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ABSTRACT

Wage-Specific Search Intensity*

I propose a model in which agents decide on job search intensity for each possible wage, unlike the usual setup of constant search intensity over wage draws. The proposed framework entails efficiency gains in that agents do not waste effort to searching for low paying unacceptable jobs or less offered high paying jobs. The proposed framework generates accepted wages distributions that differ substantially from the truncated distributions stemming from the usual setup. These different empirical implications are exploited for building two nonparametric tests, which reject constant search intensity over wages, using NLSY97 data. I further estimate the identifiable structural parameters of the two models resulting in better fit for the wage-specific setup. I quantify the increased effectiveness of wage-specific search in more total search intensity, faster transitions to the upper tail of the wage distribution, and higher wages, in particular, more than 25% increase in accepted wages after unemployment.

JEL Classification: J64, E24
Keywords: job search, search intensity, unemployment

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1 Introduction

Agents who look for a job do not just decide on the search intensity of their general job search, but also on the search intensity for specific types of jobs characterized by their location, wages, benefits and other job atributes. Moreover, there is evidence that people direct their job search. In this article, I propose a model of wage-specific search intensity which I compare to its restricted model of non-wage specific general job search. I present empirical evidence that the wage-specific model is a more accurate representation of the job search process.

In the proposed model of job search agents optimally choose the search intensity for each possible wage, unlike the classic model in which search intensity is the same for all posible wages. Agents do not need to exert any effort in searching for low wages or high wages that are unlikely to be offered. Wage-specific search intensity is found to be empirically more important especially for the upper segments of the actual wage distribution.

The classic model of search intensity or endogenous search effort was proposed originally by Burdett and Mortensen (1978). This work established the theoretical connection between search effort and arrival rates (see also Burdett 1979, Benhabib and Bull 1983, Pissarides 2000, Shimer 2004, Christensen et al., 2005). Since then there has been intense research on how search effort should be incorporated into the job search model, as well as how it should be measured. Examples of this research focus on the impact of search intensity on wage dispersion (Gautier and Moraga-González 2016, 2018, Faberman, Mueller, Åhahin, and Topa 2017, Bagger and Lentz 2019), on the reduction of search intensity or discouragement during unemployment (DeLoach and Kurt 2013), or on the impact of search methods on job duration and transitions (Bloemen 2005, Faberman and Kudlyak 2019). In recent decades the evidence that individuals choose which jobs to apply for has substantiated several models of directed search and wage posting: agents direct their search to the most attractive alternatives (Moen 1997, Shimer 1996, Rogerson, Shimer and Wright 2005, Albrecht, Gautier and Vroman 2006, Menzio 2007, Lester 2010).

In the job search literature there is a consensus that there are two kinds of models. One kind is undirected or random search in which workers have no ability to seek or direct their search towards different parts of the wage distribution, or toward different types of jobs. The other kind is directed search in which workers do not randomly search among all possible jobs, but apply for jobs that are more likely to match their skills and interests. This latter framework is partly motivated by evidence that people choose where to search, but also by that directed search entails markert optimality, which is not necessarily present in random search frameworks.

In this paper, I propose a relatively simple model to modify the random search framework to allow for targeted search. The contribution of this paper to the economic literature is to propose a tractable model of wage-specific search intensity with two non-parametric tests that show that the proposed model is a good representation of the data, and further assess quantitatively its efficiency gains over the classic setup. I find that allowing unemployed agents to choose search intensity for each possible wage increases their total search intensity, their transitions to the upper tail of the wage distribution, and their average accepted wage, by more than 25%.

The rest of this document is organized as follows. The next section describes and characterizes the theoretical model. Section 3 explains the computation of the steady-state distribution for employment status and wages. Section 4 describes the data; Section 5 and 6 perform non-parametric tests for the restricted and unrestricted model. Section 7 describes the estimation procedure to recover the model's parameters and discusses the identification strategy. Section 8 presents the empirical results and several goodness of fit tests. Section 9 evaluates the efficiency gains of wage-specific search versus constant search intensity over wages. Finally, Section 10 summarizes the main conclusions of this paper. Further details on this paper are provided in an Appendix.

2 Model

Consider a discrete time framework in which individuals look for a job both when they are unemployed and employed. In both employment statuses they receive wage offers from a known wage offer distribution F defined over $x \in [\underline{w}, \overline{w}]$. The unemployed receive unemployment transfers b; the employed lose their jobs with exogenous separation rate $\theta > 0$. Agents choose search intensity $s(x)$ for each possible wage offer x; the arrival rate is linear in search intensity:

$$\lambda(s(x)) = \underline{\lambda} + \lambda_s s(x)$$

where $\lambda_s = \overline{\lambda} - \underline{\lambda}$, $0 \leq \underline{\lambda} < \overline{\lambda} \leq 1$. Accordingly, search effort is bounded: $s(x) \in [0, \overline{s}]$, $\overline{s} = \frac{1-\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}} \geq 1$. Total search cost is the infinitesimal sum of wage-specific convex search costs, specified as a power function:[1]

$$a\int s^{\alpha}\left(x\right)dx,\ a>0,\ \alpha>1.$$

The value function[2] when unemployed is

$$rV^{u}=\max_{s(x)\leq\overline{s}}\{(b-a\int_{w^{*}}^{\overline{w}}s^{\alpha}\left(x\right)dx)\left(1+r\right)+\int_{w^{*}}^{\overline{w}}\lambda\left(s\left(x\right)\right)\left[V^{e}\left(x\right)-V^{u}\right]dF\left(x\right)\},\quad (1)$$

while the value function when employed at wage w is

$$\begin{aligned} rV^{e}\left(w\right) &= \max_{s(x)\leq\overline{s}}\{(w-a\int_{w}^{\overline{w}}s^{\alpha}\left(x\right)dx)\left(1+r\right) \qquad (2)\\ &\quad +[(1-\theta)\int_{w}^{\overline{w}}\lambda\left(s\left(x\right)\right)\left[V^{e}\left(x\right)-V^{e}\left(w\right)\right]dF\left(x\right)-\theta\left(V^{e}\left(w\right)-V^{u}\right)]\}. \end{aligned}$$

Clearly, $V^{e}\left(w\right)$ is increasing in w, so that the reservation wage is defined by $w^{*}=\{w\mid V^{e}\left(w\right)=V^{u}\}$. Nobody can be employed by less than this reservation wage.

Proposition 1 *$w^{*}>b$.*

Proof: Suppose that $w^{*}\leq b$, then because $\theta>0$, the RHS of (2) is always less than the RHS of (1), which is not possible by the definition of the reservation wage■

The reservation wage is greater than the unemployment transfers b, a feature of the model that emerges from the assumption that layoffs imply moving directly to unemployment, without on-the-job search at the same time.

The power specification for search costs makes it possible to obtain explicit wage-specific search intensities from the two Euler equations by employment status:

$$\begin{aligned} s^{*}\left(x,0\right) &\equiv \min\left[\left(\gamma\left[V^{e}\left(x\right)-V^{u}\right]f\left(x\right)\right)^{\frac{1}{\alpha-1}},\overline{s}\right],\ x\geq w^{*},\\ s^{*}\left(x,w\right) &\equiv \min\left[\left(\left(1-\theta\right)\gamma\left[V^{e}\left(x\right)-V^{e}\left(w\right)\right]f\left(x\right)\right)^{\frac{1}{\alpha-1}},\overline{s}\right],\ x\geq w, \end{aligned}$$

where $\gamma=\frac{\lambda_{s}}{\alpha a(1+r)}$. These search intensities balance returns to search over its marginal

[1]This is common specification, for instance Yashiv (2000), Christensen et al. (2005), Gomme and Lkhagvasuren (2015).

[2]In discrete time the reward per period is adjusted by the term $(1+r)$, unlike in continuous time setups.

cost for each possible wage. Search intensity may be nonmonotonic in x, as it depends on the shape of $f(x)$.

Proposition 2 $s_x^*(x,w) > (=,<)0$, $\forall x, \forall w \in 0 \cup [\underline{w}, \overline{w}]$, $s^* < \overline{s}$, *iff* $\frac{V_x^e(x)}{V^e(x)-V^e(w)} > (=,<) -\frac{f_x(x)}{f(x)}$.[3]

Proof: In Appendix■

When the wage offer distribution is nondecreasing in wage offers, $f_x \geq 0$, search intensity is increasing in targeted wages, $s_x^* > 0$. In general, when the marginal value of targeted wages relative to the excedent in values is larger than the decrease of the wage offer density relative to its density level, search intensity is increasing; otherwise it is decreasing. If the wage offer density function over targeted wages x is unimodal, the search intensity function will be also unimodal, peaking at a higher wage than the former. Moreover, because $\frac{V_x^e(x)}{V^e(x)-V^e(w)}$ is increasing in w, the peak of the search intensity function is also increasing in wage w.

Proposition 2 also implies that search intensity peaks at the same targeted wage whether unemployed or employed at the reservation wage.

Corollary 1 $s_x^*(x,w^*) = 0 \Longleftrightarrow s_x^*(x,0) = 0$, *because* $V^e(w^*) = V^u$.

These peaks occur at the same wage x; however, at different levels of search intensity.

Proposition 3 $s^*(x,0) \geq s^*(x,w^*)$, $\forall x$.

Proof: From the definition of $s^*(x,0)$ *and* $s^*(x,w)$*, when* $V^e(w) = V^u$*, then* $s^*(x,w) = (1-\theta)^{\frac{1}{\alpha-1}} s^*(x,0)$*, thus* $s^*(x,0) > s^*(x,w^*)$*, or* $s^*(x,0) = s^*(x,w^*) = \overline{s}$■

Proposition 4 *i)* $s_w^*(x,w) < 0$, $\forall x$, $\forall w \in [\underline{w}, \overline{w}]$, $s^* < \overline{s}$.

Proof: It follows from the derivative s_w^**. See Appendix for explicit expressions*■

Proposition 3 implies that search intensity is higher when unemployed than when employed at the reservation wage, whereas Proposition 4 means that search intensity is decreasing in current wages. It follows that wage-specific search intensity and thus arrival rates are always higher when umemployed than when employed at any wage:

[3] For $s_x^*(x,0)$, the corresponding condition is $\frac{V_x^e(x)}{V^e(x)-V^u} > -\frac{f_x(x)}{f(x)}$.

Corollary 2 $s^*(x,0) \geq s^*(x,w^*) \geq s^*(x,w)$,$\forall x$,$\forall w \geq w^*$.

Corollary 3 $\lambda(s^*(x,0)) \geq \lambda(s^*(x,w^*)) \geq \lambda(s^*(x,w))$,$\forall x$,$\forall w \geq w^*$.

The usual framework consists of the same search intensity and arrival rate for all wages, thus therein total search costs are $a\int_{\underline{w}}^{\overline{w}} s^\alpha dx = a_R s^\alpha$, where $a_R = a(\overline{w} - \underline{w})$. Henceforth, the usual framework is called the 'restricted' model, as it imposes a unique search intensity over wages to the 'unrestricted' model. In a restricted framework, search intensity depends on the average of marginal weighted gains over costs:

$$\begin{aligned} s^*(0) &\equiv \min\left[\left(\gamma_R \int_{w^*}^{\overline{w}} [V^e(x) - V^u]\, dF(x)\right)^{\frac{1}{\alpha-1}}, \overline{s}\right], \\ s^*(w) &\equiv \min\left[\left((1-\theta)\gamma_R \int_{w}^{\overline{w}} [V^e(x) - V^e(w)]\, dF(x)\right)^{\frac{1}{\alpha-1}}, \overline{s}\right], \end{aligned}$$

where $\gamma_R = \frac{\lambda_s}{\alpha a_R(1+r)}$. The restricted model shares some features with the unrestricted model, such as $w^* > b$, that search intensity is decreasing in current wages, $s_w^*(w) < 0$, and that search intensity is always greater when unemployed than when employed, $s^*(0) \geq s^*(w)$, $\forall w \geq w^*$, which reflect diminishing returns to search over current wages. To illustrate the properties of the wage-specific search model with respect to the restricted model, I solved the model numerically by value function iteration until convergence to a reasonable tolerance is attained.[4]

[Figure 1 here]

Figure 1 illustrates search intensity over targeted wages by employment status for the restricted and unrestricted framework. Search intensity functions approximately mimic the unimodal shape of the lognormal wage offer distribution. As detailed in Proposition 2, search intensity curves peaks on targeted wages are increasing in prior wages. This figure also shows that wage-specific search intensity does not need to waste search effot in wages that will be rejected anyway, that is, below the reservation wage, as it occurs with the restricted model search intensity, which is a horizontal line around the middle of targeted search intensity.

[Figure 2 here]

[4] I assume a lognormal wage offer distribution: $\ln w \sim N(\mu, \sigma)$, and parameter values r=0.02, b=1300, μ=8.1, σ=0.5, $\underline{\lambda}$=0.01, $\overline{\lambda}$=0.95, θ=0.01, a=1, α=2.6.

Figure 2 shows total costs by current wages for the restricted and unrestricted models. Decreasing returns to search by current wages explain that both total costs are decreasing. Taking search costs as a measure of search effort, there is more total effort when agents can target their search to specific wages.

[Figure 3 here]

Figure 3 presents the hazard rate for job-to-job transitions by current wages. Being able to choose wage-specific search effort implies a higher hazard for transitioning to higher paying jobs, but a lower hazard to transitioning to lower wages.

[Figure 4 here]

Figure 4 compares the wage offer distribution with the accepted wage distribution for the two models, when individuals were previously unemployed. For the restricted model, the accepted wage distribution is a truncated wage offer distribution; for the unrestricted model, it is a displaced and rescaled version thereof and is thus associated with higher mean wage offers.

[Figure 5 here]

Finally, Figure 5 shows the wage distribution for the restricted and unrestricted models, when individuals change employers, conditional on three prior employment statuses: unemployed, employed at a medium wage, and employed at a high wage. As in Figure 4, in the restricted model accepted wage distributions are truncated wage offer distributions, while in the unrestricted model accepted wage distributions emerge from the interaction of truncated wage offer distributions and wage-specific search efforts.

In sum, by removing the restriction that search efforts are constant over targeted wages, there are efficiency gains that allow individuals to focus their job search and increase their transition to jobs in the upper segments of the wage distribution. These efficiency gains also incentivize individuals to exert more effort than when they cannot target their job search to more highly paying jobs.

Unrestricted search intensity has hace different empirical implications than restricted search intensity. In the latter, the distribution of accepted wages, regardless of the previous employment status or wage, coincides with the wage offer distribution once truncation is accounted for. By contrast, in the unrestricted model accepted wage offer distributions do variate depending on previous employment status and wages. We use these different implications to build tests to distinguish empirically between these two models.

3 Steady-State distribution

The steady-state distributions of wages and employment status $h(\cdot)$ is defined over $0 \cup [w^*, \overline{w}]$ and can be computed recursively, exploiting the model's feature that job separations are exogenous. Let $h(0)$ be the fraction of individuals that are unemployed, a masspoint, and $h(x)$ the fraction of individuals that are employed at wage x. Evidently, $h(0) + \int_{w^*}^{\overline{w}} h(x)\,dx = 1$, and clearly $h(x) = 0$ for $x < w^*$.

The steady-state unemployment rate is then defined by

$$0 = \underbrace{\theta(1 - h(0))}_{\text{Job separations}} - \underbrace{h(0)\int_{w^*}^{\overline{w}} \lambda(x, 0)\,dF(x)}_{\text{Job entrants}},$$

and has this explicit expression:

$$h(0) = \frac{\theta}{\int_{w^*}^{\overline{w}} \lambda(x, 0)\,dF(x) + \theta}.$$

This expression is computed directly from the model's parameters. Then, the steady-state density for wage x is given by

$$\begin{aligned} 0 \;=\; & \underbrace{(1-\theta) f(x) \int_{w^*}^{x} \lambda(x, w)\,dH(w)}_{\text{Entrants from lower wages}} + \underbrace{h(0)\,\lambda(x, 0)\, f(x)}_{\text{Job entrants}} \\ & \underbrace{- h(x)(1-\theta)\int_{x}^{\overline{w}} \lambda(z, x)\,dF(z)}_{\text{Quitters to higher wages}} - \underbrace{h(x)\,\theta}_{\text{Job separations}}. \end{aligned}$$

This expression defines explicitly the steady-state the density function over wages x :

$$h(x) = \frac{(1-\theta)\int_{w^*}^{x} \lambda(x, w)\,dH(w) + h(0)\,\lambda(x, 0)}{(1-\theta)\int_{x}^{\overline{w}} \lambda(z, x)\,dF(z) + \theta} f(x).$$

This formula requires first the computation of the steady-state unemployment rate $h(0)$ and the steady-state density for wages w lower than x: $h(w), w < x$. Given this configuration, there is no need for any iterative procedure for computing this steady-state distribution. The whole distribution can be calculated recursively, first computing the steady-state unemployment rate, then the lowest acceptable wage, $x = w^*$, and then higher wages until wage $\overline{w}$ is reached. In practice, a discretized approximation is computed, which makes the solution straightforward.

4 Data

Data come from the United States National Longitudinal Survey Youth Cohort of 1997 (NLSY97). This is a nationally representative sample of 8,984 youths who were 12 to 16 years old as of December 31, 1996. The dataset consists of 682,784 observations for the time period 1997-2015. Interviews were conducted annually up to 2011, and bi-annually afterward. The dataset includes a weekly employment history from which we take the last week in the quarter as representative for the whole quarter. The survey keeps track of employment transition including employer changes and weekly wages, which are derived from hourly wage rates and hours worked weekly.

In the sample there are only individuals who have turned 18 after they obtain their highest reported education level. They are considered employed if their weekly wage is at least \$100 per week; otherwise, they are classed as unemployed. The sample is organized in unemployment or employment spells, which can be complete or incomplete (right-censored). The spell ends with an employment change, which can be job-finding, when the individual is unemployed, or job separation or a job-to-job transition, when the individual is employed. The final sample consists of 154,874 observations contained in 9,344 spells.

5 Testing for equal conditional wage distributions

The two models' empirical implications can be tested using nonparametric approaches. Let $g(x \mid w)$ be the empirical density function for wages x conditional on past wages w or past unemployment $g(x \mid 0)$. There is a masspoint for unemployment denoted by $g(0 \mid w)$. Then the observed transitions can be related to the transitions generated by the model. For legibility, in this section arrival rates are expressed shortly as $\lambda(x, 0) = \lambda(s^*(x, 0))$ and $\lambda(x, w) = \lambda(s^*(x, w))$.

Given that the separation rate is exogenous and constant, it is straightforward to identify this parameter from the data: $g(0 \mid w) = \theta$. The density function for targeted wages conditional on current unemployment is $g(x \mid 0) = \lambda(x,0) f(x)$, $x \geq w^*$ and conditional on current employment at wage w is $g(x \mid w) = \lambda(x,w) f(x)$, $x \geq w$. Thus, the job-finding transition is $1 - G(w^* \mid 0) = \int_{w^*}^{\overline{w}} \lambda(x,0)\, dF(x)$ and the job-to-job transition from employment at wage w is $1 - G(w \mid w) = \int_{w}^{\overline{w}} \lambda(x,w)\, dF(x)$. Then, there is a clear empirical distinction between the restricted model and the unrestricted model for the employed:

$$g(x \mid w,\ x > w) = \frac{g(x \mid w)}{1 - G(w \mid w)} = \frac{f(x)}{1 - F(w)}, \text{ restricted model, and} \tag{3}$$

$$= \frac{\lambda(x,w) f(x)}{\int_{w}^{\overline{w}} \lambda(x,w)\, dF(x)}, \text{ unrestricted model.} \tag{4}$$

Transitions from unemployment, $g(x \mid 0, x > w^*)$, have similar expressions. As discussed in Section 2, in the restricted model all conditional truncated accepted wage distributions coincide with the truncated wage offer distribution. In the restricted model search intensity does not build the shape of the conditional accepted wage distribution. By contrast, in the unrestricted model the conditional accepted wage distribution is smooth, not truncated, and different by current employment status and targeted wages. This difference between models can be exploited to build a statistical test. For the restricted model, if we truncate a conditional distribution at the same wage w_1, the resulting conditional distributions should be exactly the same regardless of whether the prior wage was w_1 or w_2. Thus we have a null and an alternative hypothesis.

$$\begin{aligned} H_0 : g(x \mid w_1,\ x > w_1) &= g(x \mid w_2,\ x > w_1),\ w_1 > w_2, \\ H_1 : \qquad\qquad &\neq \qquad\qquad . \end{aligned}$$

This test is also applicable when the prior status is unemployment: $g(x \mid 0, x > w_1)$, $w_1 > w^*$. Accepting the hypothesis of equality of truncated conditional distributions lends empirical support to the restricted model, while rejection thereof works in favor of the unrestricted model.

A Kolmogorov-Smirnov (KS) test determines equality between conditional distributions by prior wages and unemployment. Wage distributions are conditioned on being unemployed or employed in one of the four quartiles of the wage distribution in the previous period

[Figure 6 here]

Figure 6 presents these distributions both for wages (left panel) and logwages (right panel). Even though by construction their support is truncated by the assumption that wages have to be higher than prior wages, none of them looks like a truncated distribution; they all start smoothly from the truncation point. For a proper comparison of these conditional distributons, however, we have to truncate each distribution at the same point, as shown in the next figures, a pairwise graphical comparison of conditional distributions with a KS-test.

[Figures 7-10 here]

Figure 7 compares two wage distributions, one is conditional on being unemployed in the previous period and the other is conditional on being employed at wages from the first quartile of the prior wage distribution. This comparison is made for wages in the left panel and logwages in the right panel. KS-tetsts in both panels reject the hypothesis that these two distributions are equal. This test is repeated with a comparison for wage distributions conditional on unemployment, first and second quartile in Figure 8, for unemployment, first, second and third quartile, in Figure 9, and for unemployment and the four quartiles in Figure 10. All of these tests rejects that any pair of conditional wage distributions are equal, thus implying a sound rejection of the restricted search intensity model. Moreover, the Kolmogorov D-statistic increases with the distance between quartiles, that is, the combined D-statistic for the first and second wage quartile is 0.21, for the first and third quartile it is 0.29, and for the first and fourth quartile it is 0.44.

6 Testing for constant relative search effort

A second nonparametric test is for constant relative search effort over current wages, as predicted by the restricted model. Certainly, the reason for the coincidence of conditional accepted wage distributions in the restricted model is that therein there is a constant search intensity over targeted wages. Thus, another way to check for the restricted model is to build a ratio of conditional densities by accepted wage:

$$\begin{aligned}\frac{g\left(x|w_1\right)}{g\left(x|w_2\right)} &= \frac{\lambda\left(w_1\right)}{\lambda\left(w_2\right)}, \text{ restricted model, and} \\ &= \frac{\lambda\left(x,w_1\right)}{\lambda\left(x,w_2\right)}, \text{ unrestricted model.}\end{aligned}$$

These ratios of conditional densities by prior unemployment, $\frac{g(x|w)}{g(x|0)}$, have similar expressions. For the restricted model, a ratio of two empirical conditional distributions is constant over accepted wages x, unlike the unrestricted model. We can build then a hypothesis test for the restricted model

$$\begin{aligned} H_0 : \quad & d\frac{\frac{g(x|w)}{g(x|0)}}{dx} = 0 \\ H_1 : \quad & \qquad\quad \neq \end{aligned}$$

and similarly for $d\frac{\frac{g(x|w_1)}{g(x|w_2)}}{dx} = 0$. Accepting the hypothesis of constant relative search intensity implies a clear empirical support for the restricted model.

This test is similar in essence to the KS-test, but it requires some previous steps for its empirical implementation:

1. Smooth the empirical bivariate distribution of current wages and prior wages $g(x, w)$ by a bivariate kernel density estimation.

2. Build several ratios of conditional wage distributions, for each possible combination of prior wages and truncations points: $r(x, w_1, w_2, z) = \frac{g(x, w_1 \mid x > z)}{g(x, w_2 \mid x > z)}$. These ratios are built for truncation points z that are greater than or equal to prior wages w: $z \geq w$ and current wages have to be greater truncation points: $x > z$.

3. Run fixed effect regressions that control for several possible levels of these ratios, which are assumed to be fixed-effect intercepts for prior wages and truncation points: $a(w_1, w_2, z)$. The parameter of interest is the slope b over current wages x.
$$r(x, w_1, w_2, z) = a(w_1, w_2, z) + bx$$

4. Perform a significance test for b, which is really a test for $d\frac{\frac{g(x|w)}{g(x|0)}}{dx} = 0$, explained above.

Wages are discretized in 100 points that are truncated over the first 60 points, so that there are always at least 40 points of current wages available as observations of the estimation. Certainly, for lower current wages there are fewer prior wage and truncation points, but there are more current wage points. For higher wages, there are more prior wages and truncation points for conditioning the distribution of x, but there are fewer current wages, so that the estimation is less precise.

[Figure 11 here]

The results for the slopes of these estimations by truncation points with confidence intervals as upper and lower bounds are reported in Figure 11. The slope of x is generally statistically significant: for almost all prior wages we reject the hypothesis that there is a constant search intensity over targeted wages. This rejection is clearer for higher wages than for lower wages, which coincides with the previous KS-tests shown in indicating larger distances between conditional wage distributions for larger distances between prior wages.

In sum, from the empirical distributions of current wages conditional on prior wages and unemployment I build two tests for constant wage-specific search intensities. The first is a KS-test of equality of accepted wage distributions conditional on different prior wages and the second is a test that search intensities are constant ratios over targeted wages. Both of these tests reject the implications of the restricted model, namely that the conditional distribution of accepted wages is a truncated wage offer distribution and that search intensity is constant over targeted wages.

7 Recovering model's parameters

Besides these nonparametric tests, it is possible to recover the theoretical model parameters from the data. We do that by means of a General Method of Moments (GMM) Estimation. This procedure relates a parameter set to a weighted measure of distance between sample and predicted moments: $G(\Theta) = (m_a - m_p)' W^{-1} (m_a - m_p)$, where m_a is a vector of actual moments, m_p is a vector of simulated moments, and W is a weighting matrix. The moments used in this estimation are the following:

1. wage segment-unemployment distributions
2. transitions between wage segments, and
3. employment status transitions.

I compute the predicted moments from the steady-state employment and wage distribution as well as the employment and wage segment transitions predicted by the theoretical model. For W I use a diagonal matrix consisting of each element of the vector m_p. Since the moments are probabilities of being in a wage segment or of transitions between segments, the criterion function is in fact a sum of χ^2-statistics, that is, $\chi^2 = \frac{(m_a - m_p)^2}{m_p}$. The estimated behavioral parameters are thus $\widehat{\Theta} = \arg\min G(\Theta)$.

The identification of wage-specific search intensity is only possible if some parameters are fixed. It is not possible to identify $\underline{\lambda}$, $\overline{\lambda}$, a, and α at the same time. With data on out-of-the labor force, one could identify the lower bound for the arrival rate $\underline{\lambda}$ from the transitions from out-of-the labor force to employment (see Flinn and Heckman 1983). With data on time spent in job search, one could identify the scale parameter of the cost function a. Accordingly, to reinforce the identification of some parameters, unemployment transfers b, the interest rate r and thus the discount rate, the bounds of the arrival rates, and the scaling factor a of the search intensity cost function are fixed. More concretely, the interest rate is fixed at 4% annually, which is 0.985% for a quarter; transfers while unemployment are set at a low value of $b = 200$ and $\underline{\lambda} = 0$ and $\overline{\lambda} = 1$, and the search cost scale parameter is set at $a = 1000$. As discussed previously, the separation rate is definitely well identified from observed average job separations, so we fix this parameter, $\widehat{\theta} = g(0 \mid w) = 3.64\%$.

We concentrate the identification power of the data on the curvature parameter α for both restricted and unrestricted models. We also estimate the mean and standard deviation of logwages and the separation rate. The parameters to estimate are then $\Theta = \{\mu, \sigma, \alpha\}$.

For identification of the remaining model's parameters we can use the criteria by Flinn and Heckman (1982a, 1982b), matching the employment transitions and conditional wage distributions to their empirical counterparts. A key feature of these authors' identification strategy is the coincidence between the accepted wage distribution and a truncated wage offer distribution. This feature also characterizes the restricted model and makes the identification of the wage offer distribution straightforward. As discussed in the previous sextion on nonparametric tests, the restricted model implies a similar shape of all accepted wage distributions conditional on prior wages or unemployment (Equation 3). In the data, however, we may have different accepted wage distributions by prior wages and unemployment. The unrestricted model consists of diferent wage-specific arrival rates by previous employment status and a common wage offer distribution with two parameters (Equation 4), which can then be properly identified.

We optimize both criterion functions by means of the Powell algorithm, as in Press et al. (1992). For the likelihod function asymptotic standard errors are computed from first derivatives computed by a smoothing polynomial interpolation around their estimated value. For the GMM estimation standard errors are computed from 20 bootstrap computations of the parameter set.

8 Empirical Results

This section presents the estimated parameters and an assessment of the two models's accuracy in replicating the data.

[Table 1 here]

Table 1 reports the results of the estimation for both models. The unrestricted model has a mean of logwages of 7.10 and a standard deviaton of 0.85 which reflect a less dispersed wage offer distribution than the restricted model, whose logwage mean is 7.35 and standard deviation is 0.82. Hoever, the unrestricted model has lower search costs than the restricted model, captured by a higher curvature parameter for the cost function, 7.56, than the estimated parameter for the restricted model, 5.16. This suggests the the predicted shape of accepted wages is relatively more influenced by wage-specific arrival rates in the unrestricted framework. These parameters generate an overall better fit of the unrestricted model, as measured by the criterion function of the GMM procedure.

[Table 2 here]

Table 2 shows unemployment rates and transition rates from unemployment and from employment for the two models. Both models overpredict the unemployment rate of 7.33%, which comes from their underprediction of the actual job-finding rate of 41.74%. For these statistics the unrestricted model performs somehow better. Also, both models underpredict the job-to-job transition rate of 5.89, with some more accuracy for the restricted model, measured by the goodness of fit test, a χ^2-statistics. Both models have the same job separation rates, as this is determined by a fixed parameter that comes from descriptive statistics.

[Table 3 here]

Table 3 reports the actual and predicted wage segment distributions for the two models, computed at steady-state. The predictions of the two models are barely different from each other and both of them are close to the actual distribution in the middle segments. Altogether, the goodness of fit metric indicates a fairly good replication of the actual data by both models. However, there is underprediction in the lower tails and overprediction in the upper tails of the distribution. This can be visualized in the following figures.

[Figure 12 here]
[Figure 13 here]

The predicted and actual steady-state accepted wage distributions and the estimated wage offer distributions are illustrated in Figure 12. The corresponding image for log-wages is reported in Figure 13. The wage offer distribution of unrestricted model is clearly more concentrated around lower wages than the restricted model, which presents a larger dispersion, as we anticipated in the discussion of Table 1. The difference in shapes of accepted wage distributions is sensible, because wage-specific search intensity creates more dispersion in the distribution of accepted wages from the same wage offer distribution than the model with a constant search intensity, as shown in Section 2. Figure 13 shows these same graphic information in log-wages and is particularly useful to illustrate that both models' underprediction of the lower tail of the accepted wage distribution. The predicted distribution has the reservation wage as truncation point, which implies that the model does not account for very low wages. Nevertheless, the models' replication of the main shape of the accepted wage distribution is acceptable, both visually and by formal goodness of fit tests.

[Table 4 here]

Table 4 presents the transition matrix between wage segments.[5] In general, the restricted model replicates slightly better the actual transition for lower wages, while the unrestricted model does better with medium and upper wage segments, and unemployment. This is in line with the results of the non-parametric tests that show that the unrestricted model performs particularly better for the upper tail of the conditional wage distributions.

In summary, with a few identifying assumption the structural parameters of the restricted and unrestricted models are identified and estimated. By goodness of fit metrics both models are able to replicate fairly well the main observables, with the

[5]Because these model do not generate wage-specific job separation rates, I just report transitions between wage segments, conditional on not transitioning to unemployment.

unrestricted model performing better altogether, especially for transitions from unemployment and the upper segments of the wage distribution and the restricted model performing relatively better for the lower segments of the wage distribution.

9 Efficiency gains in job search

Can we measure the efficiency gains of wage-specific job search over the classical setup of constant search intensity for all wages? After recovering the behavioral parameters of the unrestricted job search model, we calculate the counterfactual outcomes if we restrict search intensity to be equal across wages. The quantitative comparison of these two outcomes measures the efficiencty gains of the wage-specific search model over its classic countepart.

[Table 5 here]

In Table 5 I compare three outcomes of the unrestricted and restricted model using the estimates of the unrestricted model. Outcomes are accepted mean wages, search costs, and hazard rates from the current employment or wage status. Allowing for wage-specific job search increases reservation wages by 166%, implying wage gains by 22% when people were unemployed. These wage gains are decreasing in current wages. Total search costs are higher for wage-specific search and their gains are increasing in current wages. That is, wage-specific search intensity is higher than classic search intensity mainly for upper wage segments, about 35% higher. Hazard rates in the unrestricted setup are lower for unemployent, by 29%, and for lower wage segments, but up to 26% higher for upper wage segments. This is because wage-specific search is targeted to the upper wage segments and, accordingly, transitions therefrom are faster in contrast with slower transitions from lower segments.

[Table 6 here]

Table 6 is similar to the previous table, but there we compare outcomes between the counterfactual unrestricted model and the estimated restricted model. Efficiency gains are higher than in the previous table. Reservation wages increase by 325% and accepted mean wages after unemployment increase by 50%. As in the previous table, wage gains are decreasing, search costs increasing, and hazard rates increasing in current wages. Once again, transitions are slower from unemployment and lower wage segments and faster from upper wage segments.

The analysis of the two estimated models deliver a similar message about the efficiency gains of wage-specific search. Compared with constant search intensity over wages, wage-specific search implies higher reservation wages and thus wage gains that are decreasing over current wages, whereas search intensity is increasing in current wages. However, hazard rates from unemployment and lower current wages are lower for wage-specific search, by more than 25%, but higher by more than 25% from higher current wages than for constant search intensity over wages.

10 Conclusions

In this paper I have proposed a model of job search wih optimal choice of wage-specific search intensity. The proposed framework is based on random search, yet it it extended to allow for a *controlled* random process, so that search is no longer "undirected". The proposed model allows agents to concentrate their search effort in the upper wage segments, so that they do not have to spend any effort in wage offers that they will not accept anyhow, as in the usual framework in which search effort is not targeted.

I have shown that wage-specific search intensity generates a distribution of accepted wages that differs substantially from the truncated distribution that emerges from the usual search intensity setup. In the latter a constant search intensity over targeted wages is related to the same truncated accepted wage offer distribution by prior wages or unemployment. When search intensity is allowed to be wage-specific, accepted wage distribution differ by prior wage. These different empirical implications between these two setups are the basis to build two nonparametric tests. Once they are performed, both these tests reject the implications of the widely used model of search intensity of a constant search intensity for all possible wage draws.

I also have estimated the main behavioral parameters of the wage-specific search model, which is able to replicate the data fairly. Usual goodness of fit tests coincide with the nonparametric tests in indicating that the wage-specific setup presents a better fit to the data than the usual constant search intensity setup. I calculate the efficiency gain of wage-specific search over classic search in up to 25% higher wages, more search activity mainly for higher wages up to 35% increase in total search cost, and slower transitions to lower wage segments but faster transitions to upper wage segments.

The proposed framework of wage-specific search intensity has been estimated with a relatively conventional dataset. In future research datasets that contain observed metrics of search intensity can certainly bring more precision to disentangle between the two models. A fruitful extension of the proposed framework would be also to allow for equilibrium.

Appendix

A Proof of Proposition 1

The derivates of search intensities over current and targeted wages when $s < \overline{s}$ are:

$$\begin{aligned} s_w^* &= \frac{1}{\alpha - 1} s^* (x, w) \frac{-V_w^e (w)}{V^e (x) - V^e (w)}, \\ s_x^* &= \frac{1}{\alpha - 1} s^* (x, w) \left[\frac{V_x^e (x)}{V^e (x) - V^e (w)} + \frac{f_x (x)}{f (x)} \right]. \end{aligned}$$

The first derivate is always negative, while the second derivate is positive if $\frac{V_x^e(x)}{V^e(x) - V^e(w)} > -\frac{f_x(x)}{f(x)}$, which defines the condition for Proposition 1. Certainly, for $s^* (x, 0)$, the corresponding condition is $\frac{V_x^e(x)}{V^e(x) - V^u} > -\frac{f_x(x)}{f(x)}$.

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Table 1. Parameter Estimates and Standard Errors
Unrestricted and Restricted Models

Parameter	Θ	GMM	
		Unrest	Rest
Mean of logwages	μ	7.09875	7.35161
		0.019687	0.026771
S D of logwages	σ	0.84621	0.82067
		0.007437	0.010945
Search cost	α	7.56240	5.16086
		0.199205	0.138046
Criterion Function	S	0.562579	0.613192

Table 2. Actual and Predicted Employment Status and Transitions
Restricted and Unrestricted Models. In Percent

Transitions	Actual	Unrestricted	Restricted
Unemployment Rate	7.33	10.79	10.93
χ^2-statistic		0.0328	0.0355
p-value		0.85631	0.85054
Job Finding	41.74	30.07	29.64
χ^2-statistic		0.0560	0.0602
p-value		0.81299	0.80618
Job Separations	3.64	3.64	3.64
Job-to-Job	5.89	4.61	4.93
χ^2-statistic		0.0030	0.0017
p-value		0.99852	0.99917

Table 3. Actual and Predicted Wage Distributions
Unrestricted and Restricted Models. In Percent

Wage segment	Actual	Unrestricted	Restricted
-2000	23.78	12.32	13.42
2000-4000	30.50	30.35	28.81
4000-6000	20.85	24.47	24.90
6000-8000	11.48	15.42	16.73
8000+	13.38	17.43	16.14
χ^2-statistic		0.0873	0.0836
p-value		0.99907	0.99915

Table 4. Actual and Predicted. Wage Segment Transitions
Unrestricted and Restricted Models. In Percent

Wage Segment		Unemp	-2000	2000 -4000	4000 6000	6000 8000	8000+	χ^2 statistic	p value
	Actual	58.26	7.49	13.75	10.64	4.95	4.92		
Unemployment	Unrest	69.93	17.06	9.21	2.30	0.84	0.67	1.7301	0.88509
	Rest	70.36	18.30	7.70	2.14	0.86	0.64	2.0454	0.84283
	Actual		89.71	5.30	2.94	1.13	0.92		
-2000	Unrest		86.40	9.49	2.48	0.91	0.73	0.0360	0.99984
	Rest		86.41	9.23	2.56	1.03	0.77	0.0312	0.99988
	Actual			92.15	4.12	2.07	1.66		
2000-4000	Unrest			96.11	2.29	0.88	0.72	0.0220	0.99914
	Rest			96.07	2.31	0.93	0.69	0.0216	0.99916
	Actual				96.14	2.15	1.71		
4000-6000	Unrest				98.51	0.80	0.69	0.0151	0.99247
	Rest				98.62	0.79	0.59	0.0166	0.99172
	Actual					97.56	2.44		
6000-8000	Unrest					99.37	0.63	0.0137	0.90680
	Rest					99.53	0.47	0.0163	0.89850

Table 5. Accepted Wages, Search Intensity and Hazard gains of Wage-Specific Search
Estimates of Unrestricted Model and Counterfactuals of Restricted Model

	Accepted Mean Wages			Search Costs			Hazard		
	Unrest	Rest	Diff %	Unrest	Rest	Diff %	Unrest	Rest	Diff %
Reservation Wage	**713**	**268**	**165.9**						
Unemployed	2365	1763	22.4	200	200	0.0	30.1	42.4	-29.1
Wages: 1000	2715	2504	8.4	474	432	9.9	26.2	27.2	-3.8
2000	3923	3717	5.5	353	310	13.8	12.6	12.1	4.3
3000	5134	4913	4.5	252	211	19.2	6.5	5.8	11.2
4000	6293	6048	4.1	173	137	26.4	3.6	3.0	17.9
5000	7357	7096	3.7	116	86	35.1	2.1	1.7	25.7

Table 6. Accepted Wages, Search Intensity and Hazard gains of Wage-Specific Search
Estimates of Restricted Model and Counterfactuals of Unrestricted Model

	Accepted Mean Wages			Search Costs			Hazard		
	Unrest	Rest	Diff %	Unrest	Rest	Diff %	Unrest	Rest	Diff %
Reservation Wage	**1055**	**248**	**325.2**						
Unemployed	3248	2159	50.4	200	200	0.0	18.9	29.6	-36.2
Wages: 1000	3210	2779	15.5	845	726	16.3	24.6	25.9	-5.0
2000	4343	3921	10.7	660	534	23.7	13.8	13.0	6.4
3000	5519	5076	8.7	494	372	32.6	7.9	6.7	17.6
4000	6646	6177	7.6	357	246	44.8	4.7	3.6	29.9
5000	8574	8162	5.1	159	88	81.7	1.9	1.1	66.0

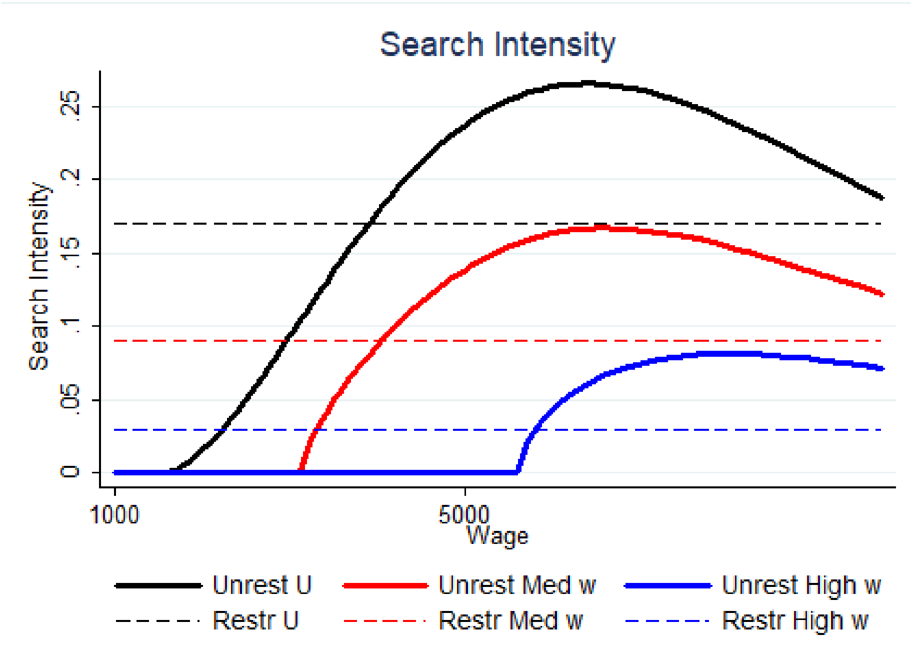

Figure 1. In Wage-specific and Classic, search intensity decreases in the current wage

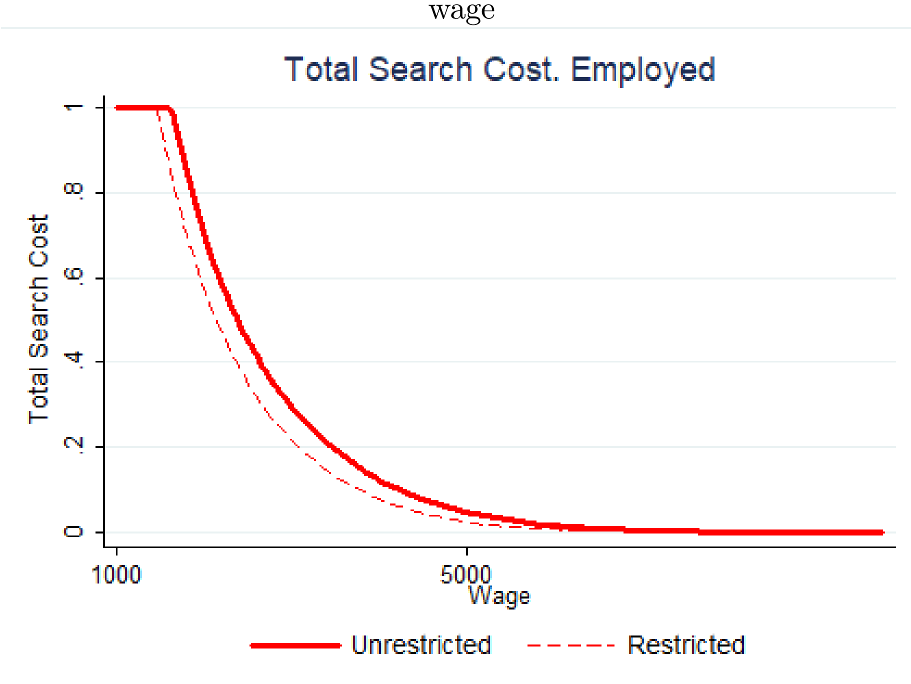

Figure 2. Wage-specific implies more total intensity than Classic

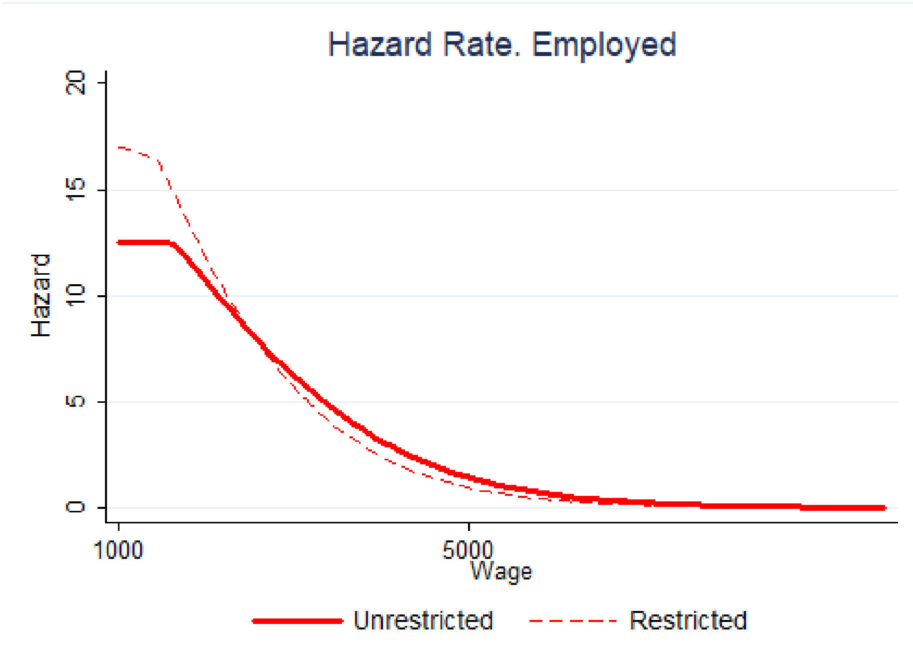

Figure 3. Hazard rate while unemployed and for lower wages is higher in Classic

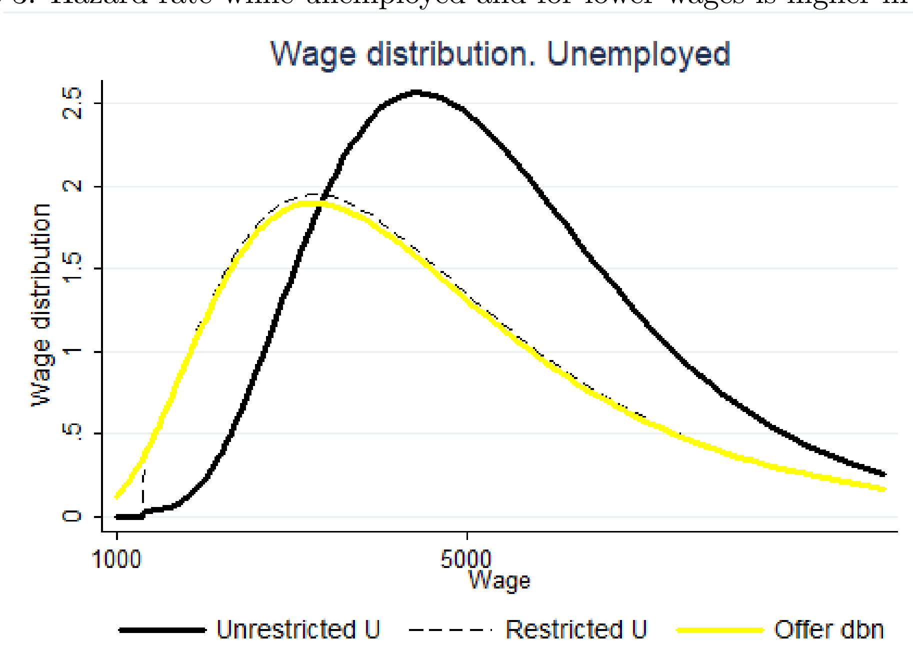

Figure 4. Offer and accepted wage distribution for the unemployed

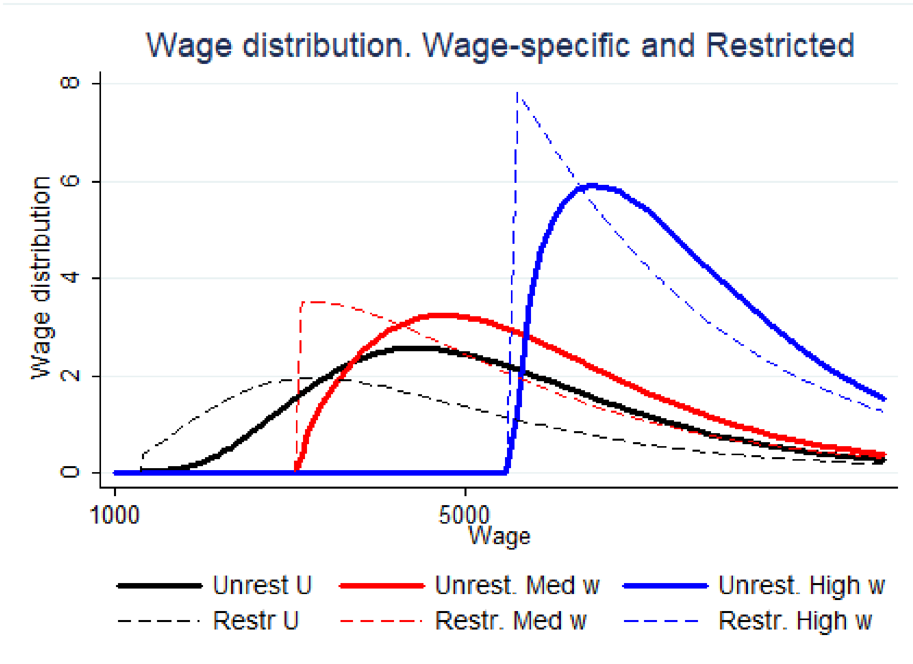

Figure 5. Accepted wage distribution for the employed by wage level: Classic (m) and Wage-specific (s)

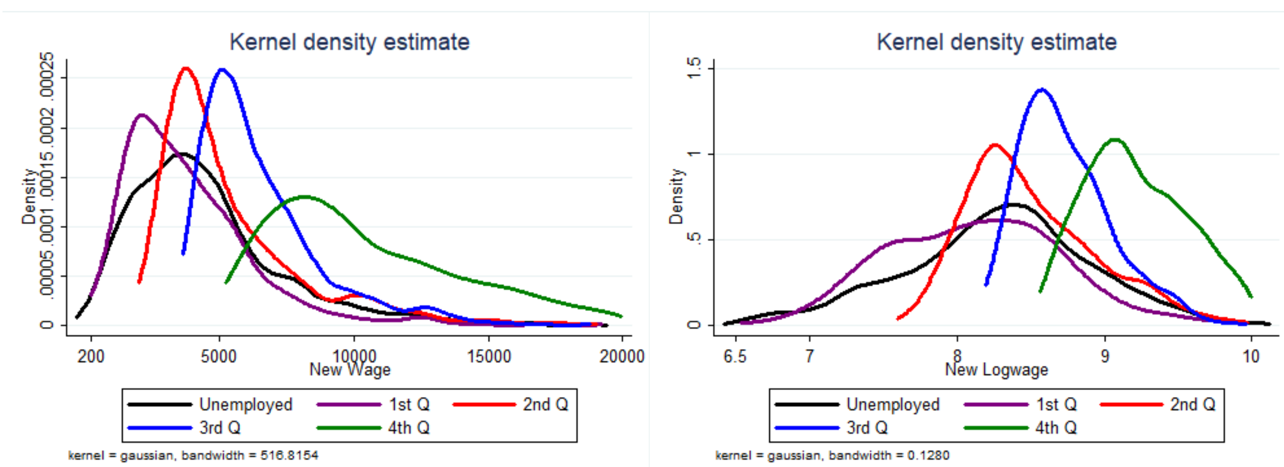

Figure 6. Wage and Logwage Distributions by Prior Wage Quartile

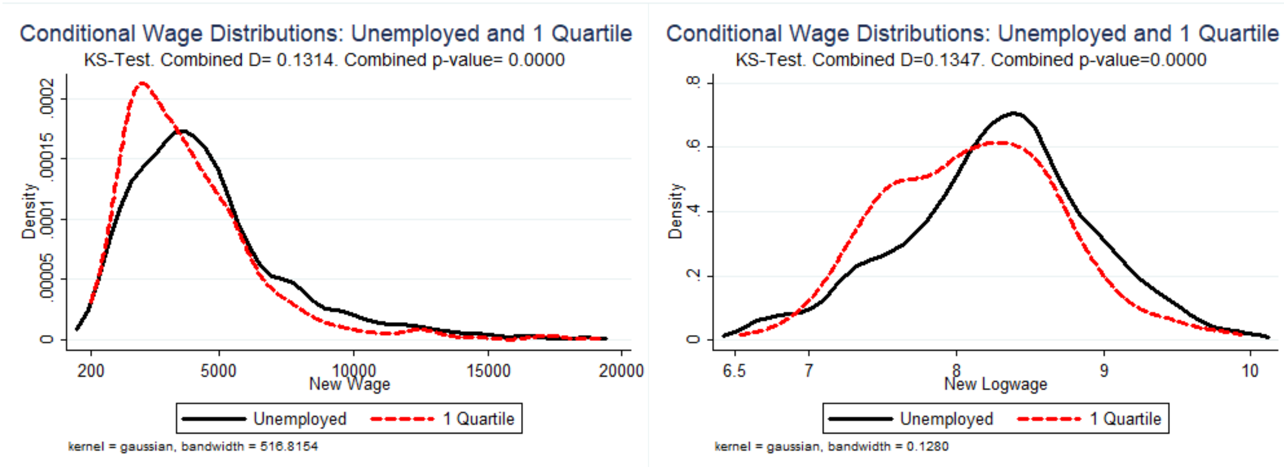

Figure 7. Wage and Logwage Distribution conditional by prior 1st quartile and unemployment. KS-test

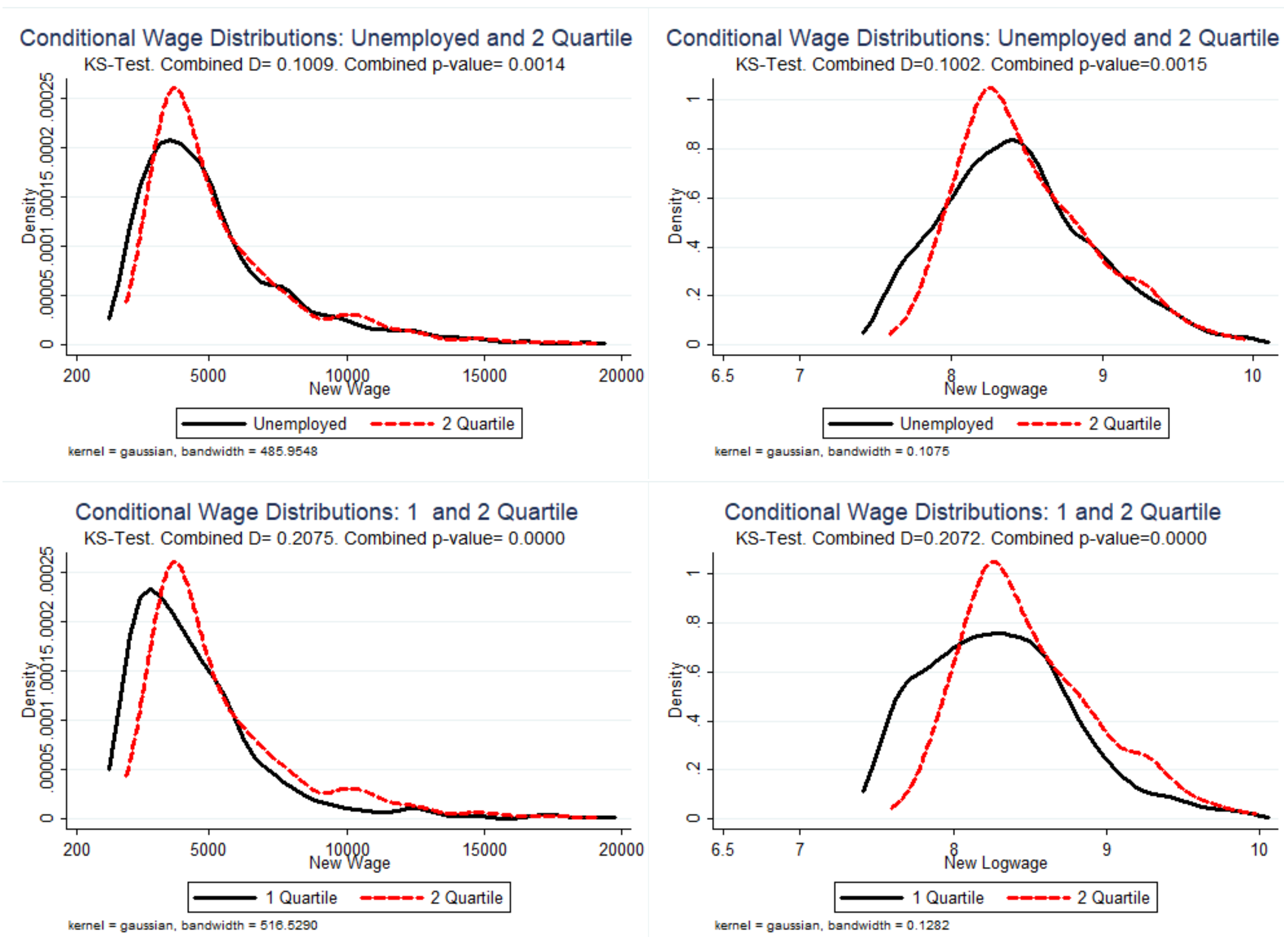

Figure 8. Wage and Logwage Distributions by prior 2nd and 1st quartile and unemployment. Same truncation point. KS-test

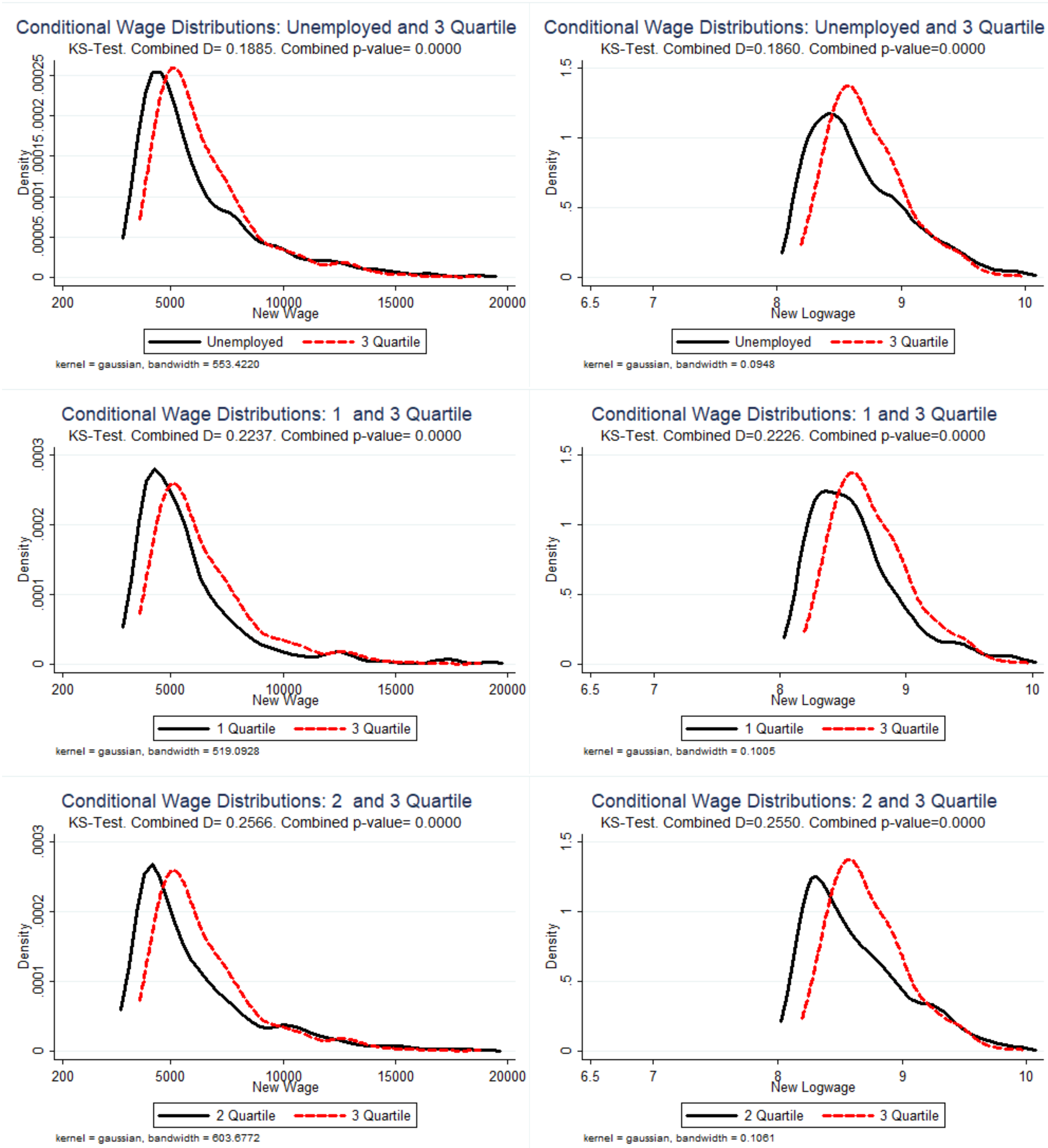

Figure 9. Wage and Logwage Distributions by prior 3rd, 2nd and 1st quartile and unemployment. Same truncation point. KS-test

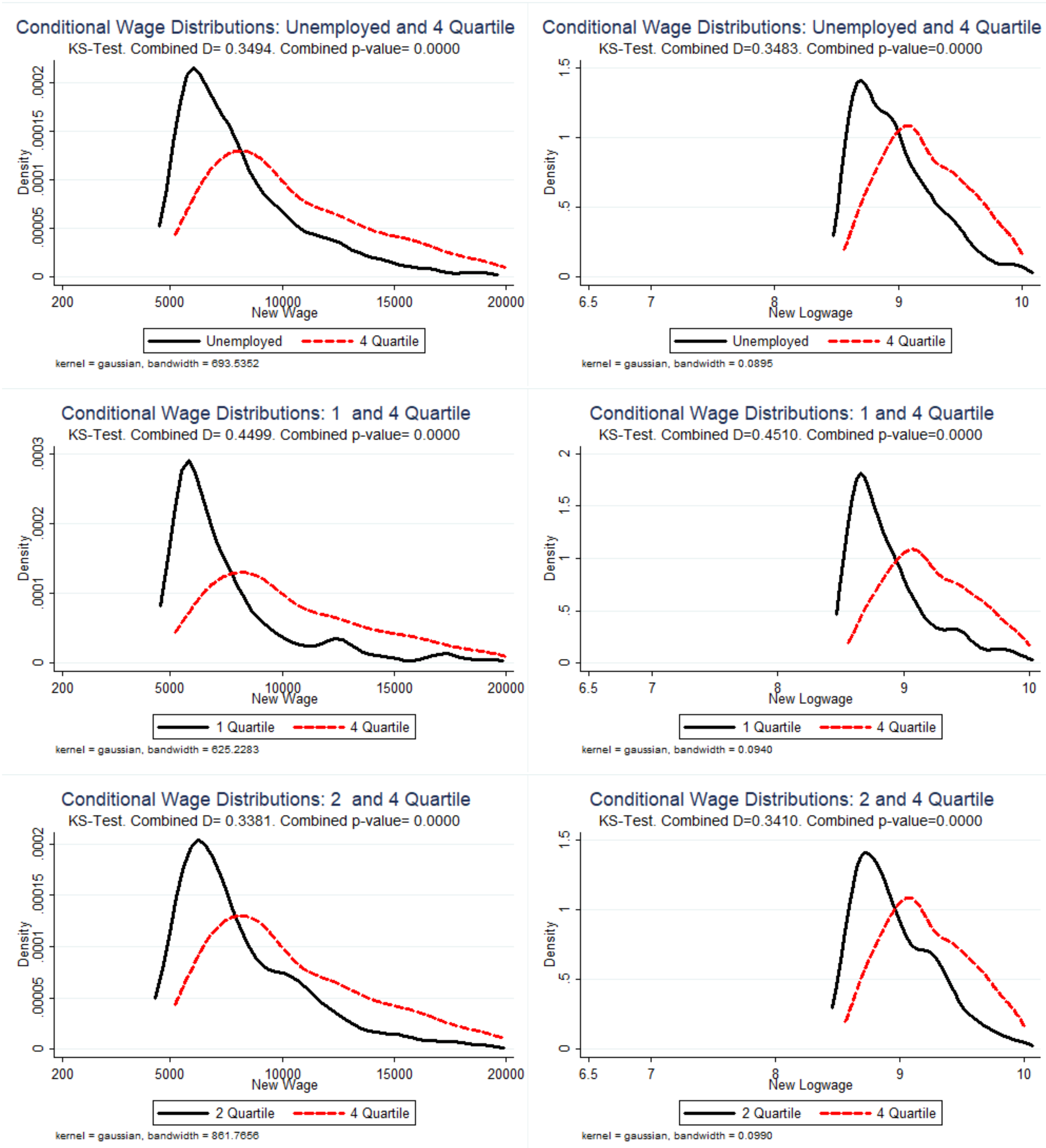

Figure 10. Wage and Logwage Distributions by prior 4th, 3rd, 2nd and 1st quartile and unemployment. Same truncation point. KS-test

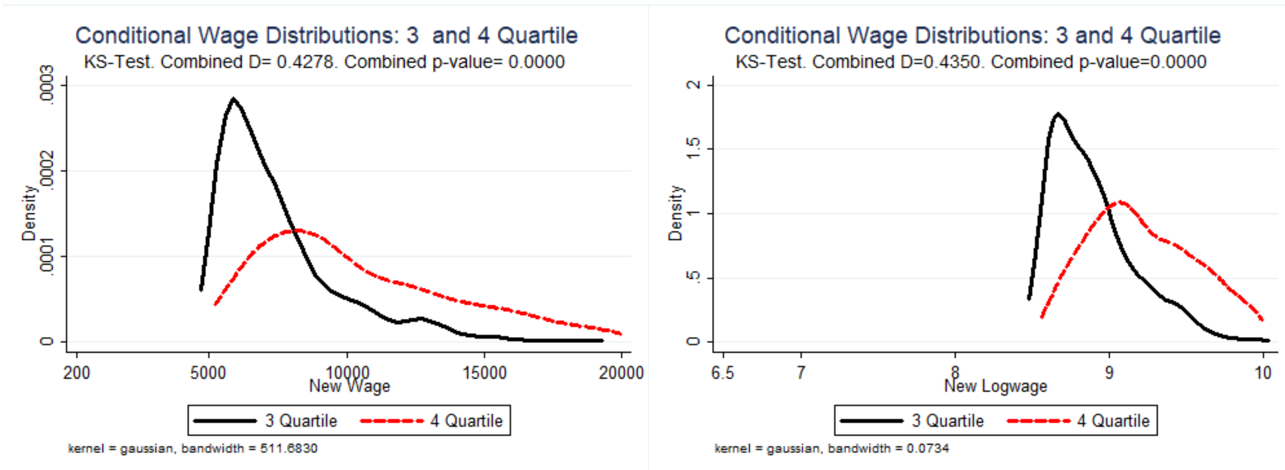

Figure 10 (cont.). Wage and Logwage Distributions by prior 4th, 3rd, 2nd and 1st quartile and unemployment. Same truncation point. KS-test

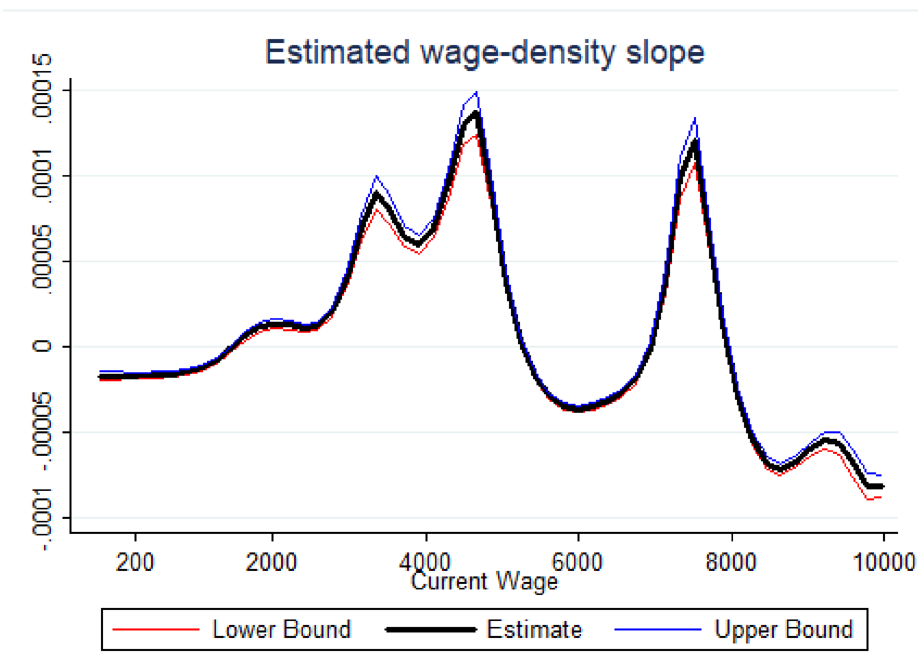

Figure 11. Wage-density slope

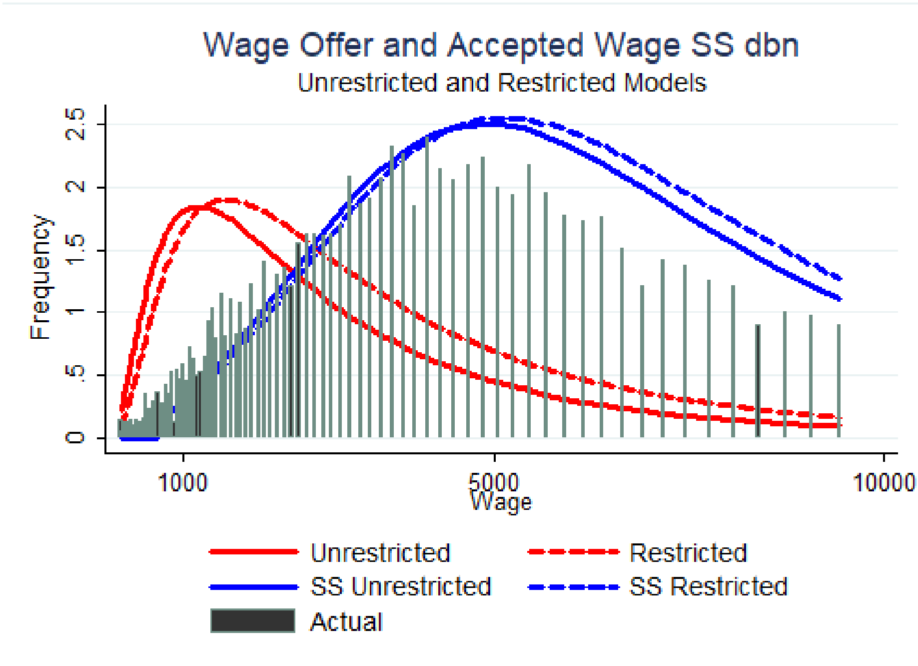

Figure 12. Wage Offer and Steady-State Accepted Wage Distributions, Actual and Predicted

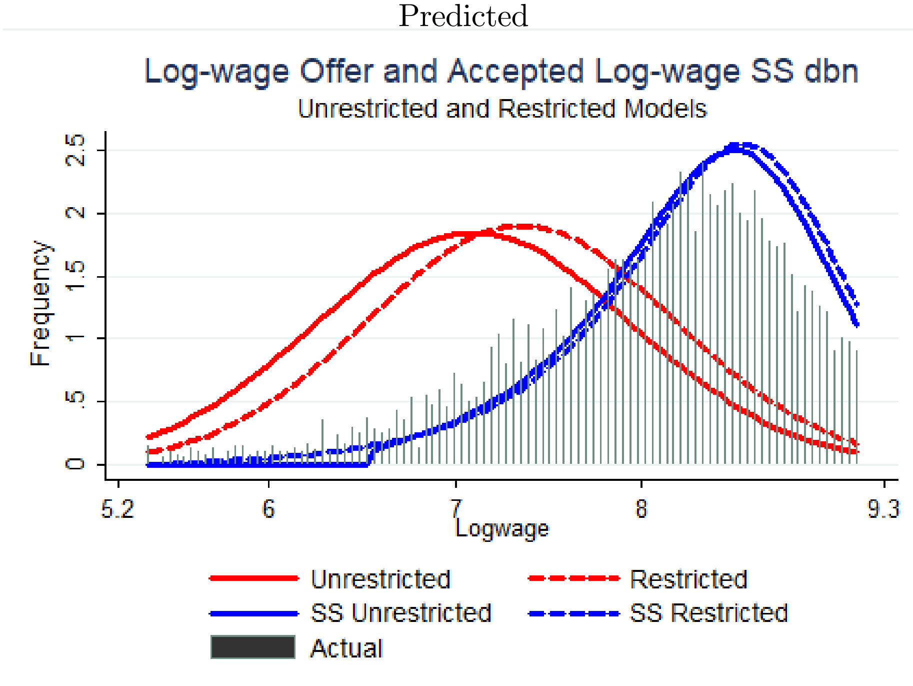

Figure 13. Log-wage Offer and Steady-State Accepted Log-wage Distributions, Actual and Predicted