|\alpha)}\frac{\mathbb{P}(\beta|a)}{\mathbb{P}(\beta|b)}\frac{\left(\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}\frac{\mu}{1-\mu} - 1\right)}{\left(\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} - 1\right)},$$

with $R > 0$ if the conditions from Proposition 5 fail. The proof shows that the protest is one-sided for all parameters for which $R \neq 1$, which holds generically.

The proof proceeds as follows. We first note that every monotone equilibrium will be in cutoffs, with citizens with signal a participating in $\tilde{A}$ if $c \leq c_a$, and citizens with signal b participating in $\tilde{B}$ if $c \leq c_b$. Then, given an arbitrary pair of interior cutoffs (c_a, c_b) and the policy maker's best response to it, a citizen's best response is determined by R: specifically, the ratio of the best-response cutoffs is $R\frac{c_a}{c_b}$. Hence, if $R > 1$ (which holds, for example, if $\sigma_{\tilde{B}}$ is large), then starting from cutoffs of any given ratio, the best response will imply a strictly larger ratio; thus, along any sequence of iterated best responses, since c_a is bounded, c_b must vanish.

Intuitively, if $\sigma_{\tilde{B}}$ is large, then the policy maker is less responsive to turnout in $\tilde{B}$, which reduces participation incentives. As participation in $\tilde{B}$ decreases, the policy maker becomes even less responsive to it, which further decreases participation in $\tilde{B}$, eventually unraveling the protest.

Thus, the proof shows not only that protests must be one-sided, but also that if $R > 1$, then in every equilibrium that is "stable" in the sense of being approached by iterating best responses, the active protest must be $\tilde{A}$ (and vice versa: if $0 < R < 1$, then in every "stable" equilibrium, the active protest is $\tilde{B}$).

Proposition 6 assumes that participation costs are positive. If participation can also be beneficial, with $\underline{c} < 0$, and if tipping separates incentives, then there is always an informative equilibrium in which both protests are active.

Proposition 7. *Suppose $\underline{c} < 0 < \bar{c}$. Then there exists a two-sided equilibrium if $R > 0$, that is, if tipping separates incentives.*

The proof of Proposition 7 constructs an equilibrium in which citizens with signal a participate in $\tilde{A}$ if their costs are below some cutoff $c_a \geq 0$, and those with signal b participate in $\tilde{B}$ if their costs are below some cutoff $c_b \geq 0$. Thus, whenever $R > 0$, there is an informative equilibrium in which all citizens who enjoy participation as such will participate in "their" respective protests, revealing their signals.

However, based on Proposition 6, it is natural to conjecture that when there are few such citizens (that is, if $\underline{c}$ is close to 0), the protest sizes become increasingly asymmetric, with one protest attracting significantly more participation. We show in the appendix that this is indeed the case. Suppose that costs are uniformly distributed on $[-\epsilon, 1-\epsilon]$ and $c_b^\epsilon \geq 0$ is a corresponding equilibrium cutoff. If $R > 1$, then we show that for $\epsilon \to 0$, the cutoff $c_b^\epsilon \to 0$. This adds to the previous stability argument, which also selected $\tilde{A}$ when $R > 1$.

Our extension with multiple messages also models many of the other informal political processes described in the introduction, such as polls (see Morgan and Stocken (2008)) and nonbinding voting (a common format for shareholder elections; see Levit and Malenko (2011)). In particular, Propositions 5 and 6 show that many of our insights extend directly to such settings.

When applied to elections, our results also identify an important difference between the effects of costs in binding versus nonbinding elections: in a nonbinding election, Proposition 5 shows that the presence of costs may be helpful—indeed, sometimes necessary—for information transmission. This is in contrast to binding elections: as shown in Krishna and Morgan (2012), for a standard election setup with majority rule and voters having common interests, costs inhibit information transmission by reducing participation.

7 Citizens' Participation Incentives

This section discusses our assumptions about the citizens' participation and their motivation. For this, we fix the policy maker's behavior (the tipping point).

7.1 Reduced-Form Model

A sufficient reduced-form specification of the participation payoff in our setup is simply

$$u * S'_\omega(\lambda_\omega)) - c,$$

where S_ω is the probability that the policy successfully matches the state, with $S_\alpha(\lambda) = 1 - \Phi\left(\frac{T-\lambda}{\sigma}\right)$ and $S_\beta(\lambda) = \Phi\left(\frac{T-\lambda}{\sigma}\right)$. A citizen with signal $\theta \in \{a, b\}$ participates if $c \leq c_\theta = u\mathbb{E}\left[S'_\omega(\lambda_\omega)|\theta\right]$. Our specification reflects the motivation to join a protest in which participation "matters", in the sense that the *effectiveness* of additional participation, S', is high.

In existing models of large protests, Cantoni et al. (2019) note that the net payoff from participation depends directly on the participation level and the probability of success, so the net payoff is $U(\lambda_\omega, S_\omega(\lambda_\omega))$. In these models, complements and substitutes arise from assumptions on the partial derivatives of U, such as a desire to join large and successful protests. In our model, we shut down these other channels. Instead, we assume that the instrumental benefits of participation are captured by S'_ω, and the (endogenous) shape of S'_ω determines whether and when participation decisions are substitutes and complements.[14] We will now see how these participation incentives arise with pro-social citizens.

7.2 Finite Population and Altruism

We first consider a setting with a finite population of n citizens, who are altruistic, obtaining a payoff of $n \times u$ when the policy matches the state (and 0 otherwise). We show that the citizens' optimality condition (5) arises in a large population; in particular, the density at the tipping point corresponds to the probability of being pivotal. This discussion follows ideas of Evren (2012).

To start, if each citizen uses a symmetric cutoff c_θ, then the expected turnout in state ω is $\lambda(\omega) = n\mathbb{P}(a|\omega)F(c_a) + n\mathbb{P}(b|\omega)F(c_b)$. When the policy maker uses a pure strategy and chooses A iff turnout is above T, the cutoff c_θ is optimal if

$$c_\theta = nu[\mathbb{P}(\alpha|\theta)\mathbb{P}(\text{Piv}|\alpha) - \mathbb{P}(\beta|\theta)\mathbb{P}(\text{Piv}|\beta)], \tag{12}$$

where the probability of being pivotal is $\mathbb{P}(\text{Piv}|\omega) = \mathbb{P}(t = T|\omega)$.

Next, suppose that there is also a random number of activists that is normally distributed with mean 0 and standard deviation $n \times \sigma$. We denote the implied c.d.f. of the distribution of total turnout by $G(\cdot|\omega)$.[15] Given the tipping point T,

$$\mathbb{P}(\text{Piv}|\omega) = G(T|\omega) - G(T-1|\omega).$$

We now consider a large population with $n \to \infty$. For this, we normalize the tipping point to be k standard deviations above the mean in α: $T_n = \lambda_n(\alpha) + kn\sigma$. Tedious but straightforward arguments following Evren (2012) show that

$$\lim_{n\to\infty} n\mathbb{P}(\text{Piv}|\alpha; n) = \lim_{n\to\infty} n[G(T_n|\alpha; n) - G(T_n - 1|\alpha; n)] = \frac{1}{\sigma}\phi(k). \tag{13}$$

The critical observation is that the standard deviation of the total turnout is $n\sigma + \sqrt{n\lambda(\omega)(1-\lambda(\omega))}$, which becomes dominated by the normal distribution.

[14] Thus, a general specification of participation payoffs may be $U(\lambda_\omega, S_\omega(\lambda_\omega), S'_\omega(\lambda_\omega))$.

[15] Thus, $G(t|\omega) = \sum_{i\le n}\binom{n}{i}\lambda(\omega)^i(1-\lambda(\omega))^{n-i}\Phi(\frac{t-i}{n\sigma})$.

Let $p = \lim_{n\to\infty} \frac{\lambda_n(\alpha)-\lambda_n(\beta)}{n\sigma}$ be the limit informativeness. Then, from (13),

$$\lim_{n\to\infty} c_{\theta,n} = \lim_{n\to\infty} nu \left[\mathbb{P}(\alpha|\theta)\mathbb{P}(\mathrm{Piv}|\alpha; n) - \mathbb{P}(\beta|\theta)\mathbb{P}(\mathrm{Piv}|\beta; n)\right] \tag{14}$$
$$= u \left[\mathbb{P}(\alpha|\theta)\phi(k) - \mathbb{P}(\beta|\theta)\phi(p+k)\right].$$

To summarize, (13) shows that the probability of being pivotal is of the order of $1/n$. Therefore, by (14), a substantial share of the altruistic citizens are willing to participate, even in a large population. Finally, for large n, the cutoff condition (12) becomes equivalent to the cutoff condition of the continuum model, (5), where the density ϕ takes the place of the probability of being pivotal.

More generally, citizens may have payoff $(1-\delta)u + n\delta u$ if the policy matches the state and 0 otherwise, where $\delta \in (0, 1]$ parametrizes altruism.[16] In that case,

$$\lim_{n\to\infty} c_{\theta,n} = \delta u \left[\mathbb{P}(\alpha|\theta)\phi(k) + \mathbb{P}(\beta|\theta)\phi(p+k)\right],$$

and so the basic structure of our previous results extends. However, when δ decreases, free-riding incentives increase as we approach the case of fully selfish agents. Thus, while quantitatively important, the level of altruism per se does not change the basic economic forces at work.

The comparative statics in δ are analogous to those in u in our main model. In particular, when δ is very small, informative equilibria will generally not exist. Equivalently, δ determines the quantitative level of noise σ that is compatible with information transmission. However, for larger δ, the comparative statics of information transmission are nontrivial. In particular, Remark 1 immediately implies that large δ may actually limit the equilibrium informativeness when signals are uninformative or the policy maker's bias is large. That is, being too motivated can backfire by undermining protesters' credibility.

7.3 Relationship to Ethical Voting

Our model of participation can be interpreted as "act utilitarianism" (Harsanyi, 1980), because citizens take the behavior of others as given when maximizing welfare. Consequently, our model admits a game-theoretic foundation based on individual choices. A stronger notion is "rule utilitarianism", according to which citizens choose the rule that, *when followed collectively*, maximizes welfare. Feddersen and Sandroni (2006) apply this idea of group-level choice in their "ethical voting" model. Importantly, the first-order conditions for the citizen-optimal rule coincide with (5),

[16] The altruism parameter allows us to capture varying degrees of other-regarding preferences. Moreover, as in Feddersen and Sandroni (2006), we can allow for a limited or uncertain share of the population to be other-regarding (analogously to the noise in our main model). Finally, Ali and Lin (2013) note that selfish citizens will behave similarly if they attempt to appear altruistic by mimicking the actions of ethical citizens.

but not all such cutoffs satisfy the sufficient conditions for a global optimum; thus, our solution concept is more permissive than that of Feddersen and Sandroni (2006). The exact relationship is discussed in Section A.6 of the online appendix.

7.4 Finite Population and No Altruism

In Ekmekci and Lauermann (2019), we study the finite-population model of the previous section, but without altruism or exogenous activists. Here, when the probability of being pivotal is small, the expected number of informative citizens is also small, implying a small difference between the mean turnouts in the two states. On the other hand, the noise is also small: it arises only from the randomness of citizens' participation decisions, stemming from their random costs. In particular, when the population n is large, equilibrium cost cutoffs (c_a, c_b) are close to 0, so the expected number of informative citizens is approximately $nf(0)(c_a - c_b)$, and the noise is given by the standard deviation of the binomial distribution, $\sigma(\omega) = \sqrt{\lambda(\omega)(1-\lambda(\omega))} \approx \sqrt{nF(0)(1-F(0))}$.

In Ekmekci and Lauermann (2019), we invoke the central limit theorem to show that the turnout is approximately normal and that the distance between the means, relative to the standard deviation, is again the appropriate measure of informativeness. Moreover, we show that the equilibrium informativeness of a large protest is again a fixed point of an analogous mapping, namely,

$$\hat{p}_0(p) = f(0)\frac{u}{F(0)(1-F(0))} Z\phi(\kappa(p)), \tag{15}$$

where Z and κ are exactly those defined in Lemma 1. The term $F(0)(1-F(0))$ corresponds to σ^2, and $f(0)$ corresponds to the mass of citizens in our continuum model, which we previously normalized to be 1.[17]

In particular, even without any altruism, equilibrium informativeness is characterized by essentially the same trade-offs, and the functional forms of $\hat{p}_0$ and $\hat{p}$ differ only in a minor way. Together with our previous observation on the effects of γ, this emphasizes that altruism and other-regarding motivations have an important quantitative effect—determining how much noise is compatible with information transmission—but less of a qualitative effect than one might expect.

8 Related Literature

Our paper is related to several strands of the literature: (i) communication between multiple senders and a receiver, (ii) information aggregation in elections,

[17] We are presenting a specific version of the model from Ekmekci and Lauermann (2019). The version in the main body of that paper is slightly different. In particular, we normalize to $u = 1$, and we consider a Poisson-distributed number of citizens. Hence, $\sigma(\omega) = \sqrt{\lambda(\omega)} \approx \sqrt{nF(0)}$, and so the analogous mapping is $f(0)\frac{1}{F(0)}Z\phi(\kappa(p))$.

(iii) costly voting, and (iv) communication with money-burning.[18]

Informal election models resemble communication models involving multiple senders with coarse messages. Unlike in formal voting models, the receiver does not commit to a particular voting rule ex ante. One of the first such models is that of Wolinsky (2002), where a decision-maker receives advice from multiple experts with dispersed information. Wolinsky (2002) shows that information transmission is impossible if the experts' preferences are not sufficiently aligned with the receiver's. Morgan and Stocken (2008) study a model of polling in which the receiver makes a policy choice after polling a group of experts with heterogeneous preferences. They show (in Proposition 13) that when there is a conflict of interest between the policy maker and the experts, information transmission is limited regardless of the number of experts. Levit and Malenko (2011) show that for nonbinding elections in the context of shareholder voting, when the conflict between the shareholders and the board is sufficiently large, the unique equilibrium is babbling. Battaglini (2017) introduces the protest model on which ours is built. He shows that when the citizens' information is poor relative to the policy maker's bias, no information can be transmitted, again regardless of population size.[19]

We contribute to this literature by providing a tractable model with a continuum of citizens, and by adding participation costs and noise. We connect our model to previous work via a benchmark scenario where costs are concentrated around 0 (Proposition 4). Here we recover the result that babbling is the unique equilibrium when the policy maker's bias is large relative to the citizens' information. In contrast to most previous work, we provide a tight condition for information transmission that is necessary and sufficient. We show that these previous contributions are closely linked, by considering one-sided and two-sided protests within the same framework. Our main contributions are our analyses of the roles of costs and noise in information transmission, showing how their presence transforms the strategic problem into a public good problem shaped by strategic substitutes and complements.

Information aggregation has been studied extensively in formal elections. Among numerous contributions, Feddersen and Pesendorfer (1997) show that large elections aggregate information under any supermajority voting rule except unanimity. In these models, voting is costless. Krishna and Morgan (2012) study costly voting in a pure common-value model. They show that costs reduce participation, and hence informativeness, but information still aggregates in the limit, as the number of potential voters grows large. As we have noted, in the formal elections of Krishna and Morgan (2012), costs are generally detrimental, while, in our informal election setup, costs may be helpful or even necessary for information transmission.

[18] We have already discussed related ethical voting models in Section 7.

[19] Battaglini et al. (2020) test the protest model experimentally.

There is also a relation to the effects of participation costs in private-value voting models. When voting is costless, the outcome depends on the citizens' ordinal preferences; for example, a simple majority rule leads to the outcome preferred by the median voter. When there are participation costs, their effect within the voting mechanism is akin to that of monetary transfers, and so the outcome can reflect cardinal preferences. For example, Ledyard (1984) and Krishna and Morgan (2015) provide conditions under which costly voting with majority rule leads to the utilitarian outcome. Thus, costly voting implies a different selection among Pareto-efficient social choice functions. More generally, participation costs are just a particular form of payment for the right to vote. Eguia and Xefteris (2021) provide a complete characterization of the implementable social choice functions for a class of payment rules.

In our model, costs improve information transmission (as shown in Section 5) by screening citizens' beliefs; that is, costs induce differential participation across citizens with different interim beliefs. This resembles the screening property of participation costs with private values. However, in our model, the effects are different, and costs may yield a strict Pareto improvement by enabling information transmission, rather than selecting among social choice functions.

Finally, our framework is an example of a sender–receiver game, first modeled by Crawford and Sobel (1982) with a single sender.[20] In sender–receiver games, the existence of purely dissipative signals—that is, the possibility of "money-burning"—may increase the equilibrium amount of information transmission (see Austen-Smith et al., 2000 and Kartik, 2007). This is similar to our finding that costs may improve information transmission.[21]

9 Extensions, Discussion, and Conclusion

9.1 Discussion of Assumptions

Signal- or state-dependent costs. In our model, costs are independent of the citizens' signals and the state; they reflect participation incentives that are orthogonal to informational incentives. In practice, however, costs may be related to signals or to the state. For example, citizens may enjoy participating in a protest that supports a cause they consider worthy. Our model already captures some of this intuition through the assumption $c_a > c_b$. In addition, there may be further participation benefits that depend directly on a citizen's own beliefs. These could be captured by assuming that the participation costs of citizens with signal a are

[20] Austen-Smith (1993) studies cheap-talk models with multiple senders, as does Battaglini (2002). However, these papers focus on the case of a small number of senders, and they typically assume that the senders' information is not dispersed, which sustains information transmission through cross-checking.

[21] Related papers on polling and protests (that either address different questions or make different assumptions) are Olson (1965), Lohmann (1994), Banerjee and Somanathan (2001), and Battaglini and Benabou (2003).

discounted by some w—formally, the cutoff for signal a is shifted down by w (while the cutoff for signal b is unchanged). The main effect of this would be akin to that of an upward shift of $\hat{p}$, implying that an informative equilibrium always exists.

Similarly, the distribution F may depend directly on the state, with $F(c|\alpha) < F(c|\beta)$ for all c. Again, this would imply that there is always some information transmission (moreover, a citizen's cost draw would provide additional information about the state). We have excluded these possibilities from our main model because, even if realistic, they may be somewhat immediate—at least, if modeled ad hoc via an exogenous relationship. We leave it to future research to develop and study richer models with an endogenous relationship between participation costs and signals/state, analogous to the endogenous participation incentives in our analysis.

Normally distributed noise. The assumption that the number of activists is normally distributed simplifies our analysis. Furthermore, by the central limit theorem, a normal distribution arises naturally with large protests (e.g. in the limit of finite populations; see Ekmekci and Lauermann (2019)). It may also be a reasonable assumption for counting errors. A literal interpretation, however, requires one to allow a negative number of activists. This can be avoided by considering a noise distribution with positive support. Indeed, our results for general cost distributions extend qualitatively if noise can have only positive realizations, provided that the noise distribution implies that updating is monotone.[22] Finally, note that several sources of noise are not additive, such as noise that stems from uncertainty about the mass of (ethical) citizens; see Section 9.2.

Further discussion. In our working paper, Ekmekci and Lauermann (2019), we discuss other extensions, including (i) citizens with general, heterogeneous preferences, (ii) nonbinary signals, and (iii) a privately informed policy maker.

9.2 Modeling the Sources of Noise

Our model reflects the ubiquity of "noise" in the context of protests. For many reasons, it may be difficult to correctly measure or interpret protest turnout, and this uncertainty limits the effectiveness of protests. The additive normal noise in our model is a convenient way to capture various sources of uncertainty in a reduced form. The most immediate interpretation of this noise is as counting uncertainty: even the participants in a protest may have trouble estimating the crowd size, and actual turnouts are often subject to considerable debate.[23]Another source of noise is uncertainty about participants' motivations: does a large turnout mean that many

[22]For a general c.d.f. G of the noise, updating is monotone if G admits a density that is log-concave. Note that, for general G, we can no longer reduce the equilibrium characterization to a one-dimensional fixed-point problem. Of course, turnouts below $\lambda(\alpha)$ now perfectly reveal the state β.

[23]The feature article McPhail and McCarthy (2004) provides numerous examples where demonstration organizers have argued with police and the media about attendance.

participants had substantial information, or merely that they derived entertainment or monetary value from participation (e.g., as in astroturfing)?[24]

As noted in Section 7, in a model with a finite population, Ekmekci and Lauermann (2019) show that normal noise emerges from the uncertainty inherent in the binomial distribution of the participants, with the noise determined by the share of citizens with costs below 0. In the current model with a continuum of citizens, there is no uncertainty in the mass of participating citizens if the distribution of costs F is known. However, uncertainty about the participation costs, modeled as aggregate uncertainty about F, would have similar consequences. For example, suppose that F is randomly drawn from a family of parametrized distributions, and suppose citizens again participate according to cutoffs c_a and c_b with $c_a > c_b$. A large turnout may then occur either if many citizens receive signal a, or if $F(c_b)$ is large for the realized distribution F. In an alternative approach, Feddersen and Sandroni (2006) and Evren (2012) assume uncertainty in the share of altruistic citizens. Here, we could capture these conditions through uncertainty in (the distribution of) the participation payoffs u or the altruism weight γ, respectively. In general, aggregate uncertainty allows the model to reflect structural uncertainty about the environment. Especially for political events, it seems fundamentally difficult to predict turnout, even for professional forecasters, and "surprises" happen frequently.

9.3 Conclusion

Citizens can shape policies through formal political processes, such as elections, or through informal processes such as protests, polls, petitions, and referenda. These informal processes have previously been studied in isolation, primarily using a cheap-talk approach. Here, we analyze them within a unified framework, providing a tractable model of informal political processes with many participants. We replicate the earlier finding that, because of limited commitment, even a large number of participants cannot overcome a bias on the part of the receiver (who could be a policy maker or the general public). Unlike the earlier cheap-talk approach, our model allows for participation costs and noise. We show that costs can make informal political processes more effective by lending credibility to participation. More generally, we show that participation costs put the focus on the public good nature of the communication problem, giving rise to strategic substitution and complementarity effects, with the scope for information transmission determined by the receiver's burden of proof (the level of information required to change his decision). Moreover, complementarities imply equilibrium multiplicity and make information

[24]Anecdotally, for many people who protested against the Vietnam War, the opportunity to socialize with friends was a major motivation. There are many examples of astroturfing by actors such as the tobacco industry, McDonald's, and Walmart; see https://en.wikipedia.org/wiki/Astroturfing for details.

transmission fragile.

Our model of a large protest with a continuum of citizens, which in many respects is more tractable than a finite-population model, may be helpful in various other settings. In contrast to existing continuum models, the effectiveness of the protest in our model is endogenous, and the participation incentives are derived from the instrumental effect of a marginal change in participation. Previous models often assume functional relationships between turnout and participation incentives, but do not derive them; for a literature overview and empirical evidence, see Cantoni et al. (2019, Section 5) and our own "reduced-form" model in Section 7.1.

One potential direction for future research is the comparison of formal and informal elections. In a formal election, the policy maker is bound by the voting rule, whereas in an informal election he retains some flexibility. This flexibility may let him take his own information into account or adjust the decision rule to the circumstances, such as the level of noise or the informativeness of the citizens; however, it may also upset information transmission, particularly when the policy maker is biased and protest participation is cheap. It may be interesting to systematically study the relative benefits of formal versus informal elections.

Variations of our model could be applied to numerous instances of informal elections. One example is shareholder voting. Here, nonbinding elections have increased in importance because the Dodd–Frank Act requires all publicly traded companies to hold a nonbinding vote on executive compensation. As another example, social media platforms have made it easier for citizens to voice their opinions, organize petitions, and coordinate protests (see Enikolopov et al. (2020)). However, some argue that ease of community mobilization has not made protests and mass movements more effective; in fact, their success has declined (see Tufekci (2017)). Finally, concerns about "fake news" on social media abound. Can informative users in sufficient numbers overcome the noise from fake news, or will fake news ultimately drown out reliable information?

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A Online Appendix—Not for Publication

A.1 Proof of Lemma 1 (Equilibrium Characterization)

Suppose (c_a, c_b, T) is an equilibrium. We show that then the implied informativeness of the protest, p, must be a fixed point of $\hat{p}$, proving the necessity; sufficiency is already shown in the main text.

The policy maker's optimality condition (2) requires

$$\frac{\frac{1}{\sigma}\phi(k)}{\frac{1}{\sigma}\phi(k+p)} = \frac{1-q}{q}\frac{\mu}{1-\mu}, \tag{16}$$

where $k = \frac{T-\lambda(\alpha)}{\sigma}$, $p = \frac{\lambda(\alpha)-\lambda(\beta)}{\sigma}$, and $k+p = \frac{T-\lambda(\beta)}{\sigma}$. Using the analytic expression for ϕ to solve for k yields

$$\kappa(p) := -\frac{p}{2} + \frac{1}{p}\ln\left(\frac{1-q}{q}\frac{\mu}{1-\mu}\right). \tag{17}$$

The citizens' optimal cutoffs are given by (5), which, if we substitute $\kappa(p)$ for $\frac{T-\lambda(\alpha)}{\sigma}$ and $\kappa(p)+p$ for $\frac{T-\lambda(\beta)}{\sigma}$, becomes

$$c_\theta = uP(\alpha|\theta)\frac{1}{\sigma}\phi\left(\kappa(p)\right) - uP(\beta|\theta)\frac{1}{\sigma}\phi\left(\kappa(p)+p\right) \text{ for } \theta = a, b. \tag{18}$$

Using (16) to substitute for $\phi\left(\kappa(p)+p\right)$ in (18), we get

$$c_a - c_b = u\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right)\left(1 + \frac{q}{1-q}\frac{1-\mu}{\mu}\right)\frac{1}{\sigma}\phi(\kappa(p)). \tag{19}$$

From here, (1) implies that the difference between the expected turnouts in states α and β is

$$\lambda(\alpha) - \lambda(\beta) = \left(F\left(c_a\right) - F\left(c_b\right)\right)\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right).$$

Observe that the assumption $u < \frac{\sigma}{2\phi(0)}$ implies that $c_a, c_b \in (-1/2, 1/2)$. Hence, using the uniform distribution on $[-1/2, 1/2]$,

$$\lambda(\alpha) - \lambda(\beta) = \left(c_a - c_b\right)\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right). \tag{20}$$

Dividing both sides by σ and using (19) to substitute $(c_a - c_b)$, we obtain that

$$p = \frac{\lambda(\alpha)-\lambda(\beta)}{\sigma} = \frac{u}{\sigma}\,Z\,\frac{1}{\sigma}\phi(\kappa(p)), \tag{21}$$

with

$$Z = \left(1 + \frac{q}{1-q}\frac{1-\mu}{\mu}\right)\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right)\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right);$$

that is, p is a fixed point of $\hat{p}$.

A.2 Proofs for Section 4 (Existence and Comparative Statics)

Proof of Proposition 1.

It is useful to parametrize the family of functions $\hat{p}$ with $\gamma = \frac{1-q}{q}\frac{\mu}{1-\mu}$, so we define

$$\hat{p}(p;\gamma) = \frac{u}{\sigma^2}\, Z(\gamma)\, \phi\left(\kappa(p;\gamma)\right),$$

where

$$\kappa(p;\gamma) = -\frac{p}{2} + \frac{1}{p}\ln\gamma,$$

and

$$Z(\gamma) = \left(1+\frac{1}{\gamma}\right)\left(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)\right)\left(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b)\right).$$

For now we assume that $\gamma \geq 1$.

The following preliminary observations will be used repeatedly:

1. The function $\kappa(p;\gamma)$ is decreasing in p and increasing in γ.

2. The function $\hat{p}(p;\gamma)$ is single-peaked in p. This follows from the first property and ϕ being single-peaked. For $p>0$, $\hat{p}(p;\gamma)$ is continuous in both arguments, and $\lim_{p\to\infty}\hat{p}(p;\gamma)=0$.

3. Let $p^+(\gamma)$ be the location of the peak, that is, $p^+(\gamma) = \arg\max_{p\geq 0}\hat{p}(p;\gamma)$. Observe that $p^+(\gamma) = \sqrt{2\ln\gamma}$ because $\phi(\kappa)$ is maximized at $\kappa=0$. This also implies that the peak is $\hat{p}(p^+(\gamma);\gamma) = \frac{u}{\sigma^2}\, Z(\gamma)\, \phi(0)$.

Pick $1<\gamma'<\gamma''$. Then, using the formula of $p^+(\gamma)$ in Property 3 above,

$$p^+(\gamma') < p^+(\gamma''). \tag{22}$$

Moreover,

$$\hat{p}(p;\gamma') > \hat{p}(p;\gamma'') \text{ for all } 0<p\leq p^+(\gamma'), \tag{23}$$

because $Z(\gamma)$ is decreasing in γ, $\kappa(p;\gamma)$ is increasing in γ, and $\kappa(p;\gamma'')>\kappa(p;\gamma')\geq 0$ for all $p\in(0,p^+(\gamma')]$.

Claim 1: If $\hat{p}(p_1;\gamma)>p_1$ for some $p_1>0$, then $\hat{p}(p_2;\gamma)=p_2$ for some $p_2>p_1$. This follows from the intermediate value theorem (IVT) because $\hat{p}$ is continuous and vanishes in the limit as $p\to\infty$.

Claim 2: If the equation $\hat{p}(p;\gamma')=p$ has no positive solution, then the equation $\hat{p}(p;\gamma'')=p$ also has no positive solution. By Claim 1, if $\hat{p}(p;\gamma')=p$ has no positive solution, then $\hat{p}(p_1;\gamma')<p_1$ for all $p_1>0$. The inequality (23) implies that $\hat{p}(p_1;\gamma'')<\hat{p}(p;\gamma')<p_1$ for all $p_1\in(0,p^+(\gamma')]$. We will now show that $\hat{p}(p_1;\gamma'')<p_1$ for all $p_1>p^+(\gamma')$. Because $\max_p\hat{p}(p;\gamma')=\hat{p}(p^+(\gamma');\gamma')>\max_p\hat{p}(p;\gamma'')$, we have

$\hat{p}(p^+(\gamma'); \gamma') > \hat{p}(p_1; \gamma'')$ for all $p_1 > 0$. Because $p^+(\gamma') > \hat{p}(p^+(\gamma'); \gamma')$, we have $p^+(\gamma') > \hat{p}(p_1; \gamma'')$ for all $p_1 > 0$. Therefore, $p_1 > \hat{p}(p_1; \gamma'')$ for all $p_1 > p^+(\gamma')$.

Claim 3: $\hat{p}(p; 1) = p$ has a positive solution $p^*(1)$. To see this, note that $\kappa(p; 1) = -\frac{p}{2}$; therefore $\hat{p}(0; 1) > 0$, and $\hat{p}(p; 1)$ is continuous and decreasing. Hence, by the IVT, $\hat{p}(p; 1) = p$ has a positive solution.

Claim 4: There exists $\epsilon > 0$ such that $\hat{p}(p; \gamma) = p$ has a positive solution for all $\gamma \in [1, 1 + \epsilon)$. By Claim 3 and the monotonicity of $\hat{p}(p, 1)$, we have $\hat{p}\left(\frac{1}{2}p^*(1), 1\right) > \frac{1}{2}p^*(1)$. By the continuity of $\hat{p}(p; \gamma)$ in γ for $p > 0$, there is $\epsilon > 0$ such that $\hat{p}\left(\frac{1}{2}p^*(1), \gamma\right) > \frac{1}{2}p^*(1)$ for all $\gamma \in [1, 1 + \epsilon)$. Hence, by Claim 1, $\hat{p}(p; \gamma) = p$ has a positive solution for all $\gamma \in [1, 1 + \epsilon)$.

Claim 5: There exists $\bar{\gamma} > 1$ such that for all $\gamma > \bar{\gamma}$, $\hat{p}(p; \gamma) = p$ has no positive solution. To obtain a contradiction, suppose that there is a sequence $\{\gamma_n\}_{n=1,2,..}$ such that $\lim_n \gamma_n = \infty$, and for every $n \geq 1$, $\hat{p}(p_n; \gamma_n) = p_n$ for some $p_n > 0$. Because $Z(\gamma_n) \to 0$, the functions $\hat{p}(\cdot; \gamma_n)$ uniformly converge to 0, implying $p_n \to 0$. This leads to a contradiction, because (i) $\lim_{p \to 0} \frac{\partial}{\partial p}\hat{p}(p; 2) = 0$, which means there is some $\epsilon \in (0, \hat{p}(p^+(2); 2))$ such that $\hat{p}(p; 2) < p$ for all $p \in (0, \epsilon)$, and (ii) $\hat{p}(p; \gamma) < \hat{p}(p; 2)$ for all $\gamma > 2$, $p < p^+(2)$.

To complete the proof for $\gamma \geq 1$, Claims 3 and 4 imply that there is a $d_2 > 0$ such that $0 \leq \mu - q < d_2$ implies the existence of an informative equilibrium. Combining this with Claims 2 and 5, we see that d_2 can be chosen to be less than $1 - q$, so that $0 \leq \mu - q < d_2$ implies that there is an informative equilibrium and $\mu - q > d_2$ implies that there is no informative equilibrium. Finally, there exists an informative equilibrium when $\mu - q = d_2$ because $\hat{p}(p; \gamma)$ is continuous in γ.

We omit the proof for $\gamma < 1$ because it is analogous. To see why, note that for $\gamma < 1$,

$$\hat{p}(p; \gamma) = \hat{p}\left(p; \frac{1}{\gamma}\right).$$

This holds because $(1+1/\gamma)\phi\left(-p/2 + 1/p \ln \gamma\right) = (1+\gamma)\phi\left(-p/2 - 1/p \ln \gamma\right)$, using the functional form of ϕ.

The thresholds d_2 and $-d_1$ increase with u/σ^2 or as signals become more informative: Let $\hat{p}(p; d_2; u/\sigma^2) = p$ for some $p > 0$. If $k > 1$, then $\hat{p}(p; d_2; ku/\sigma^2) = kp > p$. By the continuity of $\hat{p}$ in γ, there is some $\epsilon > 0$ such that for all $d \in (d_2, d_2 + \epsilon)$, $\hat{p}(p; d; ku/\sigma^2) > p$. Hence, by the IVT, for every $d \in (d_2, d_2 + \epsilon)$ there is a $p_2 > p$ such that $\hat{p}(p_2; d; ku/\sigma^2) = p_2$. The proof is analogous for $-d_1$.

If signals become more informative, then the expression $(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta))\,(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b))$ increases. This follows because the posterior beliefs are more spread out, so that the term $(\mathbb{P}(\alpha|a) - \mathbb{P}(\alpha|b))$ increases. The term $(\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta))$ also increases, since $\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta) = \mathbb{P}(b|\beta) - \mathbb{P}(b|\alpha)$ implies that

$$\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta) = \mathbb{P}(a|\beta)\left(\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} - 1\right) = \mathbb{P}(b|\beta)\left(1 - \frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\right).$$

For a more informative signal, $\left(\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} - 1\right)$ and $\left(1 - \frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\right)$ both increase. Since either $\mathbb{P}(a|\beta)$ or $\mathbb{P}(b|\beta)$ increases as well, the left side must increase, too. Hence, when signals are more informative, $Z(\gamma)$ is higher for every γ. The result now follows analogously to the proof for an increase in u/σ^2. **QED.**

Proof of Proposition 2.

The claim that the maximal informativeness increases (strictly so when an informative equilibrium exists) with u/σ^2 or with the informativeness of signals was proved in the last part of the proof of Proposition 1. **QED.**

A.3 Proofs for Section 5 (General Costs)

Proof of Proposition 3. We state two preliminary steps.

Step 1. The informativeness p^* is an equilibrium if and only if $\hat{p}_F(p^*) = p^*$ for

$$\hat{p}_F(p) = \frac{\lambda(\alpha) - \lambda(\beta)}{\sigma} = \frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\sigma}\left[F(\hat{c}_a) - F(\hat{c}_b)\right], \tag{24}$$

with

$$\hat{c}_\theta(p) := \frac{u}{\sigma}\phi(\kappa(p))\,\mathbb{P}(\alpha|\theta)\left[1 - \frac{\mathbb{P}(\theta|\beta)}{\mathbb{P}(\theta|\alpha)}\frac{1-\mu}{\mu}\right], \tag{25}$$

and $\kappa(p) = -\frac{p}{2} + \frac{1}{p}\ln\left(\frac{1-q}{q}\frac{\mu}{1-\mu}\right)$, as before.

This follows immediately from the proof of Lemma 1, because the proof uses the uniform distribution only in the step giving the closed-form expression, $F(\hat{c}_a) - F(\hat{c}_b) = \hat{c}_a - \hat{c}_b$.

It will be convenient to use that the cutoff ratio is independent of p:

$$\frac{\hat{c}_a(p)}{\hat{c}_b(p)} = \frac{\mathbb{P}(\alpha|a)}{\mathbb{P}(\alpha|b)}\frac{1 - \frac{\mathbb{P}(\beta|a)}{\mathbb{P}(\alpha|a)}\frac{q}{1-q}\frac{1-\mu}{\mu}}{1 - \frac{\mathbb{P}(\beta|b)}{\mathbb{P}(\alpha|b)}\frac{q}{1-q}\frac{1-\mu}{\mu}} =: R_0. \tag{26}$$

In general, $\hat{p}_F$, $\hat{c}_\theta$, and κ are also functions of the parameters $(\mathbb{P}(\theta|\omega), \sigma, u, \mu)$. In the following, we will selectively include these as additional arguments when discussing parameter changes.

The mapping $\hat{p}_F$ shares the basic properties of $\hat{p}$. In particular, $\hat{p}(p,\mu)$ inherits continuity in p from that of $\kappa(p,\mu)$ for all μ. It is also continuous in μ for all $p > 0$; just like $\hat{p}$, it fails to be continuous only at $p = 0$ and $\mu = q$. Moreover, $\hat{p}_F(p,\mu) \to 0$ for $p \to \infty$ since $\kappa(p,\mu) \to \infty$.

Step 2. There exist $\Delta^m \geq \Delta^n > 0$ such that, for all $p \in [0, \frac{1}{\sigma}]$ and all $\mu \in (0,1)$,

$$\Delta^n \hat{p}(p) \leq \hat{p}_F(p) \leq \Delta^m \hat{p}(p).$$

To see why, note that from (9), $\hat{c}_a$ and $\hat{c}_b$ are uniformly bounded. Therefore, given that f is continuous and strictly positive, there exist $\Delta^n > 0$ and $\Delta^m \geq \Delta^n$ such that

$$\Delta^n[\hat{c}_a - \hat{c}_b] \leq F(\hat{c}_a) - F(\hat{c}_b) \leq \Delta^m[\hat{c}_a - \hat{c}_b]$$

for all p and μ. Hence, for $\hat{p}(p)$ defined in Lemma 1,

$$\Delta^n \hat{p}(p) \leq \hat{p}_F(p) \leq \Delta^m \hat{p}(p). \tag{27}$$

Proof of Item 1 (equilibrium existence and burden of proof).

An informative equilibrium exists when $\mu = q$, for all parameters: Let us consider $\hat{p}(p)$ and $\hat{p}_F(p)$ when $\mu = q$. In this case, $\kappa(p) = -p/2$, so $\hat{p}(0) > 0$, and (27) implies $\hat{p}_F(0) > 0$. Therefore, $\hat{p}_F(p)$ has a fixed point by the IVT, since it is also continuous and vanishes to 0 for large p.

An informative equilibrium exists for μ close to q: Pick $c_\theta^m = \hat{c}_\theta(p = 0, \mu = q)$. Let $p^m := \hat{p}_F(p = 0, \mu = q)$. Since $\hat{p}_F(p = 0, \mu = q) > 0$, and $\hat{p}_F$ is continuous in p for $\mu = q$ and vanishes to 0 for $p \to \infty$, there is some $p_1 < \frac{1}{2}p^m$ such that $\hat{p}_F(p_1, \mu = q) > \frac{1}{2}p^m > p_1$. Moreover, $\hat{p}_F(p_1, \mu)$ is continuous in μ (including $\mu \to q$), and so there is some cutoff $\epsilon > 0$ such that $\hat{p}_F(p_1, \mu) > p_1$ for all $|\mu - q| \leq \epsilon$. Now, existence follows from the IVT and the fact that $\hat{p}_F(p, \mu)$ vanishes to 0 for all p large enough.

An informative equilibrium does not exist for μ close to 0 or 1. From (27),

$$\hat{p}_F(p, \mu) \leq \Delta^m \hat{p}(p, \mu, u) = \hat{p}(p, \mu, u\Delta^m)$$

for all $p \in [0, \frac{1}{\sigma}]$ and all $\mu \in (0,1)$. We know from Proposition 1 that $p = 0$ is the unique fixed point of $\hat{p}(\cdot, \mu, u\Delta^m)$ for μ large enough. In particular, for all μ larger than some μ^m, it must be that $\hat{p}(p, \mu, u\Delta^m) < p$ for all $p > 0$ (otherwise, the IVT would imply the existence of a fixed point). Hence, $\hat{p}_F(p, \mu) < p$ for all $\mu > \mu^m$ and all $p > 0$. Thus, no informative equilibrium exists. An analogous argument establishes the existence of a threshold μ^n close to 0 such that no equilibrium exists when $\mu < \mu^n$.

Proof of Item 2 (existence extends with higher σ, higher u, and more precise signals).

Existence extends if σ decreases: Suppose $\hat{p}_F(p_1, \sigma_1) = p_1$ for some σ_1 and $p_1 > 0$. Since we are done otherwise, suppose that $\mu \neq q$. We show that there exists some $p_2 < p_1$ such that $\hat{p}_F(p_2, \sigma_2) = p_1 > p_2$, and so existence follows from the IVT. To construct such a p_2, let $c_1 = \hat{c}_b(p_1, \sigma_1)$ and recall that $\hat{c}_a(p) = R\hat{c}_b(p)$. Take $0 < \sigma_2 < \sigma_1$. Suppose $0 < R$ (the proof extends analogously if $R < 0$), and note that $R > 1$. Pick $0 < c_2 < c_1$ such that $\frac{F(Rc_2)-F(c_2)}{\sigma_2} = \frac{F(Rc_1)-F(c_1)}{\sigma_1}$, where such a c_2 exists because F is continuous and $F(Rc_2) - F(c_2) \to 0$ if $c_2 \to 0$. Now, since $\phi(\kappa(p,\mu)) \to 0$ for $p \to 0$, there exists some $0 < p_2 < p_1$ such that $\hat{c}_b(p_2, \sigma_2) = c_2$. Hence,

$$\begin{aligned}\hat{p}_F(p_2, \sigma_2) &= \frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\sigma_2}\left[F(\hat{c}_a(p_2, \sigma_2)) - F(\hat{c}_b(p_2, \sigma_2))\right] \\ &= \frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\sigma_1}\left[F(\hat{c}_a(p_1, \sigma_1)) - F(\hat{c}_b(p_1, \sigma_1))\right] = p_1,\end{aligned}$$

as claimed.

<u>Existence extends if u increases:</u> Analogously to the previous argument, suppose $\hat{p}_F(p_1, u_1) = p_1$ for some σ_1 and $u_1 > 0$, and pick some $u_2 > u_1$. Since $\phi(\kappa(p,\mu)) \to 0$ for $p \to 0$, there exists some $0 < p_2 < p_1$ such that $\hat{c}_b(p_2, u_2) = \hat{c}_b(p_1, u_1)$. Hence, $\hat{c}_a(p_2, u_2) = \hat{c}_a(p_1, u_1)$, and so $\hat{p}_F(p_2, u_2) = \hat{p}_F(p_1, u_1) > p_2$. Existence of an equilibrium again follows from the IVT.

<u>Existence extends if signal informativeness increases:</u> If the signal informativeness increases from $\mathbb{P}_1(\cdot|\cdot)$ to $\mathbb{P}_2(\cdot|\cdot)$, then for every p, we have $\hat{c}_a(p, \mathbb{P}_2(\cdot|\cdot)) > \hat{c}_a(p, \mathbb{P}_1(\cdot|\cdot))$ and $\hat{c}_b(p, \mathbb{P}_2(\cdot|\cdot)) < \hat{c}_b(p, \mathbb{P}_1(\cdot|\cdot))$; see (9). Thus, $F(c_a(p, \mathbb{P}_i(\cdot|\cdot))) - F(c_b(p, \mathbb{P}_i(\cdot|\cdot)))$ is larger for $i = 2$. So

$$p_1 = p_F(p_1, \mathbb{P}_1(\cdot|\cdot)) < \hat{p}_F(p_1, \mathbb{P}_2(\cdot|\cdot)). \tag{28}$$

Therefore, the IVT implies the existence of some $p_2 > p_1$ such that $p_2 = \hat{p}_F(p_2, \mathbb{P}_2(\cdot|\cdot))$.

Proof of Claim 3 (informativeness increases with signal precision).

Suppose $\mathbb{P}_2(\cdot|\cdot)$ is more informative than $\mathbb{P}_1(\cdot|\cdot)$ and $p_1 = p_F(p_1, \mathbb{P}_1(\cdot|\cdot))$ for $p_1 > 0$. From the inequality (28), there exists $p_2 > p_1$ such that $p_2 = \hat{p}_F(p_2, \mathbb{P}_2(\cdot|\cdot))$.

Proof of Claim 4 (informativeness is unbounded with small σ).

For σ small enough, there exists some $p_\sigma > 1$ such that $-\frac{1}{2}[\kappa(p_\sigma)]^2 = \ln\sigma$. Hence, for all σ,

$$\begin{aligned}\hat{c}_\theta(p, \sigma) &= \frac{u}{\sigma}\phi(\kappa(p))\,\mathbb{P}(\alpha|\theta)\left[1 - \frac{\mathbb{P}(\theta|\beta)}{\mathbb{P}(\theta|\alpha)}\frac{1-\mu}{\mu}\right] \\ &= \frac{u}{\sigma}\frac{1}{2\pi}e^{-\frac{1}{2}[\kappa(p_\sigma)]^2}\mathbb{P}(\alpha|\theta)\left[1 - \frac{\mathbb{P}(\theta|\beta)}{\mathbb{P}(\theta|\alpha)}\frac{1-\mu}{\mu}\right] = \frac{u}{2\pi}\mathbb{P}(\alpha|\theta)\left[1 - \frac{\mathbb{P}(\theta|\beta)}{\mathbb{P}(\theta|\alpha)}\frac{1-\mu}{\mu}\right] := c^m_\theta.\end{aligned}$$

It follows that

$$\hat{p}_F(p_\sigma, \sigma) = \frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\sigma}\left[F(c^m_a) - F(c^m_b)\right].$$

Thus, $\hat{p}_F(p_\sigma,\sigma)$ grows proportionally to σ^{-1}. By the restriction $p_\sigma > 1$, the value p_σ is also diverging to ensure that $-\frac{1}{2}[\kappa(p_\sigma)]^2 = \ln\sigma$, given that $\kappa(p) = -\frac{p}{2} + \frac{1}{p}\ln\left(\frac{1-q}{q}\frac{\mu}{1-\mu}\right)$. However, p_σ is growing much more slowly than σ^{-1} (if $\lim\sigma p_\sigma > 0$, then $\lim\frac{-\frac{1}{2}[\kappa(p_\sigma)]^2}{\ln\sigma} = \infty$). Hence, for small enough σ, we have $\hat{p}_F(p_\sigma,\sigma) > p_\sigma$. If we pick σ small enough so that $p_\sigma > m$, it follows from the IVT that the map $\hat{p}_F$ has some fixed point larger than m. **QED.**

Proof of Proposition 4.

Case 1: $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} < 1 < \frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}\frac{\mu}{1-\mu}$.

Take some $p^m < \bar{p} := \frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}$. We show that there is some η^m such that $\hat{p}_F(p^m) > p^m$ if $F(\eta^m) - F(-\eta^m) > 1-\eta^m$. For this, let $c^m_\theta = \hat{c}_\theta(p^m)$. The hypothesis of the case implies that $c^m_b < 0 < c^m_a$. Since $p^m < \bar{p}$, there is some η^m_1 small enough so that

$$\frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}(1-\eta^m_1) > p^m.$$

Now $\eta^m = \min\{\eta^m_1, -c^m_b, c^m_a\}$ satisfies the claim, since $\hat{c}_b(p^m) \leq -\eta^m < 0 < \eta^m \leq \hat{c}_a(p^m)$.

Given that $\hat{p}_F(p^m) > p^m$, the IVT implies that there is some fixed point $p > p^m$, because $\lim_{p\to\infty}\hat{p}_F(p) = 0$ and $\hat{p}_F$ is continuous.

Case 2a: $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} > 1$.

Take any $\epsilon > 0$. We show that there is some η^m such that $\hat{p}_F(p) < \epsilon$ for all $p \in [\epsilon,\bar{p}]$ if $F(\eta^m) - F(-\eta^m) > 1-\eta^m$. Let $c^m_b = \min\{\hat{c}_b(p) : p \in [\epsilon,\bar{p}]\}$, which exists by the continuity of $\hat{c}_b$. By the hypothesis of the case, $c^m_b > 0$. Take η^m_1 small enough so that $\frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}\eta^m_1 < \epsilon$. Then $\eta^m = \min\{\eta^m_1, c^m_b\}$ satisfies the claim. (This is because, for all $p \in [\epsilon,\bar{p}]$, we have $c^m_b \leq \hat{c}_b(p)$. Therefore, $F(\hat{c}_a(p)) - F(\hat{c}_b(p)) \leq 1-F(c^m_b) \leq 1-F(\eta^m) \leq \eta^m$, which implies that $\frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}F(\hat{c}_a(p))-F(\hat{c}_b(p)) < \epsilon$, by our choice of η^m_1.)

The remaining case (Case 2b), with $1 < \frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}\frac{\mu}{1-\mu}$, follows from an analogous argument and is therefore omitted. **QED.**

A.4 Proofs for Section 6 (Multiple Messages)

Preliminaries. A participation strategy is a measurable function $\sigma : \{a,b\} \times [\underline{c},\bar{c}] \to \Delta\{\tilde{A},\tilde{B},\emptyset\}$, denoting the probability that a citizen with a given signal and cost will participate in either protest or will abstain (this is denoted by $\emptyset$). Each participation strategy induces turnout distributions for protests $\tilde{A}$ and $\tilde{B}$ in state ω, with means $\lambda_{\tilde{A}}(\omega)$ and $\lambda_{\tilde{B}}(\omega)$, respectively, given by

$$\lambda_{\tilde{A}}(\omega) = \mathbb{E}\left[\sigma(s,c)(\tilde{A})|\omega\right] = \int_{\underline{c}}^{\bar{c}} \sum_{s\in\{a,b\}} \mathbb{P}(s|\omega)\sigma(s,c)(\tilde{A})f(c)dc,$$

and $\lambda_{\tilde{B}}(\omega) = \mathbb{E}\left[\sigma(s,c)(\tilde{B})|\omega\right]$. As in our basic model, we add independent, normally distributed noise, with standard deviations $\sigma_{\tilde{A}}$ and $\sigma_{\tilde{B}}$, respectively. Hence, the joint density of the turnouts of $\tilde{A}$ and $\tilde{B}$ in state ω given $\lambda_{\tilde{A}}(\omega), \lambda_B(\omega)$ is

$$g(x, y|\omega; \lambda_{\tilde{A}}(\omega), \lambda_{\tilde{B}}(\omega)) = \phi\left(\frac{x - \lambda_{\tilde{A}}(\omega)}{\sigma_{\tilde{A}}}\right)\phi\left(\frac{y - \lambda_{\tilde{B}}(\omega)}{\sigma_{\tilde{B}}}\right),$$

where ϕ is the density of the standard normal (we will often drop $\lambda_{\tilde{A}}(\omega), \lambda_{\tilde{B}}(\omega)$ from the arguments of g).

Given turnout realizations $t_{\tilde{A}}, t_{\tilde{B}}$, evaluating ϕ gives that the posterior likelihood ratio of the states is

$$L(t_{\tilde{A}}, t_{\tilde{B}}) = \frac{q}{1-q} \cdot \frac{g(t_{\tilde{A}}, t_{\tilde{B}}|\alpha)}{g(t_{\tilde{A}}, t_{\tilde{B}}|\beta)} = \frac{q}{1-q} \cdot \frac{e^{-\frac{1}{2}\left(\frac{t_A - \lambda_{\tilde{A}}(\alpha)}{\sigma_{\tilde{A}}}\right)^2}}{e^{-\frac{1}{2}\left(\frac{t_{\tilde{A}} - \lambda_{\tilde{A}}(\beta)}{\sigma_{\tilde{A}}}\right)^2}} \cdot \frac{e^{-\frac{1}{2}\left(\frac{t_{\tilde{B}} - \lambda_{\tilde{B}}(\alpha)}{\sigma_{\tilde{B}}}\right)^2}}{e^{-\frac{1}{2}\left(\frac{t_{\tilde{B}} - \lambda_{\tilde{B}}(\beta)}{\sigma_{\tilde{B}}}\right)^2}}.$$

The policy maker chooses A if $L(t_{\tilde{A}}, t_{\tilde{B}}) > \frac{\mu}{1-\mu}$ and B if $L(t_{\tilde{A}}, t_{\tilde{B}}) < \frac{\mu}{1-\mu}$. Recall that we consider monotone equilibria with $\lambda_{\tilde{A}}(\alpha) \geq \lambda_{\tilde{A}}(\beta)$ and $\lambda_{\tilde{B}}(\alpha) \leq \lambda_{\tilde{B}}(\beta)$, which implies that L is monotone in both arguments. Specifically, the identity $L(t_{\tilde{A}}, t_{\tilde{B}}) \equiv \frac{\mu}{1-\mu}$ gives us the set of the pivotal turnouts. Upon taking the natural log, we obtain

$$t_{\tilde{A}}\underbrace{\left(\frac{\lambda_{\tilde{A}}(\alpha) - \lambda_{\tilde{A}}(\beta)}{\sigma_{\tilde{A}}^2}\right)}_{=:\rho_{\tilde{A}} \geq 0} - t_B \underbrace{\left(\frac{\lambda_{\tilde{B}}(\beta) - \lambda_{\tilde{B}}(\alpha)}{\sigma_{\tilde{B}}^2}\right)}_{=:\rho_{\tilde{B}} \geq 0} = \tag{29}$$

$$\underbrace{\ln\left(\frac{1-q}{q}\frac{\mu}{1-\mu}\right) + \frac{1}{2}\left(\frac{\lambda_{\tilde{A}}(\alpha)^2 - \lambda_{\tilde{A}}(\beta)^2}{\sigma_{\tilde{A}}^2} + \frac{\lambda_{\tilde{B}}(\alpha)^2 - \lambda_{\tilde{B}}(\beta)^2}{\sigma_{\tilde{B}}^2}\right)}_{=:C};$$

that is, $t_{\tilde{A}}\rho_{\tilde{A}} + t_{\tilde{B}}\rho_{\tilde{B}} \equiv C$. The probability that the policy maker chooses A in state ω is

$$P(A|\omega; \lambda_{\tilde{A}}(\omega), \lambda_{\tilde{B}}(\omega)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1_{\{x\rho_{\tilde{A}} - y\rho_{\tilde{B}} \geq C\}} g(x, y|\omega; \lambda_{\tilde{A}}(\omega), \lambda_{\tilde{B}}(\omega)) dxdy.$$

Taking the partial derivative of P with respect to $\lambda_{\tilde{A}}(\omega)$ (taking the policy maker's response as given), we obtain that a marginal increase in turnout in protest $\tilde{A}$ increases the probability of the outcome A in state ω by

$$\begin{aligned} Pr\left(Piv_{\tilde{A}}|\omega\right) &= \frac{\partial}{\partial \lambda_{\tilde{A}}(\omega)} P(A|\omega; \lambda_{\tilde{A}}(\omega), \lambda_{\tilde{B}}(\omega)) \\ &= \rho_{\tilde{A}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1_{\{x\rho_{\tilde{A}} - y\rho_{\tilde{B}} = C\}} g(x, y|\omega) dxdy. \end{aligned} \tag{30}$$

Taking the partial derivative with respect to $\lambda_{\tilde{B}}(\omega)$, we find that a marginal increase in turnout in protest $\tilde{B}$ increases the probability of the outcome A in state ω by

$$\begin{aligned} Pr\left(Piv_{\tilde{B}}|\omega\right) &= -\rho_{\tilde{B}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1_{\{x\rho_{\tilde{A}}-y\rho_{\tilde{B}}=C\}} g(x,y)dxdy \\ &= -\frac{\rho_{\tilde{B}}}{\rho_{\tilde{A}}} Pr\left(Piv_{\tilde{A}}|\omega\right), \end{aligned} \tag{31}$$

the latter equality holding when $\rho_{\tilde{A}} > 0$. (Note that this "probability" is negative because an increase in turnout for $\tilde{B}$ decreases the probability that A is implemented.) For a citizen with signal θ, the marginal benefit of participating in Ã (excluding the cost of participation) is

$$MB_{\tilde{A}}(\theta) = u\left(\mathbb{P}(\alpha|\theta)Pr\left(Piv_{\tilde{A}}|\alpha\right) - \mathbb{P}(\beta|\theta)Pr\left(Piv_{\tilde{A}}|\beta\right)\right),$$

while the marginal benefit of participating in $\tilde{B}$ is

$$\begin{aligned} MB_{\tilde{B}}(\theta) &= u\left(\mathbb{P}(\alpha|\theta)Pr\left(Piv_{\tilde{B}}|\alpha\right) - \mathbb{P}(\beta|s)Pr\left(Piv_{\tilde{B}}|\beta\right)\right) \\ &= -u\frac{\rho_{\tilde{B}}}{\rho_{\tilde{A}}}\left(\mathbb{P}(\alpha|\theta)Pr\left(Piv_{\tilde{A}}|\alpha\right) - \mathbb{P}(\beta|\theta)Pr\left(Piv_{\tilde{A}}|\beta\right)\right). \end{aligned}$$

Because $\frac{q}{1-q}\frac{g(x,y|\alpha)}{g(x,y|\beta)} = \frac{\mu}{1-\mu}$ when $x\rho_{\tilde{A}} - y\rho_{\tilde{B}} = C$ (by the definition of the policy maker's indifference curve), we have that $\frac{Pr(Piv_{\tilde{A}}|\alpha)}{Pr(Piv_{\tilde{A}}|\beta)} = \frac{Pr(Piv_{\tilde{B}}|\alpha)}{Pr(Piv_{\tilde{B}}|\beta)} = \frac{1-q}{q}\frac{\mu}{1-\mu}$. Hence,

$$\begin{aligned} MB_{\tilde{A}}(\theta) &= uPr\left(Piv_{\tilde{A}}|\alpha\right)\left(\mathbb{P}(\alpha|\theta) - \mathbb{P}(\beta|\theta)\frac{q}{1-q}\frac{1-\mu}{\mu}\right), \\ MB_{\tilde{B}}(\theta) &= -u\frac{\rho_{\tilde{B}}}{\rho_{\tilde{A}}}Pr\left(Piv_{\tilde{A}}|\alpha\right)\left(\mathbb{P}(\alpha|\theta) - \mathbb{P}(\beta|\theta)\frac{q}{1-q}\frac{1-\mu}{\mu}\right). \end{aligned} \tag{32}$$

Note that the bracketed terms can be written as

$$\begin{aligned} \mathbb{P}(\alpha|\theta) - \mathbb{P}(\beta|\theta)\frac{q}{1-q}\frac{1-\mu}{\mu} &= \mathbb{P}(\beta|\theta)\left(\frac{q}{1-q}\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)} - \frac{q}{1-q}\frac{1-\mu}{\mu}\right) \\ &= \mathbb{P}(\beta|\theta)\frac{q}{1-q}\frac{1-\mu}{\mu}\left(\frac{\mathbb{P}(\theta|\alpha)}{\mathbb{P}(\theta|\beta)}\frac{\mu}{1-\mu} - 1\right). \end{aligned} \tag{33}$$

Extending the definition of an ethical equilibrium to two-sided protests, an equilibrium is described by the policy maker's strategy, summarized by $\rho_{\tilde{A}}, \rho_{\tilde{B}}$ satisfying (29), and the citizens' strategy σ, satisfying the corresponding optimality conditions, namely, that $\sigma(\theta,c)(\tilde{A}) > 0$ implies $MB_{\tilde{A}}(\theta) \geq \max\{MB_{\tilde{B}}(\theta), 0\}$ and $\sigma(\theta,c)(\tilde{B}) > 0$ implies $MB_{\tilde{B}}(\theta) \geq \max\{MB_{\tilde{A}}(\theta), 0\}$.

Proof of Proposition 5 (conditions under which protests must be one-sided). Suppose that $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}\frac{\mu}{1-\mu} < 1$, and, on the way to a contradiction, suppose that the protest is two-sided, meaning $\lambda_{\tilde{A}}(\alpha) > \lambda_{\tilde{A}}(\beta)$ and $\lambda_{\tilde{B}}(\alpha) < \lambda_{\tilde{B}}(\beta)$. Then

$\rho_{\tilde{A}}, \rho_{\tilde{B}} > 0$ by (29). Given $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu} < 1$, (32) and (33) imply that $MB_{\tilde{A}}(\theta) < 0 < MB_{\tilde{B}}(\theta)$ for $\theta \in \{a, b\}$. Hence, optimality requires $\sigma(\theta, c)(\tilde{A}) = 0$ for all θ, c. Therefore, $\lambda_{\tilde{A}}(\omega) = 0$, in contradiction to $\lambda_{\tilde{A}}(\alpha) > \lambda_{\tilde{A}}(\beta) \geq 0$. Now, suppose that the protest is one-sided, with $\lambda_{\tilde{A}}(\alpha) > \lambda_{\tilde{A}}(\beta)$ and $\lambda_{\tilde{B}}(\alpha) = \lambda_{\tilde{B}}(\beta)$. Then $\rho_{\tilde{A}} > \rho_{\tilde{B}} = 0$. It follows that $MB_{\tilde{A}}(\theta) < 0 = MB_{\tilde{B}}(\theta)$. Hence, $\sigma(\theta, c)(\tilde{A}) = 0$ in equilibrium, which again implies a contradiction to $\lambda_{\tilde{A}}(\alpha) > \lambda_{\tilde{A}}(\beta)$.
So, if a protest is informative, it must be that $\lambda_{\tilde{A}}(\alpha) = \lambda_{\tilde{A}}(\beta)$ and $\lambda_{\tilde{B}}(\alpha) < \lambda_{\tilde{B}}(\beta)$. Arguing as before, $MB_{\tilde{A}}(\theta) < 0 = MB_{\tilde{B}}(\theta)$ implies that, indeed, $\lambda_{\tilde{A}}(\alpha) = \lambda_{\tilde{A}}(\beta) = 0$, proving the claim for the case $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu} < 1$. The case $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)} \frac{\mu}{1-\mu} > 1$ follows analogously.

The arguments in the proof do not depend on the cost distribution; hence, the claim of the proposition is true for every cost distribution. **QED.**

Proof of Proposition 6 (if participation is costly, protests are one-sided). Now, assume that $\underline{c} = 0$. Recall the definition

$$R = -\frac{\sigma_{\tilde{B}}^2}{\sigma_{\tilde{A}}^2} \frac{\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta)}{\mathbb{P}(b|\beta) - \mathbb{P}(b|\alpha)} \frac{\mathbb{P}(\beta|a)}{\mathbb{P}(\beta|b)} \frac{\left(\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu} - 1\right)}{\left(\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)} \frac{\mu}{1-\mu} - 1\right)}.$$

Suppose that $\frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)} \frac{\mu}{1-\mu} > 1 > \frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)} \frac{\mu}{1-\mu}$, which implies that $R > 0$. Moreover, suppose that the protest is two-sided (in contradiction to the claim). Then $Pr(Piv_j|\omega) > 0$ for $j = \tilde{A}, \tilde{B}$, and (32) implies that $MB_{\tilde{A}}(a) > 0 > MB_{\tilde{B}}(a)$ and $MB_{\tilde{A}}(b) < 0 < MB_{\tilde{B}}(b)$. Therefore, citizens with signal a (respectively, b) participate in $\tilde{A}$ (respectively, $\tilde{B}$) if their cost is below a positive cutoff c_a (respectively, c_b), with $c_a = MB_{\tilde{A}}(a)$ and $c_b = MB_{\tilde{B}}(b)$.
Dividing c_a by c_b and using the definitions from (32), we obtain that

$$\frac{c_a}{c_b} = \frac{MB_{\tilde{A}}(a)}{MB_{\tilde{B}}(b)} = -\frac{\rho_{\tilde{A}}}{\rho_{\tilde{B}}} \frac{\mathbb{P}(\alpha|a) - \mathbb{P}(\beta|a) \frac{q}{1-q} \frac{1-\mu}{\mu}}{\mathbb{P}(\alpha|b) - \mathbb{P}(\beta|b) \frac{q}{1-q} \frac{1-\mu}{\mu}}. \tag{34}$$

The uniform distribution of costs implies that $\lambda_{\tilde{A}}(\omega) = \frac{1}{\bar{c}} c_a \mathbb{P}(a|\omega)$ and $\lambda_{\tilde{B}}(\omega) = \frac{1}{\bar{c}} c_b \mathbb{P}(b|\omega)$. We obtain that

$$-\frac{\rho_{\tilde{A}}}{\rho_{\tilde{B}}} = -\frac{\frac{\lambda_{\tilde{A}}(\alpha) - \lambda_{\tilde{A}}(\beta)}{\sigma_{\tilde{A}}^2}}{\frac{\lambda_{\tilde{B}}(\beta) - \lambda_{\tilde{B}}(\alpha)}{\sigma_{\tilde{B}}^2}} = -\frac{\sigma_{\tilde{B}}^2}{\sigma_{\tilde{A}}^2} \frac{\frac{1}{\bar{c}} c_a (\mathbb{P}(a|\alpha) - \mathbb{P}(a|\beta))}{\frac{1}{\bar{c}} c_b (\mathbb{P}(b|\beta) - \mathbb{P}(b|\alpha))}.$$

Substituting this expression into (34), we obtain that

$$\begin{aligned}\frac{c_a}{c_b} &= -\frac{c_a}{c_b}\left(\frac{\sigma_{\tilde{B}}}{\sigma_{\tilde{A}}}\right)^2 \frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\mathbb{P}(b|\beta)-\mathbb{P}(b|\alpha)} \frac{\mathbb{P}(\alpha|a)-\mathbb{P}(\beta|a)\frac{q}{1-q}\frac{1-\mu}{\mu}}{\mathbb{P}(\alpha|b)-\mathbb{P}(\beta|b)\frac{q}{1-q}\frac{1-\mu}{\mu}} \\ &= R\frac{c_a}{c_b},\end{aligned}$$

where we use (33) to rewrite the right side.

For $R \neq 1$, the above equality will have no solution with $c_a, c_b > 0$. Thus, there cannot exist a two-sided protest for generic parameters. This proves the claim. **QED.**

Remark 4. **(Best-response dynamics)** Suppose $R > 1$ and consider some interior cutoffs $c_a, c_b > 0$. Let $\rho_{\tilde{A}}$ and $\rho_{\tilde{B}}$ be the coefficients of the policy maker's best response. It follows from the previous observations that, given these cutoffs and coefficients, the citizens' best responses $\hat{c}_a > 0$ and $\hat{c}_b > 0$ (defined via (32)) imply that $\frac{\hat{c}_a}{\hat{c}_b} = R\frac{c_a}{c_b} > \frac{c_a}{c_b}$. Now, given any pair of interior cutoffs c_a^0, c_b^0, define a sequence $(c_a^i, c_b^i)_{i=0}^{\infty}$ by iterating the best-response mapping. Since c_a^i is bounded, it follows that $c_b^i \to 0$. Moreover, if c_a^*, c_b^* is a pair of equilibrium cutoffs with $(c_a^i, c_b^i)_{i=0}^{\infty}$ converging to it, it must be that $c_b^* = 0$. Hence, if c_a^*, c_b^* is "stable", in the sense that the iterated best-response dynamic converges to it, then it must be an equilibrium with a protest in favor of A.

Proof of Theorem 7 (two-sided protests exist if there are benefits).

Define the function $G : [0, \bar{c}]^2 \to [0, \bar{c}]^2$ as follows: given $c_a, c_b \in [0, \bar{c}]$, let

$$\begin{aligned}\lambda_{\tilde{A}}(c_a, \omega) &:= \frac{1}{\bar{c} - \underline{c}}(c_a + \underline{c})\,\mathbb{P}(a|\omega), \\ \lambda_{\tilde{B}}(c_b, \omega) &:= \frac{1}{\bar{c} - \underline{c}}(c_a + \underline{c})\,c_b\mathbb{P}(b|\omega).\end{aligned}$$

Let $\rho_{\tilde{A}}(c_a, c_b) := \frac{\lambda_{\tilde{A}}(c_a,\alpha)-\lambda_{\tilde{A}}(c_a,\beta)}{\sigma_{\tilde{A}}}$, $\rho_{\tilde{B}}(c_a, c_b) := \frac{\lambda_{\tilde{B}}(c_b,\beta)-\lambda_{\tilde{B}}(c_b,\alpha)}{\sigma_{\tilde{B}}}$. Define the marginal benefits $MB_{\tilde{A}}(a; c_a, c_b), MB_{\tilde{B}}(a; c_a, c_b)$ using (32). Observe that $\frac{\mathbb{P}(b|\alpha)}{\mathbb{P}(b|\beta)}\frac{\mu}{1-\mu} < 1 < \frac{\mathbb{P}(a|\alpha)}{\mathbb{P}(a|\beta)}\frac{\mu}{1-\mu}$ implies that $MB_{\tilde{A}}(a; c_a, c_b), MB_{\tilde{B}}(a; c_a, c_b) \geq 0$, and the assumption $\frac{u}{\sigma_{\tilde{A}}}, \frac{u}{\sigma_{\tilde{B}}} < \frac{\bar{c}}{\phi(0)}$ implies that $MB_{\tilde{A}}(a; c_a, c_b), MB_{\tilde{B}}(a; c_a, c_b) \leq \bar{c}$. Let

$$G(c_a, c_b) = (MB_{\tilde{A}}(a; c_a, c_b), MB_{\tilde{B}}(b; c_a, c_b)).$$

This function G is continuous and maps a compact and convex set to itself. Hence, by Brouwer's fixed-point theorem, a fixed point exists. The fixed point corresponds to a two-sided equilibrium. **QED.**

Remark 5. **(Selection of equilibrium with small benefits.)** Suppose that costs are uniformly distributed on $[-\epsilon, 1-\epsilon]$, and observe that the same arguments as before imply that

$$-\frac{\rho_{\tilde{A}}}{\rho_{\tilde{B}}} = -\frac{\frac{\lambda_{\tilde{A}}(\alpha)-\lambda_{\tilde{A}}(\beta)}{\sigma^2_{\tilde{A}}}}{\frac{\lambda_{\tilde{B}}(\beta)-\lambda_{\tilde{B}}(\alpha)}{\sigma^2_{\tilde{B}}}} = -\frac{\sigma^2_{\tilde{B}}}{\sigma^2_{\tilde{A}}}\frac{c_a+\epsilon\left(\mathbb{P}\left(a|\alpha\right)-\mathbb{P}\left(a|\beta\right)\right)}{c_b+\epsilon\left(\mathbb{P}\left(b|\beta\right)-\mathbb{P}\left(b|\alpha\right)\right)}.$$

Plugging this into (34), we obtain that

$$\begin{aligned}\frac{c_a}{c_b} &= -\frac{c_a+\epsilon}{c_b+\epsilon}\frac{\sigma^2_{\tilde{B}}}{\sigma^2_{\tilde{A}}}\frac{\mathbb{P}\left(a|\alpha\right)-\mathbb{P}\left(a|\beta\right)}{\mathbb{P}\left(b|\beta\right)-\mathbb{P}\left(b|\alpha\right)}\frac{\mathbb{P}(\alpha|a)-\mathbb{P}(\beta|a)\frac{q}{1-q}\frac{1-\mu}{\mu}}{\mathbb{P}(\alpha|b)-\mathbb{P}(\beta|b)\frac{q}{1-q}\frac{1-\mu}{\mu}}\\ &= R\frac{c_a+\epsilon}{c_b+\epsilon}.\end{aligned}$$

Now, for every ϵ pick some cutoffs $c_a^\epsilon, c_b^\epsilon$. Suppose that $R>1$. Then, for $\epsilon \to 0$, it follows from $c_a^\epsilon \leq 1-\epsilon$ that

$$\lim_{\epsilon\to 0} c_b^\epsilon = 0.$$

Hence, participation in $\tilde{B}$ vanishes. Moreover, consider a setting in which there are only costs; that is, $\underline{c}=0$. As noted before, we cannot exclude the possibility that there is a one-sided protest in favor of B. However, the previous arguments show that adding just a small benefit to the participation will select the protest in favor of A. This is another argument that "selects" between the two protests.

A.5 Protests in Favor of B

In the main model, we considered equilibria where protests are in favor of A, that is, $c_a \geq c_b$. As noted, this assumption is without loss of generality, in the sense that all results hold verbatim if we consider protest in favor of B. We now ask whether and when it is also without loss of generality for the equilibrium magnitudes, especially the informativess. To do so, we keep using the same definition of informativeness, that is, $p = \frac{\lambda(\alpha)-\lambda(\beta)}{\sigma}$, allowing for negative values now.

Lemma 2. *If there is an equilibrium in favor of A with cutoffs (c_a, c_b) and informativeness $p^* > 0$, then there is an equilibrium in favor of B with informativeness $-p^*$ if and only if $F(c_a) - F(c_b) = F(-c_b) - F(-c_a)$.*

Proof. Consider an equilibrium in favor of B, that is, with $\lambda(\beta) > \lambda(\alpha)$. Then the policy maker's inference problem is the same as before, and is given by (2). Solving for T, we again obtain that $\kappa(p) = \frac{T-\lambda(\alpha)}{\sigma} = -p/2 + 1/p\ln(\gamma)$. The citizens' optimal participation cutoffs are given by

$$c_\theta^B = u\left(-\mathbb{P}(\alpha|\theta)\phi(\kappa(p)) + \mathbb{P}(\beta|\theta)\phi(\kappa(p)+p)\right), \tag{35}$$

because additional participation tips the decision to B, with probability $\phi(\kappa(p))$ in state α for a loss $(-u)$, and with probability $\phi(\kappa(p)+p)$ in state β for a gain (u).

If there is an equilibrium in favor of A with cutoffs (c_a, c_b) and informativeness $p^* > 0$, then

$$c_\theta = u\left(\mathbb{P}(\alpha|\theta)\phi(\kappa(p^*)) - \mathbb{P}(\beta|\theta)\phi(\kappa(p)+p)\right).$$

If citizens' cutoffs are $(-c_a, -c_b)$ and if $F(c_a) - F(c_b) = F(-c_b) - F(-c_a)$, then the implied informativeness is $-p^*$. Since $\kappa(-p) = -\kappa(p)$, and using the symmetry of the density of the normal distribution around its mean, $\phi(x) = \phi(-x)$, we obtain that

$$-c_\theta = u\left(-\mathbb{P}(\alpha|\theta)\phi(\kappa(-p^*)) + \mathbb{P}(\beta|\theta)\phi(\kappa(-p^*) - p^*)\right);$$

that is, citizens' participation cutoffs satisfy (35). Then the cutoffs $(-c_a, -c_b)$ for the citizens and the tipping point $T = \lambda(\alpha) + \sigma\kappa(-p^*)$ for the policy maker, where $\lambda(\alpha)$ is derived from $(-c_a, -c_b)$, constitute an equilibrium.

Conversely, pick an equilibrium in favor of B with informativeness $-p^*$. Equation (35) implies that the cost cutoffs are $(-c_a, -c_b)$.

Finally, if $F(c_a) - F(c_b) \neq F(-c_b) - F(-c_a)$, then (24) implies that the informativeness is $\frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}\left[F\left(-c_a\right) - F\left(-c_b\right)\right] \neq -\frac{\mathbb{P}(a|\alpha)-\mathbb{P}(a|\beta)}{\sigma}\left[F\left(c_a\right) - F\left(c_b\right)\right] = -p^*$. $\square$

A.6 Ethical Voting: Details

In the ethical voting model introduced by Feddersen and Sandroni (2006), voters belonging to a group follow a symmetric cutoff strategy that maximizes the total welfare of all voters in the group. In our model, ex-ante the citizens have common preferences, so they all belong to the same group; ethical citizens will therefore choose participation cutoffs (c_a, c_b) that maximize the payoff of all citizens for a given tipping point T of the policy maker. The ex-ante expected payoff of a representative citizen is

$$uq\left(1 - \Phi\left(\frac{T - \lambda(\alpha; c_a, c_b)}{\sigma}\right)\right) + u\left(1-q\right)\Phi\left(\frac{T - \lambda(\beta; c_a, c_b)}{\sigma}\right) - \mathbb{E}\left(\mathbf{1}_{\{c<c_\theta\}}c\right), \tag{36}$$

where $\mathbb{E}\left(\mathbf{1}_{\{c<c_\theta\}}c\right) = \sum_{\theta,\omega\in\{a,b\}\times\{\alpha,\beta\}}\mathbb{P}(\omega)\mathbb{P}(\theta|\omega)\int_{-\infty}^{c_\theta} cf(c)dc$ is a citizen's ex-ante expected participation cost (before observing the signal), and $\lambda(\omega; c_a, c_b)$ is given by (1). Evaluating the first-order conditions of the payoff (36) with respect to c_a

and c_b, we obtain the system of equations[25]

$$c_\theta = u\mathbb{P}(\alpha|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\alpha;c_a,c_b)}{\sigma}\right) - u\mathbb{P}(\beta|\theta)\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\beta;c_a,c_b)}{\sigma}\right). \tag{37}$$

Equation (37) looks similar to Equation (5). However, there is an important difference. In (37), the cost cutoffs (c_a, c_b) affect the turnouts $\lambda(\omega; c_a, c_b)$, whereas in (5), the cost cutoffs are found by taking the turnouts $\lambda(\omega)$ as fixed. This reflects an important behavioral difference between our model and the ethical voting model: in our model, each citizen's preferences put weight on the other citizens' welfare, but her participation decision takes the others' participation behavior as given (that is, given $\lambda(\omega)$). In Feddersen and Sandroni (2006) the ethical citizens act collectively and follow the cutoff strategy that globally maximizes their joint payoff, leading to a stronger solution concept than ours. Our solution concept corresponds to Harsanyi's "act utilitarianism", while that of Feddersen and Sandroni (2006) corresponds to "rule utilitarianism"; see Harsanyi (1980).[26] Hence, our concept may better reflect decentralized protests, while theirs may capture more centralized, "coordinated" protests.

[25]Increasing c_θ increases the participation costs in Equation (36) by

$$\frac{\partial}{\partial c_\theta}\mathbb{E}\left(\mathbf{1}_{\{c<c_\theta\}}c\right) = c_\theta f(c_\theta)\mathbb{P}(\theta) = c_\theta f(c_\theta)\left(q\mathbb{P}(\theta|\alpha) + (1-q)\mathbb{P}(\theta|\beta)\right).$$

The marginal benefit of increasing c_θ is given by the derivative of the first two terms of (36) ,

$$qu\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\alpha)}{\sigma}\right)\mathbb{P}(\theta|\alpha)f(c_\theta) - (1-q)u\frac{1}{\sigma}\phi\left(\frac{T-\lambda(\beta)}{\sigma}\right)\mathbb{P}(\theta|\beta)f(c_\theta),$$

where we use $\frac{\partial}{\partial c_\theta}\lambda(\omega) = \mathbb{P}(\theta|\omega)f(c_\theta)$. Equating marginal costs and benefits, collecting terms, and using Bayes' formula gives Equation (5).

[26]A second difference is that Equation (37) solves for (c_a, c_b) implicitly, and hence may have multiple solutions, some of which do not satisfy sufficiency conditions. Equation (5), however, solves for (c_a, c_b) explicitly.