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IZA DP No. 16771

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ABSTRACT

Greenhouse Gas Mitigation and Price-Driven Growth in a Solow-Swan Economy with an Environmental Limit*

The existence of an environmental limit in the Solow-Swan economy changes the nature of economic growth, but does not preclude it. When atmospheric greenhouse gases reach a predetermined absolute threshold, further growth requires a permanently expanding, resource-intensive mitigation effort. If the rate of technical progress in mitigation is too low, it becomes the effective constraint on economic growth. Yet growth in both quantities and relative prices remains a robust feature of this class of economies. It also characterizes the social planner's optimum that anticipates the costs of reaching the environmental limit abruptly.

JEL Classification: O44, Q01, Q54

Keywords: Solow-Swan growth model, Baumol cost disease, anthropogenic climate change, mitigation, price-driven economic growth, Ramsey optimal policy

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1 Introduction

The Solow-Swan growth model (Solow, 1956; Swan, 1956) continues to serve as the workhorse of long-run macroeconomic analysis for students and researchers alike. It provides immediate intuition for the role of neoclassical principles in long-run economic growth and highlights important limitations of the framework. Yet concerns about the compatibility of indefinitely expanding economic activity with environmental quality necessary for humanity's survival cast doubt on the usefulness of the neoclassical growth model (Dasgupta, 2021). Such concerns were first raised by the "limits to growth" literature a half-century ago (Forrester, 1971; Meadows et al., 1972) but were widely dismissed as alarmist (Solow, 1973). Since then, the emergence of anthropogenic climate change as an existential threat has sparked a large macroeconomic literature on its interaction with economic growth beginning with Nordhaus (1977, 1992). If continuing unmitigated growth leads to environmental collapse, the steady state of the Solow-Swan model cannot exist.

Like Mankiw et al. (1992), this paper takes Solow (1956) and Swan (1956) seriously. We adapt the canonical model to account for anthropogenic climate change as a hard environmental constraint, and apply its minimal set of assumptions to trace out strong predictions about the economy's long-run growth path. As a by-product of economic activity, greenhouse gas accumulates in the atmosphere, yet cannot exceed a critical threshold. Based on this assumption, the economy must either scale back production or mitigate its impact on atmospheric greenhouse gas concentration at an exponentially increasing rate. Mitigation can occur in two distinct ways: 1) decarbonization of production processes through technical innovation, and 2) deployment of a resource-intensive abatement technology. Costly abatement is necessary if decarbonization does not progress quickly enough to avert an environmental disaster.

Under these conditions, the overall rate of technical progress in greenhouse gas mitigation becomes a key determinant of the long-run behavior of this economy. If this rate exceeds the rate of technical progress in goods production, the economy can grow without limits related to the environmental constraint.[1] In the long run, output, consumption, and capital per capita expand at a common rate converging to that of technical progress in the production of goods. Limits to growth emerge, however, if productivity in mitigation progresses less rapidly than the technology of goods production. In this case, growth in material production converges to the rate of technical progress in mitigation.

The limited growth scenario contains new and yet unexplored implications for the Solow-Swan model. First, if technical progress in mitigation is the limiting factor, the share of inputs used for abatement in this "Baumol regime" must approach one hundred percent, while a vanishing share of inputs are used in the production of goods. The reallocation of production factors towards abatement is accompanied by exponential growth of its relative price. For similar reasons, these are familiar and central features of Baumol's (1967) "service disease" and its effects on consumption

[1] In this paper, "material production" or "goods production" refers to marketed output of both goods and services.

and production patterns. According to a recent estimate by Fixler et al. (2023), output in the environmental goods and services sector (EGSS) already represents 2% of US GDP. [2] A rapid reduction in greenhouse gas emissions makes growth in this share unavoidable, unless productivity in mitigation outgrows that of goods over the next century.

Second, a secularly increasing price of abatement is a source of price-driven growth if the government produces it with factors purchased in competitive markets. In this scenario, the growth rate of GDP measured in terms of the produced good exceeds the growth rate of goods production. To raise necessary resources for supplying abatement, the government taxes factor incomes, which grow at the same rate. This price-driven growth does not occur in a "mandate" economy that compels the private sector to produce its own mitigation or purchase it in the market. Instead, GDP growth measured in terms of the numeraire equals growth in produced quantities. Currently, the share of EGSS output produced by the government is close to 30%. If this fraction remains approximately constant, growth outcomes will lie between the two extremes.

Third, we revisit some classical results of Solow-Swan growth model with a binding environmental constraint. We show the constraint does not imply the cessation of capital accumulation. Moreover, "golden rule" savings is unaffected in a regime of secularly expanding mitigation effort. We confirm that population growth exacerbates the impact of the environmental limit. Higher population growth accelerates the transition to the Baumol regime and reduces long-run per-capita income. If production requires a fixed factor like land, population growth can drive consumption per capita to zero in the long-run limit.

Mitigation effort is dictated by the need to respect the environmental constraint. In the period before the constraint is reached, there are degrees of freedom in the choice of mitigation. The assumed policy implies an abrupt transition from the Solow-Swan to Baumol growth regime that is not optimal. In the penultimate section of the paper, we assess the welfare costs of a myopic path to the environmental constraint through the lens of Ramsey (1928) optimal policy. Modest welfare losses implied by "business as usual" climate policy may signal limitations of the standard growth model in assessing costs of inaction.

2 Growth and Climate Change in the Literature

The reduction of global greenhouse gas emissions is necessary to avoid the most disastrous consequences of climate change in this century (IPCC, 2022). If the production of goods and services is to grow at all, the attendant production of greenhouse gases must be mitigated. Mitigation can take the form of *decarbonization*, the reduction of greenhouse gas emissions intensity of economic activity through structural change in production favoring less energy-intensive activities like services, improvements in energy efficiency or a rising share of renewable energy. A second

[2] This share includes not only the mitigation of climate change but also of other forms of environmental damage caused by economic activity.

form of mitigation is the use of resource-intensive *abatement* technologies such as carbon dioxide removal through afforestation, bioenergy with carbon capture and storage (BECCS) and direct air CO_2 capture and storage (DACCS).[3] Figure 1 displays the path of decarbonization, as emissions intensity of global GDP has fallen steadily. Despite this progress, overall global emissions continue to rise and according to current projections, business as usual will not suffice to meet global climate goals. Consequently, all pathways described in IPCC (2022) for limiting global warming to below 2°C include large-scale use of abatement technologies. Decarbonization and abatement have similar but distinct implications for economic growth. While decarbonization progresses in a factor-neutral fashion, as in Nordhaus (2017),[4] abatement requires the use of a resource-intensive technology which diverts factors of production from conventional economic activities. Abatement is present in Nordhaus (2017) only in reduced form, but has been incorporated explicitly into a number of structural models of growth and climate change (Kalkuhl et al., 2015; van der Ploeg and Rezai, 2019). As we show below, this subtle difference has important consequences for GDP measurement and relative price dynamics.

From a modeling perspective, these two approaches to mitigation are closely related to an earlier literature on economic growth and pollution. Brock and Taylor (2010) propose a growth model with polluting and non-polluting technologies in which the potential for long-run growth depends on the productivity growth of the non-polluting technology. They find that their setup is isomorphic to one in which all production causes pollution but there is a cleanup technology. Long-run growth is then determined by the growth rate of that technology.[5] In our model, technical progress in both decarbonization and abatement determine long-run growth of consumption possibilities.

Our research relates to a literature on integrated assessment models (IAMs) that explicitly address bidirectional feedback between economic activity and the climate system. In these models, goods production is associated with an externality of greenhouse gas emissions that accumulate in the atmosphere. By decreasing the earth's reflectivity, accumulated emissions cause climate change, which in turn feeds back into economic activity through a damage function. The optimal policy response to this externality should balance the costs of avoiding present emissions against future expected discounted damages caused by climate change, but results are particularly sensitive to two modeling choices: the social discount rate and the damage function.[6] The social discount rate

[3] We use these definitions of decarbonization and abatement throughout. Not all technologies can be cleanly separated into one of these categories. Van der Ploeg and Rezai (2019) make a similar distinction between sequestration and the substitution of renewable for fossil energy sources. Another form of mitigation not considered here is adaptation, as considered by Fried (2021).

[4] For example, capital and labor can both be employed in a solar collector rather than a coal fired-power plant. Directed technical change studied in Acemoglu et al. (2012) can replicate this transition as an endogenous phenomenon. Allocating research effort to the production of clean inputs reduces the emissions intensity of output over time. Sufficient substitutability of clean and dirty inputs is necessary for the possibility of long-run growth without environmental collapse. See also Acemoglu et al. (2016) and Aghion et al. (2019).

[5] Stokey (1998) finds similar results in an optimal growth model. Aghion et al. (1998) extend the analysis to a Schumpterian growth model and provide an detailed discussion of the literature investigating pollution and sustainable growth.

[6] The literature on IAMs includes parsimonious models that capture relevant interactions with a small number of parameters and allow (partial) analytical characterizations of the model dynamics. Among others, see DICE

involves trading off the well-being of current against future generations and raises a philosophical question without a clear answer (Ramsey, 1928; Stern, 2007). Specification and parameterization of an appropriate damage function require assumptions regarding the path of climate change and its effects on economic activity over many decades. In the presence of exponential growth, damages based on historical data often appear trivial compared with future incomes. At the same time, uncertainty associated with climate change is significant, and many growth scenarios include catastrophic and irreversible changes.[7] We sidestep these questions and focus on the consequences of mitigation and mitigation policy on economic growth by imposing an exogenous upper bound on the concentration of greenhouse gases in the earth's atmosphere.[8] Moreover, since we study a Solow-type economy, the savings rate is constant and the social discount rate does not influence model behavior.

Our modeling approach is inspired by the literature on resource economics. The distance to the environmental limit can be understood as a depletable natural resource. Building on Hotelling's (1934) seminal contribution, Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974) establish that one necessary condition for positive long-run growth is an elasticity of substitution between the scarce resource and production factors that exceeds unity. The emissions intensity of output is exogenous in our framework, which implies that no substitution away from the natural resource is possible. Yet positive production and sustained growth can occur at the binding environmental limit, because the natural resource can be replenished through abatement. This option does not exist in standard resource economics settings. More recently, Hassler et al. (2021) study directed technical improvements in input efficiency with fossil fuels as a depletable resource. In the absence of directed technical change, the factor share of fossil fuels approaches 100 percent in the limit and their result resembles our finding for resource-intensive abatement. Directing technical change towards improvements in fuel efficiency, however, returns the economy to a balanced growth path.

This research is also related to a literature on unbalanced economic growth. Baumol and coauthors emphasized the effects of unequal rates of technical progress in different sectors on relative prices and long-run growth.[9] If sectoral output shares are stable over time, Baumol's cost disease arises; the share of total inputs employed in technologically stagnant sectors approaches unity and the long-run growth rate declines. Nordhaus (2008) finds strong empirical support for Baumol's cost disease in advanced economies. Subsequent research has established this effect in the structural

(Nordhaus, 2017), Golosov et al. (2014) and van der Ploeg and Rezai (2019). A second strand of the literature focuses on larger quantitative models with more complex climate systems, multiple sectors, different types of emissions, or spatial differences. See for example FUND (Waldhoff et al., 2014), PAGE (Hope, 2011), GCAM (Calvin et al., 2019) and REMIND (Luderer et al., 2015). For discussions of policy relevance see Stern (2007), Pindyck (2013) and Heal (2017).

[7] For example, non-linear effects (Burke et al., 2015), growth effects of damages (Piontek et al., 2019) or different impacts on investment versus consumption (Casey et al., 2021). On the role of uncertainty itself, see Aengenheyster et al. (2018).

[8] The assumption of an upper bound on cumulative emissions follows Kalkuhl and Brecha (2013) and Llavador et al. (2011).

[9] See Baumol and Bowen (1965), Baumol (1967) and Baumol et al. (1985).

transformation from manufacturing to services during the second half of the last century.[10] A central assumption driving this result is an elasticity of substitution in utility between manufactured goods and services that is less than one. In our framework, the elasticity of substitution between abatement and physical production is effectively zero, giving rise to a strong form of Baumol's cost disease. Acemoglu and Guerrieri (2008) study how unequal output elasticities with respect to production factors across sectors cause unbalanced growth with secular trends in factor shares. In the analysis that follows, we will emphasize the role of technical progress by imposing identical output elasticities in material production and abatement sectors.

3 A Solow-Swan-Baumol Growth Model with Greenhouse Gases

3.1 Technology of Goods and Mitigation Production

The Solow-Swan model (Solow 1956, Swan 1956) serves as the analytic framework. In the spirit of Baumol (1967), we assume a two-good economy: produced goods in time period t, Q_t serve as either private consumption or investment, while *abatement* B_t is a pure public good that reverses the negative environmental consequences of that production - i.e. the emission of greenhouse gases. Goods and abatement compete for scarce capital K_t and labor L_t, using constant-returns-to-scale Cobb-Douglas production technologies identical up to a multiplicative factor:

$$Q_t = K_{Qt}^{\alpha} \left(A_{Qt} L_{Qt}\right)^{1-\alpha} \quad \text{and} \quad B_t = K_{Bt}^{\alpha} \left(A_{Bt} L_{Bt}\right)^{1-\alpha}. \tag{1}$$

Harrod-neutral technical progress characterizes both production functions, and A_Q and A_B grow at exogenous and nonnegative rates a_Q and a_B, respectively.[11] Factors of production can be deployed costlessly in sector Q or sector B and are fully employed:[12]

$$K_t = K_{Qt} + K_{Bt} \quad \text{and} \quad L_t = L_{Qt} + L_{Bt}. \tag{2}$$

[10] A large literature addresses the relative importance of non-homothetic preferences and unequal technical growth rates. See Kongsamut et al. (2001), Herrendorf et al. (2013), Herrendorf et al. (2015) and Boppart (2014). López et al. (2003) point out that the cost of pollution can induce structural change favoring the service sector. Ngai and Pissarides (2007) study approximate balanced growth in economies with unequal rates of technical progress across sectors.

[11] Identical output elasticities in the two sectors highlight the importance of technical change and are not essential for the results we present. Allowing for different Cobb-Douglas technologies implies an additional factor bias in the sense of the Rybczynski Theorem (see Jones, 1965) for which we have no a priori intuition. See Acemoglu and Guerrieri (2008) for an analysis of a two-sector growth model with different output elasticities.

[12] In his original analysis of the service disease, Baumol modeled a single factor of production, labor.

3.2 Households

Households supply labor L_t inelastically at any point in time, which grows at exogenous rate $n \geq 0$. The capital stock they own evolves as the difference between gross investment and depreciation:[13]

$$\dot{K}_t = I_t - \delta K_t, \tag{3}$$

where the rate of depreciation of capital δ lies in the unit interval. Investment in this closed economy equals private savings, a fixed fraction of after-tax income:[14]

$$I_t = s(Y_t - T_t). \tag{4}$$

Y_t is GDP measured at factor costs and T_t stands for lump-sum taxes to be determined below. The remainder is consumed:

$$C_t = (1 - s)(Y_t - T_t). \tag{5}$$

3.3 Factor Markets and Factor Price Determination

Factor prices are determined in competitive markets for labor and capital rental services. With the produced good as a numeraire, competitive remuneration of these production factors implies that

$$w_t = (1 - \alpha)K_{Qt}^{\alpha} A_{Qt}^{1-\alpha} L_{Qt}^{-\alpha} \quad \text{and} \quad r_t = \alpha K_{Qt}^{\alpha-1} \left(A_{Qt} L_{Qt}\right)^{1-\alpha} \tag{6}$$

where w_t and r_t are the wage and the rental rate of capital in period t, respectively.

Because abatement is a pure public good, it will not be supplied voluntarily by the private sector. In the benchmark version of this model, the government hires workers and rents capital at market prices to produce it. In practice, not all abatement will be provided by the government and enter GDP as public consumption; instead, private firms may face mandates to offset their emissions, giving rise to a private demand for abatement as an intermediate production input. We consider this case in Section 4.4. If the government chooses factor inputs to minimize cost of producing abatement B_t, capital intensity will be equal in both sectors:

$$\frac{K_{Bt}}{L_{Bt}} = \frac{K_{Qt}}{L_{Qt}}, \tag{7}$$

and the economy will scale without any structural implications across sectors. If γ_t and $1 - \gamma_t$ are the fractions of production factors dedicated to abatement and material output, respectively, then

$$Q_t = (1 - \gamma_t)K_t^{\alpha} \left(A_{Qt} L_t\right)^{1-\alpha} \quad \text{and} \quad B_t = \gamma_t K_t^{\alpha} \left(A_{Bt} L_t\right)^{1-\alpha}. \tag{8}$$

In this closed economy, GDP equals the sum of total expenditures of private agents and the

[13] As is standard, the "dot notation" refers to the first derivative with respect to time.
[14] We show below that after-tax income equals Q_t.

government, measured in units of the produced good:[15]

$$Y_t = Q_t + p_t B_t, \tag{9}$$

where p_t is the price of abatement in terms of goods. Abatement is produced by the government and enters GDP at factor cost:

$$p_t = \frac{w_t \gamma_t L_t + r_t \gamma_t K_t}{B_t} = \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha}. \tag{10}$$

The last equality follows from the fact that the marginal rate of transformation under the assumed conditions is $\frac{dQ_t}{dB_t} = \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha}$.[16] The government sets a lump-sum tax T_t to finance its expenditures

$$T_t = p_t B_t, \tag{11}$$

establishing that after-tax income indeed equals Q_t.

3.4 Environmental Limit and Mitigation

On the environmental side of the model, greenhouse gas G_t accumulates in the atmosphere according to[17]

$$\dot{G}_t = \sigma_t^{-1} Q_t - B_t \tag{12}$$

where σ_t is the marginal output associated with an additional unit of greenhouse gas emissions. Conversely, σ_t^{-1} measures the marginal CO_2 emissions intensity of produced output.

In addition to abatement, the economy can mitigate its impact on atmospheric CO_2 through *decarbonization*. By decarbonization we mean the reduction of emissions intensity of output over time. This can be due to improvements in energy efficiency or substitution to less CO_2-intensive production processes. This measure of "CO_2 efficiency" of economic output, σ_t grows at exogenous rate a_σ, which we call the rate of decarbonization. We define *mitigation* as the combination of abatement and decarbonization. The cost of abating emissions caused by one unit of output is $\frac{p_t}{\sigma_t}$ and grows at rate $(1-\alpha)(a_Q - a_B) - a_\sigma$. It may decline even as the cost of abatement per kilogram of CO_2 is increasing over time.

As in the standard Solow-Swan model, preferences are not modeled explicitly. We simply assume

[15] Baumol (1967) employed an alternative, nonstandard definition: $Y_t = W_1 Q_t + W_2 B_t$ where W_1 and W_2 are arbitrary and constant weights. Our definition highlights the effect that he stressed: the value of the good produced by the low-productivity sector increases secularly relative to the other. We discuss issues related to the measurement of GDP and its growth rate in Section 4.4.

[16] This is a direct consequence of Cobb-Douglas technology and factor price equalization.

[17] We ignore here, for simplicity, natural sinks for greenhouse gases. This assumption has no qualitative consequences for the model's implications.

a strict upper bound $\bar{G}$ that we call the *environmental limit*:

$$G_t \leq \bar{G} \quad \forall t. \tag{13}$$

In this simple form, the environmental limit represents the maximum sustainable level of greenhouse gases consistent with avoiding environmental collapse (Dasgupta, 2021). Alternatively, it might stand for some exogenous and possibly socially optimal level of greenhouse gases, or the outcome of an arbitrary political agreement. The government is assumed to ensure that the environmental limit is not violated:

$$B_t = \begin{cases} 0 & if \quad G_t < \bar{G} \\ \sigma_t^{-1} Q_t & if \quad G_t = \bar{G}. \end{cases} \tag{14}$$

The government does not engage in any abatement until it becomes binding. Once the limit is reached, the government implements the minimal amount of abatement necessary to ensure $G = \bar{G}$.[18] In the spirit of the Solow-Swan model, we are silent on the optimality of the transition between the two regimes and focus on long-run outcomes. Characterization of this transition requires an explicit Ramsey analysis, which we present in Section 5.

4 Equilibrium Growth Paths

We now exploit the simplicity of the Solow-Swan-Baumol setup and characterize two regimes which govern the economy's law of motion. The *Solow-Swan regime* obtains as long as the environmental limit is not binding. Upon reaching the constraint, the economy enters the *Baumol regime* and production of goods is only possible with commensurate abatement: $Q_t = \sigma_t B_t$. The two technologies grow at respective exogenous rates a_Q and a_B, with levels in t given by $A_{Qt} = A_{Q0} e^{a_Q t}$ and $A_{Bt} = A_{B0} e^{a_B t}$. In the Baumol regime, the levels of these indicators determine the level of sustainable consumption and investment (residual output) going forward.

4.1 Growth Path for $G_t < \bar{G}$: Balanced Growth in the Solow-Swan Regime

In the Solow-Swan regime, $B_t = 0$ and the model above is identical to the standard model. Greenhouse gas accumulates according to (12) but does not affect dynamics. Physical output, the capital stock, and GDP all grow at rate $a_Q + n$ along the balanced growth path:

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{Y}_t}{Y_t} = a_Q + n, \tag{15}$$

with all factors employed in the material production sector:

$$K_{Qt} = K_t; \quad L_{Qt} = L_t; \quad \gamma_t = L_{Bt} = K_{Bt} = B_t = 0. \tag{16}$$

[18] Judging from the current state of policy inertia, this might not actually be a bad approximation to reality.

The rate of greenhouse gas accumulation is proportional to the CO_2 emissions intensity of material production:

$$\frac{\dot{G}_t}{G_t} = \sigma_t^{-1} \frac{Q_t}{G_t} \tag{17}$$

and the regime is in place as long as $G_t < \bar{G}$.

In the following, we assume $a_Q + n \geq a_\sigma$.[19] In this case, exponential growth in Q_t in the Solow-Swan regime implies that G_t grows without bound, but this is prevented in the long run by the environmental limit. After $\bar{t}$, the constraint $G_t = \bar{G}$ is binding, unbounded growth is no longer possible, and the Solow-Swan regime ends.

4.2 Growth Path when $G_t = \bar{G}$: Unbalanced Growth à la Baumol

At time $\bar{t}$, the government abatement effort commences and $\sigma_t B_t = Q_t$ for all $t \geq \bar{t}$. Produced output in $\bar{t}$ falls discretely by $100\gamma_t\%$ to free up capital and labor necessary for abatement. Thereafter the economy assumes characteristics familiar from Baumol's (1967) economy under conditions of differential sectoral productivity growth. A sufficient condition for Baumol's cost disease to arise is that the output share of the sector with slower technical progress does not converge to zero.[20] Our environmental interpretation of the model provides economic foundations for government intervention which ensures that this condition is met.[21]

To see this, first note that the condition $\sigma_t B_t = Q_t$ pins down γ_t, the share of resources devoted to abatement:

$$\gamma_t \sigma_t K_t^\alpha \left(A_{Bt} L_t\right)^{1-\alpha} = (1-\gamma_t) K_t^\alpha \left(A_{Qt} L_t\right)^{1-\alpha} \tag{18}$$

or

$$\gamma_t = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}} \tag{19}$$

Define $a_M \equiv a_B + \frac{a_\sigma}{1-\alpha}$, the rate of technical progress in mitigation, which reflects both abatement and decarbonization.[22] Throughout the rest of this paper, a_M will play a central role. Three distinct cases emerge in the Baumol regime:

1. $a_M = a_Q$: the factor share in abatement γ_t is constant and determined by the initial relative productivities.

2. $a_M > a_Q$: the factor share in abatement converges to zero, $\gamma_t \to 0$.

[19] This condition is sufficient but not necessary for the Solow-Swan regime to end in finite time.
[20] Baumol et al. (1985) find that sectoral output shares were approximately constant and unrelated to the rate of productivity growth. Generally, an elasticity of substitution in demand for goods and services below unity is sufficient to generate Baumol's cost disease (Boppart, 2014).
[21] Baumol (1967) points out that a government intervention could impose constant output shares, but offers no reasons why this might be desirable.
[22] An isomorphism between ex-post cleanup and emissions reduction at the source can also be found in Brock and Taylor (2010). Abatement is driven by Harrod-neutral (labor-augmenting) technical progress, while decarbonization is equivalent to Hicks-neutral technical progress. With Cobb-Douglas technologies, they are equivalent up to a scaling factor.

3. $a_M < a_Q$: the factor share in abatement converges to one, $\gamma_t \to 1$.

In the first and second cases, the environmental limit imposes no long-run constraints on growth and its effects on economic dynamics are unremarkable. For details see Appendix A. The second and third cases reflect Baumol's (1967) conclusion that in the long run, all factors of production are ultimately diverted to the low-growth sector if its output share does not vanish. Because it has the most salient implications for the economy's growth path, we focus on the third case ($a_M < a_Q$).

At $\bar{t}$, the price of abatement is given by equation (10). Changing relative productivity levels imply that the price of abatement grows at a constant rate:

$$\frac{\dot{p}_t}{p_t} = (1-\alpha)(a_Q - a_B). \tag{20}$$

It is useful to decompose GDP growth at time t, g_{Yt}, into components deriving from goods production and abatement valued at market prices, in turn the sum of growth rates of the valuation and the quantity of that abatement:

$$g_{Yt} \equiv \frac{\dot{Y}_t}{Y_t} = \frac{Q_t}{Y_t}\left[\frac{\dot{Q}_t}{Q_t}\right] + \frac{p_t B_t}{Y_t}\left[\frac{\dot{p}_t}{p_t} + \frac{\dot{B}_t}{B_t}\right]. \tag{21}$$

Below, we use (21) to derive long-run limits of economic growth measured under different GDP weighting schemes.

4.3 When Mitigation is a Limit to Growth: $a_M < a_Q$

In the most interesting case, productivity growth in mitigation technologies is exceeded by productivity growth in the produced goods sector.[23] In the extreme case of $a_B = a_\sigma = 0$, as argued byDasgupta (2021) and the "degrowth" literature more generally (e.g. Hickel and Kallis, 2020), there is a strict upper bound on the possibility of mitigating environmental damages caused by economic activity.[24] As we have seen, the case $a_M < a_Q$ implies that abatement absorbs all factors of production in the long run. This powerful result follows from a hard environmental limit combined with slow technical progress in mitigation.

Under these conditions, the aggregate dynamics of goods production, the capital stock, abatement

[23] One motivation for this assumption is that many negative emissions technologies, like afforestation or BECCS, are limited by the efficiency of natural processes involved and the amount of available land, see Dasgupta (2021). The calibration presented in Section 5 and Appendix C will assume this to be the case.
[24] Georgescu-Roegen (1971) argues for an even tighter constraint on total cumulative output that earth can sustain.

and the factor share devoted to producing it are determined by the following system:

$$Q_t = (1-\gamma_t) K_t^{\alpha} (A_{Qt} L_t)^{1-\alpha}, \tag{22}$$

$$\dot{K}_t = sQ_t - \delta K_t, \tag{23}$$

$$B_t = \gamma_t K_t^{\alpha} (A_{Bt} L_t)^{1-\alpha}, \tag{24}$$

$$\gamma_t = \frac{1}{1+\sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}. \tag{25}$$

For purposes of visualization, we express model variables in terms of efficiency units of labor used in abatement, i.e. by expressing X_t as $x_t = \frac{X_t}{\sigma_t^{\frac{1}{1-\alpha}} A_t^B L_t}$.[25] We denote the long-run limit of a variable x_t with a star, i.e. $x^* = \lim_{t\to\infty} x_t$. We emphasize that x^* is a limiting point and *not* a steady state. Unlike a steady state, the behavior of the economy at x^* is not defined, because $\gamma = 1$ implies zero material output. As the economy approaches the limit, output does not converge to zero, but continues to grow at a positive rate.

Material output per effective capita q_t available for consumption or capital formation is given by

$$q_t = \frac{Q_t}{A_{Bt} L_t \sigma_t^{\frac{1}{1-\alpha}}} = (1-\gamma_t) k_t^{\alpha} \sigma_t^{-1} \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha} = \gamma_t k_t^{\alpha}. \tag{26}$$

Capital in terms of efficiency units of labor in abatement accumulates according to:

$$\dot{k}_t = \gamma_t s k^{\alpha} - (\delta + n + a_M)\, k_t. \tag{27}$$

This is the familiar accumulation equation of the standard Solow-Swan model, with a_M representing the overall technical growth rate and augmented by the term $\gamma_t = \frac{1}{1+\sigma_t \left(A_{Bt}/A_{Qt}\right)^{1-\alpha}}$, which shifts productivity over time.

4.3.1 Long-run Limit of the Limited Growth Regime

We now study the behavior of our economy in the long run as γ_t approaches unity. From equation (26), we can see that the intensive-form production function of labor measured in units of mitigation efficiency in the "Baumol-Solow-Swan model" converges to

$$q^* = (k^*)^{\alpha} \tag{28}$$

with the associated accumulation equation:

$$\dot{k}^* = s\,(k^*)^{\alpha} - (\delta + n + a_M)\, k^*. \tag{29}$$

[25] If mitigation technology grows faster than technology of material production, efficiency units have to be defined in terms of the productivity of the material sector to obtain finite limits.

At this long-run limit, $\dot{k}^* = 0$ and K_t grows at rate $g_K = n + a_M$. Figure 2 depicts this modified Baumol-Solow-Swan model using the familiar textbook diagram. The long-run limit of this economy is given by the intersection of the concave intensive-form savings function with the capital widening line with slope $a_M + \delta + n$. Output in intensive form (measured per efficiency unit of labor employed in mitigation) can be directly read off the intensive-form production function. Comparative statics correspond closely to those of the Solow-Swan model. Material output per capita is $\sigma_t^{\frac{1}{1-\alpha}} A_{Bt} (k^*)^\alpha$ and grows at rate a_M, as do consumption and capital per capita.

Figure 3 displays an illustrative growth path for material output. Upon reaching the environmental limit, the economy experiences an immediate and permanent decline in sustainable consumption and goods production levels relative to the Solow-Swan regime, as resources are redeployed to the abatement sector. The extent of this decline depends on the relative levels of productivity at the point of regime change, which we quantify in Section 5.2. In the aftermath, overall material output growth is lower and declines even more over time, as efficiency gains in the material sector have less and less impact. In the long run, the growth rate of material production output per capita reaches a_M, reflecting technical progress in mitigation.

The growth rates of key exogenous and endogenous variables in the model are summarized in Table 1 and constitute a central contribution of this paper. Growth in quantities is augmented by price-driven GDP growth, that is, growth in the value of GDP in terms of the produced good. As a general result, the long-run limit growth rate of GDP at market prices is:

$$\begin{aligned} g_Y^* \equiv \lim_{t\to\infty} g_{Yt} &= \lim_{t\to\infty} \left\{ \frac{Q_t}{Y_t}(a_M + n) + \frac{p_t B_t}{Y_t} \left[(1-\alpha)(a_Q - a_B) + a_B + \frac{\alpha}{1-\alpha} a_\sigma + n \right] \right\} \\ &= (1-\alpha)(a_Q + n) + \alpha\,(a_M + n) \\ &= g_p + g_B^*. \end{aligned} \tag{30}$$

In contrast, the growth rate of GDP with weights W_1 and W_2 converges to

$$\begin{aligned} \hat{g}_Y^* &\equiv \lim_{t\to\infty} \left\{ \frac{W_1 Q_t}{\bar{Y}_t}(a_M + n) + \frac{W_2 B_t}{\bar{Y}_t} \left(a_B + \frac{\alpha}{1-\alpha} a_\sigma + n \right) \right\} \\ &= 1 * (a_M + n) + 0 * \left(a_B + \frac{\alpha}{1-\alpha} a_\sigma + n \right) = g_Q, \end{aligned} \tag{31}$$

a result identical to Baumol's (1967) finding. When $a_Q > a_M$, the limit growth rate of GDP at market prices exceeds the growth rate of material production. Technical progress in goods production has two effects. First, it increases productivity. Second, it leads to more greenhouse gas emissions. To respect the environmental limit, the government must divert a growing share of production factors to the abatement sector and pay ever-increasing prices for them. Because abatement enters GDP as public consumption at cost, secularly rising factor prices lead to long-run GDP growth.

4.3.2 Savings, Mitigation and the Golden Rule

If $a_Q > a_M$, all resources are diverted in the limit to the abatement effort, while a vanishing share of resources is devoted to the production of goods. Since it is the production of goods that makes the abatement effort necessary, this growth path might appear inefficient; a "degrowth" position might maintain that it is better to scale back production instead by reducing the savings rate, capital accumulation and productive capacity. It is thus crucial to understand the effects of savings and abatement on long-run consumption possibilities in our model.

In the long-run limit, consumption *per capita* is given by:

$$\sigma_t^{\frac{1}{1-\alpha}} A_{Bt}(1-s)\left(\frac{s}{a_M+n+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Material output and consumption per capita are increasing in productivity in the abatement sector (A_{Bt}) and "CO_2 efficiency" of production (σ_t). Levels of A_{Bt} and σ_t jointly determine effective total factor productivity $A_t \equiv A_{Bt}\sigma_t^{\frac{1}{1-\alpha}}$, which grows at rate a_M. In a growing economy with a binding environmental constraint, the wealth of nations is determined by productivity in mitigation activities.

As in the Solow-Swan model, the savings rate plays a dual role for per-capita consumption. A higher savings rate decreases consumption directly, but increases capital accumulation and future consumption. As in Phelps (1961), a golden rule savings rate characterizes maximal consumption in effective per capita units. Upon reaching the environmental limit, the abrupt need for abatement leads to a decline in effective total factor productivity. Does this reduction in effective productivity imply that the economy should reduce the savings rate? The answer is no. Given that the government supplies all abatement, the golden rule savings rate s_{Gold} is equal to α, the same consumption-maximizing savings rate as in the standard Solow-Swan model. The golden rule consumption level requires accumulation for both material growth as well as abatement, and price-driven growth is a necessary outcome if consumption per capita is to be maximized while purchasing production factors necessary for abatement in competitive markets.[26] In Appendix B and C, we show that Ramsey-optimal long-run savings rates with and without an environmental limit differ analytically, but these differences are not economically significant for conventionally calibrated economies.

4.4 Alternative Implementation: Emission Mandates

In this section, we outline an alternative arrangement that implements the environmentally sustainable path in the Baumol regime. Instead of producing abatement itself, the government compels private firms to purchase abatement services from other firms or produce them in-house.[27] The

[26] In our setup, government spending on abatement would be recorded as consumption in the income and product accounts, but our result would also be consistent with a broader interpretation of investment to include abatement, in which the savings rate would be expanded to include this activity as well.

[27] In the following, we treat these two alternatives as equivalent.

production side of the economy remains identical to the economy described above. This "mandate economy" is capable of achieving the same path for material production and abatement as described above. The difference in measured GDP at market prices, however, could not be more striking.

Consider the static optimization problem of a firm operating under a government mandate to offset its emissions. Its profits are

$$\Pi_t^Q = \max K_t^\alpha (A_{Qt}L_t)^{1-\alpha} - w_t L_t - r_t K_t - p_t B_t \quad \text{s.t.} \quad K^\alpha (A_{Qt}L_t)^{1-\alpha} \leq \sigma_t B_t. \tag{32}$$

This yields factor demands characterized by:

$$w_t = (1-\alpha)(1-\sigma_t^{-1}p_t)K_t^\alpha A_{Qt}^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = \alpha(1-\sigma_t^{-1}p_t)K^{\alpha-1}(A_{Qt}L_t)^{1-\alpha}. \tag{33}$$

The firm purchases B_t units of abatement from a distinct economic sector producing under competitive conditions. The objective function of an abatement-producing firm is:

$$\Pi_t^B = p_t K_t^\alpha (A_{Bt}L_t)^{1-\alpha} - w_t L_t - r_t K_t. \tag{34}$$

Optimality implies:

$$w_t = (1-\alpha)p_t K_t^\alpha (A_{Bt})^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = \alpha p_t K_t^{\alpha-1}(A_{Bt}L_t)^{1-\alpha}. \tag{35}$$

Using the fact that factor shares are equal in both sectors and combining equations (33) and (35), cost of abatement per unit of output is:

$$\sigma_t^{-1}p_t = \frac{1}{1+\sigma_t\left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}. \tag{36}$$

In the case that $\sigma_t\left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha} \to 0$, $\sigma_t^{-1}p_t \to 1$. Conversely, if $\sigma_t\left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha} \to \infty$, then $\sigma_t^{-1}p_t \to 0$.

Consider GDP defined as the sum of value added in each sector:

$$Y_t = (Q_t - p_t B_t) + p_t B_t. \tag{37}$$

When the rate of technical progress in goods production exceeds that in abatement ($a_Q > a_B$), the price of abatement rises secularly; yet it no longer enters GDP directly, but only as an intermediate input. In fact, as $\sigma_t^{-1}p_t \to 1$, $Y_t = Q_t = \sigma_t^{-1}p_t Q_t = p_t B_t$. In the limit, all value added originates in the abatement sector. While measured GDP growth remains positive, it is entirely driven by technical progress in mitigation, a_M.[28] This result contrasts starkly with price-driven GDP growth that emerges when the government purchases abatement in the market. This outcome resembles the ambiguous role played by investment expenditures in the national and income accounts, see Barro (2021).

Fixler et al. (2023) estimate the government's share of output in the environmental goods and services sector (EGSS) at 27% in 2019. This suggests that real-world dynamics will lie somewhere between the two cases studied here. Indeed it is straightforward to construct a model in which a constant fraction of abatement must be paid for by the firm as an intermediate, leading to an aggregate growth rate lying between the growth rate in quantities, a_M, and the price-driven growth rate, g_Y^*, defined in equation (30).

4.5 Population Growth and a Fixed Factor in Production

Central to the analysis thus far is the assumption that abatement is produced at constant returns to scale in capital and labor. In reality, the feasibility of large-scale deployment of negative emissions technologies has been called into question. In particular, those related to biomass production, traditional afforestation and BECCS require land (or ocean) surface, which is in fixed supply. A limit to the absolute quantity of abatement has dramatic consequences in the presence of population growth: If abatement cannot be scaled with population, the amount of emissions per head must decline. If decoupling of production and emissions proceeds too slowly, output and consumption must decline to zero in the long-run.

We formalize these results by introducing land D as a fixed factor of production:

$$Q_t = K_{Qt}^{\alpha} \left(A_{Qt} L_{Qt}\right)^{1-\alpha-\xi} D_{Qt}^{\xi} \quad \text{and} \quad B_t = K_{Bt}^{\alpha} \left(A_{Bt} L_{Bt}\right)^{1-\alpha-\xi} D_{Bt}^{\xi}. \tag{38}$$

The total supply of land is fixed at unity: $D_{Qt} + D_{Bt} = 1$. Apart from this extension, the model is unchanged. The assumption of equal factor shares guarantees that both technologies are scaled without factor bias on the equilibrium growth path.

We continue to focus on the case in which conventional technical progress in material goods production exceeds that of mitigation ($a_Q > a_M$). To highlight the main result of this section, we limit attention to the long run as $\gamma_t \to 1$.[29] The limiting dynamics of the capital stock are determined by

$$\dot{K}_t = s\sigma_t K_t^{\alpha} (A_{Bt} L_t)^{1-\alpha-\xi} - \delta K_t. \tag{39}$$

[28] This result holds irrespective of the numeraire chosen and is thus immune from the "Gerschenkron effect" (Gerschenkron, 1947). It is easy to show that despite the secular rise of the price of mitigation, growth of the Fischer exact index of economic activity in either economy in time t is given by $g_{Yt} = (1-\gamma_t)g_{Qt} + \gamma_t g_{Bt}$ and $\lim_{t\to\infty} g_{Yt} = a_M$. See Duernecker et al. (2021) for a related discussion.

[29] An exposition of the standard Solow-Swan model with land as a production factor can be found in Romer (2012) or Weil (2012).

This is the same accumulation equation as in Romer's (2012) version of the Solow-Swan model, with land in production and productivity in mitigation replacing productivity in conventional production. It follows that the common growth rate of capital, output and consumption in levels along the balanced growth path is given by[30]

$$g_K = g_Q = g_C = \frac{(1-\alpha-\xi)(n + a_B + \frac{1}{1-\alpha-\xi}a_\sigma)}{1-\alpha}. \tag{40}$$

Equation (40) implies that consumption per capita grows at the rate

$$\frac{(1-\alpha-\xi)(a_B + \frac{1}{1-\alpha-\xi}a_\sigma) - \xi n}{1-\alpha}. \tag{41}$$

If $n > \frac{1-\alpha-\xi}{\xi}(a_B + \frac{1}{1-\alpha-\xi}a_\sigma)$, consumption per capita falls at a constant rate along the long-run growth path. Productivity growth in mitigation places an upper bound on sustainable population growth in the presence of a fixed factor.

5 The Ramsey Perspective

The standard Solow-Swan growth model modified for abatement implies positive steady state economic growth, regardless of the numeraire and despite a binding environmental constraint. The model also predicts a sharp decline in GDP and consumption per capita as the economy transits from the Solow-Swan to Baumol regime. This decline will not be optimal if periodic utility is concave. How large are the implied welfare losses? Because it does not track utility directly, the Solow-Swan model cannot answer this question. In the following, we sketch the socially optimal response to the goods versus abatement allocation problem using Ramsey's (1928) model of optimal growth and investment. For a treatment of the standard model without an environmental limit see Barro and Sala-i Martin (2004).

5.1 The Ramsey Problem

The social planner chooses paths of consumption per capita, $\tilde{c}_t = C_t/L_t$, investment I_t and abatement B_t as well as the share of capital and labor allocated to the production of abatement (γ_{Kt} and γ_{Lt}), for $t \geq 0$ to maximize a time-separable utilitarian objective discounted at constant instantaneous rate ρ:

$$\int_0^\infty e^{-\rho t} L_t u(\tilde{c}_t) dt \tag{42}$$

where utility takes the standard isoelastic form $u(c) = \frac{\varepsilon}{\varepsilon-1}c^{\frac{\varepsilon-1}{\varepsilon}}$ with $\varepsilon > 0$.[31] It is straightforward to show that optimal choice compels the social planner to equate factor shares across material production and abatement as in previous sections, so $\gamma_{Kt} = \gamma_{Lt} = \gamma_t$. Substituting investment

[30] See Romer (2012) for a proof.

[31] To guarantee that the objective function is bounded, we impose $\rho > n + \frac{\epsilon-1}{\epsilon}a_M$.

$I_t = Q_t - C_t$ and using the production function for Q_t, capital evolves according to

$$\dot{K}_t = (1-\gamma_t)K_t^{\alpha}\left(A_{Qt}L_t\right)^{1-\alpha} - \tilde{c}_t L_t - \delta K_t. \tag{43}$$

The accumulation equation for atmospheric greenhouse gases is

$$\dot{G}_t = \sigma_t^{-1}(1-\gamma_t)K_t^{\alpha}\left(A_{Qt}L_t\right)^{1-\alpha} - \gamma_t K_t^{\alpha}\left(A_{Bt}L_t\right)^{1-\alpha} \tag{44}$$

subject to the environmental limit (13) as in the Solow-Swan economy. The share γ_t is restricted to the closed unit interval. As before, A_{Qt}, A_{Bt}, L_t and σ_t grow at rates a_Q, a_B, n and a_σ respectively, with initial conditions A_{Q0}, A_{B0}, L_0 and σ_0. The initial conditions for state variables capital stock and cumulative emissions are K_0 and G_0 respectively.

Transformation into stationary variables. We focus on the case $a_M < a_Q$. and write all quantities in intensive form per unit of abatement-efficiency labor. We denote the transformed variables in lower case letters, e.g. $k_t = \frac{K_t}{A_{Bt}\sigma_t^{\frac{1}{1-\alpha}}L_t}$. As we verify in Appendix B, this implies that c_t, q_t and k_t approach finite limits in the long run. To simplify notation, we continue to use $p_t = \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha}$, the exogenous (cost-based) price of abatement in terms of material output.

We can then rewrite the objective as

$$A_{B0}^{\frac{\varepsilon-1}{\varepsilon}} L_0 \int_0^{\infty} e^{-\left[\rho - n - \frac{\varepsilon-1}{\varepsilon}a_M\right]t} \frac{\varepsilon}{\varepsilon-1} c_t^{\frac{\varepsilon-1}{\varepsilon}} dt$$

Equations (43) and (44) become

$$\dot{k}_t = \left(1-\gamma_{Kt}\right)k_t^{\alpha}\frac{p_t}{\sigma_t} - c_t - \left(a_M + n + \delta\right)k_t,$$

and

$$\dot{g}_t = \frac{1}{\sigma_t}(1-\gamma_t)k_t^{\alpha}\frac{p_t}{\sigma_t} - \frac{1}{\sigma_t}\gamma_t k_t^{\alpha}\left(a_M + n\right)g_t.$$

The environmental limit (13) can be written as

$$A_{B0}L_0\sigma_0^{1-\alpha}e^{(a_M+n)t}g_t \leq \overline{G}.$$

The Hamiltonian. The Hamiltonian for the optimization problem described above is

$$
\begin{aligned}
\mathcal{H}_t =& e^{-\left[\rho - n - \frac{\varepsilon-1}{\varepsilon}(a_M)\right]t} \\
& \times \left\{
\begin{array}{l}
\frac{\varepsilon}{\varepsilon-1}\left(c_t\right)^{\frac{\varepsilon-1}{\varepsilon}} \\
\\
+ \ \lambda_t \ \left[(1-\gamma_t)\, k_t^{\alpha} \frac{p_t}{\sigma_t} - c_t - (a_M + n + \delta)\, k_t\right] \\
\\
- \ \frac{\nu_t}{\sigma_t} \ \left[(1-\gamma_t)\, k_t^{\alpha} \frac{p_t}{\sigma_t} - \gamma_t k_t^{\alpha} - \sigma_t\,(a_M + n)\, g_t\right] \\
\\
+ \ \ \mu_t \ \left[\overline{G} - A_{B0} L_0 \sigma_0^{1-\alpha} e^{(a_M+n)t} g_t\right]
\end{array}
\right\},
\end{aligned}
\tag{45}
$$

λ_t, ν_t, and μ_t are costate variables associated with the physical capital stock, the greenhouse gas stock expressed in abatement-efficiency units, and the absolute environmental limit $\bar{G}$.

At the optimum, λ_t is the shadow value in utility terms of a marginal unit of output, ν_t is the shadow value of a marginal unit of mitigation and μ_t is the shadow value of a relaxation of the environmental limit at time t. We omit the constraints $0 \leq \gamma_t \leq 1$ for readability, ruling out corner solutions. In model simulations we present, the constraints do not bind.

Economic interpretation of the optimality conditions In Appendix B, we present the full set of optimality conditions and characterize the optimal growth path. In particular, we focus on welfare-maximizing choices of abatement and greenhouse gas emissions before the environmental limit is reached highlighting differences with the Solow-Swan economy in which no abatement occurs until $\bar{t}$.

The first order necessary condition for γ_t, the share of resources deployed in abatement, can be written as:

$$
\nu_t = \left[\frac{1}{\sigma_t} + \frac{1}{p_t}\right]^{-1} \lambda_t. \tag{46}
$$

Optimal policy implies that the value of a marginal reduction of atmospheric greenhouse gases equals the cost in utility of achieving this reduction through an increase in abatement activities γ_t. The term $\left[\frac{1}{\sigma_t} + \frac{1}{p_t}\right]^{-1}$, which is twice the harmonic mean of σ_t and p_t, captures two effects of a marginal increase in γ_t. First, it reduces emissions through a decrease in material goods production. Higher σ_t reduces the emissions intensity of material goods production which attenuates this effect. Second, raising γ_t increases abatement which reduces atmospheric greenhouse gases. As p_t rises, material goods production becomes more efficient relative to abatement and the amount of forgone material output for an extra unit increases. Both effects imply that the output cost of reducing greenhouse gases increases over time, which causes ν_t to rise relative to λ_t.

The first order necessary condition for g_t links the marginal value of reducing atmospheric greenhouse gases to its benefit, a marginal relaxation of the environmental constraint. When the environmental limit is not binding ($\mu_t = 0$), this condition reads:

$$\frac{\dot{\nu}_t}{\nu_t} = \rho + \frac{1}{\varepsilon}\left(a_B + \frac{a_\sigma}{1-\alpha}\right). \tag{47}$$

Thus, a reduction in g_t brings no immediate benefit. Still, the shadow value of emissions reduction ν_t is positive because the planner anticipates that the limit will bind in the future. As the economy approaches the constraint, ν_t grows at a constant rate, as utility losses associated with the constraint become more imminent.

Taken together, equations (46) and (47) determine the dynamics of γ_t before the economy reaches the environmental limit. As the economy approaches the constraint, ν_t rises exponentially, guiding the planner to increase γ_t.

5.2 Solow Meets Ramsey: Welfare Costs of Sub-optimal Climate Policy

Numerical simulations will provide perspective on the transition paths in both economies and quantify the sub-optimality of the Baumol-Solow-Swan growth path compared with Ramsey-optimal policy. We parameterize the model using standard values from growth and climate literatures. Details can be found in Appendix C.

Figure 4 contrasts the growth paths under Solow-Swan and Ramsey-optimal policies. Most importantly, the social planner initiates abatement immediately at $t = 0$, so γ_0 is positive. Yet measured GDP Y_t is almost identical across the two economies throughout, concealing compositional differences with respect to material production and abatement. While the path of GDP is similar in both economies, the capital-output ratio in the Ramsey economy is significantly lower than in the Solow-Swan economy before hitting the environmental limit. The social planner "invests" in a reduction of G (abatement) at the expense of lower growth in the capital stock. The Solow-Swan economy reaches the environmental limit sooner with $\bar{t}^{Solow} = 2029$ and $\bar{t}^{Ramsey} = 2035$. Both of these dates are earlier than current estimates for the transition to net zero emissions, which follows from our calibration to a tight carbon budget, aimed at limiting global warming to 1.5°C.

In the Solow-Swan economy, consumption exceeds the Ramsey-optimal level before $\bar{t}^{Solow}$, and declines afterwards by 20%. Abatement jumps from zero to the value necessary to offset all additional emissions going forward. In contrast, the Ramsey planner chooses a smooth path for consumption around $\bar{t}^{Ramsey} = 2035$ and a positive amount of abatement before $\bar{t}^{Ramsey}$. The earlier deployment of abatement technology implies that emissions are reduced more gradually, enhancing consumption smoothing and increasing total utility.

After the transition phase, the two economies converge to the same growth path and by 2050 all variables are almost indistinguishable across the two economies, except for the slow-moving capital stock.[32] The growth path is characterized by unbalanced growth, as both p_t and p_t/σ_t

[32] By 2100 capital stocks in the respective economies have converged as well.

grow without bound and γ_t approaches one. However, this process occurs at a very slow rate, as can be seen in the third and last panels of Figure 4. This slow convergence to the long-run limit is due to the similar rates of technical progress in goods production ($a_Q = 2\%$) and mitigation ($a_B + \frac{a_\sigma}{1-\alpha} = 0.53\% + 1.43\% = 1.96\%$), which stems largely from Nordhaus (2017)'s parameterization.[33] If one takes a less optimistic stance on decarbonization and abatement, results are more dramatic. For example, in Appendix C.1, we present an alternative, pessimistic scenario of zero technical progress in decarbonization. Under these assumptions, the fraction of resources devoted to abatement rises from 20% of GDP to more than 35% over the next century.

Despite the short-run differences, the Solow-Swan and Ramsey-optimal paths are qualitatively similar. How large is the welfare loss of following sub-optimal policy? Define Λ as the percentage increase in annual consumption of inhabitants of the Solow-Swan economy until 2050 that compensates them for their inability to implement optimal policy, i.e.

$$\int_{2022}^{2050} e^{-\rho t} L_t \left(\left(1 + \frac{\Lambda}{100}\right) \frac{C_t^{Solow}}{L_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-28\rho} V_{2050}(K_{2050}^{Solow}, G_{2050}^{Solow}) = \tag{48}$$
$$\int_{2022}^{2050} e^{-\rho t} L_t \left(\frac{C_t^{Ramsey}}{L_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-28\rho} V_{2050}(K_{2050}^{Ramsey}, G_{2050}^{Ramsey}),$$

where $V_{2050}(K_{2050}^i, G_{2050}^i)$ is the continuation value of starting in 2050 from capital K_{2050}^i and greenhouse gas stock G_{2050}^i and following Ramsey-optimal policies thereafter. The interval is chosen to focus on the welfare cost of the sub-optimal Solow-Swan transition, and this is essentially complete by mid-century.[34] For our benchmark parameterization, we compute a remarkably modest loss of $\Lambda = 0.128$ percent of consumption over the next 28 years. Despite the abrupt and significant one-time drop in consumption implied upon reaching the environmental limit at time $\bar{t}^{Solow}$, the integral over the two paths is very similar. This is because households consume significantly more relative to the Ramsey path before the environmental limit is reached. This result is similar to Lucas' (1987) finding that business cycle fluctuations in consumption imply only small welfare losses in representative agent economies.

Table 2 displays the sensitivity of the welfare cost with respect to key parameters. A lower value of the elasticity of intertemporal substitution increases the cost of eschewing Ramsey optimal policy, but it remains relatively small for plausible values. For $\varepsilon = 0.5$ and all else held equal, the welfare loss roughly doubles to $\Lambda = 0.299$. For two different reasons, the social discount rate cannot account for low welfare costs of postponed adjustment. First, the loss of consumption occurs within the first decade, a point at which utility flows are hardly discounted. In later periods, when the discount rate matters more, consumption is nearly identical across the two economies.

[33] Note that Nordhaus (2017) is even more optimistic about the costs of full abatement, which start out a lower level (7.5% of output in 2015) and then decline towards zero.

[34] Computing welfare costs in terms of permanent consumption equivalents scales down the welfare losses but does not change the relative welfare losses across the different parameterizations. Our welfare measure ensures that different levels of the endogenous state variables at the end of the considered window do not bias results.

Second, the discount rate is already low ($\rho = 0.5\%$ per annum). With a population growth rate of $n = 0.37\%$, utility flows are effectively discounted at a rate of 0.13% annually. Reducing the time preference rate further to $\rho = 0.4\%$ per annum, raises the welfare cost to only $\Lambda = 0.134$.[35] More pessimistic assumptions regarding the rate of technical progress can increase welfare costs of the Solow-Swan-Baumol economy, but they retain the same order of magnitude. The pessimistic scenario presented in Appendix C.1 assumes $a_\sigma = 0$, implying significantly more resource diversion and active CO_2 removal to accommodate material growth. In this case, the required consumption variation is about 0.151%. Our final experiment is sets $\varepsilon = 0.5$ and $a_\sigma = 0$, in which case the loss rises to 0.618 percent of consumption.

These remarkably modest costs of business as usual should not be seen as trivializing a policy path of inaction. Rather, they highlight the limitations of this class of models for assessing welfare costs of climate change and climate policy in general. A number of additional features are available that could generate significant welfare losses. The first is adjustment costs, external or internal, associated with the movement of factors across sectors. In contrast to our model, reallocation of production factors in reality is not frictionless and a rapid ramping up of abatement is likely to be costly or economically infeasible. Similarly, in a model with income heterogeneity, welfare losses for the poorest in the population are likely to be considerably larger. We have intentionally abstracted from these mechanisms to highlight the lack of plausible welfare effects in a minimalist setting generally used by growth researchers.

6 Conclusion

The utility of the Solow-Swan model derives from highlighting the implications of minimal assumptions on technology and behavior for long-run economic growth. Our adaptation of the Solow-Swan model demonstrates the importance of resource-intensive mitigation as a central feature of economic growth under a binding environmental limit and provides a benchmark for understanding economic growth under such circumstances.

Price-driven growth is essential for enabling government supplied mitigation, as it directly reflects market consequences of government usage of production factors for public good provision. A typical form of Baumol's cost disease emerges, as the slowly growing abatement sector employs a growing share of production factors. Directed technical change could thus play a central role in altering the long-run growth path of these economies and is a subject for future research. The power of Baumol's message extends to the choice of mandates versus markets. Paradoxically, the market solution requires active engagement of government - as opposed to a "mandate economy" that simply forces firms to adopt the desired level of mitigation and depresses measured economic growth in the long run.

[35] If the discount rate is reduced further, the objective function becomes unbounded and the optimization problem is no longer well-defined.

Appendix

A Alternative Cases in the Baumol Regime

In this section we characterize the dynamics in the Baumol regime in two cases not discussed in the main text: i) equal growth rates in both sectors and ii) faster productivity growth in the mitigation

A.1 Costly Green Growth: $a_Q = a_B + \frac{a_\sigma}{1-\alpha} = a$

Consider now the case in which the two technologies grow at the same rate. In this case the ratio of the two productivities and the share of resources devoted to each sector are constant as soon as the economy reaches the environmental constraint:

$$\gamma = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}. \tag{A1}$$

The economy exhibits balanced growth in this case, with steady state level of capital per efficiency unit of labor

$$k^* = \left(\frac{\gamma s}{a + \delta + n}\right)^{\frac{1}{1-\alpha}}. \tag{A2}$$

Furthermore, all quantities grow asymptotically at rate $a+n$. That is, the economy behaves like a standard Solow-Swan economy, except for the presence of the parameter γ in steady state value of capital. This parameter captures the fact that in the steady state, a positive fraction of resources must be devoted to the abatement sector. The relative price of abatement is constant.

Reaching the environmental constraint corresponds to a one-time downward shift in the production function, as resources are redeployed immediately for abatement. Asymptotically the economy reaches its previous growth rate. The new growth path permanently lies below the growth path before the environmental constraint was reached.

A.2 Outgrowing Environmental Constraints: $a_Q < a_M$

Now consider the case in which the mitigation technology grows at a faster rate than the production technology. The share of resources used in the abatement sector approaches zero in the long run.

$$\lim_{t\to\infty} \gamma_t = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}} = 0. \tag{A3}$$

Capital per efficiency unit of labor converges to the same limit as a standard Solow-Swan economy

$$k^* = \left(\frac{s}{a_Q + \delta + n}\right)^{\frac{1}{1-\alpha}} \tag{A4}$$

As above, the environmental constraint shifts back the production function, as resources are now required for abatement. However, as technical progress reduces the resources necessary to mitigate the environmental damage and the production function gradually shifts back. As a result, output falls temporarily as γ_t% of resources are used for abatement. As the mitigation technology becomes more productive, however, the growth rate of produced output recovers and output attains once again its initial growth path.

B The Ramsey-Optimal Path

Here we present the optimality conditions, analytically characterize the limit behavior of the economy and show that the planner optimally chooses to reach the environmental limit at a finite time T.

Optimality Conditions First order necessary conditions for the problem stated in (45) for $t \in [0, \infty)$ are

$$\frac{\partial \mathcal{H}_t}{\partial c_t} = 0 \qquad \Leftrightarrow \qquad c_t^{-\frac{1}{\varepsilon}} = \lambda_t, \tag{A5}$$

$$\frac{\partial \mathcal{H}_t}{\partial \gamma_t} = 0 \qquad \Leftrightarrow \qquad \lambda_t p_t = \nu_t \left[\frac{p_t}{\sigma_t} + 1\right], \tag{A6}$$

$$\begin{aligned} \frac{\partial \mathcal{H}_t}{\partial k_t} = -\frac{d\left(e^{-\left[\rho - n - \frac{\varepsilon - 1}{\varepsilon} a_M\right]t} \lambda_t\right)}{dt} \qquad \Leftrightarrow \\ \lambda_t \left[\alpha\left(1 - \gamma_t\right) k_t^{\alpha - 1} \frac{p_t}{\sigma_t} - \left(a_M + n + \delta\right)\right] - \frac{\nu_t}{\sigma_t}\left[\alpha\left(1 - \gamma_t\right)\frac{p_t}{\sigma_t} - \alpha\gamma_t\right] k_t^{\alpha - 1} \\ = \left[\rho - n - \frac{\varepsilon - 1}{\varepsilon} a_M\right] \lambda_t - \dot{\lambda}_t, \end{aligned} \tag{A7}$$

and

$$\frac{\partial \mathcal{H}_t}{\partial g_t} = \frac{d\left(e^{-\left[\rho - n - \frac{\varepsilon - 1}{\varepsilon} a_M\right]t} \nu_t\right)}{dt} \qquad \Leftrightarrow \qquad \dot{\nu}_t = \left[\rho + \frac{1}{\varepsilon} a_M\right] \nu_t - \mu_t A_{Bt} L_t \sigma_t^{1-\alpha}. \tag{A8}$$

Finally, $\mu_t \geq 0$ and the complementary slackness condition

$$\mu_t \left[\overline{G} - A_{B0} L_0 \sigma_0^{1-\alpha} e^{(a_M + n)t} g_t\right] = 0 \tag{A9}$$

apply.

Long-run limit behavior Note that $\frac{\dot{p}_t}{p_t} = (1 - \alpha)\left(a_Q - a_B\right)$. The assumption $a_M < a_Q$ implies that $\lim_{t \to \infty} \frac{p_t}{\sigma_t} = \infty$ which will be useful below.

The intensive-form greenhouse gas accumulation equation is:

$$(1-\gamma_t)\, k_t^{\alpha} \frac{p_t}{\sigma_t} = \gamma_t k_t^{\alpha} \sigma_t g_t + \dot{g}_t$$

In the long-run limit, g_t must shrink at exponential rate $a_B + \frac{a_\sigma}{1-\alpha} + n + \delta$ to satisfy the environmental constraint while σ_t grows at rate a_σ, implying $\lim_{t\to\infty} \dot{g}_t = 0$ and $\lim_{t\to\infty} g_t \sigma_t = 0$. Thus [36]

$$\lim_{t\to\infty} (1-\gamma_t) \frac{p_t}{\sigma_t} = \lim_{t\to\infty} \gamma_t = 1.$$

The last equality follows from the fact that $\lim_{t\to\infty} \frac{p_t}{\sigma_t} = \infty$.

Applying the limit to the Euler equation for capital yields

$$\lim_{t\to\infty} -\frac{\dot{\lambda}_t}{\lambda_t} = \lim_{t\to\infty} \alpha k_t^{\alpha-1} - \rho - \delta - \frac{1}{\varepsilon} a_M.$$

Similarly, the capital accumulation equation becomes

$$\lim_{t\to\infty} \dot{k}_t = \lim_{t\to\infty} k_t^{\alpha} - c_t - (a_M + n + \delta)\, k_t.$$

In the limit, both of these equations are approaching the respective equations of a standard Ramsey economy with a rate of technical progress of a_M. The limiting behavior of all variables is therefore identical to the Ramsey economy, which implies in particular that the system approaches a unique stable point. As in the Solow-Swan economy, all quantities grow at rate $a_M + n$ in the long run. The long-run level of the capital stock in intensive form is $k^* = \left(\frac{\alpha}{\delta+\rho+\frac{1}{\varepsilon}a_M}\right)^{\frac{1}{1-\alpha}}$ with a long-run savings rate of $s^* = (\delta + n + a_M)(k^*)^{1-\alpha}$.

The economy reaches $\bar{G}$ at a finite time T. To see that the economy must reach the environmental limit in finite time, assume the opposite. This in turn implies $\mu_t = 0\ \forall t$. From equation (A8), it follows that

$$\frac{\dot{\nu}_t}{\nu_t} = \rho + \frac{1}{\varepsilon} a_M.$$

That is, ν_t grows at a constant positive rate. Rewriting equation (A6), we get

$$\lambda_t = \nu_t \left[\frac{1}{\sigma_t} + \frac{1}{p_t}\right]$$

and by logarithmic differentiation

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{\nu}_t}{\nu_t} - \left(\frac{\sigma_t}{p_t+\sigma_t}\right)\frac{\dot{p}_t}{p_t} - \left(\frac{p_t}{p_t+\sigma_t}\right)\frac{\dot{\sigma}_t}{\sigma_t}$$

[36] Here we have also used that $\lim_{t\to\infty} k_t > 0$ which is guaranteed as the production function satisfies standard Inada conditions.

As noted above $\lim_{t\to\infty}\frac{p_t}{\sigma_t}=\infty$, which implies $\lim_{t\to\infty}\frac{\sigma_t}{p_t+\sigma_t}=0$ and $\lim_{t\to\infty}\frac{p_t}{p_t+\sigma_t}=1$. It follows that

$$\lim_{t\to\infty}\frac{\dot{\lambda}_t}{\lambda_t}=\rho+\frac{1}{\varepsilon}a_M-a_\sigma$$

As we have shown above, however, $\lim_{t\to\infty}\frac{\dot{\lambda}_t}{\lambda_t}=0$ which is a contradiction unless $\rho+\frac{1}{\varepsilon}a_M-a_\sigma=0$. For generic parameter values, this proves the fact that $\mu_T>0$ for some finite T.

C Numerical Solution and Calibration Details

Ramsey Economy For the numerical simulations, we discretize time to one-year intervals. We solve the model using the extended path method, imposing initial and terminal conditions on K_t and G_t. For a given $\bar{t}^{Ramsey}$, we guess the paths of K_t and γ_t and use a numerical solver on the system of optimality conditions. We start with a low guess for $\bar{t}^{Ramsey}$ and iteratively increase it until the complementary slackness conditions are satisfied.

Our simulation starts in 2022 and continues until 2422, a simulation interval that is large enough to ensure that terminal conditions do not affect results in the time interval of interest. We set the terminal value for capital in the Ramsey economy equal to the last value in the simulation of the Solow-Swan economy described below. The terminal value of G_{2422} is $\bar{G}$.

Parameter values along with the calibration targets are given in Table A1. We rely on literature conventions to choose values for the parameters present in standard growth models, including δ, α and a_Q. We choose a elasticity of intertemporal substitution $\epsilon=1$, to focus on the consequences of sub-optimal climate change mitigation and minimize other sources of differences between the Solow-Swan and Ramsey economies.[37] We set the pure rate of time preference ρ to 0.5%, a value that is compromise between the choices of Nordhaus (2017) and Stern (2007). In combination with the rate of technical progress, this yields a rate of return $\rho+\frac{1}{\varepsilon}a_Q=2.5\%$ annually. Considering the falling trend in global interest rates, this value appears reasonable. The population growth rate is set to match the UN projections of an increase in population from 8bn in 2022 to 10.7 bn in 2100, which implies $n=0.00373$.

We choose the initial level of capital to lie on the unbalanced growth path of the economy, to prevent a large initial adjustment in capital.[38] The CO_2-efficiency parameter σ_{2022} is set to match a target of global emissions of 40.5 $GtCO_2$ in 2022. We set $\bar{G}$ to 380 $GtCO_2$ using up-to-date projection for a 50% chance of limiting global warming to 1.5 degrees Celsius.[39] Productivity of the abatement technology A_{2022} is set to generate initial output costs of full sequestration of 20% following van der Ploeg and Rezai (2019). We choose the rate of decarbonization a_σ of 1%,

[37] It is well known that the savings rate in a Ramsey economy with $\epsilon=1$ is constant and independent of the initial level of capital. Without the environmental limit, a Solow-Swan economy with the appropriate savings rate yields an identical growth path to the Ramsey economy.
[38] This level can be found by eliminating γ_t from equation (A7) and solving for k_t.
[39] For these two targets see https://www.ipcc.ch/sr15/chapter/chapter-2/

which is slightly below Nordhaus (2017)'s choice of an initial rate of 1.5%. However, in Nordhaus (2017), this rate is declining over time, while it is constant in our setting. The final parameter to set is a_B, the rate of progress in abatement, which is difficult to map to the stylized formulation of abatement costs in Nordhaus (2017). As this growth rate determines the timing of abatement decisions, we set it to target initial abatement of 3% of emissions, also following Nordhaus (2017). Our calibration implies a growth rate of a_B of 0.053% annually.

Solow-Swan economy To highlight the sub-optimal timing of abatement decisions as the source of welfare loss, we minimize the differences that arise from different capital accumulation decisions in the two economies. Because we set the elasticity of intertemporal substitution to 1, the Ramsey planner would choose a constant savings rate in the absence of the environmental limit. In the Solow-Swan economy, we set the savings rate s_1 before time $\bar{t}^{Solow}$ equal to the Ramsey optimal savings rate in an economy without the environmental constraint. After time $\bar{t}^{Solow}$, we set the savings rate s_2 to the Ramsey optimal savings rate in the long-run limit derived in Appendix B. The two savings rates are virtually identical at $s_1 = 28.465\%$ and $s_2 = 28.459\%$. All other parameters and initial conditions are identical to the Ramsey economy.

To simulate the Solow-Swan economy, we iterate forward using the initial savings rate s_1 and $\gamma_t = 0$ until $G_t > \bar{G}$. We then discard the last simulated period and continue simulations with the long-run savings rate and choosing γ_t such that $G_t = \bar{G}$.

C.1 Alternative Parameterization

We consider a second parameterization, which is identical to the one given in Table A1, with the only difference that $a_\sigma = 0$. The results can be seen in Figure 5. In contrast to Figure 4, we show the transition paths until 2100 instead of 2050 here in order to illustrate the growth in γ_t.

Tables

TABLE 1
Growth Rates in the Baumol Regime ($a_M < a_Q$).

Growth rate	Value
g_B^* (abatement)	$a_B + \frac{\alpha}{1-\alpha} a_\sigma + n$
g_Q^* (produced goods)	$a_M + n$
g_Y^* (GDP)	$g_p + g_B^*$
$g_{\bar{Y}}^*$ (GDP, constant weights)	g_Q^*
g_p (price of abatement)	$(1-\alpha)(a_Q - a_B)$

Starred values indicate long-run limits.

TABLE 2
Welfare Costs of Following Sub-optimal Solow Policies

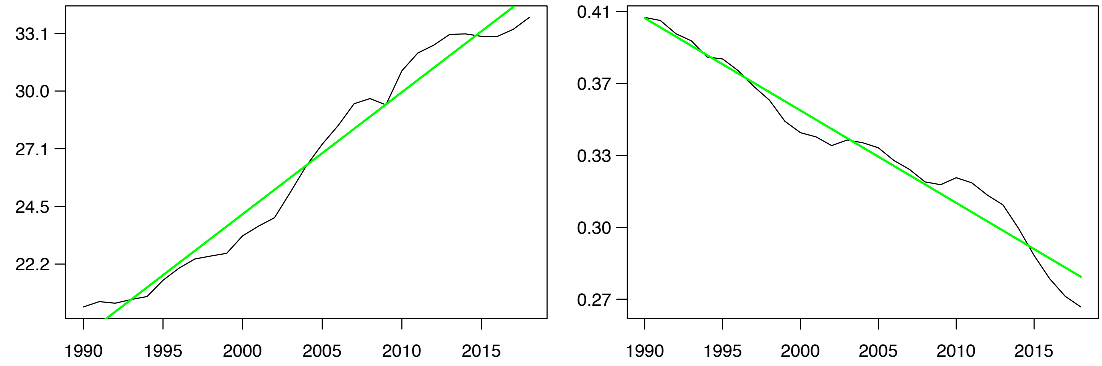
Parameterization	Baseline	I	II	III	IV
	-	$\epsilon = 0.5$	$\rho = 0.4\%$	$a_\sigma = 0$	$\epsilon = 0.5$ & $a_\sigma = 0$
Λ	0.128	0.299	0.151	0.134	0.618

In each of columns I-IV, the indicated parameter values are changed relative to the baseline parameterization. The parameter values in the baseline are $\epsilon = 1$, $\rho = 0.5\%$ and $a_\sigma = 1\%$ respectively.

TABLE A1
Parameterization

Parameter	Symbol	Value	Target
Time preference rate	ρ	0.005	see text
Intertemp. elasticity of substitution	ϵ	1	see text
Output elasticity wrt. capital	α	0.3	standard value
Capital depreciation rate	δ	7.5%	standard value
Techn. progress in material production	a_Q	2%	standard value
Population growth rate	n	0.37%	UN pop. growth (proj.) 2022-2100
Techn. progress in abatement	a_B	0.05%	Initial abatement 3% of emissions
Rate of decarbonization	a_σ	1.0%	Nordhaus (2013)
Efficiency of material production	$A^Q_{Q,2022}$	1	Normalization
Efficiency of abatement	$A_{B,2022}$	15.7	20% of GDP cost full abatement
Emissions intensity of output	σ_{2022}	0.51	2022 emissions
Environmental constraint	$\bar{G}$	380	2022 remaining carbon budget
Savings rate $t < \bar{t}^{Solow}$ (Solow)	s_1	0.2850	optimal savings rate w/o env. lim.
Savings rate $t > \bar{t}^{Solow}$ (Solow)	s_2	0.2850	long-run optimal savings rate

Figures



(a)
Annual CO_2 equivalent emissions (Gt).

(b)
Kg of CO_2 equivalent per USD of GDP.

FIGURE 1
Annual Global Greenhouse Gas Emissions, 1990-2020.
GDP is measured in purchasing power parity of 2017 US-Dollars. Both figures have logarithmic scales. Source: World Bank

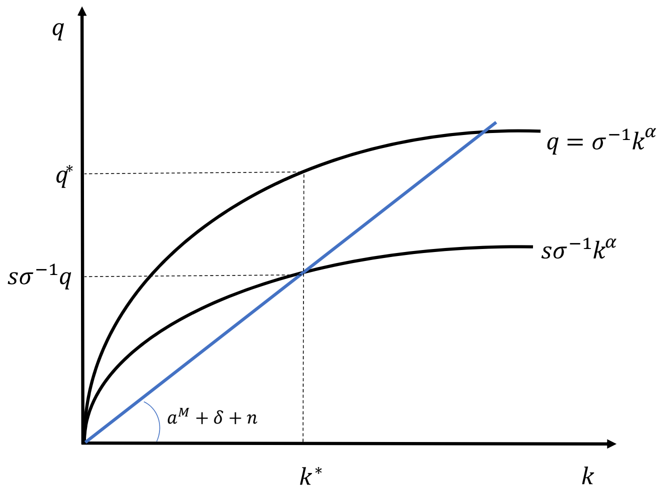


FIGURE 2
The Long-Run Limit in the Solow-Swan-Baumol Model

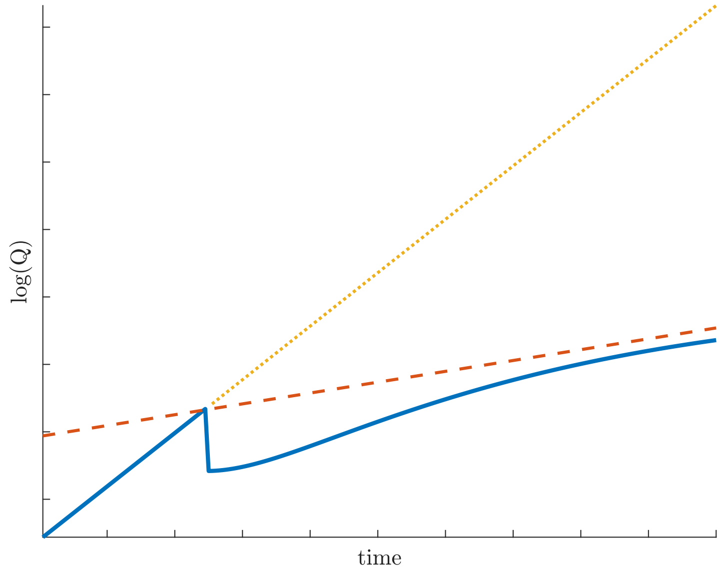


FIGURE 3
Illustrative Growth Path for Q_t.

The solid path depicts the log of output of an economy that transits from the Solow-Swan to Baumol regime. The steeper dotted yellow line is the balanced growth path in the absence of an environmental limit. The flatter red dashed line is the long-run growth path under the environmental limit.

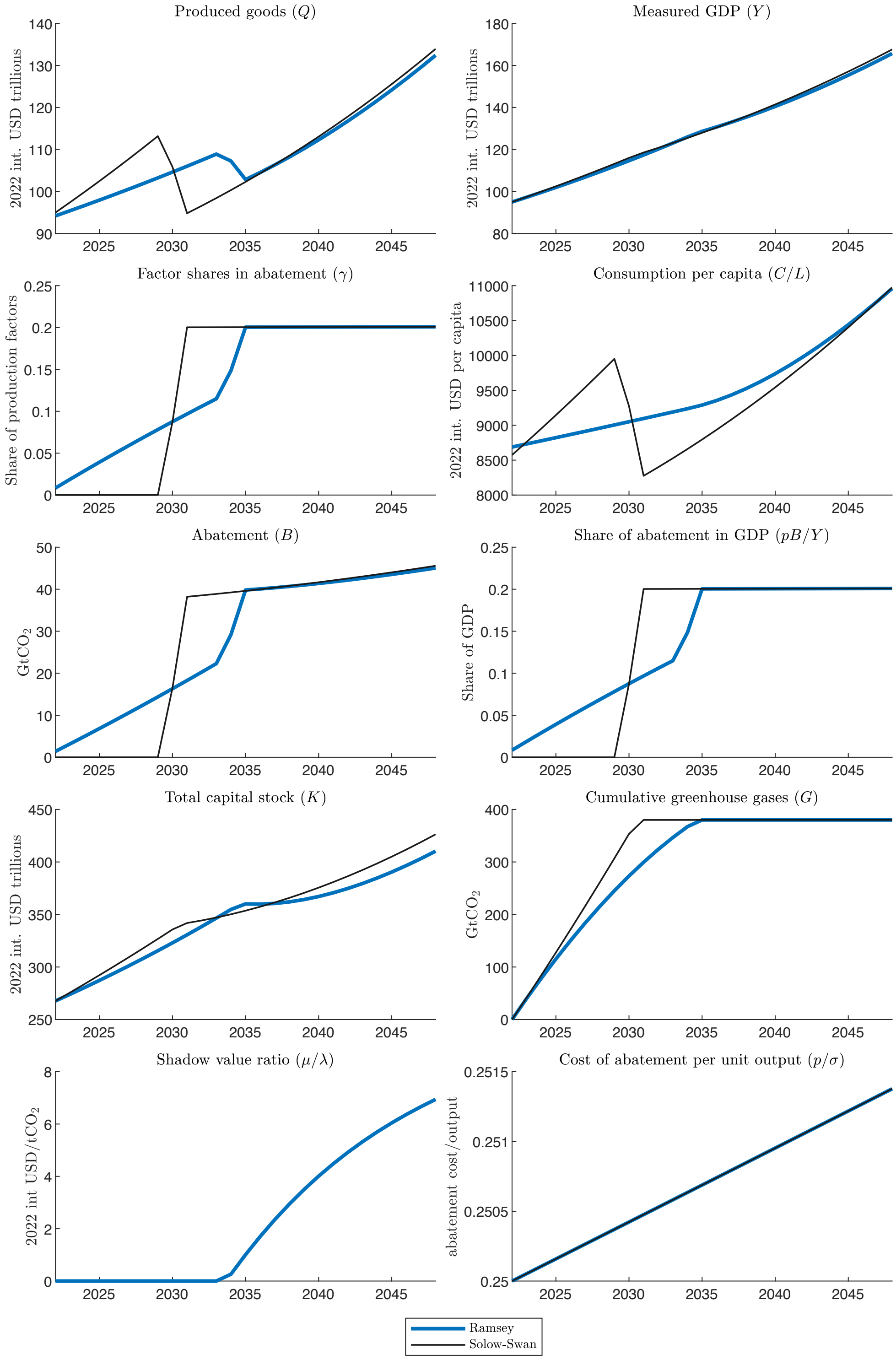


FIGURE 4
Transition Paths in the Solow-Swan and Ramsey Economy.

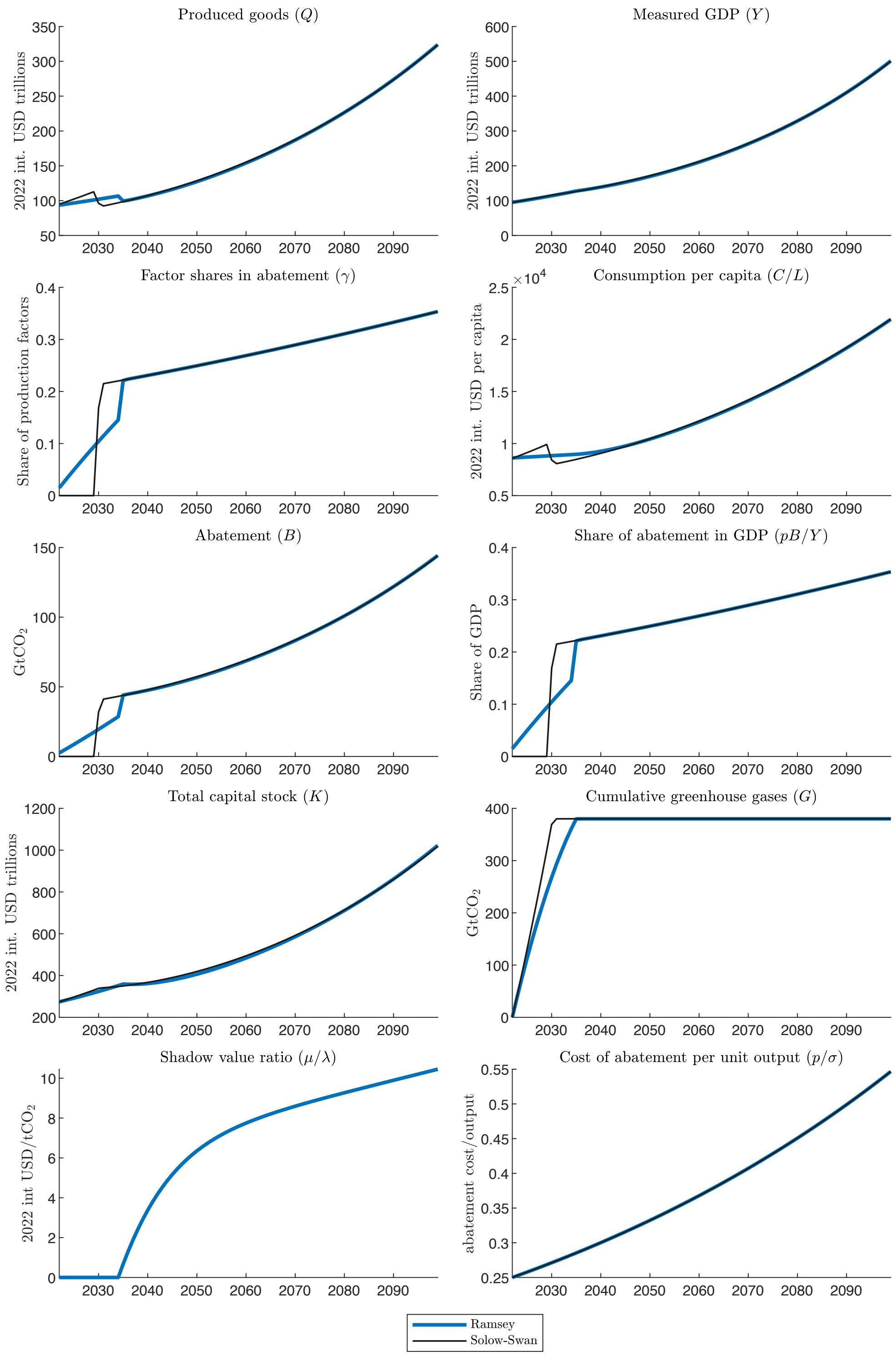


FIGURE 5
Transition Paths in the Solow-Swan and Ramsey Economy with $a_\sigma = 0$.

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