— Online Appendix —

Appendix A: Single-firm setup

This appendix provides the one-firm setup alluded to in footnote 2.4.2 of the main text. We first write out the optimization problem of this firm. Then we compare the resulting optimality conditions to the ones collected in Appendix B that characterize the behavior of firms in the model in the main text. By showing that they are equal, we conclude that the two setups are equivalent.

The setup is as described in the general text, but instead of renting labor and capital services from other firms, each intermediate good producer owns its capital stock, chooses utilization rates, and hires new workers. It still sells its good as a monopolist to a final good producer, who is described in Subsection 2.4.1.[A1] We denote the value function of the firm by J^F. To distinguish the individual choices and state variables of a firm from aggregate equilibrium quantities, we index them by j. The firm chooses its own utilization rate for capital v^j, vacancy postings V^j, investment spending i^j, and the price of its good P^j each period. It makes these decisions given the following state variables: the firm's capital stock at the beginning of the period, K^j, its investment in the last period i^j_{-1}, the firm's mass of workers of different skill s, skill-loss state l, and education e it starts the period with, denoted by $N^j(s,l,e)$,[A2] and the last period price, P^j_{-1}, it charged. In addition, $\widetilde{X}$ is the current aggregate state of the economy.[A3]

The producer's optimization problem is:

$$
\begin{aligned}
& J_F(K^j_{-1}, i^j_{-1}, (N^j_{-1}(s,l,e))_{\forall(s,l,e)}, P^j_{-1}, \widetilde{X}) \\
= \quad & \max_{v^j, V^j, i^j, P^j} (1-\tau^d)\left(y_j\left(\frac{P^j}{P(X)}\right) - i^j - \kappa(\widetilde{X})V^j - \sum_{s,l,e} es(1-\rho l)w(X)N^j(s,l,e) \right. \\
& \left. -\frac{\psi}{2}\left(\frac{P^j}{P^j_{-1}} - \overline{\Pi}\right)^2 y(X) + \mathbb{E}_\zeta\left[Q(X,X')J_F((K^j)', i^j, (N^j(s,l,e))_{\forall(s,l,e)}, P^j, \widetilde{X}')\right]\right) \\
\text{s.t.} \quad & y_j = \left(\frac{P^j}{P(X)}\right)^{-\vartheta exp(\zeta_P)} y(X) \\
& y_j = \zeta_{TFP}\left(K^j v^j\right)^\theta \left(\sum_{s,l,e} es(1-\rho l)N^j(s,l,e)\right)^{1-\theta} \\
& (K^j)' = \big(1-\delta(v^j)\big)K^j + \zeta_I\left(1-\Gamma(i^j/i^j_{-1})\right)i^j \\
& N^j(s,l,e) = \sum_{s_{-1}\in\mathcal{S}_+}\sum_{l_{-1}} \pi_S(s_{-1}, s|s\neq s_0)\pi_L^{emp}(l_{-1},l)\big(1-\lambda_x(e)-\lambda_n(e)\big)N^j_{-1}(s_{-1},l_{-1},e) \\
& +q(\widetilde{X})V^j \frac{U(\widetilde{X},s,e)\pi_L^{unemp}(l) + \lambda_n(e)N(\widetilde{X},s,l,e)}{\sum_{\hat{s},\hat{e}}\left[U(\widetilde{X},\hat{s},\hat{e}) + \lambda_n(\hat{e})\sum_{\hat{l}} N(\widetilde{X},\hat{s},\hat{l},\hat{e})\right]} \ \forall(s,e).
\end{aligned}
$$

Here, we have assumed that the firm takes the aggregate job -filling rate by group and the cost of posting a vacancy as given. This means, for example, that the quadratic adjustment costs are assumed to be an aggregate effect in line with our assumptions in the main text.

[A1]For the argument in footnote 2.4.2 of the main text the final good producer is irrelevant, as it makes no profits or intertemporal choice.

[A2]s is understood to exclude the retired here.

[A3]In the following, some aggregate variables will depend on $\widetilde{X}$, while others will depend on X. As X is a deterministic function of $\widetilde{X}$ we will follow the convention in the main text, with the silent understanding that $X \equiv X(\widetilde{X})$. In addition, given our assumption on such transitions we omit the transitions between retirement and re-birth in the law of motion for the worker types.

Having described the firm's optimization problem, we now collect the conditions characterizing an interior solution of this problem, by combining first-order conditions, constraints, and envelope conditions. To simplify notation, we suppress the dependence on X.[A4] While we substitute out y^j using the demand equation, we need to add Lagrange multipliers for the other constraints. We denote those by ν^y, ν^k, and $\nu^n(s,l,e)$. We arrive at the following equations.

$$\begin{aligned}
\text{(Optimal } P\text{):} \quad 0 = {}& (1-\tau^d)\left((1-\vartheta exp(\zeta_P))\frac{y}{P}\left(\frac{P^j}{P}\right)^{-\vartheta exp(\zeta_P)} - \psi\left(\frac{P^j}{P^j_{-1}}-\bar{\Pi}\right)\frac{y}{P^j_{-1}}\right)\\
&+\vartheta exp(\zeta_P)\frac{y}{P}\left(\frac{P^j}{P}\right)^{-\vartheta exp(\zeta_P)-1}\nu^y + (1-\tau^d)\mathbb{E}_\zeta Q\psi\left(\frac{P^j_{+1}}{P^j}-\bar{\Pi}\right)\frac{P^j_{+1}y_{+1}}{(P^j)^2}
\end{aligned}$$

$$\text{(Optimal } v\text{):} \quad \nu^y\theta\frac{y^j}{v^j} = \nu^k\delta'(v^j)K^j$$

$$\text{(Optimal } V\text{):} \quad (1-\tau^d)\kappa = q\sum_{s,l,e}\nu^n(s,l,e)\frac{U(s,e)\pi_L^{unemp}(l)+\lambda_n(e)N(s,l,e)}{\sum_{\hat{s},\hat{e}}\left[U(\hat{s},\hat{e})+\lambda_n(\hat{e})\sum_{\hat{l}}N(\hat{s},\hat{l},\hat{e})\right]}$$

$$\begin{aligned}
\text{(Optimal } I\text{):} \quad 0 = {}& -(1-\tau^d)+\nu^k\left(\zeta_I\left(1-\Gamma(i^j/i^j_{-1})\right)-\zeta_I\Gamma'(i^j/i^j_{-1})i^j/i^j_{-1}\right)\\
&+\mathbb{E}_\zeta Q\nu^k_{+1}\zeta_I\Gamma'(i^j_{+1}/i^j)\left(i^j_{+1}/i^j\right)^2
\end{aligned}$$

$$\begin{aligned}
\text{(Optimal } N\text{):} \quad & \forall(s,l,e)\ \nu^n(s,l,e) = (1-\tau^d)es(1-\rho l)w + \nu^y(1-\theta)\frac{y^j es(1-\rho l)}{\sum_{s,l,e}es(1-\rho l)N^j(s,l,e)}\\
&+\mathbb{E}_\zeta Q\sum_{s\in\mathcal{S}_+}\sum_l \pi_S(s,s_{+1}|s_{+1}\neq s_0)\pi_L^{emp}(l,l_{+1})\big(1-\lambda_x(e)-\lambda_n(e)\big)\nu^n_{+1}(s_{+1},l_{+1},e)
\end{aligned}$$

$$\text{(Optimal } K\text{):} \quad \nu^k = \mathbb{E}_\zeta\nu^y_{+1}\theta\frac{y^j_{+1}}{(K^j)'} + \mathbb{E}_\zeta\nu^k_{+1}\big(1-\delta(v^j_{+1})\big)$$

$$\text{(Production):} \quad \left(\frac{P^j}{P}\right)^{-\vartheta exp(\zeta_P)}y = \zeta_{TFP}\left(K^jv^j\right)^\theta\left(\sum_{s,l,e}es(1-\rho l)N^j(s,l,e)\right)^{1-\theta}$$

$$\text{(LoM } K\text{):} \quad (K^j)' = \big(1-\delta(v^j)\big)K^j + \zeta_I\left(1-\Gamma(i^j/i^j_{-1})\right)i^j$$

$$\begin{aligned}
\text{(LoM } N\text{):} \quad & \sum_{s_{-1}\in\mathcal{S}_+}\sum_{l_{-1}}\pi_S(s_{-1},s|s\neq s_0)\pi_L^{emp}(l_{-1},l)\big(1-\lambda_x(e)-\lambda_n(e)\big)N^j_{-1}(s_{-1},l_{-1},e)\\
&+qV^j\frac{U(s,e)\pi_L^{unemp}(l)+\lambda_n(e)N(s,l,e)}{\sum_{\hat{s},\hat{e}}\left[U(\hat{s},\hat{e})+\lambda_n(\hat{e})\sum_{\hat{l}}N(\hat{s},\hat{l},\hat{e})\right]} = N^j(s,l,e)\ \forall(s,l,e).
\end{aligned}$$

We focus on a symmetric equilibrium where all firms had the same initial conditions and where the initial distribution of skills equals the ergodic distribution. Given a unique solution to the above equations this also implies that all firms make the same choices. Imposing the definition of inflation, and $U(s,e)=\pi_S(s|s\in\mathcal{S}_+)U(e)$, $N(s,l,e)=\pi_S(s|s\in\mathcal{S}_+)N(l,e)$ we arrive at the system of equations below.

$$\begin{aligned}
\text{(Optimal } P\text{):} \quad 0 = {}& (1-\tau^d)\left((1-\vartheta exp(\zeta_P))y - \psi\left(\Pi-\bar{\Pi}\right)y\Pi\right)\\
&+\vartheta exp(\zeta_P)y\nu^y + (1-\tau^d)\mathbb{E}_\zeta Q\psi\left(\Pi_{+1}-\bar{\Pi}\right)y_{+1}\Pi_{+1}
\end{aligned}$$

$$\text{(Optimal } v\text{):} \quad \nu^y\theta\frac{y}{v} = \nu^k\delta'(v)K$$

$$\text{(Optimal } V\text{):} \quad (1-\tau^d)\kappa = q\sum_{s,l,e}\nu^n(s,l,e)\frac{\pi_S(s|s\in\mathcal{S}_+)U(e)\pi_L^{unemp}(l)+\lambda_n(e)\pi_S(s|s\in\mathcal{S}_+)N(l,e)}{\sum_{\hat{s},\hat{e}}\left[\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)U(\hat{e})+\lambda_n(\hat{e})\sum_{\hat{l}}\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)N(\hat{l},\hat{e})\right]}$$

$$\begin{aligned}
\text{(Optimal } I\text{):} \quad 0 = {}& -(1-\tau^d)+\nu^k\left(\zeta_I\left(1-\Gamma(i/i_{-1})\right)-\zeta_I\Gamma'(i/i_{-1})i/i_{-1}\right)\\
&+\mathbb{E}_\zeta Q\nu^k_{+1}\zeta_I\Gamma'(i_{+1}/i)\left(i_{+1}/i\right)^2
\end{aligned}$$

[A4] The superscript j still allows us to distinguish between variables at the firm and aggregate levels.

(Optimal N): $$\forall(s,l,e)\ \nu^n(s,l,e)=(1-\tau^d)es(1-\rho l)w+\nu^y(1-\theta)\frac{yes(1-\rho l)}{\pi_S(s|s\in\mathcal{S}_+)N(l,e)}$$
$$+\mathbb{E}_\zeta Q\sum_{s\in\mathcal{S}_+}\sum_l \pi_S(s,s_{+1}|s_{+1}\neq s_0)\pi_L^{emp}(l,l_{+1})\big(1-\lambda_x(e)-\lambda_n(e)\big)\nu^n_{+1}(s_{+1},l_{+1},e)$$

(Optimal K): $$\nu^k=\mathbb{E}_\zeta\nu^y_{+1}\theta\frac{y_{+1}}{K'}+\mathbb{E}_\zeta\nu^k_{+1}\big(1-\delta(v_{+1})\big)$$

(Production): $$y=\zeta_{TFP}\,(Kv)^\theta\left(\sum_{s,l,e}es(1-\rho l)\pi_S(s|s\in\mathcal{S}_+)N(l,e)\right)^{1-\theta}$$

(LoM K): $$K'=\big(1-\delta(v)\big)K+\zeta_I\,(1-\Gamma(i/i_{-1}))\,i$$

(LoM N): $$\sum_{s_{-1}\in\mathcal{S}_+}\sum_{l_{-1}}\pi_S(s_{-1},s|s\neq s_0)\pi_L^{emp}(l_{-1},l)\big(1-\lambda_x(e)-\lambda_n(e)\big)\pi_S(s_{-1}|s_{-1}\in\mathcal{S}_+)N_{-1}(l_{-1},e)$$
$$+qV^j\frac{\pi_S(s|s\in\mathcal{S}_+)U(e)\pi_L^{unemp}(l)+\lambda_n(e)\pi_S(s|s\in\mathcal{S}_+)N(l,e)}{\sum_{\hat{s},\hat{e}}\big[\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)U(\hat{e})+\lambda_n(\hat{e})\sum_{\hat{l}}\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)N(\hat{l},\hat{e})\big]}=\pi_S(s|s\in\mathcal{S}_+)N(l,e)\ \forall(s,l,e).$$

We are now in a position to compare the equations here to the ones for the model description in the main text. Obviously, comparing the last two equations, labeled (LoM K) and (LoM N), to the main text we see that the laws of motion for capital and various employment groups are the same. Given the market clearing conditions for labor and capital services, we also see that the expressions for the production functions are the same between the main text and the model in this appendix. Therefore, it remains to compare the optimality conditions. To simplify this we define the following new variables: $\hat{J}(s,l,e):=\nu^n(s,l,e)$, $\hat{r}=\frac{\theta y}{Kv(1-\tau^d)}$, $\hat{h}=\frac{(1-\theta)y\nu^y}{(1-\tau^d)\sum_{s,l,e}es(1-\rho l)\pi_S(s|s\in\mathcal{S}_+)N(l,e)}$. Substituting them into our optimality conditions we arrive at the expressions below:

(Optimal P): $$0=(1-\tau^d)\left((1-\vartheta exp(\zeta_P))y-\psi\left(\Pi-\bar{\Pi}\right)y\Pi\right)$$
$$+\vartheta exp(\zeta_P)y\nu^y+(1-\tau^d)\mathbb{E}_\zeta Q\psi\left(\Pi_{+1}-\bar{\Pi}\right)y_{+1}\Pi_{+1}$$

(Optimal v): $$\nu^y\hat{r}(1-\tau^d)K=\nu^k\delta'(v)K$$

(Optimal V): $$(1-\tau^d)\kappa=q\sum_{s,l,e}\hat{J}(s,l,e)\frac{\pi_S(s|s\in\mathcal{S}_+)U(e)\pi_L^{unemp}(l)+\lambda_n(e)\pi_S(s|s\in\mathcal{S}_+)N(l,e)}{\sum_{\hat{s},\hat{e}}\big[\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)U(\hat{e})+\lambda_n(\hat{e})\sum_{\hat{l}}\pi_S(\hat{s}|\hat{s}\in\mathcal{S}_+)N(\hat{l},\hat{e})\big]}$$

(Optimal I): $$0=-(1-\tau^d)+\nu^k\left(\zeta_I\left(1-\Gamma(i/i_{-1})\right)-\zeta_I\Gamma'(i/i_{-1})i/i_{-1}\right)$$
$$+\mathbb{E}_\zeta Q\nu^k_{+1}\zeta_I\Gamma'(i_{+1}/i)\left(i_{+1}/i\right)^2$$

(Optimal N): $$\forall(s,l,e)\ \hat{J}(s,l,e)=es(1-\rho l)(\hat{h}-w)(1-\tau^d)$$
$$+\mathbb{E}_\zeta Q\sum_{s\in\mathcal{S}_+}\sum_l\pi_S(s,s_{+1}|s_{+1}\neq s_0)\pi_L^{emp}(l,l_{+1})\big(1-\lambda_x(e)-\lambda_n(e)\big)\hat{J}_{+1}(s_{+1},l_{+1},e)$$

(Optimal K): $$\nu^k=\mathbb{E}_\zeta\nu^y_{+1}\hat{r}_{+1}v_{+1}(1-\tau^d)+\mathbb{E}_\zeta\nu^k_{+1}\big(1-\delta(v_{+1})\big).$$

Now, we can compare (Optimal N) and (Optimal V) to the definition of the value of a match and the free-entry condition for labor service producers in the main text and see that they are identical once we equate μs and νs and hatted variables with their un-hatted cousins. We can conclude their equivalence. The same applies if we compare the remaining optimality conditions here with the ones for the differentiated goods producers and capital services producers in Appendix B. So we can conclude that the equations describing the firm behavior conditional on a discount factor Q are the same and imply the same allocations, proving the equivalence of the two setups in terms of aggregates.

Appendix B: The firms' optimization problems

This appendix collects the equations characterizing the solution to the optimization problem of all the firms in the model, after taking first-order conditions, applying envelope conditions, and simplifying. We describe the resulting equations collected under the respective firm's name. We suppress the dependence on X to keep the notation simple.

B.1. Final goods and adjustment services

Since all differentiated goods producers in equilibrium will set the same price, final goods firms and adjustment services firms have isomorphic demand functions. So $y = y_f + y_a$.

B.2. Differentiated goods producers

The problem of the differentiated goods producer is characterized by the following set of equations, in which μ^y is the multiplier on the production function.

First, there is the optimality condition for inflation, where $\Pi = \frac{P}{P_{-1}}$:

$$\psi\Pi\left(\Pi - \bar{\Pi}\right) = \psi\mathbb{E}_\zeta\left[Q(X,X')\Pi'\left(\Pi' - \bar{\Pi}\right)y'/y\right] + [\vartheta\exp\{\zeta_P\}mc - (\vartheta\exp\{\zeta_P\} - 1)].$$

Where the optimality conditions for inputs imply that marginal costs, mc, are given by

$$mc = \left(\frac{1}{\theta}\right)^\theta\left(\frac{1}{1-\theta}\right)^{1-\theta}\frac{r^\theta h^{1-\theta}}{\zeta_{TFP}}.$$

The optimality conditions for capital and labor services input imply

$$\frac{\theta}{1-\theta}\frac{\ell}{k} = \frac{r}{h}.$$

Finally, there is the production function:

$$y = \zeta_{TFP}\,(k)^\theta\,(\ell)^{1-\theta}.$$

B.3. Labor services producers

The problem of employment agencies does not involve any decision beyond the one contained in the free-entry condition. Therefore, the relevant equations are already given in the main text. When solving the model numerically, we make use of its structure to simplify. The only decision of the employment agency that is influenced by the value of various types of matches is the vacancy posting condition. Job-finding and separation rates do not depend on a household's idiosyncratic skills s. Therefore, the share of households of skill s in each education-employment status subgroup follows the constant ergodic distribution of skills. For the free-entry condition, it is, therefore, enough to track the expected value of J_L with respect to s. Define $\widetilde{J}_L(X,l,e) = \sum_{s\in\mathcal{S}_+}\pi_S(s|s\in\mathcal{S}_+)J_L(X,l,e,s)$. Using that $\sum_{s\in\mathcal{S}_+}\pi_S s(s|s\in\mathcal{S}_+) = 1$ (the calibration assumption that average skills are equal to 1) and using that $\pi_S(s|s\in\mathcal{S}_+)$ is ergodic, we obtain that

$$\begin{aligned}\widetilde{J}_L(X,l,e) \quad = \quad & (1-\tau_d)[h(X) - w(X)]\cdot e(1-\varrho l) \\ & + \textstyle\sum_{l'}\pi_L^{emp}(l,l')\cdot\mathbb{E}_\zeta\left[Q(X,X')\big(1-\lambda_x(e)-\lambda_n(e)\big)\widetilde{J}_L(X,l',e)\right].\end{aligned}$$

With this, one can re-write the free-entry condition as

$$\begin{aligned}&\textstyle\sum_e\sum_l\left[U(\widetilde{X},e)\pi_L^{uemp}(l) + \lambda_n(e)N(\widetilde{X},l,e)\right]\widetilde{J}_L(X,l,e) \\ &\quad\cdot\left[\textstyle\sum_e[U(\widetilde{X},e) + \lambda_n(e)\sum_l N(\widetilde{X},l,e)]\right]^{-1} = (1-\tau_d)\kappa(\widetilde{X})/q(\widetilde{X}).\end{aligned}$$

B.4. Capital services producers

Denoting the Lagrange multiplier on the law of motion for capital by μ_k we obtain three optimality conditions for the producer of capital services . First, there is the intra-temporal condition for utilization:

$$(1-\tau^d)r = \mu_k\delta_1\delta_2 v^{\delta_2-1}.$$

Second, we get an Euler equation for investment:

$$(1-\tau^d) = \mu_k \zeta_I \left[1 - \frac{\phi_K}{2}\left(\frac{i}{i_{-1}} - 1\right)^2 - \phi_K \left(\frac{i}{i_{-1}} - 1\right)\frac{i}{i_{-1}}\right] + \mathbb{E}_\zeta \left[Q(X,X')\mu_k' \zeta_I' \phi_K \left(\frac{i'}{i} - 1\right)(i'/i')^2\right].$$

Third, we get an optimality condition for capital:

$$\mu_k = (1-\tau^d)\mathbb{E}_\zeta \left[Q(X,X') r' v'\right] + \mathbb{E}_\zeta \left[Q(X,X')\mu_k' \big(1 - \delta_1 (v')^{\delta_2}\big)\right].$$

For completeness we repeat the law of motion for capital:

$$K' = \big(1 - \delta_1 (v_t)^{\delta_2}\big)K + \zeta_I \left(1 - \frac{\phi_K}{2}\left(\frac{i}{i_{-1}} - 1\right)^2\right) i.$$

Appendix C: Discount factor

In this appendix, we describe the modeling of the mutual funds' stochastic discount factor Q. The discount factor governs how the mutual funds evaluate profits across time and states of nature. If asset markets were complete, or if there would be a representative household, there would be a unique stochastic discount factor. In our model, instead, asset markets are incomplete. There are, therefore, several possible choices for the discount factor, and different households may not agree on the dynamic decisions that the firms should make.[A5] Different choices of the discount factor could have different implications for the behavior of the model.[A6] That said, one restriction on the discount factor appears to be natural and easy to implement. The starting point is that for all households that hold shares in the mutual fund, the consumption Euler equation takes the form[A7]

$$\mathbb{E}_{X',S'|X,S}\left[\beta(S')\frac{u'(c(X',S'))}{u'(c(X,S))}\left(p_a(X')+d_a(X')\right)\right]=p_a(X).$$

Here S summarizes the idiosyncratic states of the household and X denotes the aggregate state as in the main text. The expectation operator $\mathbb{E}$ incorporates both the idiosyncratic and aggregate transition probabilities. $\beta(.)$ and $c(.)$ are the household's time discount factor and the household's consumption function, respectively. This equation says that in equilibrium all households holding shares have to agree on the valuation of the mutual fund. Therefore, there is a discount factor Q such that

$$\mathbb{E}_{X'|X}\left[Q(X,X')\left(p_a(X')+d_a(X')\right)\right]=p_a(X).$$

Clearly, different discount factors fulfill the restriction, including any weighted average (with non-negative weights) of individual equity holders' discount factors. Evaluating the discount factor Q starting from a set of weights of households is computationally burdensome, however.[A8] Rather, we use the fact that the asset price is determined by asset-market clearing. Therefore, in equilibrium, the price incorporates time preferences and risk premiums. We postulate that the individual mutual fund i observes the pricing functions $p(X)$, $p(X')$ that the market applies to the *other* (representative) mutual funds' cash flows. Further, mutual fund i observes the dividend policies $d_a(X)$ of the other (representative) mutual funds. A mutual

[A5] Even under incomplete markets there can be cases in which a natural candidate for the discount factor emerges. In Carceles-Poveda and Coen-Pirani (2010) and Krusell et al. (2010), while households face idiosyncratic risk and incomplete asset markets, the asset structure is rich enough to generate a unique choice of discount factor along the dimensions relevant for the firms' choices. Other papers with incomplete markets and idiosyncratic risk do not need to specify the firm's discount factor in the first place. In Krusell and Smith (1998) the only long-lived asset is capital. Households invest directly in capital. Firms are competitive and only have static rental decisions to make. A similar argument applies in den Haan et al. (2017), where firms make decisions once and for all. Neither paper has sticky prices (and so dynamic decisions by firms). See also Carceles-Poveda and Coen-Pirani (2010) for results regarding investor unanimity with heterogeneous households.

[A6] Had we solved our model using a first-order approximation or along a transition path with perfect foresight, instead, it would have been enough to specify the dynamics of the real interest rate, which then could have been uniquely derived from the households' Euler equation.

[A7] To keep the notation simple we surpress the role of the bequest term here. The same argument applies once we add the additional marginal utility that old agents receive from the value of bequest.

[A8] In an earlier draft of the current paper we used the asset-weighted mean of households' marginal utilities to define the discount factor. Implementing this discount factor in our perturbation solution would require tracking more policy functions, which would have increased the numerical burden substantially. We have verified in a simpler version of our model that up to second-order dynamics and long-run properties were comparable in both approaches.

fund i then applies to its own cash flows the market discount factor

$$Q_i(X, X') = \frac{p_a(X)}{p_a(X') + d_a(X')}.$$

To repeat, the variables entering Q_i are the other funds' equity price and the other funds' dividend policies. In equilibrium, due to symmetry, all mutual funds then apply the same discount factor $Q(X, X') = Q_i(X, X')$, and that discount factor is consistent with households' decisions.[A9]

[A9] As a robustness check, we have also solved the model using the risk-free rate $R^f(X)$ defined as

$$\frac{1}{R^f(X)} \mathbb{E}_{X'|X} \left[\left(p_a(X') + d_a(X') \right) \right] = p_a(X)$$

to discount instead of using Q. Dynamics and long-run moments were almost indistinguishable from the results reported in the main text (detailed results available upon request). This may not be surprising. Up to first order, any discount factor fulfilling the above conditions would lead to the same real interest rate dynamics. Even higher-order approximations would not be expected to change much, since the modeling assumptions we make will not generate large risk premia, or significant dynamics in these, that could drive a wedge between the risk free-rate and other reasonable discount factors.

Appendix D: Definition of equilibrium

This appendix spells out the full definition of a recursive equilibrium in our setting.

Definition (Recursive Equilibrium). A recursive equilibrium is
a set of value functions $W(.,n,a,l,e,b,s)$, $J_K(.)$, $J_D(.;j)$, $J_L(.,l,e,s)$, a set of private-sector policy functions $c(.,n,a,l,e,b,s)$, $a(.,n,a,l,e,b,s)$, y_f, $y_{f,j}(.)$, $y_a(.)$, $y_{a,j}(.)$, $d_a(.)$, $v(.)$, $i(.)$, $K(.)$, $l_j(.)$, $k_j(.)$, $V(.)$, $P_j(.)$, $y(.)$, a set of prices and discount factors $w(.)$, $p_a(.)$, $h(.)$, $r(.)$, $Q(.,.)$, $P(.)$, $\Pi(.)$, a set of labor-market variables $Pr(.|.,e)$, $f(.)$, $N(.,l,e)$, $U(.,e)$, $q(.)$, $N(.)$, $U(.)$, $\kappa(.)$, a set of government policies $\tau(.,.)$, $R(.)$, and a set of transition functions $T(.)$, $\widetilde{T}$, such that $X = \widetilde{T}(\widetilde{X})$ and $\widetilde{X}' = T(X)$, for all aggregate states X, $\widetilde{X}$ idiosyncratic states n,a,l,e,b,s, and firm indexes j such that

1. (Households' problems) given asset price $p_a(.)$, dividends $d_a(.)$, wage $w(.)$, job-finding rate $f(.)$, taxes $\tau(.,.)$, transition probabilities $Pr(.|.,e)$, and transition functions $T(.)$, and $\widetilde{T}(.)$, the value functions $W(.,n,a,l,e,b,s)$ solve the households' Bellman equations in Section 2.3, and $c(.,n,a,l,e,b,s)$ and $a(.,n,a,l,e,b,s)$ are the resulting optimal policy functions for consumption and assets;
2. (Final goods) given $P(.)$ and $P_j(.)$, policy functions $y_f(.)$ and $y_{f,j}(.)$ solve the problem of the final goods producers in Section 2.4.1;
3. (Differentiated goods) given demand function $y_j(.)$, and given prices $r(.)$, $h(.)$, $P(.)$, discount factor $Q(.,.)$, and transition functions $T(.)$ and $\widetilde{T}(.)$, $J_D(.;j)$ solves the differentiated goods producers' Bellman equation given in Section 2.4.2, and $k_j(.)$, $l_j(.)$, and $P_j(.)$ are the corresponding optimal policy functions;
4. (Labor services) given prices $h(.)$, wage $w(.)$, discount factor $Q(.,.)$, unemployment $U(.,e)$, employment $N(.,l,e)$ and transition functions $T(.)$, and $\widetilde{T}(.)$, $J_L(.,l,e,s)$ solves the employment agencies' valuation equation in Section 2.4.2. ; given $J_L(.,l,e,s)$, $U(.,e)$, $N(.,l,e)$, and $\kappa(.)$, $q(.)$, solves the free-entry condition in the same section; given $M(.,.)$ and $q(.)$, $V(.)$ conforms with the definition of the job-filling rate in the same section; given $M(.,.)$ and $q(.)$, vacancy posting costs $\kappa(.)$ follow the form given in the same section; given $V(.)$, $U(.,e)$, and $N(.,l,e)$, $M(.,V(.))$ is given by the matching function spelled out in the same section; given $V(.)$, $M(.,V(.))$, $U(.,e)$ and $N(.,l,e)$, $f(.)$, the job-finding rate $f(.)$ is as defined in Section 2.4.2.
5. (Capital services) given rental rate $r(.)$, discount factor $Q(.,.)$, and transition functions $T(.)$ and $\widetilde{T}(.)$, $J_K(.,K,i)$ solves the Bellman equation of the representative producer of capital in Section 2.4.2, and $i(.)$, $K(.)$ and $v(.)$ are the resulting optimal policy functions for investment, capital, and utilization, respectively;
6. (Adjustment services goods) given $P(.)$ and $P_j(.)$, policy functions $y_a(.)$ and $y_{a,j}(.)$ solve the problem of the adjustment-services producers in Section 2.4.2;
7. (Financial firms) given interest rates $R(X)$, inflation $\Pi(X)$, and transition function $T(.)$ and $\widetilde{T}(.)$, the discount factor satisfies the Euler equation in Section 2.5; given final output $y_f(.)$, investment policy $i(.)$, wage $w(.)$, dividends are as described in the same section; for given dividends $d_a(X)$ and the discount factor $Q(.,.)$, the asset price $p_a(.)$ is consistent with the definition of the discount factor in the same section.
8. (Central bank) given inflation $\Pi(.)$ and unemployment $U(.)$, the interest rate $R(.)$ follows the Taylor rule given in Section 2.6.
9. (Fiscal authority) given dividends $d_a(.)$, consumption policies $c(.,n,a,l,e,b,s)$, and wage $w(.)$, $\tau(.,.)$ balances the government budget in every period (Section 2.6);.
10. (Wage) given $y(.)$, the wage follows the wage rule spelled out in Section 2.4.2;
11. (Birth) $Pr(.|.,e)$ is consistent with the flows into retirement and new birth described in Section 2.3.3;

12. (Consistency, demand function) $y_j(.) = y_{f,j}(.) + y_{a,j}(.)$ is the demand for good j.
13. (Symmetry) for all j, $P_j(.) = P(.)$, $y_{f,j}(.) = y_f(.)$, $y_{a,j}(.) = y_a(.)$, and $y_j(.) = y(.)$.
14. (Market clearing, final goods)
$$y_f(.) = \int_{\mathcal{M}} c(., n, a, l, e, b, s) d\mu(.) + i(.) + g;$$
15. (Market clearing, adjustment services goods)
$$y_a(.) = \frac{\psi}{2}\left(\Pi(.) - \bar{\Pi}\right)^2 y(.) + \kappa(.)V(.);$$
16. (Market clearing, differentiated goods) $y(.) = y_f(.) + y_a(.)$.
17. (Market clearing, capital) $\int_0^1 k_j(.)dj = K_{-1}(.)v(.)$;
18. (Market clearing, labor services) $\int_{\mathcal{M}} se(1 - \varrho l)1_{n=1} d\mu = \int_0^1 l_j dj$;
19. (Market clearing, shares) $\int_{\mathcal{M}} a(X, n, a, l, e, s) dmu = 1$;
20. (Consistency, capital flow)
$$K(.) = [1 - \delta_1 v(.)^{\delta_2}] \cdot K_{-1}(.) + \zeta_I [1 - \phi_K/2\left(\frac{i(.)}{i_{-1}(.)} - 1\right)^2] i(.)$$
21. (Consistency, employment flow) Employment flows have to be consistent with the evolution of μ and $\widetilde{\mu}$ and the respective definition of the employment aggregates, $N, N(., l, e)$, $U(., e)$, U.
22. (Consistency, aggregate state transition) $T, \widetilde{T}$ are consistent with $K_{-1}(T(X)) = K(X)$, $w_{-1}(T(X)) = w(X)$, $i_{-1}(T(X)) = i(X)$, $R_{-1}(T(X)) = R(X)$ and $K_{-1}(\widetilde{T}(\widetilde{X})) = K_{-1}(\widetilde{X})$, $w_{-1}(\widetilde{T}(\widetilde{X})) = w_{-1}(\widetilde{X})$, $i_{-1}(\widetilde{T}(\widetilde{X})) = i_{-1}(\widetilde{X})$, $R_{-1}(\widetilde{T}(\widetilde{X})) = R_{-1}(\widetilde{X})$.
23. $T, \widetilde{T}$ are consistent with the law of motion for the distribution (described in Section D.1).

D.1. Law of motion of distributions μ, $\widetilde{\mu}$

It remains to state the law of motion for μ and $\widetilde{\mu}$. Let A be a measurable subset of $[0, 1]$, the set of feasible asset holdings. We need to describe the updating of $\widetilde{\mu}$ to μ and of μ to $\widetilde{\mu}'$ for all feasible combinations of (n, A, l, e, b, s).

D.1.1. Transitions from $\widetilde{\mu}$ to μ

We start with the transition from $\widetilde{\mu}$ to μ, that is from the beginning of the period (after shocks to the exogenous idiosyncratic states (e, b, s) and the aggregate states have been realized, but before employment transitions of working-age households have occurred and before earnings-loss transitions have materialized) to the end of the period (the production stage).

The retired can neither lose nor find a job, and the earnings-loss state does not matter for their income. Therefore, for $s = s_0$ we have $\mu(n, A, l, e, b, s_0) = \widetilde{\mu}(n, A, l, e, b, s_0)$. For $s \in \mathcal{S}_+$ (working-age households, including those that have just been reborn), we have for $n = 1$ at the production stage
$$\begin{aligned}\mu(1, A, l, e, b, s) \quad &= \sum_{\hat{l}\in\{0,1\}} \left(1 - \lambda_x(e) - \lambda_n(e)(1 - f(\widetilde{X}))\right)\pi_L^{emp}(\hat{l}, l)\widetilde{\mu}(1, A, \hat{l}, e, b, s) \\ &+ \sum_{\hat{l}\in\{0,1\}} f(\widetilde{X})\pi_L^{uem}(l)\widetilde{\mu}(0, A, 0, e, b, s),\end{aligned}$$
and for $n = 0$ at the production stage
$$\begin{aligned}\mu(0, A, 0, e, b, s) \quad &= \sum_{\hat{l}\in\{0,1\}} \left(\lambda_x(e) + \lambda_n(e)(1 - f(\widetilde{X}))\right)\widetilde{\mu}(1, A, \hat{l}, e, b, s) \\ &+ \left(1 - f(\widetilde{X})\right)\widetilde{\mu}(0, A, 0, e, b, s).\end{aligned}$$

D.1.2. Transitions from μ to $\widetilde{\mu}'$

We now turn to the transition from μ to $\widetilde{\mu}'$. We start with households that end up in the labor force next period. For $s \in \mathcal{S}_+$, we have

$$\begin{aligned}\widetilde{\mu}'(n, A, l, e, b, s) \quad = \quad & \sum_{\hat{s}\in\mathcal{S}_+} \pi_S(\hat{s}, s) \int_{\hat{a}:a(X,1,\hat{a},l,e,b,\hat{s})\in A} d\mu(X, n, \hat{a}, l, e, b, \hat{s}) \\ & +\pi_S(s_0, s)\mathrm{Pr}(n, l|X, e) \sum_{\hat{e}} \sum_{\hat{b}} \pi_E(\hat{e}, e)\pi_{\Delta\beta}(b) \\ & \cdot \int_{\hat{a}:a(X,0,\hat{a},l,\hat{e},\hat{b},0)\in A} d\mu(X, 0, \hat{a}, l, \hat{e}, \hat{b}, 0).\end{aligned}$$

For transitions into old age, the following rule applies:

$$\begin{aligned}\widetilde{\mu}(\widetilde{X}', 0, A, 0, e, b, s_0)' \quad = \quad & \sum_l \sum_n \sum_{\hat{s}\in\mathcal{S}_+} \pi_S(\hat{s}, s_0) \\ & \cdot \int_{\hat{a}:a(X,n,\hat{a},l,e,b,\hat{s})\in A} d\mu(X, n, \hat{a}, l, e, b, \hat{s}) \\ & +\pi_S(s_0, s_0) \int_{\hat{a}:a(X,0,\hat{a},0,e,b,s_0)\in A} d\mu(X, 0, \hat{a}, 0, e, b, s_0).\end{aligned}$$

Appendix E: TANK model variant

This appendix spells out the TANK model variant. There are two groups of households. A mass π^{saver} of the population are savers. Savers have access to the mutual fund. Savers all have the same discount factor, β^{saver}. Savers live in a family that pools all incomes of its members. Thus, although savers' incomes depend on education and fluctuate with employment and retirement, their consumption is not exposed to idiosyncratic income risk or to their life-cycle income profile. The remaining households are excluded from asset markets (the spenders). Spenders have discount factor $\beta^{spend} < \beta^{saver}$. Spender households' incomes directly translate into their consumption. Incomes differ by education, skill-loss, employment status, and retirement status. For both savers and spenders, we abstract from fluctuations of skills s during working age. We also abstract from a bequest motive. The education status is assumed to be permanent. In order to simplify notation, we no longer explicitly highlight the dependence on aggregate state variables. Rather, a subscript t is used to index dependence on the period t state of the economy.

E.1. Spenders

Spenders have either permanently low or permanently high education. Spenders in each education group can be in one of four idiosyncratic states: they can be unemployed, employed with an earnings loss, employed without an earnings loss, or retired. In order to preserve on notation, here we discuss incomes and welfare of low-educated spenders only. The formulae for high-education spenders are analogous. In the following, $\pi_{RET} = \pi_{s_0}$ marks the probability of retiring. π_{born} marks the probability of leaving retirement.

E.1.1. Spenders' consumption

When employed, the spender consumes $c^L_{0,t}$ if it does not suffer an earnings loss and $c^L_{1,t}$ if it does. Superscript L marks low education. Without an earnings loss, the spender consumes:

$$c^{L,spend}_{0,t}(1+\tau_c) = w_t e_L[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_L)],$$

where $\tau_t(\cdot)$ is the progressive income tax function. With an earnings loss, the spender consumes:

$$c^{L,spend}_{1,t}(1+\tau_c) = w_t e_L(1-\varrho)[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_L(1-\varrho))].$$

When unemployed, the spender consumes $c^L_{U,t}$, where the U marks the unemployment state:

$$c^{L,spend}_{U,t}(1+\tau_c) = b_{UI}(e_L)[1-\tau_t(b_{UI}(e_L))].$$

Last, when retired, the spender household consumes

$$c^{L,spend}_{R,t}(1+\tau_c) = b_{RET}(e_L)[1-\tau(b_{RET}(e_L))].$$

E.1.2. Welfare of spenders

With this, and the labor-market transitions, the welfare of the spender household is as follows. If employed, without an earnings loss

$$\begin{aligned} W^{L,spend}_{0,t} \quad = \quad & (1-\beta^{spend})\frac{(c^{L,spend}_{0,t})^{1-\sigma}}{1-\sigma} \\ & +\beta^{spend}\pi_{RET}\mathbb{E}_t\{W^{L,spend}_{R,t+1}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{(1-\lambda^L_x-\lambda^L_e(1-f_{t+1}))W^{L,spend}_{0,t+1}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{(\lambda^L_x+\lambda^L_e(1-f_{t+1}))W^{L,spend}_{U,t+1}\}. \end{aligned}$$

If employed, with an earnings loss

$$\begin{aligned} W^{L,spend}_{1,t} \quad = \quad & (1-\beta^{spend})\frac{(c^{L,spend}_{1,t})^{1-\sigma}}{1-\sigma} \\ & +\beta^{spend}\pi_{RET}\mathbb{E}_t\{W^{L,spend}_{R,t+1}\} \\ & +\beta^{spend}(1-\pi_{RET})\pi^{emp}_L(1,0)\mathbb{E}_t\{(1-\lambda^L_x-\lambda^L_e(1-f_{t+1}))W^{L,spend}_{0,t+1}\} \\ & +\beta^{spend}(1-\pi_{RET})\pi^{emp}_L(1,1)\mathbb{E}_t\{(1-\lambda^L_x-\lambda^L_e(1-f_{t+1}))W^{L,spend}_{1,t+1}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{(\lambda^L_x+\lambda^L_e(1-f_{t+1}))W^{L,spend}_{U,t+1}\}. \end{aligned}$$

If unemployed

$$\begin{aligned} W_{U,t}^{L,spend} = & (1-\beta^{spend})\frac{(c_{U,t}^{L,spend})^{1-\sigma}}{1-\sigma} \\ & +\beta^{spend}\pi_{RET}\mathbb{E}_t\{W_{R,t+1}^{L,spend}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{f_{t+1}\pi_L^{uem}(0)W_{0,t+1}^{L,spend}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{f_{t+1}\pi_L^{uem}(1)W_{1,t+1}^{L,spend}\} \\ & +\beta^{spend}(1-\pi_{RET})\mathbb{E}_t\{(1-f_{t+1})W_{U,t+1}^{L,spend}\}. \end{aligned}$$

If retired

$$\begin{aligned} W_{R,t}^{L,spend} = & (1-\beta^{spend})\frac{(c_{R,t}^{L,spend})^{1-\sigma}}{1-\sigma} \\ & +\beta^{spend}(1-\pi_{born})\mathbb{E}_t\{W_{R,t+1}^{L,spend}\} \\ & +\beta^{spend}\pi_{born}\mathbb{E}_t\{(\Pr_{0,t}^L(1-\lambda_x^L-\lambda_e^L(1-f_{t+1}))+\Pr_{U,t}^L f_{t+1}\pi_L^{uem}(0))W_{0,t+1}^{L,spend}\} \\ & +\beta^{spend}\pi_{born}\mathbb{E}_t\{(\Pr_{1,t}^L(1-\lambda_x^L-\lambda_e^L(1-f_{t+1}))+\Pr_{U,t}^L f_{t+1}\pi_L^{uem}(1))W_{1,t+1}^{L,spend}\} \\ & +\beta^{spend}\pi_{born}\mathbb{E}_t\{[(\Pr_{0,t}^L+\Pr_{1,t}^b)(\lambda_x^L+\lambda_e^L(1-f_{t+1}))+\Pr_{U,t}^L(1-f_{t+1})]W_{U,t+1}^{L,spend}\}. \end{aligned}$$

Where the probabilities $\Pr^L$ of being reborn into the respective group are defined as

$$\Pr_{0,t}^L = (N_{0,t}^L + N_{1,t}^L\pi^{emp}(1,0))/(N_t^L+U_t^L),$$

$$\Pr_{1,t}^L = (N_{1,t}^L\pi^{emp}(1,1))/(N_t^L+U_t^L),$$

$$\Pr_{U,t}^L = U_t^L/(N_t^L+U_t^L).$$

The same relations above hold for the highly educated, replacing index L with H.

E.2. Savers

Savers are exposed to the same income risk as spenders. They are not exposed to idiosyncratic consumption risk, however. Rather, savers live in a representative family that encompasses all the different household types (low/high education; employed with/without earnings loss; unemployed; retired). The family pools the incomes of its members.
Saver families maximize expected lifetime utility

$$\max_{\{c_t,a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t(c_t^{saver})^{1-\sigma}/(1-\sigma)\right\}$$

subject to the family's budget constraint

$$\begin{aligned} (1+\tau_c)c_t^{saver}+p_{a,t}a_{t+1} = & (p_{a,t}+d_{a,t})a_t \\ & +\pi^{\text{saver}}U_t^L b_{UI}(e_L)[1-\tau_t(b_{UI}(e_L))] \\ & +\pi^{\text{saver}}U_t^H b_{UI}(e_H)[1-\tau_t(b_{UI}(e_H))] \\ & +\pi^{\text{saver}}N_{0,t}^L w_t e_L[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_L)] \\ & +\pi^{\text{saver}}N_{0,t}^H w_t e_H[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_H)] \\ & +\pi^{\text{saver}}N_{1,t}^L w_t e_L(1-\varrho)[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_L(1-\varrho))] \\ & +\pi^{\text{saver}}N_{1,t}^H w_t e_H(1-\varrho)[1-\tau_{RET}-\tau_{UI}-\tau_t(w_t e_H(1-\varrho))] \\ & +\pi^{\text{saver}}(1-\pi^{\text{labforce}})\pi_E(e_L)b_{RET}(e_L)[1-\tau_t(b_{RET}(e_L))] \\ & +\pi^{\text{saver}}(1-\pi^{\text{labforce}})\pi_E(e_H)b_{RET}(e_H)[1-\tau_t(b_{RET}(e_H))], \end{aligned}$$

where $\pi^{\text{labforce}}(=\pi_S(\mathcal{S}_+))$ is the share of households in the labor force. The exposition above assumes that savers trade shares in the mutual fund. We make this assumption so as to keep the exposition close to the heterogeneous-household's baseline. We could as well have had each saver family decide directly over a portfolio of non-financial firms.

E.3. Financial firms

The firm side of the TANK variant is identical to the heterogeneous-agent version, with the exception that only saver families own shares in the representative mutual fund. The mutual funds trade the equity of all firms. Being owned by savers only, the mutual funds' discount factor is $Q_{t,t+1} = \beta^{saver}(c_{t+1}^{saver}/c_t^{saver})^{-\sigma}$. The central bank steers the inter-fund interest rate, R_t, resulting in the consumption Euler equation $1 = \mathbb{E}_t\{Q_{t,t+1}R_t/\Pi_{t+1}\}$. The mutual fund distributes to shareholders all income that it does not reinvest or use for paying adjustment costs. After-tax dividends are given by

$$\begin{aligned} d_{a,t} \quad = \quad (1-\tau_d)\Big(\quad & y_{f,t} - i_t \\ & -N_{0,t}^L w_t e_L \\ & -N_{0,t}^H w_t e_H \\ & -N_{1,t}^L w_t e_L(1-\varrho) \\ & -N_{1,t}^H w_t e_H(1-\varrho)\Big) \end{aligned}$$

E.4. Non-financial firms

Non-financial firms are identical to the firms in the heterogeneous-agent model.

E.4.1. Final goods

There is a representative competitive final-goods firm that transforms differentiated intermediate goods into homogeneous final goods. Final goods can be used for personal consumption expenditures, government consumption, and physical investment. The firm solves

$$\max_{y_{f,t},(y_{f,j,t})_{j\in[0,1]}} (1-\tau_d)\left(P_t y_{f,t} - \int_0^1 P_{j,t} y_{f,j,t} dj\right) \text{ s.t. } y_{f,t} = \left(\int_0^1 y_{f,j,t}^{\frac{\vartheta\exp\{\zeta_{P,t}\}-1}{\vartheta\exp\{\zeta_{P,t}\}}} dj\right)^{\frac{\vartheta\exp\{\zeta_{P,t}\}}{\vartheta\exp\{\zeta_{P,t}\}-1}},$$

where $\vartheta > 1$. $y_{f,t}$ marks the output of final goods. $P_{j,t}$ marks the price of differentiated input j and $y_{f,j,t}$ denotes the quantity demanded of that input by final-goods firms. P_t is the consumer price index.

E.4.2. Intermediate inputs

Next to final goods, there are intermediate inputs, as in the heterogeneous-household model.

Differentiated goods producers. There is a unit mass of producers of differentiated goods. Producer $j \in [0,1]$ solves

$$\begin{aligned} \max_{\{P_{j,t},\ell_{j,t},k_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0\Bigg\{ \sum_{t=0}^{\infty} Q_{0,t}(1-\tau_d) \quad \Bigg(\quad & y_{j,t}\left(\tfrac{P_{j,t}}{P_t}\right) - \Xi - r_t k_{j,t} - h_t \ell_{j,t} \\ & -\tfrac{\psi}{2}\left(\tfrac{P_{j,t}}{P_{j,t-1}} - \overline{\Pi}\right)^2 y_t\Bigg)\Bigg\} \\ s.t. \qquad y_{j,t} &= \zeta_{TFP,t} k_{j,t}^{\theta} \ell_{j,t}^{1-\theta}, \\ y_{j,t} &= \left(\tfrac{P_{j,t}}{P_t}\right)^{-\vartheta\exp\{\zeta_{P,t}\}} y_t, \\ & P_{j,-1} \text{ given.} \end{aligned}$$

Labor services. Labor services are homogeneous. They are produced by employment agencies, under constant returns to scale. Workers come in four types to the agency: low/high education each with/without skill loss. The value to the agency of a household to the employment agency depends on the household's characteristics. The value the agency of a low-educated worker without skill loss is

$$J_{L,0,t}^L \quad = \quad (1-\tau_d)(h_t - w_t)e_L + \mathbb{E}_t\big\{Q_{t,t+1}\big(1-\lambda_x(e_L) + \lambda_n(e_L)\big)J_{L,0,t+1}^L\big\}.$$

The value to the agency of a low-educated worker with skill loss is

$$J^L_{L,1,t} = (1-\tau_d)(h_t - w_t)e_L(1-\varrho) + \mathbb{E}_t\big\{Q_{t,t+1}\big(1-\lambda_x(e_L)+\lambda_n(e_L)\big)\big(\pi^{emp}_L(1,0)J^L_{L,0,t+1} + \pi^{emp}_L(1,1)J^L_{L,1,t+1}\big)\big\}.$$

The value to the agency of a high-educated worker without skill loss is

$$J^H_{L,0,t} = (1-\tau_d)(h_t - w_t)e_H + \mathbb{E}_t\big\{Q_{t,t+1}\big(1-\lambda_x(e_H)+\lambda_n(e_H)\big)J^H_{L,0,t+1}\big\}.$$

The value to the agency of a high-educated worker with skill loss is

$$J^H_{L,1,t} = (1-\tau_d)(h_t - w_t)e_H(1-\varrho) + \mathbb{E}_t\big\{Q_{t,t+1}\big(1-\lambda_x(e_H)+\lambda_n(e_H)\big)\big(\pi^{emp}_H(1,0)J^H_{L,0,t+1} + \pi^{emp}_L(1,1)J^H_{L,1,t+1}\big)\big\}.$$

After separations have occurred, and before production, employment agencies can recruit new households. Let V_t be the aggregate number of vacancies posted and M_t the mass of new matches. The job-filling probability is identical for all vacancies, and given by $q_t = M_t/V_t$. Letting κ_t/q_t be the average cost per hire, the free-entry condition for recruiting is given by

$$\begin{aligned}&[\widetilde{U}^L_t\pi^{uem}_L(0)+\lambda_n(e_L)\widetilde{N}^L_{0,t}]J^L_{L,0,t}\\ &+[\widetilde{U}^L_t\pi^{uem}_L(1)+\lambda_n(e_L)\widetilde{N}^L_{1,t}]J^L_{L,1,t}\\ &+[\widetilde{U}^H_t\pi^{uem}_L(0)+\lambda_n(e_H)\widetilde{N}^H_{0,t}]J^H_{L,0,t}\\ &+[\widetilde{U}^H_t\pi^{uem}_L(1)+\lambda_n(e_H)\widetilde{N}^H_{1,t}]J^H_{L,1,t} = (1-\tau_d)\kappa_t/q_t\cdot\left[\widetilde{U}^L_t+\lambda_n(e_L)\widetilde{N}^L_t+\widetilde{U}^H_t+\lambda_n(e_L)\widetilde{N}^H_t\right].\end{aligned}$$

Recruiting costs are given by

$$\kappa_t = (\kappa_H q_t + \kappa_v)\left(\frac{M_t/\widetilde{N}_t}{\overline{M}/\overline{\widetilde{N}}}\right)^2.$$

Matches emerge according to matching function

$$M_t = \left[\left(\widetilde{U}_t+\lambda_n(e_L)\widetilde{N}^L_t+\lambda_n(e_H)\widetilde{N}^H_t\right)V_t\right]\Big/\left(\left(\widetilde{U}_t+\lambda_n(e_L)\widetilde{N}^L_t+\lambda_n(e_H)\widetilde{N}^H_t\right)^{\alpha}+{V_t}^{\alpha}\right)^{\frac{1}{\alpha}}$$

with $\alpha > 0$. The job-finding rate is

$$f_t = \frac{M_t}{\widetilde{U}_t+\lambda_n(e_L)\widetilde{N}^L_t+\lambda_n(e_H)\widetilde{N}^H_t}.$$

The wage rule is

$$\log(w_t/\overline{w}) = \phi_w\ \log(w_{t-1}/\overline{w}) + \phi_w\ \log\left(\frac{y_t}{\overline{y}}\right) + \zeta_{w,t}.$$

Capital services. The representative producer of capital services faces the following problem

$$\max_{\{v_t,i_t,K_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0\Big\{\textstyle\sum_{t=0}^{\infty}(1-\tau_d)(r_tK_{t-1}v_t - i_t)\Big\}$$
$$\text{s.t.}\quad K_t = \big[1-\delta(v_t)\big]\cdot K_{t-1} + \zeta_{I,t}\cdot[1-\Gamma(i_t/i_{t-1})]i_t.$$

Depreciation of capital evolves as

$$\delta(v_t) = \delta_1\, v_t^{\delta_2},\ \delta_1 > 0, \delta_2 > 1.$$

The transformation function that governs how investment is transformed into physical capital is given by

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \phi_K/2\left(\frac{i_t}{i_{t-1}}-1\right)^2,\ \ \phi_K \geq 0.$$

Adjustment services. The competitive representative adjustment-services firm solves

$$\max_{y_{a,t},(y_{a,j,t})_{j\in[0,1]}} (1-\tau_d)\left(P_t y_{a,t} - \int_0^1 P_{j,t}y_{a,j,t}dj\right) \text{ s.t. } y_{a,t} = \left(\int_0^1 y_{a,j,t}^{\frac{\vartheta\exp\{\zeta_{P,t}\}-1}{\vartheta\exp\{\zeta_{P,t}\}}}dj\right)^{\frac{\vartheta\exp\{\zeta_{P,t}\}}{\vartheta\exp\{\zeta_{P,t}\}-1}},$$

where $y_{a,t}$ are total adjustment services produced and $y_{a,j,t}$ is demand for differentiated good j by the adjustment-services firm.

E.5. Central bank and fiscal authority

The central bank sets the gross nominal interest rate according to Taylor rule

$$\log\left(\frac{R_t}{\overline{R}}\right) = \phi_R \log\left(\frac{R_{t-1}}{\overline{R}}\right) + (1-\phi_R)\left[\phi_\Pi \log\left(\frac{\Pi_t}{\overline{\Pi}}\right) - \phi_u\left(\frac{U_t-\overline{U}}{\pi^{\text{labforce}}}\right)\right] + \log\zeta_{R,t}.$$

The fiscal authority's budget constraint is given by

$$\begin{aligned}
& g \\
& +U_t^L b_{UI}(e_L)[1-\tau_t(b_{UI}(e_L))] \\
& +U_t^H b_{UI}(e_H)[1-\tau_t(b_{UI}(e_H))] \\
& +(1-\pi^{\text{labforce}})\pi_E(e_L) b_{RET}(e_L)[1-\tau_t(b_{RET}(e_L))] \\
& +(1-\pi^{\text{labforce}})\pi_E(e_H) b_{RET}(e_H)[1-\tau_t(b_{RET}(e_H))] \\
= {} & \tau_c c_t \\
& +\tau_d\Bigg(y_t - \Xi - \left(\frac{\psi}{2}(\Pi_t-\overline{\Pi})^2 y_t\right) - (\kappa_H q_t + \kappa_v)\left(\frac{M_t/\widetilde{N}_t}{\overline{M}/\overline{\widetilde{N}}}\right)^2 V_t - i_t - \\
& \quad w_t e_L[N_{0,t}^L + N_{1,t}^L(1-\varrho)] - w_t e_H[N_{0,t}^H + N_{1,t}^H(1-\varrho)]\Bigg) \\
& +N_{0,t}^L w_t e_L[\tau_{RET}+\tau_{UI}+\tau_t(w_t e_L)] \\
& +N_{1,t}^L w_t e_L(1-\varrho)[\tau_{RET}+\tau_{UI}+\tau_t(w_t e_L(1-\varrho))] \\
& +N_{0,t}^H w_t e_H[\tau_{RET}+\tau_{UI}+\tau_t(w_t e_H)] \\
& +N_{1,t}^H w_t e_H(1-\varrho)[\tau_{RET}+\tau_{UI}+\tau_t(w_t e_H(1-\varrho))]
\end{aligned} \tag{E.1}$$

E.6. Laws of motion (un)employment

We list both (un)employment at the beginning of the period and at the end of the period.

E.6.1. (Un)employment at the beginning of the period

Low-education, no skill loss employment at the beginning of the period evolves as

$$\widetilde{N}_{0,t}^L = N_{0,t-1}^L + \pi^{emp}(1,0)N_{1,t-1}^L.$$

With skill loss, the corresponding law of motion is

$$\widetilde{N}_{1,t}^L = \pi^{emp}(1,1)N_{1,t-1}^L.$$

Total low-education employment at the beginning of the period is

$$\widetilde{N}_t^L = \widetilde{N}_{0,t}^L + \widetilde{N}_{1,t}^L.$$

High-education, no skill loss employment at the beginning of the period evolves as

$$\widetilde{N}_{0,t}^H = N_{0,t-1}^H + \pi^{emp}(1,0)N_{1,t-1}^H.$$

With skill loss, the corresponding law of motion is

$$\widetilde{N}_{1,t}^H = \pi^{emp}(1,1)N_{1,t-1}^H.$$

Total high-education employment at the beginning of the period is

$$\widetilde{N}_t^H = \widetilde{N}_{0,t}^H + \widetilde{N}_{1,t}^H.$$

Total employment at the beginning of the period evolves as

$$\widetilde{N}_t = \widetilde{N}_t^L + \widetilde{N}_t^H.$$

Unemployment at the beginning of the period is defined as

$$\widetilde{U}_t^L = \pi_E(e_L)\pi^{\text{labforce}} - \widetilde{N}_t^L,$$

$$\widetilde{U}_t^H = \pi_E(e_H)\pi^{\text{labforce}} - \widetilde{N}_t^H,$$

and

$$\widetilde{U}_t = \widetilde{U}_t^L + \widetilde{U}_t^H.$$

E.6.2. (Un)employment at the end of the period

Low-education employment at the end of the period evolves as

$$N_{0,t}^L = [1 - \lambda_x(e_L) - \lambda_e(e_L)(1 - ft)]\widetilde{N}_{0,t}^L + f_t \pi_L^{uem}(0) U_{t-1}^L.$$
$$N_{1,t}^L = [1 - \lambda_x(e_L) - \lambda_e(e_L)(1 - ft)]\widetilde{N}_{1,t}^L + f_t \pi_L^{uem}(1) U_{t-1}^L.$$
$$N_t^L = N_{0,t}^L + N_{1,t}^L.$$

High-education employment at the end of the period evolves as

$$N_{0,t}^H = [1 - \lambda_x(e_H) - \lambda_e(e_H)(1 - ft)]\widetilde{N}_{0,t}^H + f_t \pi_H^{uem}(0) U_{t-1}^H.$$
$$N_{1,t}^H = [1 - \lambda_x(e_H) - \lambda_e(e_H)(1 - ft)]\widetilde{N}_{1,t}^H + f_t \pi_H^{uem}(1) U_{t-1}^H.$$
$$N_t^H = N_{0,t}^H + N_{1,t}^H.$$

Employment at the end of the period evolves as

$$N_t = N_{0,t}^L + N_{1,t}^L + N_{0,t}^H + N_{0,t}^H.$$

Unemployment at the end of the period evolves as

$$U_t^L = \pi_E(e_L)\pi^{\text{labforce}} - N_t^L,$$
$$U_t^H = \pi_E(e_H)\pi^{\text{labforce}} - N_t^H,$$

and

$$U_t = U_t^L + U_t^H.$$

E.7. Aggregates

Total per-capita consumption of spenders is

$$\begin{aligned} c_t^{\text{spend}} \quad = \quad & (1 - \pi^{\text{labforce}})\pi_E(e_L) c_{R,t}^{L,spend} + (1 - \pi^{\text{labforce}})\pi_E(e_H) c_{R,t}^{H,spend} \\ & + U_t^L c_{U,t}^{L,spend} + U_t^H c_{U,t}^{H,spend} \\ & + N_{0,t}^L c_{0,t}^{L,spend} + N_{1,t}^L c_{1,t}^{L,spend} \\ & + N_{0,t}^H c_{0,t}^{H,spend} + N_{1,t}^H c_{1,t}^{H,spend}. \end{aligned}$$

Total consumption is

$$c_t = \pi^{\text{saver}} c_t^{\text{saver}} + (1 - \pi^{\text{saver}}) c_t^{\text{spend}}.$$

E.8. Market clearing and equilibrium

The market for adjustment services clears if production equals use of adjustment services for price adjustment and labor adjustment,

$$y_{a,t} = \frac{\psi}{2}\left(\Pi_t - \overline{\Pi}\right)^2 y_t + \kappa_t V_t.$$

The market for capital services clears if (with $k_{j,t} = k_t$ for all j)

$$v(X)K_{t-1} = k_t.$$

The market for labor services clears if all labor services supplied are used in the production of differentiated goods (with $\ell_{j,t} = \ell_t$ for all j),

$$N_{0,t}^L e_L + N_{1,t}^L e_L(1 - \varrho) + N_{0,t}^H e_H + N_{1,t}^H e_H(1 - \varrho) = \ell_t.$$

Total demand for differentiated goods is given by

$$y_t = y_{f,t} + y_{a,t}.$$

The market for differentiated goods clears if demand equals production (using symmetry in both price setting and demand for each differentiated good j), so

$$y_t = \zeta_{TFP,t} k_t^\theta \ell_t^{1-\theta}.$$

The market for final goods clears if

$$y_{f,t} = c_t + i_t + g.$$

Normalizing the supply of shares to unity, the market for shares in the mutual fund clears if

$$a_t = 1.$$

Appendix F: Solution algorithm

This appendix outlines our solution algorithm. We extend the perturbation method developed by Reiter (2009) and Reiter (2010a) to compute a second-order approximation with a parameterized law of motion for the distribution of households.[A10] The parameterized law of motion is obtained from a principal-component decomposition of the first-order dynamics of the distribution of wealth. This step is necessary as, on the one hand, a full second-order solution is infeasible given currently available RAM and the size of the model, and, on the other hand, we need to compute a second-order solution to study welfare along the business-cycle dimension. In earlier versions of the paper we used an approach closer to Krusell and Smith (1998), in which we forecasted the expectation terms in the firms' Euler equations and asset prices, and later a method based on Reiter (2010b). While these methods allow for a global solution of the model, they suffer from a strong curse of dimensionality, limiting the number of aggregate states one can take into account. The algorithm described here overcomes this constraint.

We use splines to approximate households' decision rules along their asset dimension, and approximate the distribution of households as a histogram on the product of skill state, discount factor, education, employment state and a grid on the wealth distribution. All agents use this approximation to construct their forecasts about the evolution of the economy. The solution algorithm takes the following steps.

1. Solve for the model's steady state without aggregates shocks. Collect the values of aggregate variables, the households' decision rules, value functions,[A11] and distributions on their respective grids.

2. Collect all equations characterizing the solution of the model economy. Take first derivatives of these equations with respect to aggregate variables, households' policy functions on the grids, and the mass of agents in each bin of the approximated histogram at the steady state.[A12] Solve for the first-order policy and transition matrices using the algorithm described in Schmitt-Grohé and Uribe (2004).

3. Use the first-order solution to compute the variance-covariance matrix of the deviations of the distribution of assets (the mass points in the respective bins of the histogram) from the steady state. Compute a principal-component decomposition of the matrix. Keep the first n principal-component vectors so that these n components explain x percent of the total variation of the distribution, for example, 99.9 percent, based on the principal-component analysis. Use the principal-component vectors to compute two projection matrices: one, D, from $\mathbb{R}^n$ into the linear space spanned by the principal-component vectors, and the other, H, which projects back from this space onto $\mathbb{R}^n$, such that $D \cdot H$ equals the identity matrix in $\mathbb{R}^n$.[A13]

[A10] See also Winberry (2018) and Ahn et al. (2017) for closely related solution strategies.

[A11] In practice, we approximate the expected marginal utility of the households and the value function at the beginning of the period before idiosyncratic uncertainty is resolved. This makes it easier to deal with the borrowing constraint when we apply some of the dimension reductions discussed below. The beginning-of-period value function is the object used in the calculation of the welfare effects of policy changes so we approximate it directly.

[A12] In practice, it helps to drop one of the bins from the set of histograms and to use the fact that the total mass has to be 1. In addition, we are keeping track of the aggregate mass of agents in employment with and without skill loss by education, and use this to reduce the number of bins to track further. Here, we are utilizing the fact that the shares of agents in different skill and discount factor states are constant over time and that the job-finding and -loss rate does not depend on these characteristics.

[A13] This procedure follows Reiter (2010a). More details and motivation can be found there. In practice, we found that adding the asset price relative to the steady state as another variable in the decomposition increased both numerical efficiency and stability as it directly relates to the dynamics of the stochastic discount factor in our setting.

4. Collect all equations characterizing the solution of the model economy again. To reduce the number of state variables we now use the sum of the distribution in the steady state, μ^{SS}, and $D \cdot p$ in place of tracking the mass of agents in each bin of the approximated histogram at each point in time separately. Here p is a vector in $\mathbb{R}^n$ weighting the different principal components. Its changes over time to allow us to track, approximately, changes in the distribution.[A14] In practice, in our system of equations, the economy starts the period with a vector p as part of the state. We then assume $\mu^{ss} + D \cdot p$ as the beginning-of-period distribution, use it in all equations involving the distribution and update it using the law of motion for the distribution. Finally, we subtract μ^{ss} from the resulting end-of-period distribution and project the difference on H to obtain p', as part of the new state vector. Add these adjustments to the model equations. Take first and second derivatives of these equations with respect to aggregate variables, households' policy and value functions on the grids, and the new state variables just introduced at the steady state. Solve for the first- and second-order policy and transition matrices.

To reduce the computational complexity of the problem further, we follow Ahn et al. (2017) and Bayer et al. (2019) and approximate the deviations of households' policy and value functions from the steady state using a piece-wise linear spline of a smaller order than the grid used to solve for steady-state policy functions. We verified that increasing the degree of the spline and the number of principal components did not change our conclusions.[A15] We implement the algorithm in MATLAB using a modified version of the codes provided with Schmitt-Grohé and Uribe (2004). We adjust them to allow us to handle the very large derivative matrices resulting from the second-order approximation. We need around $300GB$ of RAM to perform the calculations; this is so despite making use of the sparsity of the matrices and the reduction in the number of policy functions. We use a spline of order ten, and four principal components for the results in the text and have verified that both the model dynamics and other implications are robust to changes in these numbers.

[A14] Given the derivation of H and D we expect the approximate dynamics to be fairly close to the full ones, and indeed, we verified that the first-order dynamics for aggregate variables of the model with and without this reduction in the state space are extremely close. We can think of $D \cdot p$ as describing the most likely state of the distribution if a projection of the histogram using H results in p.

[A15] We also experimented with Chebyshev polynomials and a smoother spline. In the end, all methods gave similar answers and we used the piece-wise linear spline as it gave us the best trade-off between precision and number of parameters to approximate.

Appendix G: Data on wealth and income

Our data source for wealth and income is the U.S. Survey of Consumer Finances. We use the Summary Extract Public Data of the SCF 2004. Acronyms in bold below correspond to the variable names in the dataset. In the construction of different income categories we split business income between labor and financial income under the following assumptions:

- Income from non-actively managed businesses is financial income.
- Income from actively managed businesses is 60 percent labor income and 40 percent financial income, based on the average labor-income share in 2004 according to the BEA.
- As the SCF only provides the value of actively and non-actively managed businesses, but does not provide the income from both types of ventures separately, we split total business income into the two categories using the value shares as weights.

We then arrive at the following definitions of income categories:

- "Labor Income" is the sum of wage income plus the labor share of business income constructed as described above. ($WAGEINC + 0.6 \cdot \frac{ACTBUS}{BUS} \cdot BUSSEFARMINC$)
- "Social security" is social security and pension income net of withdrawals from pension accounts ($SSRETINC$ - $PENACCTWD$). We exclude pension account withdrawals, since such pensions will be treated as equity in the model. Withdrawals, therefore, will not be income. Rather, we adjust financial income each period by the putative returns to retirement accounts (see below).
- "Non-SocSec transfer income" is ($TRANSFOTHINC$). This includes among other items unemployment benefits.
- "Financial income," then is computed as the financial part of business income ($(0.4 \cdot \frac{ACTBUS}{BUS} + \frac{NONACTBUS}{BUS}) \cdot BUSSEFARMINC$), interest and dividend income ($INTDIVINC$), realized capital gains ($KGINC$), and imputed income on other assets (labeled IMP_FININC below).

Category IMP_FININC we compute ourselves. This is necessary so as to map financial income in the model to the SCF. The SCF does not cover the rents and the service flow from owner-occupied housing. Neither does it capture financial gains on retirement accounts. Rather, both interest and dividend income and realized capital gains in the SCF are taken from IRS Form 1040. We impute the income flow from these categories. Toward this end, we first derive the average rate of return on financial assets for which we have income information, by dividing the sum of the financial part of business income, of interest and dividend income, and of realized capital gains by the stock of wealth generating them. We define this stock as the sum of the value of businesses (BUS) and total financial wealth (FIN) excluding quasi-liquid retirement accounts (RETQLIQ). The resulting real rate of return is $rret = 4.31$ percent per year (using data for all households ageds 25-99).[A16] We use this rate to impute the missing financial income by the return with the value of houses (HOUSES,ORESRE,NRESRE), other non-financial assets (OTHNFIN), and quasi-liquid retirement accounts (RETQLIQ). In addition, we also use $rret$ to impute negative income from debt secured by a primary residence (MRTHEL), debt secured by other residential property (RESDBT), credit card balances after last payment (CCBAL), other lines of credit (OTHLOC), and other debt (ODEBT). To compute total net worth we sum the value of all asset categories listed above and subtract the value of all debts listed above.

The after-tax real rate of return of 3.2 percent per year, to which we calibrate in Section 3.2.1, is derived as follows. Financial income as reported in the SCF is pre-tax income, where pre-tax refers to taxes paid by households. So as to get the households' after-tax returns, we split the capital tax rate of 36 percent into a part paid by households and a part paid by firms, the

[A16] Here we make the implicit assumption that both income and wealth are measured at the end of the period so that the ratio can be treated as the real rate of return.

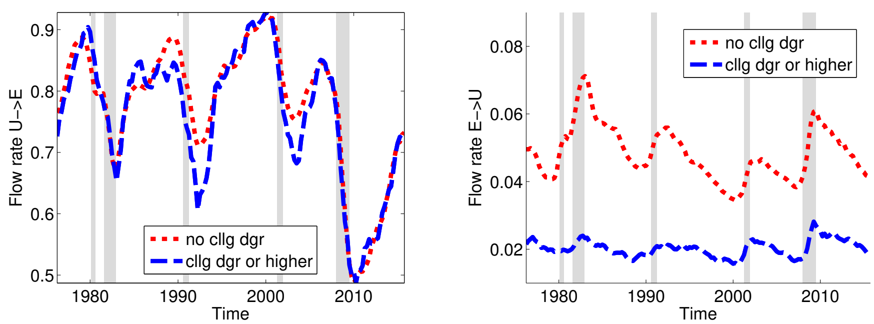
latter of which we proxy by the average corporate tax rate over the sample period. Using the same calculations as in Fernández-Villaverde et al. (2015), the average corporate tax rate over the sample is 10 percent. This leaves a 26 percent capital tax rate to be paid by households, meaning that households' after-tax return is $4.31 \cdot (1 - 0.26) \approx 3.2$.

Appendix H: Labor-market flow rates by education

This appendix provides details on the construction of the labor-market flow rates shown in Section 3.1. The flow rates are based on the Current Population Survey (CPS). We follow the methodology described in Cairó and Cajner (2018). We first compute unemployment rates and monthly flow rates from the survey. From this, we construct quarterly time series. The quarterly flow rate from unemployment to employment (the "job-finding rate") that we construct is defined as one minus the probability that a worker who was unemployed at the end of a quarter also is unemployed at the end of the next. The flow rate from employment to unemployment is constructed so as to ensure that, combined with the job-finding rate defined above, the flow rate into unemployment replicates the evolution of the unemployment rate in each education group.

Figure A1 shows the resulting quarterly flow rates from unemployment to employment (left panel) and from employment to unemployment (right panel) for working-age individuals. In the low-education group are all those workers with less education than a college degree (red dots), including workers who never went to college as well as college dropouts. The high-education group is composed of workers with a college degree, or more education (blue dashed).

Figure A1: Data. Flow rates into (un)employment



Notes: Flow rate from unemployment to employment (left panel) and flow rate from employment to unemployment (right panel). Quarterly frequency. Based on the Current Population Survey. Workers ages 25 to 65. Red dotted line: workers without a college degree. Blue dashed: workers with a college or higher degree.

As alluded to in the main text, the level and volatility of flow rates into employment is rather similar for the two education groups. The flow rates into unemployment, instead, differ notably by education. The flow rate into unemployment for the low-educated on average is about twice the level of the flow rate for the high-educated. And it is about twice as volatile. What this means is that the low-educated are exposed to cyclical and average unemployment risk to a larger extent.

Appendix I: Calibration details and model fit

This appendix provides further details on the calibration and on how well the model fits the data.

I.1. List of targeted moments

This appendix lists all data moments that were targeted during the calibration of the model's steady state and the resulting values in the steady state of the model. The list is presented in two tables. The first table lists those targets that we match exactly.[A17] The second table contains moments that were included as targets in the minimum distance criterion described in Footnote 22 of the main text.

TABLE A1

Moments matched exactly

Target Description	Target	Model	Source
Post-Tax Real Rate	3.2%	3.2%	SCF.
Wealth Share Less Educated	30%	30%	SCF.
Wealth Share Poorest 20% Workers	0%	0%	SCF.
Wealth Share Poorest 50% retired	5.25%	5.25%	SCF.
Wealth Share Low Educated	30%	30%	SCF.
Average Working Life	40 years	40 years	Sample choice.
Average Retirement Length	12 years	12 years	SCF.
Share Low Educated Worker	60%	60%	SCF.
Intergen. Elasticity of Income	0.5	0.5	Solon (1992), Mazumder (2005).
Initial earnings loss	25%	25%	Couch/Placzek (2010), Altonji *et al.* (2013).
Loss six years later	14%	14%	Couch and Placzek (2010).
Stand. Dev. Residual Earnings	0.508	0.508	Floden and Lindé (2001).
Wealth Gini Working Age	82.4	82.4	SCF.
College Wage Premium	50%	50%	Mukoyama and Sahin (2006).
Capital Depreciation Rate	1.5%	1.5%	NIPA.
Utilization	1	1	Normalization.
Share of exog. separations e_L	69.5&	69.5%	Calculations based on Table II.
Share of exog. separations e_L	65.7&	65.7%	Calculations based on Table II.
Rel. Unempl. Rate by Educ.	44%	44%	CPS.
Unemployment Rate	6%	6%	BLS.
$\frac{\text{Total Cost per hire}}{\text{Average Wage}}$	50%	50%	Silva and Toledo (2009).
Share Fixed Hiring Cost	94%	94%	Christiano et al. (2016).
Job-Finding Rate	0.82	0.82	CPS.
Job-Filling Rate	0.71	0.71	den Haan et al. (2000).
Labor Share	66%	66%	NIPA.
$\frac{\text{Investment}}{\text{GDP}}$	18%	18%	NIPA.
Inflation Rate	2% p.a.	2% p.a.	Inflation target.
$\frac{\text{Government Spending}}{\text{GDP}}$	19%	19%	NIPA.

Notes: 'Target Description' explains what moment was target. 'Target' provides the targeted value. 'Model ' lists the corresponding value from the calibrated steady state of the HANK model. 'Source' adds information on the source of 'Target Value.' This table contains moments that were, by design, matched exactly.

[A17]We omit targets in cases where we simply set a parameter to match values found in previous research from this appendix. An example of such a parameter is relative risk aversion, which we target to be 2.5 based on Blundell et al. (2016). The same goes for parameters we found through the maximum likelihood estimation of the RANK version of the model.

TABLE A2

Fit of percentiles of the wealth distribution

Target Description	Target	Model
Poorest 5%	-0.1%	0%
Poorest 10%	-0.1%	0%
Poorest 15%	-0.1%	0%
Poorest 20%	-0.1%	≈ 0%
Poorest 25%	≈ 0%	0.01%
Poorest 30%	0.21%	0.07%
Poorest 35%	0.43%	0.15%
Poorest 40%	0.91%	0.30%
Poorest 45%	1.50%	0.55%
Poorest 50%	2.27%	0.96%
Poorest 55%	3.29%	1.72%
Poorest 60%	4.62%	2.90%
Poorest 65%	6.32%	4.75%
Poorest 70%	8.56%	7.40%
Poorest 75%	11.46%	10.92%
Poorest 80%	15.27%	15.49%
Poorest 85%	20.48%	21.62%
Poorest 90%	27.80%	30.53%
Poorest 95%	39.53%	44.25%

Notes: In the calibration of $(\beta_{e_L}, \beta_{e_H}, \Delta_\beta, \gamma_1, \gamma_2)$, among other targets, we seek to minimize the distance between percentile of the distribution of networth in the data and model; see Footnote 22 of the main text. This table reports the moments used and the fit. All moments in this table are based on the SCF and were rounded to two digits.

I.2. Calibration of skills

Table A3 provides the targets for calibrating the skills and lists how many restrictions each target delivers. See page 25 of the main text for a detailed discussion of these targets.

TABLE A3

Temporary skills. Targets and Parameterization.

	Targets	# restrictions
Assumptions on skill levels		
(i)	Average length of a working life: 40 years	1
(ii)	Average length of retirement: 12 years	1
(iii)	Length of working life independent of skill level	2
(iv)	Skills after birth according to ergodic distribution	2
(v)	Ergodic mass super-skilled in working-age pop. 1%	1
(vi)	Prob. remain super-skilled if not retiring 0.975	1
(vii)	Probability of becoming s_3 independent of s_1 and s_2	1
(viii)	Ergodic distrib. determines transition from s_3 to s_1, s_2	1
(ix)	Equal ergodic mass of low- and medium-skill agents	1
(x)	Persistence of residual earnings of 0.975	1
(xi)	Ergodic standard deviation of residual earnings of 0.51	1
(xii)	Normalize average skill of workers to 1	1
(xiii)	Normalize $s_0 = 0$ (no labor income in retirement)	1
(xiv)	Target 0.825 for Gini index of wealth of the working aged	1

Notes: Calibration strategy for skills. Section 3.2.1 of the main text provides further details.

I.3. Wealth distribution in the model

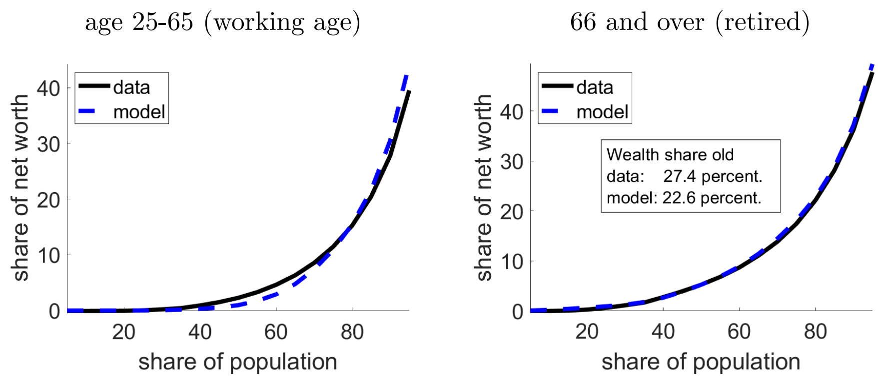
Figure A2 plots the Lorenz curves for net worth and income from the model, for working-age households and households of retirement age.

Note that the wealth Lorenz curves were a target in the calibration. The model matches these well. The share of aggregate net worth held by the old is 27.4 percent in the data. In the model, the retired households hold 22.6 percent of aggregate wealth.

I.4. Income shares

Table A4 provides the model-based counterpart to Table I in the main text. The calibrated model matches the overall pattern of the shape of the distribution of incomes. The bottom part of the wealth distribution in the model has a somewhat lower share of financial income than the data. We wish to note here, however, that our measure of financial income in the data includes imputed income from housing and retirement savings. Also for the retired, the model matches the rough split.

Figure A2: Model vs. data. Wealth distribution



Notes: Wealth Lorenz curves within each age group. Comparing model (blue dashed line) and data (solid line). Left: working age (25-65). Right: old age (66 and over).

TABLE A4

MODEL. INCOME SOURCES BY NET WORTH (PERCENT OF TOTAL INCOME)

	Percentile of net worth						
working age	0-20	20-40	40-60	60-80	80-95	Top 5	Top 1
Labor income	96.5	97.0	96.6	87.7	79.4	58.0	32.3
Financial income	0.0	0.3	1.9	10.3	19.6	41.6	67.4
Transfers	3.5	2.7	1.5	2.0	1.0	0.3	0.2
retired							
Financial income	0.6	2.0	7.0	12.1	32.1	73.9	89.3
Social security	99.4	98.0	93.0	87.9	67.9	26.1	10.7

Notes: Based on the model calibrated in Section 3. For the respective statistics in the data, see Table I in Section 3.

I.5. Second moments

For comparing the fit of the model with the business-cycle facts, we rely on the disaggregated data described in Section 3.1 of the main text and further aggregate data. The data are either quarterly to start with or transformed from monthly to quarterly frequency. Unless noted otherwise, this transformation from monthly is done by averaging the monthly data over the quarter. The data are seasonally adjusted where applicable. Unless noted otherwise, the source of the aggregate data is the St. Louis Fed's FRED II database. We start with the series that cover the period 1977Q1 to 2015Q4. After HP-filtering (HP-weight 1600), we drop observations at the beginning and end to arrive at a sample of HP-filtered observations covering the period 1984Q1 to 2008Q3. Nominal variables are deflated by the GDP deflator, which we also use as our measure of inflation. Personal consumption expenditures, c, include total durable and non-durable consumption expenditures as well as services. Investment, i, is gross private domestic investment. Government consumption is government consumption and gross investment.

Capacity utilization, v, is measured by the quarterly average of the Board of Governors' headline index of industrial capacity utilization. We measure vacancies V using Barnichon's (2010) composite help-wanted index. The wage, $W(X)$, is computed as wage and salary accruals from the national accounts divided by the GDP deflator divided by total nonfarm payrolls. The interest rate, R, is the quarterly average of the effective federal funds rate. The unemployment and separation rates are taken from Table II.

Table A5 presents the corresponding moments and compares them to the moments in the model.

A result of economic substance is that allowing for heterogeneity implies notable changes to the business cycle. The heterogeneous-agent model has more procyclical consumption than both the saver-spender variant and the representative-agent model. To some extent, this comes at the expense of making investment less volatile. What is important to note is that employment in the model results from firms making an investment in employment relationships. Just as the investment in physical capital is less volatile in the heterogeneous-agent economy (in spite of output being more volatile), labor-market activity is somewhat *dampened*. Unemployment, vacancies, and the job-finding rate all are somewhat less volatile than in the representative-agent economy. This result holds unconditionally. Section I.6 in this same appendix shows impulse responses to all shocks. For four of them, the heterogeneous-agent economy shows a stronger response of employment than the representative-agent economy. For the wage shock, the response is dampened notably. Before going there, however, Section I.8 discusses the forecast error decomposition of the three model variants, that is, which of the shocks accounts for how much of the fluctuations in each variable.

TABLE A5

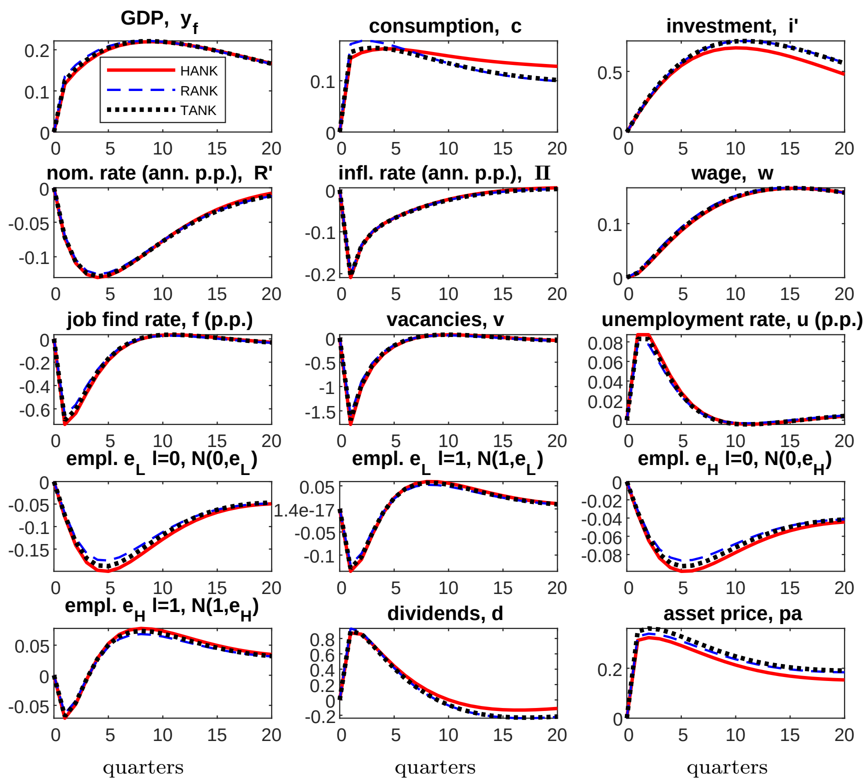
MODEL VS. DATA – FILTERED SECOND MOMENTS

	Model									Data		
	heterog. hh.			TANK			RANK			1984Q1-2008Q3		
	Std	Corr	AR1	Std	Corr	AR1	Std	Corr	AR1	Std	Corr	AR1
Output												
GDP, y_f	0.93	1.00	0.70	0.90	1.00	0.70	0.91	1.00	0.70	1.06	1.00	0.90
Consumption, c	0.57	0.59	0.67	0.51	0.55	0.66	0.51	0.53	0.66	0.90	0.88	0.87
Investment, i	4.27	0.92	0.70	4.27	0.93	0.69	4.34	0.94	0.70	5.05	0.90	0.89
Capacity util., v	2.22	0.63	0.24	2.14	0.59	0.22	2.16	0.60	0.22	2.21	0.80	0.93
Labor market												
Unempl. rate (e_L)	0.81	-0.79	0.77	0.82	-0.77	0.77	0.84	-0.77	0.77	0.63	-0.84	0.97
Unempl. rate (e_H)	0.40	-0.79	0.78	0.40	-0.78	0.78	0.41	-0.77	0.78	0.33	-0.82	0.97
Employment	0.69	0.79	0.77	0.70	0.77	0.77	0.71	0.77	0.77	0.57	0.84	0.97
Flow rate $U \to E$ $f(X)$	4.92	0.79	0.73	5.00	0.77	0.73	5.07	0.77	0.73	4.07	0.83	0.97
Flow rate $E \to U$ e_L	0.47	-0.79	0.72	0.48	-0.77	0.73	0.49	-0.77	0.73	0.31	-0.88	0.96
Flow rate $E \to U$ e_H	0.23	-0.79	0.72	0.23	-0.77	0.83	0.23	-0.77	0.73	0.15	-0.77	0.93
Vacancies, V	9.94	0.73	0.55	10.09	0.71	0.54	10.19	0.71	0.55	11.18	0.86	0.93
Prices												
Wage, W	1.02	0.23	0.70	1.01	0.21	0.54	1.02	0.20	0.70	0.86	0.34	0.78
Inflation, Π [1]	0.76	0.25	0.56	0.77	0.23	0.54	0.77	0.25	0.55	0.62	0.31	0.43
Nominal rate, R [1]	0.91	-0.20	0.66	1.10	-0.25	0.63	1.10	-0.24	0.63	1.24	0.65	0.92

Notes: The table compares moments of the data and two variants of the model (heterogeneous households, representative households). Appendix I describes the source of data and construction of data-based moments. The model moments follow the construction of the data. They are based on 100 repeated simulations of the model. Each simulation is initialized with 500 periods of simulations that are dropped for the computation of the moments. The next 156 periods are kept. In each case, we take the natural log of the data and compute the cyclical component of the data multiplied by 100 so as to have percentage deviations from trend. The trend is an H-P-trend with weight 1,600. We then drop the first 28 and last 29 observations and compute moments of interest. Finally, we average across the simulations. The left block shows the model's moments, the block on the right the data's. The first column ("Std.") reports the standard deviation of each series. The second column ("Corr") shows the correlation of the series with GDP. The final column ("AR1") shows the autocorrelation of the series. [1]: the nominal interest rate and inflation are reported in annualized percentage points.

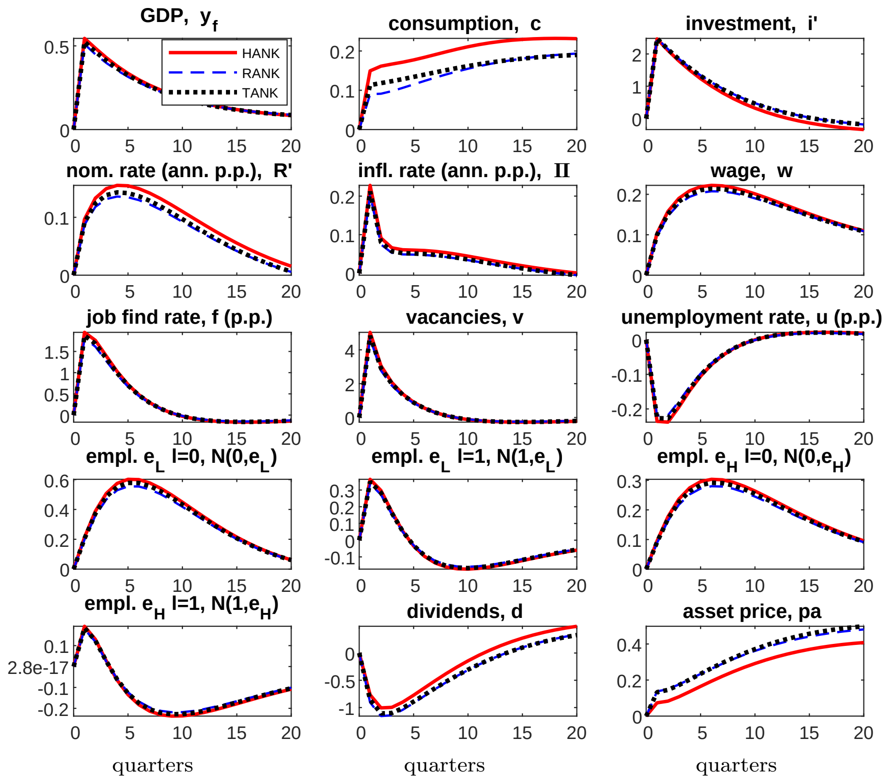
I.6. Impulse responses, aggregate variables

Figure A3: Impulse response to TFP shock, ζ_{TFP}



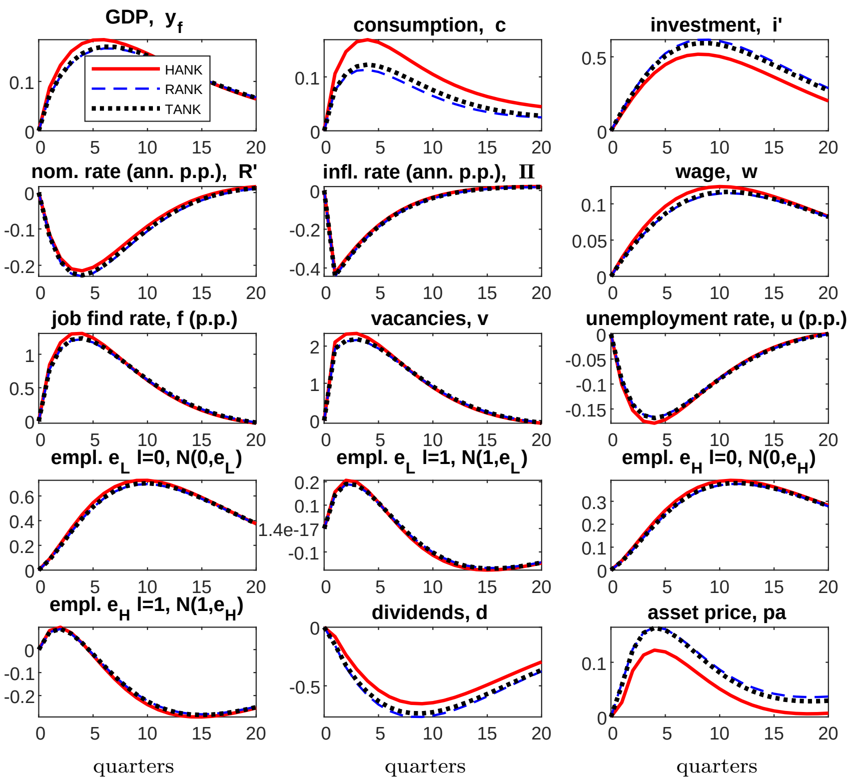
Notes: Impulse response to a one-standard-deviation TFP shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

Figure A4: Impulse response to MEI shock, ζ_I



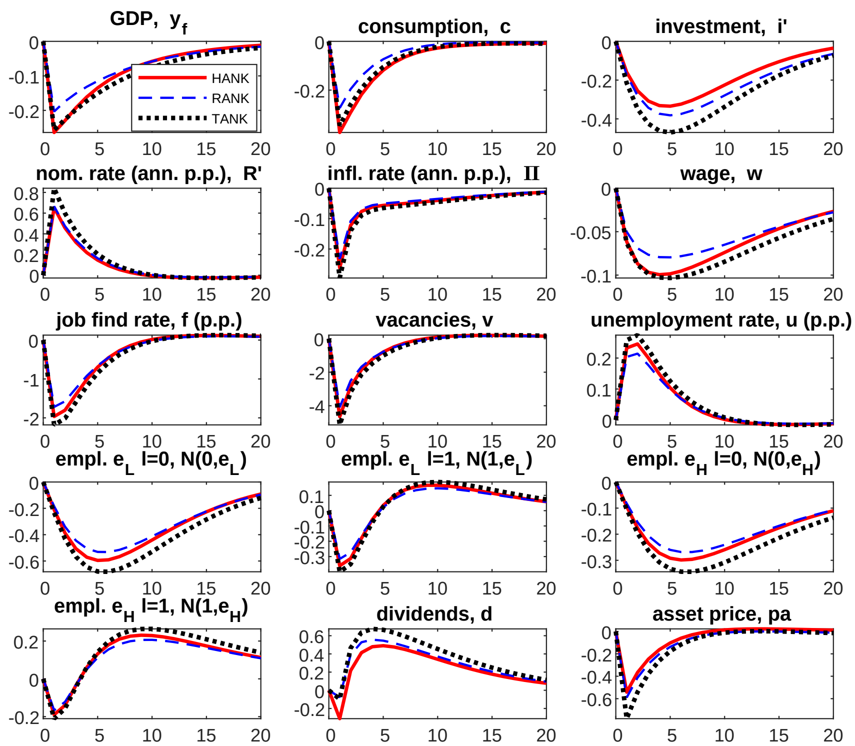
Notes: Impulse response to a one-standard-deviation MEI shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

Figure A5: Impulse response to demand-elasticity shock (a negative price-markup shock), ζ_P



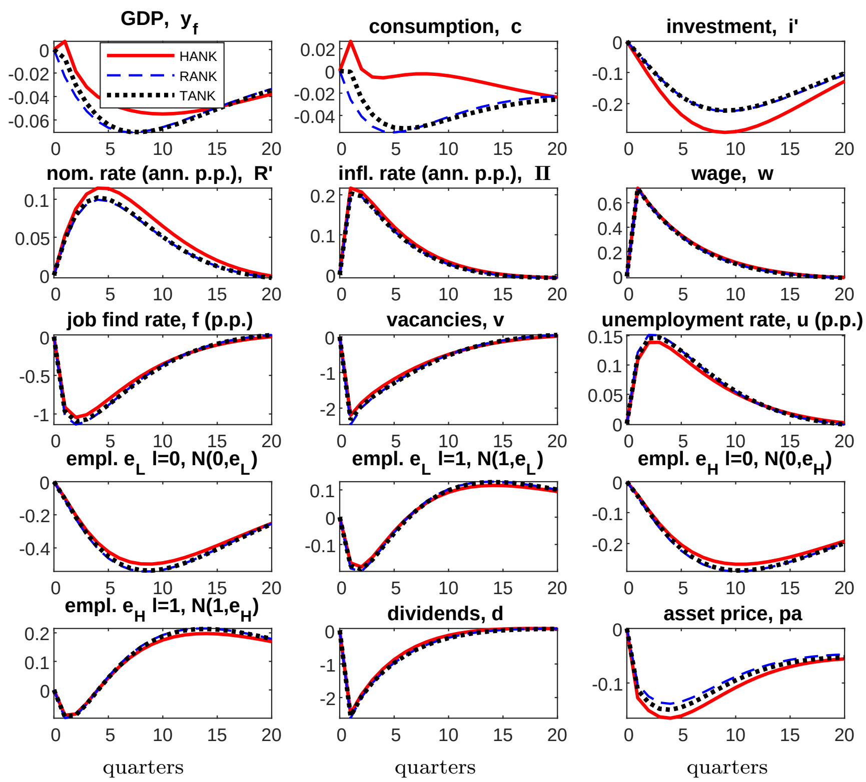
Notes: Impulse response to a one-standard-deviation price -markup shock, starting at the stochastic steady state. The shock compresses price markups. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

Figure A6: Impulse response to monetary shock, ζ_R



Notes: Impulse response to a one-standard-deviation monetary shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

Figure A7: Impulse response to wage shock, ζ_w



Notes: Impulse response to a one-standard-deviation wage shock, starting at the stochastic steady state. All variables expressed as percent deviations, unless noted otherwise. ann. p.p.: expressed in annualized percentage points. p.p.: expressed in percentage points.

I.7. Impulse responses cross-sectional inequality

The following figures report the response of inequality in the cross-section of households, as measured by the Gini coefficients. The shocks (and, thus, the aggregate dynamics) are as in the previous graphs that showed the aggregate impulse responses for the HANK model.

Figure A8: HANK: Impulse response of Ginis to TFP shock, ζ_{TFP}

Notes: Impulse response of Gini coefficients to a one-standard-deviation TFP shock, starting at the stochastic steady state. The impulse responses are scaled such that a "1" on the y-axis means an increase of the Gini coefficient by one percentage point; say from a Gini coefficient of 0.50 to 0.501.

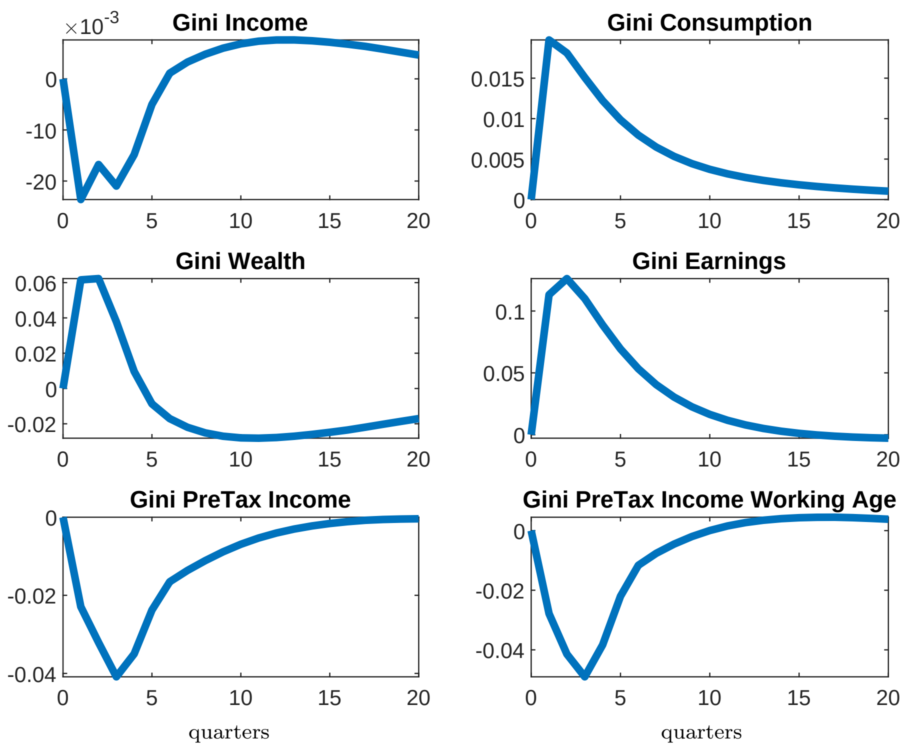
Figure A9: HANK: Impulse response of Ginis to MEI shock, ζ_I

Notes: Impulse response of Gini coefficients to a one-standard-deviation MEI shock.

Figure A10: HANK: Impulse response of Ginis to demand-elasticity shock (a negative price-markup shock), ζ_P

Notes: Impulse response of Gini coefficients to a one-standard-deviation price-markup shock.

Figure A11: HANK: Impulse response of Ginis to monetary shock, ζ_R



Notes: Impulse response of Gini coefficients to a one-standard-deviation monetary shock.

Figure A12: HANK: Impulse response of Ginis to wage shock, ζ_w

Notes: Impulse response of Gini coefficients to a one-standard-deviation wage shock.

I.8. Variance decomposition

Table A6 reports the unconditional variance decomposition for the heterogeneous-agent economy in the main text. To compute these moments we solve for the second-order policy functions with all shocks as described in the calibration and compute the variances of the variables we are interested in, for example, consumption.[A18] We then compute the variances of each variable when we force the realizations of all but one type of shock to zero, while we keep using the policy functions we computed in the model with all the shocks. The ratio of the variances gives us the share of the variance they explain. As the model is non-linear given the second-order solution, these shares will not add up exactly to one as there are potential interaction effects. In practice, we find that for most series listed below most of the variance is explained by the individual shock types.[A19] Therefore, as an approximation, we simply scale the individual shares proportionally to sum up to 1.

For comparison, Tables A7 and A8 report the same results for the representative-agent model and the saver-spender variant. Differences are modest in size. However, focusing for concreteness on consumption, we see that the price-markup shock and the MEI shock play a larger role in driving its dynamics than in the representative-agent model, while the role of TFP shocks is reduced.[A20]

TABLE A6

VARIANCE DECOMPOSITION HANK

variable	ζ_{TFP}	ζ_I	ζ_P	ζ_R	ζ_w
GDP, y_f	28.00	52.47	10.41	6.63	1.50
Consumption, c	22.95	62.75	5.64	6.95	1.71
Investment, i	14.66	76.04	5.66	1.57	2.07
Unemployment, U	2.90	32.06	27.01	24.04	13.98
Flow rate $U \to E$, f	3.00	33.83	25.29	24.37	13.51
Vacancies, V	3.31	36.28	22.11	26.08	12.21
Wage, W	19.20	26.98	6.52	2.87	44.42
Inflation, Π	8.68	16.16	48.08	10.68	16.39
Nominal rate, R	7.67	23.65	17.02	46.41	5.25

Notes: Forecast error variance decomposition for the heterogeneous-agent model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

[A18] We use pruning.

[A19] More than 90 percent.

[A20] The overall volatiltiy of consumption is slightly higher in the heterogeneous-agent model.

TABLE A7

Variance decomposition RANK

variable	ζ_{TFP}	ζ_I	ζ_P	ζ_R	ζ_w
GDP, y_f	30.58	53.46	9.53	4.54	1.90
Consumption, c	23.71	67.61	3.66	3.44	1.59
Investment, i	16.93	72.09	7.79	2.08	1.11
Unemployment, U	2.55	30.94	27.33	21.05	18.14
Flow rate $U \to E$, f	2.63	32.77	25.60	21.33	17.66
Vacancies, V	2.96	35.50	22.51	22.83	16.21
Wage, W	20.83	28.35	6.08	2.14	42.60
Inflation, Π	8.92	15.06	53.17	8.52	14.33
Nominal rate, R	7.71	20.97	19.25	48.48	3.60

Notes: Forecast error variance decomposition for the representative-agent model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

TABLE A8

Variance decomposition TANK

variable	ζ_{TFP}	ζ_I	ζ_P	ζ_R	ζ_w
GDP, y_f	28.79	53.07	9.23	7.15	1.76
Consumption, c	23.07	65.83	3.84	5.73	1.54
Investment, i	17.02	71.51	7.22	3.14	1.11
Unemployment, U	2.49	28.81	23.85	29.83	15.03
Flow rate $U \to E$, f	2.57	30.49	22.35	29.97	14.62
Vacancies, V	2.85	32.66	19.45	31.88	13.17
Wage, W	20.13	28.51	6.00	3.45	41.91
Inflation, Π	8.69	15.30	49.08	13.19	13.74
Nominal rate, R	6.20	17.44	14.38	59.06	2.92

Notes: Forecast error variance decomposition for the saver/spender model. Contribution of respective shock (TFP, MEI, price-markup, monetary, wage) to the variance. Based on second-order dynamics with pruning. Entries in percent. Rows may not sum to 100 because of rounding error.

I.9. Marginal propensities to consume

An oft-referenced statistic that helps us to understand the transmission of shocks in many models is household marginal propensity to consume (MPC). To document the MPCs for our model, we perform the following experiment. We are interested in the individual household's consumption response to an exogenous one-time increase in income. A household is characterized by its state (n, a, l, e, b, s). At the beginning of the quarter, we give the model-equivalent of \$500 in wealth to a household (and to that household only).[A21] Then, we record the cumulative increase in consumption expenditure as a share of the initial gift over the next quarter, the next two quarters, three quarters, and four quarters. We focus on the economy's non-stochastic steady state. That is, for the experiment shown here, all aggregate shocks ζ are known to be zero in all time periods. Endogenous aggregate variables have settled to their long-run value. While the aggregate state of the economy remains fixed in this experiment, the household still faces idiosyncratic risk (for example, idiosyncratic income and employment risk). We allow the household's individual states to change over time. Table A9 summarizes group averages. It shows the average MPCs for different groups of households (first four columns, in percent of the initial increase in income), where households are grouped by their characteristics at the time of the transfer. The last column reports the share of these households in the population.

The first line shows the average MPC for the entire economy, giving equal weight to all households. In the first quarter, the average MPC is 15.4 percent. After a year, on average households have spent – for consumption – a third of the increase in income (an MPC of 33.5 percent). This is in line with the evidence summarized in Carroll et al. (2017), for example. The former conclude that most empirical estimates in the literature find aggregate annual MPCs of 20 to 60 percent over a yearly horizon.[A22]

The average MPC may be an incomplete guide to the effect of shocks, however. The reason is that shocks impact different sources of income differentially, so that households in different idiosyncratic states might respond differently to a given shock. For example, a rise in wages does not benefit the retired or unemployed directly, while it raises the labor earnings of the employed. Therefore, the table also shows the MPCs for different subgroups of the population. As before, households are assigned to groups on the basis of their characteristics at the time of the gift. But we allow households to transit to other states thereafter. The first set of subgroups splits the population by wealth quartile. Next, we separate households by employment and life-cycle state. The third block shows the MPCs for different levels of idiosyncratic productivity, while the final block looks at MPCs by different time preferences.

The results in the table can be summarized as follows. As is typical in models like ours the MPC decreases in wealth. The wealth-poorest 25 percent of households ("$\leq$ Wealth25") spend about 81 percent of the gift within a year. The wealth-richest 25 percent ("Wealth75+"), instead, convert only roughly 9 percent of the gift into consumption within a year. Looking over the employment states we see that employed households without skill loss ("working age, $n = 1$, $l = 0$") spend about 1/3 of the gift within a year as they tend to be wealth-richer and tend to save both to insure against employment risk and for retirement. Employed households with a skill loss ("working age, $n = 1$, $l = 1$") and unemployed households ("working age, $n = 0$") have a significantly higher MPC. These households are wealth-poorer on average, as they have earlier used some of their wealth to stabilize consumption. In addition, they hope for a likely rise in income in case their employment or skill-loss state improves. Finally, retired households supplement their low but secure social security income. The same analysis can be applied to the effects of household skills. Finally, the MPC is decreasing in a household's patience. This is so because of both the direct effect of more forward-looking behavior and the induced stock of savings. Our calibration to the wealth distribution implies that lower-educated households tend to be less patient, and so tend to have higher MPCs.

[A21] We use year-2000 US\$, in line with the cut-off date for the tax functions employed in the model.

[A22] The heterogeneity in discount factors is an important factor for the average MPC, as discussed in Carroll et al. (2017).

TABLE A9

Cumulative Marginal Propensity to Consume (in percent)

	Cumul. MPC after quarter				Fraction of
	1	2	3	4	households
All	15.4	24.0	29.3	33.5	100
By wealth					
$\leq$ Wealth25	46.4	67.3	75.8	80.6	25.0
Wealth25-Wealth50	9.5	17.9	25.3	31.8	25.0
Wealth50-Wealth75	3.3	6.6	9.7	12.7	25.0
Wealth75+	2.2	4.4	6.6	8.7	25.0
By age and employment					
working age, $n = 1$, $l = 0$	13.7	22.6	28.6	33.1	25.5
working age, $n = 1$, $l = 1$	21.2	32.3	38.1	42.6	46.8
working age, unempl., $n = 0$	25.8	36.4	42.0	46.0	4.6
retired (s_0)	3.3	6.5	9.6	12.8	23.1
Working age, by skill					
s_1 (low)	28.3	41.3	46.9	50.4	38.1
s_2 (medium)	9.9	17.8	24.1	30.0	38.1
s_3 (high)	4.2	8.2	11.8	15.3	0.8
By education and patience					
β_{e_L}, low edu., patient	4.3	8.1	11.6	14.4	30.0
β_{e_H}, high edu., patient	2.0	3.9	5.9	7.8	20.0
$\beta_{e_L} - \Delta\beta$, low edu., impat.	28.9	44.4	52.9	58.9	30.0
$\beta_{e_H} - \Delta\beta$, high edu., impat.	25.1	37.6	44.1	49.0	20.0

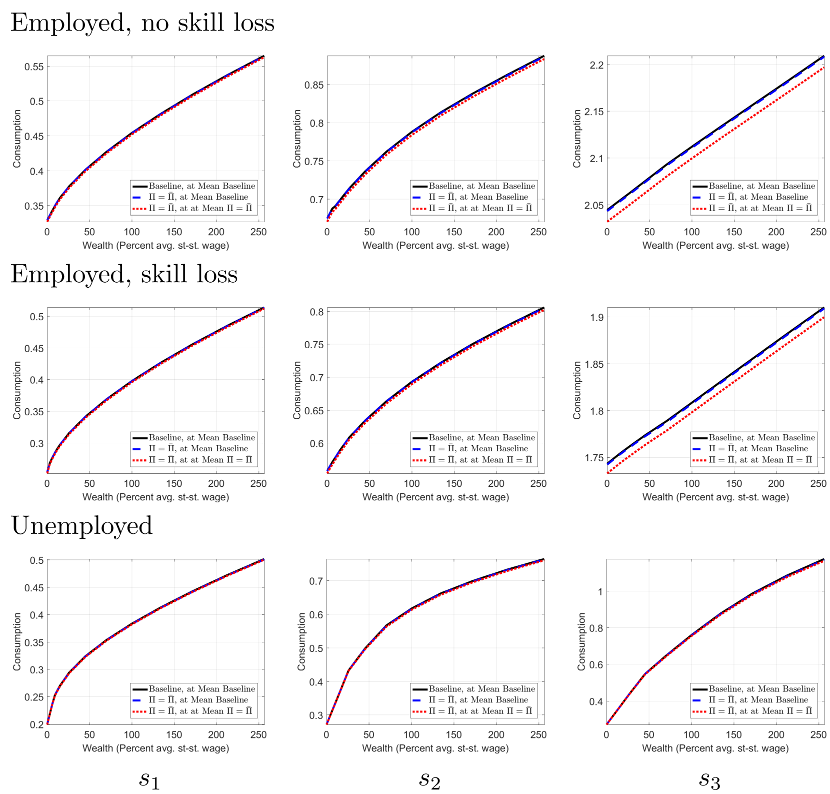
Notes: The table shows the share of a \$500 gift to an individual household that is spent after one quarter, two quarters, three quarters, and four quarters in the deterministic steady state. Column 'Fraction of households' denotes the percentage of the population in the respective group at the beginning of the first quarter. As per the rows, 'All' contains all households and reports the aggregate average MPC. Rows 'By wealth' split households into four equally sized groups by wealth and reports their respective MPCs with '$\leq$Wealth25' being the lowest wealth quartile and 'Wealth75+' being the highest. 'By age and employment' reports average MPCs for the retired (s_0), and three groups of working-age households. The first are the employed without skill loss ($n = 1, l = 0, s \in \mathcal{S}_+$), the next the employed with skill loss ($n = 1, l = 1, s \in \mathcal{S}_+$). Last, there are the unemployed households ($n = 0, s \in \mathcal{S}_+$). 'retired' contains all the retired households ($s = s_0$). 'Working age, by skill' groups the working-age households by their skill level (from s_1 (lowest) to s_3 (highest)). 'By education and patience' reports results for the different time preferences. 'β_{e_L}' summarizes the low-educated households with high β. 'β_{e_H}' describes the high-educated households with high β. The next two rows contain the same education groups but with lower β (impatient). For each of the groups, membership is determined at the beginning of the first quarter, before the gift is given. Households are allowed to transit to different states thereafter. Shares do not necessarily add up to 100 due to rounding.

I.10. Consumption policies in HANK

Figures A13 through A17 plot the consumption policies as a function of the wealth of a household, where the wealth is expressed as a percent of the average steady-state wage in the economy. For readability, we focus on the lower end of the policy functions. Each panel shows three lines. The blue solid line reports consumption policies under the baseline monetary policy rule, at the aggregate stochastic mean of that economy. A blue dashed line shows the consumption policies under strict inflation targeting, but evaluating the policies at the stochastic mean of the baseline economy. The red dotted line evaluates the consumption policy functions under strict inflation targeting at the stochastic mean state that arises under that policy. That is, differences between the blue solid and the blue dashed line are due to a change in policy only. Differences between blue dashed lines and blue dotted lines have the same policy but evaluate it at a different state.

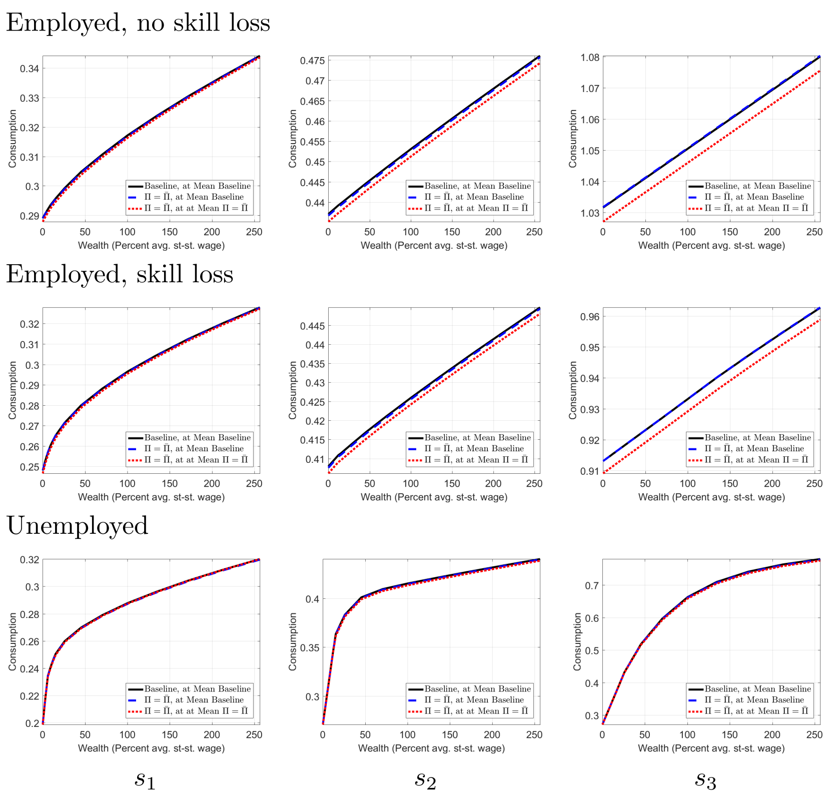
The change toward inflation targeting shifts the policy functions down for lower wealth levels, as households' expected labor income falls. For sufficiently rich individuals the lines cross and consumption increases in response to the rise in dividends and real rates (these wealth levels not shown here). One important point to keep in mind when interpreting the figures is that the wealth of households actually changes with the policy change; everybody with positive wealth becomes richer, as asset prices rise. However, the figure holds wealth constant.

Figure A13: Consumption policies of the working-aged, by skill: impatient households of low education



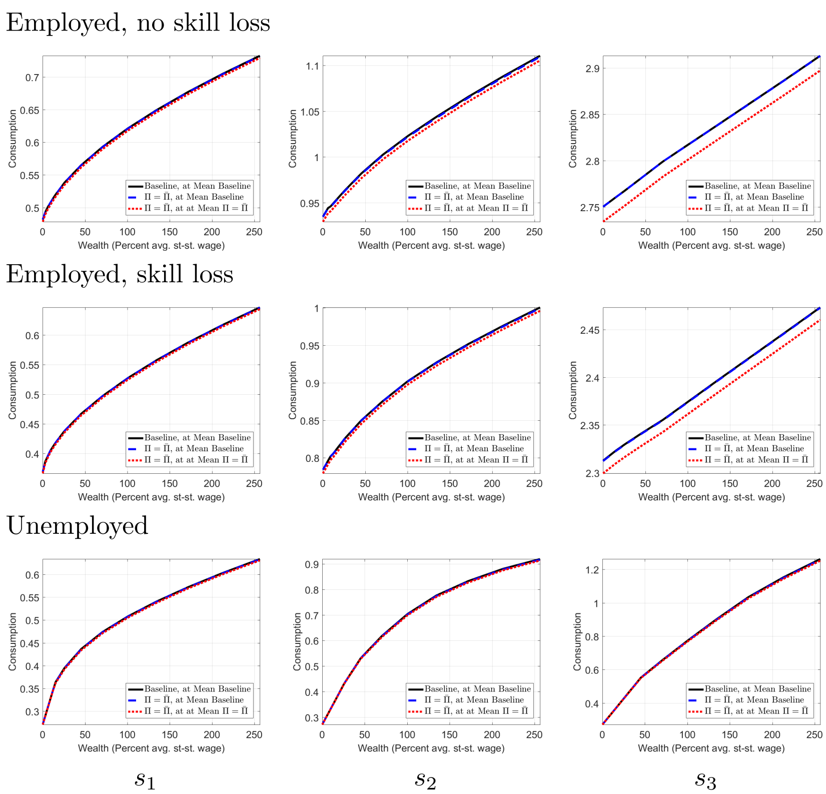
Notes: Consumption policies for households of working age that have low education and are impatient.

Figure A14: Consumption policies of the working-aged, by skill: patient households of low education



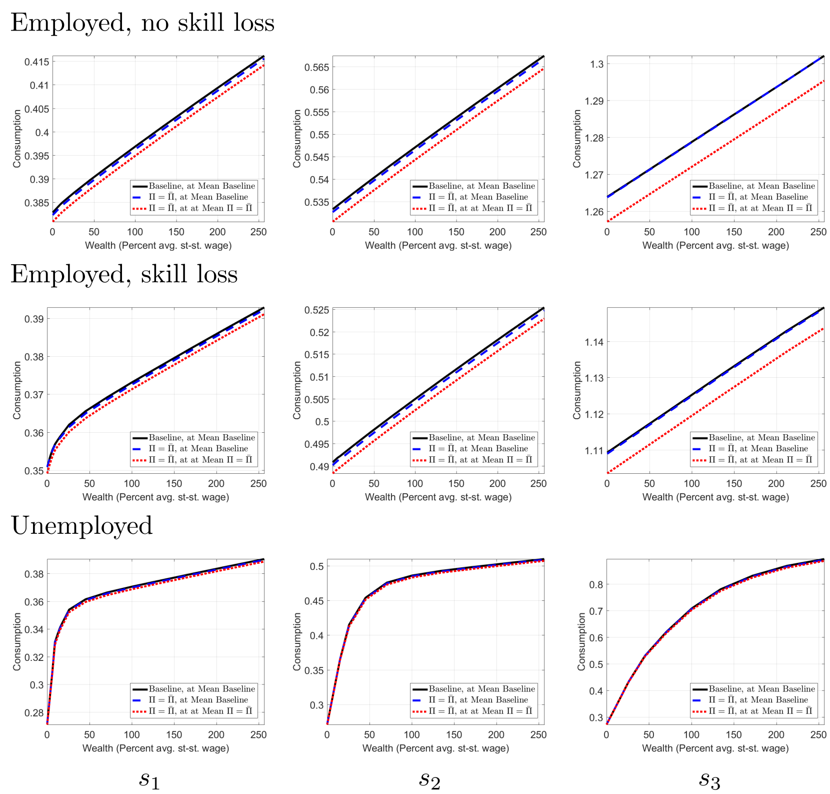
Notes: Consumption policies for households of working age that have low education and are patient.

Figure A15: Consumption policies of the working-aged, by skill: impatient households of high education



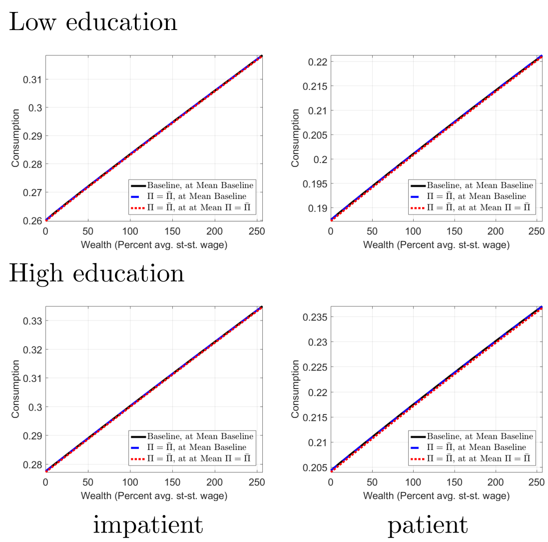
Notes: Consumption policies for households of working age that have high education and are impatient.

Figure A16: Consumption policies of the working-aged, by skill: patient households of high education



Notes: Consumption policies for households of working age that have high education and are patient.

Figure A17: Consumption policies of the retired, by education and patience



Notes: Consumption policies for households of retirement age, by education and patience.

I.11. Countercyclical skewness in earnings

Guvenen et al. (2014) document for the United States that cross-sectional logged labor earnings growth becomes more left-skewed during recessions, while the cross-sectional standard deviation of logged labor earnings growth remains fairly stable on average. We would expect this prediction to also hold in our model as the fall in the job-finding rate during a recession implies that more workers fall into or remain in unemployment and a larger share of the population is affected by earnings loss. Conversely, in a cyclical expansion the flows toward employment rise and workers slowly regain their productivity. All these factors should lead to a procyclical skewness in earnings. Furthermore, we would expect a smaller influence of the business cycle on the cross-sectional standard deviation.

In the following, we document model statistics that are consistent with this idea. To be consistent with their yearly data, we focus on yearly logged labor earnings growth of households that are in the labor force, and that do not retire.[A23] We compute the *cross-sectional* standard deviation and skewness of these earnings growth rates for a 5000-periods-long panel simulation from the HANK model for one million agents.[A24] This gives us 5000 observations of the cross-sectional standard deviation and the cross-sectional skewness of earnings growth. We then compute the correlation of these series with log GDP growth and log employment growth, the latter two being used as cyclical indicators.

The average cross-sectional skewness is -0.24, so the left tail of earnings growth (drops in earnings) is longer than the right tail (rising earnings). The average standard deviation of the cross-sectional skewness is 0.0437, meaning that the cross-sectional skewness fluctuates notably over time. Instead, the cross-sectional standard deviation of earnings growth turns out to be rather stable over time. The average standard deviation of the cross-sectional standard deviation of earnings growth turns out to be one orders of magnitude smaller (0.0033 with a mean of 0.22) than the standard deviation of cross-sectional skewness, consistent with the evidence reported in Guvenen et al. (2014).

The skewness of cross-sectional earnings growth in the model is not only volatile; it is also procyclical. The correlation between GDP growth and the skewness of earnings growth in the model is 0.53 and the correlation of skewness with employment growth is 0.61.[A25] In other words, in booms the left tail of the earnings growth distribution is less pronounced, the opposite in recessions, in line with the above intuition. The smaller correlation with GDP growth than with employment is explained as follows. Unemployment is the main driver of skewness, linking skewness tightly to employment growth. A rise in GDP growth, instead, need not always have a positive effect on employment in a given period, depending on the innovations affecting the economy.[A26]

[A23]We include unemployment benefits in our earnings measure as they constitute taxable income in the U.S..

[A24]We allow for 1000 periods of burn-in in each simulation. The simulation uses the version of the model with monetary policy shocks and without adjustment to average inflation - the version we compared the data to.

[A25]When we compute the same correlations for the U.S. using the data in Guvenen et al. (2014) for 1984-2008 we find values of 0.58 and 0.78 for GDP and employment growth, respectively.

[A26]For example, if innovations to TFP lead to a rise in GDP growth in a period, this would coincide with a temporary fall in the job-finding rate, while a monetary policy shock would imply positive comovement of GDP growth and employment. Compare the impulse responses in Appendix I.6.

Appendix J: Adjusting for the effect on average inflation

The effect on average inflation may or may not be a desirable feature of the baseline model. This section reports, for the representative-agent economy, the same results on the inflation unemployment trade-off and on the welfare gains from policy changes that we showed before, with one difference: for each parameter ϕ_U, we now adjust the Taylor rule such that average inflation always is at the steady-state level. That is, it designs policy such that there is no effect on average inflation. Technically, this is done as follows. We adjust Taylor rule (2.4) by a term that shifts the nominal rate in the stochastic economy (but leaving the non-stochastic steady state in place). Let $\epsilon_t^{\text{adjust}}$ be a white noise standard normal shock. The adjusted Taylor rule takes the form:

(J.1)

$$\begin{aligned} \log\left(\frac{R(X)}{\overline{R}}\right) \quad = \quad & \phi_R \log\left(\frac{R_{-1}(X)}{\overline{R}}\right) \\ & +(1-\phi_R)\left[\phi_\Pi \log\left(\frac{\Pi(X)}{\overline{\Pi}}\right) + \phi_\epsilon E_t\left\{\left(\epsilon_{t+1}^{\text{adjust}}\right)^2\right\} - \phi_u\left(\frac{U(X)-\overline{U}}{\pi_S(\mathcal{S}_+)}\right)\right]. \end{aligned}$$

Note that the term involving the expectation is a constant that appears only in the stochastic version of the model, but not in the non-stochastic steady state. For each value of ϕ_U, we choose a ϕ_ϵ such that the average inflation rate stays at (very close to) the target level of 2 percent annualized throughout. In other words, whenever we change ϕ_U, we also change ϕ_ϵ.

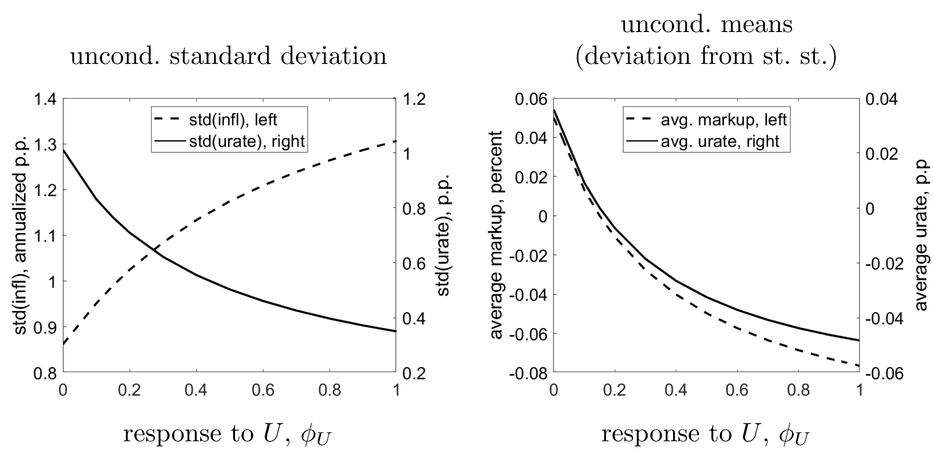
Appendix K: Inflation-unemployment trade-off in RANK/TANK

The current section documents the trade-off between inflation and unemployment that is inherent in the RANK/TANK economies. It complements the results for the HANK economy in Section 4.2 of the main text.

K.1. Inflation-unemployment trade-off and markups — RANK

The current section presents the inflation-unemployment trade-off in the RANK model.

Figure A18: RANK: Inflation-unemployment trade-off

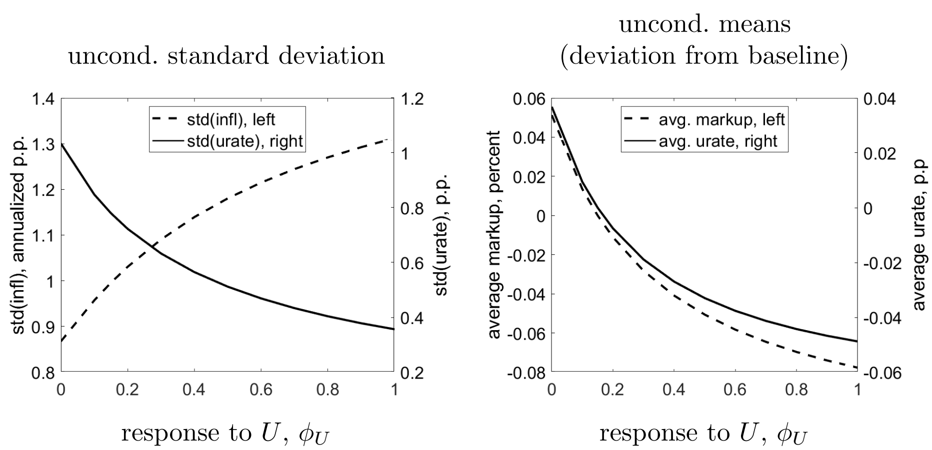


Notes: Same as Figure 2 in the main text, but for the RANK economy.

K.2. Inflation-unemployment trade-off and markups – TANK

The current section presents the inflation-unemployment trade-off in the TANK model.

Figure A19: TANK: inflation-unemployment trade-off



Notes: Same as Figure 2 in the main text, but for the TANK economy.

Appendix L: Welfare gains by shock

This appendix reports welfare results in HANK and RANK/TANK if only one shock is present at a time. Note that this change of scenario changes both the cyclical fluctuations in the economy and the mean of the economy under the baseline policy and, therefore, the starting point for the transition. Table A10 shows the distribution of welfare gains from changing policy if only the TFP shock is present in both the baseline and under the alternative monetary policy. On impact, unemployment rises after a productivity shock, and inflation falls: the Keynesian transmission mechanism. The calibration of the baseline means that real wages are not only rigid, but that they propagate the productivity shock. In spite of the real wage rigidity, though,

TABLE A10

WELFARE EFFECTS OF POLICY CHANGE - TFP SHOCK ONLY

		Response to unemployment, ϕ_u							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = 0$
HANK		Consumption-equivalent welfare gain (in percent)							
Wealth percentile	0-20	-0.004	-0.001	—	0.001	0.003	0.004	0.005	**0.019**
	20-40	-0.004	-0.001	—	0.001	0.003	0.004	0.005	**0.018**
	40-60	-0.005	-0.001	—	0.002	0.004	0.005	0.006	**0.018**
	60-80	-0.006	-0.001	—	0.002	0.005	0.006	0.008	**0.027**
	80-95	-0.008	-0.002	—	0.002	0.005	0.008	0.010	**0.049**
	95+	-0.007	-0.002	—	0.002	0.005	0.008	0.009	**0.051**
		Utilitarian sum of welfare (change)							
		-0.004	-0.001	—	0.001	0.003	0.004	0.005	**0.021**
		Share of households in favor over baseline							
		0.000	0.000	—	1.000	1.000	1.000	1.000	**1.000**
RANK		-0.003	-0.001	—	0.001	0.002	0.002	0.003	**0.009**
TANK	spender	-0.002	0.000	—	0.000	0.001	0.001	0.002	**0.005**
	saver	-0.003	-0.001	—	0.001	0.002	0.003	0.004	**0.011**

Notes: Welfare effects of a permanent policy change from the baseline policy, if the only shock is the TFP shock. The new policy either has a different response to unemployment, ϕ_u (central panels) or is characterized by strict inflation targeting ($\Pi = \overline{\Pi}$, last column). From top to bottom: HANK, lifetime consumption-equivalent welfare gains (in percent of consumption) by wealth, utilitarian consumption-equivalent welfare gains, share of votes in favor of the policy change, all taking the baseline as the alternative. Then, RANK and TANK spender/saver households.

households' assessment of optimal stabilization policy is virtually unanimous if the economy is driven by TFP only. Locally (for a small change in the rule), households prefer a switch toward a stronger unemployment response. Globally, households unanimously favor strict inflation targeting. Virtually all households would be willing to move toward strict inflation targeting if productivity were the only source of fluctuations. We conclude that failure of divine coincidence as in Faia (2009) and Ravenna and Walsh (2011) is not the central driving force of our results. Indeed, for the TFP shock, qualitatively, the HANK models' households' policy assessments are remarkably similar to RANK and to TANK (for both savers and spenders). The stakes are somewhat higher in HANK, however.

TABLE A11

Welfare Effects of Policy Change - MEI Shock only

		Response to unemployment, ϕ_u							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = 0$
		Consumption-equivalent welfare gain (in percent)							
Wealth percentile	0-20	0.000	-0.002	—	0.001	-0.002	-0.001	**0.001**	-0.004
	20-40	-0.004	-0.002	—	0.002	0.001	0.002	0.004	**0.008**
	40-60	-0.008	0.000	—	0.007	0.010	0.014	0.017	**0.033**
	60-80	-0.013	0.002	—	0.013	0.022	0.028	0.031	**0.084**
	80-95	-0.024	-0.006	—	0.006	0.015	0.022	0.028	**0.130**
	95+	-0.021	-0.005	—	0.005	0.013	0.019	0.025	**0.128**
		Utilitarian sum of welfare (change)							
		-0.008	0.000	—	0.006	0.010	0.013	**0.015**	**0.046**
		Share of households in favor over baseline							
		0.102	0.128	—	0.933	0.707	0.724	**0.893**	**0.772**
RANK		-0.012	-0.003	—	0.003	0.007	0.011	0.013	**0.034**
TANK	spender	-0.013	-0.003	—	0.003	0.008	0.011	0.014	**0.029**
	saver	-0.014	-0.004	—	0.003	0.008	0.012	0.015	**0.039**

Notes: Same as Table A10, but the only shock is the MEI shock.

Table A11 shows the welfare gains of changing the systematic monetary policy rule if only the MEI shock causes business-cycle fluctuations. Under the baseline policy, the MEI shock works like a demand shock, driving inflation and employment in the same direction. Stabilizing unemployment can, therefore, be conducive to stabilizing inflation. This explains why wealth-richer households benefit both from a bigger weight ϕ_u in the Taylor rule, and from strict inflation targeting. For poorer households, the gains from strict inflation targeting are less pronounced. The majority of households favor strict inflation targeting, but roughly 23 percent do not, namely, those in the lower wealth percentiles. For the MEI shock, there is disagreement in HANK, while RANK and TANK do not show any. In TANK, in particular, there is no disagreement between savers and spenders. In this sense, TANK is missing the disagreement present in HANK. Here, too, the stakes of individual groups in HANK can be notably bigger than RANK and TANK would signal.

TABLE A12

Welfare Effects of Policy Change - Price-Markup Shock only

		Response to unemployment, ϕ_u							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = 0$
		Consumption-equivalent welfare gain over baseline							
Wealth percentile	0-20	-0.012	-0.002	—	0.001	**0.002**	0.002	0.002	-0.136
	20-40	-0.007	0.000	—	0.000	-0.002	-0.005	-0.007	-0.136
	40-60	**0.014**	0.006	—	-0.006	-0.016	-0.026	-0.035	-0.060
	60-80	**0.028**	0.010	—	-0.009	-0.026	-0.041	-0.053	-0.011
	80-95	0.039	0.012	—	-0.011	-0.031	-0.048	-0.063	**0.044**
	95+	0.038	0.012	—	-0.011	-0.029	-0.045	-0.059	**0.061**
		Utilitarian sum of welfare (change)							
		0.007	0.003	—	-0.003	-0.010	-0.017	-0.023	-0.055
		Share of households in favor over baseline							
		0.494	**0.554**	—	0.313	0.279	0.259	0.245	0.260
RANK		-0.031	-0.007	—	0.005	0.010	0.013	**0.014**	-0.239
TANK	spender	-0.050	-0.012	—	0.009	0.021	0.028	**0.033**	-0.289
	saver	-0.028	-0.006	—	0.004	0.008	0.009	**0.009**	-0.227

Notes: Same as Table A10, but the only shock is the price-markup shock.

Next, Table A12 looks at the welfare gains if the price-markup shock is the only shock in the economy. The price-markup shock presents the monetary authority with a trade-off between stabilizing inflation and stabilizing unemployment. Next to this, fluctuations in inflation cause price adjustment costs, which our modeling imparts directly to the owners of capital. The HANK model suggests that households strongly disagree about how the monetary authority should handle cost-push shocks. Roughly 26 percent of households favor strict inflation targeting to the baseline policy. These households are in the top of the wealth distribution. Households at the bottom of the wealth distribution, instead, would be strongly opposed to this. What is important to note is that for the price-markup shock the HANK policy advice notably differs from the advice that the simpler variants give. Both RANK and TANK do not see any support for inflation targeting. And they favor a change to a rule with a larger response to unemployment ($phi_u = 0.5$). Note that, in HANK, this is a policy change that all but the poorest 20 percent of households would dislike. So, here too HANK is capturing disagreement that RANK/TANK would miss. And HANK leads to different policy conclusions. In HANK, a majority of households would wish to see slightly *more* inflation-centric policy; RANK/TANK, instead, favor a stronger inflation focus in the rule.

TABLE A13

Welfare Effects of Policy Change - Wage Shock only

		Response to unemployment, ϕ_u							
		0	0.1	0.2	0.25	0.3	0.4	0.5	$\Pi = \overline{\Pi}$
		Consumption-equivalent welfare gain over baseline							
Wealth percentile	0-20	0.004	0.001	—	-0.001	-0.004	-0.006	-0.007	**0.015**
	20-40	0.006	0.002	—	-0.002	-0.005	-0.008	-0.011	**0.013**
	40-60	0.010	0.003	—	-0.003	-0.010	-0.015	-0.020	**0.016**
	60-80	0.013	0.005	—	-0.005	-0.013	-0.020	-0.026	**0.016**
	80-95	0.017	0.005	—	-0.005	-0.014	-0.022	-0.029	**0.022**
	95+	0.016	0.005	—	-0.005	-0.013	-0.020	-0.026	**0.022**
		Utilitarian sum of welfare (change)							
		0.008	0.003	—	-0.003	-0.007	-0.012	-0.015	**0.011**
		Share of households in favor over baseline							
		1.000	1.000	—	0.000	0.000	0.000	0.000	**1.000**
RANK		**0.002**	0.001	—	-0.001	-0.004	-0.007	-0.009	-0.004
TANK	spender	-0.002	0.000	—	0.000	-0.001	-0.001	-0.002	-0.010
	saver	**0.003**	0.002	—	-0.002	-0.005	-0.008	-0.011	-0.003

Notes: Same as Table A10, but the only shock is the wage shock.

Last, Table A13 looks at the welfare gains from policy changes if the wage-markup shock is the only shock in the economy. The pattern in HANK that emerges from Table A13 is unanimous. The wage-markup shock not only presents the central bank with a trade-off between output and inflation stabilization. It also directly distorts poor households' consumption plans. Even though the wage-markup shock in our model works like a cost-push shock, households unanimously favor a more inflation-centric approach (including strict inflation targeting). This is not the case in RANK or TANK. The RANK households favor a somewhat more inflation-centric approach, but falling short of strict inflation targeting. The preferred policy of TANK spenders is the baseline policy. The preferred policy of TANK savers is the RANK model's optimal policy. In sum, for wage-markup shocks, too, the HANK policy advice differs from the advice that simpler models would give.

Appendix M: Welfare effects of a one-sided monetary shock

As a point of reference, the current section reports the welfare gains or losses from a one-time monetary shock. This serves as a reference both for the magnitude of welfare gains and so as to discuss the magnitude of gains. In the baseline, a contractionary monetary shock reduces

TABLE A14

WELFARE GAINS FROM A MONETARY TIGHTENING – UNDER BASELINE POLICY

		Baseline model		Portfolio variant	
		c equiv.	US$ (2004)	c equiv.	US$ (2004)
Wealth percentile	0-20	-0.264	-5,426	-0.264	-5,416
	20-40	-0.231	-5,142	-0.225	-5,170
	40-60	-0.172	-4,305	-0.269	-6,907
	60-80	-0.071	-1,755	-0.103	-2,635
	80-95	-0.036	-1,255	-0.021	-826
	95-100	-0.019	-1,064	0.008	1,573
	top 1% only	0.003	982	0.056	10,148
	Utilitarian	-0.067	—	-0.081	—
	Vote	0.01	—	0.114	—
	All $	—	-3,567		-9,232

Notes: Welfare gains from a one-time monetary shock (negative numbers are welfare losses), 100 bps annualized. Shown are two cases:without accounting for the portfolio composition (the baseline) and with accounting for the composition as in Section 5.2 of the main text. And for each of these, the table shows the consumption-equivalent welfare gain (in percent of lifetime consumption) and the dollar-equivalent welfare gain (in 2004 US$).

lifetime welfare for all but the richest 1 percent of households, that is, for the large majority of households. The monetary contraction induces a persistent recession; compare Figure A6. This is particularly costly in terms of consumption-equivalents for low-wealth households for two reasons. On the one hand, these households have few assets to self-insure against the unemployment risk that comes with the recession. On the other, these households tend to be relatively impatient in the first place. For impatient households, the near future gets stronger weight than for patient households; so for the impatient, the welfare losses of a persistent recession in the near term are particularly steep. For the lowest 20 percent of the wealth distribution, the welfare costs of a contractionary one-time monetary shock are twice as large as the welfare costs of systematically more hawkish policy; compare Table IX. For the wealth-richest instead, a change in systematic monetary stabilization policy easily carries the day, relative to the welfare effects of a one-time monetary shock. Indeed, the top 1 percent gain from a contractionary monetary policy shock, but only 0.0033 percent of lifetime consumption, or $982 in total.

Another important result emerges from Table A14. In particular, compare the columns on the right to column “Leveraged portfolio” of Table XVI in the main text. In both cases, households hold a mix of nominal and real assets, as described in Section 5.2 of the main text. The role that the household portfolio plays in allocating gains and losses is fundamentally different, however. To see this, focus on the middle class in both tables (households in the 40th-60th percentile of net worth). These households tend to hold rather leveraged portfolios, being long in real assets and having nominal debt. With such portfolios, a surprise monetary tightening is particularly costly for this group of households because their nominal debt (which we assume is short-term debt) exposes them to the ensuing higher real rates and a higher real debt burden; next to the higher risk of losing employment. The role of leverage is considerably different, instead, when considering a move to *systematically* more hawkish policy. Namely, in the latter case leverage exposes the middle class to the windfall gains to financial wealth that, in our model, are associated with a move toward inflation targeting.

The portfolio composition also plays a considerable role for the welfare gains that the richest households have from a one-time monetary tightening. Even then, however, a change toward systematically more hawkish policy brings larger welfare gains than the one-time monetary tightening for the richest households.

Appendix N: Welfare gain by dimension of heterogeneity

The current appendix provides the welfare gains of a policy change grouping households by idiosyncratic states. The first block of Table A15 reproduces the entries of Table X in Section 4.4 in the main text. The remaining blocks select households based on other idiosyncratic states.

TABLE A15

ONE-TIME DOLLAR-EQUIVALENT GAIN BY DIMENSION OF HETEROGENEITY

		Response to unemployment, ϕ_u							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = \overline{\Pi}$
skills	s_0 (retired)	1,672	665	—	-276	-1,087	-1,807	-2,528	**7,815**
	s_1 (low)	-284	-24	—	**38**	-28	-134	-206	-162
	s_2	-337	-12	—	**35**	-72	-232	-343	**166**
	s_3 (super)	2,048	753	—	-742	-2,279	-3,548	-4,786	**19,636**
empl., loss	$n = 0$ (unemp.)	-360	-30	—	**54**	-6	-128	-185	-905
	$n = 1, l = 1$	-318	-20	—	**41**	-37	-169	-253	-127
	$n = 1, l = 0$	-220	**10**	—	1	-149	-321	-468	**992**
educ.	e_L	-45	**87**	—	34	-118	-308	-440	**87**
	e_H	443	221	—	-150	-572	-971	-1,377	**4,637**
pati.	$b = 1$ (impat.)	-112	**23**	—	-15	-166	-352	-418	-856
	$b = 0$	421	262	—	-64	-436	-800	-1,223	**4,756**

Notes: Same as first block of Table IX, but sorting the population by dimensions of heterogeneity other than net worth. From top to bottom: residual skill (retired, low skill, medium skill, super-skill), current employment status (unemployed, employed with skill loss, employed without skill loss), education status (low, high), patience (less patient than average comparably educated, more patient than average comparably educated). Average dollar-equivalent gains for each group (2004 US$).

Table A16 provides the corresponding consumption-equivalent welfare gains in percent.

TABLE A16

Consumption-equivalent welfare gain by dimension of heterogeneity

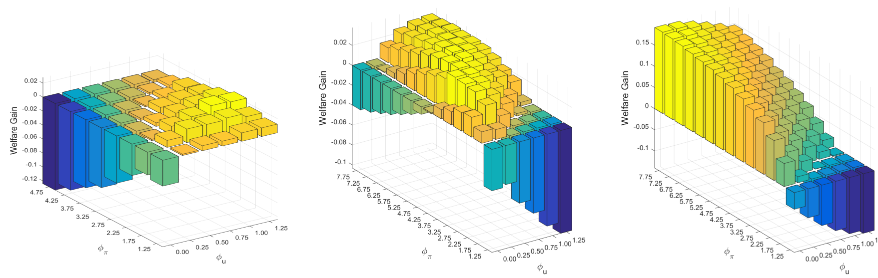
		Response to unemployment, ϕ_u							
		0	0.1	0.15	0.2	0.3	0.4	0.5	$\Pi = \overline{\Pi}$
skills	s_0 (retired)	**0.063**	0.026	—	-0.009	-0.040	-0.069	-0.096	**0.276**
	s_1	-0.019	-0.003	—	0.004	**0.004**	0.001	0.000	-0.056
	s_2	-0.018	-0.002	—	**0.003**	0.001	-0.003	-0.005	-0.041
	s_3	**0.015**	0.006	—	-0.007	-0.020	-0.032	-0.043	**0.151**
educ.	e_L	-0.006	**0.003**	—	0.002	-0.003	-0.011	-0.016	-0.018
	e_H	**0.008**	0.006	—	-0.003	-0.013	-0.024	-0.035	**0.092**
pati.	$b = 1$	-0.007	**0.000**	—	-0.000	-0.006	-0.014	-0.016	-0.046
	$b = 0$	0.007	**0.008**	—	0.001	-0.009	-0.019	-0.032	**0.100**
empl., loss	$n = 0$	-0.021	-0.003	—	0.004	**0.004**	0.001	0.001	-0.083
	$n = 1, l = 1$	-0.019	-0.003	—	**0.004**	0.003	0.000	-0.001	-0.054
	$n = 1, l = 0$	-0.016	-0.002	—	**0.002**	0.000	-0.004	-0.006	-0.025

Notes: Same as Table A15, but reporting average consumption-equivalent welfare gains (in percent).

Appendix O: Optimal simple rules

For the three wealth percentiles defined in Section 4.5, Figure A20 reports the consumption-equivalent welfare gains for alternative combinations of systematic monetary policy, (ϕ_Π and ϕ_u). The left panel shows the assessment of the bottom 20 percent by wealth, the middle panel

Figure A20: Welfare gains from switching Taylor rule



Wealth percentile 0-20 | Wealth percentile 40-60 | Wealth percentile 95-100

Notes: Welfare gains from alternative simple policies for different wealth percentiles. Welfare gains from change to new combination of ϕ_Π and ϕ_u. We used grids $\phi_\Pi \in \{1.25, 1.5, ..., 7.75\}$ and $\phi_u \in \{0, 0.25, ..., 1.25\}$ to search for the preferred policy. For better readability, the left panel shows a smaller range of responses to inflation. Gains are expressed in percent of lifetime consumption. Negative gains are welfare losses

that of the central wealth percentiles, and on the right is the assessment by the wealthiest 5 percent of households.

Appendix P: Long-run policy assessment

This appendix collects information on the long-run welfare counterfactuals, and the effect of policy on average employment, and wages in the long run.

P.1. Long-run policy assessment — HANK

Table A17 reports the HANK welfare gains in the long run. Here we ask a household: "At your current idiosyncratic state (before the policy change), how much would you pay for an alternative monetary policy if you were to jump (with your shares and other idiosyncratic states) to an economy with that policy, the initial aggregate state of which is that economy's long-run stochastic mean?"[A27] In the long run, in HANK the vast majority of households

TABLE A17

Long-run Welfare Effects of changing policy – HANK

		Response to unemployment, ϕ_u								
		0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	$\Pi = \overline{\Pi}$
		Consumption-equivalent welfare gain (in percent)								
Wealth percentile	0-20	-0.141	-0.034	—	0.028	0.068	0.094	0.115	**0.129**	-0.646
	20-40	-0.131	-0.033	—	0.025	0.060	0.082	0.100	**0.114**	-0.598
	40-60	-0.110	-0.023	—	0.027	0.059	0.079	0.094	**0.105**	-0.495
	60-80	-0.085	-0.015	—	0.026	0.054	0.070	0.080	**0.087**	-0.329
	80-95	-0.066	-0.016	—	0.015	0.036	0.052	0.058	**0.065**	-0.125
	95+	-0.042	-0.011	—	0.009	0.021	0.033	0.038	**0.041**	**0.002**
		Utilitarian sum of welfare (change)								
		-0.070	-0.015	—	0.018	0.039	0.053	0.061	**0.068**	-0.284
		Share of households in favor over baseline								
		0.004	0.082	—	0.986	0.995	0.997	0.998	**0.995**	0.054

Notes: Same as Table VIII, but looking only at the long run, that is, abstracting from the transition.

would favor notably more accommodative monetary policy. The stakes are high. Namely, the poorest 20 percent of househoulds would be willing to give up almost 0.65 percent of their (already lower) lifetime consumption to avoid a move toward strict inflation targeting (top row, right-most column). And even the upper middle class (the 80-95th percentile of wealth) would be willing to pay 0.12 percent of lifetime consumption to avoid that policy change. Of the groups shown here, only the wealth-richest 5 percent of households would marginally favor strict inflation targeting. In contrast to the RANK and TANK models, in HANK putting the focus on the long run only, therefore, does change the policy evaluation notably. In the long run, also in HANK, the utilitarian planner would implement a monetary policy that is focused on unemployment stabilization (see row Utilitarian sum of welfare). Votes would support such a policy by a wide margin (bottom row). The support for inflation targeting, instead, would run at barely 5 percent of the vote.

[A27] In the computation here, households take their share-holding with them, not the market value of net worth. Rather, the long-run share price will differ for different policies, for reasons that we discuss in the text.

P.2. Long-run policy assessment — RANK and TANK

Table A18 focuses on the long run in the RANK and TANK economies. The table computes welfare gains asking a household: "How much would you be willing to pay to jump to an economy with an alternative monetary policy, and starting in that economy at its ergodic mean?" Compare this with Table XIV in the main text, which accounts for both the long run

TABLE A18

LONG-RUN. CONSUMPTION-EQUIVALENT GAIN FROM CHANGING POLICY – RANK/TANK

		Response to unemployment, ϕ_u								
		0.0	0.1	0.15	0.2	0.4	0.6	0.8	1.0	$\Pi = \overline{\Pi}$
RANK		-0.056	-0.013	—	0.009	0.028	**0.033**	**0.033**	0.031	-0.279
TANK	saver	-0.055	-0.012	—	0.009	0.024	**0.027**	0.026	0.023	-0.265
	spender	-0.086	-0.021	—	0.017	0.058	0.078	0.090	**0.097**	-0.391

Notes: Same as Table XIV, but looking only at the long run, that is, abstracting from the transition.

and the transition path. All households would prefer slightly more accommodative monetary policy than with the transition phase. And welfare gains or losses are somewhat larger than accounting for the transition in Table XIV, but qualitatively and in terms of substance, little changes. There is little disagreement across households, and there is no support for inflation targeting.

Appendix Q: Transitional dynamics

This appendix details the computation of average transitional dynamics. We build on Andreasen et al. (2018), the notation of which we use below. The first-order dynamics of the state equations are given by

$$x^f_{t+1} = h_x x^f_t + \eta\epsilon_{t+1}$$

The state equation's second-order dynamics are:

$$x^s_{t+1} = h_x x^s_t + \frac{1}{2}H_{xx}\left(x^f_t \otimes x^f_t\right) + \frac{1}{2}h_{\sigma\sigma}$$

The jump variables' policy function is:

$$y^s_t = g_x(x^f_t + x^s_t) + \frac{1}{2}G_{xx}\left(x^f_t \otimes x^f_t\right) + \frac{1}{2}g_{\sigma\sigma}.$$

We want to find the mean change from a point $(\bar{x}^f, \bar{x}^s)$

Observation 1: If we can find the component terms for the means of x we get the mean of y "for free." That is, we can focus on x. The mean dynamics for $E_0\left(x^f_h\right)$ are given by

$$E_0\left(x^f_h\right) = h^h_x \bar{x}^f$$

The dynamics for $\mathbb{E}_0(x^f_h \otimes x^f_h)$ are given by

$$\begin{aligned}
\mathbb{E}_0(x^f_h \otimes x^f_h) &= \mathbb{E}_0\left(\left(h^h_x\bar{x}^f + \sum_{j=1}^{h} h^{h-j}_x \eta\epsilon_j\right) \otimes \left(h^h_x\bar{x}^f + \sum_{j=1}^{h} h^{h-j}_x \eta\epsilon_j\right)\right) \\
&= \left(h^h_x\bar{x}^f\right) \otimes \left(h^h_x\bar{x}^f\right) + \mathbb{E}_0\left(\sum_{j=1}^{h} h^{h-j}_x \eta\epsilon_j \otimes \sum_{j=1}^{h} h^{h-j}_x \eta\epsilon_j\right) \text{ (as corr}(\epsilon_h, \bar{x}^f)\text{=0)} \\
&= \left(h^h_x\bar{x}^f\right) \otimes \left(h^h_x\bar{x}^f\right) + \sum_{j=1}^{h} \mathbb{E}_0\left(h^{h-j}_x \eta\epsilon_j \otimes h^{h-j}_x \eta\epsilon_j\right) \text{ (as corr}(\epsilon_h, \epsilon_k)\text{=0)},\ h \neq k \\
&= \left(h^h_x\bar{x}^f\right) \otimes \left(h^h_x\bar{x}^f\right) + \sum_{j=1}^{h} \mathbb{E}_0\left(h^{h-j}_x \eta \otimes h^{h-j}_x \eta\right)(\epsilon_j \otimes \epsilon_j) \\
&= \left(h^h_x\bar{x}^f\right) \otimes \left(h^h_x\bar{x}^f\right) + \sum_{j=1}^{h} \mathbb{E}_0\left(h^{h-j}_x \eta \otimes h^{h-j}_x \eta\right) vec(I_{nshocks}).
\end{aligned}$$

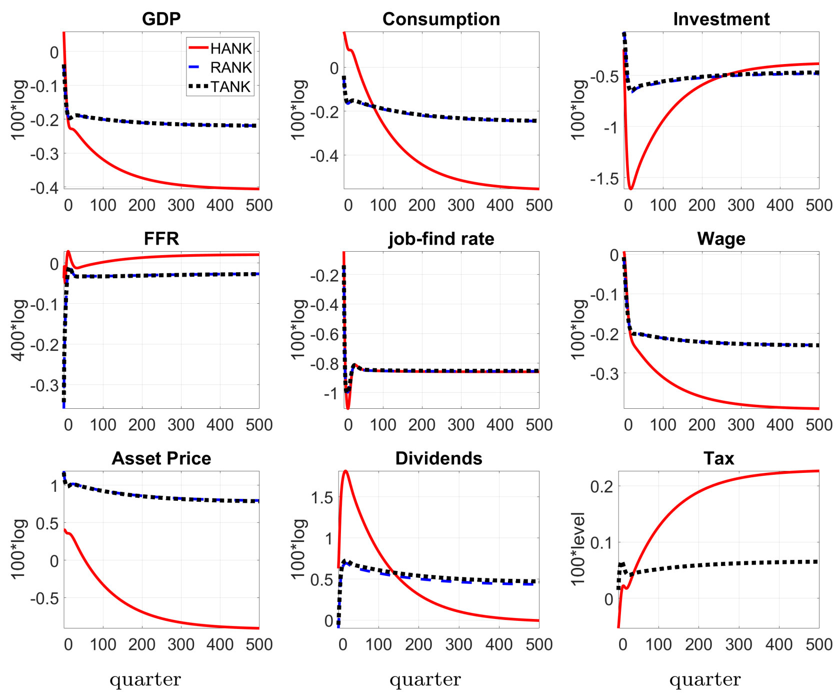
Dynamics for $\mathbb{E}_0(x^s_h)$:

$$\mathbb{E}_0(x^s_h) = h^h_x \bar{x}^s + 0.5\sum_{j=1}^{h} h^{h-j}_x \left(H_{xx}\mathbb{E}_0(x^f_{j-1} \otimes x^f_{j-1}) + h_{\sigma\sigma}\right)$$

Appendix R: Transitions plotted over longer horizon

Where Figure 3 in the main text has reported transition dynamics for 40 quarters, Figure A21 below sketches the entire transition phase by plotting 500 quarters.

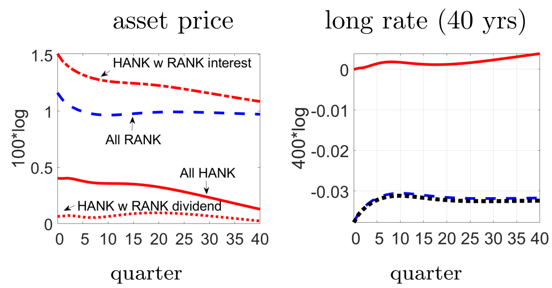
Figure A21: Transition toward policy of $\Pi = \overline{\Pi}$ − 125 years



Notes: Transition toward strict inflation targeting. Same as Figure 3 in the main text, but plotting 500 quarters of transition.

Appendix S: Decomposition of the asset price

Figure A22: Long rate, decomposition of the asset price



Notes: The right panel shows the long federal funds rate along the transition path; shown here is a 40-year long rate (inflation is constant by assumption, so the nominal rate equals the real rate of interest). The left panel decomposes asset price dynamics for the HANK model, based on present-value calculations. The red solid line gives the asset price transition in HANK. The red dotted line uses the dividend stream for the RANK economy, but the discount rate from HANK. The red dash-dotted line discounts dividends in HANK using the RANK discount rate. The blue dashed line uses both RANK dividends and the RANK discount rate.

The asset price in HANK rises less sharply than in RANK/TANK. The left panel of Figure A22 decomposes the differences in the response of asset prices in HANK and RANK into two sources: discounting and different dividend streams. The panel computes a present-value approximation for the asset price. Namely, we compute the present value of average dividends along the transition path, discounting by the approximate real rate. The red solid line is the approximation for the asset-price in HANK that we obtain; it closely matches the path shown in Figure 3. The same goes for the blue dashed line, which matches RANK asset price dynamics. The asset price in the HANK economy differs from that in the RANK economy for two reasons. First, the discounting differs. Toward this end, the right panel of the figure plots the transition of a 40-year long nominal rate. Since by the nature of the exercise inflation is constant after the policy change (strict inflation targeting), the trajectory of the nominal interest rate also describes the trajectory of the real interest rate. In line with the reduced savings by the wealthier households, documented in Table XIII, inflation targeting raises the real rate of interest in the HANK economy. Indeed, in the long run, the effect of a move toward inflation targeting sends the average federal funds rate 4.7 bps (annualized) higher in HANK than in the RANK or TANK counterpart. Second, the dividend stream differs; recall Figure 3. We introduce each of these two elements one by one. The red dashed-dotted line (the one on top) in the left panel of Figure A22 shows the effect of the interest rate alone. It uses the dividend stream from the HANK model alone, but discounts as in the RANK economy. Since dividends are higher in HANK, the dividend effect alone would let the asset price rise by more than in RANK, and much more (about three-fold) than in HANK. The discount-rate effect is quantitatively important. The dotted line at the bottom looks at the asset-price change that would result using the HANK discounting, but the RANK dividends. Since dividends are lower in HANK, the asset-price effect of the policy change would be lower, too. Combining the dividend and the discounting effect, the RANK price effect results.

Appendix T: Further results for the TANK economy

T.1. TANK welfare when spenders do not pool

This section provides the consumption-equivalent welfare gains for spenders when spender households do not pool incomes across idiosyncratic labor-market state, education, and age. Shown are results for the baseline calibration of the TANK model with 15 percent of spender households.

TABLE A19

SPENDERS – NO POOLING – CONSUMPTION-EQUIVALENT GAINS IN TANK

Spender type	Response to unemployment, ϕ_u								$\Pi = \overline{\Pi}$
	0.0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
	Throughout: Spender welfare only								
	Low education								
retired	-0.024	-0.005	—	0.004	0.009	0.011	0.013	**0.014**	-0.096
unemployed	-0.088	-0.021	—	0.016	0.040	0.055	0.065	**0.073**	-0.358
employed, skill loss	-0.076	-0.018	—	0.014	0.034	0.047	0.056	**0.062**	-0.325
employed, no loss	-0.089	-0.022	—	0.017	0.041	0.057	0.067	**0.075**	-0.375
	High education								
retired	-0.024	-0.005	—	0.004	0.009	0.011	0.013	**0.013**	-0.097
unemployed	-0.103	-0.025	—	0.019	0.047	0.064	0.077	**0.085**	-0.412
employed, skill loss	-0.077	-0.019	—	0.014	0.034	0.047	0.056	**0.062**	-0.331
employed, no loss	-0.091	-0.022	—	0.017	0.041	0.057	0.068	**0.075**	-0.389

Notes: Same as Table XIV, but calculating welfare based on the assumption that spenders do not pool their incomes.

T.2. Effects of varying the mass of spenders

The baseline TANK model categorizes 15 percent of the population as spenders. This is based on the strict notion of liquidity-constrained households. Table XV shows the welfare effects for savers in TANK when, instead, wealth is more concentrated, that is, when we assume that a bigger and bigger share of households are spenders. Table A20 reports the corresponding welfare gains for spender households.

TABLE A20

WELFARE GAINS FOR SPENDERS IN TANK BY SHARE OF SPENDERS

Share of spenders	Response to unemployment, ϕ_u								$\Pi = \overline{\Pi}$
	0.0	0.1	0.15	0.2	0.3	0.4	0.5	0.6	
50	-0.073	-0.017	—	0.013	0.031	0.042	0.050	**0.055**	-0.273
70	-0.081	-0.019	—	0.014	0.033	0.045	0.053	**0.059**	-0.265
75	-0.084	-0.019	—	0.015	0.034	0.047	0.055	**0.060**	-0.262
80	-0.087	-0.020	—	0.015	0.036	0.049	0.057	**0.063**	-0.258

Notes: Same as Table XV in the main text, but reporting the welfare gains for spenders.

Appendix U: Model with bonds and stocks

This section provides the write-up for the economy with bonds and stocks. We wish to analyze the political economy behind systematic monetary stabilization policy. At the same time, we wish to keep the setup tractable. The household portfolio has two components: the share of the mutual fund and short-term deposits (or loans, which are negative deposits). The latter are issued by competitive banks, which in turn are held by the mutual fund. Households decide how much net worth to accumulate. They do so knowing that the composition of the portfolio into stock-market wealth and nominal savings is determined by their net worth. In particular, let nw' be the real net worth that the household decides to accumulate for next period. Then, a share $\phi(nw'; e, s)$ of that wealth will flow into investing in stocks (so $p_a a' = \phi(nw'; e, s) \cdot nw'$); the remaining share will be invested in deposits ($deposit(nw') = (1 - \phi(nw'; e, s))nw'$). The portfolio compositions by net worth depend on education (e_L, and e_H) and retirement status ($s = s_0, s \neq s_0$). The former is meant to capture the effect of permanent income on the portfolio structure. The latter captures age effects on the portfolio structure (in particular, younger households having mortgages, whereas older households tend not to). Mutual funds have to provide those deposits at interest rate $R(X)$.

We first walk through the calibration of functions $\phi(nw'; e, s)$. Then, we walk through those problems that change relative to Section 2.

U.1. Definition of share wealth and bond wealth

We build the analysis on the SCF (in 2004). In particular, we define stock-market wealth as containing all non-nominal assets other than the household's vehicles (VEHIC) and SCF category other non-financial assets (OTHNFIN, which includes, among other items, furniture). We exclude these two components, since they include consumer durables. We do include housing in our measure of stock-market wealth, though. That is, we define stock-market wealth as

$$swth = EQUITY + BUS + HOUSES + ORESRE + NNRESRE.$$

$EQUITY$ is the total value of financial assets invested in stock. BUS is the value of businesses in which the household has an active interest. $HOUSES$ is the value of the primary residence. $ORESRE$ is the value of other residential real estate. $NNRESRE$ is the total value of net equity in non-residential real estate held by household.

Turning to savings, we have to make a decision. Namely, the nominal assets that households hold include nominal claims on the government. In keeping with the structure of the model (no government debt, balanced budget), in the modeling we assign all nominal claims by households to private-sector counterparties (the mutual funds). From the nominal claims, we subtract the nominal liabilities. We treat all nominal claims and liabilities as if they were short-term in nature. As in the baseline calibration, we do not include education loans when computing liabilities. With this, the net nominal savings (bond wealth) position of the households is

$$bwth = (FIN - EQUITY) - (MRTHEL + RESDBT + CCBAL + ODEBT + OTHLOC).$$

Nominal assets are financial assets net of equity ($FIN - EQUITY$). Nominal liabilities are mortgages on the primary residence ($MRTHEL$), other residential debt ($RESDBT$), credit card debt $CCBAL$, other debt ($ODEBT$, for example, loans against pensions or life insurance, margin loans) and other lines of credit not secured by real estate ($OTHLOC$).

A household's net worth then is defined as $nwth = swth + bwth$.

U.1.1. Portfolio composition by net worth

As to determining functions $\phi(nw'; e, s)$, we proceed as follows. We split the sample into two education states (high and low) and two age states (working age – age 25-65 ($s \in S_+$), and retired – age 66 and older ($s = s_0$), each by head of household as before).

For each of the four resulting groups, we wish to have a relationship that gives the portfolio composition by net worth. We focus on households with positive net worth. For working age households, we drop households that receive social security income. We are interested in the "typical" evolution of portfolio shares by net worth and so want to guard against outliers.

Toward this end, for each group, we split households into bins, by group-specific percentile of net worth. We use the same bins as in the main text (Table I), but split the lowest net worth bin in two, so as to have finer information. That is, we look at the seven percentile bins 0-10, 10-20, 20-40, 40-60, 60-80, 80-95, 95-100. For each of the these, we compute the interquartile means of $swth/nwth$ and of $nwth/annlinc$. Here $annlinc$ scales net worth by economy-wide average labor income of households ages 25-65.

Table A21 reports the resulting values. The lowest net worth households in each education-

TABLE A21

DATA. PORTFOLIO SPLIT BY NET WORTH, EDUCATION, AND AGE

	Group-specific percentile of net worth						
	0-10	10-20	20-40	40-60	60-80	80-95	Top 5
ages 25-65, low education							
$swth/nwth$ (in percent)	0	20	208	173	133	103	97
$nwth/annlinc$ (in percent)	1	5	31	94	232	644	1880
ages 25-65, high education							
$swth/nwth$ (in percent)	78	234	189	133	108	99	94
$nwth/annlinc$ (in percent)	10	51	144	350	792	2048	10351
ages 66+, low education							
$swth/nwth$ (in percent)	0	79	98	91	80	78	71
$nwth/annlinc$ (in percent)	2	28	103	243	466	1165	2932
ages 66+, high education							
$swth/nwth$ (in percent)	46	104	92	76	79	81	82
$nwth/annlinc$ (in percent)	23	168	342	747	1428	2988	12268

Notes: Based on SCF 2004. Households with heads ages 25 to 65 and households with heads ages 66 to 99, low education or high education. For each group, households are placed into the net worth percentile of the group. For each percentile shown: interquartile mean share of stock-market wealth in total net worth (in percent), first row; and interquartile average net worth scaled by economy-wide labor income of households ages 25-65 (in percent), second row.

age bin tend to hold none, or only a smaller share of their net worth in what we measure as stock-market wealth. That is, their savings are largely nominal and the real value of these savings is directly subject to fluctuations in inflation. For all age-education groups, the share of wealth in net worth is hump-shaped. Beyond the lowest net worth groups, as net worth rises, the young households tend to become indebted. For the 20th to 40th percentile, for example, the ratio of stock-market wealth (including housing in the data) to net worth is 2.08. That is, the ratio of bond wealth to net worth is -1.08. Using an average net worth to labor-income ratio of 0.31 for that group, the average debt in that percentile is worth $1.08 \cdot 0.31 = 0.33$, or about a third of the economy's average annual labor income. For higher-educated households the peak in nominal debt occurs earlier. Generally, as households grow wealthier, they grow out of debt. For each age-education group, the richest 5 percent of households in terms of net worth hold, on net, nominal savings. For high-education retirement-age households, for example, stock-market wealth accounts for 82 percent of net worth, so that nominal assets account for the remaining 18 percent.

U.2. Representing portfolio shares as a function of net worth

From Table A21, for each of the bins of the net worth distribution in the four education-age groups, we have observations on share of stock-market wealth in net worth and of net worth itself, the latter relative to economy-wide average labor income. Since households' savings choice is continuous, we need to have the portfolio composition for all feasible (that is, non-negative) values of net worth. Therefore, we fit a function to the observations in each bin that has the following properties:

- it is continuously differentiable.
- the share of stock-market wealth at zero net worth is zero.
- for low levels of net worth, the share of stock-market wealth can rise rapidly in net worth.
- there is a finite asymptote for the share of stock-market wealth in net worth as net worth rises.

Differentiability is important for the algorithm. The functional form at low net worth is important to fit the data summarized in Table A21. The asymptote is important if we want to be able to extrapolate. In particular, we choose a functional form that folds two differentiable functions f_1 and f_2 as follows

$$\begin{aligned} stockshare &= f(\log(1 + networth/annlinc); \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4; \vartheta_5, \vartheta_6) \\ &= f_2(\log(1 + networth/annlinc); \vartheta_5, \vartheta_6) \\ &\qquad \cdot f_1(\log(1 + networth/annlinc); \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4). \end{aligned}$$

All parameters $\vartheta_1, ..., \vartheta_6$ are positive. Here f_1 is chosen to resemble the functional form of the log-logistic probability density function, other than that we add a constant ϑ_1 that will serve as the asymptote for the stock share, and scale net worth by scaling parameter ϑ_4:

$$f_1(x; \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4) = \vartheta_1 + \frac{\vartheta_3/\vartheta_2 \cdot (\vartheta_4 \cdot x/\vartheta_2)^{\vartheta_3 - 1}}{(1 + \vartheta_4 \cdot x/\vartheta_2)^{2 \cdot \vartheta_3}}.$$

That part of f can deliver the skewness in the data and the asymptote. f_2 is responsible for the other two properties, with

$$f_2(x; \vartheta_5, \vartheta_6) = \frac{1}{1 + (x/\vartheta_5)^{-\vartheta_6}}$$

being the cumulative distribution function of the log-logistic distribution. ϑ_5 determines where the function bends, and ϑ_6 determines the steepness of f_2 at that point. For $\vartheta_6 \to \infty$, f_2 would resemble an indicator function with the step at ϑ_5. This part of the functional form allows the stock share to rise rapidly in net worth at the low end of the net worth distribution.

For each education/age group, we choose parameters $\vartheta_1, ..., \vartheta_6$ to minimize the sum of absolute deviations (summing over the bins) between the data, and the stock wealth implied by f. Table A22 reports the fitted parameters.

TABLE A22

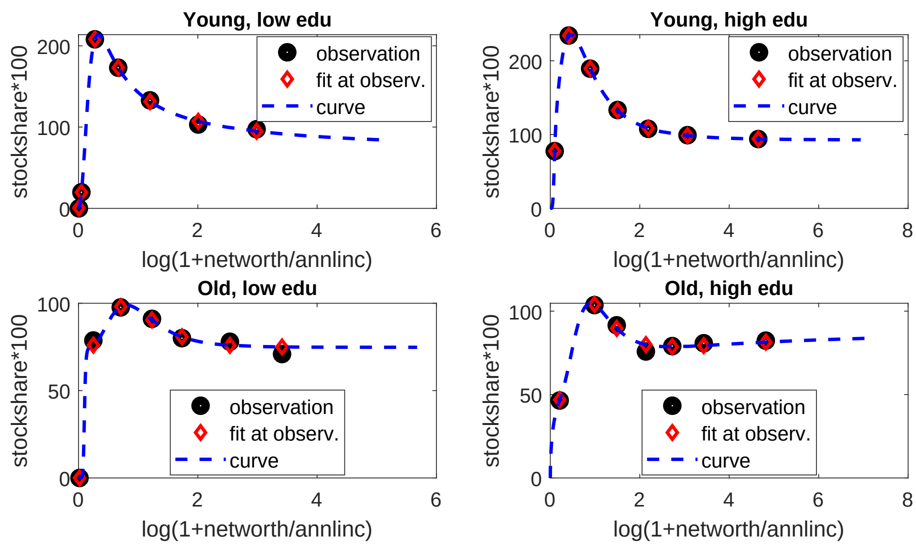
Data. Portfolio split by net worth, education, and age

Fitted param value	ϑ_1	$\vartheta_2 \cdot 1000$	ϑ_3	$\vartheta_4 \cdot 1000$	ϑ_5	ϑ_6
low-edu young	0.6865	2240.1273	0.0238	27.9758	0.2319	2.7564
high-edu young	0.9261	19.5075	3.7261	24.4083	0.0872	5.1419
low-edu old	0.7480	0.0856	9.4790	0.0863	0.0990	8.7097
high-edu old	0.9406	0.0264	9.7277	0.0263	0.2197	0.6025

Notes: Fitted parameters for stock shares.

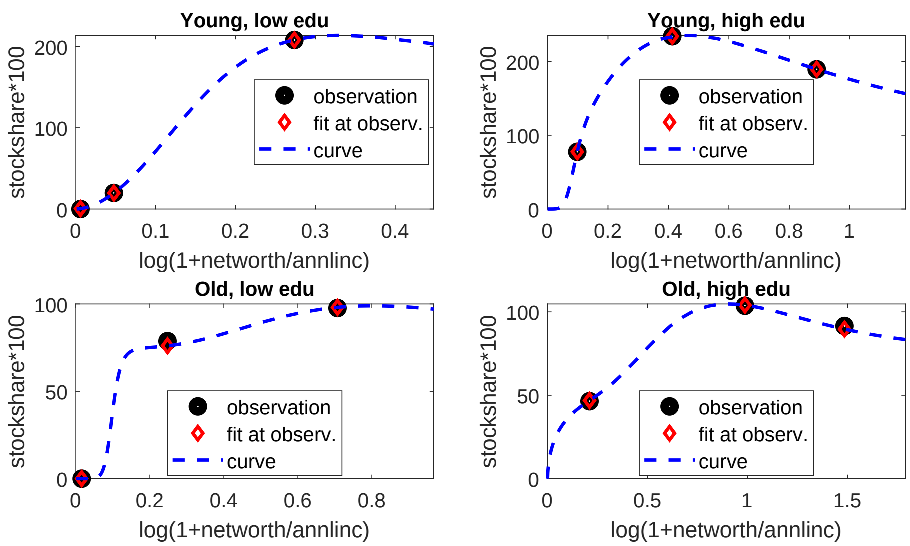
Figure A23 plots the resulting distributions and the fit. Figure A23 zooms in on the implied representation for the lower tail of the distribution.

Figure A23: Stock share by net worth, education, and age



Notes: This plots the fitted share of net worth invested in shares for the four education/age groups. In each panel, the y-axis gives the percent of net worth invested in shares. In each panel, the x-axis plots $\log(1 + networth/annlinc)$, where $networth$ is the household's net worth and $annlinc$ is the average annual labor income of working-age households (where the average is taken over all education groups).

Figure A24: Stock share – focusing on low net worth percentiles



Notes: Same as Figure A23, but plotting only the lower tail of net worth. For each group, the maximum value on the x-axis corresponds to net worth at the 40th percentile of the respective group's distribution of net worth.

Next, we turn to changes in the modeling implied by introducing the portfolio dimension.

U.3. States

To be able to track household portfolios without introducing more idiosyncratic states at the household level, we need to keep track of the past price of the stock $p_{a,-1}$ (so that $p'_{a,-1} = p_a$). Let nw' denote the net worth that the household carries into the next period. Note that nw' and p_a are sufficient to compute the household's asset allocation in stocks a, and *deposits* (positive or negative) today. At the same time, for allocating returns from the portfolio, we need the same information to also be available next period. This is why the problem with household portfolios introduces one additional aggregate state.

U.4. Households' problems

Relative to the household problem in Section 2.3 of the main text, the household problem changes as it has to account for the borrowing and lending by households. We assume that the household chooses net worth and that household-level net worth chosen last period and the aggregate states are sufficient to uniquely pin down the asset allocation.

U.4.1. Employed households

Let $W(X, n, nw, l, e, b, s)$ be the value of a household at the time of production. Here, nw' is the net worth that the household chooses to invest in the current period. The employed household's Bellman equation ($n = 1$, $s \in \mathcal{S}_+$) is given by

$$\begin{aligned} W(X,1,nw,l,e,b,s) = \max_{c,nw'\geq 0,a',deposits'} \Big\{ & u(c) + \pi_{s_0}\,\mathbb{E}_\zeta\big[\beta(e,b)W(X',0,nw',0,e,b,s_0)\big] \\ & + \textstyle\sum_{s'\in\mathcal{S}_+} \pi_S(s,s')\beta(e,b) \cdot \\ & \mathbb{E}_\zeta\Big[\, [1-\lambda_x(e)-\lambda_n(e)(1-f(\widetilde{X}'))] \textstyle\sum_{\hat{l}} \pi_L^{emp}(l,\hat{l}) \cdot \\ & \qquad W(X',1,nw',\hat{l},e,b,s') \\ & + [\lambda_x(e)+\lambda_n(e)\big(1-f(\widetilde{X}')\big)]W(X',0,nw',0,e,b,s')\,\Big]\Big\} \\ & \text{s.t.} \end{aligned}$$

$$\begin{aligned} (1+\tau_c)c + p_a(X)a' + deposits' &= [p_a(X)+d_a(X)]\,a \\ &\quad + deposits \cdot R_{-1}(X)/\Pi(X) \\ &\quad + w(X)es(1-l\varrho)\,[1-\tau_{RET}-\tau_{UI}] \\ &\quad - w(X)es(1-l\varrho)\tau(X, w(X)es(1-l\varrho)). \\ p_a a' &= \phi(nw';e,s)\cdot nw'. \\ nw' &= p_a a' + deposits'. \end{aligned}$$

When choosing to accumulate net worth, the household acquires a certain share of stocks and of deposits, governed by function $\phi(nw'; e, s)$. Deposits can be positive or negative. The interest rate applied to deposits is the same as the interest rate for loans. The nominal return on deposits is $R(X)$ next period. Similarly, with knowledge of the past price of the asset, the net worth carried into the period can be split into stocks and deposits and returns allocated accordingly.

U.4.2. Unemployed households

The unemployed household's Bellman equation ($n = 0$, $s \in \mathcal{S}_+$) is given by

$$\begin{aligned}
W(X,0,nw,0,e,b,s) = \max_{c,nw'\geq 0,a',deposits'} \Big\{ & u(c) + \pi_{s_0}\,\mathbb{E}_\zeta[\beta(e,b)W(X',0,nw',0,e,b,s_0)] \\
& + \textstyle\sum_{s'\in\mathcal{S}_+} \pi_S(s,s')\beta(e,b)\cdot \\
& \mathbb{E}_\zeta\Big[\quad f(\widetilde{X}')[\quad \pi_L^{uem}(1)W(X',1,nw',1,e,b,s') \\
& \qquad +\pi_L^{uem}(0)W(X',1,nw',0,e,b,s')] \\
& \quad +\big(1-f(\widetilde{X}')\big)W(X',0,nw',0,e,b,s')\Big] \Big\}
\end{aligned}$$

s.t.

$$\begin{aligned}
(1+\tau_c)c + p_a(X)a' + deposits' = & \; [p_a(X)+d_a(X)]\,a \\
& + deposits\cdot R_{-1}(X)/\Pi(X) \\
& + b_{UI}(es)[1-\tau(X,b_{UI}(es))], \\
p_a a' = & \; \phi(nw';e,s)\cdot nw', \\
nw' = & \; p_a a' + deposits'.
\end{aligned}$$

U.4.3. Retired households

The retired household's Bellman equation ($s = s_0$) is given by

$$\begin{aligned}
W(X,0,nw,0,e,b,s_0) = \max_{c,nw'\geq 0,a',deposits'} \Big\{ & \quad u(c) \\
& + \pi_S(s_0,s_0)\beta(e,b)\mathbb{E}_\zeta\left[W(X',0,nw',0,e,b,s_0)\right] \\
& + (1-\pi_S(s_0,s_0))\mathbb{E}_\zeta\left[\gamma_1\cdot(p_a(X')a+\gamma_2)^{1-\sigma}/(1-\sigma)\right] \\
& + \beta(e,b)\textstyle\sum_{s'\in\mathcal{S}_+}\sum_{e'}\sum_{b'}\sum_{l}\pi_S(s_0,s')\pi_E(e,e')\pi_{\Delta_\beta}(b')\cdot \\
& \quad \Pr(n=1,l|X,e')\mathbb{E}_\zeta\Big[[1-\lambda_x(e')-\lambda_n(e')(1-f(\widetilde{X}'))]\cdot \\
& \qquad \textstyle\sum_{\hat{l}}\pi_L^{emp}(l,\hat{l})W(X',1,nw',\hat{l},e',b',s') \\
& \quad +[\lambda_x(e')+\lambda_n(e')\big(1-f(\widetilde{X}')\big)]W(X',0,nw',0,e',b',s')\Big] \\
& + \beta(e,b)\textstyle\sum_{s'\in\mathcal{S}_+}\sum_{e'}\sum_{b'}\pi_S(s_0,s')\pi_E(e,e')\pi_{\Delta_\beta}(b')\cdot \\
& \quad \Pr(n=0|X,e')\mathbb{E}_\zeta\Big[f(\widetilde{X}')\big[\pi_L^{uem}(1)W(X',1,nw',1,e',b',s') \\
& \qquad +\pi_L^{uem}(0)W(X',1,nw',0,e',b',s')\big] \\
& \quad +[1-f(\widetilde{X}')]W(X',0,nw',0,e',b',s') \quad \Big]\Big\}
\end{aligned}$$

s.t.

$$\begin{aligned}
(1+\tau_c)c + p_a(X)a' + deposits' &= \; [p_a(X)+d_a(X)]\,a \\
&\quad + deposits\cdot R_{-1}(X)/\Pi(X) \\
&\quad + b_{RET}(e)[1-\tau(X,b_{RET}(e))], \\
p_a a' &= \phi(nw';e,s)\cdot nw', \\
nw' &= p_a a' + deposits'.
\end{aligned}$$

U.5. Non-financial firms

The block of the model that describes the non-financial firms does not change relative to the exposition in the main text. Non-financial firms continue to be owned by competitive mutual funds, the latter to be described next.

U.6. Financial firms

What needs to be adapted relative to the baseline in Section 2.5 of the main text is the financial sector.

U.6.1. Banks

Banks are owned by the mutual fund sector. In the current paper, we do not seek to provide a deeper modeling of the banking sector as a source of propagating macroeconomic shocks. Instead, we assume that there are representative, perfectly competitive banks. On the liability side, banks can issue demand deposits ($hhdeposits' > 0$) to households, or attract wholesale funding from mutual funds $mfdeposits'$, the former at gross nominal interest rate R^D, the latter at gross nominal interest rate $R(X)$. On the asset side, banks can lend to mutual funds ($mfloans$) at funding rate R, or they can issue one-period nominal loans to the household sector ($hhloans' > 0$). Lending to households has a one-period rate of return R^L. We assume that there is a fixed per-unit cost of issuing loans of $\phi_L > 0$ (say, for monitoring the loan) and that there is a fixed per-unit cost of issuing demand deposits $\phi_D > 0$. Let $mf' := mfdeposits' - mfloans'$ mark the bank's net borrowing from the mutual fund sector. Assuming that the bank does not have net worth to start with, the bank's cash-flow constraint today is

$$hhdeposits' + mf' = hhloans' + (\phi_D hhdeposits' + \phi_L hhloans').$$

The left-hand side is deposits raised from households plus lending by the mutual fund sector. These proceeds are used for (right-hand side) making loans to mutual funds or households, and for covering the costs of issuing deposits or loans. At the beginning of the next period, cash flows in real terms are given by

$$loans' \cdot R^L/\Pi' - \left(deposits' \cdot R^D/\Pi' + mf' \cdot R/\Pi'\right)$$

Since banks are owned by the mutual funds, the optimality conditions for the banks are as follows. The indifference condition for loans is

$$R^L = (1 + \phi_L)R.$$

The indifference condition for issuing deposits is

$$R^D = (1 - \phi_D)R.$$

With this, it is trivial to show that banks, in equilibrium (due to perfect competition and constant returns to scale), make zero profits. In what follows, we will focus on the case $\phi_L \to 0$ and $\phi_D \to 0$, so that $R^D(X) = R^L(X) = R(X)$.

U.6.2. Mutual funds

The mutual funds' problem is the same as in the baseline model, with the one exception that we now have to account for financial investment by the mutual funds $mf(X)$. The banks owned by the mutual fund sector make zero profits. To the extent, however, that the banking system's exposure to mutual funds (or vice versa) $mf(X)$ is not equal to zero, the mutual fund will earn interest income We continue to apply the cashless limit assumption. The mutual fund distributes to the households all income that is not reinvested in physical or financial capital, after paying taxes to the government. After-tax dividends are given by

$$\begin{aligned} d_a(X) \quad = \quad & (1-\tau_d)\big(y_f(X) - i(X) - \textstyle\int_{\mathcal{M}} w(X)se(1-\varrho l)\mathbb{1}_{n=1}\, d\mu \\ & + mf \cdot R_{-1}/\Pi \\ & - mf'(X) \qquad\qquad \big), \end{aligned}$$

where $\mathbb{1}$ marks the indicator function, meaning $\mathbb{1}_{n=1}$ marks employment of the household.

U.7. Central bank and fiscal authority

The descriptions of the central bank and the fiscal authority are exactly as in the baseline.

U.8. Market clearing and equilibrium

To close the description of the model, in the following we list the market-clearing conditions. We list only those market clearing conditions that differ from the baseline model.

Let $nw'(X, n, nw, l, e, b, s)$ be the net worth policy function. Total demand for assets is given by $a'(X, n, nw, l, e, s)$

$$\int_{\mathcal{M}} p_a(X) a' \; d\mu = \int_{\mathcal{M}} \phi(nw'(X, n, nw, l, e, b, s); e, s) \; d\mu$$

With the mass of shares normalized to unity, that is, $\int_{\mathcal{M}} p_a(X) a' \; d\mu = 1$, the market-clearing price for the asset follows

$$p_a(X) = \int_{\mathcal{M}} \phi(nw'(X, n, nw, l, e, b, s); e, s) \; d\mu.$$

Note that we do not model the frictions or considerations that lead households to opt for a certain split in portfolios. Rather, we assume a fixed relationship between net worth and portfolio shares. This means that the price will have to move to make demand and supply mutually consistent. It also means that *ex-ante* returns on stocks and bonds need not coincide (households cannot arbitrage).

Last, the bond market clears if funding provided by mutual funds makes up for the shortfall of funding provided by deposits, that is, if

$$mf(X) = hhloans(X) - hhdeposits_t(X) := -\int_0^1 deposits' d\mu.$$

U.9. Calibration

The calibration is the same as in the baseline model, with one difference: we need the functions $\phi(nw'; e, s)$. They follow from Appendix U.2.