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Relational Contracts: Public versus Private Savings*

Francesc Dilmé[†] **Daniel F. Garrett**[‡]

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Abstract

Work on relational employment agreements often predicts low payments or termination for poor performance. The possibility of saving can, however, limit the effectiveness of monetary incentives in motivating an employee with diminishing marginal utility for consumption. We study the role of savings and their observability in optimal relational contracts. We focus on the case where players are not too patient, and hence the constant first-best effort cannot be implemented. If savings are hidden, the relationship eventually deteriorates over time. In particular, both payments and effort decline. On the other hand, if savings are public, consumption is initially high, so the agent's savings fall over time, and effort and payments to the agent increase. The findings thus suggest how tacit agreements on consumption can forestall the deterioration of dynamic relationships in which the agent can save.

JEL Classification: C73, J30

Keywords: relational contracts, consumption smoothing preferences, private savings

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1 Introduction

Work on relational contracts has examined the role of commitment in employment relationships. Yet, the role of employee savings in this context is still not well understood. From the perspective of incentive provision, the possibility that the employee accumulates wealth creates a potential difficulty. If the employee can become wealthy, he will be less reliant on the employer's pay, as his options outside the relationship are then more attractive. We study how this difficulty is managed in optimal relational contracts, focussing on the role of the observability of savings and consumption.

In our model, an employer (the principal) is in a repeated relationship with a worker (the agent). The agent provides effort determining output and also chooses how much to consume and to save. The principal has linear preferences over money, and the agent has concave utility for consumption implying a preference for smooth consumption streams. The agent cannot commit to future effort or consumption and the principal cannot commit to future pay. Effort and pay are publicly observable, and we contrast the situation where savings/consumption are private to the agent with that in which these decisions are public information. We characterize relational contracts that maximize the principal's discounted payoff.

Private savings. We begin by considering the case where the agent can privately consume and save. The "private savings problem" has by now a long tradition in work on dynamic contracting where the principal commits to the contract. Particularly relevant to our paper is work on private savings in dynamic moral hazard, where there is imperfect monitoring of effort (see a detailed discussion of the literature in Section 6.2).[1] There is little work, however, on agreements with private savings but without commitment. To our knowledge, the main exception is Ábrahám and Laczó (2018) who study constrained-efficient agreements between two risk-averse agents in a setting of mutual insurance (without effort) when there is both private and public savings, and which we discuss below.

We contribute to the understanding of relationships with private savings and without commitment by characterizing optimal relational contracts in the employment setting described above. It is without loss to consider punishments for public deviations that involve autarky; this means the cessation of effort and pay. Two types of incentive compatibility conditions are then relevant. The first is standard in work on relational contracts: the principal's payment cannot be larger than the continuation value the relationship has to her. This sets a limit on how much the principal can credibly pay to the agent. The second type of compatibility condition is new and concerns the incentive of the agent to follow the consumption and effort specified in the agreement. While there are many ways in which the agent can deviate jointly in effort and consumption, we identify the critical deviations to be those where the agent

[1] Contributions include Werning (2002), Kocherlakota (2004), Ábrahám and Pavoni (2005), Mitchell and Zhang (2010), Ábrahám et al. (2011), Edmans et al. (2012), He (2012), and Di Tella and Sannikov (2021).

(a) follows the agreed effort up to any given date and then shirks by supplying zero effort forever, and (b) reduces consumption from the beginning so that he perfectly smooths over his lifetime the income derived up to the date at which he begins to shirk. These deviations thus involve the agent secretly saving more in anticipation of a public defection on the agreement. Such deviations represent the optimal ones for the agent: provided they are not profitable, agent incentive compatibility is assured.

Now consider the principal's optimal contract and in particular the role of the constraints preventing the agent's critical deviations. Were the principal able to commit to payments, she would ask for a constant (first-best) level of effort and pay to the agent would rise over time. The reason the agent's pay must be increasing is the following. As the agent obediently works more periods, he values additional payments less. This is because the agent can smooth his earnings over his lifetime and because of diminishing marginal utility for consumption. It follows that the agent must be paid more at later dates to induce the same level of effort.

When the players are sufficiently patient, the first-best effort can be sustained. Otherwise, we find that effort is initially constant, and then eventually declines over time. The reason is related to the need to compensate the agent more at later dates for any given level of effort, as identified above. As time passes, the agent's higher pay relative to effort reduces the profits of the principal. Because future profits are lower, the principal can then only credibly promise lower levels of pay. This in turn depresses the sustainable effort and profits, which creates a feedback loop.

The effects described above are new. The dynamics of the optimal contract are driven by the constraints ensuring the agent does not engage in the "double deviations" of secretly increasing savings and later publicly defecting. This is not the case in Ábrahám and Laczó (2018). They solve a relaxed problem that omits constraints related to double deviations where agents secretly increase savings and then later quit the agreement for autarky. They then check numerically that this is justified when the return on savings is not too high, concluding that private savings in the constrained-efficient agreements are zero in this case. They therefore argue that the "characteristics of the constrained-efficient allocations ... are the same" (p. 17) whether or not the agent can privately save. When the return on savings is higher, however, agents' double deviations cannot be ignored, and so no results are provided.

Due to the forces explained above, pay to the agent is eventually strictly declining with time whenever the principal cannot achieve the first-best payoff. Because equilibrium consumption is constant, the agent's savings are eventually increasing. These predictions contrast with what occurs in the dynamic moral hazard literature with private savings where the players fully commit. In particular, in situations where the agent can borrow as well as save, the principal is unconstrained in the timing of payments and the optimal timing of pay is indeterminate. For instance, the principal can always delay payments, effectively "saving for the agent". Given the indeterminacy in pay, the convention has been to consider pay that requires the agent to have zero savings in equilibrium. Papers where pay is indeterminate and this convention is invoked include Ábrahám et al. (2011), Edmans et al. (2012) and Williams (2015) (an early reference is Cole and Kocherlakota, 2001, although this studies an insurance setting).

Public savings. We compare our results with the case where consumption and savings by the agent are observable by the principal. This case is also new to the principal-agent literature. For the public-consumption case, first-best effort and consumption is again sustainable when the players are sufficiently patient. Otherwise, the dynamics of the relationship stand in sharp contrast to what occurs for private savings. First, the agent's consumption is distorted: it is high in the initial periods and lower in later ones. Also, the relational contract induces the agent to dissave, worsening his outside option from exiting the contract. As the agent becomes poorer, he is more willing to trade high effort for pay and the relationship becomes more profitable for the principal. The level of pay and effort that can be sustained increases with time. The advantage of a relationship in which the agent becomes impoverished with time is that the principal's higher profitability at later dates relaxes her credibility constraint in early periods. This increases the pay that is credible early on and increases the sustainable level of effort. Impoverishment is shown to continue indefinitely, with the balance on the agent's account approaching a level at which the first best is sustainable. That is, we obtain convergence to efficiency in the long run.

The fact that the agent becomes poorer over time is reminiscent of immiseration results such as Thomas and Worrall (1990) where the agent's utility declines without bound with probability one. However, note that the classical immiseration results are driven by the provision of incentives for information revelation, rather than the absence of commitment which is responsible for the agent's impoverishment in our paper.

Broader implications. The contrast between private and public savings may have broader implications for settings with limited commitment where agents can invest in their outside options. Savings is one possible investment, but other possibilities include physical capital accumulation – e.g. Kehoe and Perri (2002) – or investment in human capital – e.g. Voena (2015). In such settings, lower outside options tend to enhance the efficiency of the relationship, and optimal investments must be determined in light of such effects. As noted, the role of private investment in outside options has been explored little to date. With private investments, optimal relationships can be shaped by agents' abilities to gradually and secretly invest from the beginning. In this sense what can matter is agents' *potential* outside options, i.e. the ones they can access if investing more from the beginning than they do on path. We demonstrate this in our setting where the possibility of private investment in outside options hampers efficiency and causes the relationship to deteriorate over time. To our knowledge, we are the first to establish such effects.

Applications. Our results establish that the principal benefits from a relationship with high consumption, as low savings prevent it from deteriorating in the future. In Section 6.1 we discuss different examples of how high consumption is induced in practice, including perks to CEOs, corporate cultures with high spending, fashion in Louis XIV's court, supply relationships, and the "trucking system" that required workers to buy from the company store in the industrial era. In the case of corporate cultures of high spending, for example, Henderson and Spindler (2004) have argued in the law literature that high consumption expenditures may be an implicit requirement of an employer. They write: "Contrary to the conventional wisdom that agents wear expensive clothes and drive fancy cars in order to impress principals, it may well be that principals *require* their agents to engage in such consumption, because spending money on these items increases an agent's reliance upon the future relationship with her principal" (p. 1869).[2] Since such requirements would usually be only implicit, it suggests the usefulness of the relational contracting framework, certainly above a formal contracting one.

The organization of the rest of the paper is as follows. Section 2 introduces the setting. Section 3 solves the case where both principal and agent can commit. Section 4 solves the case with limited commitment where savings are private, and Section 5 the case with limited commitment where savings are public. Section 6 provides details on the applications and a detailed literature review. Section 7 concludes. Appendix A and an Online Appendix contain the proofs of all results.

2 Setting

Environment and preferences. A principal and agent meet in discrete time at dates $t = 1, 2, \ldots$. Letting $r > 0$ be the interest rate that will apply to the balance on the agent's savings account, we suppose the players have a common discount factor $\delta = \frac{1}{1+r}$. In every period t, first the agent exerts an effort e_t and consumes an amount c_t. Then, the principal makes a discretionary payment w_t to the agent. These variables are all restricted to be non-negative.

The agent has initial savings balance $b_1 > 0$ as well as access to a savings technology (with the interest rate r as specified above). The initial balance will be common knowledge between the principal and agent, including in our model of private savings in Section 4. The agent's balance at time $t + 1 > 1$ then satisfies

$$b_{t+1} = \frac{b_t + w_t - c_t}{\delta} = b_1 \delta^{-t} + \sum_{s=1}^{t} \delta^{s-t-1} (w_s - c_s). \quad (1)$$

[2] While savings decline over time in our model with observable consumption, we view the key empirical prediction for employment settings as excessive consumption rather than declining balances. There are various reasons why most workers tend to accumulate wealth over time that we do not model. However, the forces we document may act to slow accumulation.

Balances can, in principle, be negative (i.e., the agent can borrow). We say that the agent's intertemporal budget constraint is satisfied in case

$$\sum_{t=1}^{\infty} \delta^{t-1} c_t \leq b_1 + \sum_{t=1}^{\infty} \delta^{t-1} w_t. \tag{2}$$

The agent's felicity from consumption c_t in any period t is denoted $v(c_t)$, where $v : \mathbb{R}_+ \to \mathbb{R} \cup \{-\infty\}$. We assume that $v(c)$ is real-valued for $c > 0$, and takes value $-\infty$ at $c = 0$. We further assume that v, when evaluated on positive consumption values, is twice continuously differentiable, strictly increasing and strictly concave. In addition, v is onto all of $\mathbb{R}$, implying $\lim_{c \searrow 0} v(c) = -\infty$.

The agent's disutility of effort e_t is $\psi(e_t)$. We assume that ψ is continuously differentiable, strictly increasing, strictly convex, and such that $\psi(0) = \psi'(0) = 0$, and that $\lim_{e \to \infty} \psi'(e) = \infty$.

The agent's period-t payoff will be $v(c_t) - \psi(e_t)$, while the principal's will be $e_t - w_t$; hence, we interpret effort as equal to the output enjoyed by the principal.

Relational contracts. We focus for tractability on *deterministic* relational contracts.[3] We identify relational contracts with their outcomes; denote them $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t \geq 1}$. We restrict attention to contracts that satisfy the following feasibility constraints.

Definition 2.1. A *feasible relational contract* is a sequence $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t \geq 1}$ satisfying the following conditions:

1. **Positivity:** $\tilde{e}_t, \tilde{w}_t, \tilde{c}_t \geq 0$ for all t.
2. **Balance dynamics and constraint:** Conditions (1) and (2) hold.
3. **Bounded consumption:** The sequences of consumption, pay and effort $((\tilde{c}_t)_{t \geq 1}$, $(\tilde{w}_t)_{t \geq 1}$, and $(\tilde{e}_t)_{t \geq 1})$ are bounded.

While the first and second conditions reflect features of the environment introduced above, the third condition guarantees that the players' payoffs are well-defined in a feasible contract.

3 First best and full commitment to the contract

Consider first the problem of maximizing the principal's payoff by choice of a feasible relational contract subject only to the constraint that the agent is initially willing to participate (in effect, supposing commitment to the contract on both the side of the principal and agent). If the

[3] This assumption is in common with some other work such as Ray (2002). In examining contracts that are optimal for the principal, whether random contracts can improve on deterministic ones might be expected to depend on the nature of risk aversion (e.g., whether v exhibits increasing or decreasing risk aversion). Our results for deterministic contracts do not depend on these considerations.

agent does not participate, a possibility we describe as "autarky", we stipulate that he consumes $(1-\delta)b_1$ per period. This is the optimal consumption for the agent among consumption streams satisfying the intertemporal budget constraint in Equation (2) given that all payments are set to zero. Therefore, we consider maximizing the principal's payoff over feasible relational contracts $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ such that the payoff of the agent

$$\sum_{t=1}^{\infty} \delta^{t-1} \left(v(\tilde{c}_t) - \psi\left(\tilde{e}_t\right) \right) \tag{3}$$

is no lower than his autarky value, $\frac{1}{1-\delta} v((1-\delta)b_1)$.

Proposition 3.1. *Consider maximizing the principal's discounted payoff by choice of feasible contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$, subject to ensuring the agent a payoff at least his autarky value $\frac{1}{1-\delta} v((1-\delta)b_1)$. In any optimal feasible contract, effort and consumption are constant at $e^{FB}(b_1) > 0$ and $c^{FB}(b_1) > (1-\delta)\, b_1$, respectively, being the unique solutions to:*

1. ***First order condition:*** $\psi'(e^{FB}(b_1)) = v'(c^{FB}(b_1))$, *and*
2. ***Agent's indifference condition:*** $v(c^{FB}(b_1)) - \psi(e^{FB}(b_1)) = v((1-\delta)b_1)$.

Furthermore, the payoff of the principal is $V^{FB}(b_1) \equiv \frac{1}{1-\delta}(e^{FB}(b_1) - (c^{FB}(b_1) - (1-\delta)b_1))$, which is a strictly decreasing function of b_1.

Note that the first-best policies depend on both b_1 and δ, since they depend on the value of autarky consumption $(1-\delta)b_1$ (see Condition 2). However, we reduce the notational burden by making dependence only on b_1 explicit. Note also that the proposition does not specify the timing of payments. The only requirements on payments is that they are feasible and satisfy the agent's budget constraint (2) with equality. Payments may be constant, in which case they equal $c^{FB}(b_1) - (1-\delta)b_1$ in each period. Sections 4.1 and 5.1 discuss how, when the principal fully commits but the agent cannot, sufficient backloading of payments is enough to ensure the agent's continued obedience to a first-best contract.

4 Unobservable consumption

We now suppose the principal can observe the agent's effort, but not the consumption choices nor the agent's balance. Given the absence of commitment, we are interested to determine feasible relational contracts $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ which coincide with outcomes of a perfect Bayesian equilibrium (PBE) of a dynamic game. These represent the outcomes that are sustainable by a relational contract, and among which we can consider optimizing the principal's payoff.

We begin by defining the histories in our game. For $t \geq 0$, a t-history for the agent is $h_t^A = (e_s, w_s, c_s)_{1\leq s<t}$, which gives the observed effort, payments and consumption up until

time $t-1$. The set of such histories at date $t \geq 1$ is $\mathcal{H}_t^A = \mathbb{R}_+^{3(t-1)}$ (with the convention that $\mathbb{R}_+^0 = \emptyset$). Note that, given h_t^A and the agent's initial balance b_1, we can completely determine the evolution of the balance up to date t using Equation (1). We denote the date-t balance by $b(h_t^A)$. A t-history for the principal is $h_t^P = (e_s, w_s)_{1 \leq s < t}$. The set of such histories at date $t \geq 1$ is $\mathcal{H}_t^P = \mathbb{R}_+^{2(t-1)}$.

A strategy for the agent is then a collection of functions

$$\alpha_t : \mathcal{H}_t^A \to \mathbb{R}_+^2, \ t \geq 1,$$

and a strategy for the principal is a collection of functions

$$\sigma_t : \mathcal{H}_t^P \times \mathbb{R}_+ \to \mathbb{R}_+, \ t \geq 1.$$

Here, α_t maps the t-history of the agent to a pair (e_t, c_t) of effort and consumption. Also, σ_t maps the t-history of the principal, together with the agent's effort choice e_t, to a payment w_t.

As noted above, we will restrict attention to equilibria whose outcomes coincide with a feasible relational contract. However, we do not restrict the strategies that are available to the players. Certain strategies imply, for instance, the violation of the agent's intertemporal budget constraint in Equation (2). To ensure that the agent finds it optimal to satisfy this constraint, we make the following assumption on payoffs. While the principal's payoff is as specified above (and so given by $\sum_{t=1}^{\infty} \delta^{t-1} (e_t - w_t)$), the agent obtains the payoff $\sum_{t=1}^{\infty} \delta^{t-1} (v(c_t) - \psi(e_t))$ if the constraint in Equation (2) is satisfied, and obtains payoff $-\infty$ otherwise.[4]

To obtain the set of feasible relational contracts $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t \geq 1}$ that are PBE outcomes, we consider PBE where publicly observed deviations from the agreed outcomes are punished by "autarky". This means that, if the agent deviates from the agreed effort $\tilde{e}_t$, or if the principal deviates from the agreed payment $\tilde{w}_t$, the principal makes no payments and the agent exerts no effort from then on; the agent perfectly smoothing the balance of his account over the infinite future.[5] If the agent's balance is negative when autarky begins, the intertemporal budget constraint in Equation (2) is necessarily violated (as the agent receives no further payments), the agent must earn payoff $-\infty$, and so we can specify for instance that the agent consumes zero in every period. Note that deviations by the agent from the specified consumption, provided they are not accompanied by any deviation in effort, go unpunished (i.e., the principal continues to adhere to the payments specified by the agreement).

If the agent plans to always choose effort in accordance with the contract, he optimally

[4] Alternative assumptions can be made which yield the same results as documented below. For instance, another possibility involves permitting negative consumption (assigning it a value $-\infty$ in the agent's payoff), but limiting the extent the agent can draw down the balance on his account (i.e., imposing a hard lower bound on $b_t - c_t$).

[5] The reason we can consider autarky punishments is that they deliver the lowest possible individually rational payoffs for the players.

consumes

$$\bar{c}_\infty \equiv (1-\delta)\left(b_1 + \sum_{s=1}^{\infty} \delta^{s-1}\tilde{w}_s\right)$$

in every period. Clearly, any contract to which the agent is willing to adhere must then specify $\tilde{c}_t = \bar{c}_\infty$ for all t. To conclude that the agent does not want to deviate from the contract, it is then enough to show that he does not gain by planning to shirk on effort for the first time at any given date t, while making all other choices optimally. Suppose then that the agent plans to shirk for the first time at some date t, and so puts effort equal to $\tilde{e}_s$ for all $s < t$, and then optimally sets it equal to zero at all later dates. Then the agent optimally sets consumption equal to

$$\bar{c}_{t-1} \equiv (1-\delta)\left(b_1 + \sum_{s=1}^{t-1} \delta^{s-1}\tilde{w}_s\right) \tag{4}$$

at all dates, so as to completely smooth consumption and exhaust lifetime earnings. Note that this corresponds to the double deviation mentioned in the Introduction.

Given the above, the maximum payoff the agent achieves when deviating in choice of effort for the first time at date t is

$$\frac{1}{1-\delta}v(\bar{c}_{t-1}) - \sum_{s=1}^{t-1}\delta^{s-1}\psi(\tilde{e}_s).$$

Hence, the agent does not want to deviate from the agreement if and only if, for all $t \geq 1$,

$$\frac{1}{1-\delta}v(\bar{c}_{t-1}) - \sum_{s=1}^{t-1}\delta^{s-1}\psi(\tilde{e}_s) \leq \frac{1}{1-\delta}v(\bar{c}_\infty) - \sum_{s=1}^{\infty}\delta^{s-1}\psi(\tilde{e}_s). \tag{AC$_t^{\text{un}}$}$$

This is the incentive compatibility condition described in the Introduction.

The principal remains willing to continue abiding by the agreement if and only if, at each time t, the payment $\tilde{w}_t$ that is due is less than her continuation payoff in the agreement. The exact requirement is that, for all $t \geq 1$,

$$\tilde{w}_t \leq \sum_{s=t+1}^{\infty}\delta^{s-t}(\tilde{e}_s - \tilde{w}_s). \tag{PC$_t$}$$

The following result states that the above constraints determine whether a feasible relational contract is the outcome of a PBE.

Proposition 4.1. *Fix a feasible contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$. It is the outcome of a PBE if and only if, for all $t \geq 1$, Conditions* (AC$_t^{\text{un}}$) *and* (PC$_t$) *are satisfied, and $\tilde{c}_t = \bar{c}_\infty$.*

Necessity of the conditions in the proposition follow for the reasons described above. To obtain sufficiency, we completely specify PBE strategies and beliefs.

From now on we refer to a contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ that satisfies the conditions of Proposition 4.1 as "self-enforceable".[6] Our task reduces to characterizing feasible contracts $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ that maximize the principal's payoff subject to the requirement of being self-enforceable. We term such contracts "optimal".

To determine the properties of optimal contracts, we first show that we can restrict attention to contracts with a particular pattern of payments over time. This pattern involves paying the agent as early as possible, subject to satisfying the agent's incentive constraints. This requires that the agent's obedience constraints in Condition ($\mathrm{AC}_t^{\mathrm{un}}$) hold with equality for all $t \geq 1$. Inspired by the terminology of Board (2011), we refer to this condition as "fastest payments". We show the following.

Lemma 4.1. *For any optimal contract, there is another optimal contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ with the same sequence of efforts and consumption such that, for all $t \geq 1$,*[7]

$$\frac{v\left(\bar{c}_{t-1}\right)}{1-\delta} - \sum_{s=1}^{t-1} \delta^{s-1} \psi(\tilde{e}_s) = \frac{v\left(\left(1-\delta\right)b_1\right)}{1-\delta}. \tag{$\mathrm{FP}_t^{\mathrm{un}}$}$$

An explanation for the result is as follows. First, note that it is optimal to hold the agent to his outside option, and hence

$$\frac{v\left(\bar{c}_{\infty}\right)}{1-\delta} - \sum_{t=1}^{\infty} \delta^{t-1} \psi(\tilde{e}_t) = \frac{v\left(\left(1-\delta\right)b_1\right)}{1-\delta}. \tag{5}$$

If Condition (5) does not hold, $\tilde{e}_1$ can be slightly increased (keeping the rest of the contract the same) so that the constraints ($\mathrm{AC}_t^{\mathrm{un}}$) and ($\mathrm{PC}_t$) continue to hold for all t. Second, when ($\mathrm{FP}_t^{\mathrm{un}}$) holds for all t, the agent is paid as early as possible while preserving the constraints ($\mathrm{AC}_t^{\mathrm{un}}$). The agent cannot be paid earlier, otherwise he will prefer to work obediently for a certain number of periods, save his income at a higher rate than specified in the agreement, and then quit by exerting no effort. It is easily seen that moving payments earlier in time only relaxes the "principal's constraints" (PC_t).

Concerning "fastest payments", we have the following result.

Lemma 4.2. *Consider a feasible relational contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ that satisfies Condition ($\mathrm{FP}_t^{\mathrm{un}}$) at all dates. For any t, if $\tilde{e}_t > 0$, then*

$$\tilde{w}_t \in \left(\frac{\psi\left(\tilde{e}_t\right)}{v'\left(\bar{c}_{t-1}\right)}, \frac{\psi\left(\tilde{e}_t\right)}{v'\left(\bar{c}_t\right)} \right). \tag{6}$$

[6] We favor this term to "self-enforcing" because that latter would refer to a complete specification of strategies.

[7] The conclusion of Lemma 4.1 also follows if we permit the agent to make payments to the principal, alongside his choice of effort. The same argument as for Lemma 4.1 then establishes that, in an optimal contract, the agent never makes any payment to the principal, so results on optimal contracts are unaffected.

Since $\bar{c}_t$ is increasing in t, the lemma implies that the ratio $\frac{\tilde{w}_t}{\psi(\tilde{e}_t)}$ increases with t. This result translates the agent's incentive constraints ($\mathrm{AC}_t^{\mathrm{un}}$) into the conclusion that the agent becomes more expensive to compensate with time. One explanation is as follows. The longer the agent obediently works, the more he is paid in total. Since he can smooth his consumption of these payments over his entire lifetime, and since he has concave utility of consumption, he values additional payments less. Therefore, the payments needed to keep the agent obediently in the relationship, relative to the disutility of effort incurred, increase with time. As explained in the Introduction, this observation will be useful for understanding the shape of optimal relational contracts.

Apart from the observation in Lemma 4.2, the usefulness of Lemma 4.1 is that it permits the design of the relational contract to be reduced to the choice of an effort sequence $(\tilde{e}_t)_{t\geq 1}$. From $(\tilde{e}_t)_{t\geq 1}$ we can obtain $(\bar{c}_t)_{t\geq 1}$ using ($\mathrm{FP}_t^{\mathrm{un}}$) (so the corresponding consumption $\tilde{c}_t = \bar{c}_\infty$ is also pinned down). Then $(\tilde{w}_t)_{t\geq 1}$ is obtained from Equation (4), and $(\tilde{b}_t)_{t\geq 1}$ from Equation (1). We next discuss the implementation of first-best contracts (Section 4.1), before moving to optimal contracts when there is no first-best contract that is self-enforceable (Section 4.2).

4.1 Implementing the first-best outcome

Lemma 4.1 is also useful for understanding the conditions under which the principal obtains the first-best payoff. For instance, we can observe that the first-best effort and consumption, which are constant over time and equal to $e^{FB}(b_1)$ and $c^{FB}(b_1)$, can be implemented when the principal can commit to the agreement, but the agent cannot commit. For this, we simply suppose the principal agrees to payments satisfying the conditions in Equation ($\mathrm{FP}_t^{\mathrm{un}}$), provided the agent chooses effort obediently. Any deviation by the agent from the required effort is met with zero payments from then on. Because first-best effort is constant, and by Lemma 4.2, the payments determined by Equation ($\mathrm{FP}_t^{\mathrm{un}}$) are increasing over time. Since these represent the *earliest* payments that satisfy the agent's incentive constraints, the result makes clear that backloading of pay is essential to achieving first-best outcomes when the principal can fully commit.[8]

Now consider whether the principal can attain the first-best payoff when neither the principal nor agent can commit; i.e., whether there is a first-best contract that is self-enforceable. We can restrict attention to payments that satisfy the conditions in Equation ($\mathrm{FP}_t^{\mathrm{un}}$). As mentioned, Lemma 4.2 then implies that these payments increase over time. In the long run, payments approach

$$\frac{\psi\left(e^{FB}(b_1)\right)}{v'\left(c^{FB}(b_1)\right)}.$$

[8]Delaying payments relative to the ones determined by Equation ($\mathrm{FP}_t^{\mathrm{un}}$), while holding their NPV constant, only relaxes the agent's constraints ($\mathrm{AC}_t^{\mathrm{un}}$). Given the principal is assumed able to commit to these payments, such delayed payments also constitute an optimal implementation of first-best outcomes.

Because the principal's constraints (PC_t) tighten over time, verifying they are always satisfied amounts to verifying that

$$\frac{\psi\left(e^{FB}(b_1)\right)}{v'\left(c^{FB}(b_1)\right)} \leq \frac{\delta}{1-\delta}\left(e^{FB}(b_1) - \frac{\psi\left(e^{FB}(b_1)\right)}{v'\left(c^{FB}(b_1)\right)}\right). \tag{7}$$

The right-hand side is the limiting value of the principal's future discounted profits in the agreement, while the left-hand side is the limiting value of the payment to the agent. We have the following result.

Proposition 4.2. *Suppose that neither the principal nor agent can commit to the terms of the agreement and that consumption is unobservable. Then the principal attains the first-best payoff in an optimal contract if and only if Condition* (7) *is satisfied.*

While understanding the parameter range for which Condition (7) holds is clearly important for understanding optimal contracts, this is complicated by the dependence of the first-best policy on both b_1 and δ. Nonetheless, if we vary δ while allowing b_1 to adjust, holding $b_1(1-\delta)$ constant, then the first-best consumption and effort remain constant. There is then a threshold value of δ above which Condition (7) is satisfied, and below which it fails.

4.2 Main characterization for unobservable consumption

We now state our main result for the unobservable consumption case, which is a characterization of optimal effort when the first-best effort cannot be sustained.

Proposition 4.3. *An optimal relational contract exists. Suppose the principal cannot attain the first-best payoff in a self-enforceable contract (i.e., Condition* (7) *is not satisfied). Then, for any optimal contract, there is a date $\bar{t} \geq 1$ such that effort is constant up to this date, and is subsequently strictly decreasing. Effort converges to a value $\tilde{e}_\infty > 0$ in the long run. There exist b_1 and δ such that, for any optimal contract, the value $\bar{t}$ is strictly greater than one; in particular, effort can indeed be constant in the initial periods.*[9]

The dynamics of optimal effort when the principal cannot attain the first-best payoff can be explained as follows. There may be some initial periods when the effort is constant. This occurs if the principal's constraint (PC_t) is initially slack. Given that we consider "fastest payments", the payments rise over these periods for the reasons discussed in relation to Lemma 4.2. Given the principal cannot achieve the first-best payoff, it turns out that the principal's constraint eventually binds, and so payments must be reduced. This is only possible by reducing the level of effort. Note that how much effort can be asked without violating the principal's constraint

[9] The proof shows that effort is initially constant in an optimal contract for values of b_1 and δ close to those for which the principal can attain the first-best payoff.

depends on the future profitability of the relationship. Profitability declines over time, both because higher payments must be made relative to the agent's disutility of effort (see Lemma 4.2), and because the effort that can be requested is less. The fact that profitability declines contributes to the decline in effort, which creates a feedback loop.

Our approach to proving Proposition 4.3 relies on variational arguments. For contracts that fail to exhibit the dynamics described in the proposition, we construct more profitable contracts satisfying all the constraints in Proposition 4.1. We demonstrate some of these arguments below.

One useful result towards establishing Proposition 4.3 links the dynamics of effort to the dates at which the principal's constraint (PC_t) is slack (rather than holding with equality).

Lemma 4.3. *Suppose that* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ *is an optimal relational contract. Suppose that the principal's constraint is slack at* t^**, i.e.* $\tilde{w}_{t^*} < \sum_{s=t^*+1}^{\infty} \delta^{s-t^*} (\tilde{e}_s - \tilde{w}_s)$*. Then,* $\tilde{e}_{t^*+1} \leq \tilde{e}_{t^*}$*; also, if* $t^* > 1$*, then* $\tilde{e}_{t^*-1} \leq \tilde{e}_{t^*}$*.*

The proof (in the Appendix) proceeds by showing that, if the conclusion of the lemma fails, then effort can be smoothed raising the principal's profits. Such smoothing is profitable given that the disutility of effort is strictly convex (so that differences in effort across periods are inefficient). An immediate implication of the lemma is that effort is constant over any sequence of periods for which the principal's constraint is slack, explaining why effort may be constant in the initial periods.

A further key part of the proof of Proposition 4.3 is to show that effort strictly decreases from a finite date $\bar{t}$ onwards. The main steps of this argument can be explained as follows. Building on Lemma 4.3, we are able to show (in Lemma A.5 in the Appendix) that effort is weakly decreasing with time. Lemma A.6 then establishes that, if the principal's constraint (PC_t) holds with equality at some date $\hat{t}$, then $\tilde{e}_{\hat{t}+1} < \tilde{e}_{\hat{t}}$ and the constraint holds with equality also at $\hat{t}+1$. Hence effort strictly decreases from $\hat{t}$ onwards.

The argument for Lemma A.6 can be summarized as follows. By assumption, the principal's constraint (PC_t) at date $\hat{t}$ holds as an equality, i.e.

$$\tilde{w}_{\hat{t}} = \sum_{s=\hat{t}+1}^{\infty} \delta^{s-\hat{t}} (\tilde{e}_s - \tilde{w}_s) .$$

We are able to show that $\tilde{e}_{\hat{t}+1} - \tilde{w}_{\hat{t}+1} > \tilde{e}_s - \tilde{w}_s$ for all $s > \hat{t}+1$. This follows because $\psi'(\tilde{e}_t) \leq v'(\bar{c}_\infty)$ for all t (as established in Lemma A.1), because effort is weakly decreasing over time (as noted above), and making use of Lemma 4.2 (which recall implies that the ratio of payments to disutility of effort increases with time). Therefore,

$$\tilde{w}_{\hat{t}} = \sum_{s=\hat{t}+1}^{\infty} \delta^{s-\hat{t}} (\tilde{e}_s - \tilde{w}_s) > \sum_{s=\hat{t}+2}^{\infty} \delta^{s-\hat{t}-1} (\tilde{e}_s - \tilde{w}_s) \geq \tilde{w}_{\hat{t}+1}$$

where the second inequality is the principal's constraint (PC_t) at date $\hat{t}+1$. Hence, (again using Lemma 4.2) effort is *strictly* lower in period $\hat{t}+1$ (i.e., $\tilde{e}_{\hat{t}+1} < \tilde{e}_{\hat{t}}$). In turn, using Lemma 4.3, the principal's constraint must hold again with equality at $\hat{t}+1$. So we have shown that, if the principal's constraint holds with equality at a given date, it must hold with equality from then on, and so effort strictly decreases with time.

The above argument assumes that the principal's constraint (PC_t) holds with equality at some date. To show this must in fact be the case when the principal cannot attain the first-best payoff, assume to the contrary that these constraints are always slack. Then Lemma 4.3 implies that optimal effort is constant at all dates, say at a value $\tilde{e}_\infty$ (using the notation of the proposition). Letting the payments and the equilibrium consumption $\bar{c}_\infty$ be determined by Equation (FP_t^{un}), payments increase over time, and converge to $\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}$. The principal's constraint (PC_t) is then satisfied at all dates if and only if

$$\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} \leq \frac{\delta}{1-\delta}\left(\tilde{e}_\infty - \frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}\right),$$

where the left-hand side can be read as the limiting payment to the agent, while the right-hand side is the limiting NPV of future profits to the principal. For the most profitable choice of a constant effort $\tilde{e}_\infty^*$, this inequality holds as equality. The principal's constraints (PC_t) tighten over time, but never hold with equality.

Because effort is below the first-best level, we have $\psi'(\tilde{e}_\infty^*) < v'(\bar{c}_\infty^*)$, with $\bar{c}_\infty^*$ the level of agent consumption that corresponds to a contract with constant effort $\tilde{e}_\infty^*$. It follows that any sufficiently small adjustment to the effort policy that raises the NPV of effort, together with a change in payments and consumption that leaves the agent's payoff in the contract unchanged, raises profits. We therefore suggest a perturbation to the constant-effort contract (see Lemma A.7 in the Appendix) that increases the NPV of effort, but (assuming that payments continue to satisfy (FP_t^{un})) leaves the principal's constraints (PC_t) intact. To be more precise, we consider increasing effort by a little at date one and lowering it by a constant amount in future periods. If we only raise effort at date one, leaving other effort values unchanged and assuming that payments are adjusted to satisfy (FP_t^{un}) at all dates, the principal's constraint (PC_t) is eventually violated (since v is strictly concave and total pay increases, it becomes more costly to compensate the agent for his effort; in particular, payments must increase in all periods). Therefore the reduction in effort at future dates is a "correction" intended to relax the principal's constraint (PC_t) when it is tightest.

We have established then that, when the first-best payoff is not attainable, the principal's constraint (PC_t) holds with equality from some date onwards. At these dates, the principal is indifferent between paying the agent and reneging. This feature is the same as in the optimal contracts of Ray (2002) (although his model is quite general, it does not include the possibility of savings or investments).

It remains to translate the findings of Proposition 4.3 into predictions for payments and the agent's balance. Note however, that while Lemma 4.1 tells us it is optimal for Condition ($\mathrm{FP}_t^{\mathrm{un}}$) to hold at all dates, other contracts with a different timing for payments may be optimal. We therefore provide a partial converse for Lemma 4.1.

Proposition 4.4. *Suppose the principal cannot attain the first-best payoff in a self-enforceable contract. Fix any optimal contract* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ *and let* $\bar{t}$ *be the date from which effort is strictly decreasing (see Proposition 4.3). Then, Condition* ($\mathrm{FP}_t^{\mathrm{un}}$) *holds for all* $t > \bar{t}$. *Payments to the agent strictly decrease from date* $\bar{t}+1$ *onwards, while the agent's balances strictly increase.*

The reason payments satisfying Condition ($\mathrm{FP}_t^{\mathrm{un}}$) are strictly decreasing from date $\bar{t}+1$ is explained above. The fact that the agent's balance increases over time then follows straightforwardly from Equation (1) and from Equation (2) taken to hold with equality. In particular, note that

$$\tilde{b}_t = \frac{\bar{c}_\infty}{1-\delta} - \sum_{\tau=t}^{\infty} \delta^{\tau-t} \tilde{w}_\tau$$

which strictly increases with t when payments fall over time.

Note that, when $\bar{t} > 1$, the principal's constraint (PC_t) is initially slack. In this case, Condition ($\mathrm{FP}_t^{\mathrm{un}}$) need not hold at $t < \bar{t}$, and so payments before date $\bar{t}$ are not uniquely determined. When this "fastest payments" condition is nonetheless taken to hold, payments in fact increase over time up to date $\bar{t}$ (as was mentioned above).

5 Observed consumption

We now study the case where, at each time t, before making the payment w_t, the principal can observe the agent's past and present-period effort choices $(e_s)_{s=1}^t$ as well as past and present-period consumption choices $(c_s)_{s=1}^t$. Since payments and consumption are commonly observed, the balance b_t at the beginning of each period t is also commonly known (as deduced from Equation (1)).

The game is now one of complete information, and we consider sub-game perfect Nash equilibrium (SPNE). Both players observe at date t the history $h_t = (e_s, w_s, c_s)_{1\leq s<t}$. The set of such histories at each date t is $\mathcal{H}_t = \mathbb{R}_+^{3(t-1)}$. Re-using notation from Section 4 introduces no confusion, so we describe a strategy for the agent as a collection of functions

$$\alpha_t : \mathcal{H}_t \to \mathbb{R}_+^2,\ t \geq 1,$$

and a strategy for the principal as a collection of functions

$$\sigma_t : \mathcal{H}_t \times \mathbb{R}_+^2 \to \mathbb{R}_+,\ t \geq 1.$$

Here, α_t maps the public t-history to a pair (e_t, c_t) of effort and consumption. Also, σ_t maps the public t-history, together with the observed effort and consumption choices (e_t, c_t) of the agent, to a payment w_t. We assume that payoffs are as specified in Section 4 (i.e., the agent earns a payoff $-\infty$ in case his intertemporal budget constraint (2) is violated).

Again we identify a relational contract with the equilibrium outcomes, and we want to characterize contracts that maximize the principal's payoff. A first step is then to determine equilibrium outcomes $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ that are feasible relational contracts. Analogous to the arguments made in the previous section, we begin supposing deviations from the agreed outcomes are punished by "autarky". That is, when either player deviates from the contract, all future effort and payments cease, and the agent perfectly smooths his balance over time. In autarky, the agent consumes $b_t\,(1-\delta)$ when his balance is $b_t > 0$, and we specify zero consumption in case the balance is $b_t \leq 0$ (in the latter case, the agent can only obtain a payoff of $-\infty$ since violating the intertemporal budget constraint in Equation (2) implies this payoff; hence we might as well set consumption to zero). Now, autarky follows not only deviations in effort and payments, but also in consumption.

Suppose that the agreed contract is $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$, and deviations are punished by autarky. The agent's payoff, if complying until date $t-1$ and optimally failing to comply from t onwards, is now

$$\sum_{s=1}^{t-1} \delta^{s-1}(v(\tilde{c}_s) - \psi(\tilde{e}_s)) + \delta^{t-1}\frac{v(\max\{0,(1-\delta)\tilde{b}_t\})}{1-\delta}.$$

This takes into account that the agent who deviates at date t optimally exerts no effort from then on, and consumes $\max\{0,(1-\delta)\tilde{b}_t\}$ per period as explained above. Thus, the agent is willing to follow the prescription of the contract if and only if, at all dates t,

$$\frac{v(\max\{0,(1-\delta)\tilde{b}_t\})}{1-\delta} \leq \sum_{s=t}^{\infty} \delta^{s-t}(v(\tilde{c}_s) - \psi(\tilde{e}_s)). \qquad (\mathrm{AC}_t^{\mathrm{ob}})$$

The reason for the difference to Condition $(\mathrm{AC}_t^{\mathrm{un}})$ is that publicly honoring the agreement up to date $t-1$ ensures that the agent begins period t with the specified balance $\tilde{b}_t$, which in turn determines the wealth he has available to spend in autarky. Condition $(\mathrm{AC}_t^{\mathrm{un}})$, on the other hand, takes into account that the agent who plans to publicly deviate at date t (by shirking on effort) can save in advance for this event, because consumption is not observed.

We can characterize equilibrium outcomes as follows.

Proposition 5.1. *Fix a feasible contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$. It is the outcome of an SPNE in the environment where consumption is observed if and only if, for all $t \geq 1$, Conditions* $(\mathrm{AC}_t^{\mathrm{ob}})$ *and* (PC_t) *are satisfied.*

Notice here that the principal's constraint (PC_t) is the one in Section 4. A feasible contract

$(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ satisfying the conditions in the proposition is again termed "self-enforceable" and a self-enforceable contract that maximizes the principal's payoff is "optimal". We can now state a result similar to Lemma 4.1.

Lemma 5.1. *For any optimal contract, there exists another optimal contract* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ *with the same effort and consumption, with* $\tilde{b}_t > 0$ *for all* t*, and where the timing of payments ensures that agent constraints hold with equality in all periods; that is, for all* $t \geq 1$*,*

$$\frac{v(\tilde{b}_t(1-\delta))}{1-\delta} = \sum_{s=t}^{\infty} \delta^{s-t} \left(v\left(\tilde{c}_s\right) - \psi\left(\tilde{e}_s\right)\right). \tag{8}$$

Lemma 5.1 implies that we can focus on relational contracts where, for all $t \geq 1$,

$$\tfrac{1}{1-\delta} v((1-\delta)\tilde{b}_t) = v(\tilde{c}_t) - \psi(\tilde{e}_t) + \tfrac{\delta}{1-\delta} v((1-\delta)\tilde{b}_{t+1}). \tag{FP$_t^{\text{ob}}$}$$

This says that the agent is indifferent between quitting at date t (i.e., ceasing to exert effort) and smoothing the balance $\tilde{b}_t$ optimally over the infinite future, and instead working one more period, exerting effort $\tilde{e}_t$ and consuming $\tilde{c}_t$, before quitting at date $t+1$ and smoothing the balance $\tilde{b}_{t+1}$ over the infinite future.

5.1 Implementing the first-best outcome

Let us turn now to the question of when the principal can attain the first-best payoff in a self-enforceable relational contract. As for the case with private savings, we can begin by asking how the principal implements the first-best outcomes if she can fully commit to payments (but the agent cannot commit). We can again answer this question by focusing on the earliest payments, where Equation (8) is satisfied at all dates, noting that delayed payments (with the same NPV) will also do the job. Any deviation in effort or consumption leads to a cessation of pay. Given effort and consumption constant at the first-best levels $e^{FB}(b_1)$ and $c^{FB}(b_1)$, the agent's balance under the specified payments is constant and equal to b_1. Therefore, the payment is constant over time and equal to $w^{FB}(b_1) \equiv c^{FB}(b_1) - (1-\delta)b_1$. This shows an important difference between the solutions to the principal's full-commitment problem when the agent's consumption is observed rather than unobserved. With observed consumption, payments can be made earlier without the agent quitting the agreement; in particular, they are constant rather than rising over time. This is because agent deviations of secretly saving and then quitting are not available as any deviation in consumption is observed and so punished by a cessation of pay.

Now turn to the question of when the principal is able to obtain the first-best payoff when she cannot commit. Note that the principal's continuation payoff in a first-best contract with

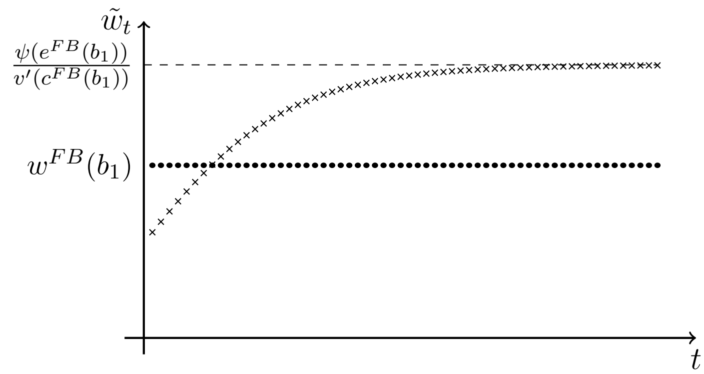

Figure 1: Payments for optimal relational contracts satisfying fastest payments when Equation (7) holds (and so the the principal obtains her first-best payoff), in the unobservable case (crosses) and observable case (circles).

the earliest payments is

$$V^{FB}(b_1) = \frac{e^{FB}(b_1) - w^{FB}(b_1)}{1-\delta}.$$

Using the above observations, we have the following result.

Proposition 5.2. *Suppose that consumption is observable. Then the principal attains the first-best payoff in an optimal relational contract if and only if*

$$w^{FB}(b_1) \leq \frac{\delta}{1-\delta}\big(e^{FB}(b_1) - w^{FB}(b_1)\big). \tag{9}$$

Condition (9) is more easily satisfied than Condition (7) (the condition for the unobservable consumption case). This follows immediately from showing that

$$w^{FB}(b_1) < \frac{\psi\left(e^{FB}(b_1)\right)}{v'\left(c^{FB}(b_1)\right)}. \tag{10}$$

Here, $w^{FB}(b_1)$ is the constant payment to the agent in the observed-consumption case, as specified above. On the other hand, $\frac{\psi(e^{FB}(b_1))}{v'(c^{FB}(b_1))}$ is the limiting payment for the unobserved-consumption case (assuming that payments satisfy the "fastest payments" condition in Equation (FP_t^{un})).

The key insight is that, in the observed-consumption case, the principal's constraints (PC_t) are identical in every period, since payments remain constant. In contrast, in the unobserved-consumption case, we saw that they tighten over time. After enough time, the payments in the unobserved-consumption case exceed the constant payments in the observed-consumption case (note that the NPV of payments in both cases is the same), which makes the principal's constraints more difficult to satisfy. Figure 1 illustrates the payments in optimal contracts achieving the first-best payoff for the principal in the unobserved and observed consumption cases.

To derive the inequality (10) formally, observe that by concavity of v, and because $c^{FB}(b_1) > (1-\delta)\, b_1$, we have

$$v\big(c^{FB}(b_1)\big) - v\left((1-\delta)\, b_1\right) > v'\big(c^{FB}(b_1)\big)\big(c^{FB}(b_1) - (1-\delta)\, b_1\big) = v'\big(c^{FB}(b_1)\big)w^{FB}(b_1).$$

The result then follows because the first-best effort and consumption satisfy $v\big(c^{FB}(b_1)\big) - v\left((1-\delta)\, b_1\right) = \psi\big(e^{FB}(b_1)\big)$ by Condition 2 of Proposition 3.1.

5.2 Optimal contract with observed consumption

Now consider the problem of characterizing an optimal contract when the principal's first-best payoff is not attainable. We can consider the "fastest payments" as given in Lemma 5.1. It is convenient to write the principal's problem recursively, with the balance $\tilde{b}_t$ a state variable for the relationship, applying the principle of optimality. Indeed, suppose for some date t that the continuation contract $(\tilde{e}_s, \tilde{w}_s, \tilde{c}_s, \tilde{b}_s)_{s\geq t}$ does not maximize the continuation value of the relationship to the principal $\sum_{s=t}^{\infty} \delta^{s-t}(e_s - w_s)$, subject to it being self-enforceable; i.e., there is some more profitable self-enforceable continuation contract $(\tilde{e}'_s, \tilde{w}'_s, \tilde{c}'_s, \tilde{b}'_s)_{s\geq t}$ with $\tilde{b}'_t = \tilde{b}_t$, which can be taken to satisfy the agent's indifference conditions (8) at all dates. Then this contract can be substituted, increasing the continuation value $\sum_{s=t}^{\infty} \delta^{s-t}(e_s - w_s)$ (and hence the principal's payoff in the contract overall), maintaining the agent indifference conditions (8) at all dates, and continuing to satisfy the principal's constraints (PC_t).

Since an optimal contract maximizes the principal's continuation profits, an optimal contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ must solve a sequence of sub-problems with value $V(\tilde{b}_t)$ given by

$$V(\tilde{b}_t) = \max_{e_t, b_{t+1}, c_t \in \mathbb{R}_+} \big(e_t - (\delta b_{t+1} - \tilde{b}_t + c_t) + \delta V(b_{t+1})\big) \tag{11}$$

subject to the agent's indifference condition

$$v(c_t) - \psi(e_t) + \tfrac{\delta}{1-\delta} v((1-\delta)b_{t+1}) = \tfrac{1}{1-\delta} v((1-\delta)\tilde{b}_t). \tag{12}$$

and to the principal's constraint

$$\delta b_{t+1} - \tilde{b}_t + c_t \leq \delta V(b_{t+1}). \tag{13}$$

The left-hand side of (13) can be understood as the date-t payment w_t, which is divided into date-t consumption $c_t \in \mathbb{R}_+$ and savings $\delta b_{t+1} - b_t \in \mathbb{R}$. Non-negativity of the payment $\delta b_{t+1} - \tilde{b}_t + c_t$ is assured by the equality (12) and the concavity of v. The same equality ensures that, given $\tilde{b}_t$ is strictly positive, optimal c_t and b_{t+1} must be strictly positive also.

We show that any optimal policy for the principal can be characterized as follows.

Proposition 5.3. *An optimal contract exists. Suppose that, given the balance b_1, an optimal contract $(\tilde{e}_t, \tilde{w}_t, \tilde{c}_t, \tilde{b}_t)_{t\geq 1}$ fails to obtain the first-best payoff $V^{FB}(b_1)$. Then the agent's balance $\tilde{b}_t$ and consumption $\tilde{c}_t$ decline strictly over time, with $\tilde{b}_t \to \tilde{b}_\infty$ for some $\tilde{b}_\infty > 0$. Effort $\tilde{e}_t$ and the payments $\tilde{w}_t$ determined by the Conditions* (8) *increase strictly over time. We have $V(\tilde{b}_t) \to V^{FB}(\tilde{b}_\infty)$ as $t \to \infty$, and effort and consumption converge to first-best levels for the balance $\tilde{b}_\infty$.*

A heuristic account of the forces behind this result is as follows. When the principal's constraint ((PC$_t$) or equivalently (13)) binds, effort is suppressed. That is, if the principal could increase effort and credibly increase payments to keep the agent as well off, she would gain by doing so. Also, the principal's value function $V(\cdot)$ is strictly decreasing; intuitively, because a lower balance makes the agent cheaper to compensate to keep him in the agreement. Therefore, for any date t, reducing the balance b_{t+1} increases the principal's continuation payoff $V(b_{t+1})$ and relaxes the principal's date-t constraint (13). Therefore, the principal asks the agent to consume earlier than he would like, driving the balance down over time. This continues to a point where, given the revised balance, the contract is close to first best, and so the value of continuing to distort consumption vanishes.

It is worth pointing out here that the dynamics of $V(\tilde{b}_t)$ are determinative of both the dynamics of effort and payments. When there is no self-enforceable first-best contract, $V(\tilde{b}_t)$ strictly increases with t, and moreover the principal's constraint (13) binds. The latter implies that, for all t, both $\tilde{w}_t = \delta V(\tilde{b}_{t+1})$ and $V(\tilde{b}_t) = \tilde{e}_t - \tilde{w}_t + \delta V(\tilde{b}_{t+1}) = \tilde{e}_t$.[10]

A further part of our analysis worth highlighting is an Euler equation

$$1 - \frac{v'((1-\delta)\tilde{b}_{t+1})}{v'(\tilde{c}_t)} = \frac{v'(\tilde{c}_{t+1})}{\psi'(\tilde{e}_{t+1})}\left(1 - \frac{v'((1-\delta)\,\tilde{b}_{t+1})}{v'(\tilde{c}_{t+1})}\right) \tag{14}$$

which must hold for an optimal contract at all dates t, and which we use to derive several key properties. This condition is derived (in Lemma A.11) by fixing the contract at and before $t-1$, and from date $t+2$ onwards, and then requiring the contractual variables at t and $t+1$ to be chosen optimally. The equation captures the relationship between a static distortion in effort and a dynamic distortion in consumption. In particular, when the principal's first-best payoff cannot be attained, we are able to show that $\psi'(\tilde{e}_{t+1}) < v'(\tilde{c}_{t+1})$ for all t (reflecting a static (downward) distortion in effort), and correspondingly $(1-\delta)\,\tilde{b}_{t+1} < \tilde{c}_{t+1} < \tilde{c}_t$ (i.e., consumption strictly decreases over time, which is a dynamic distortion). A trade-off between the static and dynamic distortions should be anticipated, since asking the agent to consume

[10]Note that the conclusion the the principal's constraint (13) binds is obtained under the assumption that Conditions (FP$_t^{\text{ob}}$) hold at all dates; but we establish in Proposition 5.4 below that the satisfaction of Conditions (FP$_t^{\text{ob}}$) is necessary for optimality.

excessively early in the relationship increases the agent's marginal utility of consumption later on, which makes him easier to motivate and permits higher effort and profits at later dates. In turn, this relaxes the principal's credibility constraint (PC_t), permitting higher payments and therefore effort also early in the relationship. As $\tilde{b}_t \to \tilde{b}_\infty$, consumption falls to its lower bound, becoming almost constant, so $\frac{v'(\tilde{c}_{t+1})}{\psi'(\tilde{e}_{t+1})} \to 1$, which accords with convergence of effort and consumption to first-best levels.

Finally, analogous to Proposition 4.4, we provide a result on the uniqueness of the timing of payments.

Proposition 5.4. *Suppose the principal cannot attain the first-best payoff in a self-enforceable relational contract. Then, in any contract that is optimal for the principal, Condition* (FP_t^{ob}) *holds at all dates. Hence payments to the agent strictly increase over time.*

The logic of this result is that, if the Condition (FP_t^{ob}) fails, then payments can be made earlier in time, while maintaining the agent constraints (AC_t^{ob}). This induces slack in the principal's constraint (PC_t), which can then be exploited by increasing payments, consumption and effort, increasing profits. As noted for the case of unobservable consumption, such an observation is related to arguments in Ray (2002).

6 Applications and literature review

6.1 Applications

The comparison between private and public savings suggests firms might want to monitor consumption and promote high spending, since it can avoid the deterioration of the relationship that is associated with the accumulation of wealth. In our framework with observable consumption, high consumption is maintained through the threat of termination of the agreement. There could be a range of ways firms encourage high consumption and low savings in practice. This is the thesis of the legal scholars Henderson and Spindler (2004) who argue that firms seek to reduce the savings especially of top employees through "payment-in-kind (perks), deferred compensation (corporate loans), and the encouragement of employees' conspicuous consumption" (p. 1835). They argue that these tools are being used precisely to resolve agency problems: "Employees who reduce savings are more reliable over the long term than employees who do not, since reduced savings makes employees more dependent on remaining employed into the future" (p. 1835). Therefore high consumption can serve a useful purpose in the agency relationship with senior management and it is not necessarily the case that high perks are simply a sign of corporate excess and poor governance.[11]

[11] While Henderson and Spindler develop their argument in detail, they do not offer an economic model as we do here. Without such a model, it is difficult to know whether distortions in consumption are worthwhile.

While our theory can also explain the use of perks or corporate loans to encourage dependency, the model most directly captures the encouragement of conspicuous consumption.[12] Henderson and Spindler (2004) argue precisely that high consumption expenditures may be an implicit requirement of an employer (see the quote in the Introduction). To develop their argument, they note ideas such as in Fournier (1991) that "products can help in the creation and management of identities at the group and society levels [...] by serving as unambiguous announcements of role and position" (p. 14). This suggests that the acquisition of certain types of goods can seem close to being necessary for maintaining a certain role or position in a firm;[13] Henderson and Spindler argue that such a perception can benefit the firm by preventing top employees from accumulating wealth. Among their examples are the cars driven by corporate employees: "BMW makes a line of automobiles of gradated expense that are meant to be marketed to those at various stages of the corporate ladder; entry-level employees in the "executive segment" are meant to purchase, of course, "entry level" BMWs. Or there may be certain posh suburbs, expensive restaurants, or fashion designers that an employee is expected to spend her money on" (p. 1869). Another of their examples is historical, coming from Louis XIV of France: "Louis adopted extravagantly expensive fashions, which his courtiers were expected to emulate. The courtiers thus spent all of their money on these fashions and became entirely dependent upon Louis' allowance to them. In that case, as in the above examples, the employee destroys value through extravagant and wasteful consumption, which serves to bind herself to the firm (or sovereign, as the case may be)" (p. 1870). The case of Enron is then compared to Louis XIV in providing an example of corporate culture of high spending developed through leadership: Chairman Ken Lay and CEO Jeff Skilling "created a "culture of excess" that, according to one executive, "could spoil you pretty well." Lay and Skilling drove fancy cars and built mansions in Aspen, Colorado and tiny Houston neighborhoods. Their minions followed suit.... According to the special report prepared by the board of directors after Enron was wiped out, Enron's senior leadership created a culture of spending to excess that permeated the ranks of top executives..." (p. 1870). Henderson and Spindler thus argue that even the infamous case of Enron can prove their point.

To add our own examples, similar ideas may apply to supply relationships between small firms and large procurers. A possible case in point relates to the US poultry industry, where chicken farmers supply to a few large firms that dominate the industry. An article at *The*

On the one hand, the principal seems set to benefit from the dependence of an employee with low savings. On the other hand, from an ex-ante perspective, the employee needs to be compensated for the lack of discretion in consumption/savings decisions.

[12] Our findings suggest that, if consumption is otherwise hidden, the principal could benefit by paying the agent partly in kind (perks), effectively forcing the agent to consume. Also, loans to the employee could relax borrowing constraints to allow the employee to consume at a higher level.

[13] The view that established norms may require high spending is similar to an observation of Postlewaite (1998). He argues that excessive consumption might be sustained due to a need to meet cultural norms rather than necessarily being a result of signaling.

Guardian illustrates what it sees as a common situation through the example of a farmer that contracted with chicken producer Tyson Foods.[14] The farmer in question entered an exclusive agreement with Tyson. After some time, Tyson began demanding additional expenditures on equipment such as extra feed bins and chicken houses the farmer believed unnecessary. The farmer commented: "If we are independent contractors, then why does the company have the right to tell us what equipment to use?" After the farmer failed to comply with the demands, the relationship deteriorated and, in the end, it was terminated. The connection to our theory is that Tyson asked for expenditures by the farmer that (according to the farmer) were to a degree superfluous, but as in our theory could have served the purpose of making him financially dependent. In fact, another farmer in the article commented precisely that such financial dependence is the producers' objective: "As long as they keep us in debt we have to keep raising their chickens. They don't want farmers to pay off their farms."

Still a further example of an institution encouraging high employee consumption and low savings is the "trucking system" that operated for instance in eighteenth and nineteenth century Britain. According to Hilton (1957), "in the nineteenth century the truck system consisted mainly of compulsion to deal with the employer's grocery store at risk of reprimand or discharge" (p. 237). The requirement to shop at the company store was typically only backed by an implicit threat. As Bailey (1859) writes about the typical worker under the system: "He is not obliged to go to the tommy-shop or the butty collier's drinking-shop,– of course not, – none of the workmen ever were; it is of their own choice to go there, – choice between that and having no work to do" (p. 17). As for the principal's optimal contract in our model, failure to consume at the specified level is associated with an implicit threat of termination. Bailey also describes the impoverishment of workers under this system. While the conventional explanation of the system is the expropriation of profits through elevated prices of goods, our theory suggests the impoverishment of the worker through the demand of a certain level of consumption could be a benefit in itself. The idea of impoverishment and dependence is perhaps best captured by the song Sixteen Tons by Merle Travis, first recorded in 1946, about a coal miner:

You load sixteen tons, what do you get?
Another day older and deeper in debt
St. Peter, don't you call me 'cause I can't go
I owe my soul to the company store

[14]See "Fowl play: The chicken farmers being bullied by big poultry," by Alison Moodie, published at *The Guardian* on April 22nd, 2017. https://www.theguardian.com/sustainable-business/2017/apr/22/chicken-farmers-big-poultry-rules

6.2 Literature review

The literature on relational contracts has been reviewed in MacLeod (2007) and Malcomson (2015). Classic references include Bull (1987), MacLeod and Malcomson (1989) and Levin (2003). In many papers – for instance, Fuchs (2007), Chassang (2010), Halac (2012), Li and Matouschek (2013), Yang (2013), Andrews and Barron (2016), Malcomson (2016), Fong and Li (2017), Li et al. (2017), and Barron and Powell (2019) – exogenous uncertainty plays a key role and the dynamics of relationships are studied in light of this uncertainty. In our paper we are concerned with relationship dynamics when the environment is deterministic. The center of our analysis is then the interaction between dynamic enforcement constraints and the agent's consumption-smoothing preferences and ability to save.

Our paper has an important connection to settings with limited commitment and some form of savings or storage. Perhaps most closely related in terms of the model, Bull (1987) studies a setting where the principal faces overlapping generations of agents who each work for two periods. Like our setting, agents exert productive effort, and have concave utility from consumption and can save. The paper focuses on providing conditions under which efficient effort can be sustained, with the agent free to optimally determine consumption/savings. It does not examine the dynamics when efficient outcomes are not sustainable, which is a central objective of our paper.

Other papers where agents face a consumption/storage decision, and where there is limited commitment, include Kehoe and Perri (2002), Ligon et al. (2000), Wahhaj (2010), and Voena (2015).[15] Kehoe and Perri consider international lending in a setting where countries both consume and invest. Ligon et al. and Wahhaj consider a setting of mutual insurance among agents who can consume and save. Voena considers married couples who can accumulate both financial assets and human capital, and who can divorce. In all these papers, the level of investment or savings affects the value of the outside option that is accessed through default, since it affects the payoffs available in autarky. Ligon et al. find that savings in the optimal agreement could be either higher or lower than privately optimal due to agent outside options, and are unable to sign the direction of the effect. On the other hand, Wahhaj provides some conditions under which savings are distorted downwards. He links these effects to social pressure towards low savings in tribal societies. There are perhaps three main points of distinction from our work. First, we examine a setting with agent effort, so our model can naturally represent for instance employment relationships. Second, we provide more complete analytical characterizations.[16]

[15]Also related is Marcet and Marimon (1992, their fourth case) which considers a lender and a manager, where the manager can choose between consumption and investment. Note however that the lender is able to fully commit.

[16]Kehoe and Perri and Ligon et al. rely on numerical simulations. Wahhaj provides conditions for downward distortions in savings that are on endogenous variables, though analytically verifies his claim for a case with CARA utility. Voena focuses on a structural estimation of her model rather than analytic results.

For instance, we provide analytical results on whether pay, effort, consumption and savings are increasing or decreasing, and we establish convergence to the first best in the case of public savings. The third and crucial distinction is that we explore the situation where storage/savings is private to the agent, giving rise to the concern of double deviations as explained in the Introduction. We noted there that this is in common with Ábrahám and Laczó (2018), a further setting with savings and limited commitment.[17] The Introduction explained our important differences to this work.

Our paper also connects to work on outside options in relational contracts more generally. For instance, we can compare to settings where outside options are either private but exogenous or endogenous but public. With respect to the former, Halac (2012) considers a model where the principal's outside option is her private information and is exogenous and persistent. Examples of the latter include Englmaier and Fahn (2019) and Malcomson (2021), where initial one-time public investments affect payoffs both inside and outside the agreement (see also Halac, 2015, for a related model). Similarly, Fahn et al. (2019) consider a setting where the up-front one-shot decision is the capital structure of the firm and this decision influences payoffs when the relational agreement breaks down.[18]

Our work on private savings is related to the literature on moral hazard with private savings where the principal has commitment power; see Footnote 1 for references. In our deterministic setting where monitoring of effort is perfect, we show that principal commitment implies effort that is constant and efficient. This contrasts with work on imperfect monitoring where there is a trade-off between incentives for more efficient effort and the additional riskiness of pay. We show that, when savings are private and the principal commits, payments must be backloaded to avoid double deviations in effort and savings (see Section 4.1). On the other hand, if players are impatient enough and the principal cannot commit, then the principal is constrained in what she can credibly pay, and effort and pay eventually decline. These effects are absent from the moral hazard literature with full commitment, where there are no credibility constraints on what the principal can pay, and where there is often indeterminacy in the timing of pay. However, there, the timing of consumption is still uniquely determined. In work on moral hazard with private savings, the marginal utility of consumption must be a martingale if the agent can both save and borrow (or it must be a supermartingale if the agent can only save). This can manifest in a trade-off between committing to riskier future consumption and reducing the extent to which consumption is backloaded (see, for example, Di Tella and Sannikov, 2021). In contrast, given perfect monitoring of effort and deterministic pay, we predict constant consumption, allowing

[17] Thomas and Worrall (1994, Section 4) and Garicano and Rayo (2017) are still further papers where agents' outside options are determined through variables that accumulate with time, but modeling differences make these papers more difficult to compare. Again accumulation is public information in these settings whereas a key part of our analysis instead focuses on private accumulation.

[18] Other papers that consider the role of debt in a relational contracting setting include Hennessy and Livdan (2009) and Barron et al. (2021).

us to establish that savings are eventually monotonically increasing on path.

A key goal of the literature on moral hazard with private savings has been to obtain analytic characterizations of optimal contracts. This has been challenging, however, because of the complexity of potential agent deviations involving saving more and shirking.[19] Most of the papers that characterize contracts analytically do so via a first-order approach where complex deviations can at first instance be ignored in the study of a "relaxed optimization program" that addresses only local deviations in effort and consumption (examples include Ábrahám et al., 2011, Edmans et al., 2012, Williams, 2015, and Di Tella and Sannikov, 2021). An exception is Mitchell and Zhang (2010) where, similar to our paper, the binding constraints relate to global deviations of shirking and saving. A key simplification in our optimization program is that relevant agent deviations involve pairing public deviations in effort with a choice of consumption that is optimal in light of the agent's reduced pay. This simplification is not available when effort is imperfectly observed. The tractability of our model permits predictions on the optimal contract without strong restrictions on model primitives such as functional form assumptions on preferences which have been common in the literature.[20]

Similar to our paper, an objective of the moral hazard literature with private savings has been comparison to the case where savings are public. Given full commitment, if the principal observes consumption/savings, then there is no loss in taking pay to equal consumption and having the principal in this sense "save for the agent". It has then been understood since Rogerson (1985) that optimal contracts in repeated moral hazard force the agent to consume more upfront than privately optimal; if the agent could save privately and consume later, he would do so.[21] Early consumption is driven here by the optimal provision of incentives for imperfectly monitored effort, rather than the absence of commitment as in our paper. The optimal timing of consumption in formal contracting has sometimes been related to applications, as for instance in Chien and Song (2014) who relate it to the oversupply of perks in employee contracts. Their theory, however, seems less suited to accounting for excessive consumption that is not stipulated by a formal contract. As mentioned, this appears an important advantage of the relational contracting framework.

[19] Ábrahám and Pavoni (2005) comment: "a deviation on [savings] at period t typically generates wealth effects that most likely will induce further future deviations of effort and consumption. Thus, a "brute force" approach to solving [the problem] requires keeping track of all these off equilibrium deviations that makes ... direct procedures typically infeasible in practice" (p. 373).

[20] For instance, Mitchell and Zhang (2010) study CARA utility for consumption and linear disutility of effort, Edmans et al. (2012) study CRRA preferences with financial disutility of effort, He (2012) imposes a restriction that permits CARA and power utility of consumption as well as considering only three distinct levels of effort, Williams (2015) studies CARA preferences with a quadratic and financial disutility of effort, and Di Tella and Sannikov (2021) study a linear stealing technology together with CRRA consumption preferences.

[21] Other papers where the principal controls the agent's consumption or savings include Lambert (1983), Spear and Srivastava (1987), Fudenberg et al. (1990), Rey and Salanie (1990), Phelan and Townsend (1991), Sannikov (2008), and Garrett and Pavan (2015). See also the contributions on private savings comparing to what happens with public savings: e.g. Edmans et al. (2012), He (2012), Williams (2015) and Di Tella and Sannikov (2021).

7 Conclusions

This paper has studied optimal relational contracts in a simple deterministic setting where the agent has consumption-smoothing preferences and can save. We contrasted the case where the agent's consumption is unobservable to the principal and where consumption is observed. In the case where consumption is unobservable, we found that the relationship eventually becomes less profitable with time, implying that the payments the principal can credibly offer must decline. Hence effort eventually declines with time. When consumption is instead observable, the agent consumes inefficiently early (i.e., saves too little), the balance on his savings account gradually declines, the relationship becomes more profitable as the agent grows easier to incentivize, payments to the agent gradually increase, and the agent's effort increases. The contract when the principal observes the agent's consumption is a Pareto improvement on the one when it is not observed. This is in spite the fact there is an additional source of distortion, namely in the timing of consumption. This distortion is more than offset by an improvement in the provision of incentives.

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A Appendix: Omitted proofs

Proof of Proposition 3.1. The proof is standard and so relegated to the Online Appendix.

A.1 Proofs of the results in Section 4.

Proof of Proposition 4.1. Necessity follows the arguments in the main text. Sufficiency is obtained by explicitly constructing equilibria. See the Online Appendix.

Proof of Lemma 4.1

Proof. Fix an optimal relational contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$; that is, a feasible contract that maximizes the principal's discounted payoff subject to the conditions of Proposition 4.1. Then Condition (5) holds, as explained in the main text. Hence, if

$$\frac{v\left(\bar{c}_{t-1}\right)}{1-\delta} - \sum_{s=1}^{t-1} \delta^{s-1} \psi(\tilde{e}_s) \tag{15}$$

exceeds $\frac{v(b_1(1-\delta))}{1-\delta}$ at any date t, then the inequality (AC_t^{un}) must not be satisfied; i.e., the conditions of Proposition 4.1 are not satisfied.

Finally, suppose that the expression (15) is strictly less than $\frac{v(b_1(1-\delta))}{1-\delta}$ at some increasing sequence of dates $(t_n)_{n=1}^N$, where N may be finite or infinite. For each n, there is $\varepsilon_n > 0$ such that

$$\frac{1}{1-\delta} v(\bar{c}_{t_n-1} + \delta^{t_n-2}\varepsilon_n(1-\delta)) - \sum_{s=1}^{t_n-1} \delta^{s-1}\psi(\tilde{e}_s) = \frac{v\left(b_1\left(1-\delta\right)\right)}{1-\delta}.$$

Increase $\tilde{w}_{t_n-1}$ by ε_n, and reduce $\tilde{w}_{t_n}$ by $\frac{\varepsilon_n}{\delta}$; note that this leads to a change in $\bar{c}_{t_n-1}$, but does not affect $\bar{c}_t$ for $t \neq t_n$. After this adjustment has been made for each n, we have a relational contract for which the expression (15) is equal to $\frac{v(b_1(1-\delta))}{1-\delta}$ at all dates t. Also, because ψ is non-negative, $\bar{c}_t$ must be a non-decreasing sequence, and hence all payments $\tilde{w}_t$ in the new relational contract are non-negative. Hence, the new contract is feasible, and we have observed that the agent's constraints (AC_t^{un}) are satisfied. Also, the principal's constraints (PC_t) are satisfied. To see the latter, note that these constraints are affected by the adjustments to the original contract only at dates satisfying $t = t_n$ for some n. At such dates the principal's constraint is *slackened* by the amount $\frac{\varepsilon_n}{\delta}$. □

Proof of Lemma 4.2

Proof. Observe from Condition (FP_t^{un}) evaluated at consecutive dates, we have

$$\frac{v\left(\bar{c}_{t-1} + (1-\delta)\,\delta^{t-1}\tilde{w}_t\right) - v\left(\bar{c}_{t-1}\right)}{1-\delta} = \delta^{t-1}\psi\left(\tilde{e}_t\right).$$

By the Fundamental Theorem of Calculus, we have

$$\int_0^{\tilde{w}_t} v'\left(\bar{c}_{t-1} + (1-\delta)\,\delta^{t-1}x\right)dx = \psi\left(\tilde{e}_t\right)$$

and hence $k\tilde{w}_t = \psi\left(\tilde{e}_t\right)$ for $k \in \left(v'\left(\bar{c}_t\right), v'\left(\bar{c}_{t-1}\right)\right)$, which proves the result. □

Proof of Lemma 4.3

Proof. **Proof that $\tilde{e}_{t^*+1} \leq \tilde{e}_{t^*}$.** Suppose, for the sake of contradiction, that $\tilde{e}_{t^*+1} > \tilde{e}_{t^*}$. We can choose a new contract with efforts $(\tilde{e}'_t)_{t\geq 1}$, and payments $(\tilde{w}'_t)_{t\geq 1}$ chosen to satisfy Equation ($\mathrm{FP}_t^{\mathrm{un}}$), such that they coincide with the original policy except in periods t^* and t^*+1. In these periods, $\tilde{e}'_{t^*}$ and $\tilde{e}'_{t^*+1}$ are such that $\tilde{e}_{t^*} < \tilde{e}'_{t^*} \leq \tilde{e}'_{t^*+1} < \tilde{e}_{t^*+1}$ and

$$\psi\left(\tilde{e}'_{t^*}\right) + \delta\psi\left(\tilde{e}'_{t^*+1}\right) = \psi\left(\tilde{e}_{t^*}\right) + \delta\psi\left(\tilde{e}_{t^*+1}\right),$$

which implies (by convexity of ψ) that $\tilde{e}'_{t^*} + \delta\tilde{e}'_{t^*+1} > \tilde{e}_{t^*} + \delta\tilde{e}_{t^*+1}$. We then have also that $\tilde{w}_{t^*} < \tilde{w}'_{t^*}$ and $\tilde{w}'_{t^*} + \delta\tilde{w}'_{t^*+1} = \tilde{w}_{t^*} + \delta\tilde{w}_{t^*+1}$ (since the NPV of payments does not change, equilibrium consumption does not change in any period t; so the balance at date t^*+1 is larger under the new contract). Provided the changes are small, the principal's constraint (PC_t) at t^* remains satisfied. The above observations imply $\tilde{w}'_{t^*+1} < \tilde{w}_{t^*+1}$, so the principal's constraint is relaxed at date t^*+1. Since the NPV of output goes up, the principal's constraint is relaxed at all periods before t^*.[22] The contract after date t^*+1 is unaffected. The modified contract is thus self-enforceable, and it is strictly more profitable than the original, establishing a contradiction.

Proof that $\tilde{e}_{t^*-1} \leq \tilde{e}_{t^*}$. Analogous and omitted. □

Proof of Propositions 4.2 and 4.3

Proof. The remaining steps in the proof of Proposition 4.3 are divided into nine lemmas. The proof of Proposition 4.2 is provided in the process, in Lemma A.7. Throughout, we restrict attention to payments determined under the restriction to "fastest payments", i.e. satisfying Condition ($\mathrm{FP}_t^{\mathrm{un}}$). Proofs of Lemmas A.4-A.7 are provided in the main text, while the others are in the Online Appendix.

The following result provides a bound on effort in an optimal contract.

Lemma A.1. *In an optimal contract, $\psi'\left(\tilde{e}_t\right) \leq v'\left(\bar{c}_\infty\right)$ for all t.*

[22] Note that, for the new contract, the principal's constraint at any date $\hat{t}$ may be written as $\sum_{t=\hat{t}}^{\infty}\delta^{t-\hat{t}}\tilde{w}'_t \leq \sum_{t=\hat{t}+1}^{\infty}\delta^{t-\hat{t}}\tilde{e}'_t$. For $\hat{t} < t^*$ this inequality is satisfied strictly since $\sum_{t=\hat{t}}^{\infty}\delta^{t-\hat{t}}\tilde{w}'_t = \sum_{t=\hat{t}}^{\infty}\delta^{t-\hat{t}}\tilde{w}_t$, while $\sum_{t=\hat{t}+1}^{\infty}\delta^{t-\hat{t}}\tilde{e}'_t > \sum_{t=\hat{t}+1}^{\infty}\delta^{t-\hat{t}}\tilde{e}_t$.

This observation is used to prove existence of an optimal contract.

Lemma A.2. *An optimal relational contract exists.*

We then establish the following regarding the non-degeneracy of optimal contracts.

Lemma A.3. *The principal obtains a strictly positive payoff in any optimal contract* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$*. Moreover,* $\tilde{e}_t$ *and* $\tilde{w}_t$ *are strictly positive at all dates* t*.*

We now establish an important property of relational contracts: they become (approximately) stationary in the long run.

Lemma A.4. *Suppose that* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ *is an optimal relational contract satisfying* (FP$_t^{\text{un}}$)*. Then, there exists an effort/payment pair* $(\tilde{e}_\infty, \tilde{w}_\infty)$ *such that* $\lim_{t\to\infty} (\tilde{e}_t, \tilde{w}_t) = (\tilde{e}_\infty, \tilde{w}_\infty)$*.*

Proof. **Step 0.** In this step we observe that, for an optimal contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ satisfying (FP$_t^{\text{un}}$),

$$\lim_{t\to\infty} \left(\tilde{w}_t - \frac{\psi(\tilde{e}_t)}{v'(\bar{c}_\infty)} \right) = 0.$$

This follows from Lemma 4.2, after noticing that $(\tilde{e}_t)_{t\geq 1}$ is bounded in an optimal contract.

Step 1. Define $\bar{e} \equiv \limsup_{t\to\infty} \tilde{e}_t$, which we know from Lemma A.1 is no greater than $z(v'(\bar{c}_\infty))$, where z is the inverse of ψ'. We now show that, for any $e \in [0, \bar{e}]$,

$$\frac{\psi(e)}{v'(\bar{c}_\infty)} \leq \frac{\delta}{1-\delta} \left(e - \frac{\psi(e)}{v'(\bar{c}_\infty)} \right). \tag{16}$$

Note, by convexity of ψ, if the inequality (16) is satisfied at $\bar{e}$, then it is satisfied for all $e \in [0, \bar{e}]$. Assume now for the sake of contradiction that the inequality (16) is not satisfied for some $e \in [0, \bar{e}]$. Then we must have

$$\frac{\psi(\bar{e})}{v'(\bar{c}_\infty)} > \frac{\delta}{1-\delta} \left(\bar{e} - \frac{\psi(\bar{e})}{v'(\bar{c}_\infty)} \right). \tag{17}$$

Observe then that there is a sequence $(\varepsilon_t)_{t=1}^{\infty}$ convergent to zero such that, for all $t \geq 1$,

$$\tilde{e}_t - \tilde{w}_t \leq \bar{e} - \frac{\psi(\bar{e})}{v'(\bar{c}_\infty)} + \varepsilon_t.$$

This follows because $\tilde{w}_t - \frac{\psi(\tilde{e}_t)}{v'(\bar{c}_\infty)} \to 0$ as $t \to \infty$ (by Step 0), because $e - \frac{\psi(e)}{v'(\bar{c}_\infty)}$ increases over effort levels e in $[0, \bar{e}]$ (since $\psi'(\bar{e}) \leq v'(\bar{c}_\infty)$ by Lemma A.1), and by definition of $\bar{e}$ as $\limsup_{t\to\infty} \tilde{e}_t$.

We therefore have that

$$\limsup_{t\to\infty}\sum_{s=t+1}^{\infty}\delta^{s-t}\left(\tilde{e}_s-\tilde{w}_s\right)\leq\frac{\delta}{1-\delta}\left(\bar{e}-\frac{\psi\left(\bar{e}\right)}{v'\left(\bar{c}_\infty\right)}\right)<\frac{\psi\left(\bar{e}\right)}{v'\left(\bar{c}_\infty\right)},$$

where the last inequality holds by Equation (17). However, Step 0 implies that the superior limit of payments to the agent must be $\frac{\psi(\bar{e})}{v'(\bar{c}_\infty)}$, which implies that the principal's constraint (PC_t) is not satisfied at some time t. This contradicts the definition of $\bar{e}$ as $\limsup_{t\to\infty}\tilde{e}_t$.

Step 2. We complete the proof by showing that $\liminf_{t\to\infty}\tilde{e}_t=\bar{e}$. This is immediate if $\bar{e}=0$, so assume $\bar{e}>0$. Assume, for the sake of contradiction, that $\liminf_{t\to\infty}\tilde{e}_t<\bar{e}$. In this case, there exists some $t'>1$ such that $\tilde{e}_{t'}<\min\left\{\bar{e},\tilde{e}_{t'+1}\right\}$.

Step 2a. We have

$$\tilde{w}_{t'}\leq\frac{\delta}{1-\delta}\left(\tilde{e}_{t'+1}-\frac{\psi\left(\tilde{e}_{t'+1}\right)}{v'\left(\bar{c}_\infty\right)}\right). \tag{18}$$

This follows because (i) $\tilde{w}_{t'}\leq\frac{\psi(\tilde{e}_{t'})}{v'(\bar{c}_\infty)}$ by Lemma 4.2 and the assumption that payments satisfy condition ($\mathrm{FP}_t^{\mathrm{un}}$); (ii) $\frac{\psi(\tilde{e}_{t'})}{v'(\bar{c}_\infty)}\leq\frac{\delta}{1-\delta}\left(\tilde{e}_{t'}-\frac{\psi(\tilde{e}_{t'})}{v'(\bar{c}_\infty)}\right)$, by assumption that $\tilde{e}_{t'}<\bar{e}$ and by Step 1; and (iii) $\tilde{e}_{t'}-\frac{\psi(\tilde{e}_{t'})}{v'(\bar{c}_\infty)}\leq e_{t'+1}-\frac{\psi(\tilde{e}_{t'+1})}{v'(\bar{c}_\infty)}$ because $z\left(v'\left(\bar{c}_\infty\right)\right)\geq\tilde{e}_{t'+1}>\tilde{e}_{t'}$ (recall that the inequality $z\left(v'\left(\bar{c}_\infty\right)\right)\geq\tilde{e}_{t'+1}$ is established in Lemma A.1).

Step 2b. We now show that the principal's constraint (PC_t) is slack at t'. Note first that, for any $t\geq1$, we have

$$\begin{aligned}\tilde{w}_{t+1}-\tilde{w}_t&=\frac{\bar{c}_{t+1}-\bar{c}_t}{\delta^t(1-\delta)}-\frac{\bar{c}_t-\bar{c}_{t-1}}{\delta^{t-1}(1-\delta)}\\&\geq\frac{v(\bar{c}_{t+1})-v(\bar{c}_t)}{\delta^t(1-\delta)v'(\bar{c}_t)}-\frac{v(\bar{c}_t)-v(\bar{c}_{t-1})}{\delta^{t-1}(1-\delta)v'(\bar{c}_t)}\\&=\frac{\psi(\tilde{e}_{t+1})-\psi(\tilde{e}_t)}{v'(\bar{c}_t)},\end{aligned}$$

where we used that v is concave and Lemma 4.2. Hence, we have that $\tilde{e}_{t+1}>\tilde{e}_t$ implies $\tilde{w}_{t+1}>\tilde{w}_t$.

Since t' was chosen so that $\tilde{e}_{t'+1}>\tilde{e}_{t'}$, we have $\tilde{w}_{t'+1}>\tilde{w}_{t'}$. Hence,

$$\begin{aligned}\tilde{w}_{t'}&<\left(1-\delta\right)\tilde{w}_{t'}+\delta\tilde{w}_{t'+1}\\&\leq\delta\left(\tilde{e}_{t'+1}-\frac{\psi\left(\tilde{e}_{t'+1}\right)}{v'\left(\bar{c}_\infty\right)}\right)+\delta\sum_{s=t'+2}^{\infty}\delta^{s-t'-1}\left(\tilde{e}_s-\tilde{w}_s\right)\\&\leq\sum_{s=t'+1}^{\infty}\delta^{s-t'}\left(\tilde{e}_s-\tilde{w}_s\right),\end{aligned}$$

where the second inequality uses (i) Equation (18) from Step 2a, and (ii) the principal's constraint (PC_t) in period $t'+1$. The third inequality uses that $\tilde{w}_{t'+1} \leq \frac{\psi(\tilde{e}_{t'+1})}{v'(\bar{c}_\infty)}$, which follows from Lemma 4.2.

Step 2c. We finish the proof with the following observation. The fact the principal's constraint (PC_t) is slack at time t' (proven in Step 2b) contradicts Lemma 4.3, since effort is strictly higher at $t'+1$ than at t'. □

The following lemma determines that, in an optimal contract, effort is weakly decreasing (as mentioned in the main text).

Lemma A.5. *In an optimal contract, the effort policy $(\tilde{e}_t)_{t\geq 1}$ is a weakly decreasing sequence. Therefore, for all t, $\tilde{e}_t \geq \tilde{e}_\infty \equiv \lim_{s\to\infty} \tilde{e}_s$.*

Proof. By Lemma A.4, $(\tilde{e}_t)_{t=1}^\infty$ is a convergent sequence, so using the notation in its proof, we have $\tilde{e}_\infty = \bar{e}$. Step 2 in the proof of Lemma A.4 proves that there is no time t' such that $\tilde{e}_{t'} < \min\{\bar{e}, \tilde{e}_{t'+1}\}$. Hence, there is no t' such that $\tilde{e}_{t'} < \tilde{e}_\infty$.

Now suppose, for the sake of contradiction, that $(\tilde{e}_t)_{t=1}^\infty$ is not a weakly decreasing sequence. Thus, there exists a date t' where $\max_{t>t'} \tilde{e}_t > \tilde{e}_{t'}$ (the maximum exists by the first part of this proof, and because $\lim_{t\to\infty} \tilde{e}_t = \tilde{e}_\infty$ by Lemma A.4). Let $t^*(t')$ be the smallest value $t > t'$ where the maximum is attained, that is, $\tilde{e}_{t^*(t')} = \max_{t>t'} \tilde{e}_t$.

For any $s > t^*(t')$,

$$\tilde{e}_{t^*(t')} - \tilde{w}_{t^*(t')} > \tilde{e}_{t^*(t')} - \frac{\psi(\tilde{e}_{t^*(t')})}{v'(\bar{c}_{t^*(t')})} \geq \tilde{e}_{t^*(t')} - \frac{\psi(\tilde{e}_{t^*(t')})}{v'(\bar{c}_{s-1})} \geq \tilde{e}_s - \frac{\psi(\tilde{e}_s)}{v'(\bar{c}_{s-1})} > \tilde{e}_s - \tilde{w}_s. \tag{19}$$

The first inequality follows from Lemma 4.2; the second inequality follows because $\bar{c}_{s-1} \geq \bar{c}_{t^*(t')}$. The third inequality follows because $e - \frac{\psi(e)}{v'(\bar{c}_{s-1})}$ is increasing in e over $[0, z(v'(\bar{c}_\infty))]$, and because $\tilde{e}_s \leq \tilde{e}_{t^*(t')}$ for $s > t^*(t')$ by definition of $t^*(t')$. The fourth inequality follows because $\tilde{w}_s > \frac{\psi(\tilde{e}_s)}{v'(\bar{c}_{s-1})}$ by Lemma 4.2.

Equation (19) implies that

$$\tilde{e}_{t^*(t')} - \tilde{w}_{t^*(t')} > (1-\delta) \sum_{s=t^*(t')+1}^{\infty} \delta^{s-t^*(t')-1} (\tilde{e}_s - \tilde{w}_s),$$

so that

$$\begin{aligned}\sum_{s=t^*(t')}^{\infty} \delta^{s-t^*(t')} (\tilde{e}_s - \tilde{w}_s) &= \tilde{e}_{t^*(t')} - \tilde{w}_{t^*(t')} + \delta \sum_{s=t^*(t')+1}^{\infty} \delta^{s-t^*(t')-1} (\tilde{e}_s - \tilde{w}_s) \\ &> (1-\delta) \sum_{s=t^*(t')+1}^{\infty} \delta^{s-t^*(t')-1} (\tilde{e}_s - \tilde{w}_s) + \delta \sum_{s=t^*(t')+1}^{\infty} \delta^{s-t^*(t')-1} (\tilde{e}_s - \tilde{w}_s) \\ &= \sum_{s=t^*(t')+1}^{\infty} \delta^{s-t^*(t')-1} (\tilde{e}_s - \tilde{w}_s). \qquad (20)\end{aligned}$$

Recall from Lemma 4.3 that the principal's constraint must hold with equality at $t^*(t') - 1$ (since $\tilde{e}_{t^*(t')} > \tilde{e}_{t^*(t')-1}$ by the definition of $t^*(t')$). The inequality (20) then implies (given satisfaction of the principal's constraint (PC_t)) that $\tilde{w}_{t^*(t')-1} > \tilde{w}_{t^*(t')}$. But then, recalling Lemma 4.2, we have

$$\frac{\psi\left(\tilde{e}_{t^*(t')-1}\right)}{v'\left(\bar{c}_{t^*(t')-1}\right)} > \tilde{w}_{t^*(t')-1} > \tilde{w}_{t^*(t')} > \frac{\psi(\tilde{e}_{t^*(t')})}{v'\left(\bar{c}_{t^*(t')-1}\right)}.$$

Hence, $\tilde{e}_{t^*(t')-1} > \tilde{e}_{t^*(t')}$, contradicting the definition of $t^*(t')$. □

Having shown that the effort is weakly decreasing in an optimal relational contract (Lemma A.5) we now show that it is strictly decreasing when the principal's constraint holds with equality.

Lemma A.6. *If the principal's constraint* (PC_t) *holds with equality at some date t^*, then $\tilde{e}_{t^*} > \tilde{e}_{t^*+1}$. Hence, by Lemma 4.3, the principal's constraint also holds with equality at t^*+1.*

Proof. The same arguments we used in Lemma A.5 to establish the inequalities in (19) imply that $\tilde{e}_{t^*+1} - \tilde{w}_{t^*+1} > \tilde{e}_s - \tilde{w}_s$ for all $s > t^* + 1$. In turn, this means that, if the principal's constraint (PC_t) holds with equality at t^*, then $\tilde{w}_{t^*} > \tilde{w}_{t^*+1}$. Indeed, because the principal's constraint holds with equality at t^*,

$$\begin{aligned}\tilde{w}_{t^*} &= \delta\left(\tilde{e}_{t^*+1} - \tilde{w}_{t^*+1} + \delta \sum_{s=t^*+2}^{\infty} \delta^{s-t^*-2}(\tilde{e}_s - \tilde{w}_s)\right) \\ &> \delta\left((1-\delta) \sum_{s=t^*+2}^{\infty} \delta^{s-t^*-2}(\tilde{e}_s - \tilde{w}_s) + \delta \sum_{s=t^*+2}^{\infty} \delta^{s-t^*-2}(\tilde{e}_s - \tilde{w}_s)\right) \\ &= \sum_{s=t^*+2}^{\infty} \delta^{s-t^*-1}(\tilde{e}_s - \tilde{w}_s) \\ &\geq \tilde{w}_{t^*+1}.\end{aligned}$$

The final inequality follows from the principal's constraint (PC_t) at date t^*+1. Using Lemma 4.2, we have $\tilde{e}_{t^*+1} < \tilde{e}_{t^*}$. □

Lemma A.6 implies that, given payments satisfy condition (FP_t^{un}), if the principal's constraint (PC_t) holds with equality at some date, then effort is strictly decreasing forever after (and the principal's constraints (PC_t) hold with equality forever after). Our next goal is therefore to establish the condition under which the principal attains the first-best payoff, and, when this condition fails, establish that there is necessarily a date at which the principal's constraint is satisfied with equality.

Lemma A.7. *An optimal contract achieves the first-best payoff of the principal if and only if Condition (7) holds. If this condition is not satisfied, then there is a time $t^* \in \mathbb{N}$ such that the principal's constraint is slack if and only if $t < t^*$. Hence, effort is constant up to date t^*-1 and strictly decreases from date t^*.*

Proof. Consider payments satisfying (FP_t^{un}), for all t, and determined given the first-best effort (this is $e^{FB}(b_1)$ in Proposition 3.1). Lemma 4.2 shows that the payments increase over time and tend to $\frac{\psi(e^{FB}(b_1))}{v'(c^{FB}(b_1))}$. On the other hand, per-period profits fall over time towards $e^{FB}(b_1) - \frac{\psi(e^{FB}(b_1))}{v'(c^{FB}(b_1))}$. This establishes Condition (7) is both necessary and sufficent for implementation of the first best.

Assume now that Condition (7) fails, and fix an optimal contract that is not first best. Lemma A.6 established that there are two possibilities. First, we might have a finite date $t^* \in \mathbb{N}$, with the principal's constraint (PC_t) holding with equality at t^*, and every subsequent date, but slack at dates t^*-1 and earlier. In this case, effort is constant from the initial date up to t^*-1 (by Lemma 4.3) and strictly decreases from date t^*. Second, we might have that the principal's constraint (PC_t) is slack at all dates. Effort is then constant over all periods (by Lemma 4.3), but not first-best. The result in the lemma is established if we can show this second case does not occur; so assume for a contradiction that it does. Letting $\tilde{e}_\infty$ be the constant effort level and $\bar{c}_\infty$ equilibrium consumption, Proposition 3.1 then implies that $v'(\bar{c}_\infty) \neq \psi'(\tilde{e}_\infty)$. By Lemma A.1, we have $v'(\bar{c}_\infty) > \psi'(\tilde{e}_\infty)$. By Lemma A.3, we have $\tilde{e}_\infty > 0$.

Note that $\tilde{w}_t$ increases over time to $\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}$ (from Lemma 4.2). We claim then that

$$\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} = \frac{\delta}{1-\delta}\left(\tilde{e}_\infty - \frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}\right). \tag{21}$$

If instead $\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} > \frac{\delta}{1-\delta}\left(\tilde{e}_\infty - \frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}\right)$, then, for large enough t we must have

$$\tilde{w}_t > \sum_{s=t+1}^{\infty} \delta^{s-t}\left(\tilde{e}_\infty - \tilde{w}_t\right),$$

so the principal's constraint is violated at t. If instead $\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} < \frac{\delta}{1-\delta}\left(\tilde{e}_\infty - \frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)}\right)$, we have $\tilde{w}_t$ remains bounded below $\sum_{s=t+1}^{\infty} \delta^{s-t}\left(\tilde{e}_\infty - \tilde{w}_t\right)$. Without violating ($\text{PC}_t$), effort can be increased by a small constant amount across all periods (with payments adjusted to satisfy (FP_t^{un})). This increases profits.

Note then that Condition (21) can be written as $\frac{\psi(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} = \delta\tilde{e}_\infty$. Because ψ is strictly convex, we have $\frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty)} > \delta$.

We now consider an adjusted contract in which effort increases at date 1 by $\varepsilon > 0$, raising the disutility of effort at date 1 by $\psi(\tilde{e}_\infty + \varepsilon) - \psi(\tilde{e}_\infty)$. Because payments to the agent increase at all dates under condition (FP_t^{un}), the new policy will not satisfy the principal's constraint (PC_t) if this is the only adjustment (as explained in the main text). We therefore simultaneously reduce effort from some fixed date 2 onwards by $\kappa(\varepsilon) > 0$ to be determined (i.e., effort is given by $e_t = \tilde{e}_\infty - \kappa(\varepsilon)$ for $t \geq 2$).

We let $\bar{c}_\infty(\varepsilon, \kappa(\varepsilon))$ denote equilibrium consumption under the new plan (naturally, $\bar{c}_\infty(0,0)$ is consumption under the original plan). The new consumption satisfies

$$\begin{aligned}\frac{v\left(\bar{c}_\infty\left(\varepsilon, \kappa\left(\varepsilon\right)\right)\right)}{1-\delta} - \frac{v\left(\bar{c}_\infty\left(0,0\right)\right)}{1-\delta} =&\psi\left(\tilde{e}_\infty + \varepsilon\right) - \psi\left(\tilde{e}_\infty\right)\\ &- \frac{\delta}{1-\delta}\left(\psi\left(\tilde{e}_\infty\right) - \psi\left(\tilde{e}_\infty - \kappa\left(\varepsilon\right)\right)\right)\end{aligned}$$

or

$$\bar{c}_\infty(\varepsilon, \kappa(\varepsilon)) = v^{-1}\begin{pmatrix} (1-\delta)(\psi(\tilde{e}_\infty + \varepsilon) - \psi(\tilde{e}_\infty)) \\ -\delta(\psi(\tilde{e}_\infty) - \psi(\tilde{e}_\infty - \kappa(\varepsilon))) + v(\bar{c}_\infty(0,0)) \end{pmatrix}$$

To determine the value for $\kappa(\varepsilon)$, define the following function

$$f(\varepsilon, k) \equiv \frac{\psi\left(\tilde{e}_\infty - k\right)}{v'(\bar{c}_\infty(\varepsilon, k))} - \delta\left(\tilde{e}_\infty - k\right). \tag{22}$$

We then define $\kappa(\varepsilon)$ by $f(\varepsilon, \kappa(\varepsilon)) = 0$ for positive ε in a neighborhood of 0. We will use the implicit function theorem to show that such a local solution $\kappa(\varepsilon)$ exists.

To apply the implicit function theorem, note that $f(\varepsilon, k)$ is continuously differentiable in a neighborhood of $(\varepsilon, k) = (0,0)$. The derivative of $f(\varepsilon, k)$ with respect to k, evaluated at $(\varepsilon, k) = (0,0)$, is

$$f_2\left(0,0\right) =\delta - \frac{\psi'\left(\tilde{e}_\infty\right)}{v'\left(\bar{c}_\infty\left(0,0\right)\right)} + v''\left(\bar{c}_\infty\left(0,0\right)\right)\left(\frac{\delta\psi'\left(\tilde{e}_\infty\right)}{v'\left(\bar{c}_\infty\left(0,0\right)\right)^3}\right)\psi\left(\tilde{e}_\infty\right).$$

This is strictly negative, using that $\frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))} > \delta$. The derivative $f(\varepsilon, k)$ instead with respect to ε, evaluated at $(\varepsilon, k) = (0,0)$, is

$$f_1(0,0) = -v''(\bar{c}_\infty(0,0)) \left(\frac{(1-\delta)\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))^3} \right) \psi(\tilde{e}_\infty).$$

The implicit function theorem then gives us that κ is locally well-defined by $f(\varepsilon, \kappa(\varepsilon)) = 0$ on some interval around 0, unique, and continuously differentiable, with derivative approaching

$$\begin{aligned} \kappa'(0) &= -\frac{f_1(0,0)}{f_2(0,0)} \\ &= \frac{v''(\bar{c}_\infty(0,0)) \left(\frac{(1-\delta)\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))^3} \right) \psi(\tilde{e}_\infty)}{\delta - \frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))} + v''(\bar{c}_\infty(0,0)) \left(\frac{\delta\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))^3} \right) \psi(\tilde{e}_\infty)} \\ &< \frac{1-\delta}{\delta} \end{aligned} \tag{23}$$

as $\varepsilon \to 0$ (the strict inequality follows because $\frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))} > \delta$).

For small enough ε, the new effort policy and payments defined by condition (FP_t^{un}) satisfy the principal's constraints (PC_t). This follows by observing that, when ε is small, the constraint (PC_t) remains slack at date $t = 1$. For all other dates, the satisfaction of the constraint (PC_t) follows from $f(\varepsilon, \kappa(\varepsilon)) = 0$, and by Lemma 4.2.

It remains to show that, for small enough positive ε, the principal's profits strictly increase. The NPV of effort increases by

$$\varepsilon - \frac{\delta}{1-\delta}\kappa(\varepsilon) = \left(1 - \frac{\delta}{1-\delta}\kappa'(0)\right)\varepsilon + o(\varepsilon)$$

(where $o(\varepsilon)$ represents terms that vanish faster than ε as $\varepsilon \to 0$). From the inequality (23) we have $1 - \frac{\delta}{1-\delta}\kappa'(0) > 0$, and so the increase in effort is strictly positive for ε small enough. Using that payments continue to satisfy Condition (FP_t^{un}), a marginal increase in the NPV of effort is compensated by an increase in the NPV of payments to the agent by $\frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))}$. Therefore, the principal's payoff under the new policy increases by

$$\left(1 - \frac{\psi'(\tilde{e}_\infty)}{v'(\bar{c}_\infty(0,0))}\right)\left(1 - \frac{\delta}{1-\delta}\kappa'(0)\right)\varepsilon + o(\varepsilon)$$

which is strictly positive for small enough ε, recalling that $v'(\bar{c}_\infty(0,0)) > \psi'(\tilde{e}_\infty)$. □

We have established that, for any optimal contract that does not attain the first-best payoff of the principal, there is a date $\bar{t} \geq 1$ such that effort is constant up to this date, and subsequently strictly decreasing to a value $\tilde{e}_\infty$, as stated in the proposition. Our next result establishes that $\tilde{e}_\infty > 0$, which requires only ruling out $\tilde{e}_\infty = 0$.

Lemma A.8. *Suppose the principal cannot attain the first-best payoff. In any optimal contract, the limiting value of effort* $\tilde{e}_\infty \equiv \lim_{t\to\infty} \tilde{e}_t$ *is strictly positive.*

Our final lemma states that, for some configurations of the problem, $\bar{t} > 1$. In this case, effort is constant in the initial periods, before strictly decreasing.

Lemma A.9. *For any v and ψ admitted in the model set-up, there exists a discount factor δ and initial balance b_1 such that (i) the principal's payoff in an optimal contract is less than the first-best payoff, and (ii) for any optimal contract, the principal's constraint* (PC_t) *is slack for at least $t = 1, 2$.*

(End of the proof of Proposition 4.3.) □

Proof of Proposition 4.4. See the Online Appendix.

A.2 Proofs of the results in Section 5

Proof of Proposition 5.1. The proof follows standard arguments. See the Online Appendix.

Proof of Lemma 5.1. The proof is similar in spirit to that for Lemma 4.1. See the Online Appendix.

Proof of Proposition 5.2. Follows from the arguments in the main text.

Proof of Proposition 5.3

Proof. It will be useful to write the recursive problem in the main text by substituting out agent effort. To this end, define a function $\hat{e}$ by

$$\hat{e}(c_t, b_t, b_{t+1}) \equiv \psi^{-1}\Big(v(c_t) + \tfrac{\delta}{1-\delta} v\left((1-\delta)\, b_{t+1}\right) - \tfrac{1}{1-\delta} v((1-\delta) b_t)\Big) \tag{24}$$

for $c_t, b_t, b_{t+1} > 0$ and $v(c_t) + \frac{\delta}{1-\delta} v\left((1-\delta)\, b_{t+1}\right) - \frac{1}{1-\delta} v((1-\delta) b_t) \geq 0$. We will focus throughout on relational contracts that satisfy the "fastest payments" condition ($\mathrm{FP}_t^{\mathrm{ob}}$). Hence, given contractual variables $\tilde{c}_t$, $\tilde{b}_t$ and $\tilde{b}_{t+1}$, the date-t effort must be given by $\tilde{e}_t = \hat{e}(\tilde{c}_t, \tilde{b}_t, \tilde{b}_{t+1})$.

We can then write the principal's optimal payoff given balance $\tilde{b}_t > 0$ (which we establish below can be attained by a self-enforceable contract) as follows:

$$V(\tilde{b}_t) = \max_{c_t, b_{t+1} > 0} \left(\hat{e}(c_t, \tilde{b}_t, b_{t+1}) - (\delta b_{t+1} - \tilde{b}_t + c_t) + \delta V(b_{t+1})\right) \tag{25}$$

subject to the principal's constraint

$$\delta b_{t+1} - \tilde{b}_t + c_t \leq \delta V\left(b_{t+1}\right) \tag{26}$$

and to the requirement that the implied effort is non-negative, i.e.

$$v(c_t) + \tfrac{\delta}{1-\delta} v\left((1-\delta)\, b_{t+1}\right) - \tfrac{1}{1-\delta} v((1-\delta)\tilde{b}_t) \geq 0. \tag{27}$$

The proof of Proposition 5.3 will now consist of eight lemmas. The proof of Lemma A.11 is provided here, while the proofs of all other lemmas are in the Online Appendix.

We begin by observing that no trivial contract can be optimal. Also, in an optimal contract, effort is (weakly) distorted downwards within each period: a marginal increase in effort, compensated by pay/consumption that keeps the agent equally well off, would (weakly) raise profits. A strict distortion occurs only if the principal's constraint is binding.

Lemma A.10. *In any optimal contract* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$*,* $V(\tilde{b}_t) \in (0, V^{FB}(\tilde{b}_t)]$ *for all* t*. Also,* $\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t > 0$ *for all* t*. In addition,* $\psi'(\tilde{e}_t) \leq v'(\tilde{c}_t)$ *for all* t*, and* $\psi'(\tilde{e}_t) < v'(\tilde{c}_t)$ *only if* $\tilde{w}_t = \delta V(\tilde{b}_{t+1})$*.*

We now establish the Euler equation in the main text and monotonicity of consumption.

Lemma A.11. *Any optimal contract* $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ *satisfies the Euler equation (14) in all periods. Furthermore,* $\tilde{c}_t \geq \tilde{c}_{t+1} > (1-\delta)\tilde{b}_{t+1}$ *for all* t*.*

Proof. We divide the proof in three steps:

Step 1: Fix an optimal contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$. Consider a contract $(\check{e}_t, \check{c}_t, \check{w}_t, \check{b}_t)_{t\geq 1}$, coinciding with the original contract in all periods except for periods t and $t+1$ (so, also, $\check{b}_t = \tilde{b}_t$). We specify that the new contract keeps the agent indifferent between being obedient and optimally deviating in all periods. This requires

$$v(\check{c}_t) - \psi(\check{e}_t) + \tfrac{\delta}{1-\delta} v\left(\tfrac{1-\delta}{\delta}(\tilde{b}_t + \check{w}_t - \check{c}_t)\right) = \tfrac{1}{1-\delta} v((1-\delta)\tilde{b}_t), \tag{28}$$

$$v\left(\tfrac{1}{\delta}(\tilde{b}_t + \check{w}_t - \check{c}_t) + \check{w}_{t+1} - \delta\tilde{b}_{t+2}\right) - \psi(\check{e}_{t+1}) + \tfrac{\delta}{1-\delta} v((1-\delta)\tilde{b}_{t+2}) = \tfrac{1}{1-\delta} v\left(\tfrac{1-\delta}{\delta}(\tilde{b}_t + \check{w}_t - \check{c}_t)\right), \tag{29}$$

which uses that consumption in period $t+1$ under the new contract is $\check{c}_{t+1} = \frac{1}{\delta}(\tilde{b}_t + \check{w}_t - \check{c}_t) + \check{w}_{t+1} - \delta\tilde{b}_{t+2}$ (guaranteeing the agent has savings $\tilde{b}_{t+2}$ at date $t+2$).

Fix $\check{e}_t = \tilde{e}_t$ and $\check{w}_{t+1} = \tilde{w}_{t+1}$. Equations (28) and (29) implicitly define $\check{e}_{t+1}$ and $\check{w}_t$ as functions of $\check{c}_t$. Let these functions be denoted $\hat{e}_{t+1}(\cdot)$ and $\hat{w}_t(\cdot)$, respectively. We can use the implicit function theorem to compute the derivatives at $\check{c}_t = \tilde{c}_t$:

$$\hat{e}'_{t+1}(\tilde{c}_t) = \frac{v'(\tilde{c}_t)\left(v'((1-\delta)\tilde{b}_{t+1}) - v'(\tilde{c}_{t+1})\right)}{\delta\psi'\left(\hat{e}_{t+1}(\tilde{c}_t)\right) v'((1-\delta)\tilde{b}_{t+1})} \quad \text{and} \quad \hat{w}'_t(\tilde{c}_t) = 1 - \frac{v'(\tilde{c}_t)}{v'((1-\delta)\tilde{b}_{t+1})}.$$

Note that the original contract is obtained by setting $\check{c}_t = \tilde{c}_t$. If $\check{c}_t$ is changed from $\tilde{c}_t$ to $\tilde{c}_t + \varepsilon$, for some (positive or negative) ε small, the total effect on the continuation payoff of the principal at time t is $(-\hat{w}_t'(\tilde{c}_t) + \delta\hat{e}_{t+1}'(\tilde{c}_t))\varepsilon + o(\varepsilon)$. Hence, a necessary condition for optimality is that $-\hat{w}_t'(\tilde{c}_t) + \delta\hat{e}_{t+1}'(\tilde{c}_t) = 0$, which is equivalent the Euler equation (14).

The Euler equation implies that if $v'(\tilde{c}_{t+1}) = \psi'(\tilde{e}_{t+1})$ we have $\tilde{c}_t = \tilde{c}_{t+1}$. From Lemma A.10 we have that, if instead $v'(\tilde{c}_{t+1}) \neq \psi'(\tilde{e}_{t+1})$, then $v'(\tilde{c}_{t+1}) > \psi'(\tilde{e}_{t+1})$. In this second case, there are three possibilities:

1. If both sides of the Euler equation are strictly positive, then $\tilde{c}_t < \tilde{c}_{t+1} < (1-\delta)\tilde{b}_{t+1}$.
2. If both sides of the Euler equation are zero, then $\tilde{c}_t = \tilde{c}_{t+1} = (1-\delta)\tilde{b}_{t+1}$.
3. If both sides of the Euler equation are strictly negative, then $\tilde{c}_t > \tilde{c}_{t+1} > (1-\delta)\tilde{b}_{t+1}$.

Step 2: We now prove that if $\tilde{c}_t \leq (1-\delta)\tilde{b}_t$ then $\tilde{c}_s \leq \tilde{c}_{s+1} < (1-\delta)\tilde{b}_{s+1}$ for all $s \geq t$. Assume first that there is a period t such that $\tilde{c}_t \leq (1-\delta)\tilde{b}_t$. Hence, since $\tilde{e}_t = \hat{e}(\tilde{c}_t, \tilde{b}_t, \tilde{b}_{t+1}) > 0$ (recall Lemma A.10) we have $\tilde{b}_{t+1} > \tilde{b}_t$. This shows that each side of the Euler equation is strictly positive, i.e.

$$1 - \frac{v'((1-\delta)\,\tilde{b}_{t+1})}{v'\left(\tilde{c}_t\right)} = \frac{v'\left(\tilde{c}_{t+1}\right)}{\psi'\left(\tilde{e}_{t+1}\right)}\left(1 - \frac{v'((1-\delta)\,\tilde{b}_{t+1})}{v'\left(\tilde{c}_{t+1}\right)}\right) > 0.$$

Since $v'\left(\tilde{c}_{t+1}\right)/\psi'\left(\tilde{e}_{t+1}\right) \geq 1$ (from Lemma A.10), $(1-\delta)\tilde{b}_{t+1} > \tilde{c}_{t+1} \geq \tilde{c}_t$. The result then follows by induction.

Step 3: We prove that $\tilde{c}_t > (1-\delta)\tilde{b}_t$ for all $t > 1$; it then follows immediately from Step 1 that consumption is (weakly) decreasing in t. Assume then, for the sake of contradiction, that there is a $t' > 1$ such that $\tilde{c}_{t'} \leq (1-\delta)\tilde{b}_{t'}$. We will construct a self-enforceable contract that is strictly more profitable than the original, contradicting the optimality of the original.

We first make some preliminary observations. From Step 2, we have that $\tilde{c}_s \leq \tilde{c}_{s+1} < (1-\delta)\tilde{b}_{s+1}$ for all $s \geq t'$. Also, since effort is strictly positive at all times (from Lemma A.10), we have

$$\sum_{s=t'}^{\infty} \delta^{s-t'} v(\tilde{c}_s) > \tfrac{1}{1-\delta} v((1-\delta)\tilde{b}_{t'}).$$

Hence, there must be a period $s \geq t'$ where $\tilde{c}_{s+1} > \tilde{c}_{t'}$. Let t'' be the earliest such period, and note that it satisfies $\tilde{c}_{t''+1} > \tilde{c}_{t''}$. Additionally, we can observe that, for all t,

$$\tilde{b}_t + \sum_{\tau=t}^{\infty} \delta^{\tau-t}\tilde{w}_\tau = \sum_{\tau=t}^{\infty} \delta^{\tau-t}\tilde{c}_\tau. \tag{30}$$

If this is not the case (the right-hand side is strictly smaller), then, applying Equation (1) repeatedly, we have $\tilde{b}_t \to \infty$ and so the agent's constraint ($\mathrm{AC}_t^{\mathrm{ob}}$) must be violated for large t (given that $(\tilde{c}_t)_{t=1}^{\infty}$ is bounded, as the contract is feasible).

Now let us construct the more profitable contract for the principal, given our assumption that $\tilde{c}_{t'} \leq (1-\delta)\tilde{b}_{t'}$. We first construct a self-enforceable contract $\left(\tilde{e}_t^{new}, \tilde{c}_t^{new}, \tilde{w}_t^{new}, \tilde{b}_t^{new}\right)_{t\geq 1}$ in which the agent obtains a strictly higher payoff than in the original, while the principal obtains the same payoff. We then show how that contract can be further adjusted to obtain one which is strictly better for the principal. In the new contract that is better for the agent, we maintain $\tilde{w}_t^{new} = \tilde{w}_t$ and $\tilde{e}_t^{new} = \tilde{e}_t$ for all t, but specify a different agreed consumption sequence $\tilde{c}_t^{new}$ (and hence different balances $\tilde{b}_t^{new}$).

The change in agent consumption is to specify *constant* consumption $\bar{c}$ in each period from t'' onwards, where

$$\bar{c} = (1-\delta)\sum_{\tau=t''}^{\infty} \delta^{\tau-t''}\tilde{c}_\tau. \tag{31}$$

That is, $\tilde{c}_t^{new} = \bar{c}$ for all $t \geq t''$, while $\tilde{c}_t^{new} = \tilde{c}_t$ for $t < t''$. Notice that, $\bar{c} < (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\tilde{c}_\tau$ for all $t > t''$.

Balances are determined recursively by Equation (1). That is, they are given by $\tilde{b}_t^{new} = \tilde{b}_t$ for $t \leq t''$, and by

$$\tilde{b}_t^{new} = \delta^{t''-t}\tilde{b}_{t''} + \sum_{\tau=t''}^{t-1}\delta^{\tau-t}\left(\tilde{w}_\tau - \bar{c}\right)$$

for all $t > t''$. Observe then that

$$\begin{aligned}
\tilde{b}_t^{new} + \sum_{\tau=t}^{\infty}\delta^{\tau-t}\tilde{w}_\tau &= \delta^{t''-t}\tilde{b}_{t''} + \sum_{\tau=t''}^{\infty}\delta^{\tau-t}\tilde{w}_\tau - \sum_{\tau=t''}^{t-1}\delta^{\tau-t}\bar{c} \\
&= \sum_{\tau=t''}^{\infty}\delta^{\tau-t}\tilde{c}_\tau - \sum_{\tau=t''}^{t-1}\delta^{\tau-t}\bar{c} \\
&= \frac{\bar{c}}{1-\delta},
\end{aligned}$$

where the second equality uses Equation (30) and the third equality uses Equation (31). Therefore, for all $t > t''$,

$$\tilde{b}_t^{new} + \sum_{\tau=t}^{\infty}\delta^{\tau-t}\tilde{w}_\tau = \frac{\bar{c}}{1-\delta} < \sum_{\tau=t}^{\infty}\delta^{\tau-t}\tilde{c}_\tau = \tilde{b}_t + \sum_{\tau=t}^{\infty}\delta^{\tau-t}\tilde{w}_\tau,$$

where the second equality follows from Equation (30). This implies that $\tilde{b}_t^{new} < \tilde{b}_t$ for all $t > t''$.

Now, we want to show that the contract $\left(\tilde{e}_t^{new}, \tilde{c}_t^{new}, \tilde{w}_t^{new}, \tilde{b}_t^{new}\right)_{t\geq 1}$ is self-enforceable. Because effort and payments are unchanged relative to the original contract, the principal's constraints (PC_t) remain intact. Consider then the agent's constraint (AC_t^{ob}) for each period $t \geq 1$. For all $t \leq t''$, the agent anticipates a strictly higher continuation payoff under the new contract, i.e.

$$\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(v\left(\tilde{c}_\tau^{new}\right) - \psi\left(\tilde{e}_\tau^{new}\right)\right) > \sum_{\tau=t}^{\infty}\delta^{s-t}\left(v\left(\tilde{c}_\tau\right) - \psi\left(\tilde{e}_\tau\right)\right).$$

The strict inequality is immediate from the strict concavity of v, and because consumption from date t'' onwards is constant in the new contract, but the NPV of this consumption is the same as in the original. Since, in addition, $v(\tilde{b}_t^{new}(1-\delta)) = v(\tilde{b}_t(1-\delta))$, the agent's constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) are satisfied at dates $t \leq t''$ as strict inequalities.

To understand how the agent's constraints change at each $t > t''$, define $\bar{c}^{(t)} \equiv (1-\delta)\sum_{\tau=t}^{\infty} \delta^{\tau-t}\tilde{c}_\tau$. Consider the original contract, and suppose that the agent's consumption is changed from date t onwards, being set equal to $\bar{c}^{(t)}$ in all such periods. The agent's payoff increases from the smoothing of consumption, and so

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}(v(\bar{c}^{(t)}) - \psi(\tilde{e}_\tau)) \geq \sum_{\tau=t}^{\infty} \delta^{\tau-t}\left(v\left(\tilde{c}_\tau\right) - \psi\left(\tilde{e}_\tau\right)\right) \geq \frac{v(\tilde{b}_t(1-\delta))}{1-\delta}, \tag{32}$$

where the second inequality follows because the agent's constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) are satisfied in the original contract.

Because ψ is non-negative, the inequalities in Equation (32) imply $\bar{c}^{(t)} \geq \tilde{b}_t(1-\delta)$. Therefore, since v is concave, we have

$$v(\bar{c}^{(t)}) - v(\bar{c}^{(t)} - (1-\delta)(\tilde{b}_t - \tilde{b}_t^{new})) \leq v(\tilde{b}_t(1-\delta)) - v(\tilde{b}_t(1-\delta) - (1-\delta)(\tilde{b}_t - \tilde{b}_t^{new})). \tag{33}$$

Note that $\bar{c} = \bar{c}^{(t)} - (1-\delta)(\tilde{b}_t - \tilde{b}_t^{new})$. Combining Equations (32) and (33), we therefore have that, for all $t > t''$,

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}\left(v\left(\bar{c}\right) - \psi\left(\tilde{e}_\tau\right)\right) \geq \frac{v(\tilde{b}_t^{new}(1-\delta))}{1-\delta}.$$

This shows that, for the contract $\left(\tilde{e}_t^{new}, \tilde{c}_t^{new}, \tilde{w}_t^{new}, \tilde{b}_t^{new}\right)_{t\geq 1}$, the agent's constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) are satisfied also at dates $t > t''$.

We have thus shown that $\left(\tilde{e}_t^{new}, \tilde{c}_t^{new}, \tilde{w}_t^{new}, \tilde{b}_t^{new}\right)_{t\geq 1}$ is a self-enforceable contract (in particular, it satisfies all the constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) and ($\mathrm{PC}_t$). Moreover, we saw that the constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) are satisfied strictly at all $t \leq t''$. We can therefore further adjust the contract by raising effort at date t'' by a small amount $\varepsilon > 0$ such that, without any other changes to the contract, all the agent's constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) remain intact. The adjusted contract then satisfies all the constraints ($\mathrm{AC}_t^{\mathrm{ob}}$) and ($\mathrm{PC}_t$), and the principal obtains a strictly higher payoff than in the original contract, contradicting the optimality of the original. □

We can then provide the key result that balances decrease over time towards $\tilde{b}_\infty$.

Lemma A.12. *In any optimal contract, $(\tilde{b}_t)_{t\geq 1}$ is a weakly decreasing sequence. It is constant if it attains the first-best payoff, and strictly decreasing towards some $\tilde{b}_\infty > 0$ otherwise. Also,*

$V(\tilde{b}_\infty) = V^{FB}(\tilde{b}_\infty)$.

We then translate the above result into implications for the dynamics of the principal's continuation payoff.

Lemma A.13. *Assume $V(b_1) < V^{FB}(b_1)$. Then $(V(\tilde{b}_t))_{t\geq 1}$ is a strictly increasing sequence.*

We then show that, if the first-best outcome is not attainable in a self-enforceable relational contract, effort is always downward distorted.

Lemma A.14. *Assume $V(b_1) < V^{FB}(b_1)$. Then, in any optimal contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$, $v'(\tilde{c}_t) > \psi'(\tilde{e}_t)$ for all t.*

Lemma A.14 implies by Lemma A.10 that, if $V(b_1) < V^{FB}(b_1)$, the principal's constraint (PC_t) holds with equality in every period. We use this to show the following.

Lemma A.15. *If $V(b_1) < V^{FB}(b_1)$, then, in any optimal contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$, effort $\tilde{e}_t$ and payments $\tilde{w}_t$ strictly increase over time, while consumption $\tilde{c}_t$ strictly declines over time.*

Finally, we establish existence.

Lemma A.16. *An optimal contract exists.*

(End of the proof of Proposition 5.3.) □

Proof of Proposition 5.4. See Online Appendix.