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ABSTRACT

A Multisector Perspective on Wage Stagnation*

Low-skilled workers are concentrated in sectors that experience fast productivity growth and yet their real wages have been stagnating. We document evidence from the U.S. to show the importance of sectoral reallocations. Key to our two-sector model is the fall in the relative price of the low-skill intensive sector caused by faster productivity growth. When outputs are complements across sectors, this leads to a reallocation of low-skilled workers to the high-skill intensive sector where their marginal product is stagnant. We show that this mechanism is quantitatively important for the stagnation of low-skill real wages and their divergence from aggregate labor productivity during 1980-2010.

JEL Classification: E24, J23, J31

Keywords: wage stagnation, wage-productivity divergence, low-skill wage, multisector model

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1 Introduction

Low-skilled workers have experienced very little wage growth, despite working mostly in sectors with fast productivity growth. In the U.S., the real wage of non-college workers increased by about 20% between 1980-2010, which is less than half the increase in aggregate labor productivity.[1] The low-skill wage "stagnation" persists even after controlling for age, race, gender, education, and occupation, so it is not due to compositional changes in low-skill employment.[2] Hours worked by these workers represent two-thirds of overall hours worked, so their wage stagnation explains why the average wage is lagging behind aggregate labor productivity, despite the real wage of college graduates growing faster than aggregate labor productivity. Taken together, they reject the view that a rising tide lifts all boats; apparently, many boats are left behind in absolute terms.

Our objective is to understand why the growth of the low-skill wage is so low and why it is lagging behind the aggregate labor productivity. We offer a novel multisector perspective through changing relative prices driven by the faster productivity growth in sectors that use low-skilled workers more intensively.

The importance of changing relative prices across sectors is motivated by differences between the real wage and the product wage. The product wage is measured as the nominal wage deflated by the sectoral value-added price, which is the same as the sectoral marginal product of labor in a perfectly competitive labor market. Three observations emerge from the data. First, low-skill real wages grew similarly across sectors, but there is a large variation in the growth of product wages, because of changes in relative prices across sectors. Second, the share of low-skill

[1] The precise increase in non-college real wage ranges from 15% to 25%, depending on the choice of price deflators, composition adjustment, the inclusion of non-wage compensation and self-employment, and whether it is only for the nonfarm business sectors. See Appendix A1.3. However, regardless of these choices, the finding that the non-college real wage has had little growth and lags behind the aggregate labor productivity growth is robust.

[2] As documented in Acemoglu and Autor (2011), low-skill wage stagnation co-exists with occupational polarization according to which the wages of low-wage occupations have been growing faster than the wages of middle-wage occupations. The low-skill wage stagnation is about a group of workers with given education qualifications whereas polarization is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within an occupation in understanding these two patterns.

hours increased in sectors with slower growth in product wages. Together they highlight that what matters is not sectoral reallocation *per se* but the reallocation of low-skilled workers into sectors with growing relative prices. The third observation is that the sectors that have increased both their hour shares and relative prices are sectors that use low-skilled workers less intensively.

To understand how the low-skill real wage is related to aggregate labor productivity, we turn to the accounting identity that the total value-added of the economy is the sum of total factor payments. This identity implies that there are three driving forces for the divergence between the low-skill wage and aggregate labor productivity: the rising skill premium, falling labor income share and the rising relative cost of living, measured by the ratio of the consumption deflator and the output deflator. The presence of capital is necessary for the last two forces to materialize. In its absence, both labor income share and the relative price of consumption are equal to one.

Motivated by these observations, we build a two-sector model in which the sectoral production functions use high-skill labor, low-skill labor and capital. The only differences between the two production functions are the skill intensity of each sector and their productivity growth. The low-skill intensive sector experiences faster productivity growth, which implies a fall in its relative price. In the presence of consumption complementarity across the sector's outputs, the faster productivity growth in the low-skill intensive sector implies a movement of low-skilled workers to the high-skilled intensive sector. As this is the sector with a slower-growing marginal product of labor, this process contributes to the low-skill wage stagnation.[3]

The same mechanism contributes to the divergence by predicting a rise in the skill premium and a rise in the relative cost of living. The shift towards the high-skill intensive sector acts like a skill-biased change, which increases the skill

[3] In other words, specializing in sectors with faster productivity growth works against the low-skilled workers, as the output they produce is getting cheaper over time. This has a similar flavor, but the mechanism is different, to the early trade literature on immiserizing growth, where faster productivity growth results in a country being worse off because of deteriorating terms of trade (Bhagwati, 1958).

premium. It also implies a rise in the relative price of consumption, because the consumption share of the high-skill sector exceeds its value-added share.[4]

To evaluate the quantitative role played by the uneven productivity growth, the model is calibrated to match key features of the US labor market from 1980 to 2010. In addition to our mechanism through uneven productivity growth, the calibration allows for four other forces that are shown to be important for understanding the skill premium and the labor share. They are the fall in the relative price of capital, the increase in the relative supply of high-skill labor and the changing production weights of low-skill labor and high-skill labor.

We find that uneven productivity growth, which is calibrated to match the observed changes in relative prices, is quantitatively important. In its absence, the divergence would have been halved and the growth in the low-skill wage would have been doubled. The falling production weight of the low-skill labor has a similar quantitative role as it is the key factor for the fall in labor share and the rise in the skill premium. However, its prediction about the low-skill wage stagnation relies on lowering the growth of the low-skill product wages in both sectors, which misses the differential trends observed in the data. The fall in the relative price of capital also contributes to the divergence by predicting a rise in the skill premium, but it also predicts a rise in the growth of the low-skill wage. These quantitative exercises demonstrate those factors that contribute to the rise in the skill premium and the divergence do not necessarily contribute to low-skill wage stagnation.

Finally, we compare the quantitative results to a one-sector model by focusing on an aggregate production function that takes the same form as the sectoral production functions. We calibrate the baseline of the one-sector model to match the same set of aggregate targets as in the multisector model. We find that increasing the productivity index of the aggregate production function will have no effect on the divergence because it would increase the aggregate labor productivity

[4]Uneven productivity growth across sectors was also the focus of Baumol (1967) in his seminal paper on aggregate growth stagnation. For our results on the low-skill wage stagnation and its divergence from aggregate productivity, we need in addition capital, heterogeneous labor and different skill intensities across sectors to interact with the uneven productivity growth.

and the low-skill wage equally. But for the multisector model, the source of the increase in aggregate productivity is important. If it is due to a balanced increase in the productivity indexes of the two sectors, then the result is similar to the one-sector model. If the increase is due to the high-skill (low-skill) intensive sector, then the increase in the low-skill wage will be larger (smaller) than the increase in aggregate labor productivity.

Related Literature The role of different price deflators and falling labor income share have been empirically documented as the sources of the decoupling of the average wage and productivity (e.g. Lawrence and Slaughter, 1993; Stansbury and Summers, 2017). This paper shows that a majority of the labor force, i.e. the low-skill workers, suffer from an even larger divergence due to the growing skill premium. Since the seminal work of Katz and Murphy (1992), there has been a large literature studying the effects of skill-biased demand and supply shifts on the skill premium, with particular focus on skill-biased technical change (see Goldin and Katz, 2009, for a review).

The skill-biased technical change that simply improves the relative productivity of high-skilled workers, however, cannot explain wage stagnation for low-skilled workers (Johnson, 1997; Acemoglu and Autor, 2011). This has partly contributed to a growing literature on automation and falling labor shares (see recent examples, Zeira, 1998; Karabarbounis and Neiman, 2014; Acemoglu and Restrepo, 2018; Martinez, 2019; Caselli and Manning, 2019; Hémous and Olsen, 2022; Moll et al., 2022, among others).[5]. There are other potential explanations for the low-skill wage stagnation, such as de-unionization and decline in the minimum wage (Lee, 1999; Dustmann et al., 2009), increasing monospony power (Manning, 2003), increasing imports (Autor et al., 2013)[6], and the decline in the urban premium for non-college workers due to region-specific occupational changes (Autor, 2019).[7]

[5]This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labor income shares (see e.g., Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020; Chen et al., 2021; Kapetaniou and Pissarides, 2020, among others)

[6]Both Autor et al. (2013) and Kehoe et al. (2018) find that trade accounts for a quarter or less of the decline in the US manufacturing. Kehoe et al. (2018) specifically show that most of the decline is due to uneven productivity growth, which is the mechanism we focus on.

[7]To the extent that most of the expansion in high-skill services happens in urban areas, our

Our contribution to this literature is to show the importance of uneven productivity growth *across sectors*.

As an alternative between-sector mechanism, one might invoke imperfect labour markets and a fall in low-skilled workers' bargaining power, which spread unevenly across sectors. Evidence for this is provided by Stansbury and Summers (2020), who found that worker bargaining power fell more in low-skill industries than in high-skill ones. However, uneven productivity growth is still needed to generate the observed changes in relative prices, as this simple equation shows. Express the low-skill wage as $w_{lj} = p_j MPL_j \pi_j$, where MPL_j is the low-skill marginal product and π_j captures the worker's bargaining power in an imperfect market. We have argued that low-skill wages increase at similar rates across sectors while the prices of high-skill industries are growing faster. For these facts to be consistent with the finding of Stansbury and Summers, the marginal product of low-skill labor in high-skill intensive industries must be falling relative to that in low-skill intensive industries. Rather than act as an alternative, changes in bargaining power reinforce our explanation.

The effect of uneven productivity growth on the skill premium has also been explored by Buera et al. (2020). Our objective is to understand its effects on the low-skill wage stagnation and its divergence from the aggregate labor productivity, which are not addressed in their paper. Capital, absent in their model, plays an important role in our paper. It accounts for 30%-50% of the divergence due to the predicted fall in the labor share and the rise in the relative price of consumption.

Section 2 presents motivating facts on low-skill wages and the importance of a multisector perspective. Section 3 presents a two-sector model and discusses the mechanism for low-skill wage stagnation through uneven productivity growth across sectors. The quantitative importance of the mechanism is presented in Section 4 when the model is calibrated to match key features of the US labor market.

mechanism is consistent with the finding of Autor (2019) on the decline of the urban premium for non-college workers.

2 Motivation

2.1 Observations on Low-skill Wages

Using BEA's Regional Economic Accounts, IMPUS Census extras and American Community Survey, we document three observations on low-skill wages across 11 sectors and 51 US states. These observations also hold for the US economy as a whole.[8]

Observation 1: Changes in sectoral relative prices imply large variations in the growth of sectoral product wages.

Figure 1A plots the growth of low-skill real wage against the growth of low-skill product wage for each sector in each state.[9] The product wage is calculated as the nominal wage divided by sectoral value-added price, which is the exact measure of marginal product of labor in a perfectly competitive labor market. The substantial changes in relative prices across sectors imply large variations in the growth of low-skill product wages, compared to small variations in the growth of low-skill real wages.

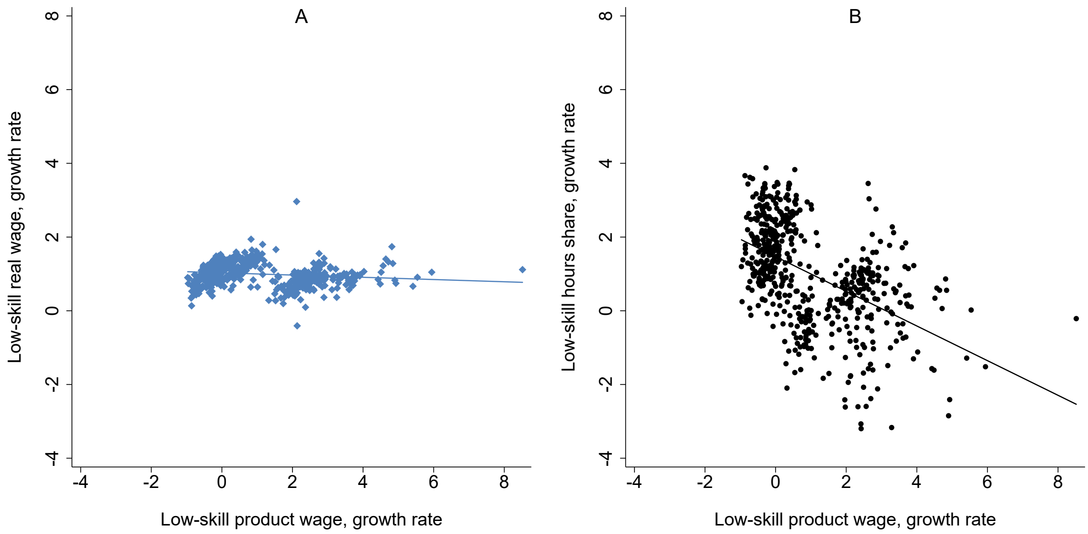
Observation 2: Low-skilled workers are moving from sectors with faster growing product wages into sectors with slower growing product wages.

Observation 1 reveals that the stagnation in the aggregate low-skill real wage does not necessarily mean that there is no growth in low-skill product wages everywhere. In fact many low-skilled workers are working in sectors with high growth in product wages. However, Figure 1B shows that low-skilled workers are leaving those sectors and moving into sectors with slower growth in product wages. These two observations also hold for the US economy as a whole, where the equivalent figures contain 11 sectors at the national level (see Figure A1).

[8] See Data Appendix A1 for the construction of variables and sectors. To begin with, we conduct a shift-share analysis to decompose the changes in the low-skill wage at the national level into within-state and across-state components. We find that the within-state component accounts for almost all the changes.

[9] All growth rates are adjusted for state fixed effects to ensure that the data pattern is not driven by variations across states. Specifically, we regress the growth rates at the state-sector

Figure 1: Growth in Low-skill Real Wage, Product Wage and Hour Shares



Notes: Each panel includes data on growth rates for 11 sectors in 51 US states. Growth rates between 1980 and 2010 are annualized. The growth of sectoral real wages on the left panel and the growth of sectoral low-skill hour shares on the right panel are plotted against the growth of low-skill product wage. Sectoral real wage is calculated as sectoral nominal wage divided by the PCE price index. Sectoral product wage is calculated as sectoral nominal wage divided by sectoral value-added price. The figure shows the pooled observations for 51 states where each variable's growth rate is adjusted for state fixed effects. Low-skill is defined as education less than a university degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix A1 for the construction of variables and sectors.
Source: BEA Regional Economic Accounts, Census, and ACS.

To illustrate the importance of the reallocation mechanism, we conduct a simple accounting exercise by expressing the aggregate low-skill real wage as a weighted sum of the sectoral low-skill product wages:

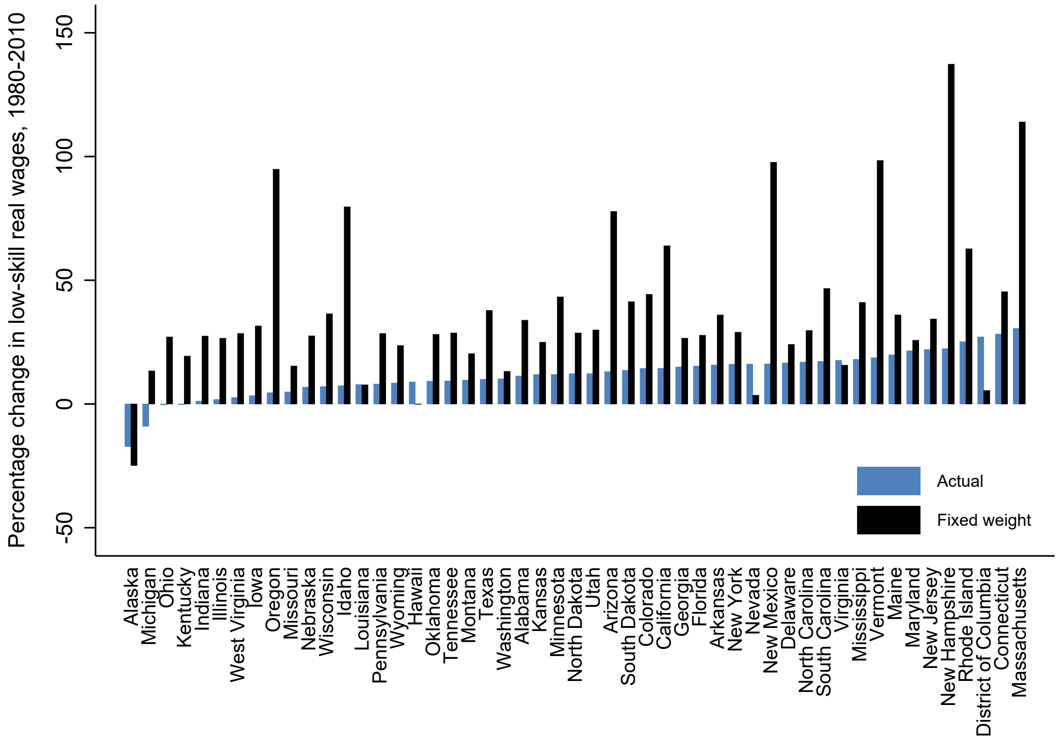
$$\frac{w_l}{P_C} = \sum_j \frac{w_{lj}}{p_j}\alpha_j; \quad \alpha_j \equiv \frac{p_j}{P_C}\frac{L_j}{L}; \tag{1}$$

where w_l is the aggregate low-skill nominal wage, P_C is the aggregate consumption price index, w_{lj} and p_j are the low-skill nominal wage and value-added price in sector j, and the weight α_j is the product of the relative price p_j/P_C and the share of low-skill labor L_j/L in sector j.

Observations 1 and 2 imply that the weights α_j are rising for sectors with slower-growing product wages because of their rising relative prices and hour shares. To see the importance of these changes, Figure 2 reports the percent-

level on state fixed effects and use the residuals scaled up by the average national growth rate.

Figure 2: Actual and Hypothetical Low-skill Real Wage Change



Notes: The figure shows the percentage change in the real wage for each state from 1980 to 2010. Wages are deflated by PCE. The blue bar shows the actual change and the black bar shows the change when the weight α_j on sector j's product wage is fixed for all sector j, see equation (1). Low-skill is defined as education less than a university degree. Composition adjusted wages are calculated as the weighted mean of 216 cells. See Data Appendix A1 for the construction of variables.
Source: BEA Regional Economic Accounts, Census, and ACS.

age change in the aggregate low-skill real wage by state when the weights α_j are fixed against the actual changes. It shows that in almost all states the hypothetical real wage with fixed weight increases more than the actual wage. The median ratio of the hypothetical relative to the actual is 2.5 which is also the ratio for the U.S. as a whole.

Observation 3: The growth of sectoral low-skill hour shares, sectoral prices, and sectoral low-skill product wage are "skill-biased".

Observation 3 is shown using the following simple regression:

$$g_{njt} = \theta s_{nj} + \gamma_n + \gamma_t + \epsilon_{njt}, \tag{2}$$

Table 1: Sectoral Growth and Skill Intensity

	Share of Low-skill Hours		Sectoral Price		Low-skill Product Wage	
	(1)	(2)	(3)	(4)	(5)	(6)
Skill Intensity						
Hours	2.24		4.98		-3.86	
	(0.39)		(0.44)		(0.39)	
Compensation		1.47		3.32		-2.49
		(0.31)		(0.29)		(0.31)

Notes: The table reports the coefficients of the skill intensity variables estimated from equation (2). The dependent variable is the annualized growth rate of the sectoral low-skill hour share in columns (1)-(2), the sectoral value-added price in columns(3)-(4), and the low-skill product wage in columns (5)-(6) in each decade from 1980 to 2010 by state. Sectoral product wage is calculated as nominal wage divided by sectoral value-added price. Skill intensity in hours is calculated as the sample mean of sectoral hours of high-skill divided by total hours in the sector. Skill intensity in labor compensation is calculated as the sample mean of sectoral compensation of high-skill divided by total compensation in the sector. Low-skill is defined as education less than a university degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix A1 for the construction of variables. All specifications include state and decade fixed effects. The number of observations is 1683. Robust standard errors are in parentheses. All reported coefficients are significant at the 1 percent level.

where g_{njt} is the growth rate of low-skill hour shares, prices or low-skill product wages of sector j in state n and decade t; s_{nj} is the long-run skill-intensity of sector j in state n, γ_n and γ_t are state and decade fixed effects that control for state-specific and decade-specific elements affecting the economy-wide growth rates and ϵ_{njt} is the disturbance term. The slope term θ indicates the strength of conditional correlation between the growth rates and skill intensity.

Table 1 reports the estimated θ from equation (2), where the three left-hand side growth variables are regressed on two alternative skill intensity measures based on hours and labor compensation. Columns (1)-(4) show that the growth in both the share of low-skill hours and value-added prices are positively correlated with skill intensity. The growth of low-skill product wages, on the other hand, is negatively correlated with skill intensity measures in columns (5)-(6). In other words, low-skilled workers are reallocating into sectors with higher skill intensity, slower growth in low-skill product wages and rising relative prices.

2.2 Low-skill Wage and Productivity Divergence

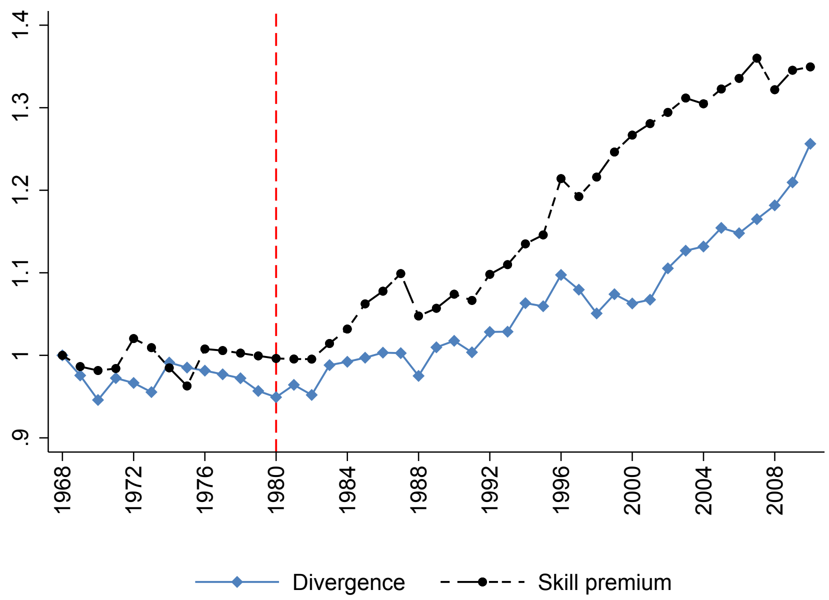
To construct a consistent measure for the divergence between labor productivity growth and the low-skill real wage, we compute the aggregate wages by merging the WORLD KLEMS data on total compensation and hours with the distribution of demographic subgroups in the CPS. The labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. Given the distribution of demographic subgroups is taken from the CPS, the implied relative wage is the same as the CPS.[10] See Data Appendix A1 for details.

Our objective is to understand the low-skill wage stagnation and its divergence from aggregate labor productivity. As shown in Figure 3, these patterns were not present prior to 1980, when the low-skill real wage was growing at about the same rate as the high-skill wage and the aggregate labor productivity. Motivated by Observation 3, we aggregate sectors into two based on the level of skill intensity to examine the potential of our mechanism in in understanding Figure 3. The high-skill intensive sector includes: finance, insurance, government, health and education services, and the low-skill intensive sector includes the remaining industries (see Data Appendix A1).

Figure 4 shows that our mechanism is consistent with the timing of the divergence reported in Figure 3. Figure 4A shows that the relative price and the relative productivity of the high-skill intensive sector were broadly constant prior to 1980. Since then, the relative price of the high-skill intensive sector was rising, mirroring the decline in its relative productivity. Figure 4B shows that the low-skill wages are similar across the two sectors, supporting the view of an integrated low-skill labor market. What is hidden behind the similarity of sectoral low-skill wages is

[10] As in Section 2.1, wages are composition adjusted (age, sex, race and education within high-skill and low-skill). We do not control for occupation for the rest of the analysis because unlike other controls, occupation is a choice variable for the worker. However, the evolution of wages is similar when controls for occupations are included.

Figure 3: The divergence and the rise in skill premium



Notes: Divergence is the ratio of aggregate labor productivity relative to the low-skill real wage. Skill premium is the ratio of the high-skill wage relative to the low-skill wage. Both ratios are normalized to 1 in 1968. Low-skill is defined as education less than a university degree. Composition adjusted wages control for age, sex, race and education within the high-skill and the low-skill. See Data Appendix A1 for the construction of variables.
Source: WORLD KLEMS and CPS.

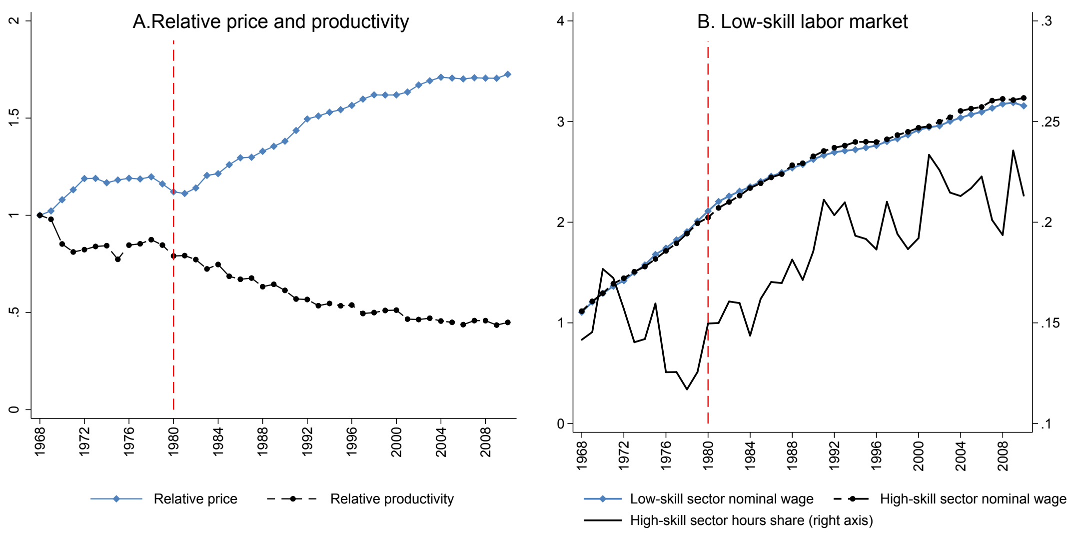
that the sectoral low-skill product wage is growing much slower in the high-skill intensive sector because of the rise in its relative price. These sector-specific trends in the low-skill product wages contribute to the low-skill wage stagnation because of a reallocation of low-skilled workers to the high-skill intensive sector shown in Figure 4B. In contrast, Appendix A1.5 shows that high-skilled workers did not experience the same reallocation.

Finally, we decompose the divergence into the three factors discussed in the Introduction using an accounting relationship between the low-skill wage and the aggregate labor productivity, starting with the definition of the labor income share:

$$\beta y = w, \quad y \equiv \frac{\sum_j p_j Y_j}{M}, \quad w \equiv \frac{\sum_i w_i M_i}{M}, \tag{3}$$

where β is the labor income share, y is the nominal aggregate labor productivity and w is the average nominal wage, p_j and Y_j are the price and real value-added of sector j, and w_i and M_i are the wage and market hours by labor input i. Let P_Y

Figure 4: Relative prices, Relative Productivity and Labor reallocation



Notes: Panel A shows the value-added price and real labor productivity of the high-skill intensive sector relative to the low-skill intensive sector, normalized to one in 1968. Panel B shows the composition adjusted log nominal wages of the low-skilled workers in the high-skill intensive sector and the low-skill intensive sector, and the share of low-skill hours in the high-skill intensive sector. See Data Appendix A1 for the construction of variables and sectors.
Source: WORLD KLEMS and CPS.

be the aggregate output price index, P_C be the consumption price index and M be the total market hours. The ratio of real productivity relative to the low-skill real wage is:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{Divergence} = \underbrace{\left(\frac{w}{w_l}\right)}_{Wage\ Inq} \underbrace{\left(\frac{1}{\beta}\right)}_{Labor\ Share} \underbrace{\left(\frac{P_C}{P_Y}\right)}_{Living\ Cost} \tag{4}$$

It shows that the divergence in the low-skill real wage and productivity is potentially attributable to three factors: (1) a rise in the relative cost of living, which reduces the real value of wage relative to productivity, (2) a decline in labor share, which reduces the share of output that goes to workers and (3) a rise in the wage inequality, measured by the ratio of average wage relative to the low-skill wage. The quantitative decomposition of these three factors depends on the choice of consumption price index. If we use PCE as a measure of P_C, then the decompositions of the three factors are 10%, 20% and 70%. If we use CPI instead, then the decompositions are 30%, 20% and 50%. The main takeaway is that all three factors are quantitatively important for understanding the divergence.

3 The Model

The economy consists of two sectors: the high-skill intensive sector and the low-skill intensive sector. There is a measure H of high-skilled households and a measure $L = 1 - H$ of low-skilled households. Each household is endowed with one unit of time which is supplied to the market inelastically. Household i derives utility from consuming output from both sectors:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi)\, c_{ih}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad i = h, l. \tag{5}$$

The budget constraint is:

$$p_h c_{ih} + p_l c_{il} = w_i, \tag{6}$$

where w_i is the wage of household i.

The representative firm in sector $j = l, h$ uses low-skill labor, high-skill labor, and capital as inputs:

$$Y_j = A_j F_j\left(G_j\left(H_j, K_j\right), L_j\right) \tag{7}$$

$$F_j\left(G_j\left(H_j, K_j\right), L_j\right) = \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[G_j\left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{8}$$

$$G_j\left(H_j, K_j\right) = \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\, H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{9}$$

where H_j and L_j are the high-skill labor and the low-skill labor used in sector j. The parameter ξ_j captures the importance of the low-skill labor in sector j with $\xi_l > \xi_h$. The parameter κ_j measures the importance of capital within the capital-skill composite.

The output of the low-skill intensive sector can be also be converted into $1/\phi$ unit of capital, where ϕ can be interpreted as the price of capital relative to the low-skill intensive goods.[11] The objective of the quantitative exercise is to compare

[11] This two-sector model can be mapped into a three-sector model where the low-skill intensive sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production functions except the sector-specific TFP index. In this environment, the relative price of capital ϕ is equal to the inverse of their relative TFP's, so a fall in ϕ is interpreted as an investment-specific technical change (Greenwood et al., 1997).

the labor market changes from 1980 to 2010 instead of studying the time path. To keep the framework simple, we assume full depreciation of capital. The market clearing conditions for goods, capital and labor are:

$$Y_l = C_l + \phi K, \quad Y_h = C_h. \tag{10}$$

$$K = K_h + K_l. \tag{11}$$

$$H_h + H_l = H; \quad L_h + L_l = L. \tag{12}$$

3.1 Household's Optimization

The optimal decision of household i implies the marginal rate of substitution across the two goods equal to their relative prices:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \tag{13}$$

thus the relative consumption share is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}. \tag{14}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{15}$$

where x_j is the consumption share of good j. Aggregating across households, the aggregate demand for goods C_j satisfies:

$$p_j C_j = x_j (H w_h + L w_l). \tag{16}$$

The aggregate relative demand and relative consumption share are derived as:

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}; \qquad \frac{p_h C_h}{p_l C_l} = x. \tag{17}$$

3.2 Firm's Optimization

The optimal decision of the representative firm implies the marginal rate of technical substitution across any two inputs is equal to the ratio of their relative prices. Across the high-skill labor and capital in sector $j = h, l$, this implies:

$$\frac{H_j}{K_j} = (\chi\delta_j)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \quad \chi \equiv \frac{w_h}{q_k}. \tag{18}$$

Define $\tilde{I}_j$ as the high-skill income relative to the sum of high-skill and capital income:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_k H_j} = \frac{1}{1+\chi^{\rho-1}{\delta_j}^{\rho}}, \tag{19}$$

where the last equality follows from the condition (18). Using the optimal condition across high-skill and low-skill labor, Appendix A2.1.1 shows that the relative skill-intensity in sector $j = h, l$ is:

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta}\,(1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}}\,\tilde{I}_j^{\,\frac{\eta-\rho}{1-\rho}}; \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}, \quad q \equiv \frac{w_h}{w_l}. \tag{20}$$

Define J_j as the income share of the low-skill labor and I_j as the income share of the high-skill labor in sector $j = h, l$:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\,(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{21}$$

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = (1-J_j)\,\tilde{I}_j. \tag{22}$$

Together they imply the total labor income share, see Appendix A2.1.2:

$$\beta_j = I_j + J_j = J_j\left[q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\,(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right], \quad j = h, l. \tag{23}$$

3.3 Equilibrium Prices and Allocation

The equilibrium low-skill wage is equal to the value of marginal product of low-skill labor, which is equal across sectors (see Appendix A2.1.3):

$$w_l = p_j \frac{\partial Y_j}{\partial L_j}; \qquad \frac{\partial Y_j}{\partial L_j} = A_j \left[J_j \xi_j^{-\eta} \right]^{\frac{1}{1-\eta}}, \tag{24}$$

The equilibrium high-skill wage can then be expressed as:

$$w_h = q w_l = p_j A_j q \left[J_j \xi_j^{-\eta} \right]^{\frac{1}{1-\eta}}. \tag{25}$$

The expression of low-skill income share in (21) implies that $J_j^{\frac{1}{1-\eta}}$ is decreasing while $qJ_j^{\frac{1}{1-\eta}}$ is increasing in q. The relative price across sectors is derived from the free mobility of labor:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h} \right) \left(\frac{\xi_l}{\xi_h} \right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h}{J_l} \right)^{\frac{1}{\eta-1}}, \tag{26}$$

which shows that a faster productivity growth in the low-skill intensive sector implies a rising relative price of the high-skill intensive sector.

The optimal conditions so far depend on the relative factor prices $\chi \equiv w_h/q_k$ and $q \equiv w_h/w_l$. As explained in footnote (11), ϕ can be mapped into the price of capital relative to the low-skill intensive goods, $\phi = q_k/p_l$. Thus, the firm's optimal decision on capital implies an equilibrium condition across χ and q :

$$\chi = q \frac{A_l}{\phi} \left(J_l \xi_l{}^{-\eta} \right)^{\frac{1}{1-\eta}}. \tag{27}$$

Substituting J_l in (21), Appendix A2.2.1 derives q as a function of χ:

$$q = \chi \left[\left(\frac{\phi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^{\eta} \left[\left(\chi^{1-\rho} + \delta_l^{\rho} \right) (1-\kappa_l)^{\rho} \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}}, \tag{28}$$

Given q is a function of χ, it follows that sectoral income shares I_j, J_j and $\tilde{I}_j$ are also functions of χ. As shown in Appendix A2.2, the equilibrium of the model can be summarized by solving for χ and the share of low-skill labor in the high-skill sector $l_h \equiv L_h/L$ using two equilibrium conditions:

$$l_h = S\left(\chi; \frac{H}{L}, \frac{\phi}{A_l}\right) \equiv \frac{\frac{H}{L} q\left(\chi; \frac{\phi}{A_l}\right)^{\eta} \sigma_l^{-\eta} \left(1-\kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l\left(\chi\right)^{\frac{\eta-\rho}{\rho-1}} - 1}{\left(\sigma_h/\sigma_l\right)^{\eta} \left(\frac{1-\kappa_l}{1-\kappa_h}\right)^{\frac{\rho(\eta-1)}{1-\rho}} \left(\frac{\tilde{I}_l(\chi)}{\tilde{I}_h(\chi)}\right)^{\frac{\eta-\rho}{\rho-1}} - 1}. \tag{29}$$

$$l_h = D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right) \equiv \left[1 + \frac{J_l\left(\chi; \frac{\phi}{A_l}\right)}{J_h\left(\chi; \frac{\phi}{A_l}\right)} \left(\frac{1}{x\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)\beta_l\left(\chi\right)} + \frac{1-\beta_h\left(\chi\right)}{\beta_l\left(\chi\right)}\right)\right]^{-1}, \tag{30}$$

where the relative consumption share x is derived from (14) and (26) as:

$$x\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right) = \hat{A}_{lh}^{1-\varepsilon} \left(\frac{J_h\left(\chi; \frac{\phi}{A_l}\right)}{J_l\left(\chi; \frac{\phi}{A_l}\right)} \left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\varepsilon}{\eta-1}} \quad ; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{31}$$

In a nutshell, the condition S is derived using the labor market clearing conditions and the firm's optimization. The condition D is derived using the goods market clearing conditions and the household's optimization. These two conditions together solve for (χ, l_h) and the skill premium q is obtained from (28). Given q and χ, the level of wages are derived from (24) and (25) and the income shares are derived from (19), (21) and (22). Appendix A2.3 derives the value-added shares as:

$$v_h \equiv \frac{p_j Y_j}{\sum_j p_j Y_j} = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \quad v_l = 1 - v_h, \tag{32}$$

and the aggregate labor income share is:

$$\beta = \beta_l v_l + \beta_h v_h. \tag{33}$$

3.4 Divergence: Low-skill Wage and Productivity

The accounting identity (4) shows that the divergence of the low-skill real wage from aggregate labor productivity is due to rising wage inequality, falling labor income shares and rising relative cost of living. Using the equilibrium conditions derived above, we now explain how the model can generate these three factors through faster productivity growth in the low-skill intensive sector.

The effect of the faster productivity growth in the low-skill intensive sector (higher $\hat{A}_{lh}$) is summarized in the expression for the relative consumption share (31): it increases the relative price of the high-skill intensive sector in (26) which in turn increases the relative consumption share in (14) given consumption complementarity ($\epsilon < 1$). This implies a higher share of low-skill labor in the high-skill intensive sector, which acts as an endogenous skill-biased shift leading to higher skill premium as in Buera et al. (2020).

The rise in the relative cost of living is measured by the rise in the price of consumption relative to output, P_C/P_Y. These two price indexes can be obtained by the Tornqvist method using the consumption shares x_j as weights for P_C and the value-added shares v_j as weights for P_Y. Given the consumption share of the high-skill intensive sector exceeds its value-added share, the faster productivity growth in the low-skill intensive sector implies a rise in the relative cost of living P_C/P_Y by predicting a rise in the relative price of the high-skill intensive sector.[12]

The effect of uneven productivity growth on the aggregate labor share β in (33) is ambiguous as it predicts opposing effects on the sectoral labor income share β_j in (23): it reduces the low-skill income share in (21) and increases the high-skill income share in (22), and generates a shift towards the high-skill intensive sector.

3.5 Skill Premium and Low-skill Wage Growth

The skill premium measures the high-skill wage relative to the low-skill wage. A rise in the skill premium does not necessarily require a slower growth in the low-skill wage. In a similar vein, factors that imply a rise in the skill premium do not always imply a slower growth in the low-skill wage. Using the optimal capital-skill ratio in (18), the production function can be expressed as a function of the

[12]The assumption that capital is only produced by the low-skill intensive sector helps to simplify the model but what is necessary for the consumption share of the high-skill intensive sector to be larger than its value-added share is that the low-skill intensive sector contributes more to the production of capital.

high-skill and the low-skill labor:

$$Y_j = \tilde{A}_j \left[\lambda_j H_j^{\frac{\eta-1}{\eta}} + (1-\lambda_j) L_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{34}$$

$$\tilde{A}_j \equiv A_j \left(\xi_j + (1-\xi_j) \left(\frac{1-\kappa_j}{\tilde{I}_j} \right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)} \right)^{\frac{\eta}{\eta-1}} ; \ \lambda_j \equiv \frac{(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}{\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \tag{35}$$

which takes a similar form as the aggregate production function used in the literature (see Katz and Murphy, 1992; Heathcote et al., 2010) where an increase in the λ of the aggregate production function represents an aggregate skill-biased shift. Our model provides two *endogenous* sources for this aggregate skill-biased shift.

First, as in Buera et al. (2020), the predicted shift towards the high-skill intensive sector implies an increase in the aggregate λ when $\lambda_h > \lambda_l$. We show in Appendix A1.4 that this between-sector skill-biased shift is an important source for the increase in the aggregate skill intensity for the U.S.. Second, as in Krusell et al. (2000), falling relative price of capital implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies an increase in λ_j acting as a within-sector skill-biased shift in both sectors.

Both shifts imply a rise in the skill premium but they have different effects on the low-skill wage. The between-sector shift induces a shift from the low-skill intensive sector with high $(1-\lambda_l)$ to the high-skill intensive sector with low $(1-\lambda_h)$, so it reduces the aggregate $(1-\lambda)$ contributing to a slow growth in the low-skill wage. The within-sector shift, through rising $\tilde{I}_j$, reduces $(1-\lambda_j)$ in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$ due to the capital-skill complementarity (i.e. $\rho < 1$, see equation (35)). Thus the falling relative price of capital contributes to a rise in the skill premium but not the low-skill wage stagnation.

There are other sources of skill-biased shifts that can be captured by changes in κ_j and ξ_j that increase λ_j. For instance, as a result of automation some tasks performed by low-skilled workers are replaced by machines (Acemoglu and Autor, 2011), or skill-biased organizational change that increases the importance of human capital (Caroli and Van Reenen, 2001).

The skill-biased shifts discussed above can be put into the three classes of technical changes discussed in Johnson (1997). The fall in κ_j is an *intensive skill-biased technical change* which raises the marginal product of high-skill labor without affecting the marginal product of low-skill labor directly, thus it contributes to the skill premium but has little effect on the growth of the low-skill wage. The fall in ξ_j is an *extensive skill-biased technical change* which increases the marginal product of high-skill labor and lowers the marginal product of low-skill labor, thus contributing to both rising skill premium and the low-skill wage stagnation. What is interesting is the rise in A_h and A_l, which are *skill-neutral technical change* at the sectoral level, becomes skill-biased at the aggregate level because of different factor intensities across sectors, affecting both the skill premium and the low-skill wage growth.

3.6 Demand Shift Towards High-skill Goods

In addition to uneven productivity growth, a demand shift towards the high-skill intensive goods can also act as a source for the between-sector skill-biased shift. This demand shift can be induced by a rising income if the high-skill intensive goods have a higher income elasticity. As shown by Comin et al. (2021), a fall in the preference parameter ψ in the homothetic CES utility function (5) can capture this income effect in a more general non-homothetic CES utility function.[13] Thus, by examining the effect of a fall in ψ, we can learn about the effect of a demand shift towards the high-skill intensive sector on the low-skill wage.

Using (31), a fall in ψ implies an increase in $\hat{A}_{lh}$ and a rise in the relative expenditure, thus it has a similar effect on the skill premium as the increase in the relative productivity A_l/A_h. But it does not have a direct effect on relative prices of the high-skill intensive sector as shown in equation (26).[14] Thus, it cannot

[13] This can be seen explicitly from comparing the relative expenditure derived in (17) with the relative expenditure derived from a non-homothetic CES utility function in Comin et al. (2021).

[14] It has an equilibrium effect on the relative price through the rise in q by changing J_h/J_l in

contribute much to the rise in the relative cost of living. Its contribution to the divergence is through the increase in the skill premium, which is similar to the effect of skill-biased shift through ξ_j. Thus, we let the calibration of ξ_j pick up its role as skill-biased demand shift.

4 Quantitative Results

We calibrate the model to match key features of the U.S. from 1980 to 2010. To evaluate the quantitative role of uneven productivity growth, the baseline also includes changes in the relative price of capital, the production weights of the low-skill labor and the high-skill labor, and the relative supply of the high-skill labor. The productivity growth parameters are calibrated to match the rise in the relative price of the high-skill intensive sector and the aggregate labor productivity growth. The input weights of the production function are set to match the sectoral income shares while the relative supply of high-skill labor is set to match the aggregate income of the high-skill labor relative to the low-skill labor.

The predictions in the baseline are driven by changes in five sets of parameters: $\hat{A}_{lh}$ in equation (31), the relative price of capital ϕ, the production weights $\{\xi_l, \xi_h, \kappa_l, \kappa_h\}$ in the production function (7) and the relative supply of the high-skill labor H/L.[15] To evaluate their contribution, we perform counter-factual exercise by setting each parameter to its 1980 value.

4.1 Data Targets

The data targets reported in Table 2 are constructed using the two-sector data described in Section 2.2, see Data Appendix A1 for details. Data from the five-year average 1978-1982 is used for 1980 and 2006-2010 for 2008. It shows that the high-skill income share (I_j) increases while the low-skill income share (J_j) falls in both sectors. The total labor income share ($\beta_j = I_j + J_j$) falls in the low-skill

(26) but the effect is small as it depends on the difference between the parameters ξ_h and ξ_l as shown in (21).

[15] Given the definition of $\hat{A}_{lh}$ in equation (14), we do not need to separate the preference parameter ψ from A_l/A_h to solve for the model.

Table 2: Calibration Data Summary

	Level							Growth (% p.a.)		
	J	J_h	J_l	I	I_h	I_l	q	y/P_Y	$\frac{p_h}{p_l}$	ϕ
1980	0.41	0.23	0.46	0.17	0.33	0.12	1.44	-	-	-
2008	0.28	0.21	0.31	0.28	0.44	0.21	1.94	1.7	1.4	-0.5

Note: J's are the low-skill income share, I's are the high-skill income share, q is the skill premium. High-skill are those with college degree. y/P_Y is the aggregate real labor productivity, p_h/p_l is the price of high-skill intensive sector relative to the low-skill intensive sector and ϕ is the price of capital relative to the low-skill intensive sector.

intensive sector, rise in the high-skill intensive sector, and falls for the overall economy. The annual growth rate of the aggregate real labor productivity is 1.7% and the relative price of the high-skill intensive sector is 1.4%. Using the ratio of P_K/P_Y from the BEA and the ratio P_Y/p_l from the KLEMS, the implied price of capital relative to the low-skill intensive sector ϕ declines at 0.5% per year.[16]

4.2 Calibration Procedures

The elasticity of substitution across high-skill and low-skill labor $\eta = 1.4$ is taken from Katz and Murphy (1992) and the elasticity of substitution across capital and high-skill labor $\rho = 0.67$ is taken from Krusell et al. (2000). There is no direct estimate of the elasticity of substitution across the high-skill intensive and the low-skill intensive goods, ϵ. The literature on the structural transformation finds that the elasticity of substitution across agriculture, manufacturing, and services is close to zero (Herrendorf et al., 2013). Given we re-group these three sectors into two sectors, this is likely to imply a higher degree of substitution. The equilibrium condition (13), on the other hand, implies that the own-price elasticity of the two goods is $-\varepsilon$. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services ranging from -0.3 to 0, this is informative but not an exact estimate for $-\varepsilon$ which is the price elasticity of the high-skill intensive sector in

[16] It is worth noting that the growth of P_Y in KLEMS is growing at 2.94% which is almost identical to that of BEA at 2.86%.

our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value for the elasticity of substitution across the two sectors.

The income share of the high-skill labor relative to the low-skill labor is:

$$\frac{I_t}{J_t} = \frac{w_{ht} H_t}{w_{lt} L_t} = q_t \frac{H_t}{L_t}, \tag{36}$$

which implies a value for the relative supply of the high-skill labor given data on the skill premium and income shares (q_t, I_t, J_t) reported in Table 2.[17]

Given a value for ϕ/A_l, equation (27) can be used together with the equations on income shares (21)-(22) to set the input weights ξ_j, κ_j to match sectoral income shares in the data. To simplify the explanation, denote 1980 as period 0 and 2008 as period T. We normalized $\phi_0/A_{l0} = 1$, this pins down all weights in period 0 (see Appendix A3.1). Using these parameters, condition (29) implies a value of l_{h0}, and condition (30) implies a value of $\hat{A}_{lh0}$ given q_0.

For a given level of A_{lT}/A_{l0}, data on the fall in ϕ_t implies a value for ϕ_T/A_{lT}, which pins down all input weights in period T. We then set the change in A_{lhT}/A_{lh0} to match the increase in the relative price of the high-skill intensive sector. Finally, we adjust A_{lT}/A_{l0} to match the change in the aggregate labor productivity deflated by the price of the low-skill intensive sector, y/p_l.

4.3 Calibrated Parameters

Table 3 reports the calibrated parameters. The implied annual growth of ϕ, A_{lh}, A_l, ζ and input weights (κ_j, ξ_j) are reported in Panel B of Table 3.[18] Matching the rise in the relative price of the high-skill intensive sector implies faster productivity growth in the low-skill intensive sector.[19] Matching the relative aggregate income

[17] The H_j and L_j are not the raw market hours by the high-skill and low-skilled workers in the data. The composition adjusted high-skill hours H_j in sector j is computed as high-skill income in sector j divided by the composition adjusted high-skill wage, similarly for L_j.

[18] The implied negative growth in κ_j does not necessarily mean a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

[19] The calibration implies that A_h is falling, which can be understood using the findings of Aum et al. (2018) and Bárány and Siegel (2021). The former paper finds negative productivity growth for the high-skill occupations (Professional and Management) while the latter finds negative growth for the abstract occupation. Their findings could be the source for the falling A_h given these occupations are concentrated in the high-skill intensive sector.

Table 3: Parameters of Calibration

A. Parameters from the literature		
Parameters	Values	Source
ε	0.2	Benchmark value, see main text
ρ	0.67	Krusell et al. (2000)
η	1.4	Katz and Murphy (1992)

B. Calibrated parameters				
Parameters	1980	2010	Growth (% p.a.)	Target
ϕ			-0.50	Price of capital relative to the low-skill sector
A_l			1.09	Aggregate real labor productivity
A_{lh}			1.82	Relative price of the high-skill sector
ξ_l	0.33	0.25	-0.93	Sectoral income share. See Appendix A3.1
ξ_h	0.20	0.19	-0.13	Sectoral income share. See Appendix A3.1
κ_l	0.74	0.69	-0.21	Sectoral income share. See Appendix A3.1
κ_h	0.41	0.33	-0.79	Sectoral income share. See Appendix A3.1
ζ	0.29	0.50	1.92	Relative aggregate labor income shares

shares of the high-skill and low-skill labor implies a rise in the relative supply of high-skill labor. Matching the sectoral income shares, on the other hand, requires changes in the input weights reflecting other sources of skill-biased shifts.

Using the calibrated parameters, the model delivers predictions on wages, allocation of labor, relative prices, and labor productivity for each sector. As shown in Appendix Table A3, the baseline does a good job on predicting labor reallocation: it accounts for 86% of the sectoral reallocation of the low-skill labor and the constant sectoral share for the high-skill labor observed in the data. Consistent with the data, it predicts a fall in the labor income share in the low-skill intensive sector and a rise in the labor income share in the high-skill intensive sector, and a decline in the aggregate labor income share. The uneven productivity growth is crucial for the low-skill labor reallocation and the rise in the value-added share of the high-skill intensive sector. The fall in ξ_j, on the other hand, is needed for the lack of the high-skill labor reallocation and the fall in the labor income share in the low-skill intensive sector. The baseline implies the labor productivity growth is 2.2% for the low-skill intensive sector and -0.2% for the high-skill intensive sector, matching the 2.3% and 0.1% observed in the data almost perfectly. [20]

[20] The sectoral real labor productivity growth in the model, y_j/p_j, is equal to $\frac{Y_j}{L_j+H_j} = A_j \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{1+H_j/L_j}\right)$.

Consistent with previous literature, Table A3 shows that all mechanisms are important for the baseline to match the rise in the skill premium almost perfectly.[21] The new findings here are their roles on the wage-productivity divergence and the growth of the low-skill real wage, which are different from their roles on the skill premium, as discussed in Section 3.5.

4.4 Predictions on Wage-Productivity Divergence

Table 4 presents the results on the wage-productivity divergence and its decomposition into the three factors: the wage inequality, the aggregate labor share and the relative cost of living, as in the accounting equation (4). Since KLEMS data does not contain information on consumption, we take P_C/P_Y as the ratio of PCE and GDP implicit deflators from the BEA. This implies P_C/P_Y increased by 2.8%. If we were to use CPI, the increase in P_C/P_Y would be at 11.5%. This alternative value will imply a larger divergence and a slower real wage growth in the data row but does not affect other rows. Due to the concern that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we use the P_C/P_Y implied by PCE deflator as the main data moment for comparison but keep those implied by CPI in bracket.

Row 1 of Table 4 provides an empirical decomposition for the accounting identity in equation (4). During this 30-year period, the negative forces imposed by the rising relative cost of living, growing wage inequality, and falling aggregate labor income share largely offset the impact of rising productivity on the low-skill real wage. The rise in the relative cost of living contributes to 10%(=2.8/27) of the divergence, the increase in the wage inequality contributes to 70% (=19/27) and the fall in the aggregate labor income share accounts for the remaining 20%.[22] If CPI is used, the contribution of the relative cost of living increases to 30% while

[21] More specifically, row 3 on uneven productivity growth is related to Buera et al. (2020), row 4 on the fall in the relative price of capital is related to Krusell et al. (2000), and row 5 and 7 are related to the skill-biased demand shift and supply shift in Katz and Murphy (1992).

[22] The literature on the average wage and productivity divergence often uses the nonfarm business sector. In Appendix A1.3 we conduct the empirical decomposition for the accounting identity in equation (4) using similar data.

Table 4: Divergence of low-skill real wage and real labor productivity, (percentage change, 1980-2008)

			Factors for Divergence		
		Divergence	Wage Inequaity	Labor Share	Living Cost
(1)	Data	27 (38)	19	-3.4	2.8 (12)
(2)	Model	34	19	-3.7	8.2
		Counterfactual (fixing each parameter to its 1980 value)			
(3)	A_l/A_h	19	15	-5.8	-2.1
(4)	ϕ	28	12	-6.1	7.1
(5)	ξ_j	11	6.4	4.7	8.9
(6)	κ_j	37	12	-9.9	9.7
(7)	H/L	46	36	2.3	9.9

Note: Divergence is measured as the percentage change in the ratio of real labor productivity divided by the low-skill real wage. The three factors for the divergence are shown in equation (4). For the data row, the low-skill real wage is calculated using PCE as P_C and the number in bracket uses CPI as P_C. Living cost is the price of consumption relative to output, P_C/P_Y. Row 3 is the relative productivity. Row 4 is the relative price of capital. Row 5 is the input weight of the low-skill labor in (7). Row 6 is the weight of capital in (9). Row 7 is the relative supply of the high-skill labor.

the contribution of the rise in wage inequality reduces to 50%.

The baseline (row 2) can account for all the rise in wage inequality and the fall in the aggregate labor share. It over-predicts (under-predicts) the relative cost of living, thus slightly over-predicts (under-predicts) the divergence, if PCE (CPI) is used as consumption deflator. The remaining rows of Table 4 examine each of the five forces that drives these changes by shutting them down one at a time: the uneven sectoral productivity growth (higher A_l/A_h) in row 3, the falling relative price of capital (lower ϕ) in row 4, the falling input weight on the low-skill labor (lower ξ_j) in row 5, the rising input weight on the high-skill labor within the capital-skill composite (lower κ_j) in row 6 and the increase in the relative supply of the high-skill labor (higher H/L) in row 7.

Rows 3 - 7 of Table 4 show that faster productivity growth of the low-skill intensive sector (row 3) and the falling production weights of the low-skill labor (row 5) are the two most important factors for the divergence. In their absence, the predicted divergence would be reduced to almost a half and a third of the baseline respectively. Their contribution are through different channels. The faster productivity growth in the low-skill intensive sector contributes mainly through predicting a higher wage inequality and a rise in the relative cost of living. The falling production weights of the low-skill labor contribute to the divergence by predicting a fall in the labor share and a rise in the wage inequality.

The increase in the relative supply of the high-skill labor in row (7) plays an important role for the wage inequality. In its absence, the increase in the wage inequality would have been doubled but the labor share would have increased.[23] The latter offsets some of the rise in the divergence implied by the higher wage inequality. The falling relative price of capital in row 2 also contributes to the divergence by predicting a rise in the wage inequality. Finally, the increasing weight of the high-skill labor through falling κ (row 6) has an insignificant impact on the divergence. It implies a rise in the wage inequality and a rise in the labor share, resulting in opposite effects on the divergence.

4.5 Predictions on Wage Stagnation

While Table (4) shows that all parameters (except κ_j) are important for the divergence, Table (5) reveals that only the faster productivity growth of the low-skill intensive sector (row 3) and the falling production weights of the low-skill labor (row 5) are responsible for the low-skill wage stagnation. In their absence, the percentage increase in the low-skill real wage would have been more than double.[24]

The key difference between row 3 and 5 is their different implications for the marginal product of the low-skill labor (MPL) in the model, which is equal to the low-skill product wage (w_l/p_j). In the data, the low-skill product wage rose by 44% in the low-skill intensive sector but fell in the high-skill intensive sector

[23]Its impact on the labor share is due to the capital-skill complementarity, where higher relative supply of the high-skill labor increases the capital income share.

[24]Interestingly, this finding that the percentage change in the low-skill real wage would be 2.75 (=55/20 from row 2 and 3) times higher in the absence of uneven productivity growth is close to the 2.5 implied by equation (1) for the U.S. reported in Section 2.1. Intuitively, this is because both methods are computing the hypothetical change in the low-skill real wage if there is no change in relative prices or the low-skill hour shares across sectors.

Table 5: Productivity and Wages, Cumulative Percentage Change, 1980-2008

		Productivity	Low-skill real wage	Low-skill product wage Sector l	Low-skill product wage Sector h	Rel. price p_h/p_l
(1)	Data	60	26 (16)	44	-3.4	49
(2)	Model	matched	20	44	-3.4	matched
	Counterfactual (fixing each parameater to its 1980 value					
(3)	$\boldsymbol{A_l/A_h}$	**85**	**55**	**48**	**66**	**-11**
(4)	ϕ	45	13	33	-6.2	41
(5)	$\boldsymbol{\xi_j}$	**65**	**49**	**84**	**17**	**57**
(6)	κ_j	68	23	51	-3.2	56
(7)	H/L	37	-6.1	18	-28	63

Note: For the data row, the low-skill real wage is calculated using PCE as P_C and the number in bracket is when CPI is used as P_C. Low-skill product wage is measured as the nominal wage divided by the sectoral value-added price. Row 3 is the relative productivity. Row 4 is the relative price of capital. Row 5 is the input weight of the low-skill labor in (7). Row 6 is the weight of capital in (9). Row 7 is the relative supply of the high-skill labor.

due to the rise in the relative price of the high-skill intensive sector. The uneven productivity growth is the main mechanism to deliver this result. In its absence, the low-skill MPL would have increased more in the high-skill intensive sector. Another difference between the two mechanisms is the timing. As we discussed in Section (2.2), the mechanism through uneven productivity growth is consistent with the beginning of the low-skill wage stagnation and its divergence from the aggregate productivity starting in 1980. On the other hand, since the production weights of the low-skill labor (ξ_l, ξ_h) are determined by the low-skill income shares (J_l, J_h), they were falling throughout 1968-2010 (see Appendix Figure A5).[25]

Finally, row 4 of Table 5 confirms the discussion in Section 3.5 that the falling relative price of capital boosts the growth in the low-skill real wage by increasing the growth of the low-skill MPL in both sectors. Thus, even though it contributes to the divergence and the wage inequality, it does not contribute to the low-skill wage stagnation. Finally, the rise in the relative supply of the high-skill labor (row

[25]More specifically, using the five-year average 1968-1972 for 1970 as in Table 2, J_l fell from 0.50 to 0.46 while J_h fell from 0.28 to 0.23 during 1970-1980.

7) increases the growth of the low-skill real wage by increasing the low-skill MPL in both sectors. In its absence, the low-skill real wage would have fallen.

4.6 Role of Multisector

Row (3) of Table (4) and (5) shut down the sector-specific productivity growth channel in the multisector model. This, however, is not the same as having a one-sector model because production parameters ξ_j and κ_j are still different across the two sectors. We now derive the quantitative results for a one-sector model to highlight the importance of having a multisector model. Dropping all subscript j, the aggregate production function is the same as (7 – 9).

The household decision problem in the one-sector model implies households spend all wage income on the final goods. The firm's problem is the same as before but there is only one goods market clearing condition (10) which specifies how the final goods is converted into consumption and capital goods. The calibration follows the same calibration procedure as before. The full derivation and calibration of the one-sector model are described in the Appendix (A3.4).

The baseline result of the one-sector model is reported in row (2) of Table (6). Given the relative supply of the high-skill labor was set to match the aggregate high-skill income share relative to the low-skill income share as in equation (36), and the production weights are set to match the levels of the two income shares, together they imply the skill premium and the labor shares are also matched. Given the price of consumption and the price of output are the same in the one-sector model, it follows from the accounting equation (4) that the one-sector model under-predicts the divergence. Appendix Table (A9) reports the roles played by ϕ, ξ, κ and H/L as in Table (4) and (5).

The remaining rows of Table (6) demonstrate an important message of a multisector model regarding the source of productivity growth. The baseline of both models were calibrated to match the increase in the real labor productivity by setting the TFP parameters A's in the two models. In the multisector model, an increase in aggregate productivity could be due to either an increase in A_l or A_h.

Our claim is that the source is important. The quantitative exercise is conducted by setting the A's in the two models so that they imply a higher increase in the aggregate labor productivity – at 70% instead of the observed 60%.

In the one-sector model, this requires setting a higher growth in A compared to the baseline. Not surprisingly, as shown in row (3), the one-sector model predicts a similar increase in both aggregate labor productivity and low-skill real wage, nothing changes in the divergence. In the multisector model, however, the source of productivity growth is important. Comparing rows (5) - (7) to the baseline, all sources of productivity growth have a positive effect on low-skill real wage, but their extent are different.

If the growth is due to a balanced increase in the sectoral TFP (row 5), then we have similar results as in the one-sector model: aggregate labor productivity and low-skill real wage increase *almost* equally, resulting in similar divergence as in the baseline. If the source is due to an increase in the TFP of the high-skill intensive sector (row 6), it predicts a much smaller divergence relative to the baseline because it implies a larger increase in the low-skill real wage. In contrast, if the source is the TFP growth of the low-skill intensive sector (row 7), it predicts a larger divergence relative to the baseline because of a smaller rise in the low-skill real wage.

4.7 Sensitivity Analysis

Appendix A3.3 provides sensitivity analysis with alternative values for the elasticity parameters (ϵ, η, ρ) and a smaller rise in the relative price of the high-skill intensive sector. The increase in the relative price of the high-skill intensive sector is key to the quantification of the uneven productivity growth. If the relative price growth is over-estimated, for example, because of the quality improvement not properly accounted for, then the increase in P_C, P_Y and p_h/p_l in the data are biased upwards. The upward bias on the increase in P_C and P_Y implies an under-estimation of the growth of the low-skill real wage and the aggregate labor productivity in the data. The upward bias on p_h/p_l implies an overestimation of

Table 6: Role of multisector, Percentage Change, 1980-2008

		Agg. productivity	Low-skill real wage	Divergence
(1)	Data	60	26 (16)	27 (38)
		One-sector Model		
(2)	Baseline	matched	29.3	23.6
(3)	Increase in A	70	37.5	23.6
		Multisector Model		
(4)	Baseline	matched	19.7	33.5
(5)	Increase in A_h and A_l	70	26.8	34.1
(6)	Increase in A_h	70	33.2	27.6
(7)	Increase in A_l	70	25.6	35.6

Note: For the data row, the divergence and low-skill real wage are calculated using PCE as P_C and the number in bracket is when CPI is used as P_C. For rows (3), (5)-(7) the increases in A's are set so that aggregate real labor productivity increased by 70% instead of the observed 60%. Row 5 keeps A_l/A_h the same as in the baseline.

the strength of uneven productivity growth in the model. The literature finds a substantial amount of inflation bias comes from goods (Boskin et al., 1996; Bils and Klenow, 2001; Gordon, 2006) but admittedly it is also difficult to correctly measure prices of services, whether they are high-skill intensive (education and health) or low-skill intensive. For the benefit of doubt, we present the quantitative performance of the model under an alternative scenario in Appendix A3.3.4, where we halved the increase in the relative price of the high-skill intensive sector. In this case, the implied growth rate of low-skill real wage in the data is slightly higher but still less than half of the increase in the aggregate labor productivity growth. The overall performance of the baseline model is similar, the contribution of the uneven productivity growth is smaller but remains quantitatively important.

5 Conclusion

Despite working mostly in sectors with fast productivity growth, the low-skill real wage is growing slowly and lagging behind the aggregate labor productivity. We argue that this is due to the changing relative prices driven by faster productivity growth in the low-skill intensive sectors.

A key message of our multisector perspective is that the source of the aggregate labor productivity growth is important. If it is originated in the low-skill intensive sectors, it will contribute to the divergence of the low-skill real wage and the aggregate labor productivity. If it is originated in the high-skill intensive sector or due to a balanced increase in both sectors, then it can boost the growth of the low-skill real wage and the aggregate labor productivity simultaneously.

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Appendix

A1 Data Appendix

A1.1 Industry Data

A1.1.1 National level data

The main industry data is the March 2017 Release of the Unites States data from the WORLD KLEMS database (Jorgenson et al., 2017), which reports industry value-added, price indexes, labor compensation, and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S.

To classify sectors into the high-skill intensive sector and the low-skill intensive sector, we use April 2013 Release of the US data from the WORLD KLEMS (Jorgenson et al., 2013) which provides a labor input file that allows the computation of the low-skilled and the high-skilled workers' shares in labor compensation and value-added. High-skill is defined as education greater than or equal to college degree. Table A1 reports the long-run (1980-2010) average of the high-skill share in the total value-added and total labor income for 15 one-digit industries. For a sector to be included in the high-skill intensive sector, we require that the long-run high-skill labor income share out of the total labor income and the total value-added to be jointly above the total economy average. The high-skill intensive sector includes finance, insurance, government, health and education services (code J ,L, M, N), and the remaining industries are grouped into the low-skill intensive sector.

Using this classification we map the 65 NAICS industries of the KLEMS 2017 Release and the three-digit *ind1990* codes of the CPS into the two broad sectors for our quantitative analysis. Sectoral value-added prices are calculated as Tornqvist indexes, where value-added shares are used as weights. For the ratio of aggregate consumption price deflator and output price deflator, we use the BEA's implicit

Table A1: High-Skill Income Shares by Industry, 1980-2010 average

		High-skill share in	
Industry	Code	Value-added	Labor income
Agriculture, Hunting, Forestry and Fishing	AtB	10	19
Mining and Quarrying	C	11	32
Total Manufacturing	D	20	31
Electricity, Gas and Water Supply	E	9	30
Construction	F	14	16
Wholesale and Retail Trade	G	22	30
Hotels and Restaurants	H	14	18
Transport and Storage and Communication	I	16	25
Financial Intermediation	**J**	**33**	**55**
Real Estate, Renting and Business Activity	K	21	55
Public Admin	**L**	**29**	**40**
Education	**M**	**58**	**77**
Health and Social Work	**N**	**39**	**49**
Other Community, Social and Personal Services	O	23	31
Private Households with Employed Persons	P	16	16
All Industries	TOT	25	40

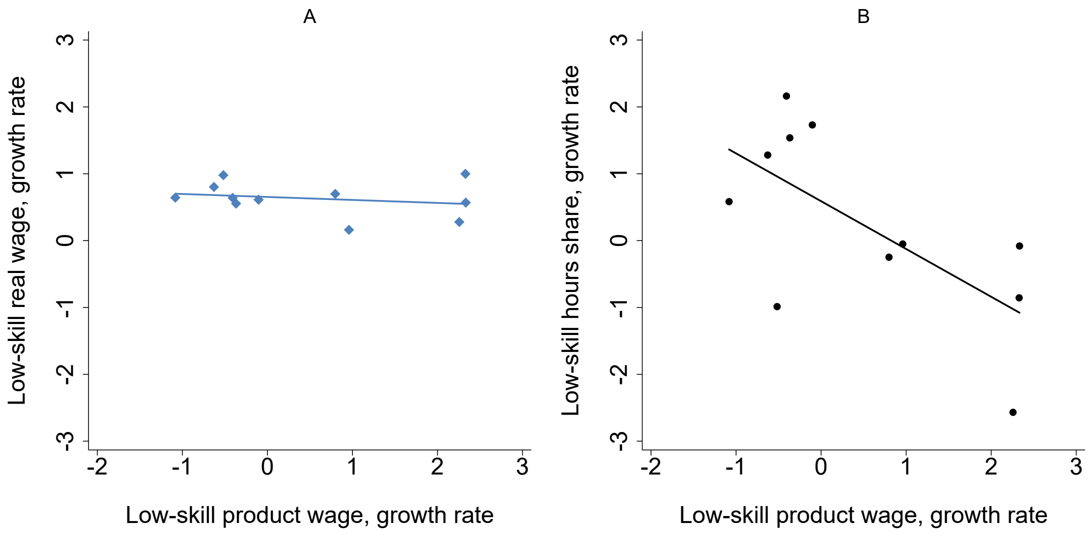
Notes: The table reports the share of high-skilled workers in total value-added and total labor income by industry. High-skill is defined as education greater than or equal to college degree. Labor income reflects total labor costs which includes compensation of employees, compensation of self-employed, and taxes on labor.
Source: April 2013 Release of the WORLD KLEMS for the U.S.

price deflators of GDP and Personal Consumption Expenditures, respectively. The price of capital is calculated as the investment in total fixed assets divided by the chain-type quantity index for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts).

Figure A1 shows a version of Figure 1 for the US economy as a whole. Due to the low number of observations in CPS we merge agriculture (AtB) with mining (C) and other services (O) with private households (P). We also regroup public administration (L), education (M), and health and social work (N) as a single industry to ensure consistency in industry definitions.[26] Our mapping across KLEMS 2013, KLEMS 2017, and CPS industries is provided in Table A2.

[26]For instance, public education is included in the general government industry in KLEMS 2017, while it is part of education in KLEMS 2013.

Figure A1: Growth in Low-skill wages and Hour Shares, National Data



Notes: Each panel includes data on growth rates for 11 sectors in the U.S.. Growth rates between 1980 and 2010 are annualized. The growth of sectoral real wages on the left panel and the growth of sectoral low-skill hour shares on the right panel are plotted against the growth of low-skill product wage. Sectoral real wage is calculated as sectoral nominal wage divided by the PCE price index. Sectoral product wage is calculated as sectoral nominal wage divided by sectoral value-added price. Low-skill is defined as education less than a university degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Appendix A1.2 for the construction of wages.
Sources: CPS, WORLD KLEMS, and authors' calculations

A1.1.2 State-level data

We use GDP by state from the BEA's Regional Economic Accounts for value-added sector prices at the state-level. BEA reports nominal and real GDP (chained at constant dollars) by industry for 51 states by SIC between 1963-1997, and by NAICS between 1997-2010. In order to calculate sectoral prices, we first aggregate the industry data into 11 sectors according to Table A2. Next, using the common year of observation 1997, we carry forward the SIC-based series by the growth rates of the NAICS-based series. Finally, we calculate sectoral price indexes as the ratio of nominal to real GDP. Our bridging strategy produces national sectoral growth rates similar to those reported in the KLEMS data. In particular, the correlation coefficients between the long-run US-level sectoral growth rates from both sources are 0.97, 0.91, and 0.90 for nominal value-added, real value-added, and prices, respectively.

A1.2 Wages, Efficiency Hours, and Productivity

For Figure A1, we use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012 (Ruggles et al., 2017). Our sample includes wage and salary workers with a job aged 16-64, who are not student, retired, or in the military. Hourly wage is calculated as annual wage income divided by annual hours worked, where the latter is the product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars (based on PCE price index) are dropped. Our treatment of Census for years 1980, 1990, 2000, and ACS for 2010 in Section 2 follows the same steps except that wages lower than the first percentile are set to the value of the first percentile following Autor and Dorn (2013).

The composition adjusted mean wages of low-skilled workers used in Section 2 are computed as follows. Within each sector, we calculate mean wages weighted by survey weights for each of 216 subgroups composed of two sexes, white and non-white categories, three education categories (high school dropout, high school graduate, some college), six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years), and three occupation categories (high-wage occupations including professionals, managers, technicians, and finance jobs, middle-wage occupations including clerical, sales, production, craft, and repair jobs, operators, fabricators, and laborers, and low-wage occupations including service jobs). The long-run hour shares of each subgroup are used as weights to calculate the low-skill wage at the industry level. Cells containing missing wages are imputed for each year of the dataset using a regression of the log of hourly wages on industry dummies and dummies including the full set of interactions of subgroups. We assign predictions from this regression to the missing wage observations while keeping the observed wages. The growth rate of sector wages with and without imputation are very close.

For the quantitative analysis used in Table 4 and 5, the aggregate wage has to be consistent with the measure of aggregate productivity, so we use the aggregate labor compensation and aggregate hour from the KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skilled worker and the average low-skilled worker, we merge KLEMS 2013 data on total labor compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sex, two race, five education, six age categories. Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours worked of each subgroup is calculated in a similar way. The wage for each subgroup is then calculated as total compensation divided by total hours. The aggregate low-skill and high-skill wages are calculated as the average of the relevant subgroups using their long-run (1980-2010) hour shares as weights. It is important to note that the labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescale the CPS total hours and total wage income to sum up to KLEMS total.

Efficiency hours, corresponding to (H, L) in the model, are computed as the labor compensation divided by composition-adjusted wage for high-the skilled worker and the low-skill workers respectively. Total efficiency hours are the sum of low- and high-skill efficiency hours. We calculate real labor productivity as total value-added divided by total efficiency hours and deflate with the output price index.

Table A2: Industry Mapping

NACE (KLEMS 2013)	NAICS (KLEMS 2017)	IND1990 (CPS)
AtB & C	Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining	Agriculture, Forestry, and Fisheries, Mining
D	Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products	Manufacturing
E	Utilities	Utilities
F	Construction	Construction
G	Wholesale Trade, Retail Trade	Wholesale Trade, Retail Trade
H	Accodomation, Food Services and Drinking Places	Hotels and Lodging Places, Eating and Drinking Places
I	Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services	Transportation, Communications
J	Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles	Finance, Insurance
K	Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services	Real Estate, Business Services, Professional Services*
L & M & N	Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises	Public Administration, Education*, Health and Social Services*
O & P	Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government	Sanitary and Personal Services, Private Households, Entertainment and Recreation Services, Museums, Art Galleries, and Zoos, Labor Unions, Religious Organizations, Membership Organizations, n.e.c.

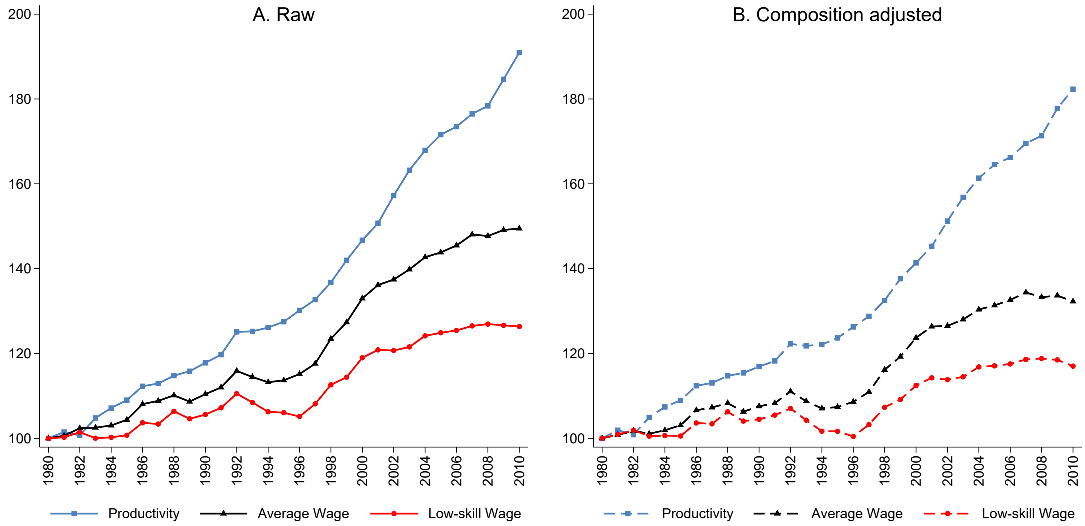
Notes: The table shows the mapping of KLEMS 2013 industries to KLEMS 2017 and CPS industries. The description of KLEMS 2013 industries is provided in Table A1. Industries marked with * do not have separate sections in CPS industry classification. They are constructed as follows. Professional Services: Engineering, architectural, and surveying services, Accounting, auditing, and bookkeeping services, Research, development, and testing services, Management and public relations services, Miscellaneous professional and related services, Legal services. Education: Elementary and secondary schools, Colleges and universities, Vocational schools, Educational services, n.e.c. Health and Social Services Offices and clinics of physicians, Offices and clinics of dentists, Offices and clinics of chiropractors, Offices and clinics of optometrists, Offices and clinics of health practitioners, n.e.c., Hospitals, Nursing and personal care facilities, Health services, n.e.c., Job training and vocational rehabilitation services, Child day care services, Family child care homes, Residential care facilities, without nursing, Social services, n.e.c.

A1.3 Divergence in the BLS Nonfarm Business Data

Bureau of Labor Statistics (BLS) nonfarm business data is often used to discuss the wage-productivity divergence (e.g. Lawrence and Slaughter, 1993; Lawrence, 2016; Stansbury and Summers, 2017), which we use to compare to findings in 2. In order to compute wages at skill-level that are consistent with the BLS productivity series' hourly compensation growth, the share of annual wage income and total hours of 120 demographic groups from March CPS are used. Demographic groups are based on six age, two gender, two race, and five education categories. Compensation (hours) for each subgroup is calculated as compensation (hours) share times BLS total compensation (hours). BLS-consistent wages for each subgroup is calculated as total compensation divided by total hours. Average and low-skill wages are then calculated as the mean wages of relevant subgroups weighted by their hour shares. This is the same procedure as before with two exceptions. First, we exclude agriculture, private households, and public administration sectors to comply with nonfarm business sector. Second, aggregate labor income and hours are rescaled to those of nonfarm business sector. For real wages Personal Consumption Expenditure price index (PCE) is used as the wage deflator. Composition adjusted hours are calculated for each skill as the total compensation divided by composition adjusted wages. The average wage for all workers is calculated as total compensation divided by total composition adjusted hours of the nonfarm business sector. Real labor productivity is the nonfarm business nominal output divided by nonfarm total composition adjusted hours and deflated by the output price deflator from BLS.

Figure A2 plots the raw and composition adjusted low-skill real wage, average real wage, and real labor productivity. From 1980 to 2010, the low-skill wage growth is around 25 percent which shrinks just below 20 percent when adjusted for compositional changes. These figures are slightly lower from those suggested by KLEMS (Table 5) and somewhat higher than those calculated directly from CPS. The former difference stems from the industry coverage that particularly affects growth rates in labor income, which is lower in the nonfarm business sector. Hours

Figure A2: Divergence in the BLS Nonfarm Business Sector Data



Notes: Raw (composition adjusted) wage and hours are used in Panel A (B). Real labor productivity is from the Bureau of Labor Statistics (BLS). Real hourly wages are calculated by merging hours and income shares in the Current Population Survey (CPS) with the total hours and labor income in BLS. Productivity is deflated by the output price index. Wages are deflated by Personal Consumption Expenditure (PCE) price index. All series are normalized to 100 in 1980. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 120 demographic groups (see Appendix A1.3).
Source: BLS nonfarm business sector multifactor productivity statistics, CPS, and authors' calculations.

grow at the same rate in both. On the contrary, the latter difference, i.e. slower wage growth in CPS, is driven by the stronger growth in CPS hours compared to those in the macro sources, despite a bit higher growth in CPS wage income.[27]

As shown in Figure A2, the low-skill real wage growth is less than a quarter of the labor productivity growth, suggesting a higher real divergence than what is calculated from KLEMS. The reason for a higher divergence is partly due to a greater decline in the labor share of nonfarm business (7 percent as opposed to 3.4 in KLEMS)and a larger increase in the BLS nonfarm business output deflator compared to the BEA's output deflator.

[27]See Stewart and Frazis (2019) for an up-to-date discussion on the hours estimated by CPS and other BLS measures. Although total annual hours estimated from CPS is seen as problematic, authors recommend the use of CPS for comparing hours across demographic groups, which is consistent with our data approach.

A1.4 Shift-share Analysis of Aggregate Skill Intensity

To show the importance of the sectoral reallocation for the rise in the aggregate skill intensity, we conduct the following shift-share decomposition:

$$\Delta s_t = \sum_j \bar{s}_j \, \Delta e_j + \sum_j \bar{e}_j \, \Delta s_j, \tag{A1}$$

where Δ denotes the change between time 0 and t, $\bar{s}_j = (s_{jt} + s_{j0})/2$ is the mean skill intensity of sector j, $\bar{e}_j = (e_{jt}+e_{j0})/2$ is the mean hours or labor compensation shares depending on which skill intensity definition is used. The first summation on the RHS of equation (A1) is the between-sector component. Figure A3 shows that the between-sector component is important, contributing to about 20% of the increase in aggregate skill intensity for the median state, which is also the contribution for the U.S. as a whole

A1.5 Hour and income shares

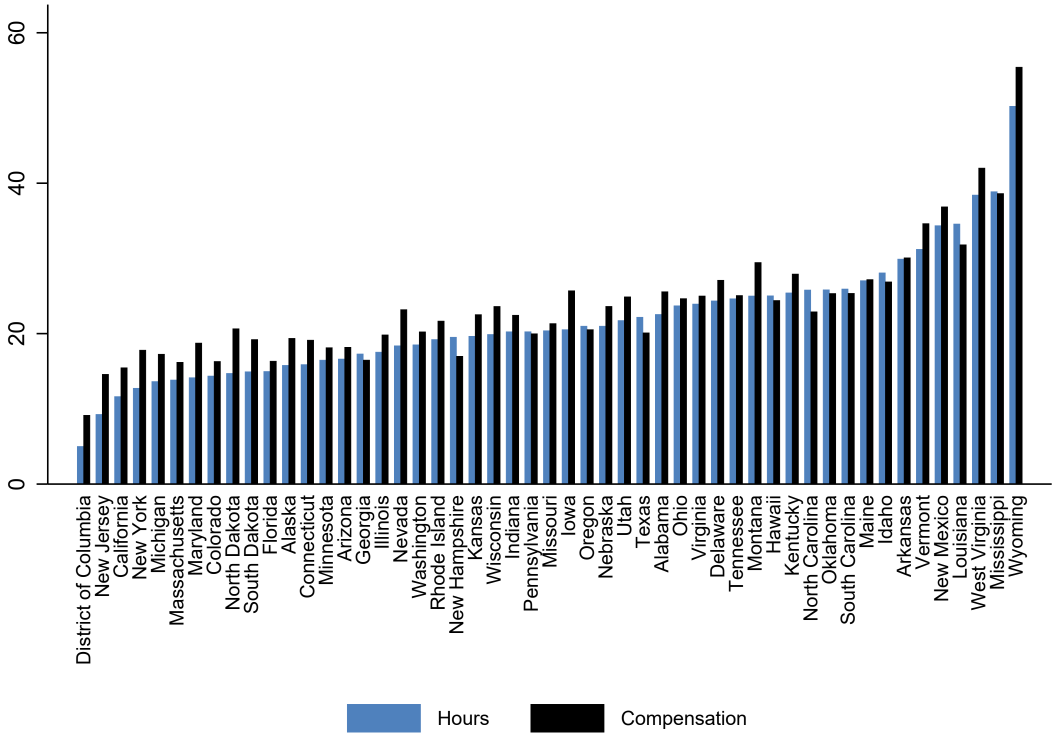
In contrast to the reallocation of low-skill hours, Figure A4A shows that the allocation of high-skill hours across the two sectors are rather constant since 1980. Figure A4B shows that the share of labor income goes to high-skilled workers are rising in both sectors.

Figure A5 plots the income share of the low-skilled workers and high-skilled workers out of the total value-added for the aggregate economy and the two sectors. It shows that the income share of the low-skilled workers have been falling while the income share of the high-skilled workers have been rising throughout the 1968-2010 period.

References

Autor, D. and D. Dorn (2013). The growth of low-skill service jobs and the polarization of the US labor market. *American Economic Review* 103 (5), 1553-97.

Figure A3: The Rise in Aggregate Skill Intensity Explained by Sectoral Shifts



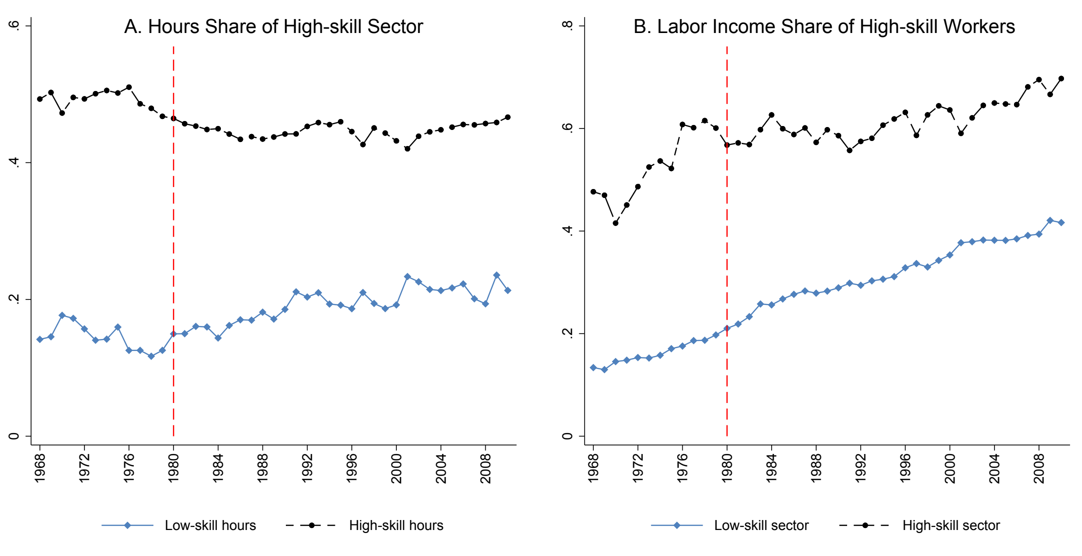
Notes: The between component is the mean change in the sectoral shares, weighted by the sectors' long-run mean skill intensity, based on equation (A1). Skill intensity in hours (compensation) is calculated as the share of high-skilled workers in total hours (compensation). The change is calculated from 1980 to 2010.
Source: Census and ACS.

Lawrence, R. Z. (2016). Does productivity still determine worker compensation? Domestic and international evidence. The US Labour Market: Questions and Challenges for Public Policy.

Ruggles, S., K. Genadek, R. Goeken, J. Grover, and M. Sobek (2017). Integrated public use microdata series: Version 7.0 [dataset].

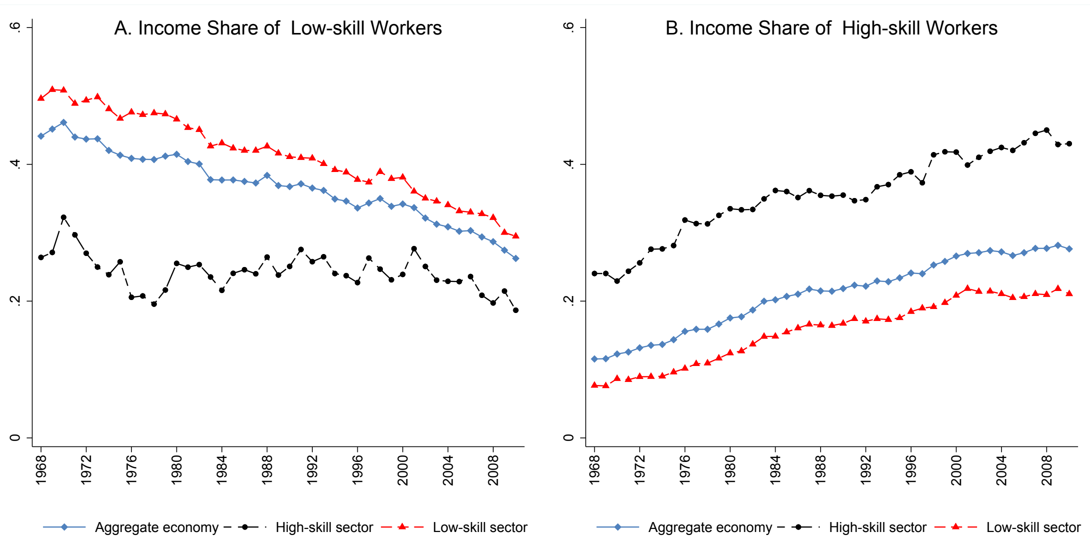
Stewart, J. and H. Frazis (2019). The importance and challenges of measuring work hours. IZA World of Labor.

Figure A4: Trends in Hours and Labor Income



Notes: Panel A shows the share of low-skill hours and the share high-skill hours in the high-skill sector. Panel B shows the labor income share of high-skilled workers in the low-skill and the high-skill sector. See Data Appendix A1 for the construction of variables and sectors.
Source: WORLD KLEMS and CPS.

Figure A5: Trends in Hours and Labor Income



Notes: Panel A (B) shows the share of low-skill (high-skill) labor income in the value-added of aggregate economy, the high-skill intensive sector, and low-skill intensive sector. See Appendix A1.1.
Source: WORLD KLEMS and CPS.

A2 Model Appendix

A2.1 Equilibrium Prices

A2.1.1 Deriving the ratio H_j/L_j

Equating MRTS across high-skill and low-skill labor to relative wages:

$$q = \frac{1-\xi_j}{\xi_j}\left(\frac{L_j}{\tilde{H}_j}\right)^{\frac{1}{\eta}}(1-\kappa_j)\left(\frac{G_j\left(H_j, K_j\right)}{H_j}\right)^{\frac{1}{\rho}},$$

which can be re-written as

$$q = \sigma_j (1-\kappa_j) \left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}} \left(\frac{G_j(H_j, K_j)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}}; \qquad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}$$

where using equation (18), we can derive:

$$\begin{aligned}
\frac{G_j(H_j, K_j)}{H_j} &= \left[\kappa_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}} \left[\delta_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + 1\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}} \left(\delta_j^{\rho} \chi^{\rho-1} + 1\right)^{\frac{\rho}{\rho-1}},
\end{aligned}$$

thus we have

$$\frac{G_j(H_j, K_j)}{H_j} = \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}}. \tag{A2}$$

Substituting (A2) into the MRTS condition across high-skill and low-skill:

$$q = \sigma_j (1-\kappa_j) \left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}} \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\eta-\rho}{(\rho-1)\eta}},$$

which implies

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta} (1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}.$$

A2.1.2 Labor income shares

The high-skill income share is

$$I_j = [1 - J_j] \tilde{I}_j, \tag{A3}$$

using (19) and (21),

$$I_j = \frac{\tilde{I}_j}{1 + q^{\eta-1} \sigma_l^{-\eta} \left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{\rho-1}}} \tag{A4}$$

The total labor income shares is

$$\beta_j = I_j + J_j = (1 - J_j)\,\tilde{I}_j + J_j = J_j \left[\frac{1 - J_j}{J_j}\tilde{I}_j + 1\right],$$

substitute (19) and (21),

$$\beta_j = J_j \left[q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\,(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right].$$

A2.1.3 Equilibrium low-skill wage $\mathbf{w_l}$

The price for low-skill efficiency labor equals to the value of its marginal product:

$$w_l = \xi_j p_j A_j \left(\frac{F_j\left(G\left(H_j, K_j\right), L_j\right)}{L_j}\right)^{\frac{1}{\eta}}$$

where using the production function

$$\begin{aligned}
\frac{F_j\left(G\left(H_j, K_j\right), L_j\right)}{L_j} &= \left[(1-\xi_j)\left[\frac{G_j\left(H_j, K_j\right)}{L_j}\right]^{\frac{\eta-1}{\eta}} + \xi_j\right]^{\frac{\eta}{\eta-1}} \\
&= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left[\frac{G_j\left(H_j, K_j\right)}{H_j}\right]^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\frac{\eta}{\eta-1}},
\end{aligned}$$

substitute (A2) and (20) to obtain

$$\begin{aligned}
\frac{F_j\left(G\left(H_j, K_j\right), L_j\right)}{L_j} &= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\,(1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}}\,\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\frac{\eta}{\eta-1}} \\
&= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j^{\eta}q^{1-\eta}\,(1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}}\,\tilde{I}_j^{\frac{\eta-1}{1-\rho}} + 1\right]^{\frac{\eta}{\eta-1}}.
\end{aligned}$$

Using the income shares (21)

$$\frac{F_j\left(G\left(H_j, K_j\right), L_j\right)}{L_j} = \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}}, \tag{A5}$$

and low-skill wage is

$$w_l = \xi_j^{\frac{\eta}{\eta-1}} p_j A_j \left[J_j\right]^{\frac{1}{1-\eta}}.$$

A2.2 Allocation of Labor

A2.2.1 Expressing q as function of χ

Using (24), the equilibrium condition for price of capital is:

$$q_k = \frac{q}{\chi} p_l A_l \left[J_l \xi_l{}^{-\eta} \right]^{\frac{1}{1-\eta}}$$

Given $\phi = q_k / p_l$,

$$\chi = q \frac{A_l}{\phi} \left[J_l \xi_l{}^{-\eta} \right]^{\frac{1}{1-\eta}} .$$

Using the definition of $J_l(\chi, q)$ in (21),

$$\begin{aligned} \chi &= q \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[1 + q^{1-\eta} \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}} \\ &= \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[q^{\eta-1} + \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}} \end{aligned}$$

rearranging

$$q^{\eta-1} + \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} = \left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{\frac{\eta}{1-\eta}}$$

so

$$q = \left[\left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\tilde{I}_l(\chi) (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}} ,$$

Given the expression for $\tilde{I}_l$ in (19),

$$\begin{aligned} q &= \left[\left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\left(1 + \chi^{\rho-1} \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}} \\ &= \chi \left[\left(\frac{\phi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\left(\chi^{1-\rho} + \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}} , \end{aligned}$$

so $q > 0$ requires

$$\begin{aligned} \left(\frac{\phi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} &> \sigma_l^\eta \left[\left(\chi^{1-\rho} + \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{1-\eta}{1-\rho}} \\ \left[\left(\chi^{1-\rho} + \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{\eta-1}{1-\rho}} &> \left(\frac{\phi}{A_l} \right)^{1-\eta} (1-\xi_l)^\eta \end{aligned}$$

which requires

$$\chi > \chi_{\min} \equiv \left[\left(\frac{A_l}{\phi} \right)^{1-\rho} (1-\xi_l)^{\frac{\eta(1-\rho)}{\eta-1}} (1-\kappa_l)^{-\rho} - \delta_l^{\rho} \right]^{\frac{1}{1-\rho}}.$$

Deriving equation for $S\left(\chi;\zeta,\frac{\phi}{A_l}\right)$**:** The labor market clearing condition for the high skill labor implies:

$$\frac{H_l+H_k}{L_l+L_k}(L_l+L_k)+\frac{H_h}{L_h}L_h=H,$$

using Lemma 2 and the low-skill labor market clearing condition,

$$\frac{H_l}{L_l}(L-L_h)+\frac{H_h}{L_h}L_h=H,$$

thus the share of low-skill labor in the high-skill sector is:

$$l_h \equiv \frac{L_h}{L} = \frac{H/L - H_l/L_l}{H_h/L_h - H_l/L_l}, \tag{A6}$$

simplify to

$$l_h = \frac{\zeta/(H_l/L_l)-1}{(H_h/L_h)/(H_l/L_l)-1},$$

substitute MRTS condition (20)

$$l_h = \frac{\zeta\sigma_l^{-\eta}q^{\eta}(1-\kappa_l)^{\frac{\rho(\eta-1)}{1-\rho}}\tilde{I}_l^{\frac{\eta-\rho}{\rho-1}}-1}{(\sigma_h/\sigma_l)^{\eta}\left(\frac{1-\kappa_h}{1-\kappa_l}\right)^{\frac{\rho(\eta-1)}{\rho-1}}\left(\frac{\tilde{I}_h}{\tilde{I}_l}\right)^{\frac{\eta-\rho}{1-\rho}}-1}$$

.

Deriving equation for $D\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right)$: The goods market clearing conditions and the relative demand implies:

$$x = \frac{p_hC_h}{p_lC_l} = \frac{P_hY_h}{P_l(Y_l-\phi K)}$$

which can be written as:

$$\frac{p_h Y_h}{p_l Y_l} = x\left(1 - \frac{\phi K}{Y_l}\right), \tag{A7}$$

where using relative price (26), x is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{\xi_h^{-\eta} J_h}{\xi_l^{-\eta} J_l}\right)^{\frac{1-\varepsilon}{\eta-1}}; \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, K is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)$$

so the relative demand equation (A7) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l}\left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right],$$

given $\phi \equiv q_k / p_l$, rewrite it in terms of low-skill income share J_j :

$$\begin{aligned}
\frac{J_l}{x J_h}\left(\frac{L_h}{L_l}\right) &= 1 - \frac{q_k J_l}{q_l L_l}\left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right] \\
&= 1 - \frac{J_l}{L_l}\left[\frac{q_k K_h}{q_l L_h} L_h + \frac{q_k K_l}{q_l L_l}\left(L - L_h\right)\right] \\
&= 1 - \frac{J_l}{L_l}\left[\frac{1-\beta_h}{J_h} L_h + \frac{1-\beta_l}{J_l}\left(L - L_h\right)\right],
\end{aligned}$$

where the last equality follows from the definition of β_j. Finally:

$$\frac{J_l}{x J_h}\left(\frac{l_h}{1-l_h}\right) = 1 - \frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h} l_h + \frac{1-\beta_l}{J_l}\left(1 - l_h\right)\right],$$

thus the demand for l_h is:

$$l_h = \frac{\beta_l}{\beta_l + \frac{J_l}{J_h}\left(\frac{1}{x} + 1 - \beta_h\right)}.$$

A2.3 Value-Added Shares

The value-added shares of the high-skill sector is:

$$v_h = \left[1 + \frac{p_l Y_l}{p_h Y_h}\right]^{-1} = \left[1 + \frac{p_l A_l F_l / L_l}{p_h F_h / L_h} \frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (26) and (A5),

$$v_h = \left[1 + \left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}} \left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}} \left(\frac{L_l}{L_h}\right)\right]^{-1}$$

simplify to

$$v_h = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1},$$

given l_h, v_h is determined.

A2.4 Skill-biased shift

The production function is

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[\kappa_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

Using the MRTS condition (18),

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[\kappa_j \left(\chi \frac{\kappa_j}{1-\kappa_j}\right)^{\rho-1} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left[\left(\chi^{\rho-1}\left(\frac{\kappa_j}{1-\kappa_j}\right)^{\rho} + 1\right)(1-\kappa_j)\right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \end{aligned}$$

A3 Quantitative Results

A3.1 Calibration

This section explains how the weight of each input is calibrated to match the sectoral income share for period 0 and period T.

A3.1.1 Normalization of $\phi/\mathbf{A}_l$

The initial $\frac{\phi}{A_l}$ can be normalized to 1. By definition of $\tilde{I}_j$

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j}\right]^{-1} \Longrightarrow \frac{K_j}{\chi H_j} = \frac{1 - \tilde{I}_j}{\tilde{I}_j},$$

which is independent of ϕ/A_l. Also by definition of J

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q \frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of ϕ/A_l as well. Therefore it follows from (A6) that l_h is independent of ϕ/A_l. So the allocation of low-skill labor is independent of ϕ/A_l. Given H_j/L_j and K_j/H_j are independent of ϕ/A_1, so the allocation of all inputs are independent of ϕ/A_1. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocation across sectors. The value of ϕ_T/A_{lT} is then determined by the growth in the relative price of capital ϕ_T/ϕ_0 and the growth in low-skill productivity A_{lT}/A_{l0}.

A3.1.2 Calibration of κ_l, ξ_l

Given ϕ/A_l, equation (27) express χ as a function of ξ_l given data on q and J_l :

$$\chi = qA_k \left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}} = qA_k J_l^{\frac{1}{1-\eta}} \xi_l^{\frac{\eta}{1-\eta}}.$$

Substitute this into $\tilde{I}_l$ in (19) to solve out δ_l explicitly:

$$\delta_l = \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}}$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of ξ_l. Thus the income share equation (21) provides an implicit function to solve for ξ_l:

$$J_l = \left[1 + q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1},$$

which can be used to solve for ξ_l given data on $\left(\tilde{I}_l, J_l\right)$. This procedure pins down χ, ξ_l and κ_l. Note that

$$\left(1-\kappa_l\right)^{-1} = 1+\delta_l = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}} = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}\right)^{1-\rho}\right]^{\frac{1}{\rho}}$$

so

$$\sigma_l^{\eta}\left[\left(1-\kappa_l\right)^{-1}\right]^{\frac{\rho(\eta-1)}{1-\rho}} = \sigma_l^{\eta}\left[1 + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(qA_kJ_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}\xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}$$

The implicit function is

$$f\left(\xi_l\right) = \left[1 + q^{1-\eta}\left[\left(\frac{1-\xi_l}{\xi_l}\right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}\left(1-\xi_l\right)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}\right]^{-1} - J_l,$$

thus we have

$$\begin{aligned} f'\left(\xi_l\right) &> 0 \\ \lim_{\xi_l\to 1} f\left(\xi_l\right) &= 1-J_l > 0 \\ \lim_{\xi_l\to 0} f\left(\xi_l\right) &= -J_l < 0 \end{aligned}$$

so there is an unique solution for $\xi_l \in (0,1)$ for any given ϕ/A_l.

Table A3: Actual and Predicted Values for Key Variables

		q	l_h	h_h	v_h	β_l	β_h
	Data 1980	1.44	0.14	0.46	0.24	0.59	0.56
(1)	Data 2008	1.94	0.21	0.46	0.29	0.53	0.65
(2)	Model 2008	1.92	0.20	0.45	0.28	0.52	0.65
Counterfactual (shutting down individual mechanism)							
(3)	A_l/A_h	1.80	0.15	0.37	0.22	0.52	0.64
(4)	ϕ	1.70	0.19	0.44	0.27	0.52	0.62
(5)	ξ_j	1.51	0.18	0.52	0.31	0.59	0.64
(6)	κ_j	1.71	0.20	0.43	0.27	0.50	0.59
(7)	H/L	3.17	0.23	0.47	0.31	0.56	0.68

A3.1.3 Calibration of $\kappa_{\mathbf{h}}, \xi_{\mathbf{h}}$

Using income share $\tilde{I}_h$ in ((19)):

$$\delta_h = \left[\frac{1-\tilde{I}_h}{\tilde{I}_h}\chi^{1-\rho}\right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}$$

given $\tilde{I}_h$ and χ, κ_h is obtained. Using J_h in (21):

$$\sigma_h = \left[\frac{1-J_h}{J_h}q^{\eta-1}\left[\tilde{I}_h\left(1-\kappa_h\right)^{-\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and q, so ξ_h is obtained.

A3.2 Results for Other Variables

The performance of the model on other key variables is summarized in Table A3.

A3.3 Sensitivity Analysis

This section considers alternative values of elasticity parameters and the growth in the price of the high-skill intensive sector. Given the calibration procedure, changing one parameter will change the values for other parameters. In the interest of space, we do not report those values. These parameter values are available upon

Table A4: Data and Model Predictions, $\varepsilon = 0.5$, 1980-2008 % Change

		Divergence	y/P_Y	w_l/P_C	w_l/p_l	w_l/p_h
(1)	data	27(38)	60	26	44	-3.4
(2)	model	33	matched	20	44	-3.6
Counterfactual (shutting down individual mechanism)						
(3)	A_l/A_h	20	84	54	46	64
(5)	ξ_j	9.3	65	51	85	18

request.

A3.3.1 Elasticity of substitution across high-skill and low-skill goods

As discussed in the main text, there is no direct estimate for ε in our model but there is evidence suggesting that it is small. We now consider a higher value of $\varepsilon = 0.5$. An increase in ε implies that the model requires a higher growth in A_{lh} to match the observed growth in relative prices.

As shown in Table A4 the baseline results in row (2) are not affected given the calibration procedure. The more important question is whether it will affect the role played by the uneven productivity growth, i.e. a rise A_{lh}. As shown in row (3), the uneven productivity growth is still important for the divergence. In its absence the divergence would reduce by a third. It has a smaller role compared to the role played by falling ξ in row (5), but its advantage remains in predicting a rise in the relative price of the high-skill intensive sector, which is needed for the sector-specific trends in the low-skill product wage and the rise in the relative cost of living.

A3.3.2 Elasticity of substitution across capital and high-skill labor

The estimate of $\rho = 0.67$ in Krusell et al. (2000) is for the aggregate economy using data for 1963-1992. We can also infer the elasticity of substitution across capital and high-skill labor ρ using the equilibrium condition (18), data on income shares and relative input prices. Using the equilibrium condition (18), the response in relative income shares to changes in relative prices of the high-skill and the capital

Table A5: Data and Model Predictions, $\rho = 0.5$, 1980-2008 % Change

		Divergence	y/P_Y	w_l/P_C	w_l/p_l	w_l/p_h
(1)	data	27	60	26	44	-3.4
(2)	model	34	matched	20	44	-3.5
Counterfactual (shutting down individual mechanism)						
(3)	A_l/A_h	20	84	53	48	61
(5)	ξ_j	10	65	50	85	17

input is:

$$\ln\left(\frac{I_{jT}/\left(1-\beta_{jT}\right)}{I_{j0}/\left(1-\beta_{j0}\right)}\right) = (1-\rho)\ln\left(\frac{\chi_T}{\chi_0}\right), \tag{A8}$$

where by definition, $\chi = w_h/q_k = \phi(w_h/p_l)$, so its growth can be obtained using data on the relative price of capital and the high-skill wage deflated by price of the low-skill intensive sector. Given the data reported in 2, equation (A8) implies ρ is 0.39 using income share of the low-skill sector and 0.59 using income share of the high-skill sector, which gives an average of 0.49. If we were to use the aggregate income share instead, equation (A8) implies $\rho = 0.48$. Thus we report the results for $\rho = 0.5$ in Table A5. It shows that the results for the full model (row 2) are almost identical to those in Table 4 and Table 5. The contribution of the uneven productivity growth (row 3) to the divergence and the low-skill wage stagnation are also similar.

A3.3.3 Elasticity of substitution across low-skill and high-skill labor

The estimate of $\eta = 1.4$ in Katz and Murphy (1992) is for the aggregate economy using data for 1963-1987. For a similar period, 1963-1992, Krusell et al. (2000) finds $\eta = 1.67$ and $\rho = 0.67$ for the nested aggregate production function including capital. Using more recent data, abstracting from capital, Acemoglu and Autor (2012) find values within the range 1.6–1.8. Higher η implies a smaller exogenous decline in ξ_l is needed to account for the decline in labor income shares in the low-skill sector. Table A6 reports the results for $\eta = 2.0$. It shows the uneven productivity growth (row 3) has a more important role in accounting for the divergence as the required decline in ξ_l reduced to -0.46% compared to -0.93% in the baseline. This has weaken the contribution the fall in the input weight of the low-skill labor (row 4) compared to the baseline in Table 4 and 5.

Table A6: Data and Model Predictions, $\eta = 2.0$, 1980-2008 % Change

		Divergence	y/P_Y	w_l/P_C	w_l/p_l	w_l/p_h
(1)	data	27 (38)	60	26	44	-3.4
(2)	model	34	matched	19	44	-3.4
	Counterfactual (shutting down individual mechanism)					
(3)	A_l/A_h	22	83	50	46	55
(5)	ξ_j	23	62	31	60	4.1

A3.3.4 Relative price of the high-skill intensive sector

We now consider the case where the increase in the relative price of the high-skill intensive sector is assumed to be half of the observed increase, i.e. the increase is only 25% instead of 49%. We assume this is entirely due to an upward bias on the growth in the price of the high-skill intensive sector while keeping the growth of the low-skill intensive sector as in the data. Using the value-added shares in KLEMS and applying the Tornqvist index, this implies a lower growth in P_Y and we assume the same effect on P_C given we do not have data on consumption. Thus, by construction, the ratio of P_C/P_Y is not affected by this alternative price.

The results are reported in Table A7. In the data (row 1), the low-skill real wage, the aggregate labor productivity and the low-skill product wage in the high-skill intensive sector are growing faster because of a slower growth in p_h. As in Table 4 and 5, the full model (row 2) matches the data row very closely. The divergence predicted by the uneven productivity growth (row 3) is smaller compared to Table 4 but still quantitatively important.

Table A7: Data and Model Predictions, lower growth in p_h, 1980-2008 % Change

		Divergence	y/P_Y	w_l/P_C	w_l/p_l	w_l/p_h
(1)	data	27	68	32	44	15
(2)	model	28	matched	31	44	15
	Counterfactual (shutting down individual mechanism)					
(3)	A_l/A_h	19	84	54	47	66
(5)	ξ_j	5.8	74	64	86	41

A3.4 One-sector model

The one-sector version of utility (5) and budget constraint (6) are:

$$U_i = \ln c_i; \quad c_i = w_i \tag{A9}$$

The household decision problem implies $c_i = w_i$, $i = h, l$. The aggregate production function follows (7)- (9) with the market clearing condition for the final good as in (10). The aggregate production function is:

$$Y = A \left[\xi L^{\frac{\eta-1}{\eta}} + (1-\xi) \left[\kappa K^{\frac{\rho-1}{\rho}} + (1-\kappa) H^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right]^{\frac{\eta}{\eta-1}}, \tag{A10}$$

and the market clearing condition:

$$Y = Hc_h + Lc_l + \phi K \tag{A11}$$

As in the multi-sector model, the representative firm's problem is to take wages and capital price $\{q_k, w_h, w_l\}$ as given to maximize profit. Thus condition (18)-(28) hold for the one-sector economy, where subscript for sectors are dropped. Thus all variables of the model can be expressed as function of the relative price $\chi = \frac{w_h}{q_k}$. The equilibrium χ can be solved from the market clearing condition together with household's budget constraint:

$$Y = Hw_h + Lw_l + \phi K \tag{A12}$$

Using the definition of the income share of the low-skill labor in (21), and ϕ as the relative price of capital is equal to q_k. Equation (A12) becomes

$$\frac{w_l L}{J} = Hw_h + Lw_l + q_k K$$

Rewrite it into relative factor prices, and substitute $\zeta \equiv \frac{H}{L}$ as the relative supply of the high-skill labor:

$$\frac{1}{J} = \zeta q + 1 + \frac{q}{\chi}\left(\frac{K}{H}\right)\zeta,$$

substituting K/H from (18) and J from (19) and (21),

$$1 + q^{1-\eta}\sigma^{\eta}\left[\frac{1}{\left(1+\chi^{\rho-1}\delta^{\rho}\right)(1-\kappa)^{-\rho}}\right]^{\frac{\eta-1}{1-\rho}} = \zeta q + 1 + \frac{q}{\chi}(\chi\delta)^{\rho}\zeta,$$

which implies to

$$q^{-\eta}\sigma^{\eta}(1-\kappa)^{\frac{(1-\eta)\rho}{1-\rho}} = \zeta\left[\left(1+\chi^{\rho-1}\delta^{\rho}\right)\right]^{\frac{\eta-\rho}{1-\rho}}$$

Finally substitute q from (28) to obtain an implicit function in χ:

$$\left[\left(\frac{\phi}{A}\right)^{\eta-1}\xi^{-\eta} - \sigma^{\eta}\left[\left(\chi^{1-\rho}+\delta^{\rho}\right)(1-\kappa)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{\eta}{1-\eta}}\chi^{-\eta}\sigma^{\eta}(1-\kappa)^{\frac{(1-\eta)\rho}{1-\rho}} = \zeta\left[\left(1+\chi^{\rho-1}\delta^{\rho}\right)\right]^{\frac{\eta-\rho}{1-\rho}}. \tag{A13}$$

Once χ is solved, the rest of equilibrium variables are obtained.

A3.4.1 Calibration of the one-sector economy

The calibration strategy is the same as before. The elasticity parameters in Table 3A are identical as before. The relative supply of the high-skill labor ζ is also the same as in the multi-sector model given it is calibrated to match the aggregate income share of the high-skill labor relative to the low-skill labor as in equation (36). The rest of the parameters are summarized as below. The driving force of the one-sector are $(A_T/A_0, \phi_T/\phi_0)$. The parameter ϕ is matched to the decline in the relative price of capital. The growth rate of ϕ is now at -0.88% per annual, which fall faster than the multisector model given this is the the price relative to the aggregate output instead of output of the low-skill sector. The growth in the aggregate TFP A is set to match the growth in the real labor productivity. The production weight of each input $\{\xi, \kappa\}$ are set to match the aggregate income shares of the high-skill labor and the low-skill labor. The new calibrated parameters are summarized in Table (A8).

Table A8: Calibrated Parameters for One-sector model

Parameters	1980	2010	Growth (% p.a.)	Target
ϕ			-0.88	Price of capital relative to the final output
A			0.74	Aggregate real labor productivity
ξ	0.29	0.24	-0.80	Aggregate income share. See Appendix A3.1
κ	0.62	0.57	-0.29	Aggregate income share. See Appendix A3.1

Table A9: One-sector model, Percentage Change, 1980-2008

			Factors for divergence		Real Prod. and wage	
		Divergence	Inequality	Labor Share	Prod.	Low-skill wage
(1)	Data	27 (38)	19	-3.4	60	26 (16)
(2)	Model	24	matched	matched	matched	29
		Counterfactual (shutting down individual mechanism)				
(3)	ϕ	18	9.7	-7.0	36	15
(4)	ξ	1.0	2.3	3.3	66	68
(5)	κ	25	14	-9.0	65	32
(6)	H/L	31	33	1.5	36	3.8

Note: Factors for divergence are shown in (4). For the data row, the low-skill real wage is calculated using PCE as P_C and number in bracket is when CPI is used as P_C. By definition, real divergence is the same as nominal divergence in the one-sector model. Row (3) fix the relative price of capital ϕ. Row (4) and (5) fix production weight of low-skill labor ξ and the weight κ in production function (7). Row (6) fix the relative supply of high-skill labor H/L.

Table (A9) reports the quantitative results related to those for the multisector model in Table (4) and Table (5). Row (1) is the baseline result. Given the calibration strategy of production weights ξ and κ, and the relative supply of the high-skill labor ζ in equation (36), they imply the skill premium q is also matched in the one-sector model. It follows that the increase in the wage inequality (w/w_l) and the fall in the labor share are matched, thus the predicted nominal divergence is the same as in the data given the accounting equation (4). Given price of consumption is the same as price of output in the one-sector model, it implies the real divergence is the same as nominal divergence. Row (3)-(6) reports the role of falling relative price of capital (ϕ), the changing production weights (ξ, κ) and the increase in the relative supply of the high-skill labor. The message is similar to the results in the multisector model reported in Table (4) and Table (5).