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Pietro Garibaldi
University of Torino and IZA

Pedro Gomes
University of London and IZA

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IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Temporary Replacement Workers in a Matching Model with Employment at Will

In the US almost 3 per cent of employees are absent from their job for reasons other than vacation, but are still technically employed. We argue that firms may find optimal to use temporary replacement workers to fill these vacant positions. We set up a matching model with directed search and double-sided heterogeneity. When a workers is temporarily forced out of the labour market, firms can freely destroy the job, put it in "mothball", or look for a temporary worker to "keep the seat warm". When the latter option is optimal, a market for temporary replacement workers emerges in equilibrium. In a quantitative application to the US labor market, replacement workers represent 2.7 per cent of total employment.

JEL Classification: J22, J40, J15, J60

Keywords: replacement workers, short-duration jobs, temporary jobs, worker heterogeneity, firm heterogeneity, employment at will

Corresponding author:
Pietro Garibaldi
Collegio Carlo Alberto
University of Torino
Italy
E-mail: pietro.garibaldi@carloalberto.org

1 Introduction

In the US in 2017 2.8 per cent of full-time employees are absent from their jobs in a given week, for reasons other than vacation. These workers are either ill or injured, or on leave for other reasons. They are accounted in official statistics as employed and are expected to return to their position. This paper studies what firms may do, in a world with employment at will, when workers go on temporarily leave. If employment can be terminated at no cost, firms might destroy the job, wait for the worker to return, or hire a temporary replacement worker to fill the position. If the latter is optimal, why would workers accept these jobs with a much shorter expected duration?

Our aim is to rationalise how temporary replacement jobs can emerge in an equilibrium matching model of the labor market without any form of employment protection. To this end, we set up a model with directed search and double-sided heterogeneity. We build upon two existing applications of matching models in the spirit of Diamond Mortensen and Pissarides, and the subsequent competitive search models of Moen (1997). First, as in Fernandez-Blanco and Gomes (2017), we assume that firms post jobs with different capital intensity to attract a particular type of worker, creating double-side heterogeneity. Capital is rented at the onset of the match and a cost of capital is paid while the job is active. Second, in the spirit of Garibaldi and Wasmer (2005), we assume that workers are subject to shocks that force them to temporarily leave the job, i.e. an illness or injury, a sabbatical, or for maternity or paternity leave.

In our setting, three submarkets emerge in equilibrium. First, low-skilled workers match with low-capital firms and jobs are destroyed when workers leave the job. Second, capital-intensive firms match with high-skilled workers. When workers are hit by a temporary shock, capital-intensive firms face a key and novel economic decision. They can freely destroy the job, they can temporarily leave the job in "mothball" incurring the costs of the idle capital while the worker is absent, or they can look for a temporary worker to "keep the seat warm" and continue using the existing capital. When the latter option is optimal, a third submarket of temporary replacement jobs emerges in equilibrium. Which workers do these jobs attract? High-skilled workers do not find it optimal to search for these short-duration jobs. Only low-skilled workers are willing to search in this market; they are over-employed, in the sense that they are paired with too much capital. In a quantitative exercise applied to the US labour market, replacement workers represent 2.7 per cent of total employment, and this share is robust to changes in key parameters. Their wages are lower than the wages in the high-skilled market, but higher than those in the low-skilled market. The value of a higher wage associated to being employed in high-capital jobs is offset by a shorter duration of the match and sometimes longer job queues.

This paper relates to two strands of the literature. First, it relates to recent research on short-duration employment spells in the US. Replacement jobs are, by nature, of shorter duration. Hyatt and Spletzer (2020) highlight the pervasiveness and increasing importance of single-quarter jobs over the last 20 years. Morchio (2020), Hall and Kudlyak (2020) and Gregory et al. (2020) associated short employment spells to the characteristics of the individuals. Our contribution, is to emphasize that also the job characteristics are important in these short employment spells. In particular, we explain how these short-duration jobs can arise endogenously, as a way for firms to minimize the cost of absent workers.[1]

Second, this paper relates to the literature on temporary workers and temporary jobs. The literature that started 25 years ago with the seminal contribution of Saint-Paul (1993) associates the existence of temporary contracts to the presence of institutional firing costs in regular labor markets. While the literature is mainly empirical, the coexistence of temporary and regular contracts is analysed by Cahuc et al. (2016) and Berton and Garibaldi (2012). One important concern is whether temporary contracts are dead end or stepping stones (Booth et al., 2002). The perverse effects of temporary contracts are studied by Blanchard and Landier (2002), while the short-run employment effects by Boeri and Garibaldi (2006). This literature focuses on the legal and contractual *de jure* definition of temporary jobs. Yet, replacement jobs are temporary jobs, and they also exist *de facto* in countries in which institutional firing costs are negligible or nonexistent, and labor markets operate under the doctrine of employment at will. For instance, the US Bureau of Labor Statistics (2018) estimates that, in May 2017, between 1.3 and 3.8 per cent of workers do not expect their jobs to last for more than a year or report them as temporary.

[1] This paper does not deal with the role played by temporary help agencies (Autor and Houseman, 2005, 2010).

2 Employee Absent from Work and Replacement workers: Some Evidence

The US Current Population Survey (CPS) provides data on the number of people who are employed but were absent from work for the entire survey reference week. The reasons why they were not at work included: vacation, illness or injury, childcare problems, other family or personal obligations (for instance taking care of a sick family member), bad weather, maternity or paternity leave, school or training, civic or military duty, and other reasons. We exclude vacation from our the list. Note that the people who report that they are temporarily absent from work are classified as employed. These are different from people who were temporarily laid off and expecting recall, who are classified among the unemployed on temporary layoff. These last group have been analysed, for instance by Fujita and Moscarini (2017).

For 2017, Bureau of Labor Statistics (2018) reports that 2.8 per cent of full-time employees were absent from their job, 1.9 per cent due to illness or injury and the remaining 0.9 per cent for other reasons.[2] This corresponds to approximately 5 million workers. Yet, there is important heterogeneity across age, gender and occupations. It is amongst the older workers, aged 55 and over, that the absentee rates are higher, about 3.1 per cent. In 2017, women were more likely to be absent from the job compared to men (3.6 per cent against 2.1 per cent), probably because of maternity leave. Across occupation, the highest rates of absent workers (about 3.4 per cent) are found in service, i.e. healthcare support occupations, and in the public sector (particularly federal government).

In labour market surveys, there is no specific category of temporary replacement workers so we cannot have a precise measure. We can think of two data counterparts where these workers would be represented: workers with short employment spells or contingent workers. The interest on short employment spells has grown in the past years. Hyatt and Spletzer (2020) report detailed evidence of the size and dynamics of single-quarter jobs. Using the Longitudinal Employer-Household Dynamics (LEHD), they show that albeit declining in medium run perspective, single-quarter jobs still account for approximately 5 per cent of total employment between 2010 and 2012.

[2]This statistic is reported in Table 46 of the Labor Force Statistics from the Current Population Survey. This value is lower than the long run average of absent from of 3.1 per cent, approximately 2.1 per cent is due to illness and another 1 per cent is due to other reasons out line above.

At a general level, Hyatt and Spletzer (2020) show that these single-quarter jobs are not stepping stones into longer and stable employment relationships. They also ask whether these jobs are the results of poor match quality or are linked to the temporary nature of the job, and conclude that the evidence from CPS data on single-quarter jobs as being low-paid high turnover is more coherent with the inherently short-term hypothesis. Morchio (2020) uses the National Longitudinal Survey of Youth and finds that a subset of youth unemployed are characterized throughout their career by a very large separation rate on the job and shorter employment spells. Morchio (2020) thus highlights that specific worker characteristics are associated to short-term employment relationships. Hall and Kudlyak (2020) study labor market dynamics across labor market states and define long and short-term jobs as a different state. Using the CPS, they show that short employment spell market often is mainly composed by a subset of individuals who move frequently between employment and unemployment. While Hall and Kudlyak (2020) estimate transition probabilities among the enlarged labor market states, their ergodic estimates suggest that short term employment accounts for 5 per cent of the male population and 6.7 per cent of the female population. All these papers highlight how these jobs are of poorer quality.

Short employment spells may arise ex-post, with both workers and firms having started a match with expectations of a longer duration. But there are many jobs that workers accept knowing that they will have a short duration. The Bureau of Labor Statistics (2018) carries out a specific survey to estimate the size of wage and salary workers who expect their jobs will last for less than a year or report them as temporary. Bureau of Labor Statistics (2018) defines these workers as "contingent". These do not include workers who do not expect to continue in their jobs for personal reasons such as retirement or returning to school. They provide three different estimates of these workers, for May 2017, and argue that they range between 1.3 per cent and 3.8 per cent of employment.[3] The BLS further reports that contingent workers are more than twice as likely to be under the age of 25 or to work part-time, when compared to non-contingent workers. Young contingent workers (16- to 24-year-olds)

[3]The larger estimate, includes wage and salary workers as well as self-employed and independent contractors, and adds up to 5.9 million workers. For clarity, these estimates of contingent workers differ from the estimates of alternative workers arrangements, which include 10.6 million independent contractor (6.8 per cent of total employment), 2.6 million on call-workers (1.7 per cent), 1.4 million temporary help agency workers (0.9 per cent), and about one million workers provided by contract firms (0.6 per cent).

are much more likely than their non-contingent counterparts to be enrolled in school (62 per cent and 36 per cent, respectively) and more likely to work in "professional and related occupations" and in "construction and extraction" than non-contingent workers. More than half of contingent workers (55 per cent) would have preferred a permanent job.

Contingent workers earn less than their non-contingent counterparts. Among full-time workers, median weekly earnings for contingent workers ($685) were 77 per cent of those of non-contingent workers ($886). The disparity in earnings likely reflects the many differences in the demographic characteristics of contingent and non-contingent workers together with those of the jobs they hold. Contingent wage and salary workers were half as likely to be covered by employer-provided health insurance. One-fourth of contingent workers had employer-provided health insurance, compared with half of non-contingent workers. Among wage and salary workers, contingent workers were about half as likely as non-contingent workers to be eligible for employer-provided pension or retirement plans (23 per cent compared with 48 per cent).

In a nutshell, both short-duration employment spells and contingent workers might include temporary replacement workers, but they are not a perfect data counterpart. As such, our research strategy is to construct a search and matching model where a market for replacement workers arises endogenously, calibrate it to the US economy, and find what is the size of replacement workers that it generates.

3 A Model of Temporary Replacement Workers

3.1 Structure of the Model

Individuals in the labor markets are risk neutral and discount the future at rate r. They are endowed with one unit of time for market activity and leisure. When they are outside of the labor force they obtain a flow utility equal to z. When they are in the labor market they can be employed or unemployed. If employed, they earn a wage and forgo their outside option. If unemployed, they enjoy their outside option for a fraction of time $(1-\phi)z$, so that $\phi \in \{0, 1\}$ is the time spent searching for a job. Clearly, $\phi = 1$ when people work and $\phi = 0$ when they are outside the labor force. People in the market are subject to random shocks to their time allocation, in the spirit of what was originally proposed in the matching model by Garibaldi and Wasmer (2005). For simplicity, we assume that at rate σ^i, individuals need to abandon their market activity and leave the labor force. This might entail leaving the labor force temporarily or, if accepted by the employer, take a leave of absence. The individual worker goes back to the market at rate σ^m.

The labor market is populated by s^h high-skilled workers and s^l low-skilled workers, adding to 1 ($s^l + s^h = 1$). The superscript l and h stand for "low-skilled" and "high-skilled" workers. Their ability is indicated by y_l and y_h. For clarity, we assume that the type of worker is observable, but as we argue below, the equilibrium would survive if the worker type was not observable to firms. Production takes place between one worker and one firm. Firms contribute to production by providing capital, k, i.e. a computer and software. When posting a vacancy, the firm announces the capital intensity of their job, and once the vacancy is paired with a worker and a job is created, it rents the announced capital at cost c_k until the job is destroyed. This assumption implies only a light form of rigidity in the capital market. Though the capital needs to be rented and cannot be re-allocated if the match stays active, the firm is no longer liable if it decided to destroy the match. We assume that capital and labor are complements in production according to a basic neoclassical production function with constant returns to scale. Assuming a Cobb-Douglas production function, per-capita output of each match, net of capital costs is:

$$p = y_j k^{\alpha} - c_k k \quad j \in \{h, l\}. \tag{1}$$

Jobs are destroyed for exogenous reasons at rate δ. If a worker must temporarily leave the labour force, firms decide whether to terminate the match, to leave it in "mothball" waiting for the worker to return, or look for a temporary replacement worker to "keep the seat warm".

High-skilled workers search and meet firms in a standard matching market with constant return to scale. The matching function is $x^h = x^h(v^h, u^h)$, where v^h is the stock of vacancies and u^h is the stock of skilled unemployed workers. $\theta^h = \frac{v^h}{u^h}$ denotes the market tightness, and $\theta^h q(\theta^h)$ the (instantaneous) matching rate. High-skilled workers can be employed or unemployed, but also absent (a^h) or out of the labor force (i^h for inactive). If unemployed, they become inactive at rate σ^i. In total:

$$s^h = u^h + n^h + i^h + a^h, \tag{2}$$

where n^h are regular employees.

There is a submarket for low-skilled workers in which they meet firms with low capital. The matching function is $x^l = x^l(v^l, u^l)$, where v^l are vacancies and u^l is the stock of unemployed workers searching in this market. $\theta^l = \frac{v^l}{u^l}$ is market tightness, and $\theta^l q(\theta^l)$ is the matching rate.

If capital-intensive firms open vacancies for temporary workers to replace absent workers, a third submarket emerges. Firms post temporary vacancies, but these only attract low-skilled workers. We call this market the replacement submarket. Let v^t be the stock of temporary vacancies, and u^t the stock of low-skilled unemployed in this submarket. The matching function is $x^t = m(v^t, u^t)$, the market tightness $\theta^t = \frac{v^t}{u^t}$, and their matching rate $\theta^t q^t(\theta^t)$. Low-skilled workers can be inactive, but in equilibrium none is absent, so:

$$s^l = \sum_{j=l;j=t} (n^j + u^j + i^j), \qquad (3)$$

where $n^j$and u^j and i^j are respectively employed, unemployed and inactive, while l and t are the two submarkets.

3.2 Capital and Match Productivity

Given the production function, there is a unique capital level most suitable for each worker, given by:

$$\tilde{k}_j = \left(\frac{\alpha y_j}{c_k}\right)^{\frac{1}{1-\alpha}} \qquad j \in \{h, l\}. \qquad (4)$$

For the net output of the match to be maximized, more skilled workers should be paired with more capital.[4] Using this optimal capital intensity, there are three possible productivities in the model:

$$\begin{aligned} p_h &= y_h \tilde{k}_h^{\alpha} - c_k \tilde{k}_h, \\ p_l &= y_l \tilde{k}_l^{\alpha} - c_k \tilde{k}_l, \\ p_t &= y_l \tilde{k}_h^{\alpha} - c_k \tilde{k}_h, \end{aligned} \qquad (5)$$

[4]Equation (4) assumes a spot market for capital without any long-tun dynamic horizon that may result from the temporary market.

where p_h is the net output of a skilled worker matched with a capital-intensive firm, p_l is the net output of the low-skilled worker matched with low-capital job, while p_t is the net output of a high-capital firm matched with an unskilled worker. A low-skilled worker when paired with a capital-intensive job produces a higher gross output than when paired with its optimal capital, but once we deduce the costs of capital, the net output is lower. A high-skilled worker never matches with a low-capital firm.

This setting was proposed by Fernandez-Blanco and Gomes (2017) who show that in an environment where the skill of the worker is unobservable, firms can design a self-selection mechanism that overcomes the problem of adverse selection. This structure, together with a commitment by firms to their announced capital stock, prevents low-skilled workers from pretending to be high-skilled. Even if they benefit from a higher job-finding rate, being paired with too much capital reduces the net output of the match, enough to make it unprofitable.

The second advantage of this setting is that it allows us to endogenize productivity differences across the three submarkets, including the productivity gap of replacement workers.

3.3 Submarket For High-Skilled Workers and High-Capital Jobs

We define the joint value of the match M^h between a skilled worker and a high-capital job as:

$$(r + \delta + \sigma^l)M^h = p^h + \sigma^i Max[V^m + A^h, V^t + A^h, V^h + I^h] + \delta(U^h + V^h). \quad (6)$$

The key joint decision is what to do when the workers must leave the labour market. They might go on leave of absence and obtain a value A^h or exit from the labour force and get I^h. V^m is the value of a vacant job when the firm puts it on "mothball", and does nothing but waiting for the worker to return.[5] As an alternative, the firm can enter the submarket of temporary replacement jobs, an option whose value we label as V^t. The last option in equation (6) is to destroy the match and post a new vacancy that yields V^h, and the worker becomes inactive. Since there is free entry, in equilibrium

[5]Fujita and Moscarini (2017) model a vacancy in "mothball" within the context of temporary layoff and unemployment recall.

such option to the firm has zero value ($V^h = 0$).

The surplus from the job is $S^h = M^h - U^h - V^h$, and the wage contract implies that the worker gets a fraction β from the surplus. The value function for a vacancy reads

$$rV^h = -c^h + q(\theta^h)(1-\beta)S^h, \tag{7}$$

where c^h is the vacancy cost. The value of a vacant job in "mothball" is

$$(r + \delta + \sigma^m)V^m = -c_k\tilde{k}^h + \sigma^m(1-\beta)S^h, \tag{8}$$

where the firm pays the rental cost of capital $c_k\tilde{k}^h$ and waits for the worker to return. The value of a temporary vacancy is

$$(r + \delta + \sigma^m)V^t = -c^t - c_k\tilde{k}^h + q(\theta^t)(1-\beta)S^t + \sigma^m(1-\beta)S^h, \tag{9}$$

where c^t is the search cost for a replacement worker and $q(\theta^t)$ is their arrival rate. Once a match is formed, the firm receives a fraction $(1-\beta)$ of the surplus. Firms thus use the replacement market as long as $V^t > V^m$ or

$$q(\theta^t) > \frac{c^t}{(1-\beta)S^t}. \tag{10}$$

Under the condition of equation (10) (to be satisfied in equilibrium) the surplus of the match is:

$$(r + \delta + \sigma^i)S^h = p^h - rU^h + \sigma^i V^t + \sigma^i A^h, \tag{11}$$

and free entry implies the standard job-creation condition and the outside option of workers:

$$\frac{c_h}{q(\theta^h)} = (1-\beta)S^h, \tag{12}$$

$$rU^h = (1-\phi)z + \theta^h q(\theta^h)\beta S^h + \sigma^i(I^h - U^h), \tag{13}$$

$$rI^h = z + \sigma^m(U^h - I^h), \tag{14}$$

$$rA^h = z + \sigma^m(U^h + \beta S^h - A^h) + \delta(I^h - A^h). \tag{15}$$

These last two equations distinguish the value of inactive and absent workers. While absent workers return immediately to their job, inactive workers must first search for one.[6] Note that the high-capital market is strictly interconnected to the submarket for replacement jobs. It is possible that, if $V^m < 0$ and $V^t < 0$, it is optimal to destroy the match upon a shock, with workers becoming inactive. In that case, a market for replacement workers would not exist.

3.4 Submarket For Low-Skilled Workers and Low-Capital Jobs

In the submarket for low-capital jobs, we assume that the parameters are such that jobs are not worth keeping if a worker is pushed into inactivity. We verify in the numerical exercise that this is the case. Hence, jobs are destroyed at rate $\delta + \sigma^i$. We define the joint value of the match M^l as:

$$(r + \delta + \sigma^i)M^l = p^l + \sigma^i(I^l + V^l) + \delta(U^l + V^l) \tag{16}$$

where U^l and I^l are the unemployment value and the value of being out of the labor force, while V^l is the value of a vacancy to a firm. The surplus of the match is defined as the joint value net of the worker and the firm outside options ($S^l = M^l - U^l - V^l$). The workers gets a fraction β of the surplus while the firm gets a fraction $(1 - \beta)$. The firm and the worker value of a vacancy and unemployment are:

$$rV^l = -c^l + q(\theta^l)(1 - \beta)S^l, \tag{17}$$

$$rU^l = (1 - \phi)z + \theta^l q(\theta^l)\beta S^l + \sigma^i(I^l - U^l), \tag{18}$$

$$rI^l = z + \sigma^m(U^l - I^l). \tag{19}$$

The value functions are standard, with the exception that conditional on the shock to their time allocation, the worker transits to inactivity. Free entry implies that vacancies are opened until $V^l = 0$. Using equation (16) and the definition of the surplus, after

[6] When workers return to a job, the value is $U^h + \beta S^h$, as they get the fraction of the surplus on top of their reservation value.

some algebra, the low productivity market solves the following system:

$$(r+\delta+\sigma^i)S^l = p^l - rU^l + \sigma^i(I^l - U^l), \tag{20}$$

$$\frac{c^l}{q(\theta^l)} = (1-\beta)S^l. \tag{21}$$

3.5 Submarket For Replacement Jobs and Workers' Sorting

When temporary vacancies are available, given the short expected duration of the job, they only attract low-skilled workers. The joint value of a temporary replacement jobs M^t is

$$(r+\delta+\sigma^i+\sigma^m)M^t = p^t + \sigma^m((1-\beta)S^h + U^t) + \delta U^t + \sigma^i I^t \tag{22}$$

The return of the high-skilled worker from leave to its high-capital job at rate σ^m, implies the end of the temporary match: the firm gets back a high productivity match the replacement workers transits into unemployment. In the replacement sub-market there is no free entry and vacancies have positive value in equilibrium as long as condition (10) is satisfied. The other three key equations for the replacement submarket are:

$$(r+\delta+\sigma^i+\sigma^m)(S^t+V^t) = p^t - (r+\sigma^i)U^t + \sigma^m S^h + \sigma^i I^t, \tag{23}$$

$$rU^t = (1-\phi)z + \theta^t q(\theta^t)(\beta S^t + \sigma^i(I^t - U^t), \tag{24}$$

$$rI^t = z + \sigma^m(U^t - I^t).$$

The worker'sorting conditions requires the low-skilled workers to be indifferent between entering the two submarkets.

$$U^t = U^l. \tag{25}$$

The variable that adjusts to guarantee this equality is the number of unemployed workers u^t. If the value of replacement jobs is low, few workers want them, so for the ones that enter this sub-market, the job-finding probability will be high and compensate for the worst jobs. Conversely, if the high-capital jobs for the replacement workers pay high wages, enough to offset the lower expected duration, many unemployed workers will be attracted to this market and the corresponding job-finding rate will be lower.[7]

The submarket for replacement jobs and high-capital jobs are interlinked, and we can solve for $\{S^h, \theta^h, U^h, I^h, S^t, V^t, \theta^t\}$ using equations (9), (11)–(13) and (23)–(25). The routine for numerically solving the system and reducing it to a single equation with a guess and verify method is outlined in the Appendix 1. Appendix 2 checks the conditions to be satisfied for avoiding that the low-skilled worker pretends to be skilled, and for skilled workers not to apply to temporary jobs.

3.6 Steady-State Stocks

For high-skilled workers the evolution of stocks is given by dynamic flow equations:

$$\begin{aligned} \dot{u}^h &= \delta n^h + \sigma^m i^h - (\sigma^i + \theta^h q(\theta^h))u^h \\ \dot{i}^h &= \delta a^h + \sigma^i u^h - \sigma^m i^h \\ \dot{a}^h &= \sigma^i n^h - (\delta + \sigma^m)a^h \\ \dot{n}^h &= \theta^h q(\theta^h)u^h + \sigma^m a^h - (\sigma^i + \delta)n^h \end{aligned} \tag{26}$$

The top panel of Figure 1 depicts these transitions. Setting $\dot{u}^h = \dot{i}^h = \dot{a}^h = \dot{n}^h = 0$, together with $s^h = u^h + i^h + a^h + n^h$, pins down the steady-state stocks for the high skilled.

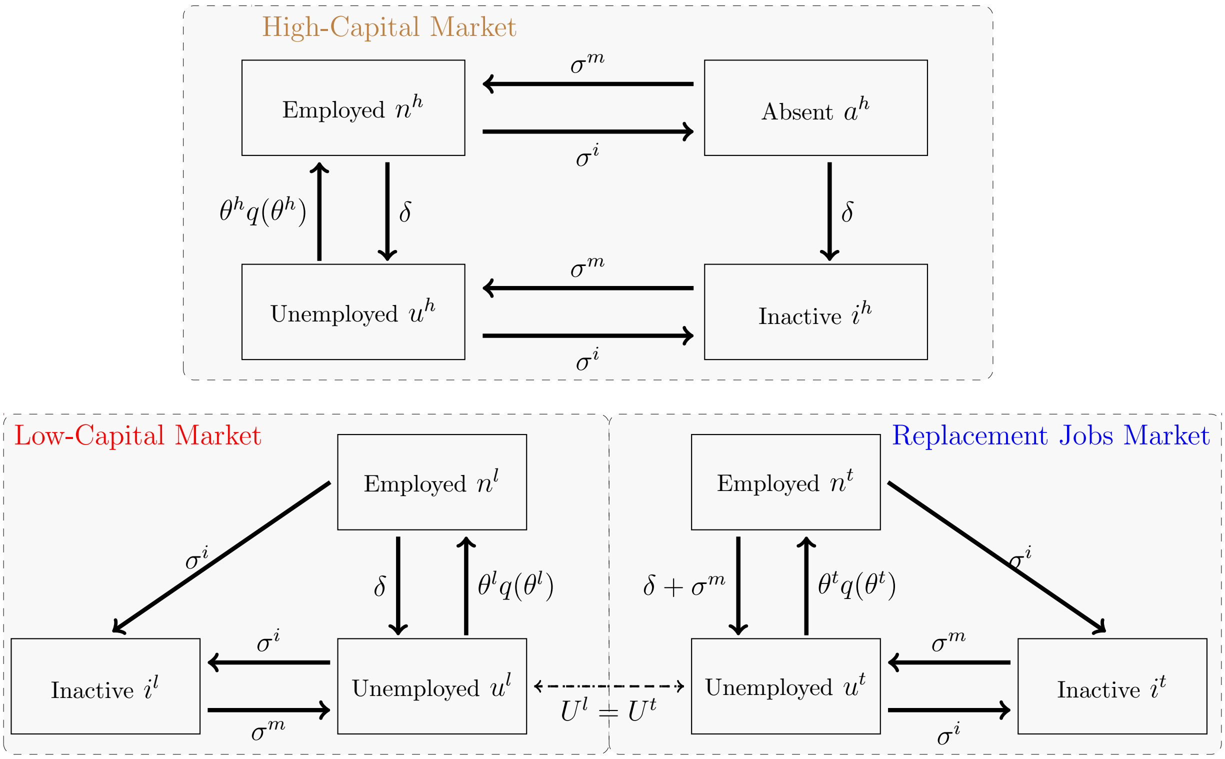
The evolution of the stock of temporary replacement workers depends on the stocks of temporary vacancies and the number of unemployed searching there. This market is different from the usual DMP market in two ways. First, the number of vacancies for replacement jobs is not a jump variable that adjusts to satisfy a free-entry condition. Instead it is a slow-moving stock that evolves according to the dynamic flow equation:

$$\dot{v}^t = \sigma^i n^h - (\delta + \sigma^m + q(\theta^t))v^t, \tag{27}$$

and its steady-state value being a constant fraction of the employment of high-skilled

[7]In principle, high-skilled workers could also apply to this market. However, given that they would compete with low-skilled workers, the job-finding rate would always be lower than the minimum level to accept jobs with such low duration. It would be possible to design an alternative separating temporary market, where firms post high-skilled and low-skilled temporary jobs, but we think it is not realistic. The nature of this market does not incentivise firms to spend much resources in posting vacancies and screening applicants.

Figure 1: Flows Rates Out of Each State



workers.

$$v^t = \frac{\sigma^i}{\delta + \sigma^m + q(\theta^t)} n^h \tag{28}$$

Second, instead of a stock, the number of unemployed in the market for temporary jobs is a jump variable, that adjusts to satisfy the arbitrage condition $U^t = U^l$. In this market, tightness adjusts not through vacancies but through unemployment. Consider a change in the high-type market that raises the number of absent workers, and leads to an increase in vacancies. The higher matching rate in the replacement market, attracts unemployment from the low-type market, until the equilibrium tightness (matching rate) is restored. This adjustment mechanism is found in models with a public sector and directed search, where vacancies in the government sector do not satisfy a free-entry condition (Gomes, 2015, 2018).

Similarly, the number of unemployed in the low-type market also jumps, but its tightness is determined by the free-entry condition of firms. When the number of unemployed in this market changes, vacancies adjust one-to-one to maintain tightness constant. The laws of motion of the remaining stocks (both low-skilled and temporary replacement markets) are given by:

$$\begin{aligned} \dot{n}^l &= \theta^l q(\theta^l) u^l - (\sigma^i + \delta) n^l, \\ \dot{n}^t &= \theta^t q^t(\theta^t) u^t - (\sigma^i + \sigma^m + \delta) n^t, \\ \dot{i}^l &= \sigma^i (u^l + n^l) - \sigma^m i^l, \\ \dot{i}^t &= \sigma^i (u^t + n^t) - \sigma^m i^t. \end{aligned} \tag{29}$$

While there is no law of motion for u^t and u^l separately because they are jump variables, together they evolve according to:

$$(u^t \dot{+} u^l) = \sigma^m (i^l + i^t) + \delta(n^l + n^t) - \theta^l q(\theta^l) u^l - \theta^t q^t(\theta^t) u^t. \tag{30}$$

The bottom panel of Figure 1 depicts these transitions. Setting $\dot{n}^l = \dot{n}^t = \dot{i}^l = \dot{i}^t = (u^t \dot{+} u^l) = 0$, together with $s^l = u^l + i^l + n^l + u^t + i^t + n^t$ and the no-arbitrage condition ($U^t = U^l$), pins down the steady-state stocks for the low-skilled workers.

The model provides some insights on the difference of job-separations for workers of different types. Despite a common job-destruction rate δ, low-skilled workers have higher job-separation rates than high-skilled workers. This happens for two reasons. First, when facing the outside-option shock, high-skilled workers go on leave-of-absence, so the employment relation remains intact; low-skilled workers see their job destroyed and become inactive. Second, a fraction of low-skilled workers are in the market for replacement jobs, and from their perspective, they transit to unemployment at rate $\delta + \sigma^m$, either when the job is destroyed or when the high-skilled worker returns to its job.

3.7 Definition

Definition 1. *An equilibrium with temporary replacement jobs is a set of productivity* $\{p_i\}$*, value functions for* $\{S^i, I^i, U^i, N^i, V^i, M^i, A^h\}$*, a set of stocks* $\{n^i, l^i, u^i, a^h\}$ *and market tightness* θ^i*, in the three submarkets* $i \in \{h, l, t\}$*, satisfying:*

1. *Capital optimality conditions (5).*

2. *Surplus conditions (equations 11, 20, 23).*

3. *Free entry in the high and low submarkets (12,21).*

4. *Unemployment sorting across replacement and low submarkets (25).*

5. *Optimal vacancy posting for temporary market (10).*

6. *Stocks are constant (equations 26, 29 and 30 are set equal to zero).*

4 Calibration and Simulations

We calibrate of the model to the long run US labor market. The main spirit of the exercise is to target the number of absent workers that we observe in the US economy and let the model deliver endogenously the stock of replacement workers and their wages, and assess whether they are consistent with the BLS estimates of between 1.3 and 3.8 per cent of workers with contingent jobs, and their wages being 23 per cent lower than non-contingent workers. The parameters are specified in Table 1 and the outcomes of the equilibrium are in Table 2.

The calibration is at the monthly level, and we thus set the monthly interest rate r to 0.003, corresponding to a yearly interest rate of approximately 3.5 per cent. The cost of capital c_k is set at the same level, so that $c_k = r = 0.003$. The share of capital in the per capita production function α is 0.333, in line with most standard macro and growth models.

Labour market parameters mostly come from Shimer (2005). The bargaining share β, and the elasticity of the matching function η are set to 0.72. Shimer (2005) normalizes the baseline productivity to 1. In our model there are two types of workers with productivities p_h and p_l respectively. The share of each group is set at $s = 0.5$. We set a value of y_h so that their productivity p_h is approximately equal to 1. This requires a value of $y_h = 0.333$. To set y_l we proceed as follows. Using the CPS March supplement, we run a regression of the log of hourly wage, on a set of controls that include dummies for age categories, state, 2-digit occupations, education, gender, part-time, and year dummies, for the period between 1998 and 2017. We calculate the inter-quantile range of the residual of this regression and find the value of 0.627. The value of y_l necessary to match these statistics is 0.21.

Shimer sets the worker outside income z to 0.4 of average productivity. Yet, since we have two types of workers, we set $z = 0.4(p_l + p_h)/2$. The time unemployed spend

Table 1: Model Parameters

Parameter	Notation	Value
Preferences, Technology and Flow Values		
Pure Discount Rate	r	0.003
Cost of Capital	c_k	0.003
Capital elasticity	α	0.300
High-Skilled Ability	y_h	0.333
Low-Skilled Ability	y_l	0.210
Elasticity w.r.t. Capital	α_l; α_h	0.300
High capital	k_h	130.553
Low capital	k_l	67.570
Share of high skilled	s_h	0.500
Productivity high-type match	p_h	1.005
Productivity low-type match	p_l	0.520
Productivity replacement match	p_t	0.475
Out of the labor force utility	z	0.336
Unemployed time	ϕ	0.100
Arrival Rates		
destruction rate	δ	0.034
leave rate	σ^i	0.028
return to market rate	σ^m	0.422
Sub Markets: Matching Functions		
Constant High Market	A_k^h	1.210
Constant Low Market	A_k^l	1.210
Constant Replacement Market	A_k^t	1.210
Matching Elasticity	η_h; η_l; η_t	0.720
Common Search Costs	c	0.211
Search Costs High-type Sub-Market	cp_h	0.212
Search Costs Low-type Sub-Market	cp_l	0.110
Search Costs Replacement Sub-Market	cp_t	0.100
Bargaining Share	β_h; β_l; β_t	0.720

Source: Authors' calculation

searching is set at $\phi = 0.1$, in line with the estimate of time use in Krueger and Mueller (2012) for the US. Search costs are modelled proportional to productivity and are thus written as $c^i = cp_i$, where $i \in \{h, l, t\}$, and c is the common search cost set as in Shimer (2005) and it is equal to 0.21. The destruction rate δ matches the monthly unemployment inflow estimated by Shimer (2005) and is set to 0.034. The matching efficiency A_h, A_l, A_t are equal and set to match the long-run monthly US unemployment rate (between 1947 and 2021) equal to 0.057.

The parameters σ^m and σ^i govern the speed at which workers become absent and return to their jobs. We calibrate them to target the overall stock of absent workers and the rate at which employed workers become absent. The value of σ^m is set so that the stock of workers in leave of absence is 2.8 per cent, according to the BLS estimate in 2017. These are defined as people who usually work 35 or more hours per week (full time) but worked less than 35 hours during the reference week for one of the following reasons: own illness, injury, or medical problems; child care problems; other

Table 2: Equilibrium Values

Symbol	Notation	Value
Value Functions		
Unemployed Value (High Skilled)	rU_h	0.937
Inactive Value (High Skilled)	rI_h	0.933
Absent Value (High Skilled)	rA_h	0.934
Unemployed Value (Low Skilled)	$rU_l;\ \ rU_t$	0.491
Inactive Value (Low Skilled)	$rL_l\ \ rL_t$	0.490
Surplus High Match	S_h	0.726
Surplus Low Match	S_l	0.299
Surplus Replacement Market	S_t	0.387
Vacancy Value (High and Low Market)	$V_h,\ V_l$	0.000
Value Vacancy (Replacement Market)	$V_t,$	0.189
Temporary Optimality Condition	$q(\theta^t) - \frac{c_t}{(1-\beta)S_t} > 0,$	3.982
Market Tightness (High Market)	$\theta_h,$	1.237
Market Tightness (Low Market)	$\theta_l,$	0.361
Market Tightness (Replacement Market)	$\theta_t,$	0.143
Equilibrium Stocks		
High skilled unemployed	u_h	0.013
High skilled out of the labor force	i_h	0.003
High skilled absent	a_h	0.028
High skilled employed	n_h	0.456
Low skilled unemployed low market	u_l	0.027
Low skilled out of the labor force	l_l	0.028
Low skilled employed	n_l	0.402
Low skilled unemployed replacement	u_t	0.016
Low skilled out of the labor force replacement	l_t	0.003
Low skilled employed replacement	n_t	0.024
Key Aggregate Statistics		
Total unemployment	u	0.057
Share of skilled employed on leave	$\frac{a_h}{n_h+n_l+n_t}$	0.031
Skilled Wage	w_h	0.997
Low Skilled Wage	w_l	0.515
Replacement Wage	w_t	0.637
Skilled Job Finding Rate	$\theta^h q(\theta^h)$	1.284
Low Skilled Job Finding Rate	$\theta^l q(\theta^l)$	0.909
Replacement Job Finding Rate	$\theta^t q(\theta^t)$	0.702
Unmatched key Aggregate Statistics		
Temporary Replacement Workers	$\frac{n_t}{n_h+n_l+n_t}$	0.027
Relative Wage of Replacement Workers	$\frac{w_t}{(w_h n_h+w_l n_l)/(n_h+n_l)}$	0.826
Source: Authors' calculation		

family or personal obligations; civic or military duty; and maternity or paternity leave. Excluded are situations in which work was missed due to vacation or personal days, holiday, labor dispute, and other reasons. For multiple jobholders, absence data refer only to work missed at their main jobs. The absence rate is the ratio of workers with absences to full time employment, and we take it to be represetnative of the entire US labor market. For σ^i, we merged CPS monthly files, following Shimer (2012) paper and method, to calculate the monthly probability that an employed worker goes on leave. The rate is the transition from"Employment - at work" to "Employment - absent", and corresponds to a value $\sigma^i = 0.028$. The value for σ^m implies that the expected duration of absent works is almost two and a half months.

The targets that we calibrated are well-matched, and include the unemployment rate, the stock of absentees, and the interquartile wage differential. The calibrated model features a stock of replacement workers equal to 2.7 per cent, a value in the range of estimates provided by the BLS for contingent workers. Regarding wages, Table 2 reports the wages of replacement workers relative to wages of all other workers, is 0.82, close to the 0.77 found in the data.

Comparative Statics on Replacement Workers

Figures 2 and 3 show a set of simulations on the share of replacement workers (panel a), wage differential (panel b) and job-finding probability (panel c) across low-skilled workers in the two submarkets. The baseline aggregate equilibrium described in Table 2 is signaled with the vertical line. In the baseline calibration, the workers in the temporary replacement market have larger wages since - for a given ability - are matched to jobs with larger stock of capital. As an equilibrium mechanism in search market, even if the duration of the match is shorter, the job-finding rate is thus lower. We vary six parameters and find that the size of replacement market, of around 2.7 per cent of total employment, is fairly robust to changes, varying in the range $\{2.5 - 2.9\}$.

In the first row of Figure 2 when we simulate an increase in the productivity $\Delta^y = y^h - y^l$ differential, the fall in wage w_t is the counterpart of large increase in the job-finding rate. Similar effects are at play when we change the cost of temporary vacancy c^t in the second row. The third row reports the results of changes in σ^m, the parameter that governs the return to market activity from leave and it was calibrated to matched the stock of workers on leave. An increase of 10 per cent in its value reduces the share of replacement workers by less than 10 per cent, with an overall elasticity below one in absolute value.

The increase in matching efficiency A_k^t is shown in the first row of Figure 3, referring to the constant in the Cobb Douglas matching function $q(\theta^t) = A_k^{t\,\alpha}$. One can think of the role of temporary agencies in improving the matching efficiency in this market. Not surprisingly, the increase in matching efficiency raises the job-finding rate that eventually becomes larger than the corresponding job-finding rate in the low-skilled market. The wages for replacement market falls as a consequence and, at the limit, tends to converge from above with the the wage in the low-skilled submarket. When the job-finding rates in the two submarkets are equal, we can infer the cost of job insecurity in the temporary market by comparing the two wages. In the second row of Figure 3 we simulate the effect of an increase in cost of capital c_k. With more expensive capital, fewer good jobs with high capital are brought to the market and the job-finding rate in the replacement submarket falls. In response, wages increase with the standard search equilibrium effect. The last simulation in Figure 3 refers to a change in the output elasticity on capital α. The effects on the general equilibrium is the opposite to the productivity differential Δ^y and acts through the effects of α on the capital stock k_h and k_l described by equation (4).

Figure 2: Comparative Statics for Key Aggregate Parameters

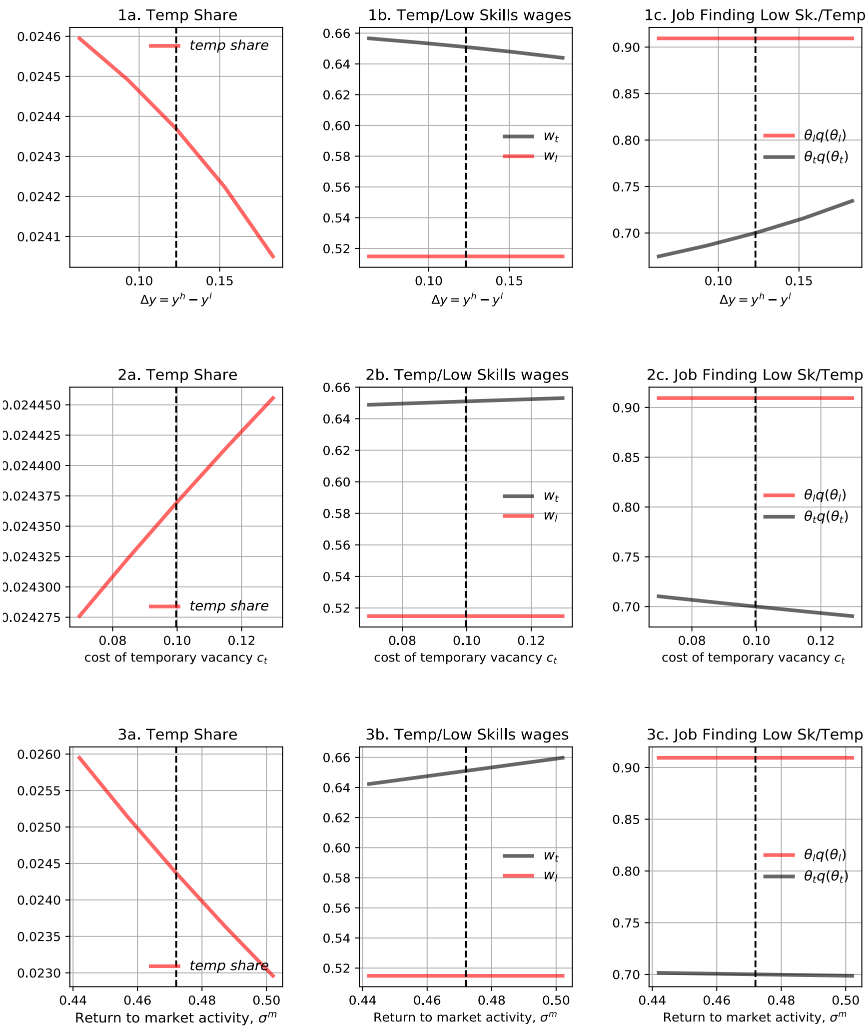
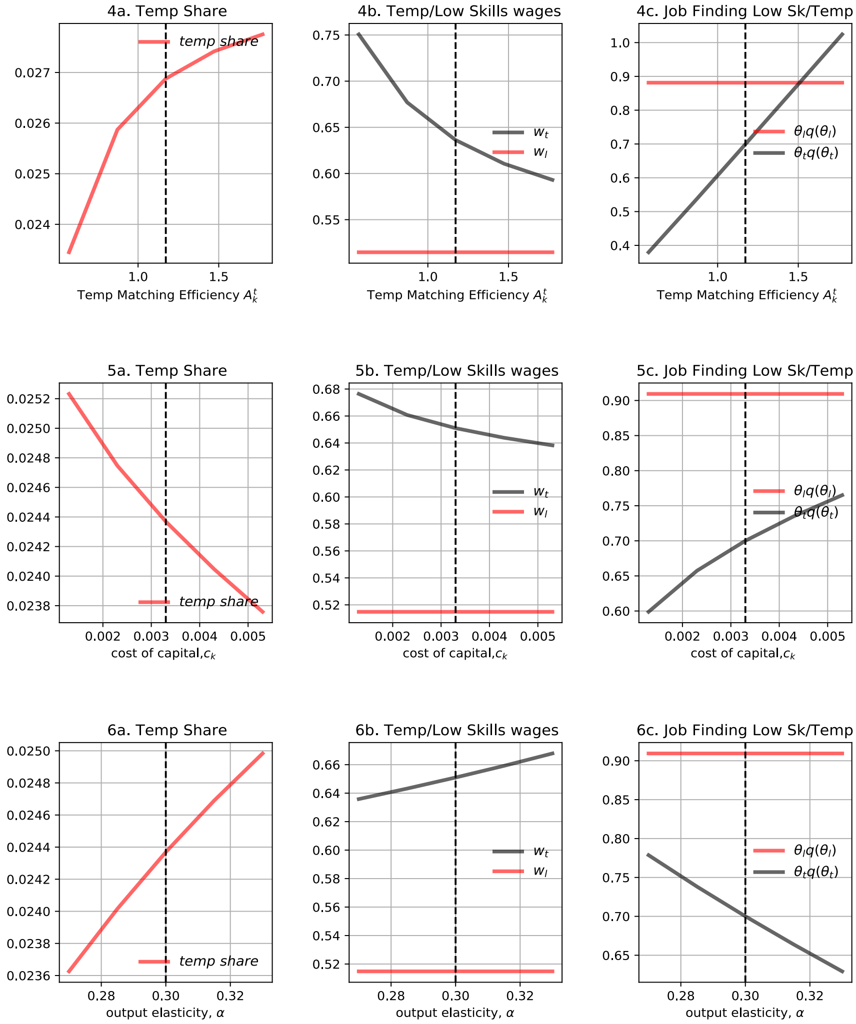


Figure 3: Comparative Statics for Key Aggregate Parameters (cont)



5 Conclusion

The phenomenon of absence from the job is a feature of the US labor market, but it has barely received academic attention. In a given month, approximately 2.8 per cent of people are employed but absent from the job and not in vacation. The paper asked theoretically what the firm does when such temporary absence takes place at the job level. It studied under what conditions firms decide to replace the absent worker with a replacement/temporary worker. It thus proposed a novel link between the size of temporary market and the workers that are absent from work. In the matching model there is no form of employment protection and firing costs. The model highlights the differences of worker's skills, and potential mismatch that occurs. In particular, it endogenously generates what the literature studying mismatch calls over-employment (or under-skilling). Finally, the implications for the literature of temporary contracts in European countries is that a significant number of temporary contracts might be due to a mechanism similar to the one presented in this paper, so only part of the duality should be attributed to more stringent employment protection.

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Online Appendix 1: Equilibrium Algorithm. *Not for Publication*

The solution to the general equilibrium requires solving for two submarket. The solution algorithm reduces both system to a single non linear equation in θ^l and θ^h that are solved through a simple guess and verify algorithm. This appendix highlight the solution of both system.

Solving for the Low-Capital Submarket

Substituting the free entry equation (21) into equation 18,and using a Cobb Douglal matching function $q(\theta^l) = A^l \theta_l^\eta$ the low submarket is the solution to the following system

$$
\begin{aligned}
rU_l &= z(1-\phi) + \frac{\theta_l \beta c}{(1-\beta)} + \frac{\sigma^i(z - z(1-t) - \frac{\theta_l \beta c}{1-\beta}}{r + \sigma^i + \sigma^m} && (31) \\
(r+\delta+\sigma^i)S_l &= p_l - rU - \sigma^i(I^l - U^l) && (32)
\end{aligned}
$$

Using again equation (21) into 32 and using 31 the equation in θ^l solves

$$
\frac{(r+\delta+\sigma^i)c_l\theta_l^\eta}{(1-\beta)A_k^l} = z - \frac{\beta c \theta_l}{1-\beta} \tag{33}
$$

which has clearly a unique solution in θ^l, since the left-hand side is increasing and the right-hand side is decreasing. Note also that this submarket is a standard job creation condition in search models. Once θ_l is determined, the stocks U_l, L_l and S_l are immediately determined.

Solving the Replacement and the High-Capital Submarkets

The large system solves for $\{U^h, I^h, \theta^h, S^h, V^t, St, \theta^t, A^h\}$

$$
\begin{aligned}
rU^h = z(1-t) + \theta^h q(\theta^h)\beta S^h + \sigma^i[I^h - U^h] && (34) \\
rI^h = z + \sigma_m[U^h - I^h] && (35) \\
c = q(\theta^h)(1-\beta)S^h && (36) \\
(r+\delta+\sigma^i)S^h = p^h - rU^h + \sigma_l(A^h - U^h) + \sigma^i V^t && (37) \\
(r+\delta+\sigma_m)V^t = -c_t - c_k k_h + q(\theta^t)(1-\beta)S_t + \sigma^m(1-\beta)S^h && (38) \\
(r+\delta+\sigma^m+\sigma^i)(S^t + V^t) = p_t - (r+\sigma^i)U^l + \sigma^m(1-\beta)S^h + \sigma^i I^l && (39) \\
rU^l = z(1-\phi) + \theta^t q(\theta^t)\beta S^t + \sigma^l(I^l - U^l) && (40) \\
(r+\delta)A^h = z + \delta I^h + \sigma^m(U^h + \beta S^h - A^h) && (41)
\end{aligned}
$$

To reduce the system to a single equation we proceed as follows

1. We guess a value $S^t = \hat{S}^t$
2. Define $\alpha(\theta^t) = Ak^t\theta^t q(\theta^t)$ and from equation 40 obtain

$$\alpha(\theta^t) = \frac{rU^l - \sigma^i(I^l - U^l) - z(1-\phi)}{\beta\hat{S}_t} \tag{42}$$

   So that we have

$$\hat{\theta}^t = \left(\frac{\alpha(\theta^t)(\hat{S}^t)}{A_k^t}\right)^{\frac{1}{1-\eta}} \tag{43}$$

3. From equation (36) obtain

$$S^h = \frac{c}{q(\theta^h)(1-\beta)} \tag{44}$$

   and plugging into equation 34 to obtain

$$rU^h = z(1-\phi) + \frac{c\beta\theta^h}{1-\beta} + \sigma^i(I^h - U^h) \tag{45}$$

   Since from equation 35

$$I^h = \frac{z + \sigma^m U^h}{r + \sigma^m} \tag{46}$$

   substituting this into 45 we obtain

$$(r + \sigma^i - \frac{\sigma^i\sigma^m}{r+\sigma^m})U^h = z + \frac{\sigma^i z}{r+\sigma^m} + \frac{c\theta^h\beta}{1-\beta} \tag{47}$$

4. Substituting (44) into equations (37), (38) and (41), and using also equation (46) we arrive at a system of 4 equations into 4 unknowns $\{\theta^h, U^h, V^t, A^h\}$

$$(r + \sigma^i - \frac{\sigma^i\sigma^m}{r+\sigma^m})U^h = z(1-\phi) + \frac{\sigma^i z}{r+\sigma^m} + \frac{c\beta\theta^h}{1-\beta} \tag{48}$$

$$(r+\delta+\sigma^i)\frac{c}{q(\theta^h)(1-\beta)} = p^h - rU^h + \sigma_l(A^h - U^h) + \sigma^i V^t \tag{49}$$

$$(r+\delta+\sigma_m)V^t = -c_t - c_k k_h + q(\hat{\theta}_t)(1-\beta)\hat{S}_t + \sigma^m\frac{c}{q(\theta^h)} \tag{50}$$

$$(r+\delta+\sigma_m)A^h = z + \delta\left(\frac{z+\sigma^m U^h}{r+\sigma^m}\right) + \sigma^m U^h + \frac{\sigma^m\beta c}{q(\theta^h)(1-\beta)}$$

The system can be simply written as follows, since $q(\theta^h) = A_k^h \theta^{h^{-\eta}}$

$$
\begin{aligned}
U^h &= a_1 + a_2\theta^h && (51)\\
\theta^{h^\eta} &= b_1 - b_2 U^h + b_3 A^h + b_4 V^t && (52)\\
V^t &= c_1(\hat{S}_t) + c_2\theta^{h^\eta} && (53)\\
A^h &= d_1 + d_2 U^h + d_3\theta^{h^\eta}. && (54)
\end{aligned}
$$

Note that we just need to defined the coefficients. Now for simplicity write θ_h^η

5. The key coefficients for the code are

$$
\begin{aligned}
a_1 &= \frac{r+\sigma_m}{r(r+\sigma^i+\sigma^m)}\left(z(1-t)+\frac{\sigma^i z}{r+\sigma^m}\right) && (55)\\
a_2 &= \frac{r+\sigma_m}{r(r+\sigma^i+\sigma^m)}\left(\frac{c}{1-\beta}\right) && (56)\\
b_1 &= \frac{A_k^h(1-\beta)}{c(r+\delta+\sigma^i)}p^h && (57)\\
b_2 &= \frac{A_k^h(1-\beta)}{c(r+\delta+\sigma^i)}(r+\sigma^i) && (58)\\
b_3 &= \frac{A_k^h(1-\beta)}{c(r+\delta+\sigma^i)}(\sigma^i) && (59)\\
b_4 &= b_3 && (60)\\
c_1 &= \frac{1}{r+\delta+\sigma^m}\left[-c_t - c_k k_h + A_k^t\left(\hat{\theta}(\hat{S}^t)\right)^\eta (1-\beta)\hat{S}_t\right] && (61)\\
c_2 &= \frac{1}{r+\delta+\sigma^m}\frac{\sigma^m c}{A_k^h} && (62)\\
d_1 &= \frac{1}{r+\delta+\sigma^m}\left(z+\frac{\delta z}{r+\sigma^m}\right) && (63)\\
d_2 &= \frac{\sigma^m}{r+\sigma^m} && (64)\\
d_3 &= \frac{1}{r+\delta+\sigma^m}\left(.\frac{\sigma^m\beta c}{A_k^h(1-\beta}\right) && (65)
\end{aligned}
$$

6. Now substitute 51 into 54 to obtain

$$A^h = d_1 + d_2 a_1 + d_2 a_2 \theta^h + d_3\theta_h^\eta \qquad (66)$$

Now plug 66 51 53 into 52 to obtain a single equation in θ^h

$$\theta_h^\eta = b_1 - b_2(a_1 + a_2\theta^h) + b_3(d_1 + d_2 a_1 + d_2 a_2\theta^h + d_3\theta_h^\eta) + b_4(c_1(\hat{S}_t) + c_2\theta^{h^\eta})$$

$$(1 - b_3 d_3 - b_4 c_2)\theta_h{}^\eta = \left(b_1 - b_2 a_1 + .b_3 d_1 + b_3 d_2 a_1 + b_4 c_1(\hat{S}_t)\right) + (b_3 d_2 a_2 - b_2 a_2)\theta_h \qquad (67)$$

which can be thought as

$$Z_1 {\theta_h}^{\eta} = Z_2(\hat{S}_t) + Z_3 \theta_h \tag{68}$$

The key coefficients are

$$\begin{aligned} Z_1 &= (1 - b_3 d_3 - b_4 c_2) \\ Z_2 &= [b_1 - b_2 a_1 + .b_3 d_1 + b_3 d_2 a_1 + b_4 c_1(\hat{S}_t)] \\ Z_3 &= (b_3 d_2 a_2 - b_2 a_2) \end{aligned} \tag{69}$$

7. Solve for $\hat{\theta}^t(\hat{S}_t)$
8. Obtain $\hat{U}^h(\hat{S}^t); \hat{V}^t(\hat{S}^t); \hat{A}^t(\hat{S}^t)$
9. Update $\hat{S}^t$ into using

$$(r + \delta + \sigma^m + \sigma^i)(S^t - V^t) = p_t - (r + \sigma^i)U^l + \sigma^m S^h + \sigma^i I^l \tag{70}$$

so that

$$\hat{S}^t{\prime} = \frac{1}{(r + \delta + \sigma^m + \sigma^i)}(p_t - (r + \sigma^i)U^l) - \hat{V}^t(\hat{S}^t) + \sigma_m S^h + \sigma^i I^l \tag{71}$$

10. Update $\hat{S}^t{\prime\prime} = (1 - \rho)\hat{S}^t + \rho \hat{S}^t{\prime}$

On Line Appendix 2: Equilibrium Deviations *Not for Publication*

Low-skilled worker pretending to be a high-skilled workers

Need to solve two equations in two unknowns, $\tilde{S}^l$ $\tilde{U}^l$, being the values of surplus of a low-skilled match in a high-capital market and the value of unemployment of a low-skilled worker pretending to be of high-skilled worker.

$$(r + \delta + \sigma^l)\tilde{S}^l = p^t - r\tilde{U}^l + \sigma^l I^l \tag{72}$$

$$(r + \sigma^l)\tilde{U}^l = z(1 - t) + \theta^h q(\theta^h)\tilde{S}^l + \sigma^l I^l \tag{73}$$

We assume that when hit by a σ^l shock, the firm would not want to keep the match, and the worker goes to inactivity. The low-skilled worker pretends to be of high time to benefit from the higher job-finding rates, but is penalized by being paired with too much capital (p^t).

After solving the two equations, we should compare $\tilde{U}^l < U^l$.

High-skilled workers applying to temporary replacement jobs

Need to solve two equations in two unknowns, $\tilde{S}^t$ $\tilde{U}^t$, being the values of surplus of a high-skilled worker in a replacement market and the value of unemployment of a high-skilled worker applying in the replacement market.

$$(r + \delta + \sigma^i + \sigma^m)(\tilde{S}^t + V^t) = p^h - (r + \sigma^i)\tilde{U}^t + \sigma^m S^h + \sigma^i I^h \tag{74}$$

$$(r + \sigma^l)\tilde{U}^t = z^l + \theta^t q(\theta^t)(\beta S^t) + \sigma^i I^h \tag{75}$$

After solving the two equations, we should compare $\tilde{U}^t < U^h$.