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Abstract

Analyzing price data from sequential German electricity markets, namely the day-ahead and intraday auction, a puzzling but apparently systematic pattern of price premiums can be identified. The price premiums are highly correlated with the underlying demand profile. As there is evidence that widespread models for electricity forward premiums are not applicable to the market dynamics under analysis, a theoretical model is developed within this article which reveals that non-convexities in only a subset of sequential markets with differing product granularity may cause systematic price premiums at equilibrium. These price premiums may be bidirectional and reflect a value for additional short-term power supply system flexibility.

Keywords: sequential market organization, electricity markets, short-term market dynamics, price premiums, arbitrage

JEL classification: C60, C62, C63, D21, D23, D24, D41, D44, D47, L11

1. Introduction and Research Question

Economic theory suggests that the limited storability of electricity may pose limits to arbitrage. Price levels in sequential markets may hence differ significantly and sudden changes in prices may be identified. At the same time, recent developments are characterized by the establishment of sequential short-term electricity markets in Germany to deal with the increasing share of highly volatile intermittent renewable electricity generation. A trend of trading shorter contracts closer to the physical delivery may be identified. These markets face an ongoing increase in trade volumes, but the economic understanding of the respective market dynamics has yet to be deepened.

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In this paper, an analysis of price premiums in the context of two sequential German short-term electricity markets is conducted. The day-ahead auction is cleared at noon one day ahead delivery and offers hourly products. It is regarded as providing the most relevant reference price for subsequent trade. Second, the intraday auction is considered which is settled three hours afterwards and allows for trading quarter-hourly contracts.

In the research area of electricity markets, the model presented in Bessembinder and Lemmon (2002) embodies a widespread explanatory approach for price premiums between forward and real-time electricity markets. However, there is evidence that the model is not applicable to price premiums between the day-ahead and intraday auction. The rapid succession of both market settlements without updated information requires the derivation of an alternative approach to decode the puzzling pattern of price premiums identified. Therefore, a theoretical model is developed to analyze price premiums in the presence of non-convexities in sequential markets with differing product granularity.

The model uncovers that non-convexities being more pronounced in only a subset of sequential markets may lead to both negative or positive price premiums. The direction of price premiums depends on the market settlement being in particular sections of the underlying merit order. Indeed, the real-world data reveals a high correlation of load and the direction as well as the value of price premiums. It may be stressed that the price premiums under analysis incorporate a value of additional short-term power system flexibility rather than reflecting a value of risk. Analyzing the cost-saving potential from smoothing these non-convexities, a proxy for the value of additional power system flexibility could be derived. On a national level, this is approximately EUR 10.2 million in 2015. The corresponding value for flexibility which is provided by neighboring countries is EUR 6.4 million in 2015. These are rather small numbers, but yet the general model framework may easily be transferred to other applications such as sequential block and single unit auctions.

It is crucial to understand the fundamental properties of the price premiums identified as they may reflect market needs or even indicate inefficiencies. As regards the day-ahead and intraday auction, the price premiums are, at least partially, triggered by restricted participation in the intraday auction. The introduction of cross-border market coupling on a sub-hourly level may be beneficial to reduce the resulting welfare losses. From a business perspective, the findings and the systematology uncovered are relevant to evaluate business strategies building on the price differences observed.

The article is organized as follows. In Section 2 the paper is positioned in the existing literature and a broad overview on possible limits to arbitrage is provided. In a next step, an empirical analysis of price premiums in the German day-ahead and intraday auction is presented in Section 3. To gain insights into the drivers of the price premiums under consideration, a theoretical analysis is conducted within Section 4. The respective results are then contextualized in Section 5. Finally, conclusions are drawn.

2. Literature Background

The Fundamental Theorem of Asset Pricing depicts conditions for arbitrage-free and complete markets (Dybvig and Ross, 1987). In particular, the coincide of stochastic processes and equivalent martingales causes markets not to exhibit unexploited arbitrage opportunities. Based on this theory, in Weber (1981) the author states that prices in sequential auctions epitomize a martingale where, on average, prices neither go systematically up nor down over time. The Law of One Price furthermore clarifies that in perfect financial markets goods should have an identical price across all locations (Isard (1977) and Richardson (1978)). However, real-world markets may require a more differentiated view.

The general impact of sequential market designs on prices has extensively been studied (see, e.g., Allaz and Vila (1993); Knittel and Pindyck (2013); Kilian and Murphy (2014); Juvenal and Petrella (2015)). In Mezzetti et al. (2007), the authors suggest a lowballing effect reducing the first stage market price. As regards the application to real markets, in Ardeni (1989) it is stressed that the Law of One Price does not hold true for sequential commodity markets, at least in the long run. Based on the example of electricity markets, empirical evidence for this hypothesis is provided in Ito and Reguant (2016). The authors identify systematic price premiums in forward markets. Taking up on the general idea of non-convergence of sequential markets' prices, a concept of equilibrium models with a certain degree of disequilibrium is developed in Grossman and Stiglitz (1980).

Economic theory suggests that prices in sequential markets may particularly differ in the case of limited arbitrage. In Grossman and Stiglitz (1980), the authors identify transaction costs as a first plausible driver of systematic price differences in sequential markets (see, e.g., Ardeni (1989); Jha and Wolak (2015)). Second, market power abuse may trigger price spreads since dominant firms may not benefit from exploiting arbitrage opportunities (Ito and Reguant, 2016). As a third factor, risk aversion may drive the Law of One Price to fail (McAfee and Vincent, 1993). For illustration purposes, in Ashenfelter (1989) the author analyzes wine auctions and observes significant differences in prices for identical goods. The respective explanatory approach is based on risk aversion, quantity constraints, and asymmetric information. Such asymmetries regarding market participants may also refer to an asymmetric valuation of goods and different preferences (Bernhardt and Scoones (1994); Salant (2010)).

One further category of explanatory approaches for limited arbitrage refers to institutional and regulatory schemes. In Borenstein et al. (2008), the authors point out that uncertainty about a regulatory change may trigger prices in sequential markets to differ and empirical evidence from Californian electricity markets is presented. Finally, newly emerging markets may trigger learning processes leading to price differences shortly after introducing a new trading opportunity (Doraszelski et al., 2016).

In this paper, focus is placed on price relations within sequential electricity markets. Following economic theory, electricity exhibits unique features that cause a need for a differentiated analysis compared to other commodities such as oil and gas. First, the limited potential to store huge amounts of electricity poses limits to arbitrage opportunities. Furthermore, limited access to capital and strict regulation for financial players may be relevant (Birge et al., 2014). In Bessembinder and Lemmon (2002), the authors develop an equilibrium model that is supposed to be tailored for sequential electricity markets. In doing so, the authors suggest that forward premiums are negatively affected by high price volatility. At the same time, they identify a positive correlation of forward premiums and the skewness of prices in the real-time market. Complementary to Bessembinder and Lemmon (2002), in Longstaff and Wang (2004) the authors present empirical evidence from US electricity markets that the theoretical model is actually applicable to real-world data. In contrast to their scope, the analysis conducted within this article focuses on electricity markets with fundamentally different market characteristics. Above all, there is essentially no informational update between both market settlements. Nevertheless, a similar pattern of price premiums is observed yielding a puzzle which is yet to be solved.

3. Empirical Analysis of Price Premiums in the German Day-ahead and Intraday Auction

An empirical analysis of price premiums between the German day-ahead and intraday auction provides first insights on the topic under analysis. The day-ahead auction is settled at noon one day ahead physical delivery. The respective products are hourly contracts for the physical delivery of electricity. Following the day-ahead auction, the intraday auction with 15-minute contract duration is settled one day ahead delivery at 3pm[1]. Both markets exhibit a uniform market price. Additionally, both market stages are settled in rapid succession such that there is no significant informational update that impacts trade (Knaut and Paschmann, 2017b)[2]. Since trade in both markets refers to physically binding contracts without pure financial clearing, the main purpose is matching supply and demand according to the contract duration offered, rather than

[1]For more details on the purpose of implementing the intraday auction complementary to continuous intraday trade see Neuhoff et al. (2016). As regards the market depth, an illustration of average trade volumes is presented in Section Appendix.7.

[2]The influence of forecast errors on the resulting market prices is negligible.

speculation and risk hedging. More precisely, in Knaut and Paschmann (2017b) the authors show that the interaction of the day-ahead and intraday auction is essentially driven by increasing product granularity (hourly vs. quarter-hourly) and restricted participation in the intraday auction. In Knaut and Paschmann (2017a), the authors furthermore clarify that the lack of sub-hourly market coupling may be regarded as the most relevant driver of restricted participation in the German intraday auction.

Since the product granularity increases from the day-ahead to the intraday auction, arbitrage refers to bundles of goods. Four evenly distributed contracts which are traded in the intraday auction may act as a perfect substitute for the respective day-ahead contract. Arbitrage is even facilitated by the rapid succession of the market settlements. As the market characteristics hence basically comply with the no-arbitrage argument, it could be expected to find mean price equivalence. Nevertheless, systematic price premiums in individual hours of the day can be identified when analyzing historical price data. For illustration purposes, the distribution of price premiums for the individual hours of the day is presented in Figure 1. The figure is based on price premiums which are defined as the difference between the day-ahead auction price and the mean price level of the corresponding four intraday auction contracts. The target figure is derived according to Equation (1). The analysis is based on historical day-ahead and intraday auction price data from January 16, 2015 until November 2, 2016 (EPEX SPOT SE, 2016a) and the respective descriptive statistics are presented in detail in Table .2 in Section Appendix.1.

$$\Delta p = p^{day-ahead} - \frac{\sum_{t=t1}^{t4} p_t^{intraday}}{4} \tag{1}$$

In Figure 1 mean values are marked in red and the black lines give the median values. The green boxes range from the second to the third quartiles, whereas the dashed lines illustrate the 10 % and 90 % percentiles. Furthermore, the dashed horizontal lines highlight the transaction costs. Based on the exact numbers which are presented in column *Mean* of Table .2, the conclusion can be drawn that in individual hours (i.e., e.g., *h2* and *h15*) positive price premiums clearly outweigh negative ones. However, in other hours (such as in *h19*) reverse relations can be identified. The direction of price differences hence varies during the course of the day. In the majority of hours these price differences even exceed the direct transaction costs for trading, which are demanded by the exchange[3]. More specifically, positive price premiums, for example, range between 0 ct/MWh and 77 ct/MWh and the respective average is 29 ct/MWh. Nevertheless, the aggregate net price premium across all hours of the day approximately equals the transaction costs.

Based on the empirical findings, it could be expected that market participants may anticipate the di-

[3]These are 0.04 EUR/MWh in the day-ahead and 0.07 EUR/MWh in the intraday auction (EPEX SPOT SE, 2016b).

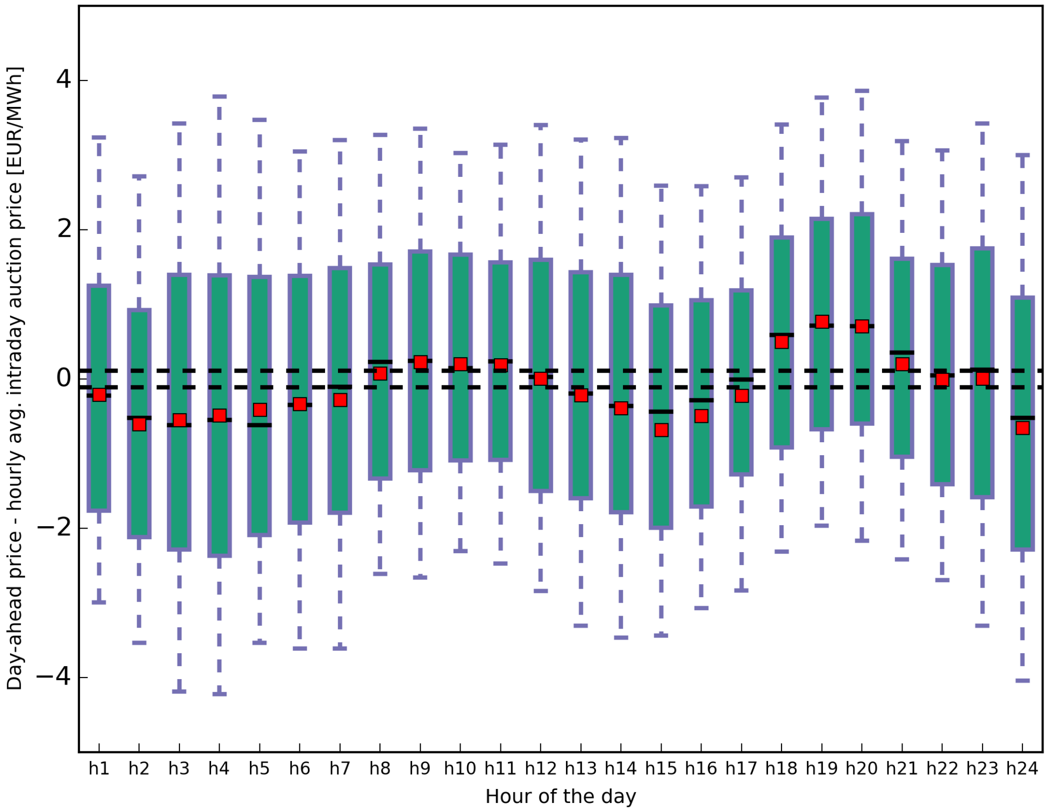

Figure 1: Distribution of differences between day-ahead and intraday auction prices compared to the respective transaction costs (January 16, 2015 - November 2, 2016)

rection of price differences and exploit additional arbitrage opportunities. As this is not reflected by the real-world data, it is relevant to deepen the understanding of the underlying drivers.

The pattern of price premiums identified, albeit less pronounced, is comparable to the findings of prior literature dealing with price differences between electricity forward and real-time markets (see, e.g., Longstaff and Wang (2004) and Viehmann (2011)). This is rather counterintuitive since some basic characteristics of the market configurations under analysis differ crucially. In particular, there is no informational update between the day-ahead and intraday auction. The lack of risk reduction between both market settlements does not comply with the assumption of risk premiums. Additionally, whereas in Longstaff and Wang (2004) the authors give empirical evidence for the applicability of the equilibrium pricing model presented in Bessembinder and Lemmon (2002), an analogous procedure is essentially not transferable to price differences between the day-ahead and intraday auction. More precisely, building on the empirical approach adopted in Longstaff and Wang (2004), a simple empirical analysis may be conducted to test for the correlation of price premiums and the variance as well as the skewness of the day-ahead spot prices. Detailed results are presented in Section Appendix.2. In short, there is empirical evidence that this explanatory approach is not applicable to the price premiums under analysis.

The analyses presented within this article follow the general idea presented in Knaut and Paschmann (2017b) and seek to analyze the impact of restricted participation on sequential commodity market prices. Whereas the respective modeling approach in Knaut and Paschmann (2017b) appears suitable to analyze general price relations on an aggregate level, a need for extending the model may be identified when analyzing the price formation in individual hours. More precisely, the analysis conducted within this paper is especially motivated through the observation of pronounced stepped shapes in real-world bid curve data as examplified in Section Appendix.3. Therefore, a theoretical framework is developed within this essay to analyze the influence of non-convexities in only a subset of sequential markets on the resulting price relations.

4. Theoretical Analysis

Two classes of suppliers (restricted and unrestricted) are distinguished, both of which interact in two simultaneously[4] settled markets that differ in terms of product granularity and market participation. Both types of suppliers participate in the first market, whereas in the second market only unrestricted suppliers are able to participate. The first stage product is split up into two identical sub-contracts for the periods t ($t \in t1, t2$) that can be traded in the second market. The sub-contracts may combined act as a perfect substitute for the first stage product.

Consumers may demand a different positive quantity D_t in each time interval t[5]. Demand is satisfied under perfect competition by both restricted and unrestricted suppliers. Both suppliers operate production units with increasing marginal costs of production.

As the quantities of both types of suppliers are chosen under perfect competition, the following optimization problem can be formulated. Simply put, the total production costs are minimized such that supply

[4]Due to the rapid succession of both market settlements, information in both markets is considered to be identical. This assumption is furthermore supported by energy trading companies confirming that there is no relevant informational update between both market settlements.

[5]Demand in electricity markets is characterized as being rather price inelastic. This is especially valid for short-term markets as considered in this paper (see, e.g., Lijesen (2007) and Knaut and Paulus (2016)).

meets demand.

$$\min z = C_r(q^r) + \sum_t [C_{u,t}(q_t^u)] \tag{2}$$

$$\text{s.t.} \qquad D_t = q^r + q_t^u \qquad \forall t. \tag{3}$$

Here $C_r(q^r)$ marks the overall costs for the production level q^r of all restricted suppliers. The costs are determined by the first market's outcome as restricted suppliers are not permitted to participate in the second market with shorter contracts. In contrast, $C_{u,t}(q_t^u)$ refers to the respective production costs for unrestricted suppliers in period t. The quantity q_t^u may vary in each time period and results from the first and second markets' settlements.

The set of restricted suppliers is characterized by an aggregate marginal cost curve. In more detail, Equation (4) depicts that a linear shape is assumed. The parameter a_0 determines a fixed offset, whereas a_1^r is the gradient of the restricted supply curve.

$$C_r'(q^r) = a_0 + a_1^r q^r \quad | \ a_0 > 0, a_1^r > 0 \tag{4}$$

With respect to unrestricted suppliers, a stepped discontinuity within their marginal cost function is considered. Hereby, it is accounted for a case in which unrestricted participation is somehow systematic and its impact on the resulting market dynamics may vary depending on the market settlement being in particular sections of the merit order (Equation (5)). For the sake of simplicity, the offset (a_0) is the same as in the case of restricted suppliers.

$$C_{u,t}'(q_t^u) = \begin{cases} a_0 + a_1^u q_t^u & q_t^u \leq Q_{disc} \quad | \ a_0 > 0, a_1^u > 0 \\ a_0 + \Delta_{disc} + a_1^u q_t^u & q_t^u > Q_{disc} \quad | \ a_0 > 0, a_1^u > 0 \end{cases} \tag{5}$$

The parameter Q_{disc} (> 0) may be regarded as the threshold which determines the discontinuous section of the merit order. Furthermore, a respective step height of Δ_{disc} is considered. For illustration purposes, Figure 2 depicts the respective relations. The lack of unrestricted suppliers within a particular section of the merit order is transformed into a marginal cost function with a stepped shape. Thereby, the aggregated merit order of both restricted and unrestricted suppliers is dynamic and depends on the demand quantities.

The analysis is based on the assumption of an increasing demand profile ($D_{t2} > D_{t1}$) and hence the resulting mixed complementarity problem can be solved on the basis of the corresponding Karush-Kuhn-Tucker (KKT) conditions.

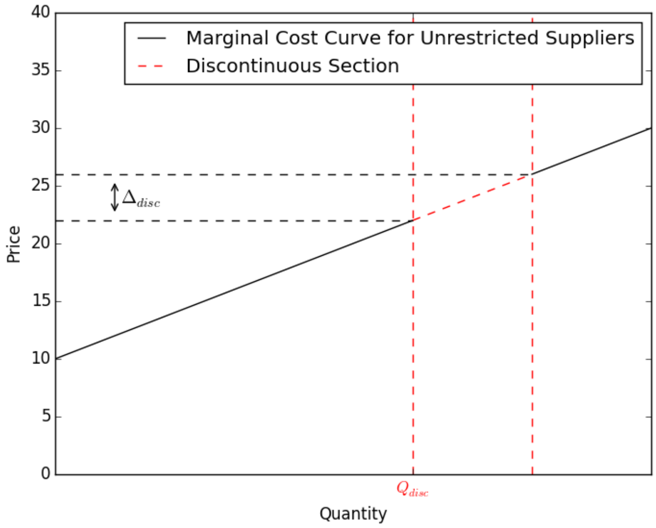

Figure 2: Illustration of the market configuration under analysis

Proposition 1. *If demand exceeds a certain threshold such that the optimal production level of unrestricted suppliers under continuous relations would exceed the non-convexity, it is cost-optimal to fix the production of unrestricted suppliers and satisfy additional demand exclusively by restricted suppliers. Thereby, excess supply by restricted suppliers compared to the case of continuous relations can be identified. This quantity choice is optimal as long as the additional costs due to exclusive supply by restricted suppliers are outweighted by avoided costs due to the discontinuous step.*

Proof. The detailed mathematical proof is outlined in Section Appendix.4. The optimal production level of restricted suppliers depending on the respective demand level may be defined according to Equation (6). The corresponding production level of unrestricted suppliers may be directly derived by the use of Equation (3).

$$q^{r*} = \begin{cases} \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} & (a1) D_{t2} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u}{2 \cdot a_1^r + a_1^u} \\ D_{t2} - Q_{disc} & (a2) \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u}{2 \cdot a_1^r + a_1^u} < D_{t2} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} + \frac{\Delta_{disc}}{2 \cdot (a_1^r+a_1^u)} & (a3) \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} < D_{t2}, D_{t1} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ D_{t1} - Q_{disc} & (a4) \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} < D_{t1} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} + \frac{\Delta_{disc}}{a_1^r+a_1^u} & (a5) D_{t1} > \frac{2 \cdot Q_{disc} \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \end{cases} \tag{6}$$

□

Furthermore, this essay aims to shed light on the respective price implications.

Proposition 2. *In the presence of non-convexities, mean price equivalence is not a necessary condition at equilibrium. Rather to the contrary, positive price premiums may be identified that range between 0 and* $0.5 \cdot \Delta_{disc}$.

Proof. As the market outcome is determined under perfect competition, the respective inverse demand functions could directly be applied to draw conclusions on prices. The equations derived in (7) have to be satisfied at equilibrium. Here m^1 refers to the first market where both restricted and unrestricted suppliers are permitted to participate. The second market with restricted participation and increased product granularity is named m_t^2. It is worth mentioning that the first market's price is directly determined by the marginal costs of restricted suppliers as the respective production may only be traded within the first market. Furthermore, $C'_r(q^r) \geq \frac{(C'_{u,t1}(q^r)+C'_{u,t2}(q^r))}{2}$ is valid.

$$\begin{aligned} p(m^1) &= C'_r(q^r) \\ p(m_t^2) &= C'_{u,t}(q^r) \end{aligned} \tag{7}$$

These price relations provide the basis to analyze whether the discontinuity may trigger price differences between the sequential markets at equilibrium[6]. Equation (8) can be used to calculate the respective price differences.

$$\Delta p = p(m^1) - \overline{p(m_t^2)} = \begin{cases} 0 & (a1) \\ (D_{t2} - Q_{disc}) \cdot a_1^r - \frac{(D_{t1}-D_{t2}+2\cdot Q_{disc})\cdot a_1^u}{2} & (a2) \\ 0 & (a3) \\ (D_{t1} - Q_{disc}) \cdot a_1^r + \frac{(D_{t1}-D_{t2}-2\cdot Q_{disc})\cdot a_1^u}{2} - \frac{\Delta_{disc}}{2} & (a4) \\ 0 & (a5) \end{cases} \tag{8}$$

Inserting the respective thresholds for D_{t2} according to (6) into the second term of (8) yields a price difference that ranges between 0 and $0.5 \cdot \Delta_{disc}$. Inserting the respective thresholds for D_{t1} into the fourth term (a4) yields analogous relations.

□

Since there are positive and negative price differences in the real-world price data, it may be worth analyzing the impact of non-convexities not only in the framework of unrestricted suppliers, but also extending the previous considerations to restricted suppliers.

The idea of considering non-convexities in the supply curve of either unrestricted or restricted suppliers may be motivated through a simplified illustration of the merit order for the German power plant fleet and its neighboring countries[7] assuming unlimited cross-border transmission capacity. The simplifying classification that unrestricted suppliers embody German power plants, whereas restricted suppliers are, in particular, located in the neighboring countries, is derived from Knaut and Paschmann (2017a). The authors stress that the lack of sub-hourly market coupling is the most relevant driver of restricted participation in the intraday auction. Figure 3 depicts the respective marginal costs depending on the underlying fuel costs as well as the CO_2 emission costs. Non-dispatchable renewable electricity generation is neglected. In addition, Figure 4 illustrates the aggregate supply curve of both types of suppliers. Hereby, it is facilitated to derive conclusions on the impact of non-convexities in either the restricted or unrestricted supply curve depending on the overall demand level. The underlying data is extracted from the fundamental electricity market model DIMENSION which is presented in more detail in Knaut and Paschmann (2017a) and (Richter, 2011). For simplification purposes and as the bidding behavior of flexible pumped storage generation units is a complex

[6]To bridge the gap to the mathematical proof in Appendix.4, it is assumed that $\epsilon \rightarrow 0$.

[7]Denmark, the Netherlands, Belgium, France, Switzerland, Austria, Poland and the Czech Republic

issue which is not in the focus of this paper, the respective generation capacities are illustrated in both edge regions of the merit order. Depending on the operational mode, pumped storage power plants may buy cheap electricity and produce electricity in periods with comparably higher prices.

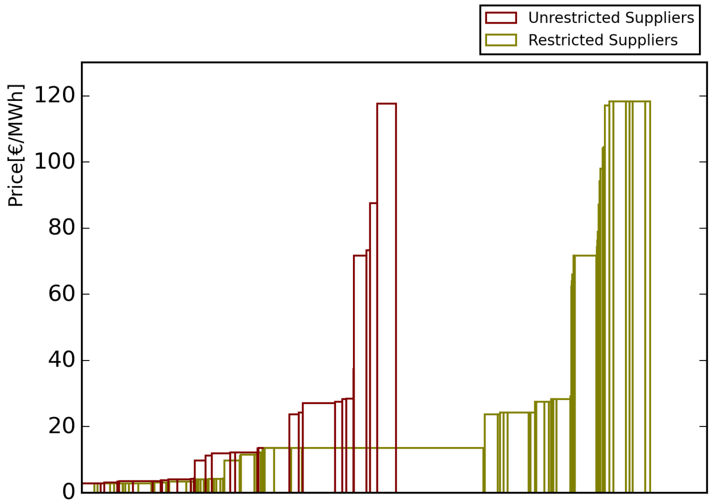

Figure 3: Stylized illustration of the unrestricted and restricted supply curves

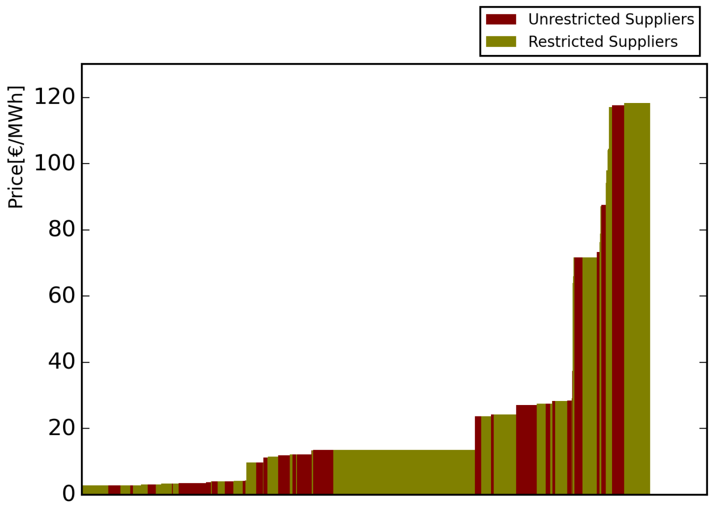

Figure 4: Stylized aggregate merit order for Germany and its neighboring countries

The figures already convey the idea that there are significant non-convexities in particular sections of the merit order. These considerations are also reflected and observable in real-world bid curve data from the German day-ahead and intraday auction (Section Appendix.3). The prevalence of non-convexities being more pronounced in either the restricted or unrestricted supply curve may vary depending on the overall demand level. Non-convexities within the German merit order tend to be especially relevant if demand is rather high.

Proposition 3. *Negative price premiums may stem from discontinuities in the marginal cost curve of restricted suppliers. The maximum price difference is bounded by* $-\Delta_{disc'}$*, which is the step height of the respective discontinuity. Overall, the frequency of positive and negative price differences depends on the market clearing being in particular sections of the merit order where non-convexities in either the unrestricted or restricted supply curve are more pronounced.*

Proof. The detailed proof is similar to the previous one and outlined in Section Appendix.6. As a result, if the merit order of restricted suppliers exhibits a non-convex section, Equation (9) depicts the optimal production level.

$$q^{r*} = \begin{cases} \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} & (b1) D_{t2} + D_{t1} \leq \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)}{a_1^u} \\ Q_{disc'} + \epsilon & (b2) \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)}{a_1^u} < D_{t1} + D_{t2} \leq \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)+2\cdot\Delta_{disc'}}{a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} - \frac{\Delta_{disc}}{a_1^r+a_1^u} & (b3) D_{t1} + D_{t2} > \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)+2\cdot\Delta_{disc'}}{a_1^u} \end{cases} \quad (9)$$

Price implications can be derived analogous to the procedure applied in Equation 8 in Proposition 2. The relation $p(m^1) = C_r'(q^r)$, however, is no longer a necessary condition as the average marginal costs of unrestricted suppliers now exceed the respective marginal costs of restricted suppliers. Depending on the trading decision of unrestricted suppliers, the first stage market price may either be $p(m^1) = C_r'(q^r)$ or $p(m^1) = \frac{(C'_{u,t1}(q^r)+C'_{u,t2}(q^r))}{2}$. However, the arbitrage-free second market's price may exceed the respective price in the first market at equilibrium since the non-convexity eliminates additional opportunities for arbitrage[8].

□

As regards the electricity markets under consideration, it may furthermore bring value added to analyze additional costs which are attributable to the non-convexities. Against this backdrop, the costs in a framework with non-convexities could be compared to a benchmark which would be a continuous merit order for both types of suppliers. Thereby, the monetary value of smoothing non-convexities is calculated. The respective difference in costs yields the value of additional flexibility from a system perspective. Once more, the classification unrestricted suppliers is linked to national generation capacity.

Proposition 4. *A naive proxy for the additional costs attributable to non-convexities within the supply curve of unrestricted suppliers may be estimated as EUR 10.2 million in 2015. Furthermore, a similar proxy for the value of additional power system flexibility in neighboring countries may be derived. The respective estimate is EUR 6.4 million.*

Proof. First, the case of a non-convexity within the cost function of unrestricted suppliers is considered. It is sufficient to analyze the respective additional costs in terms of the scenario (a2) due to symmetric relations. For the sake of simplicity, the lower threshold for D_2 in the case of (a2) may be substituted with the term $\hat{D}_t$ and Equation (10) could be defined.

$$\Delta Costs(D_t - \hat{D}_t) = \Delta Costs(x)$$
$$= \frac{4\cdot(a_1^r)^5 + 12\cdot(a_1^r)^4\cdot a_1^u + 14\cdot(a_1^r)^3\cdot(a_1^u)^2 + 8\cdot(a_1^r)^2\cdot(a_1^u)^3 + \frac{9}{4}\cdot a_1^r\cdot(a_1^u)^4 + \frac{1}{4}\cdot(a_1^u)^5}{8\cdot(a_1^r)^4 + 24\cdot(a_1^r)^3\cdot a_1^u + 26\cdot(a_1^r)^2\cdot(a_1^u)^2 + 12\cdot a_1^r\cdot(a_1^u)^3 + 2\cdot(a_1^u)^4} \cdot x^2 \quad (10)$$

[8] In the real world, the strategic rationale of agents on the demand side may support this allocation as they face an incentive not to pay the higher marginal costs of unrestricted suppliers to all restricted suppliers within a uniform price auction.

The respective cost implications are mainly driven by the gradient of the restricted supply curve (see Section Appendix.5). Analogous results can be derived for the case of a discontinuous marginal cost curve of restricted suppliers.

$$\begin{aligned} &\Delta Costs(D_t - \hat{D}_t) = \Delta Costs(x) \\ &= \frac{\frac{1}{4} \cdot a_1^r \cdot (a_1^u)^2 + \frac{1}{4}(a_1^u)^3}{2 \cdot ((a_1^r)^2 + 2 \cdot a_1^r \cdot a_1^u + (a_1^u)^2)} \cdot x^2 \end{aligned} \tag{11}$$

Here welfare losses are mainly driven by the gradient of the unrestricted supply curve.

To derive estimates from real-world price data, as x is unobservable, the relation $D_t - \hat{D}_t = x = \frac{\Delta(p(m^1), p(m_t^2))}{a_1^r}$ may be exploited . In the case of negative price premiums, a_1^r is to be substituted by a_1^u.

To derive numbers for the gradients a_1^r and a_1^u, empirical estimates presented in Knaut and Paschmann (2017b) can be used. The aggregate day-ahead merit order exhibits a gradient of approximately 0.96 EUR/GWh and is defined by $a_1^{r,u} = \frac{a_1^r \cdot a_1^u}{a_1^r + a_1^u}$. Additionally, the aggregate gradient of the unrestricted suppliers (a_1^u) may be approximated as 7.65 EUR/GWh. This yields an estimate for a_1^r which is 1.1 EUR/GWh. Inserting these estimates into (10), additional costs due to non-convexities may be calculated according to Equation (12).

$$\Delta Costs(\Delta(p(da), p(id_t))) \begin{cases} = 1145.5 \cdot \Delta(p(da), \overline{p(id_t)})^2 & \Delta(p(da), p(id_t)) < 0 \\ = 690.94 \cdot \Delta(p(da), \overline{p(id_t)})^2 & \Delta(p(da), p(id_t)) > 0 \end{cases} \tag{12}$$

The term da marks the day-ahead and id the intraday auction. Applying the respective relations to all price differences observed in 2015 (EPEX SPOT SE, 2016a), the cost terms presented above can be derived.

As regards the interpretation of these estimates, the limits of this approach should be taken into account. Nevertheless, the respective numbers yield a naive indication for the magnitude of the value of additional short-term power system flexibility.

□

4.1. Numerical Example

For illustration purposes, this section provides a simple numerical example. More precisely, the set of parameters is defined according to Table 1.

Parameter	Value
a_1^r	1
a_1^u	2
$Q_{disc'}$	4
$\Delta_{disc'}$	5
Q_{disc}	12
Δ_{disc}	10

Table 1: Numerical example: Parameter assumptions

Motivated through empirical observations in the electricity markets under analysis, it is assumed that the gradient of the unrestricted supply curve (a_1^u) exceeds the respective gradient of the restricted supply curve (a_1^r). The increment is assumed to be twice as high as in the case of restricted producers. As regards restricted suppliers, a non-convexity ($\Delta_{disc'}$) is considered if the production level is rather low ($Q_{disc'}$). In contrast, in the case of unrestricted suppliers, there is a discontinuous step (Δ_{disc}) at a higher quantity (Q_{disc}).

Demand first increases according to a linear shape and then decreases again. The resulting hourly production level over time of both restricted and unrestricted suppliers is illustrated in Figure 5.

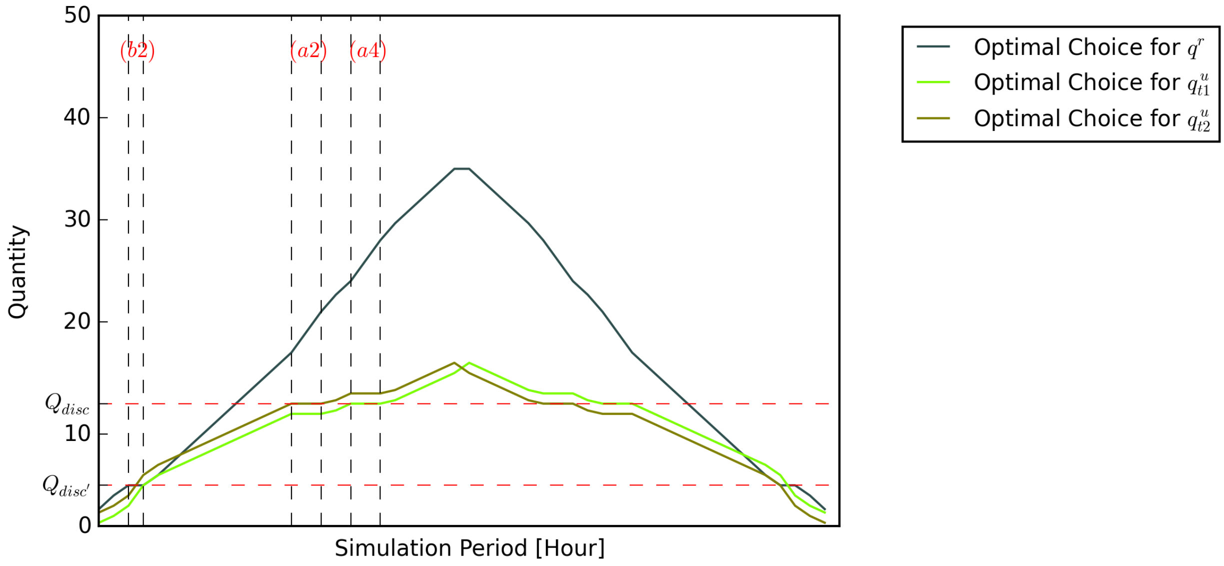

Figure 5: Optimal production level in the numerical example

If the production level of restricted suppliers approximates the non-convexity threshold ($Q_{disc'} = 5$), the respective supply is fixed and compensated by an increased production level of unrestricted suppliers ($b2$). Based on the gradient of the unrestricted supply curve ($a^u_1 = 2$), this allows for cost savings as long as the additional supply of unrestricted suppliers in both time periods does not exceed the quantity 2.5. If demand continues to increase, the overall production level of restricted suppliers is adjusted downward compared to the case of continuous relations. As a consequence, all thresholds for D_{t2}, which were presented in Equation (6), are adjusted by subtracting the term $\frac{\Delta disc'}{a^r_1 + a^u_1}$. If the optimal production level of unrestricted suppliers under continuous relations would now exceed the respective discontinuous step ($Q_{disc} = 12$), reverse relations can be identified and additional supply by unrestricted suppliers is replaced by an increased production level of restricted suppliers (($a2$) and ($a4$)).

In a next step, conclusions on the resulting price differences between both markets may be drawn. Figure 6 presents the simulated prices for the numerical example. For illustration purposes, the maximum feasible price difference for the case of a non-convexity within the restricted supply curve is considered.

Dependent on the demand level, the direction of price differences at equilibrium may vary. If the non-convexity in the restricted supply curve causes an unbalanced increase of production by unrestricted suppliers, the price in the market with shorter contracts is higher compared to the first market with unre-

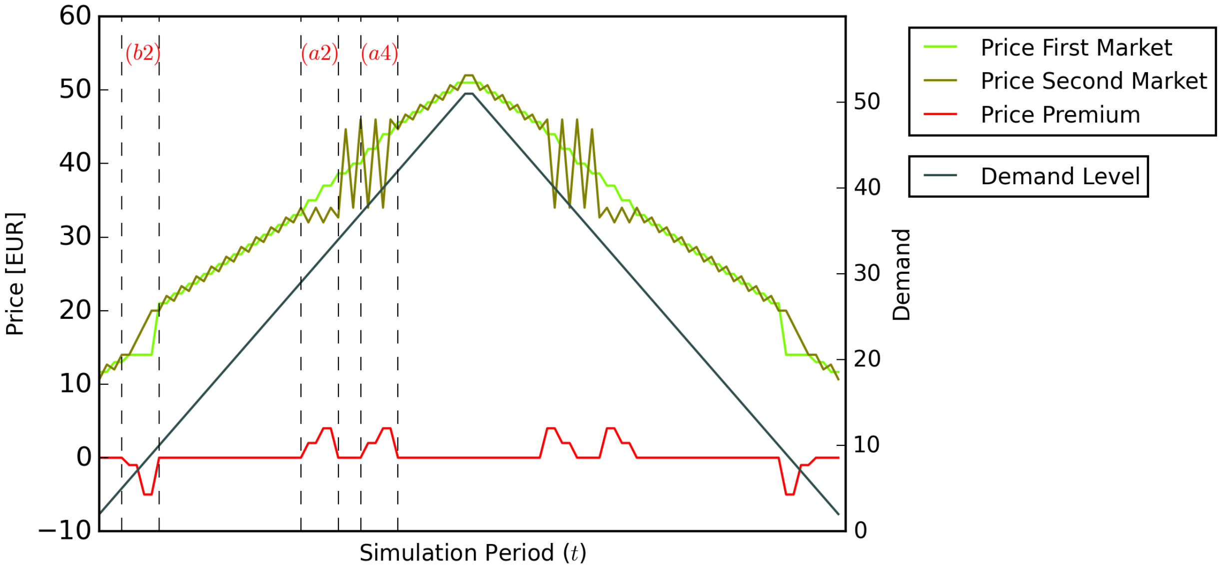

Figure 6: Price implications in the numerical example

stricted participation and vice versa. If the demand level exhibits a high temporal variability ($D_{t1} \neq D_{t2}$), the impact of a non-convexity in the unrestricted supply curve on the respective price premiums is limited to half the step height (Δ_{disc}). Finally, it is worth mentioning that price volatility may temporarily increase significantly if unrestricted supply in only one period exceeds the non-convexity.

5. Empirical Application

Following the theoretical model, price premiums in sequential markets with differing product granularity may stem from non-convexities being more pronounced in only a subset of the sequential market stages. As a prerequesite, a differing supplier structure in the German day-ahead and intraday auction can be identified which is triggered by restricted participation in the market with sub-hourly products (Knaut and Paschmann, 2017a). The sharp stepped shape of the underlying bid curves may result in a varying frequency of non-convexities dependent on the market settlement being in particular sections of the merit order. Bearing this is mind, a systematic correlation between the individual supply curves and the resulting price differences is to be expected. An analysis of the correlation of load and price premiums by the use of historical data facilitates to test for these relations.

Based on the illustration of marginal cost curves for generation units in Germany and its neighboring countries in Figure 3 and Figure 4, the following expectations could be formulated[9]:

[9]It is worth stressing that actual bidding data may not fully comply with the fundamental merit order.

1. Non-convexities are more pronounced in the right area of the German merit order. The day-ahead price may hence be expected to exceed the respective intraday auction price if demand is rather high.
2. Compared to its neighboring countries, Germany has a comparably large share of low-cost generation units, for example, due to nuclear and lignite-fired power plants. Negative price premiums, if any, would rather be expected to coincide with low demand.
3. The overall stepped shape of the merit order is less pronounced within the smoother marginal cost curve of restricted suppliers. The extremes of positive price premiums may hence exceed the maximum negative price differences.

It has to be annotated that the use of a common aggregate supply curve crucially depends on the assumption of sufficient cross-border capacity. A respective lack may trigger additional non-convexities. The empirical analysis will provide insights with respect to the validity of the three hypotheses. Since in short-term electricity markets the residual load is commonly used in order to map demand, data was gathered which is provided by ENTSO-E (2017) and EEX (2017) to derive the residual demand as the difference between the overall system load and the electricity generation from renewable energy plants. The period of observation ranges from January 16, 2015 until November 2, 2016. Figure 7 illustrates the average hourly deviation of the residual demand from its overall mean. Positive values hence embody hours with a comparably high residual demand. Apparently, daily profiles of the residual load exhibit distinct recurrent patterns. Furthermore, the corresponding average price premiums for each hour of the day along the period of observation are presented. The figure indicates a high correlation between the residual load and the resulting price differences. If the residual demand tends to be comparably high, the day-ahead price is on average higher than the respective intraday auction price. Reverse relations are applicable to hours with a tendency of lower demand.

The initial hypotheses are confirmed by the empirical observations. The historical data yield an indication that in peak hours the non-convexities in the German supply curve are more pronounced. A lack of national peak load generation units or pumped storage power plants may trigger additional electricity imports. There are incentives to target excess supply within the day-ahead market to avoid extremely high costs of purchasing additional quantities from unrestricted suppliers in the intraday auction. As a result, the day-ahead price could be above the respective average intraday auction price[10] (*Hypothesis 1*).

[10]Both types of suppliers are expected to prefer trading in the day-ahead market. In order to meet the residual demand profile, unrestricted suppliers which committed their production capacity via hourly contracts are willing to pay a price equal to their marginal production costs to reduce their electricity generation by trading sub-hourly contracts within the intraday auction.

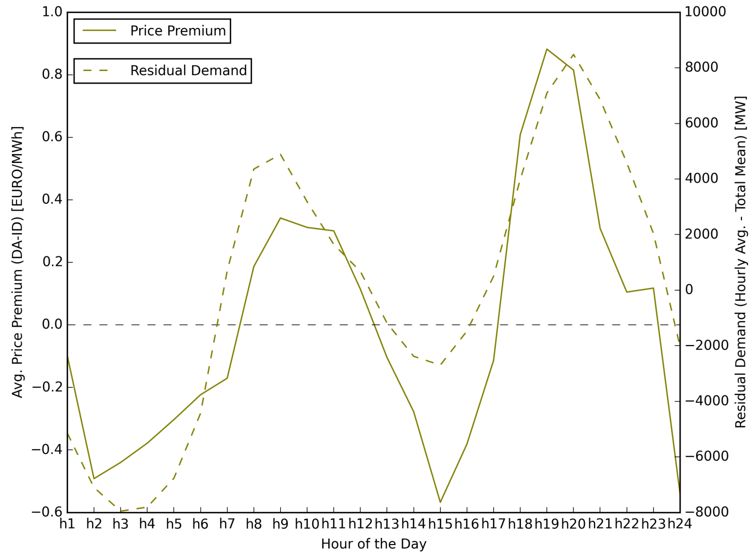

Figure 7: Average correlation of residual demand and price premiums (January 16, 2015 - November 2, 2016)

In contrast, if the residual demand is rather low, the discontinuous shape of the supply curve of restricted producers tends to drive the resulting price premiums. A comparably large share of demand is satisfied by unrestricted suppliers. The German intraday auction price may exceed the respective day-ahead price as restricted suppliers do not face the opportunity to shift their trade quantities into the intraday auction. At the same time, the respective price difference is arbitrage-free due to the non-convexity within the restricted supply curve (*Hypothesis 2*). It is finally worth mentioning that the maximum positive price premium is higher than the maximum negative price difference (*Hypothesis 3*).

To deepen the understanding of the analysis, Figure 8 presents empirical results with respect to the role of seasonality. The classification of seasons is based on metereological dates. As to be expected, the difference between the average residual load and its overall mean is more pronounced in winter than in summer periods. Accordingly, the extremes of the price premiums increase by approximately 70% ranging from 0 to 130 ct/Mwh. These relations support the hypothesis that the impact of non-convexities is especially relevant in case of extreme demand values.

6. Conclusion

This article begins with an analysis of price premiums between two sequential short-term electricity markets in Germany, namely the day-ahead and intraday auction. The framework under analysis is characterized by decreasing contract duration and differing market participation. As both markets are settled

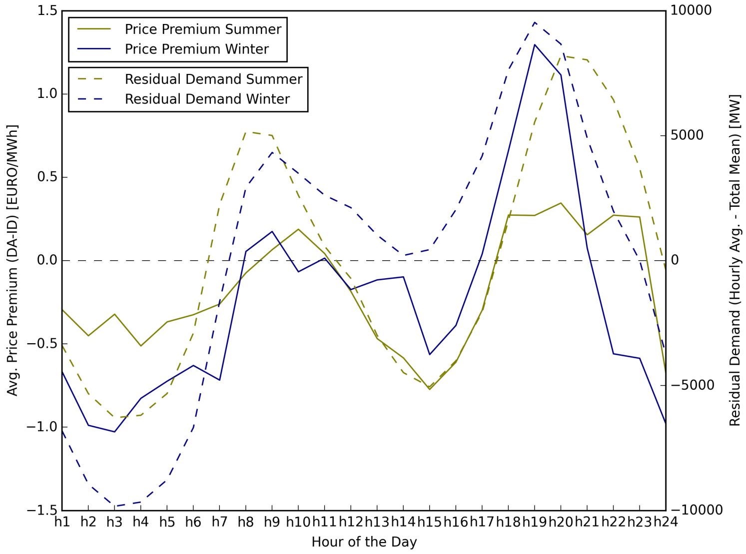

Figure 8: Average correlation of residual demand and price premiums in specific seasons(January 16, 2015 - November 2, 2016)

in rapid succession without any relevant informational update, it is initially puzzling to identify significant price premiums in specific hours of the day. Furthermore, these price premiums can be both positive or negative.

There is empirical evidence that the explanatory approach for price premiums in electricity markets, which was developed in Bessembinder and Lemmon (2002), is not applicable to the market dynamics under analysis. To address these issues, a theoretical model is developed within this article which seeks to analyze the impact of non-convexities in sequential market designs with differing market participation. The approach is motivated through the observation of pronounced stepped shapes in real-world bid curve data. Based on the model, it can be identified that if non-convexities are more pronounced in individual sequential market stages, which is feasible due to the differing supplier structure, significant price premiums may exist. Additionally, the difference in prices may be both positive or negative depending on the relevance of non-convexities in particular sections of the underlying supply curves. This article presents an empirical analysis of real-world data from German electricity markets. The respective results reveal a correlation between load and price differences what essentially complies with the underlying model.

The empirical observations suggest that non-convexities in the supply curve of neighboring countries are especially relevant if the German residual demand is rather low. As a consequence, the average intraday auction price may be significantly higher than the respective day-ahead price. Reverse relations can be observed in peak hours indicating sharp non-convexities in the German merit order due to a lack of flexible peak load generation units. There are incentives to target excess supply in the day-ahead auction and hence the respective price may go beyond the average intraday auction price.

These findings allow to draw the conclusion that the price premiums under consideration reflect a value of additional short-term power system flexibility. In more detail, numerical proxies can be derived yielding a value for smoothing non-convexities of approximately EUR 10.2 million in 2015 in the case of additional German power system flexibility. In contrast, the respective estimate for neighboring countries is EUR 6.4 million in 2015. Even if these are relatively small numbers, the inefficiencies uncovered may be exacerbated if the share of renewable energies continues to increase and if there is a lack of investment incentives for flexible generation units. It may furthermore be beneficial to urge the implementation of cross-border trade on a sub-hourly level to align the supplier structures in the day-ahead and intraday auction.

The model developed in this article supports a better understanding of price premiums and its underlying properties in short-term sequential electricity markets. However, it may also be applicable to other frameworks, for example, sequential auctions with block and single unit bids. Finally, the findings presented in this article may favor the evaluation of business strategies targeting to exploit the price differences identified. A lack of a respective business case is to be expected, as the market depth in the intraday auction is limited and since the price premiums identified do not reflect unexploited arbitrage opportunities.

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Appendices

Appendix.1. Descriptive Analysis of Historical Price Premiums

Hour	Mean	Mean (abs difference)	Probability positive	Min/Max	Percentiles (10/90)
h1	-0.2	2.0	44.7%	-18.3/13.2	-3.0/3.0
h2	-0.6	2.0	39.0%	-10.9/16.1	-3.5/2.5
h3	-0.6	2.4	41.4%	-17.8/16.2	-4.2/3.1
h4	-0.5	2.5	42.6%	-13.2/17.5	-4.3/3.6
h5	-0.4	2.3	41.2%	-12.7/11.8	-3.6/3.2
h6	-0.3	2.1	44.1%	-18.6/20.2	-3.6/2.9
h7	-0.3	2.2	48.6%	-29.5/16.3	-3.6/3.0
h8	0.1	1.8	53.1%	-21.7/9.5	-2.6/3.0
h9	0.2	1.8	55.1%	-8.6/13.2	-2.7/3.1
h10	0.2	1.7	53.7%	-13.4/9.2	-2.3/2.7
h11	0.2	1.7	55.1%	-21.3/9.7	-2.5/2.9
h12	0.0	2.0	50.4%	-44.2/13.4	-2.9/3.2
h13	-0.2	2.0	46.0%	-29.5/18.6	-3.3/3.0
h14	-0.4	2.1	44.4%	-27.0/13.5	-3.5/2.9
h15	-0.7	2.1	41.9%	-41.7/17.7	-3.5/2.3
h16	-0.5	1.9	44.0%	-37.8/10.7	-3.1/2.4
h17	-0.2	1.8	49.8%	-39.5/14.3	-2.9/2.5
h18	0.5	1.9	60.3%	-8.2/18.9	-2.4/3.3
h19	0.8	1.9	63.7%	-10.6/11.9	-2.0/3.5
h20	0.7	1.9	64.5%	-7.5/11.3	-2.2/3.7
h21	0.2	1.7	57.0%	-8.0/9.1	-2.5/3.0
h22	0.0	1.8	51.2%	-7.4/8.5	-2.8/2.8
h23	0.0	2.0	52.4%	-8.5/10.6	-3.3/3.2
h24	-0.7	2.1	41.6%	-12.4/12.7	-4.1/2.7
Total	-0.1	2.0	49.4%	-44.2/20.2	-3.1/3.0

Table .2: Descriptive statistics on price premiums between the day-ahead and intraday auction (day-ahead price - average intraday auction price) [EUR/MWh] (January 16, 2015 until November 2,2016)

The numbers presented in column *Mean (abs difference)* reveal that the mean of the four 15-minute intraday auction prices is on average 2 EUR/MWh lower or higher respectively than the corresponding hourly day-ahead auction price. Thus, there tend to be significant differences in prices in each hour. Additionally, the probability of differences in prices being positive or negative is presented in column *Probability positive.*

Appendix.2. Empirical Analysis of Price Premiums and the Underlying Drivers

Table .3 depicts descriptive statistics on price premiums between the day-ahead and intraday auction. Furthermore, the table shows the respective variance and skewness of the underlying day-ahead spot prices. The analysis is based on real-world data which is provided by (EPEX SPOT SE, 2016a) and ranges from

January 16, 2015 until November 2,2016. The Pearson's first coefficient is used as the skewness measure which is the difference between the mean and the mode divided by the standard deviation.

Hour	Mean price difference	Variance of day-ahead prices	Skewness of day-ahead prices
h1	-0.2	88.7	-1.17
h2	-0.6	84.8	-1.13
h3	-0.6	82.4	-1.55
h4	-0.5	81.3	-1.42
h5	-0.4	84.3	-1.54
h6	-0.3	102.0	-1.50
h7	-0.3	178.2	-0.80
h8	0.1	191.7	-0.30
h9	0.2	199.1	-0.23
h10	0.2	155.7	0.01
h11	0.2	153.5	0.46
h12	0.0	214.8	2.58
h13	-0.2	129.8	-0.50
h14	-0.4	159.2	-1.65
h15	-0.7	157.1	-1.85
h16	-0.5	147.2	-0.71
h17	-0.2	149.2	-0.66
h18	0.5	176.4	0.20
h19	0.8	179.9	0.38
h20	0.7	149.6	0.54
h21	0.2	107.8	-0.08
h22	0.0	75.0	-0.26
h23	0.0	73.5	-0.16
h24	-0.7	73.4	-0.54

Table .3: Descriptive statistics on price premiums between the day-ahead and intraday auction (day-ahead price - average intraday auction price) [EUR/MWh] (January 16, 2015 until November 2,2016)

To analyze the correlation between the individual figures in a condensed way, a simple Ordinary Least Squares (OLS) estimation may be applied which is following the general idea adopted in Longstaff and Wang (2004). The respective results are illustrated in Table .4. Even if there are issues linked to the small sample size, it is yet to be expected that the results provide insights on the question of basic correlations. The respective results yield an indication that there is no significant impact of the price volatility on the respective price premiums. Furthermore, the correlation between the skewness of day-ahead prices and price premiums is at least questionable. Finally, the F test does not allow for a unique conclusion on whether the model is more accurate than basically no model at all.

Dependent variable: Price Premium	
Explanatory variable	OLS
day-ahead price volatility	0.0015
	(0.0013)
day-ahead price skewness	0.248
	(0.124)
intercept (ν)	-0.19
	(0.21)
observations	24
adj. R^2	0.48
F	2.97 (p-value:0.07)

Notes to Table .4: Robust standard errors in parentheses. * / ** / *** : significant at the 0.05 /0.02 / 0.01 error level respectively. Data from January 16, 2015 until November 2,2016 is used.

Table .4: Regression of price premiums between the day-ahead and intraday auction (day-ahead price - average intraday auction price) [EUR/MWh]

Appendix.3. Exemplary Historical Bid Curves

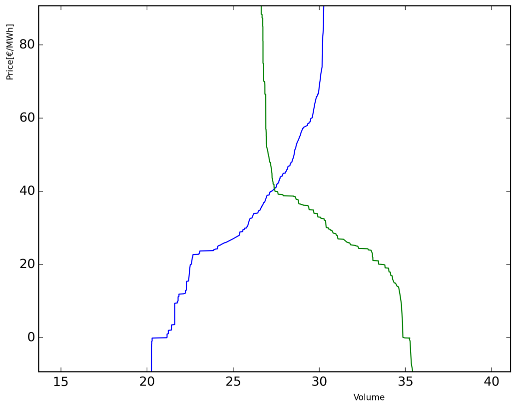
Exemplary historical bid curves are illustrated in Figure .9.

Appendix.4. Mathematical Proof (Proposition 1)

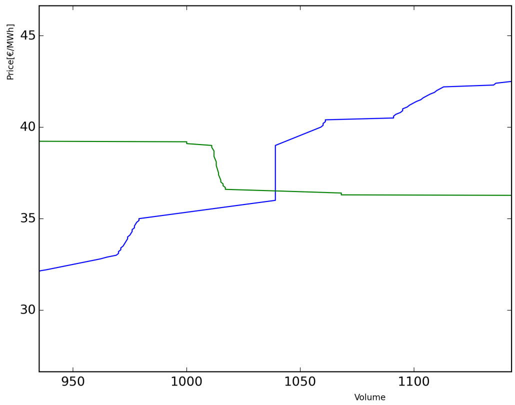
Since the procedure is based on cost minimization, the overall production costs for restricted suppliers can be calculated with the use of Equation (.1).

$$\begin{aligned} C_r(q^r) &= \int_0^{q^r} \left[a_0 + a_1^r \cdot q \right] dq \\ &= a_0 \cdot q^r + 0.5 \cdot a_1^r \cdot (q^r)^2 \end{aligned} \tag{.1}$$

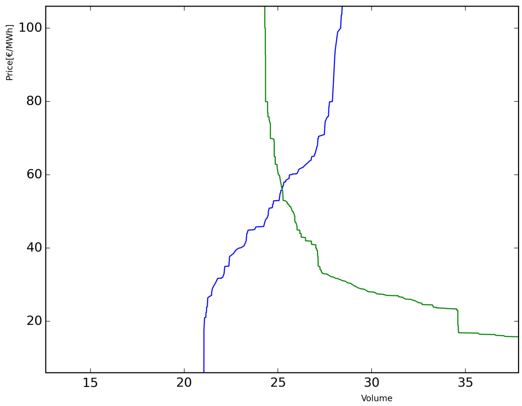
As regards the production level of unrestricted suppliers, the respective optimization variables (${q_{t1}^u}^*$ and ${q_{t2}^u}^*$) can directly be substituted according to the equilibrium condition (3). Thus, the relation ${q_t^u}^* = D_t - {q^r}^*$ may be used so that the variable q^r remains the only decision variable as demand is assumed to be inelastic. The cost function of unrestricted suppliers in period t ($C_{u,t}(q^r)$) may hence be formulated according to Equation (.2).



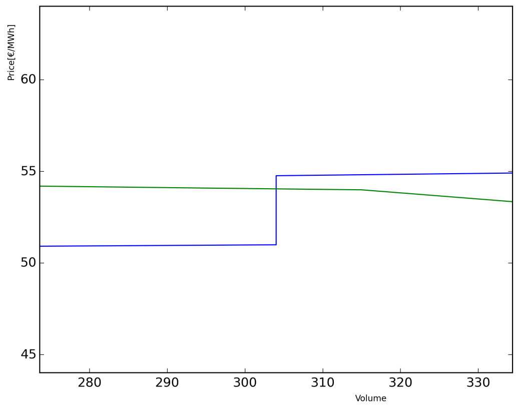
(a) Day-ahead auction May 22, 2015 (7:00/8:00)



(b) Intraday auction May 22, 2015 (7:45/8:00)



(c) Day-ahead auction October 8, 2015 (18:00/19:00)



(d) Intraday auction October 8, 2015 (18:15/18:30)

Figure .9: Exemplary bid curves observed in the day-ahead and intraday auction

$$C_{u,t}(q^r) = 0.5 \cdot \begin{cases} |1\rangle\, 0.5 \cdot \int_0^{D_t - q^r} a_0 + a_1^u q \, dq & \\ \quad = 0.5 \cdot \left[a_0 \cdot (D_t - q^r) + 0.5 \cdot a_1^u \cdot (D_t - q^r)^2 \right] & D_t - q^r \leq Q_{disc} \\ |2\rangle\, 0.5 \cdot \left[\int_0^{Q_{disc}} a_0 + a_1^u q \, dq + \int_{Q_{Disc}+\epsilon}^{D_t - q^r} a_0 + \Delta_{disc} + a_1^u q \, dq \right] & \\ \quad = 0.5 \cdot \left[a_0 \cdot (Q_{disc}) + 0.5 \cdot a_1^u \cdot (Q_{disc})^2 \right. & \\ \quad\quad + (a_0 + (Q_{disc} + \epsilon) \cdot a_1^u + \Delta disc) \cdot (D_t - q^r - Q_{disc} - \epsilon) & \\ \quad\quad \left. + 0.5 \cdot a_1^u \cdot (D_t - q^r - Q_{disc} - \epsilon)^2 \right] & D_t - q^r \geq Q_{disc} + \epsilon \end{cases} \tag{.2}$$

An infinitesimal small number ϵ is considered in order to formulate the optimization problem with weak inequalities only. The value of ϵ reflects the smallest tradeable increment of the production level. As the cost function to apply depends on the value of the decision variable, the respective Lagrangian can be set up depending on both cases i) $D_t - q^r \leq Q_{disc}$ and ii) $D_t - q^r \geq Q_{disc} + \epsilon$ for each of the time periods $t1$ and $t2$ to solve the optimization problem. For the sake of simplicity, is is assumed that the demand profile is increasing and consequently there is a reduced number of cases to be considered. That is to say, $D_{t2} > D_{t1} + \epsilon$ is valid. As the second derivative of the total cost function ($C^{total} = C_r(q^r) + \sum_t C_{u,t}(q_t^u)$) is always positive ($\frac{\partial^2 C^{total}}{\partial {q^r}^2} = a_1^r + a_1^u > 0$), all optima are local minimums.

Appendix.4.1. ***Case1:*** $D_t - q^r \leq Q_{disc} \; \forall t$

In the first case, the discontinuity is negligible due to comparably low demand. The respective Lagrangian is presented in Equation (.3).

$$\mathbb{L} = (-1) \cdot (C_r(q^r) + \sum_t C_{u,t}(q^r)) + \mu_{t1} \cdot (Q_{disc} - D_{t1} + q^r) + \mu_{t2} \cdot (Q_{disc} - D_{t2} + q^r) \tag{.3}$$

Applying the respective Karush-Kuhn-Tucker (KKT) conditions, the optimal solution has to satisfy the conditions listed in (.4).

$$\begin{aligned} (-1) \cdot \frac{\partial C_r(q^r)}{\partial q^r} + \sum_t \left[(-1) \cdot \frac{\partial C_{u,t}(q^r)}{\partial q^r} + \mu_t \cdot (Q_{disc} - D_t + q^r) \right] = 0 \\ D_t - q^r \leq Q_{disc} \; \forall t \\ \mu_t \cdot (Q_{disc} - D_t + q^r) = 0 \; \forall t \\ \mu_t \geq 0 \; \forall t \end{aligned} \tag{.4}$$

The respective marginal cost functions are formulated in Equation (.5).

$$\begin{aligned} \frac{\partial C_r(q^r)}{\partial q^r} &= a_0 + q^r \cdot a_1^r \\ \frac{\partial C_{u,t}(q^r)}{\partial q^r} &= -0.5 \cdot a_0 - 0.5 \cdot a_1^u \cdot (D_t - q^r) \end{aligned} \tag{.5}$$

There is a need to apply a distinction of cases. More precisely, the following four scenarios have to be considered to derive the optimal solution.

1. *Scenario1:* $\mu_{t1} = 0, \mu_{t2} = 0$
2. *Scenario2:* $\mu_{t1} = 0, Q_{disc} - D_{t2} + q^r = 0$
3. *Scenario3:* $Q_{disc} - D_{t1} + q^r = 0, \mu_{t2} = 0$
4. *Scenario4:* $Q_{disc} - D_{t1} + q^r = 0, Q_{disc} - D_{t2} + q^r = 0$

Due to the assumption of an increasing demand profile ($D_{t2} > D_{t1} + \epsilon$), *Scenario4* may be ignored. Furthermore, *Scenario3* is to be neglected because of the definition of *Case1*. In the following, both relevant scenarios are analyzed in detail.

Scenario1: $\mu_{t1} = 0, \mu_{t2} = 0$

Solving the KKT conditions, the optimal choice for q_r can be formulated according to Equation (.6). This is essentially the same solution as presented in Knaut and Paschmann (2017b) for the case of continuous relations.

$$q^{r\,*} = \frac{(D_{t1} + D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r + a_1^u} \tag{.6}$$

Such quantity choice yields a valid solution ($D_{t2} - q^{r\,*} \leq Q_{disc}$) if condition (.7) is met.

$$D_{t2} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r + a_1^u) + D_{t1} \cdot a_1^u}{2 \cdot a_1^r + a_1^u} \tag{.7}$$

Scenario2: $\mu_{t1} = 0, Q_{disc} - D_{t2} + q^r = 0$

According to the scenario definition, q_r is defined as follows:

$$q^{r\,*} = D_{t2} - Q_{disc}. \tag{.8}$$

Solving for μ_{t2}, Equation (.9) can be derived.

$$\mu_{t2} = (-1) \cdot \frac{(D_{t1} + D_{t2}) \cdot a1^u}{2 \cdot (a_1^r + a_1^u)} \tag{.9}$$

Equation (.9) does not yield a feasible solution as the respective condition $\mu_{t2} \geq 0$ is not satisfied. This is due to the assumption of positive values for both the gradients of the supply curves as well as the demand in the periods $t1$ and $t2$ ($D_{t1}, D_{t2}, a_1^r, a_1^u > 0$).

Appendix.4.2. ***Case2:*** $D_{t1} - q^r \leq Q_{disc}, D_{t2} - q^r \geq Q_{disc} + \epsilon$

Based on the solution for *Case1*, a threshold for D_{t2} can be derived above which the discontinuity has to be considered (Equation (.7)).

The resulting KKT conditions in *Case2* are depicted in (.10).

$$
\begin{aligned}
(-1)\cdot\frac{\partial C_r(q^r)}{\partial q^r}+\sum_t\left[(-1)\cdot\frac{\partial C_{u,t}(q_t^u)}{\partial q^r}\right]+\mu_{t1}-\mu_{t2}=0\\
D_{t1}-q^r\le Q_{disc}\\
D_{t2}-q^r\ge Q_{disc}+\epsilon\\
\mu_{t1}\cdot(Q_{disc}-D_t+q^r)=0\\
\mu_{t2}\cdot(Q_{disc}+\epsilon-D_t+q^r)=0\\
\mu_t\ge 0\ \forall t
\end{aligned}
\tag{.10}
$$

The respective marginal cost functions are listed in (.11).

$$
\begin{aligned}
\frac{\partial C_r(q^r)}{\partial q^r}&=a_0+q^r\cdot a_1^r\\
\frac{\partial C_{u,t1}(q^r)}{\partial q^r}&=-0.5\cdot a_0-0.5\cdot a_1^u\cdot(D_{t1}-q^r)\\
\frac{\partial C_{u,t2}(q^r)}{\partial q^r}&=-0.5\cdot(a_0+(Q_{disc}+\epsilon)\cdot a_1^u+\Delta_{disc})-0.5\cdot a_1^u\cdot(D_{t2}-q^r-Q_{disc}-\epsilon)
\end{aligned}
\tag{.11}
$$

A distinction of cases is to be applied.

1. *Scenario1:* $\mu_{t1}=0,\mu_{t2}=0$
2. *Scenario2:* $\mu_{t1}=0,Q_{disc}+\epsilon-D_{t2}+q^r=0$
3. *Scenario3:* $Q_{disc}-D_{t1}+q^r=0,\mu_{t2}=0$
4. *Scenario4:* $Q_{disc}-D_{t1}+q^r=0,Q_{disc}+\epsilon-D_{t2}+q^r=0$

Scenario4 is irrelevant due to the assumption of an increasing demand profile. In the following, each scenario is outlined in more detail.

Scenario1: $\mu_{t1}=0,\mu_{t2}=0$

In the case of *Scenario1*, Equation (.12) depicts the optimal choice with respect to the production level of restricted suppliers(q^r).

$$
q^{r\,*}=\frac{(D_{t1}+D_{t2})}{2}\cdot\frac{a_1^u}{2\cdot(a_1^r+a_1^u)}+\frac{\Delta_{disc}}{2\cdot(a_1^r+a_1^u)}
\tag{.12}
$$

In *Case2* the discontinuity is relevant and thus the quantity supplied by restricted suppliers is revised upwards by $\frac{\Delta_{disc}}{2\cdot(a_1^r+a_1^u)}$ to account for the stepped shape of the merit order of unrestricted suppliers. Equation (.6) may be regarded as a feasible solution ($D_{t2}-q^r\ge Q_{disc}$) if condition (.13) is fulfilled.

$$
q^{r\,*}=\frac{(D_{t1}+D_{t2})\cdot a_1^u+\Delta_{disc}}{2\cdot(a_1^r+a_1^u)}\le D_{t2}-Q_{disc}-\epsilon
\tag{.13}
$$

Condition (.13) may be transferred into three possible cases:

1. $-2 \cdot D_{t2} \cdot a_1^r + 2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) + D_{t1} \cdot a_1^u - D_{t2} \cdot a_1^u + \Delta_{disc} = 0, a_1^r + a_1^u \neq 0$
2. $a_1^u < -a_1^r, -2 \cdot D_{t2} \cdot a_1^r + 2 \cdot Q_{disc} \cdot a_1^r + D_{t1} \cdot a_1^u - D_{t2} \cdot a_1^u + 2 \cdot Q_{disc} \cdot a_1^u + \Delta_{disc} > 0$
3. $-a_1^r < a_1^u, 2 \cdot D_{t2} \cdot a_1^r - 2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) - D_{t1} \cdot a_1^u + D_{t2} \cdot a_1^u - \Delta_{disc} > 0.$

Following the case definition, the conditions $a_1^r + a_1^u \neq 0$ as well as $-a_1^r < a_1^u$ are met. However, as both gradients of the supply curves are assumed to be positive ($a_1^u, a_1^r > 0$), the second case is not feasible. The first and third case may finally be condensed into the inequality constraint which is presented in (.14).

$$2 \cdot D_{t2} \cdot a_1^r - 2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) - a_1^u \cdot (D_{t1} - D_{t2}) - \Delta_{disc} >= 0 \tag{.14}$$

Formulating inequality (.14) in terms of D_{t2}, condition (.15) can be derived.

$$D_{t2} \geq \frac{2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.15}$$

Condition (.15) embodies a threshold which reflects the trade-off between avoiding higher costs of production by unrestricted suppliers due to the step Δ_{disc} and taking losses due to both unrestricted suppliers with comparably low production costs being forced to reduce their production level to meet D_{t1} as well as higher costs of an increased production of restricted suppliers.

Besides an upper bound for D_{t2} , inequality (.16) is necessary to identify a valid solution ($D_{t1} - q_r \leq Q_{Disc}$).

$$D1 \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r + a_1^u) + D_{t2} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.16}$$

Scenario2: $\mu_{t1} = 0, Q_{disc} - \epsilon - D_{t2} + q^r = 0$

It is to be tested whether the quantity choice in *Scenario2* (Equation (.17)) yields a valid solution.

$$q^{r\,*} = D_{t2} - Q_{disc} - \epsilon \tag{.17}$$

The resulting term for μ_{t2} is defined in Equation (.18).

$$\mu_{t2} = -(D_{t2} - Q_{disc} - \epsilon) \cdot (a_1^r + a_1^u) + (D_{t1} + D_{t2}) \cdot 0.5 \cdot a_1^u + 0.5 \cdot \Delta_{disc} \tag{.18}$$

As a result, a valid solution ($\mu_{t2} \geq 0$) has to satisfy the inequality constraint which is presented in (.19).

$$D_{t2} \leq \frac{2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.19}$$

Due to the discontinuous shape of the merit order for unrestricted suppliers, the production level of unrestricted suppliers in $t2$ is held constant for a range $\Delta D_{t2} = \frac{\Delta_{disc}}{2 \cdot a_1^r + a_1^u}$ if D_{t2} increases. The lower production level is compensated by restricted suppliers that increase their production level according to the increase in D_{t2}. Due to the choice of $q^{r\,*}$, the supply of unrestricted suppliers in $t1$ never exceeds the discontinuity. Finally, as the intersection of *Scenario1* and *Scenario2* according to the inequalities (.15) and (.19) exactly yields the same choice of $q^{r\,*}$ ($q^{r\,*} = D_{t2} - Q_{disc} - \epsilon$), there is no need to compare the resulting costs in both scenarios due to the steadiness in the overlap.

Scenario3: $Q_{disc} - D_{t1} + q^r = 0, \mu_{t2} = 0$

Equation (.20) depicts the resulting term for μ_{t1}.

$$\mu_{t1} = (D_{t1} - Q_{disc}) \cdot (a_1^r + a_1^u) + (-0.5 \cdot D_{t1} - 0.5 \cdot D_{t2}) \cdot a_1^u - 0.5 \cdot \Delta_{disc} \tag{.20}$$

For this to be a valid solution ($\mu_{t1} \geq 0$), inequality (.21) has to be applicable.

$$D_{t1} \geq \frac{2 \cdot Q_{disc} \cdot a_1^r + (D_{t2} + 2 \cdot Q_{disc}) \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.21}$$

This inequality is analogous to the respective one for D_{t2} as there are symmetric relations.

Appendix.4.3. ***Case3:*** $D_{t1} - q^r \geq Q_{disc} + \epsilon, D_{t2} - q^r \geq Q_{disc} + \epsilon$

Case3 refers to a situation in which the non-convexity is relevant in both periods. The respective KKT conditions may be formulated as presented in (.22).

$$\begin{aligned} (-1) \cdot \frac{\partial C_r(q^r)}{\partial q^r} + \sum_t \left[(-1) \cdot \frac{\partial C_{u,t}(q_t^u)}{\partial q^r}\right] - \mu_{t1} - \mu_{t2} = 0 \\ D_{t1} - q^r \geq Q_{disc} + \epsilon \\ D_{t2} - q^r \geq Q_{disc} + \epsilon \\ \mu_t \cdot (Q_{disc} + \epsilon - D_t + q^r) = 0 \;\forall\, t \\ \mu_t \geq 0 \;\forall\, t \end{aligned} \tag{.22}$$

The respective marginal cost functions are presented in (.23).

$$\begin{aligned}
\frac{\partial C_r(q^r)}{\partial q^r} &= a_0 + q^r \cdot a_1^r \\
\frac{\partial C_{u,t1}(q_t^u 1)}{\partial q^r} &= -0.5 \cdot (a_0 + (Q_{disc} + \epsilon) \cdot a_1^u + \Delta_{disc}) - 0.5 \cdot a_1^u \cdot (D_{t1} - q^r - Q_{disc} - \epsilon) \\
\frac{\partial C_{u,t2}(q_t^u 2)}{\partial q^r} &= -0.5 \cdot (a_0 + (Q_{disc} + \epsilon) \cdot a_1^u + \Delta_{disc}) - 0.5 \cdot a_1^u \cdot (D_{t2} - q^r - Q_{disc} - \epsilon)
\end{aligned} \tag{.23}$$

A further distinction of cases is applied.

1. *Scenario1:* $\mu_{t1} = 0, \mu_{t2} = 0$
2. *Scenario2:* $\mu_{t1} = 0, Q_{disc} + \epsilon - D_{t2} + q^r = 0$
3. *Scenario3:* $Q_{disc} + \epsilon - D_{t1} + q^r = 0, \mu_{t2} = 0$
4. *Scenario4:* $Q_{disc} + \epsilon - D_{t1} + q^r = 0, Q_{disc} + \epsilon - D_{t2} + q^r = 0$

As has been outlined before, *Scenario4* does not play a role. Additionally, *Scenario2* does not comply with the definition of *Case3* as this would mean that the unrestricted production level in $t1$ would be below the discontinuity threshold.

Scenario1: $\mu_{t1} = 0, \mu_{t2} = 0$

Equation (.24) characterizes the optimal quantity choice ($q^{r\,*}$).

$$q^{r\,*} = \frac{(D_{t1} + D_{t2}) \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot (a_1^r + a_1^u)} \tag{.24}$$

Once more, the term presented in Equation (.24) includes an upwards adjustment ($\frac{\Delta_{disc}}{a_1^r + a_1^u}$) compared to the case of continuous relations due to the non-convexity. Inserting $D_{t1} - q^r \geq Q_{disc}$, condition (.25) may be defined as a necessary condition with respect to the optimal solution.

$$D_{t1} \geq \frac{2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.25}$$

Scenario3: $Q_{disc} + \epsilon - D_{t1} + q^r = 0, \mu_{t2} = 0$

Scenario3 yields the Lagrange multiplier which is defined in Equation (.26).

$$\mu_{t1} = -(D_{t1} - Q_{disc}) \cdot (a_1^r + a_1^u) + 0.5 \cdot a_1^u \cdot (D_{t1} + D_{t2}) + \Delta_{disc} \tag{.26}$$

This may be regarded as a valid solution ($\mu_{t1} \geq 0$) if condition (.27) is fulfilled.

$$D_{t1} \leq \frac{2 \cdot (Q_{disc} + \epsilon) \cdot (a_1^r + a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \tag{.27}$$

These relations are basically similar to *Case2*. There is steadiness with respect to the optimal solution (q^{r*}) in the intersection of the scenarios considered. To sum up, the optimal supply of restricted suppliers for different demand levels may be defined according to Equation (.28).

$$q^{r*} = \begin{cases} \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} & (a1) D_{t2} \leq \frac{2 \cdot Q_{disc} \cdot (a_1^r + + a_1^u) + D_{t1} \cdot a_1^u}{2 \cdot a_1^r + a_1^u} \\ D_{t2} - Q_{disc} - \epsilon & (a2) \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u}{2 \cdot a_1^r + a_1^u} < D_{t2} \leq \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} + \frac{\Delta_{disc}}{2 \cdot (a_1^r+a_1^u)} & (a3) \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t1} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} < D_{t2}, D_{t1} \leq \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ D_{t1} - Q_{disc} - \epsilon & (a4) \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + \Delta_{disc}}{2 \cdot a_1^r + a_1^u} < D_{t1} \leq \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} + \frac{\Delta_{disc}}{a_1^r+a_1^u} & (a5) D_{t1} > \frac{2 \cdot (Q_{disc}+\epsilon) \cdot (a_1^r+a_1^u) + D_{t2} \cdot a_1^u + 2 \cdot \Delta_{disc}}{2 \cdot a_1^r + a_1^u} \end{cases} \quad (.28)$$

Appendix.5. Decoding the Impact of Both Supply Curve Gradients on the Resulting Cost Implications

The cost factor which was derived in Equation (10) is illustrated in Figure .10. Different combinations of a_1^r and a_1^u are considered.

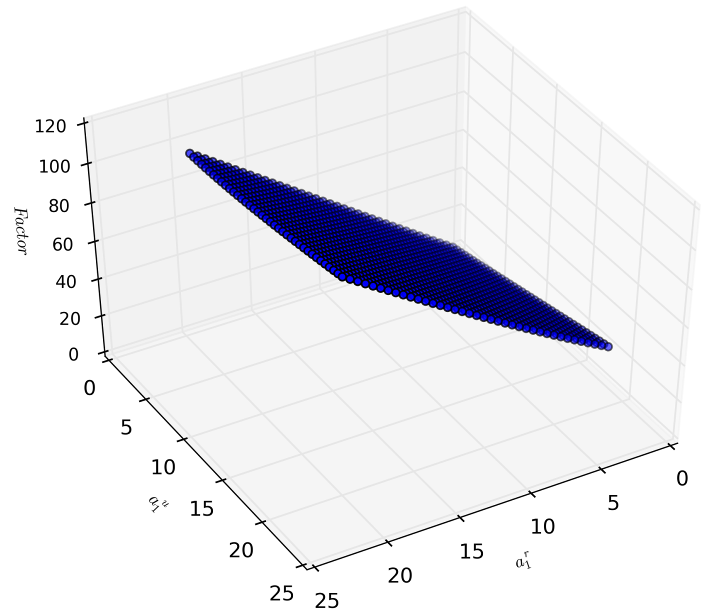


Figure .10: 3D plot for the resulting factor dependent on the parameterization of a_1^r and a_1^u

Appendix.6. Mathematical Proof (Proposition 2)

In this section, a situation in which the merit order of restricted suppliers exhibits non-convexities is considered. The following proof is a condensed formulation as it is essentially similar to the first proof which is presented in Section Appendix.4.

As the non-convexity is exclusively relevant for restricted suppliers, there are essentially two cases which have to be differentiated. In the first case (*Case1*), the discontinuity threshold is not exceeded by the production level of restricted suppliers, whereas in the second case (*Case2*) the non-convexity has to be considered. The non-convexity is addressed by a stepped shape with height $\Delta disc'$ at the threshold quantity $Q_{disc'}$.

Appendix.6.1. ***Case1:*** $q^r \leq Q_{disc}$

The previous considerations may be transferred into the Lagrangian representation of the optimization problem as presented in Equation (.29).

$$\mathbb{L} = (-1) \cdot (C_r(q^r) + \sum_t C_{u,t}(q^r)) + \mu \cdot (Q_{disc'} - q^r) \tag{.29}$$

The respective Karush-Kuhn-Tucker (KKT) conditions are defined in (.30).

$$\begin{aligned}
&(-1) \cdot \frac{\partial C_r(q^r)}{\partial q^r} + \sum_t \left[(-1) \cdot \frac{\partial C_{u,t}(q_t^u)}{\partial q^r}\right] + \mu = 0 \\
&q^r \leq Q_{disc'} \\
&\mu \cdot (Q_{disc'} - q^r) = 0 \\
&\mu \geq 0
\end{aligned} \tag{.30}$$

The relations identified within *Case1* are similar to the respective ones in the first proof. Thus, Equation (.31) depicts the optimal solution.

$$q^{r\,*} = \frac{(D_{t1} + D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r + a_1^u} \tag{.31}$$

Equation (.31) yields a valid solution as long as the inequality constraint (.32) is satisfied ($q^{r\,*} \leq Q_{disc'}$)
.

$$D_{t1} + D_{t2} \leq \frac{2 \cdot Q_{disc'} \cdot (a_1^r + a_1^u)}{a_1^u} \tag{.32}$$

Appendix.6.2. ***Case2:*** $q^r \geq Q_{disc}$

In *Case2* the production level of restricted suppliers exceeds the discontinuity threshold. The according KKT conditions are defined in (.33).

$$
\begin{aligned}
(-1) \cdot \frac{\partial C_r(q^r)}{\partial q^r} + \sum_t \left[(-1) \cdot \frac{\partial C_{u,t}(q_t^u)}{\partial q^r}\right] - \mu = 0 \\
q^r \geq Q_{disc'} + \epsilon \\
\mu \cdot (Q_{disc'} + \epsilon - q^r) = 0 \\
\mu \geq 0
\end{aligned}
\tag{.33}
$$

The respective marginal cost functions are listed as equations in (.34).

$$
\begin{aligned}
\frac{\partial C_r(q^r)}{\partial q^r} &= a_0 + \Delta_{disc'} + q^r \cdot a_1^r \\
\frac{\partial C_{u,t}(q^r)}{\partial q^r} &= -0.5 \cdot a_0 - 0.5 \cdot a_1^u \cdot (D_t - q^r)
\end{aligned}
\tag{.34}
$$

It is sufficient to consider two scenarios:

1. *Scenario1:* $\mu = 0$
2. *Scenario2:* $q^r - Q_{disc'} - \epsilon = 0$.

Scenario 1: $\mu = 0$

The optimal quantity choice with respect to q^r is derived in Equation (.35). The quantity is adjusted downwards as additional supply of unrestricted producers compensates for the discontinuous step.

$$
q^{r*} = \frac{(D_{t1} + D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r + a_1^u} - \frac{\Delta_{disc'}}{a_1^r + a_1^u} \tag{.35}
$$

Condition (.36) has to be satisfied to identify a valid solution.

$$
D_{t1} + D_{t2} \geq \frac{2 \cdot (Q_{disc'} + \epsilon) \cdot (a_1^r + a_1^u) + 2 \cdot \Delta_{disc'}}{a_1^u} \tag{.36}
$$

Scenario2: $q^r - Q_{disc'} - \epsilon = 0$

The scenario definition directly yields Equation (.37).

$$
q^{r*} = Q_{disc'} + \epsilon \tag{.37}
$$

Condition (.38) has to be satisfied to guarantee a valid solution ($\mu \geq 0$).

$$
D_{t1} + D_{t2} \leq \frac{2 \cdot (Q_{disc'} + \epsilon) \cdot (a_1^r + a_1^u) + 2 \cdot \Delta_{disc'}}{a_1^u} \tag{.38}
$$

To sum, up the optimal production level of restricted suppliers can be defined according to Equation (.39).

$$q^{r*} = \begin{cases} \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} & (b1) D_{t2} + D_{t1} \leq \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)}{a_1^u} \\ Q_{disc*} + \epsilon & (b2) \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)}{a_1^u} < D_{t1} + D_{t2} \leq \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)+2\cdot\Delta_{disc'}}{a_1^u} \\ \frac{(D_{t1}+D_{t2})}{2} \cdot \frac{a_1^u}{a_1^r+a_1^u} - \frac{\Delta_{disc}}{a_1^r+a_1^u} & (b3) D_{t1} + D_{t2} > \frac{2\cdot(Q_{disc'}+\epsilon)\cdot(a_1^r+a_1^u)+2\cdot\Delta_{disc'}}{a_1^u} \end{cases} \quad (.39)$$

Appendix.7. Average Intraday Auction Trade Volumes

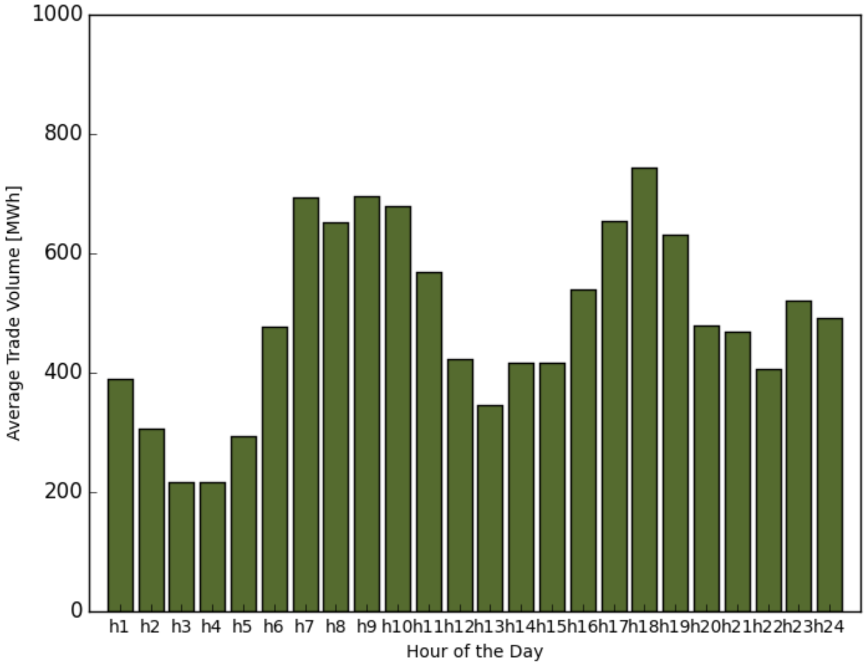


Figure .11: Average intraday auction trade volumes [MWh] in each hour of the day