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ABSTRACT

Nonlinear Propagation of Sectoral Productivity Shocks with Variable Elasticities of Substitution

This paper examines the nonlinear propagation of sectoral productivity shocks in a general equilibrium framework with intersectoral linkages characterized by allowing elasticities of substitution in sectoral outputs and sectoral productivities to vary across sector pairs. Evidence based on a sample of 38 countries and 35 sectors shows stronger roles of certain sectors in the aggregate propagation of sectoral productivity shocks with variable elasticities than with constant elasticities. The results of sectoral productivity shocks on cross-country income convergence between 2005 and 2011 are robust across the two types of elasticities in terms of the direction of change, but not the magnitude.

JEL Classification: D24, F15, F43, N10, O47, D57, Q54

Keywords: nonlinear propagation of productivity shock, Morishima elasticity of substitution, intersectoral linkages

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1. Introduction

Sectoral productivity shocks account for more than half of the variation in business cycle fluctuations in aggregate output (Atalay 2017). The welfare effects from fluctuations in aggregate output appear to be larger when shock propagation is characterized by nonlinearities in sectoral production rather than nonlinearities in risk aversion in utility (Baqaee and Farhi 2019; Lucas 1987). In a multisectoral general equilibrium framework, the Domar weights (or sectoral output shares of national income) and structural elasticities of substitution characterize the nonlinear propagation of sectoral productivity shocks to the aggregate level (Baqaee and Farhi 2019).

Structural elasticities of substitution measure the degree of complementarity of sectoral outputs, which is central to the nonlinear propagation of productivity shocks. For instance, with unitary elasticity parameters in a Cobb-Douglas production process, the Domar weights are sufficient to provide a first-order approximation of the aggregate effect of a sectoral productivity shock through a log-linear channel (Hulten 1978). In a constant elasticity of substitution (CES) production framework with nonunitary elasticities, sectoral productivity shocks explain at least four-fifths of the variation in aggregate output volatility, which is almost four times larger than with unitary elasticities (Atalay 2017).

In this study, we extend this line of research by modeling nonunitary elasticities of substitution but allowing them to vary across sector pairs. Our decision to model production networks based on variable elasticities is theoretically relevant, as output in each sector is likely to be produced with a different set of intermediate inputs (Lancaster 1966). This allows substitution elasticities in sectoral outputs to vary across sector pairs. For example, the elasticity of substitution between intermediate inputs from manufacturing and services can differ from the elasticity of substitution between intermediate inputs from construction and services. To model the variation in the substitution parameter across sector pairs, we use a general equilibrium framework with multiple sectors (more than two) and apply the Morishima gross elasticity of substitution (MGES), which is an extension of the Morishima elasticity of substitution (MES) for optimal output adjustments (Davis and Shumway 1996; Blackorby, Primont, and Russell 2007).

To assess the role of variable elasticities in the nonlinear aggregate propagation of sectoral productivity shocks, we utilize the characterization of the second-order aggregate impact of microeconomic shocks in terms of reduced-form nonparametric elasticities of production and intersectoral linkages, as formulated by Baqaee and Farhi (2019). Baqaee and Farhi use a pseudo elasticity of substitution, which is a generalization of the MGES. Both the

MGES and the pseudo elasticity of substitution measure the elasticity of the ratio of marginal rates of substitution. However, the MGES measures it with respect to two arguments, whereas the pseudo elasticity of substitution measures it with respect to one argument. In our framework, the nonlinear propagation of a sectoral productivity shock is conditioned by the MGES and a set of elasticity parameters based on sectoral productivities that also vary across sector pairs.

In our framework, the comparative statics properties of the MGES produce results that are similar to the comparative statics results arising from the pseudo elasticities in Baqaee and Farhi (2019). When sectoral outputs are gross substitutes (complements)—that is, when the MGES and the pseudo elasticities are greater (less) than unity—it amplifies (dampens) the aggregate effect of a positive sectoral productivity shock. However, the net effect in our framework depends on the sizes and signs of the sectoral productivity elasticities. This additional restriction stems from a more flexible framework that allows the substitution elasticities to vary across sector pairs.

We present evidence based on the World Input-Output Database (WIOD) (2013 release) and sectoral total factor productivity (TFP) data compiled by Fadinger, Ghiglino, and Teteryatnikova (2022). Fadinger, Ghiglino, and Teteryatnikova estimate purchasing power parity (PPP)–adjusted TFP for 35 sectors, which are comparable across 38 countries for 2005 using the WIOD. Following Fadinger, Ghiglino, and Teteryatnikova's methodology, we construct sectoral TFPs for the same set of countries and sectors in 2011 using WIOD and Socio-Economic Accounts (SEA) data. We calculate the MGES and the TFP elasticities across 34 sector pairs for the change in TFP in each sector between 2005 and 2011. These elasticity parameters vary across sector pairs but not across countries, and the aggregate effect of a TFP shock in each sector becomes a log-linear combination of the nonlinear effects in the other 34 sectors.

We compute the variable elasticities using input-output tables. These variable elasticity measures vary across sector pairs but not across countries. Evidence based on 35 sectors shows that sectoral outputs predominantly substitute across sector pairs. Using these general equilibrium elasticities and sectoral Domar weights, we compute the second-order effect of the productivity shock between 2005 and 2011 in 35 sectors, for each of the 38 countries. We consider two counterfactual cases with constant elasticities equal to 0.1 and 0.6, respectively. In most of the sectors (27 of 35 sectors), productivity shocks lead to divergence in aggregate output across countries in all models. In certain sectors, like chemicals and business services, the effect on cross-country income divergence is larger with variable elasticities than with constant elasticities. At the aggregate level, the differences in the results between variable and constant elasticities of substitution are primarily driven by the sectoral Domar weights, as our estimates of variable elasticities only vary across sector pairs but not across countries. At the sector level, the propagation channel considers the nonconstant elasticities across sector pairs. For instance, the roles of construction and inland transport are more prominent in the aggregate propagation of productivity shocks in financial and business services.

Our study contributes to a growing literature on the role of intersectoral linkages in the propagation of sectoral productivity shocks to the aggregate level (Acemoglu et al. 2012; Caliendo et al. 2018; Baqaee and Farhi 2019; Carvalho et al. 2021). Co-movement of sectoral outputs plays a crucial role in shock propagation mechanisms. Baqaee and Farhi (2019) show that a nonlinear propagation of sectoral productivity shocks closely captures the macroeconomic fluctuations in aggregate output. We extend this line of research and formulate the nonlinear propagation channel with variable elasticities of substitution in sectoral output across sector pairs. Our flexible model of intersectoral linkages allows for the possibilities of nonunitary as well as nonconstant elasticities of substitution in output across sector pairs, providing deeper insights into the varying roles of different sectors in the aggregate propagation of sectoral productivity shocks.

The remainder of the paper is organized as follows. Section 2 provides a simple general equilibrium framework and formulates the nonlinear aggregate effect of sectoral productivity shocks with variable general equilibrium elasticities of substitution. Section 3 discusses our data and empirical findings. Section 4 concludes.

2. Theory

In a multisector setup, sectoral output is typically modeled using a two-stage CES production function (Atalay 2017; Baqaee and Farhi 2019; Carvalho et al. 2021). The factor inputs are used in the first stage, and intermediate inputs (output from other sectors) are used in the second stage. A constant nonunitary elasticity of substitution parameterizes the input-output linkages through (i) the degree of substitution between the intermediate use of sectoral outputs and (ii) the degree of substitution between value added and the intermediate use of sectoral outputs. To accommodate variable substitution elasticities across sector pairs, we use a nested CES technology to model a multi-stage production process, incorporating inter-stage and intra-stage substitution between inputs (Nakano and Nishimura 2018).[1]

2.1 Nested CES Technology with a Multi-Stage Production Process

Consider a multisector (n) general equilibrium framework with labor (l) as the only factor of production. The aggregate demand is achieved through maximization of the constant returns aggregator of final demand for N sectors ($C_1, C_2, .., C_N$):

$$Y = max\, \aleph(C_1, C_2, .., C_N),$$
$$\text{subject to } \sum_i^N P_i C_i = w\bar{l} + \sum_i^N \pi_i, \quad (1)$$

where C_i is the consumption of sector i, P_i is its price, w is wages, and π_i is the profit for the producers of consumption good i. Labor is fixed in supply and is given by $\bar{l}$. The budget constraint in equation 1 shows nominal gross domestic product (GDP) from the expenditure side on the left-hand side and the income side including wages and profit on the right-hand side.

Production of output in sector i takes place in competitive firms following an implicit form of technology:

$$y_i = A_i F_i(l_i, x_{i1}, x_{i2}.., x_{iN}), \quad (2)$$

where A_i is a Hicks-neutral technology, l_i is labor used for production in sector i, and x_{ij} are intermediate inputs from sector j used for production in sector i. Output in sector i as a share of GDP is y_i/Y, which is also known as the Domar weight.

We adopt a multi-stage model, in which $n+1$ inputs (n intermediate inputs plus labor) are partitioned into K nests. We rewrite the production function in equation 2 for sector i so that $n+1$ inputs are transformed into K composite inputs across different stages of production:

[1] The production of goods and services often involves multiple stages, each of which uses a set of inputs that includes outputs from previous stages of production. For example, the production of semiconductors goes through several steps. These steps comprise silicon wafer cleaning, film deposition, resist coating, exposure, development of the pattern on the layer, etching, activation, and assembly. Each step is a different process consisting of composite intermediate inputs that are produced in one of the previous stages of production and raw intermediate inputs that are used for the first time in the production process.

$$y = AF\left(\Psi^1\left(x^{[1]}\right), \Psi^2\left(x^{[2]}\right) \dots, \Psi^K\left(x^{[K]}\right)\right). \quad (3)$$

Each nest k follows a CES production technology denoted as $\Psi^k\left(x^{[k]}\right)$, and uses a sub-vector of inputs $x^{[k]}$ through which a compound intermediate good X_k is produced, such that $x = \left(x^{[1]}, x^{[2]} \dots, x^{[K]}\right)$ and $p = \left(p^{[1]}, p^{[2]} \dots, p^{[K]}\right)$, where the input vector is x and the input price vector is p. We assume that each input market is competitive.

Equation 3 makes the partitioning of the whole production process into K subprocesses (or nests) explicit. The composite input produced in the k^{th} nest, X_k $(k = 1, 2, \dots, K)$, consists of a combination of inputs $x_1, x_2 \dots, x_{n+1}$ following two conditions: (i) $X_k \cap X_r = \phi$ for all $k \neq r$ (no input can be used in multiple nests), and (ii) $X_1 \cup X_2 \dots \cup X_K = \{x_1, x_2 \dots, x_n\}$ (all inputs are exhaustively used). The unit cost function for the final output is $c(p)$, and the unit cost function for nest k is $c_k(p_k)$. This assumption appears to be restrictive as labor cannot be used in each stage of the production process. However, the use of labor in the final stage or in one of the nests has limited bearing as we focus on how allocative efficiency between sectors affects the propagation of the change in sectoral TFP.

2.2 Morishima Gross Elasticity of Substitution

To estimate intra-stage or inter-stage substitution between two intermediate inputs in our multi-stage production process, we use a substitutability measure introduced by Morishima (1967), which Blackorby and Russell (1989) later termed the MES. Since we are interested in understanding the change in the allocative efficiency of sectoral outputs with respect to the change in sectoral productivity, we apply the MGES, a natural extension of the MES that considers optimal output adjustments (Davis and Shumway 1996). The MGES is a two-factor, one-price elasticity of substitution, which measures the percentage change in the output ratio between two sectors resulting from a 1 percent change in the price of the output in one sector (Chambers 1988).[2] The MGES is a natural multi-output generalization of the Hicksian two-input elasticity of substitution (Blackorby and Russell 1989). The MGES and the MES are equal when the production function is homothetic (Blackorby, Primont, and Russell 2007).

[2] In the presence of a CES technology, some properties of the MGES change because CES imposes a more stringent condition on the variability of the substitution parameter within a nest (Blackorby and Russell 1989).

Based on an output maximization problem (Blackorby and Russell 1989; Anderson and Moroney 1993), the MGES between inputs i and j becomes the following:

$$MGES_{ij} = \begin{cases} \eta_{ji}^{[k]} - \eta_{ii}^{[k]}, i,\ j \in X_k \\ \theta_i^{[k]} MGES_{kr} - \eta_{ii}^{[k]},\ i \in X_k, j \in X_r,\ k \neq r \end{cases}, \tag{4}$$

where $\eta_{ji}^{[k]}$ is the cross-price elasticity of conditional demand within nest k, $\eta_{ii}^{[k]}$ is the own-price elasticity of conditional demand within nest k, $\theta_i^{[k]}$ is the output share of input i in nest k, and $MGES_{kr}$ is the Morishima gross inter-stage elasticity of substitution between nests k and r.[3] The MGES captures changes in the output-maximizing optimal input ratio resulting from a percentage change in the price ratio induced by a change in p_i, holding p_j constant. The MGES holds the prices of other factor outputs constant.[4] Outputs x_i and x_j are Morishima gross complements if $MGES_{ij} < 1$, and inputs x_i and x_j are Morishima gross substitutes if $MGES_{ij} > 1$.

The MGES is a sufficient statistic to evaluate the comparative static results of the log of relative output shares with respect to the log productivity ratio (Anderson and Moroney 1993). Denoting the output share of nest k as θ_k, we can write the following:

$$(i)\ \ \frac{\partial \log\left(\frac{\theta_k}{\theta_r}\right)}{\partial \log\left(\frac{A_k}{A_r}\right)} = MGES_{kr} - 1,$$

$$(ii)\ \ \frac{\partial \log\left(\frac{\theta_i^{[k]}}{\theta_j^{[k]}}\right)}{\partial \log\left(\frac{A_i}{A_j}\right)} = MGES_{ij} - 1, \tag{5}$$

$$(iii)\ \ \frac{\partial \log\left(\frac{\theta_i^{[k]}}{\theta_j^{[r]}}\right)}{\partial \log\left(\frac{A_i}{A_j}\right)} = \theta_i^{[k]} MGES_{kr} - \eta_{ii}^{[k]} - 1,$$

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[3] Inter-nest MGES are symmetric and constant only when the intra-nest output cost shares are equal and the substitution parameters across nests are equal (Blackorby and Russell 1989; Anderson and Moroney 1993). For example, CES nesting of CES processes generates symmetric intra-nest MGES but asymmetric inter-nest MGES.
[4] As originally suggested by Pigou (1934), one way to address this issue is to hold output and other input factors, except for one of the two in the ratio, constant.

where $\frac{A_k}{A_r}$ is the TFP ratio between nest k and nest r, and $\frac{A_i}{A_j}$ is the TFP ratio between inputs i and j. If $n = k$, that is, each nest contains only one input, then (iii) in equation 5 is identical to (i) in equation 5.

2.3 First- and Second-Order Effects of Sectoral Productivity Shocks

The sectoral Domar weights can provide a first-order approximation of the aggregate effect of microeconomic shocks (Hulten 1978). Baqaee and Farhi (2019) show that the second-order approximation of productivity shocks is critical, as it can magnify or attenuate productivity shocks through disproportionate nonlinear propagation across sectors. Baqaee and Farhi also show that nonlinearities in production can generate significantly larger welfare cost arising from business cycles than the nonlinearities in risk aversion in utility shown in Lucas (1987). The reduced-form nonparametric elasticities of production, network linkages, and returns to scale at the sector level are sufficient statistics to capture the nonlinear characterization of the aggregate effect of sectoral productivity shocks (Baqaee and Farhi 2019).

Baqaee and Farhi (2019) define a pseudo elasticity of substitution (ρ_{ji}) for non-homothetic functions as $\frac{1}{\rho_{ji}} = \frac{d \log \frac{f_j}{f_i}}{d \log A_i}$ (where f is a CES aggregator at the sector level, and A denotes sectoral productivity) and deduce an expression for the derivation of the ratio of the shares of sectoral output with respect to the log ratio of sectoral productivity in sector i (A_i) as: $\frac{d \log \frac{D_i}{D_j}}{d \log A_i} = 1 - \frac{1}{\rho_{ji}}$. The MGES is the elasticity of the ratio of marginal rates of substitution with respect to the ratio of two arguments, whereas the pseudo elasticity of substitution defined by Baqaee and Farhi (2019) is the same but with one argument. The pseudo elasticity of substitution is a generalization of the MES, and they are the same when the CES aggregator is homogeneous of degree one. The input-output multiplier in Baqaee and Farhi (2019) is defined as

$$\varepsilon = \sum_{i=1}^{N} \frac{d \log Y}{d \log A_i} = \sum_{i=1}^{N} D_i, \tag{6}$$

where ε is the percentage change in aggregate output in response to a uniform change in technology. Baqaee and Farhi derive the nonlinear aggregate propagation of a productivity shock in sector i as

$$\frac{d^2 \log Y}{d \log {A_i}^2} = \frac{dD_i}{d \log A_i} = \frac{D_i}{\varepsilon} \sum_{i \neq j} D_j (1 - \frac{1}{\rho_{ji}}) + D_i \frac{d \log \varepsilon}{di} . \quad (7)$$

In equation 7, ρ_{ji} is constant across sector pairs. We revised the second-order condition to capture the varying degrees of substitution across sector pairs based on the MGES. We replace the derivative of the sectoral output shares ratio with respect to the log sectoral productivity ratio, and after following some simple algebraic steps, we derive the following expression:

$$\frac{d^2 \log Y}{d \log {A_i}^2} = \frac{D_i}{\varepsilon} \sum_{i \neq j} D_j (MGES_{ji} - 1)\, (\sigma_{ji}) + D_i \frac{d \log \varepsilon}{dA_i}, \quad (8)$$

where Y is GDP, A_i is productivity in sector i, $\varepsilon = \sum_i D_i$ is the input-output multiplier, $\sigma_{ji} = \frac{d \log \frac{A_j}{A_i}}{d \log A_i}$ is the elasticity of productivity between sector i and sector j, and $MGES_{ji}$ is the MGES between sector i and sector j. Figure 1 provides a diagram of the nonlinear propagation channel.

Equation 8 presents the nonlinear aggregate effect characterized by the MGES and σ_{ji} across sector pairs. The elasticity parameters of the sectoral TFPs do not play any role in the nonlinear characterization of sectoral productivity shocks in Baqaee and Farhi (2019). The term σ_{ji} captures how TFP in sector j responds to a change in TFP in sector i. If $\sigma_{ji} > 1$, then TFP in sector j increases (decreases) with an increase (decrease) in TFP in sector i. Similarly, if $\sigma_{ji} < 1$, then TFP in sector j decreases (increases) with an increase (decrease) in productivity in sector i. The last term in equation 8 becomes zero as we aggregate to the world level. Starting at an efficient equilibrium, reallocation effects are zero-sum distributive changes only. As such, they have no aggregate consequences (Baqaee and Farhi 2019).

If $MGES_{ji} > 1$, that is, when intermediate inputs are gross substitutes, it amplifies the aggregate effect of a positive productivity shock in sector i and dampens the aggregate effect of a negative productivity shock in sector i. Intermediate inputs are gross complements when $MGES_{ji} < 1$, which dampens the aggregate effect of a positive productivity shock in sector i

and amplifies the aggregate effect of a negative productivity shock in sector i. σ_{ji} works as a catalyst and can alter the direction of the aggregate effect due to $MGES_{ji}$. If $MGES_{ji} > 1$ and $\sigma_{ji} > 1$, then a positive productivity shock in sector i is amplified through sector j. If $MGES_{ji} > 1$ and $\sigma_{ji} < 1$, then a positive productivity shock in sector i can actually have a dampening effect on aggregate output through sector j.

When $MGES_{ji} < 1$ and $\sigma_{ji} > 1$, the aggregate effect of a positive productivity shock in sector i is attenuated through sector j. Using the same logic, we can expect the aggregate effect of a positive productivity shock in sector i to be amplified through sector j even if $MGES_{ji} < 1$, as long as $\sigma_{ji} < 0$.

To summarize, σ_{ji} plays distinct roles in the propagation of a productivity shock in sector i when it is less than zero, between zero and unity, and greater than unity.

2.4 Measuring the MGES

To measure reduced-form elasticities, we utilize the input-output framework with gross sectoral output and value-added production functions to help distinguish between value-added TFP (${A_i}^V$) and output TFP (${A_i}^O$) (Baumol and Wolff 1984; Oulton 2016). Denoting value added as ${y_i}^V$ and gross sectoral output as ${y_i}^O$, we obtain an expression for the MGES as the sum of two measures of the elasticity of substitution: the elasticity of substitution between the value-added TFP ratio and the output TFP ratio for sectors i and j, and the elasticity of substitution between the value-added ratio and the output TFP ratio for sectors i and j (equation 9). In appendix A, we describe the derivation of equation 9 in detail.

$$MGES_{ij} = \frac{d\log\frac{{y_i}^V}{{y_j}^V}}{d\log\frac{{A_i}^O}{{A_j}^O}} + \frac{d\log\frac{{A_i}^V}{{A_j}^V}}{d\log\frac{{A_i}^O}{{A_j}^O}}. \tag{9}$$

3. Data and Empirical Results

3.1 Data

We use a cross-country framework to estimate the general equilibrium elasticities and nonlinear aggregate propagation of a sectoral productivity shock. This enables us to compare the aggregate effects across countries. Data on sectoral prices that are comparable across countries are available only for certain benchmark years from the World Bank's International Comparisons Program (ICP). We use the TFP measures from Fadinger, Ghiglino, and Teteryatnikova (2022) based on ICP 2005 data, which satisfy a set of basic requirements for TFP comparisons across countries.[5] Fadinger, Ghiglino, and Teteryatnikova (2022) use the 2005 WIOD to estimate comparable country-sector-level TFP for 35 sectors across 38 countries based on Caves, Christensen, and Diewert (1982).[6] As the TFP measures in Fadinger, Ghiglino, and Teteryatnikova (2022) are constructed based on PPP-adjusted sector-level gross output, they qualify as a measure of the output TFP defined in equation A2, in appendix A.

Following Fadinger, Ghiglino, and Teteryatnikova (2022), we construct sectoral TFPs for another benchmark year, 2011, based on the sectoral price data from the ICP, the WIOD (Timmer et al. 2015), and the SEA data.[7] The SEA collected industry-level data on employment, capital stocks, gross output, and value added at current and constant prices, in millions of local currencies, for 1995–2011. We use equation A4, in appendix A, to compute the PPP-adjusted value-added TFP from the output TFP for 35 sectors and two benchmark years, 2005 and 2011. Table B1, in appendix B, compares the unweighted average TFP (across 38 countries) for these 35 sectors between 2005 and 2011.

3.2 Measuring Elasticities of Substitution

We follow three steps to compute the MGES for each sector pair resulting from the change in sectoral TFPs between 2005 and 2011. First, we calculate the differences in these ratios between 2005 and 2011: $\Delta\, log\, \frac{{y_i}^V}{{y_j}^V}$, $\Delta\, log\, \frac{{A_i}^O}{{A_j}^O}$ and $\Delta\, log\, \frac{{A_i}^V}{{A_j}^V}$. Second, we obtain the average elasticity of substitution between the value-added TFP ratio and the output TFP ratio by considering the change in TFP for each sector based on ordinary least squares (OLS)

5 See Fadinger, Ghiglino, and Teteryatnikova (2022) for further details.

6 The sectors are 1 = Agriculture, hunting, forestry and fishing; 2 = Mining and quarrying; 3 = Food, beverages, and tobacco; 4 = Textiles and textile products; 5 = Leather and footwear; 6 = Wood and products of wood and cork; 7 = Pulp, paper, printing, and publishing; 8 = Coke, refined petroleum, and nuclear fuel; 9 = Chemicals and chemical products; 10 = Rubber and plastics; 11 = Other non-metallic minerals; 12 = Basic metals and fabricated metals; 13 = Machinery, not elsewhere classified (n.e.c.); 14 = Electrical and optical equipment; 15 = Transport equipment; 16 = Manufacturing, n.e.c.; 17 = Electricity, gas, and water supply; 18 = Construction; 19 = Motor vehicle sales; 20 = Wholesale trade; 21 = Retail trade; 22 = Hotels and restaurants; 23 = Inland transport; 24 = Water transport; 25 = Air transport; 26 = Other transport activities; 27 = Post and telecommunications; 28 = Financial intermediation; 29 = Real estate activities; 30 = Other business activities; 31 = Public administration and defense; 32 = Education; 33 = Health and social work; 34 = Other social and personal services; and 35 = Private households with employed persons.

7 We first compare the sectoral TFP estimates that are available from Fadinger, Ghiglino, and Teteryatnikova (2022) with our sectoral TFP estimates from the WIOD data, and then use the discrepancies to adjust for the sectoral TFP estimates for 2011.

regressions of $\Delta\, log \frac{{y_i}^V}{{y_j}^V}$ on $\Delta\, log \frac{{A_i}^O}{{A_j}^O}$ over a sample of 38 countries for each of the 34 sector pairs. We repeat the same procedure to obtain the elasticity of substitution between the value-added ratio and the output TFP ratio for 34 sector pairs. Third, we compute the MGES by taking the sum of these two elasticity measures for each of the 34 sector pairs.

To measure σ_{ji} for the 34 sector pairs, we first compute the change in sectoral TFP ($d \log \frac{A_j}{A_i}$) between 2005 and 2011 for each sector (j). We then estimate the relationship between the change in TFP in sector i ($d \log A_i$) and the change in TFP in the other sectors by running a simple OLS regression of $d \log \frac{A_j}{A_i}$ on $d\, log\, A_i$ ($j \neq i$) over the WIOD sample of countries for each sector pair (j, i). Both the MGES and σ_{ji} do not vary across countries; however, the nonlinear propagation varies at the sector-country level due to differences in sectoral Domar weights across countries.

Panel a in figure 2 shows the unweighted average of the sectoral Domar weights across 38 countries for 2005 and 2011. The four largest sectors based on sectoral output are construction (0.16), business services (0.14), financial services (0.12), and the food industry (0.10). On average, these four sectors together produce more than 50 percent of GDP. Panel b in figure 2 shows the standard deviation of the sectoral Domar weights. The largest cross-country variation in Domar weights is observed in electrical equipment, followed by business services. Variation in the sectoral Domar weights largely determines the sectoral contribution of the nonlinear aggregate propagation of sectoral productivity shocks.

Evidence based on 35 sectors shows that sectoral outputs are predominantly substitutes across sector pairs. Figure 3 compares the relationship between the MGES and σ for the 35 sectors. Each dot in the scatter plots represents a sector pair; thus, each scatter plot has 34 dots. To put it differently, the aggregate effect of a productivity shock in a sector is the sum of its nonlinear effects on the 34 other sectors. The number in the subheading denotes the sector that experiences a productivity shock. For example, the first scatter plot shows how a productivity shock in agriculture affects output and productivity in other sectors. The linear fit is positive, suggesting a positive relationship between the MGES and σ across 34 sector pairs generated from a productivity shock in agriculture between 2005 and 2011. We find that the linear fit between the MGES and σ is positive for 19 of the 35 sectors. Overall, the findings support heterogeneity in the nonlinear sectoral effects of productivity shocks captured through the variable elasticities.

3.3 Nonlinear Aggregate Effects: MGES versus CES

Armed with the quantitative figures for the sectoral Domar weights and substitution elasticities, as a next step we non-parametrically compute the second-order aggregate effects of sectoral productivity shocks in the 35 sectors between 2005 and 2011 (following equation 12). The nonlinear aggregate effect is the sum over the nonlinear sectoral effects, which is determined by the sectoral Domar weights, the MGES, and the sectoral TFP elasticities. We consider two additional cases as counterfactuals. Following Atalay (2017), we consider $\rho_{ji} = 0.1$ and $\rho_{ji} = 0.6$. We use equation 11 to compute the nonlinear aggregate effects for these two cases of CES parameters. Once we compute the aggregate effect of a productivity shock in each of the 35 sectors for the full WIOD sample of 38 countries, we calculate the 5:1 spread as a ratio of the average aggregate effects in the fifth (top) and the first (bottom) income quintiles.

Table 1 presents the 5:1 spreads in the nonlinear aggregate effects from productivity shocks in the 35 sectors between 2005 and 2011. The first column shows the results using the MGES, the second column with CES = 0.1, and the third column with CES = 0.6. In eight of the 35 sectors, productivity shocks lead to convergence in aggregate output (that is, the 5:1 spread is less than unity) across countries, and these results are consistent across the MGES and two counterfactual scenarios. Overall, the results are robust across these models, in terms of the direction of the effects on the cross-country income gap (convergence or divergence). The magnitude of the aggregate effect varies across these models; in this regard, we note three key differences. First, the model with constant elasticities at 0.6 comes closer to matching the results from the MGES model than the model with constant elasticities at 0.1. Second, the effect on cross-country income divergence is larger with variable elasticities than with constant elasticities of 0.1 in certain sectors, like chemicals and business services. Third, productivity shocks in certain sectors (eight in total) lead to larger income divergence across countries using the model with constant elasticities of 0.1 than the model with the MGES. Since the variable elasticities of substitution do not vary across countries, we observe that the aggregate effects are heterogeneous, resulting from different models primarily due to the differences in the sectoral Domar weights across countries.

3.4 Do Variable Elasticities Offer Deeper Insights into Nonlinear Propagation?

In this section, we compare the sector-level propagation effects of sectoral productivity shocks in selected sectors. Our goal is to examine if the variable elasticities can provide additional knowledge about the role of each sector in the production network when we compare the sectoral effects to aggregate propagation of productivity shocks between variable elasticities and constant elasticities. We consider three cases: productivity shocks in inland transport (larger income divergence with CES than with the MGES), financial services (comparable rate of income divergence between CES and the MGES), and business services (larger income divergence with the MGES than with CES). Figure 4 shows the propagation results at the sector level. The construction sector has the largest Domar weight and, as a result, the average contribution of the construction sector to the aggregate effect is larger than that of the sectors across the board. For a productivity shock in inland transport, the electrical equipment, transport equipment, and health and social services sectors play a more prominent role with the MGES than CES.

By contrast, the average contributions of agriculture and textiles to the aggregate effect become negative with the MGES when a productivity shock in financial services is considered. With CES, the average effect remains positive for all sectors. Finally, when we consider a productivity shock in business services, we find that the roles of construction, inland transport, food, and social services are more prominent with the MGES compared to the case with CES. As demonstrated in equation 8, the differences in sectoral effects with CES are mainly driven by differences in the sectoral Domar weights. With the MGES, the differences in sectoral effects provide a more nuanced picture of sectoral dynamics in the production network in our flexible framework of the nonlinear characterization of productivity shocks.

4. Conclusion

Consideration of the nonlinear propagation of sectoral productivity shocks more capably captures the business cycle fluctuations in aggregate output than linear propagation relying solely on sectoral Domar weights. Intersectoral linkages are integral to the nonlinear shock propagation mechanism. In this study, we considered a more flexible production network with variable elasticities of substitution to model intersectoral linkages. At the aggregate level, the differences in the results between variable and constant elasticities of substitution are driven by the sectoral Domar weights as our estimates of variable elasticities only vary across sector pairs but not across countries. At the sector level, results based on the variable elasticity framework show the more prominent role of construction and inland transport in the aggregate propagation of a productivity shock in financial and business services.

For possible future research, it may be fruitful to apply the variable elasticity framework to understand the propagation of a nonlinear shock with elasticities that vary across sector pairs as well as across countries.

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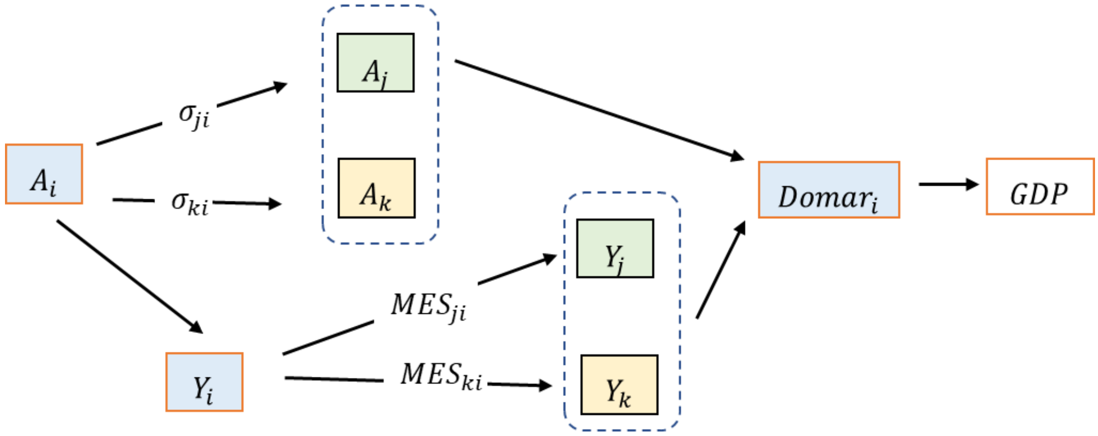
Table 1. The 5:1 Spread in the Aggregate Effect of Sectoral Productivity Shocks

Productivity shock in	MGES (1)	CES (0.1) (2)	CES (0.6) (3)
Agriculture, hunting, forestry and fishing	1.049	1.320	1.070
Mining and quarrying	3.063	2.592	3.032
Food, beverages, and Tobacco	3.211	2.937	3.192
Textiles and textile products	0.078	0.228	0.085
Leather and footwear	1.045	1.046	1.045
Wood and products of wood and cork	1.439	1.357	1.433
Pulp, paper, printing, and publishing	1.529	1.424	1.521
Coke, refined petroleum, and nuclear fuel	7.102	4.923	6.919
Chemicals and chemical products	12.294	6.374	11.701
Rubber and plastics	1.734	1.608	1.725
Other non-metallic Minerals	1.726	1.664	1.723
Basic metals and fabricated metals	6.085	4.622	5.970
Machinery, n.e.c.	3.820	2.981	3.757
Electrical and optical equipment	4.009	3.335	3.957
Transport equipment	3.569	3.041	3.531
Manufacturing, n.e.c.; recycling	1.888	1.723	1.877
Electricity, gas and water supply	1.805	1.477	1.776
Construction	0.000	0.000	0.000
Motor vehicles sales	1.402	1.297	1.394
Wholesale trade	0.698	0.932	0.714
Retail trade	2.784	2.548	2.769
Hotels and restaurants	1.409	1.429	1.412
Inland transport	0.081	0.205	0.087
Water transport	1.086	1.092	1.087
Air transport	1.390	1.306	1.385
Other transport activities	1.414	1.309	1.406
Post and telecommunications	1.794	1.684	1.788
Financial intermediation	1.441	1.459	1.443
Real estate activities	0.002	0.013	0.002
Other business activities	166.782	38.322	150.201
Public administration and defense	0.074	0.187	0.079
Education	0.300	0.496	0.312
Health and social work	2.588	1.896	2.526
Other social and personal services	0.704	0.853	0.715
Private households with employed persons	1.059	1.045	1.058

Sources: Estimates based on data from the World Input-Output Database, February 2012 release; Timmer et al. 2015; Fadinger, Ghiglino, and Teteryatnikova 2022.

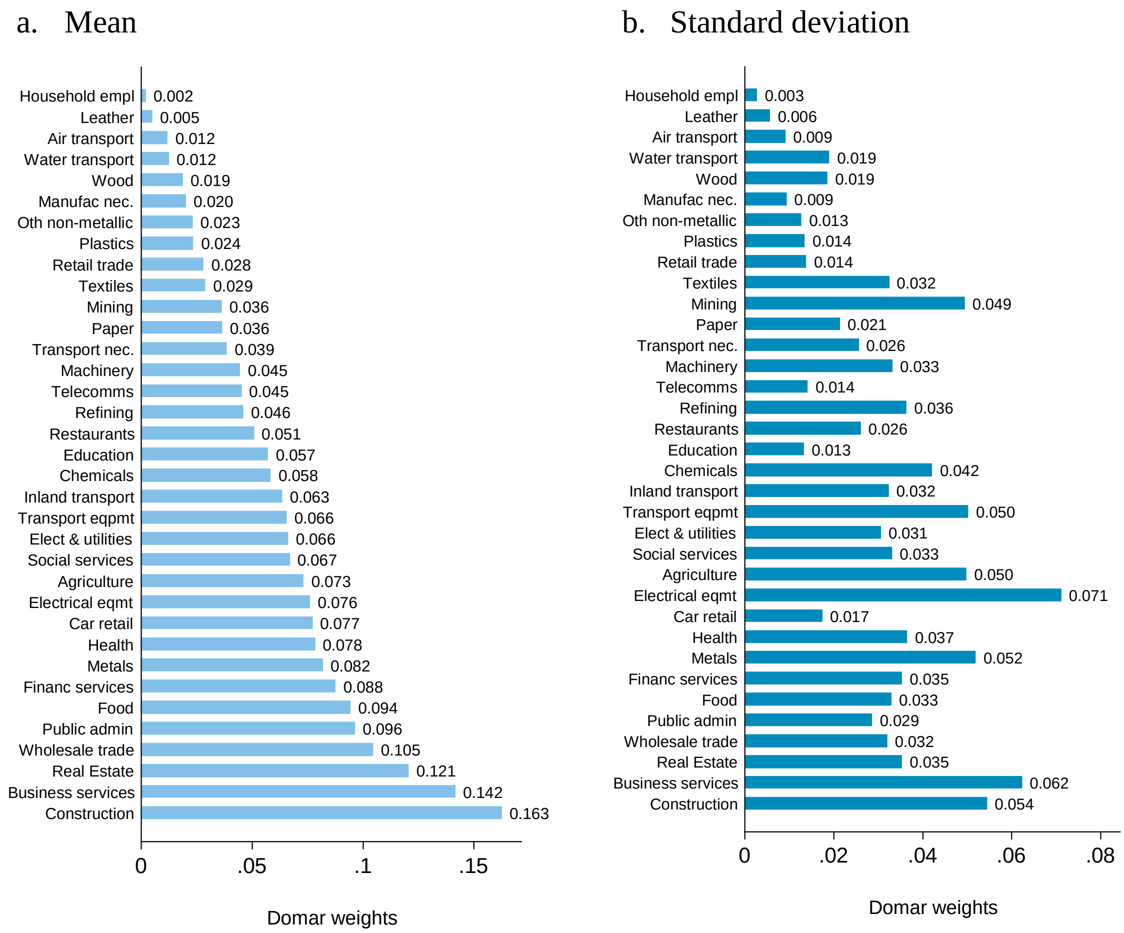
Note: We use 35-sector input-output tables. We compute the 5:1 spread as a ratio between the average aggregate effect in the fifth (top) and first (bottom) income quintiles. The aggregate effect of a productivity shock in a particular sector equals the sum of the sectoral effects across the 34 other sectors. CES = constant elasticity of substitution; MGES = Morishima gross elasticity of substitution.

Figure 1. Nonlinear Propagation with General Equilibrium Elasticities



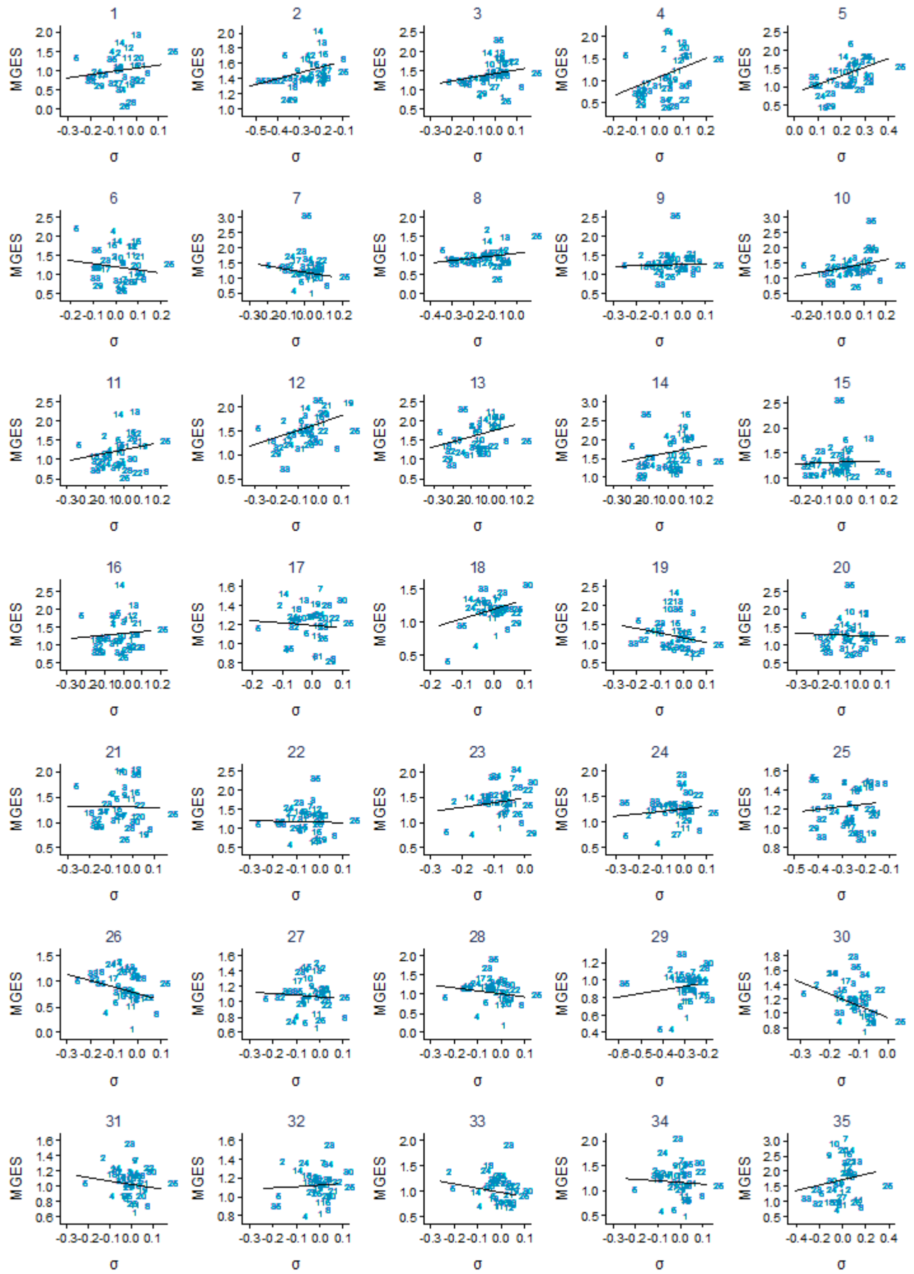
Source: Original illustration for this paper.
Note: GDP = gross domestic product; MES = Morishima elasticity of substitution.

Figure 2. Sectoral Domar Weights



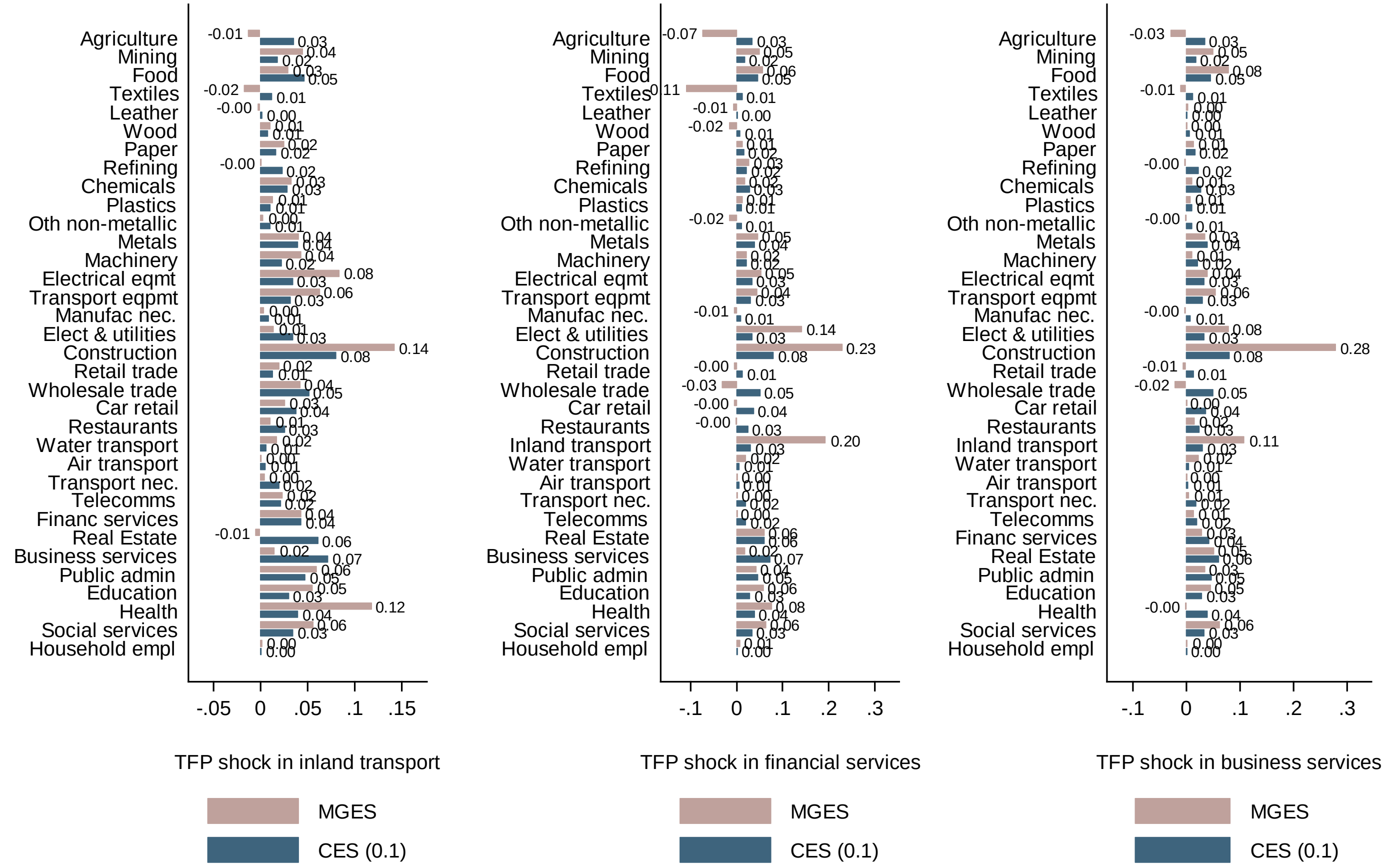
Sources: Estimates based on data from the World Input-Output Database (WIOD), February 2012 release; Timmer et al. 2015; Fadinger, Ghiglino, and Teteryatnikova 2022.
Note: Means and standard deviations of sectoral Domar weights are computed for 35 sectors based on the WIOD sample of 38 countries.

Figure 3. Relationship between the MGES and Productivity Elasticities (σ)



Sources: Estimates based on data from the World Input-Output Database, February 2012 release; Timmer et al. 2015; Fadinger, Ghiglino, and Teteryatnikova 2022.
Note: 1 = Agriculture, hunting, forestry and fishing; 2 = Mining and quarrying; 3 = Food, beverages, and tobacco; 4 = Textiles and textile products; 5 = Leather and footwear; 6 = Wood and products of wood and cork; 7 = Pulp, paper, printing, and publishing; 8 = Coke, refined petroleum, and nuclear fuel; 9 = Chemicals and chemical products; 10 = Rubber and plastics; 11 = Other non-metallic minerals; 12 = Basic metals and fabricated metals; 13 = Machinery, n.e.c.; 14 = electrical and optical equipment; 15 = Transport equipment; 16 = manufacturing, n.e.c.; 17 = electricity, gas, and water supply; 18 = Construction; 19 = Motor vehicles sales; 20 = Wholesale trade; 21 = Retail trade; 22 = Hotels and restaurants; 23 = Inland transport; 24 = Water transport; 25 = Air transport; 26 = Other transport activities; 27 = Post and telecommunications; 28 = Financial intermediation; 29 = Real estate activities; 30 = Other business activities; 31 = Public administration and defense; 32 = education; 33 = Health and social work; 34 = Other social and personal services; and 35 = Private households with employed persons. MGES = Morishima gross elasticity of substitution.

Figure 4. Nonlinear Characterization of the Aggregate Effect of Productivity Shocks



Sources: Estimates based on data from the World Input-Output Database, February 2012 release; Timmer et al. 2015; Fadinger, Ghiglino, and Teteryatnikova 2022.
Note: Each bar represents the sectoral effect of the productivity shock in a selected sector. CES = constant elasticity of substitution; MGES = Morishima gross elasticity of substitution; TFP = total factor productivity.

Appendix A

Measurement of the Morishima Gross Elasticity of Substitution

Consider an input-output framework with gross sectoral output and a value-added production function. A value-added production function exists only under the condition that real gross output per unit of real intermediate input is determined entirely by input prices and can never be reduced by technological progress (Baumol and Wolff 1984; Oulton 2016). We write the production functions for sector i by adding the superscripts "O" and "V" for output and value-added, respectively:

$$y_i^{O} = A_i^{O} F_i(l_i, x_{i1}, x_{i2}.., x_{iN}),$$
$$y_i^{V} = A_i^{V} F_i(l_i). \tag{A1}$$

A_i^{O} and A_i^{V} are the output total factor productivity (TFP) and value-added TFP, respectively. We assume marginal cost pricing based on a competitive market, and input from a particular sector receives the same price across all sectors. The output TFP, which follows from the standard definition, can be written as

$$A_i^{O} = \hat{y}_i^{O} - \alpha_L^{O} \hat{l}_i - \sum_{j=1}^{N} \alpha_{ij}^{O} \hat{x}_{ij}, \tag{A2}$$

where ∧ is the growth rate of a variable over time, and α_s^{O} is the elasticity of any factor s with respect to output, which equals the factor share of output in sector i under a competitive market. Likewise, the value-added TFP can be written as

$$A_i^{V} = \hat{y}_i^{V} - \alpha_L^{V} \hat{L}_i, \tag{A3}$$

where α_L^{V} is the elasticity of labor with respect to value added and equals the share of gross value added in sector i with competitive factor markets. Combining equations A2 and A3, we derive the following relationship between value-added TFP and output TFP:

$$y_i^{O} = \frac{A_i^{V}}{A_i^{O}} y_i^{V}.^{8} \tag{A4}$$

[8] See Gabaix (2011) and Oulton (2016) for a proof of this result.

Equation A4 is valid only if value added is measured by double deflation, one for input prices and one for output prices.[9] Based on equation A4, the Domar weight (D_i) for sector i becomes

$$D_i = \frac{{y_i}^O}{GDP} = \frac{{A_i}^V}{{A_i}^O} \frac{{y_i}^V}{GDP}. \tag{A5}$$

Taking the logs of both sides of equation A5, the log of the ratio of the Domar weights between sectors i and j can be written as the sum of three terms: (i) the log of the value-added ratio between sectors i and j, (ii) the log of the value-added TFP ratio between sectors i and j, and (iii) the log of the output TFP ratio between sectors i and j:

$$\log\frac{D_i}{D_j} = \log\frac{{y_i}^V}{{y_j}^V} + \log\frac{{A_i}^V}{{A_j}^V} - \log\frac{{A_i}^O}{{A_j}^O}. \tag{A6}$$

Differentiating both sides of equation A6 with respect to $log\frac{{A_i}^O}{{A_j}^O}$, we obtain

$$\frac{d\log\frac{D_i}{D_j}}{d\log\frac{{A_i}^O}{{A_j}^O}} = \frac{d\log\frac{{y_i}^V}{{y_j}^V}}{d\log\frac{{A_i}^O}{{A_j}^O}} + \frac{d\log\frac{{A_i}^V}{{A_j}^V}}{d\log\frac{{A_i}^O}{{A_j}^O}} - 1. \tag{A7}$$

As equation A7 shows, the elasticity of substitution between the Domar weight ratio and the output TFP ratio between sectors i and j can be estimated through the elasticity of substitution between the output TFP ratio and the value-added output ratio between sectors i and j, and the elasticity of substitution between the output TFP ratio and the value-added TFP ratio between sectors i and j. Equation A7 is analogous to the conditions for the propagation of sectoral productivity shocks through intersectoral linkages in Baqaee and Farhi (2019) and Carvalho et al. (2021). Comparing equation A7 with equation 5, we obtain an expression for the Morishima gross elasticity of substitution (MGES) as the sum of two measures of the elasticity of substitution:

[9] See Steindel and Stiroh (2001) and Oulton and O'Mahony (1994) for further discussion on this topic. Single deflation works only when the output and input prices change at the same rate, which is less likely.

$$MGES_{ij} = \frac{d\log\frac{y_i{}^V}{y_j{}^V}}{d\log\frac{A_i{}^O}{A_j{}^O}} + \frac{d\log\frac{A_i{}^V}{A_j{}^V}}{d\log\frac{A_i{}^O}{A_j{}^O}}\,. \quad \text{(A8)}$$

An illustrative example of an input-output table demonstrating how the MGES, sectoral Domar weights, and sectoral outputs are linked is provided below.

		Sectors								
		1	..	k	..	n	Intermediate demand	Final demand	Total output	Domar weights
Sectors	1	$MGES_{11}$	..	$MGES_{k1}$	..	$MGES_{n1}$	I_1	F_1	$O_1 = I_1 + F_1$	$D_1 = O_1/\ GDP$
	..	..	..	..	..	..	..	..	..	..
	k	$MGES_{1k}$	..	$MGES_{kk}$	..	$MGES_{nk}$	I_k	F_k	$O_k = I_k + F_k$	$D_k = O_k/\ GDP$
	..	..	..	..	..	..	..	..	..	..
	n	$MGES_{1n}$		$MGES_{kn}$		$MGES_{nn}$	I_n	F_n	$O_n = I_n + F_n$	$D_n = O_n/\ GDP$

Source: Original table for this paper.
Note: GDP = gross domestic product; MGES = Morishima gross elasticity of substitution.

Appendix B

Table B1. Sectoral Total Factor Productivity, 2005 and 2011

Sector	2005	2011
Agriculture, hunting, forestry and fishing	0.680	0.696
Mining and quarrying	3.235	1.570
Food, beverages, and tobacco	0.955	1.249
Textiles and textile products	0.838	0.647
Leather and footwear	0.873	–12.825
Wood and products of wood and cork	0.896	0.362
Pulp, paper, printing and publishing	0.846	1.090
Coke, refined petroleum and nuclear fuel	0.873	1.036
Chemicals and chemical products	1.206	1.228
Rubber and plastics	1.184	1.104
Other non-metallic minerals	0.850	0.804
Basic metals and fabricated metals	0.856	0.864
Machinery, n.e.c.	0.753	0.374
Electrical and optical equipment	0.842	1.143
Transport equipment	0.888	0.562
Manufacturing, n.e.c.; recycling	0.887	0.601
Electricity, gas and water supply	0.822	5.072
Construction	0.863	0.376
Motor vehicles sales	0.873	0.607
Wholesale trade	0.524	–9.791
Retail trade	0.589	1.321
Hotels and restaurants	0.672	1.201
Inland transport	1.161	0.840
Water transport	0.843	2.469
Air transport	0.382	0.627
Other transport activities	1.170	2.425
Post and telecommunications	1.160	1.456
Financial intermediation	1.041	0.932
Real estate activities	2.904	1.109
Other business activities	0.666	1.987
Public administration and defense	1.493	9.607
Education	1.941	1.944
Health and social work	1.208	2.535
Other social and personal services	0.819	3.110
Private households with employed persons	0.873	4.436

Sources: Calculations based on data from the World Input-Output Database, February 2012 release; Timmer et al. 2015; Fadinger, Ghiglino, and Teteryatnikova 2022.
Note: Unweighted averages of total factor productivities across 38 countries. n.e.c. = not elsewhere classified.