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Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements

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Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements [*]

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Abstract

This paper studies the effects of making corporate sector assets eligible as collateral for central bank borrowing. Banks are willing to pay collateral premia on assets if they become eligible as collateral. Collateral premia make debt financing cheaper for eligible firms, which respond by increasing their debt issuance. While this has a positive effect on collateral supply, firm responses also have a negative effect: higher debt issuance makes corporate bonds riskier in future periods, which in turn reduces aggregate collateral. We provide a novel analytical characterization of firm responses to eligibility requirements in a heterogeneous firm model with default risk and collateral premia paid on eligible bonds. Using a calibration of the model to euro area data, we study the impact of the ECB's collateral easing policy during the 2008 financial crisis and evaluate the quantitative relevance of firm responses. We find that firm responses substantially deteriorate collateral quality and dampen the total increase of collateral supply. Our analysis suggests that a *covenant* conditioning eligibility on leverage *and* current default risk is a potentially powerful instrument to mitigate the adverse impact of collateral premia on default risk and, thereby, to maintain a high level of collateral supply.

Keywords: Collateral Premia, Eligibility Requirements, Firm Heterogeneity, Corporate Default Risk, Collateral Policy

JEL Classification: E44, E58, G12, G32, G33

1 Introduction

Central banks typically implement monetary policy by lending to banks in exchange for collateral. This makes a sufficiently high supply of collateral essential to the financial system of most economies. During the financial crisis of 2008, it also required many central banks to expand the pool of assets they accept as collateral to facilitate the conduct of expansionary monetary policy. For example, the European Central Bank engaged in collateral easing when switching towards a full allotment regime in their Main Refinancing Operations and to enable banks to participate in Long-Term Refinancing Operations more easily. To expand the pool of eligible collateral, the ECB lowered the minimum rating requirement on assets, which added corporate sector assets of intermediate quality, such as corporate bonds and securitized bank loans, to the list of assets eligible as collateral.[1] The widespread inclusion of corporate sector assets is quantitatively relevant: corporate bonds and credit claims make up around 27% of used collateral in ECB operations.[2]

While collateral easing facilitates smooth conduct of monetary policy, a thorough assessment of such a policy must also account for endogenous responses of the corporate sector. Firm responses arise since banks increase demand for assets if they become eligible as collateral and firms cater to this demand by increasing their debt issuance. Mésonnier et al. (2021), Pelizzon et al. (2020), and Mota (2021) provide empirical evidence for this behavior. Crucially, increasing the issuance of marketable debt instruments is associated with higher leverage, in particular for high-rated borrowers, as demonstrated by Grosse-Rueschkamp et al. (2019) in the context of eligibility for QE programmes.[3] While debt supply effects are desirable in the context of monetary policy implementation, firm responses also have adverse side effects, which limit the efficacy of collateral easing: a higher amount of debt outstanding increases the risk of default in future periods. This paper presents a novel theoretical framework to study endogenous firm responses to eligibility requirements in the presence of default risk. While our framework can be applied to many situations where eligibility is specified in a discontinuous way through min-

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[1] See Wolff (2014), Heider et al. (2015), Nyborg (2017), Blot and Hubert (2018) for a discussion on the collateral eligibility of risky private sector assets and the monetary policy implementation by the ECB. We show the collateral treatment of corporate sector assets by different central banks in appendix B.

[2] As of 2020Q4, corporate bonds are the second largest asset class accepted as collateral by the ECB with a market value of EUR 1870 billion. This is only exceeded by government bonds (see ECB, link).

[3] One potential concern lies in the fact that firms could simply substitute financing by non-marketable bank loans with bond financing and leave total debt outstanding constant. Pelizzon et al. (2020) provide evidence for this substitution but still find a sizeable positive effect on leverage. A deterioration of repayment performance of assets eligible for ECB operations has been shown for Residential Mortgage Backed Security by Bekkum et al. (2018). A second potential concern would arise, if firms simultaneously increase their investment and keep leverage constant. However, Todorov (2020) and Santis and Zaghini (2021) find that QE-eligible firms primarily responded by increasing current dividend payouts.

imum rating requirements, we use this framework to quantify the importance of firm responses to the ECB's collateral easing policy.

We study endogenous firm responses to eligibility requirements through the lens of a model with heterogeneous firms that issue risky debt securities, to which we refer as corporate bonds in the following.[4] Firms are subject to idiosyncratic revenue shocks and have an incentive to issue bonds because they are more impatient than their creditors. Following Gomes et al. (2016), firms default on their bonds if revenues fall short of current repayment obligations. Bond issuance is determined by a trade-off between relative impatience and expected default costs. Bonds are held and priced by banks. We assume that banks value these bonds if they can be used to collateralize borrowing from the central bank. Consistent with actual central bank practice, only sufficiently safe corporate bonds are eligible as collateral and the central bank can freely set the minimum quality requirement as a policy instrument. The dual role of bonds as investment objects and collateral implies that spreads on *eligible* bonds contain a fundamental component and a collateral premium that - ceteris paribus - shifts the pricing schedule for corporate bonds outwards in a discontinuous way.

As our first contribution, we provide a characterization of firm responses in a setting with discontinuous demand for their bonds. We obtain analytical solutions in a simplified setting with one period bonds, i.i.d. revenue shocks, and some firms being permanently more profitable than others. Making corporate bonds eligible affects the firm's borrowing decision in a discontinuous way. We organize our discussion of firm responses around a key firm characteristic in this context, the *eligible debt capacity*, defined as the maximum amount of bonds a firm can issue without losing eligibility.

Notably, firm responses to collateral premia differ in sign above and below the discontinuity in bond demand induced by eligibility requirements. High-quality firms (with a large eligible capacity) can take advantage of banks' high valuation of corporate bonds and issue more bonds to front-load dividend payouts by issuing more bonds: an intertemporal substitution effect. On the other hand, those firms can sustain the same dividend-payout by issuing a smaller face value of bonds: an income effect. We show that, under a standard monotone hazard rate assumption on firm revenues, the former effect dominates: firms increase their *risk-taking*. In contrast, medium-quality firms (which issue bonds at or near their eligible debt capacity) may find it worthwhile to reduce their debt issuance, if this leads to a rating upgrade and makes their bonds eligible: a *disciplining* effect.[5]

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[4]These securities can also be interpreted as securitized bank loans or other marketable corporate sector assets, which are also eligible as collateral in many central bank collateral frameworks.

[5]The heterogeneous response is consistent with empirical evidence in Grosse-Rueschkamp et al. (2019). They show that in particular firms rated A or higher increase their leverage in response to the ECB's corporate sector

To evaluate the aggregate impact of eligibility requirements, we decompose collateral supply into a quantity and a quality component. While both firm-level effects - risk-taking and disciplining - increase collateral quantity, they have an opposing effect on quality. This makes a heterogeneous firm model essential to study aggregate implications, because the relative strength of both effects depends on the cross-sectional firm distribution. To illustrate the aggregate effect, consider a collateral easing, which increases the eligible capacity for all firms. The change of aggregate collateral supply contains a *mechanical component* and *endogenous firm responses*, which depend on the relative size of both firm-level effects and the mass of firms affected by each effect. In line with our decomposition of aggregate collateral supply, the quantity channel of collateral easing captures the additional debt issuance of eligible firms, holding their bond prices and rating constant. The quality channel of collateral easing in turn captures rating downgrades.

The macro level effects are related to risk-taking and disciplining effects at the firm level as follows: if risk-taking (disciplining) is the dominating force, this decreases (increases) collateral quality. In the simplified setting with one period bonds, i.i.d. revenue shocks, and permanently different revenue distributions, collateral supply effects can be characterized analytically. In particular, the market value of outstanding bonds increases also for firms which engage in risk-taking. Issuing bonds to a point where the losses of debt dilution exceed the funds raised by an additional unit of debt is never optimal. Since firm characteristics are permanent and shocks are i.i.d., this increase in bonds outstanding never results in a debt overhang in future periods. Therefore, endogenous firm responses increase collateral supply beyond the mechanical effect. We challenge this result by allowing for debt overhang using a more general specification of firm revenues in the following.

As our second contribution, we study firm responses to the ECB's collateral easing policy in 2008. Specifically, we quantify the relative importance of endogenous firm responses for collateral supply and quality in a setting with long-term debt and persistent revenue shocks rather than permanently different firm types. We solve the model using global methods and calibrate the model-implied cross-section of firms to euro area data by employing a merged dataset of corporate bonds from *IHS Markit* and corporate balance sheet data from *Compustat Global*. The calibrated model is able to replicate several important features of firm debt issuance, corporate bond spreads, and collateral premia, which are crucial to evaluate the impact of eligibility requirements.

In this setting, we study two different policies: our benchmark scenario are tight eligibility requirements, corresponding to the ECB collateral framework before the 2008 crisis, which

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purchase programmes. BBB-rated firms do not materially increase their leverage.

only accepted bonds rated A or higher. Second, we consider lenient eligibility requirements, under which all bonds rated BBB or higher are eligible for central bank borrowing, in line with ECB practice after 2008. This shifts the eligible debt capacity into regions of higher default risk. Since shocks are persistent, a lenient policy increases the probability of a firm to be eligible in the future, thereby lowering spreads for all states through the continuation value. Hence, collateral policy is not only relevant for firms that are near their eligible debt capacity in the current period, but affects all firms via the rollover value of bonds.

In all our numerical experiments, the cross-sectional firm distribution reveals that firms respond to a relaxation of eligibility requirements from A to BBB by increasing their debt issuance, which in turn leads to higher default risk. Hence, collateral quantity increases at the expense of collateral quality. Furthermore, the endogenous collateral quality channel exceeds the collateral quantity channel by a factor of 5-10, i.e. firm responses dampen the mechanical expansion of collateral supply. The macroeconomic relevance of endogenous firm responses can be illustrated by considering a reduction in eligibility requirements inducing an expansion of aggregate collateral supply by 68%, which corresponds to the increase of eligible bonds after the ECB relaxed its eligibility requirements in 2008. In our baseline calibration, collateral supply would expand by 77%, if firm behavior is kept constant. This observation is robust to varying the size of the collateral premium, firm fundamentals, and modifications to the eligibility threshold.

The dampening effect of firm responses on collateral supply is associated with a debt overhang induced by lenient eligibility requirements. While the risk-taking response at the firm level is also increasing the *current* market value of bonds outstanding, the long-term effects on collateral supply are negative: if hit by a series of adverse shocks, these previously issued bonds cease to be eligible, since leverage is sticky and default becomes more likely. This is still optimal in the current period due to the relative impatience of firm managers. Key to the large relative importance of collateral quality is the combination of persistent shocks and risky long-term debt. Similar effects have been described in the macro-finance literature (Gomes et al., 2016 or Jungherr and Schott, 2020) and in the sovereign default literature (Hatchondo et al., 2016).

Since the large adverse effect on collateral quality is associated with a debt overhang problem, it is natural to investigate an *eligibility covenant* as a potential instrument to alleviate the negative effect of endogenous firm responses. In principle, a covenant can be made dependent on current debt issuance or on current profitability. We focus on a covenant depending on *current* (beginning-of-period) leverage, which is easier to measure than profitability and public information for firms large enough to issue corporate bonds. The policy problem lies in set-

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ting a sufficiently tight covenant to provide de-leveraging incentives for risky firms, while not dis-incentivizing the issuance of bonds altogether.

On a conceptual level, default probabilities and ratings map the indebtedness and profitability of firms into a one-dimensional measure. Conditioning eligibility also on leverage (indirectly) makes use of both firm state variables. This will (weakly) improve on current collateral frameworks - which typically condition eligibility only on ratings. We show numerically that this improvement is strictly positive in our model. Restricting our attention to a simple parametric class for the covenant, we numerically demonstrate the existence of a *collateral Laffer curve* and compute the optimal covenant within this parametric class. Our numerical results suggest that conditioning eligibility on all publicly available information about firm characteristics has a sizable positive effect on collateral supply: in our baseline calibration, collateral supply expands by 82% and therefore even improves on the counterfactual with constant firm behavior. In practice, this policy could also be implemented by adding rating outlooks to the minimum rating requirements specified in collateral frameworks.

While we propose a model that is particularly well suited to study discontinuous collateral eligibility, our analysis of remains valid in many cases where firms respond endogenously to a discontinuous demand schedule for their debt. Specifically, our model can also be applied to eligibility for asset purchase programmes, where the anticipation of substantial demand increases for targeted assets may induce a different willingness to pay eligibility premia. Other applications include eligibility on private repo markets or the demand discontinuity around the lower Investment Grade rating (BBB-): many investment and pension funds are restricted to invest into Investment Grade bonds for regulatory reasons, such that bond demand exhibits a jump from BB+ to BBB-ratings.

Related Literature Our paper builds on a large strand of literature providing empirical results on the bond market impact of collateral policy and eligibility for QE programmes. Ashcraft et al. (2011) find a sizable impact of haircuts on bond prices using an event study around announcement and implementation of the Term Asset-Backed Securities Loan Facility in the US. Exploiting an unexpected policy change regarding eligibility of Chinese corporate bonds, Chen et al. (2019) identify a pledgeability premium of around 50bp for AA-bonds. Mésonnier et al. (2021) use an extension of eligibility criteria as part of the Additional Credit Claims program in 2012 and find a premium of 7 basis points relative to ineligible assets. Santis and Zaghini (2021) find that CSPP-eligible firms increase their debt issuance and use some of the funds to repurchase their own stocks. Todorov (2020) finds that issuers of QE-eligible increase their dividend payouts by four times, relative to pre-treatment levels, but do not increase investment.

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Adverse effects on firm risk-taking is presented in Grosse-Rueschkamp et al. (2019), who furthermore identify heterogeneous responses of firms in different rating brackets. This highlights potentially unintended behavior on the firm side and the role of firm heterogeneity, which are central ingredients to our model.

While the previous group of papers used surprise policy changes to identify causal effects, there are two complementary empirical approaches leading to similar findings. Pelizzon et al. (2020) document collateral eligibility premia and bond supply effects using security-level data from the euro area. Their identification relies on ECB-discretion when formally eligible bonds are actually put on the list of eligible assets. They identify collateral eligibility premia of 11-24bp. This highlights the relevance of collateral valuation also in a conventional policy regime. Building an identification strategy around the US treasury safety premium, Mota (2021) uses US corporate bond data and finds that non-financial corporate bonds carry a premium, which can be related to collateral service. The premium decreases in the default risk associated with the bond and depends on idiosyncratic firm characteristics as well as an aggregate component encompassing economy-wide collateral supply and demand factors. Regarding the cross-section, those firms enjoying the largest premia increase debt issuance and dividend payouts.

The results of our paper can be related to a group of papers studying the collateral eligibility of risky assets and implications for central bank policy. Chapman et al. (2010) propose a model where the central bank faces a trade-off between relaxing liquidity constraints and incentivizing banks to invest into illiquid and risky assets, when setting the collateral framework. Koulischer and Struyven (2014) argue that relaxing eligibility requirements can alleviate credit crunches, if collateral supply or collateral quality fall below specific levels. In their model, banks' ability to extend credit to the non-financial sector depends on both the quality and quantity of collateral. Collateral quality is defined as the difference between the valuation of first- and second-best users, which is an exogenous characteristic in their model. Cassola and Koulischer (2019) quantify a collateral policy trade-off between liquidity provision and risk-taking by the central bank, again taking collateral quality as exogenous. Choi et al. (2021) take a macroprudential approach to central bank collateral requirements. Since banks prefer to use high-quality collateral on the interbank market, central banks negatively affect liquidity creation when accepting only high-quality assets. At the same time, lending against low-quality collateral exposes the central bank to counterparty default risk. These two effects shape the central bank trade-off in their model. Costly counterparty risk for interbank market outcomes is discussed in Heider et al. (2015).[6] We view our approach as complementary to these papers in the sense that endoge-

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[6]Nyborg and Roesler (2019) document that only 1 % of total collateral pledged on the interbank market are

nous collateral supply and quality potentially interacts with bank-related frictions on money markets, from which we abstract in our setup.

Outline The paper is structured as follows. Section 2 introduces collateral premia and eligibility requirements into a corporate capital structure model and presents the main policy trade-off. We present an extended version of model in section 3, which we then calibrate to euro area data in section 4. In section 5, we conduct policy experiments regarding eligibility requirements. Section 6 concludes.

2   Simple Model of Eligibility Requirements

This section introduces a model of endogenous collateral supply and quality to flesh out the impact of eligibility requirements on firms, which will be augmented and calibrated to the data in section 3. Time is discrete and indexed by t and there are two groups of agents: a non-financial sector (*firms*) and financial intermediaries (*banks*).

Firms are endowed with a technology that generates stochastic revenues, which can be interpreted as EBIT.[7] Revenue shocks realize at the beginning of each period t and are i.i.d. across firms and over time. In addition to being subject to idiosyncratic revenue shocks, firms are ex-ante heterogeneous with respect to the probability distribution over revenue shocks: some firms are permanently more profitable than others and we denote this heterogeneity by the parameter s in the following. We will use the parameter s to index bonds and firms as well.

Each period t, firms issue debt instruments to banks. These debt instruments will be referred to as corporate bonds, but reflect all marketable debt instruments including securitized bank loans. Bonds are real one-period discount bonds, i.e. they promise to pay one unit of the all-purpose good in period $t+1$. In our model, firms are the natural borrowers, since we assume that they are more impatient than banks. Given their shock realization and bonds outstanding, firms either default or repay. Bonds have a dual role in the economy, since banks can pledge eligible bonds with the central bank to obtain funding. The demand for central bank funding can be motivated by liquidity deficits that require immediate settlement, such as net deposit outflows (see for example Bianchi and Bigio, 2021 and De Fiore et al., 2019). We follow Mota (2021) and assume a constant willingness to pay collateral premia in the baseline model. We present a robustness check where the size of collateral premia depends on collateral supply in

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corporate bonds. Since our focus is on corporate bond eligibility in central bank collateral frameworks, we do not make counterparty default risk explicit in our model.

[7]To maintain tractability we do not endogenize the investment decision.

appendix E.

Banks   There is a unit mass of perfectly competitive banks, which price bonds risk-neutrally without discounting. They purchase bonds issued by firm s, denoted by $b_{t+1}(s)$ at price $q_t(s)$. The expected bond payoff is given by the repayment probability of firm s, denoted by $1 - F_t(s)$, described below. In addition, banks value bonds for the collateral benefit they provide and are willing to pay a premium L on eligible bonds. Consistent with actual central bank policy, we assume that corporate bonds are only eligible as collateral if they are sufficiently safe, i.e. if their default probability $F_t(s)$ is below an eligibility threshold $\overline{F}$

$$\Psi(F_t(s)) = \begin{cases} 1 & \text{if } F_t(s) \leq \overline{F} \\ 0 & \text{else} \end{cases} \quad . \tag{1}$$

In the quantitative analysis, the eligibility threshold corresponds to a minimum rating requirement. Note that we model collateral policy in terms of bond eligibility thresholds, i.e. bonds either receive a 100% or a 0% haircut. In practice, eligible bonds have collateral values less than 100 % due to other risk factors, such as market illiquidity or duration risk, which are not present in our setup. Nevertheless, collateral frameworks experience large discontinuities at the eligibility thresholds, as we show in table B.2 in Appendix B. Putting these elements together, the bond pricing condition can be written as

$$q_t(s) = \left(1 - F_t(s)\right)\left(1 + \Psi(F_t(s)) \cdot L\right) . \tag{2}$$

It depends on the expected payoff, determined by the firm default decision in $t+1$, and the collateral premium L, if bond s is eligible, which in turn depends on firm default risk. In the absence of a collateral premium, bond prices would merely reflect the expected pecuniary payoff.

Firms   Firm managers are risk neutral and their time-invariant discount factor is denoted by $\beta < 1$. They operate a technology generating random revenues $\mu_t \in [\underline{\mu}, \overline{\mu}]$ with $\underline{\mu} < 0$ and $\overline{\mu} > 0$. Allowing for negative realizations of the revenue shock is consistent with the interpretation of μ_t as EBIT. We assume that μ_t is independent across firms and over time, and denote its pdf and cdf by $f(\mu_t|s)$ and $F(\mu_t|s)$, respectively. Firms are *ex-ante* heterogeneous with respect to their probability distribution over revenues, which will allow us to analytically disentangle how individual firms react to eligibility requirements and how firm heterogeneity affects aggregate

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collateral supply responses. The ex-ante heterogeneity is governed by the parameter s, which characterizes the revenue distribution in a first-order stochastic dominance sense: firms with a high s are most profitable on average. In particular, the parameter s shifts the probability mass according to $F(\mu_t|s) = F(\mu_t - s)$. We assume that s follows some continuous distribution over the open interval $[s^-, \infty]$ and that s^- is sufficiently low, such that some firms are not eligible even when they are un-levered, i.e. $F(0|s^-) = F(s^-) > \overline{F}$.

Firm managers maximize the present value of dividends. Dividends can become negative, which we interpret as equity issuance. Firms issue bonds $b_{t+1}(s)$ to banks. These bonds are subject to default risk: if firm revenues μ_t fall short of the repayment obligation b_t, the firm is unable to raise funds by issuing additional equity and mechanically defaults. In case of default, all firm revenues are lost and there is no recovery for banks.[8] The expected payoff from investing into bonds of firm s can be expressed explicitly in terms of the revenue distribution

$$\left(1 - F_t(s)\right) = \int_{b_{t+1}}^{\overline{\mu}_{t+1}} dF(\mu_{t+1}|s) \,, \tag{3}$$

as the probability of receiving a revenue draw μ_{t+1} which is larger than the repayment obligation b_{t+1}. This probability is specific to firm s and decreases in average firm profitability, holding the debt level constant.

Characterization of Debt Choices We assume that there are no delays in the restructuring of defaulted bonds. The maximization problem of firm s in period t can be written recursively as

$$W(b_t|s) = \max_{b_{t+1}} \quad V(b_{t+1}|s)$$

with

$$V(b_{t+1}|s) = \int_{b_t}^{\overline{\mu}} (\mu_t - b_t) \, dF(\mu_t|s) + q(b_{t+1}|s)b_{t+1} + \beta \mathbb{E}\left[W(b_{t+1}|s)\right] \,, \tag{4}$$

which has the first-order condition

$$\beta(1 - F(b_{t+1}|s)) = \frac{\partial q(b_{t+1}|s)}{\partial b} b_{t+1} + q(b_{t+1}|s) \,.$$

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[8]Our approach is motivated by the findings of Lian and Ma (2021), who show that most corporate borrowing is tied to the going-concern value of the firm. Allowing for a positive recovery rate would not change our qualitative results.

Plugging the expected payoff (3) into the bond pricing condition (2), we can express the derivative of the bond price as

$$\frac{\partial q(b_{t+1}|s)}{\partial b} = -f(b_{t+1}|s)(1 + \Psi(F(b_{t+1}|s)) \cdot L) \,.$$

and the first-order condition can be rearranged to

$$\beta\left(1 - F(b_{t+1}|s)\right) = \left(1 - F(b_{t+1}|s)\right) - f(b_{t+1}|s) \cdot b_{t+1} \,, \qquad\qquad \text{if } F(b_{t+1}|s) > \overline{F} \,, \qquad (5)$$

$$\beta\left(1 - F(b_{t+1}|s)\right) = \left(1 - F(b_{t+1}|s)\right) \cdot (1 + L) - f(b_{t+1}|s) \cdot b_{t+1} \cdot (1 + L) \,, \qquad \text{if } F(b_{t+1}|s) \leq \overline{F} \,. \qquad (6)$$

The eligibility requirement introduces a discontinuity into the first order condition. Non-eligible firms choose their bond issuance according to (5): the left hand side of this expression reflects discounted expected repayment obligations from issuing another unit of bonds, which have to equal the current revenues from issuing this bond net of debt dilution on the right hand side. Collateral premia distort this trade-off by making debt issuance more attractive, since it increases the amount of funds raised per unit of bonds (first term on the RHS of (6)). At the same time, Collateral premia increase the costs of debt dilution (second term on the RHS of (6)), which makes debt issuance less attractive.

Without further restrictions on the revenue distribution, the total effect of bond eligibility is ambiguous. However, guided by empirical evidence on the firm-level effects of eligibility requirements (see Pelizzon et al., 2020), we place certain restriction on the distribution. Specifically, we assume that the distribution satisfies a monotone hazard rate condition of the form $\frac{\partial \mu_{t+1} h(\mu_{t+1}|s)}{\partial \mu_{t+1}} > 0$, where $h(\mu_{t+1}|s) \equiv \frac{f(\mu_{t+1}|s)}{1 - F(\mu_{t+1}|s)}$ denotes the hazard rate.[9]

$$h(b_{t+1}|s) \cdot b_{t+1} = 1 - \beta \,, \qquad\qquad\qquad\qquad \text{if} \quad F(b_{t+1}|s) > \overline{F} \,, \qquad (7)$$

$$h(b_{t+1}|s) \cdot b_{t+1} = \frac{1 - \beta + L}{1 + L} \,, \qquad\qquad\qquad \text{if} \quad F(b_{t+1}|s) \leq \overline{F} \,. \qquad (8)$$

Since $\frac{1-\beta+L}{1+L} > 1 - \beta$, it follows that the optimal debt issuance of an eligible firm exceeds that of an otherwise identical, non-eligible firm.

Eligibility Requirements along the Cross-Section   So far, we showed that collateral premia induce additional debt issuance of eligible firms, but did not discuss which firms are able to issue eligible bonds. We define the *eligible debt capacity* $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ as the highest

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[9]This assumption is also standard in macro-finance and is for example employed in the canonical financial accelerator model of Bernanke et al. (1999).

possible debt choice for which the default probability does not exceed the threshold $\overline{F}$. We denote by $b_{t+1}^1(s)$ the debt level satisfying (7) and by $b_{t+1}^2(s)$ the debt level satisfying (8). As shown in Proposition 1, the ex-ante heterogeneous revenue distribution determines how firms select themselves into eligible and non-eligible regions, taken the eligibility threshold as given. We characterize this partitioning in Proposition 1.

Proposition 1. There are two cut-off values s_0, implicitly defined through $V^2(\widetilde{b}_{t+1}(s_0)|s_0) - V^1(b_{t+1}^1(s_0)|s_0)$, and s_2, defined through $b_{t+1}^2(s_2) = \widetilde{b}_{t+1}(s_2)$ for the shifting parameters, such that

- Firms with $s < s_0$ are *non-eligible* and choose b_{t+1} according to (7).

- Firms with $s_0 < s < s_2$ are *constrained eligible* in the sense that they borrow up to their eligible debt capacity $\widetilde{b}_{t+1}(s)$.

- Firms with $s > s_2$ are *unconstrained eligible* and choose b_{t+1} according to (8).

Proof: See appendix A.1.

The firm s_0 characterizes the indifference point between eligibility and non-eligibility whereas the firm s_2 can just issue debt according to the first-order condition under eligibility (8). The threshold firm s_1 satisfying $b_{t+1}^1(s_1) = \widetilde{b}_{t+1}(s_1)$ also levers up to its eligible debt capacity and therefore falls in the interval $s_0 < s_1 < s_2$. This threshold level will become important when characterizing risk-taking and disciplining effects of eligibility requirements. While constrained eligible firms exhaust their eligible debt capacity, it is important to note that $\widetilde{b}_{t+1}(s)$ still depends on the firm type s: within the region of constrained eligible firms, the most profitable ones are able to issue a larger amount of bonds.

In figure 1, we provide an illustration by plotting the first-order conditions (7) and (8) in solid black lines. Objective functions, for the case of non-eligibility and eligibility are denoted by V^1 and V^2 (blue dashed lines) and are obtained from evaluating (4) at the respective debt choices. There are four possible combinations of $b_{t+1}^1(s)$, $b_{t+1}^2(s)$, and $\widetilde{b}_{t+1}(s)$. Figure 1a shows the case of a firm with a high draw of s so that $\widetilde{b}_{t+1}(s) > b_{t+1}^2(s)$. The eligible debt capacity of an *unconstrained eligible* firm is sufficiently high, such that it can satisfy (8). Figure 1b shows a firm with insufficient debt capacity to satisfy (8), i.e. $b_{t+1}^2(s)$ is not feasible, whereas satisfying (7) would be possible, $b_{t+1}^1(s) < \widetilde{b}_{t+1}(s)$. However, the value of the objective $V^2(\widetilde{b}_{t+1}(s)|s)$ exceeds the value at $V^2(b_{t+1}^1(s)|s)$ because it is upward sloping for all $b < b_{t+1}^2(s)$. Thus, the firm chooses to be just eligible at a debt level $\widetilde{b}_{t+1}(s)$: such a firm is *constrained eligible*. Within the case of $\widetilde{b}_{t+1}(s) < b_{t+1}^1(s)$, there are two sub-cases: first, choosing $b_{t+1}^1(s)$ is still feasible, but the firm can be better off by choosing $\widetilde{b}_{t+1}(s)$, since $V^2(\widetilde{b}_{t+1}(s)|s) > V^1(b_{t+1}^1(s)|s)$, as in figure 1c. Such

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Figure 1: Firm Debt Choice along Revenue States s

(a) Unconstrained Eligible

(b) Constrained Eligible

(c) Constrained Eligible

(d) Non-eligible
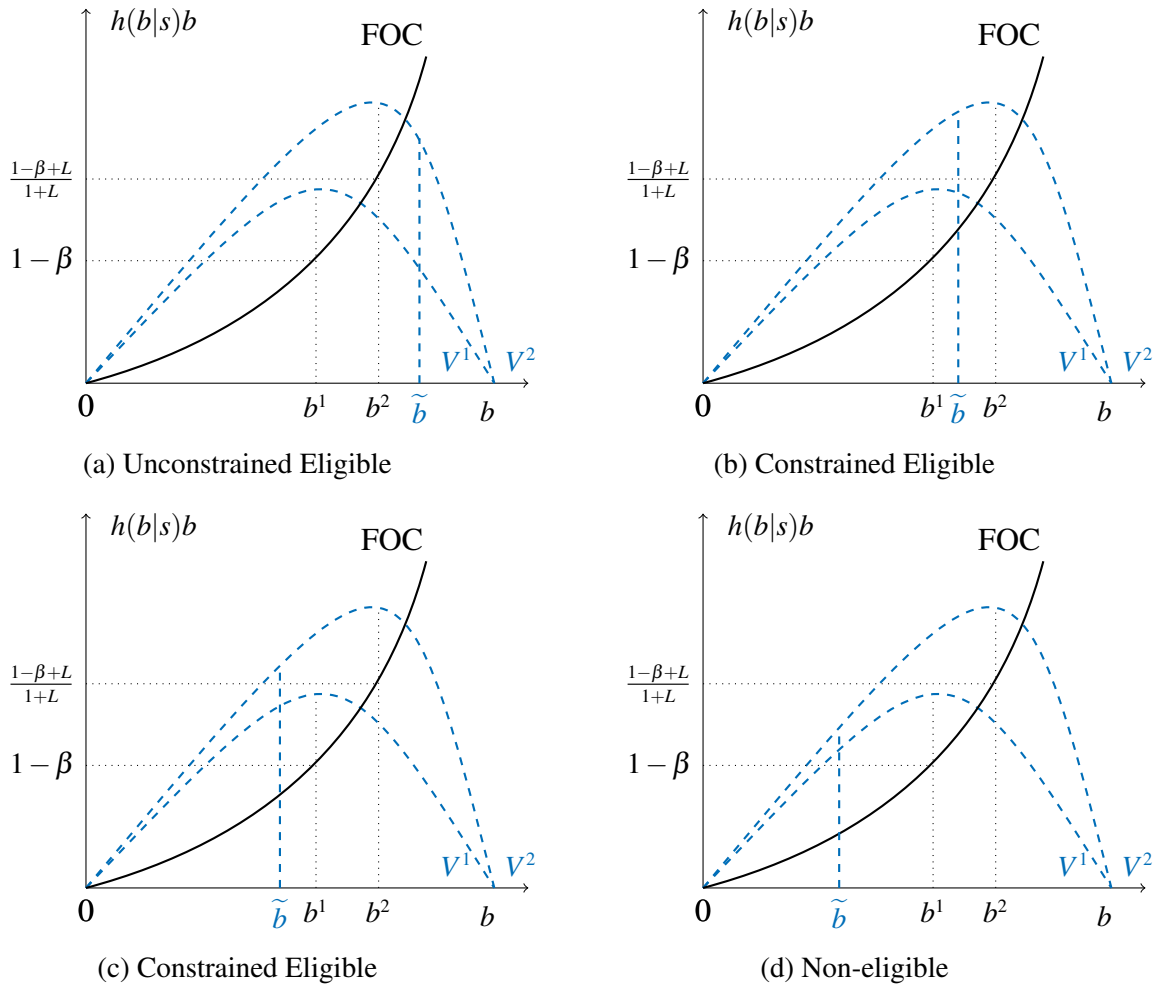
a firm chooses to be just eligible and is also classified as *constrained eligible*. Second, firms with a sufficiently low s optimally choose $b^1_{t+1}(s)$ and forgo eligibility, since the debt reduction required for eligibility is too large, as in figure 1d. These firms are *non-eligible*.

Eligibility Requirements: Macroeconomic Aggregates   Having discussed how firms along the cross-section are heterogeneously affected by eligibility requirements, we now turn to the effects of collateral easing. Specifically, we consider an increase of the threshold default probability from a low value $\overline{F}^A$ to a higher value $\overline{F}^{BBB}$, alluding to the ECB policy change in response to the 2008/09 financial crisis and also corresponding to our numerical experiments in the next section. Formally, we characterize the change of aggregate collateral $\overline{B}$ in terms of the debt choice across the firm type space S. We define the cut-off values s_0 and s_2, which determine

the partitioning of firms into constrained and unconstrained eligibility regions. Let the cut-off values associated with $\overline{F}^A$ and $\overline{F}^{BBB}$ be denoted by (s_0^A, s_2^A) and (s_0^{BBB}, s_2^{BBB}), respectively. It is straightforward to show how collateral easing affects these cut-offs:

Lemma 1. Increasing the eligibility threshold from $\overline{F}^A$ to $\overline{F}^{BBB}$ decreases the threshold levels to $s_0^{BBB} < s_0^A$ and $s_2^{BBB} < s_2^A$.

Proof: See appendix A.2.

Intuitively, the threshold productivity levels partitioning firms into different eligibility regions decrease in response to a collateral easing. Lemma 1 can be shown by observing that collateral easing increases the eligible debt capacity across the firm distribution and that $b_{t+1}^1(s)$ and $b_{t+1}^2(s)$ are independent of the eligibility thresholds. We can write the total effect on collateral supply as

$$
\begin{aligned}
\overline{B}^{BBB} - \overline{B}^A = &\int_{s_0^{BBB}}^{s_2^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right)\widetilde{b}_{t+1}^{BBB}(s)dG(s) + \int_{s_2^{BBB}}^{\infty} \left(1 - F\left(b_{t+1}^2(s)\right)\right)b_{t+1}^2(s)dG(s) \\
&- \int_{s_0^A}^{s_2^A} \left(1 - F\left(\widetilde{b}_{t+1}^A(s)\right)\right)\widetilde{b}_{t+1}^A(s)dG(s) - \int_{s_2^A}^{\infty} \left(1 - F\left(b_{t+1}^2(s)\right)\right)b_{t+1}^2(s)dG(s) .
\end{aligned}
\tag{9}
$$

Collateral supply can be divided into two parts: the two integrals over $[s_0, s_2]$ contains all constrained eligible firms, respectively, while the integrals over $[s_2, \infty)$ summarizes unconstrained eligible firms. In the following, we provide an analytical decomposition of the change in collateral supply into mechanical effects and endogenous firm responses. Mechanical effects are present if threshold levels satisfy $s_1^{BBB} < s_0^A$. This means that the firm exactly satisfying eligibility requirements after the policy change $F(b_{t+1}^1(s_1^{BBB})) = \overline{F}^{BBB}$ was not eligible before the policy change $F(b_{t+1}^1(s_1^{BBB})) > \overline{F}^A$. Put differently, there is at least one firm which was non-eligible under the tight policy, where it chooses $b_{t+1}^1(s)$, but becomes eligible while keeping its debt issuance $b_{t+1}^1(s)$ constant. To ease the exposition we also assume $s_0^A < s_2^{BBB}$, which implies that there is no firm directly switching from non-eligible (below s_0^A before the switch) to unconstrained eligible (above s_2^{BBB}).[10] We can write the mechanical effect as

$$
\overline{B}^{BBB} - \overline{B}^A\bigg|_{mech} = \int_{s_1^{BBB}}^{s_0^A} \left(1 - F\left(b_{t+1}^1(s)\right)\right)b_{t+1}^1(s)dG(s)
\tag{10}
$$

---

[10]The decomposition of the aggregate collateral supply effect does not depend on this assumption, it however eases the analytical exposition.

This reflects the additionally eligible collateral under the assumption that firms do not change their debt choice. These firms have been non-eligible under tight eligibility requirements and therefore issue bonds according to $b_{t+1}^1(s)$. The mechanical effect of collateral easing is always positive. Endogenous firm responses are residually given by

$$
\overline{B}^{BBB} - \overline{B}^A \Big|_{endo} = \int_{s_0^{BBB}}^{s_2^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right) \widetilde{b}_{t+1}^{BBB}(s) dG(s) + \int_{s_2^{BBB}}^{\infty} \left(1 - F\left(b_{t+1}^2(s)\right)\right) b_{t+1}^2(s) dG(s)
$$

$$
- \int_{s_0^A}^{s_2^A} \left(1 - F\left(\widetilde{b}_{t+1}^A(s)\right)\right) \widetilde{b}_{t+1}^A(s) dG(s) - \int_{s_2^A}^{\infty} \left(1 - F\left(b_{t+1}^2(s)\right)\right) b_{t+1}^2(s) dG(s)
$$

$$
- \int_{s_1^{BBB}}^{s_0^A} \left(1 - F\left(b_{t+1}^1(s)\right)\right) b_{t+1}^1(s) dG(s) \, .
$$

Due to the assumption $s_0^A < s_2^{BBB}$ and using $s_0^{BBB} < s_0^A$ from Lemma 1, we can split the integrals over the interval $[s_0^{BBB}, s_2^{BBB}]$ into two parts and rearrange to

$$
\overline{B}^{BBB} - \overline{B}^* \Big|_{endo} = \int_{s_0^{BBB}}^{s_1^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right) \widetilde{b}_{t+1}^{BBB}(s) dG(s)
$$

$$
+ \int_{s_1^{BBB}}^{s_0^A} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right) \widetilde{b}_{t+1}^{BBB}(s) - \left(1 - F\left(b_{t+1}^1(s)\right)\right) b_{t+1}^A(s) dG(s)
$$

$$
+ \int_{s_0^A}^{s_2^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right) \widetilde{b}_{t+1}^{BBB}(s) - \left(1 - F\left(\widetilde{b}_{t+1}^A(s)\right)\right) \widetilde{b}_{t+1}^A(s) dG(s)
$$

$$
+ \int_{s_2^{BBB}}^{s_2^A} \left(1 - F\left(b_{t+1}^{2,BBB}(s)\right)\right) b_{t+1}^{2,BBB}(s) - \left(1 - F\left(\widetilde{b}_{t+1}^A(s)\right)\right) \widetilde{b}_{t+1}^A(s) dG(s) \tag{11}
$$

The first integral is associated with firms reducing their debt issuance to benefit from being eligible. This captures the discipling effect across the firm distribution. These are graphically represented by the bottom left panel of figure 1c. All other parts of (11) are risk-taking effects: the second part corresponds to firms issuing debt at their eligible debt capacity, but above $b_{t+1}^1(s)$, which exceeds their borrowing under tight eligibility requirements, as shown in the top right of figure 1b. Likewise, the third part captures firms that remain constrained, but with a higher eligible debt capacity $\widetilde{b}^{BBB} > \widetilde{b}^A$. Lastly, the fourth integral summarizes firms that

switch from constrained to unconstrained eligible, which also increase their debt issuance by construction as we show in the top left case in figure 1a.

Disciplining and risk-taking have a positive collateral supply effect: while this is trivial for the disciplining effect, firms that engage in risk-taking will not issue debt beyond a point where debt dilution exceeds the funds raised by issuing an additional unit of debt. Differentiating the market value of bonds outstanding for any eligible debt choice $\frac{\partial(1-F(b^2_{t+1})b^2_{t+1})}{\partial b^2_{t+1}} = (1-h(b^2_{t+1}))(1-F(b^2_{t+1}))$. Using the first-order condition (8), this simplifies to $\frac{\beta}{1+L}(1-F(b^2_{t+1})) > 0$.

In contrast, effects on the corporate bond market are ambiguous. Denoting the expected revenues of firm s, conditional on not defaulting by $E(b_{t+1}(s)|s)$, aggregate default costs $\mathcal{M}_t$ can be expressed in terms of the profitability cut-offs as follows:

$$\mathcal{M}^{BBB}_t - \mathcal{M}^A_t = \int_{s^{BBB}_0}^{s^{BBB}_1} E\big(b^1_{t+1}(s)|s\big) - E\big(\widetilde{b}^{BBB}_{t+1}(s)\big)dG(s) + \int_{s^{BBB}_1}^{s^A_0} E\big(\widetilde{b}^{BBB}_{t+1}(s)\big) - E\big(b^1_{t+1}(s)\big)dG(s)$$

$$+ \int_{s^A_0}^{s^{BBB}_2} E\big(\widetilde{b}^{BBB}_{t+1}(s)\big) - E\big(\widetilde{b}^A_{t+1}(s)\big)dG(s) + \int_{s^{BBB}_2}^{s^A_2} E\big(b^{2,BBB}_{t+1}(s)\big) - E\big(\widetilde{b}^A_{t+1}(s)\big)dG(s) . \tag{12}$$

In general, the effect of collateral easing on default costs mirrors is closely related to endogenous collateral supply responses: while disciplining effects in the first part of (12) lead to a reduction in aggregate default costs, the other three parts related to risk-taking effects increase aggregate default costs. Note that by construction, there is no mechanical effect on default costs, since firm behavior is constant in this case. Putting all things together, collateral easing has a positive mechanical impact on collateral supply $\overline{B}_{t+1}$. In addition, firm responses unambiguously increase collateral quantity. The role of endogenous changes to collateral quality as measured by the default probability and, thereby, aggregate default cost $\mathcal{M}_t$ is ambiguous and depends on the relative strength of risk-taking and disciplining effects.

While the sign of mechanical and quantity effects also obtains numerically under more general specifications, the role of the collateral quality channel hinges critically on the assumption of perfectly persistent firm types and i.i.d. shocks. Indeed, we have demonstrated that the market value of aggregate collateral increases, even though the default risk of bonds increases. This relatively weak collateral quality effect does not necessarily carry over to more general specifications. When instead using an empirically plausible persistence of revenue shocks together with long-term debt, the quality effect becomes much more pronounced. Profitable firms optimally respond to collateral easing by increasing their borrowing without losing eligibility in the short run. If hit by a sequence of adverse revenue draws, firms find themselves with a large

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debt overhang, rendering them ineligible in future periods. This also increases the prevalence of default in equilibrium, as we shall see next.

3 Full Model

In this section, we extend our model of the corporate bond market to facilitate a quantitative analysis of the impact of eligibility requirements. Firm heterogeneity takes the form of persistent revenue shocks rather than permanent differences in the idiosyncratic firm revenue distribution. In addition, bonds are long-term and a firm defaults, if it cannot repay the maturing share of outstanding bonds out of its current revenues. As in the previous section, we maintain the assumption no delays in the restructuring. Consequently, the value of non-maturing bonds is not affected by a default event. This permits us to drop the credit status of firms as a state variable.

Firms   There is a continuum of competitive firms, indexed by j. Firms are endowed with random revenues $e^{\mu_t^j}$, with μ_t^j following an AR(1) process

$$\mu_{t+1}^j = \rho_\mu \mu_t^j + \sqrt{\sigma_\mu} \varepsilon_{t+1}^j \quad \text{with } \varepsilon_t^j \sim N(0,1) \ . \tag{13}$$

Throughout this section, we denote the conditional density function of μ_{t+1}^j by $f(\mu_{t+1}^j | \mu_t^j)$ and the associated cdf by $F(\mu_{t+1}^j | \mu_t^j)$. Firms issue bonds, which mature with probability π each period t, pay a coupon κ and are valued - according to the law of one price - like new issues at price q_t. Making bonds long-term enables us to generate realistic debt ratios. Firms will default on their current repayment obligation $(\pi + \kappa)b_t^j$, if they exceed current revenues $e^{\mu_t^j}$. We can write the threshold revenue level below which the firm defaults as $\widehat{\mu}_t^j = \log\left((\pi + \kappa)b_t^j\right)$ and the default probability as

$$F(b_{t+1}^j | \mu_t^j) = \Phi\left(\frac{\log\left((\kappa + \pi)b_{t+1}^j\right) - \rho_\mu \mu_t^j}{\sigma_\mu}\right) \ , \tag{14}$$

where $\Phi(\cdot)$ is the cdf of the normal distribution.

Banks and Bond Pricing   Banks are modeled in a similar way as in section 2. While they are still assumed to be risk-neutral, they discount the future at the constant rate r^{rf}. Following Gomes et al. (2016), banks incur a restructuring cost m, if firm j defaults on its bonds, such that the expected payoff is reduced by $m \cdot F_t(j)$. This gives us an additional degree of freedom in

calibrating the model to the data. Using the explicit expression of the default probability (14), the per-unit price schedule for corporate bonds can be written

$$q(b_{t+1}^j, \mu_t^j) = \frac{1 + \Psi(F(b_{t+1}^j|\mu_t^j))L}{1 + r^{rf}} \left(\left(1 - F(b_{t+1}^j|\mu_t^j)\right)(\pi + \kappa) - F(b_{t+1}^j|\mu_t^j)\pi m + (1 - \pi)\mathbb{E}_t \left[q\left(\mathcal{B}(\cdot), \mu_{t+1}^j\right)\right]\right).$$
(15)

Note that the rollover value of bonds is obtained from evaluating the bond price schedule at next period's debt choice $\mathcal{B}(b_{t+1}^j, \mu_{t+1}^j)$, which we describe below. As in the simplified model, the total payoff contains a pecuniary part and a collateral premium. The pecuniary part depends on default in $t+1$. If the firm repays, bonds pay the coupon κ, the maturing fraction π is redeemed, and the remainder $1 - \pi$ is rolled over at the next period's market price.

Characterization of Debt Choices Firms choose debt issuance b_{t+1}^j to maximize shareholder value, taken as given the bond price schedule eq. (15). The maximization problem of firm j can be represented by the Bellman equation

$$W(b_t^j, \mu_t^j) = \max_{b_{t+1}^j} \quad V(b_{t+1}^j, \mu_t^j)$$

with

$$V(b_{t+1}^j, \mu_t^j) = \mathbb{1}\{e^{\mu_t^j} > (\pi + \kappa)b_t^j\} \left(e^{\mu_t^j} - (\pi + \kappa)b_t^j \right) + q(b_{t+1}^j, \mu_t^j)\left(b_{t+1}^j - (1 - \pi)b_t^j\right) + \beta\mathbb{E}_t \left[W(b_{t+1}^j, \mu_{t+1}^j)\right].$$
(16)

Current dividends are given by EBIT, conditional on exceeding the threshold revenue level $(\pi + \kappa)b_t^j$. Note that the debt choice b_{t+1}^j does not depend on a potential default event in period t, which follows directly from our assumption of immediate restructuring. The debt choice therefore has two effects: increasing current dividends and reducing next period's dividends due to (i) higher default risk, (ii) elevated debt service conditional on no default, and (iii) increasing the roll-over burden in the next period. Plugging in the bond pricing condition (15), the first-order condition can be written as

$$\frac{\partial q(b_{t+1}^j, \mu_t^j)}{\partial b_{t+1}^j}\left(b_{t+1}^j - (1 - \pi)b_t^j\right) + q(b_{t+1}^j, \mu_t^j) = \beta\left((\kappa + \pi)(1 - F(b_{t+1}^j)) + (1 - \pi)\mathbb{E}_t\left[q_{t+1}\right]\right).$$
(17)

The derivative of the bond price schedule is given by

$$\frac{\partial q(b_{t+1}^j, \mu_t^j)}{\partial b_{t+1}^j} = \begin{cases} -F'(b_{t+1}^j)(\kappa + \pi(1+m))\frac{1}{1+r^{rf}}, & \text{if } F_{t+1}^j > \overline{F} \\ -F'(b_{t+1}^j)(\kappa + \pi(1+m))\frac{1+L}{1+r^{rf}}, & \text{if } F_{t+1}^j \leq \overline{F}. \end{cases} \tag{18}$$

Let the solution to (17) in the case without eligibility be denoted by $b_{t+1}^{j,1}$ and in the case of eligibility by $b_{t+1}^{j,2}$. As in section 2, the debt choice depends on the feasibility of $b_{t+1}^{j,2}$, i.e. the optimal bond issuance under eligibility, and the value of the objective function (16) under both candidate debt choices. The eligible debt capacity in closed form is obtained from evaluating (14) at $\overline{F}$ and re-arranging to

$$\widetilde{b}_{t+1}^j = \frac{\exp\{\sigma_\mu \Phi^{-1}(\overline{F}) + \rho_\mu \mu_t^j\}}{\pi + \kappa}. \tag{19}$$

which we can use to obtain the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$

$$\mathcal{B}(b_t^j, \mu_t^j) = \mathbb{1}\left\{V(b_{t+1}^{j,1}, \mu_t^j) \leq V(\min\{b_{t+1}^{j,2}, \widetilde{b}_{t+1}^j\}, \mu_t^j)\right\} \cdot \min\{b_{t+1}^{j,2}, \widetilde{b}_{t+1}^j\}$$
$$+ \mathbb{1}\left\{V(b_{t+1}^{j,1}, \mu_t^j) > V(\min\{b_{t+1}^{j,2}, \widetilde{b}_{t+1}^j\}, \mu_t^j)\right\} \cdot b_{t+1}^{j,1}. \tag{20}$$

If $b_{t+1}^{j,2} > \widetilde{b}_{t+1}^j$, the optimal debt choice conditional on eligibility is not feasible, such that $\mathcal{B}(b_t^j, \mu_t^j)$ depends on the value attained by exhausting the eligible debt capacity $V(\widetilde{b}_{t+1}^j, \mu_t^j)$ and the value of foregoing eligibility $V(b_{t+1}^{j,1}, \mu_t^j)$. Conversely, if $b_{t+1}^{j,2} < \widetilde{b}_{t+1}^j$, the firm can issue the optimal level of bonds without losing eligibility. Consistent with the one-period model in section 2, the firm will issue $b_{t+1}^{j,2}$ in this case, since $b_{t+1}^{j,1} < b_{t+1}^{j,2}$ and $V(b_{t+1}^{j,1}, \mu_t^j) < V(b_{t+1}^{j,2}, \mu_t^j)$ by definition. Since there is no aggregate risk and banks' pricing kernel is independent of the firm distribution, the debt choice of firms and the bond pricing condition of banks fully characterizes the equilibrium of our model. The equilibrium bond price $\mathcal{Q}(b_t^j, \mu_t^j)$ follows then form evaluating the bond price schedule (15) at the debt choice (20):

$$\mathcal{Q}(b_t^j, \mu_t^j) = q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j). \tag{21}$$

Recursive Competitive Equilibrium A competitive equilibrium is given by the bond price schedule $q(b_{t+1}^j, \mu_t^j)$, firm value function $W(b_{t+1}^j, \mu_t^j)$, the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$, such that

- Given the pricing schedules for bonds, the debt choice solves the firm problem (16).

- Bonds are priced according to (15).

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- The law of motion for the distribution of firms over credit status, bond holdings and firm-specific revenues follows

$$G_{t+1}(b_{t+1}, \mu_{t+1}) = \int \int \left[\mathbb{1}\{b_{t+1} = \mathcal{B}(b_t, \mu_t)\} \right] \times \mathbb{1}\{\mu_{t+1} = \rho_\mu \mu_t + \sigma_\mu \varepsilon_{t+1}^\mu\} \times G_t(b_t, \mu_t) f(\varepsilon_{t+1}^\mu) d\varepsilon_{t+1}^\mu db_{t+1}.$$

Numerical Solution Method   We solve the full model computationally using policy function iteration on a discrete grid for revenues and bond issuance. The algorithm contains four steps at each iteration: first, we compute both potentially optimal debt choices by solving (17), given the bond price schedule (18). Second, we compute the eligible debt capacity (19) and check whether optimal debt choice under eligibility is feasible. If this is not the case, we replace it by the eligible debt capacity $\widetilde{b}$. We randomize over the value function under both debt choices using Gumbel-distributed taste shocks as proposed by Gordon (2018) to compute the debt choice (20). Third, given these policies, we compute the distribution of firms over individual states. The fourth step of each iteration consists of updating bond price schedules. For a detailed description of the algorithm and the parameters governing our numerical approximation we refer to appendix D.

4   Calibration

We calibrate the model to euro area data between 2004Q1, the earliest data with reliable corporate bond data for the euro area, and 2008Q3, the last quarter before the ECB relaxed its collateral framework.[11] One period corresponds to one quarter. Our calibration can be broadly divided into two parts: the first set of parameters is related to firm fundamentals and the payoff profile of corporate bonds, while the second part contains parameters determining the pricing of bond payoffs and eligibility benefits by banks. These two blocks are connected by the central bank eligibility requirement, which is the main policy variable of interest. We consider two policies: the baseline calibration is associated with tight eligibility requirements (A-rating or higher), while collateral easing refers to a scenario with lenient eligibility requirements (BBB-rating or higher). These thresholds are based on the ECB policy before and after the financial crisis of 2008.

Collateral Premium and Eligibility Requirement   We begin with discussing the eligibility thresholds $\overline{F}^A$ and $\overline{F}^{BBB}$. While the ECB's collateral framework is based on ratings by external

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[11]We also consider a higher-risk period, by extending the sample to 2011Q4, which ends before the European sovereign debt crisis and large scale bond market interventions by the ECB.

credit assessment agencies, these are notoriously difficult to model parsimoniously. We therefore adopt an indirect approach based on macroeconomic aggregates. Specifically, we obtain data from *IHS Markit* on the total fixed income securities universe in Europe and extract the subset for non-financial corporate bonds. Using data from September 2008, the last month prior to the relaxation of eligibility requirements, 50% of all corporate bonds in our sample carried a rating of A or higher and were therefore formally eligible as collateral. To match this share of eligible bonds, we set the baseline eligibility threshold to $\overline{F}^A = 1.4\%$, expressed in annualized terms to allow for an easier interpretation. Similar to the baseline calibration, we choose the eligibility threshold associated with a BBB-rating such that it matches the share of available collateral in observed in the *IHS Markit* sample, which was 86% in the last month prior to the policy relaxation and corresponds to an increase in collateral supply by 72%. This share is matched when setting $\overline{F}^{BBB} = 18.5\%$.[12]

We proxy the time-invariant (real) risk-free interest rate by a short-term interbank rate from which we subtract the consumer price inflation rate. Specifically, we use the time-series average of the 3M-EURIBOR minus the euro area HCPI and obtain $r^{rf} = 0.0035$. The restructuring cost parameter will be calibrated to match the level of corporate bond spreads (described below). To calibrate the parameter L governing the collateral premium, we use empirical findings from Pelizzon et al. (2020).[13] Their paper makes use of the ECB having discretion in including bonds that formally satisfy eligibility requirements in the list of actually eligible assets. This discretion generates a randomly selected control group of bonds that eventually become eligible, but are not yet accepted by the ECB. Depending on the specification, they estimate a yield reaction to surprise eligibility of at least 11bp.[14] Our structural model permits an explicit calculation of the yield effect of a surprise inclusion. Specifically, we set $\Psi = 0$ when pricing the bond (holding firm behavior fixed) and compare this hypothetical price to the equilibrium bond price. The price of this hypothetical bond is given by

$$q^0(b^j_{t+1}, \mu^j_t) = \frac{1}{1+r^{rf}} \left(\left(1 - F(b^j_{t+1})\right)(\pi + \kappa) - F(b^j_{t+1})\pi m + (1-\pi)\mathbb{E}_t\left[q\left(b^j_{t+1}, \mu^j_{t+1}\right)\right] \right), \qquad (22)$$

and contains a collateral premium from $t+1$ onwards via the continuation value. The hypothet-

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[12]Using aggregate data on eligible corporate bonds before (2007) and after (2009) the ECB relaxed its eligibility requirements shows that eligible bonds increased by 68%. See also the time series of aggregate collateral in figure 9.

[13]In the baseline calibration, L does not depend on aggregate collateral supply. We relax this assumption in appendix E.2.

[14]Depending on the econometric specification, they report a premium between 11bp and 24bp and we pick the most conservative value of 11bp. Using a slightly different approach, Mésonnier et al. (2021) find a premium of 7bp. Appendix E.1 presents a sensitivity analysis, where we recalibrate L to match a collateral premium of 7bp. The aggregate implications of eligibility requirements are very similar.

ical ineligible asset is therefore distinct from a credit default swap. The yield-to-redemption $\tilde{r}^j$ is determined by the internal rate of return of a perpetuity with constant decay:

$$q_t^j = \sum_{t=1}^{\infty} \frac{CF_t}{(1+\tilde{r})^t} = \sum_{t=1}^{\infty} \frac{(\pi+\kappa)(1-\pi)^{t-1}}{(1+\tilde{r})^t} = \frac{\pi+\kappa}{1-\pi} \sum_{t=1}^{\infty} \left(\frac{1-\pi}{1+\tilde{r}}\right)^t = \frac{\pi+\kappa}{1-\pi} \left(\sum_{t=0}^{\infty} \left(\frac{1-\pi}{1+\tilde{r}}\right)^t - 1\right)$$

$$= \frac{\pi+\kappa}{1-\pi} \left(\frac{1}{1-\frac{1-\pi}{1+\tilde{r}}} - 1\right) = \frac{\pi+\kappa}{1-\pi} \cdot \frac{-\tilde{r}-\pi+1+\tilde{r}}{\tilde{r}+\pi} = \frac{\pi+\kappa}{\pi+\tilde{r}}$$

It follows that $\tilde{r}_t^j = \frac{\pi+\kappa}{q_t^j} - \pi$. The corporate bond spread is defined as $x_t^j \equiv \tilde{r}_t^j - r^{rf}$. Using an entirely analogous derivation, the yield on the hypothetical ineligible bond is given by $\tilde{r}_t^{j,0}$ and the collateral premium follows simply as $\tilde{r}_t^{j,0} - \tilde{r}_t^j$, which is always (weakly) positive.

Firm Fundamentals The second part of the calibration is related to firms, i.e. the parameters governing the idiosyncratic revenue process ρ_μ and σ_μ, the parameters π and κ characterizing the repayment profile of corporate bonds, and the discount factor of firm managers, which we set to the conventional value of $\beta = 0.995$. This value is smaller than the time discount factor of banks $\frac{1}{1+r^{rf}}$, which ensures that even absent collateral premia firms have an incentive to issue bonds. To inform maturity π and coupon κ parameters, we take the market-value-weighted average maturity and coupon of the *iBoxx EUR Investment Grade Non-Financials* index and the *iBoxx EUR High Yield Non-Financials ex crossover* index for all months in our sample and compute their time series averages, respectively.

We then merge our corporate bond dataset from *IHS Markit* with company data available through *Compustat Global*. A detailed description of the construction of our dataset is given in appendix C. This merged dataset forms the basis for several data moments characterizing the firm cross-section. Specifically, we target the median debt/EBIT-ratio b_t^j/μ_t^j and the bond spread distribution, characterized by its quartiles. The time-series averages over the sample period 2004Q1-2008Q3 are $Q_{0.25}(x) = 31\text{bp}$, $Q_{0.5}(x) = 51\text{bp}$, and $Q_{0.75}(x) = 81\text{bp}$.[15] Table 1 summarizes all parameters for our baseline calibration.

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[15]We conduct a second sensitivity analysis with respect to a higher level of spreads computed over an extended sample period in appendix E.1.

Table 1: Baseline Parameterization

Parameter	Value	Source
Bank discount rate r^{rf}	0.0035	EURIBOR-HCPI
Borrower discount factor β	0.995	Standard
Coupon Rate κ	0.01	*Markit* iBoxx
Maturity Parameter π	0.0625	*Markit* iBoxx
Collateral premium L	0.004	Calibrated
Bankruptcy costs m	0.2	Calibrated
Revenue persistence ρ_μ	0.93	Calibrated
Revenue shock std. dev. σ_μ	0.0375	Calibrated
(Annualized) A-eligibility threshold $\overline{F}^A$	1.4%	Calibrated
(Annualized) BBB-eligibility threshold $\overline{F}^{BBB}$	18.5%	Calibrated

In table 2 we show the targeted moments in our baseline calibration. While there is a slight discrepancy in matching the median debt/EBIT-ratio in our simplistic model of corporate indebtedness, the cross-section of corporate bond spreads and the average collateral premium $r - r_0$ closely match the data. Similarly, the share of eligible bonds before and after a relaxation of eligibility requirements are consistent with macroeconomic aggregates.

Table 2: Targeted Moments

Moment	Data	Model
Collateral premium $r - r_0$	11	11
Debt/EBIT $Q_{0.50}(b/\mu|\overline{F}^A)$	4.2	3.2
Bond spread $Q_{0.25}(x|\overline{F}^A)$	31	30
Bond spread $Q_{0.50}(x|\overline{F}^A)$	51	58
Bond spread $Q_{0.75}(x|\overline{F}^A)$	81	80
Eligible bond share $\overline{B}/(QB)|\overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB)|\overline{F}^{BBB}$	86%	83%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

5 Quantitative Analysis of Collateral Easing

With the parametrized model, we numerically examine the impact of eligibility requirements on corporate bond spreads, the firm distribution, and macroeconomic aggregates. Throughout this section, the baseline calibration (A-rating or higher) is marked in blue, while lenient eligibility requirements (BBB-rating or higher) are marked in orange.

5.1 Eligibility Requirements at the Firm Level

We begin by inspecting how lenient eligibility requirements change bond price schedules via banks' first-order condition for corporate bonds (15). Next, we evaluate how firms react to the changes in bond price schedules and study the macroeconomic effect.

Corporate Bond Spreads In figure 2, we show spreads implied in the bond price schedule. They obtain from evaluating the bond pricing condition at any candidate debt choice b_{t+1}^j, fixing revenues at their median. The discontinuity in the bond price schedule represent the location of eligibility thresholds. To the left of this point, bonds are currently eligible and investors are willing to pay collateral premia on them, resulting in lower spreads. For a debt choice to the right of the discontinuity, bonds cease to be eligible and spreads jump upwards. The effect of relaxing eligibility requirements can be inferred from the location of the discontinuities. Lenient eligibility requirements increase the eligible debt capacity, such that the discontinuity shifts to the right. Since bonds are long-term, this has also an affect on bond spreads away from the eligibility threshold: bonds are more likely to be eligible in future periods, which increases their price and lowers the spread already in the current period via the continuation value in (15). Consequently, spreads under lenient eligibility requirements are uniformly lower. The conditional densities on the right y-axis visualize the slight right-shift of the distribution of firms at the median revenue state in response to collateral easing.

Figure 2: Corporate Bond Spreads
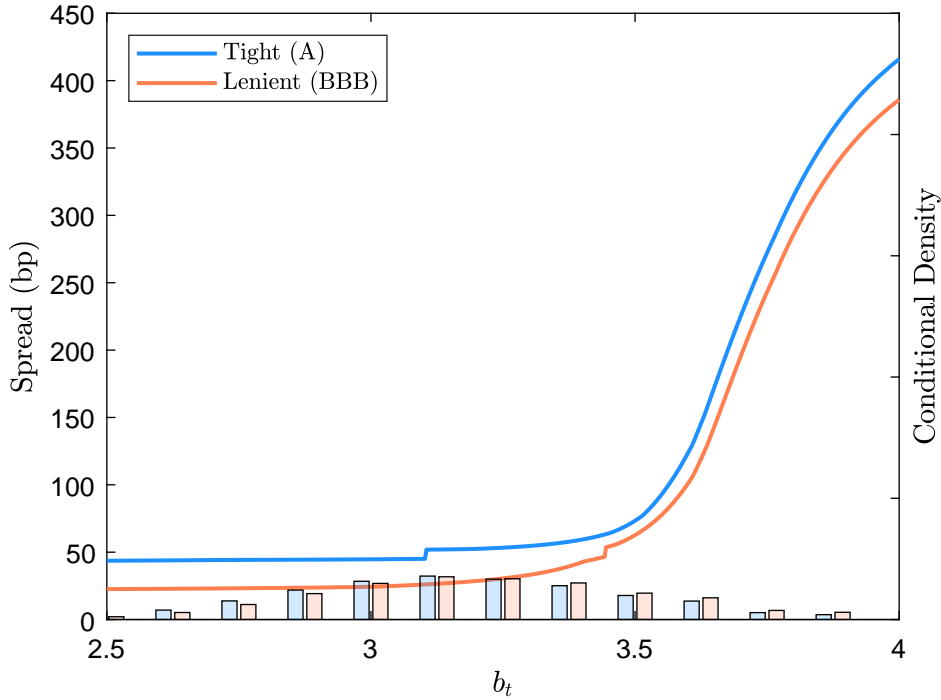
Firm Debt Choices   As a next step, we illustrate how the characterization of the firm debt choices in the simplified setting from section 2 carries over to the full model. The black solid line in figure 3 denotes the debt choice for a firm with median revenues under tight eligibility requirements. This function maps legacy bonds b_t into (gross) bond issuance b_{t+1} and exhibits two non-differentiabilities. These points are associated with the debt levels where firms switch from non-eligible first to constrained, and then to unconstrained eligible (see Proposition 1). The blue dashed line in figure 3 represents the debt choice if the firm were not eligible, while the blue dotted line shows the debt choice if the firm is eligible. Comparing the two potentially optimal debt choices, it stands out that b_{t+1}^1 is considerably higher than b_{t+1}^2 for every legacy debt stock.

   The horizontal black line displays the firms' eligible debt capacity, which is independent of legacy debt b_t. The optimal debt choice is indicated by the bold black line. It corresponds to b_{t+1}^2 until it reaches its eligible debt capacity at the first kink. For legacy debt levels between both kinks, the firm exactly exhausts its eligible debt capacity. For legacy debt levels above the second kink, firms forego eligibility. Risk-taking and disciplining effects at the firm-level are

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related to the difference between the blue dashed line b^1_{t+1}, which would be the optimal choice if there were no collateral premia, and the equilibrium debt choice $\mathcal{B}_{t+1}$. The disciplining effect is graphically represented by firms reducing their debt issuance below b^1_{t+1}, which applies to a sizeable mass of firms located near the right kink of the policy function. The risk-taking effect is reflected by all firms issuing debt according to $\mathcal{B}_{t+1} > b^1_{t+1}$ or by firms exhausting their eligible debt capacity.

Figure 3: Firm Debt Choice under Tight (A) Eligibility Requirements
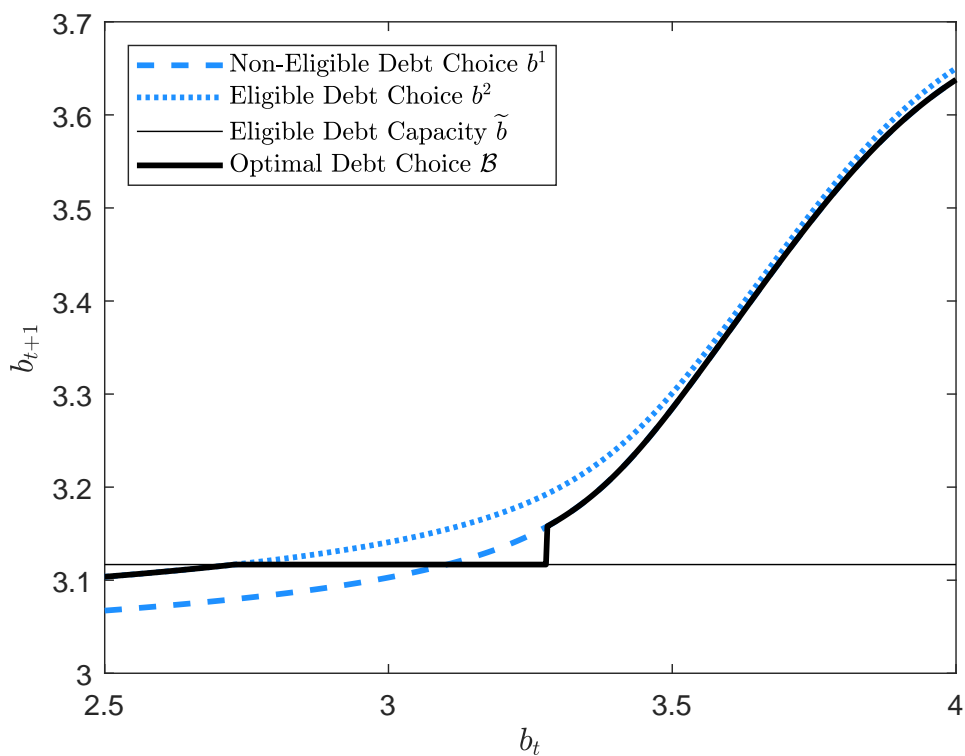
In figure 4, we show the policy functions under lenient eligibility requirements, again fixing the revenue state at the median. Level and shape of potentially optimal debt choices are very similar to figure 3, but the eligible debt capacity is markedly higher. The potentially optimal debt choices b^1_{t+1} and b^2_{t+1} slightly increase as well, which follows from the long duration of bonds: eligibility requirements also increase the rollover value of bonds, which then increases current bond issuance. The ensuing debt choice $\mathcal{B}_{t+1}$ has a much smaller region where firms are disciplined by eligibility requirements. This already suggests that the risk-taking effect is the

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dominating force for the impact of eligibility requirements on firms.

Figure 4: Firm Debt Choice, Lenient (BBB) Eligibility Requirements
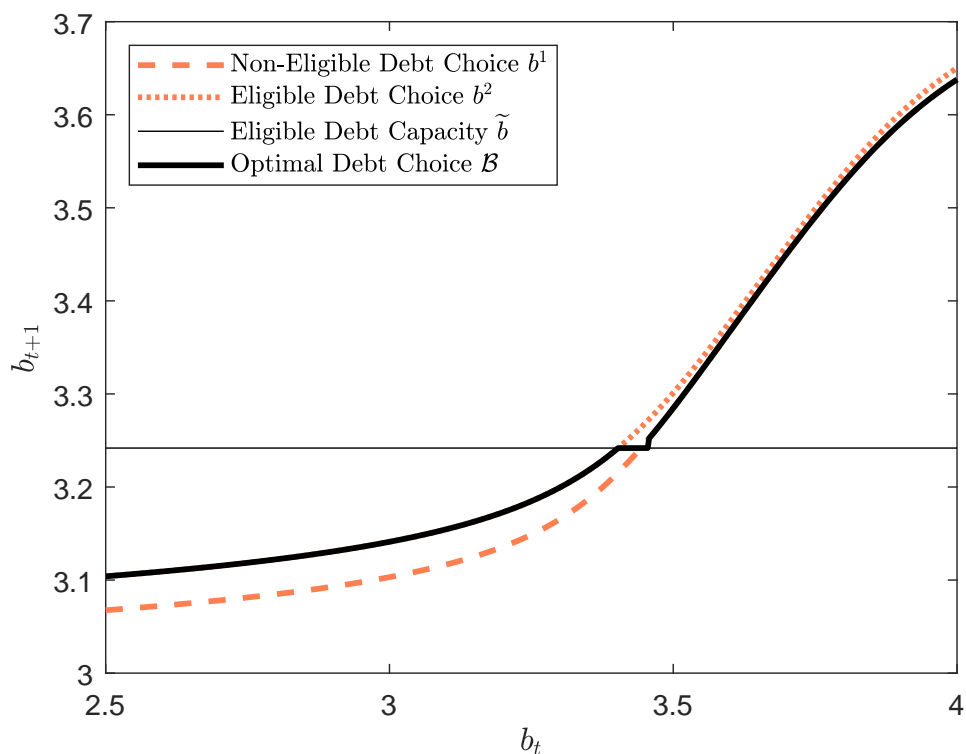
Reconciling Cross-Sectional Evidence Before discussing the cross-sectional firm distribution and macroeconomic aggregates, we test the model's capability to replicate the impact of eligibility requirements identified by several empirical papers. Therefore, we run several regressions on a simulated cross-section of firms, which is drawn from the equilibrium firm distribution of our baseline calibration. Running these regressions on the firm distribution associated with lenient eligibility requirements yields very similar results. We consider three

specifications, which differ in the outcome variable

$$r_t^j - r_t^{j,0} = \beta_0 + \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \frac{b_t^j}{\mu_t^j} + \varepsilon^j \,, \tag{23}$$

$$\mathcal{B}_{t+1}^j - b_{t+1}^{j,1} = \beta_0 + \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \frac{b_t^j}{\mu_t^j} + \nu^j \,, \tag{24}$$

$$\mathcal{D}_t^j - d_t^{j,1} = \beta_0 + \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \frac{b_t^j}{\mu_t^j} + \chi^j \,. \tag{25}$$

$$\tag{26}$$

First, we examine the bond yield reaction to surprise eligibility $r_t^j - r_t^{j,0}$ in (23), following Pelizzon et al. (2020). By controlling for firm leverage as a measure of default risk, this is similar to the approach taken in Mota (2021). As second step, we evaluate the effect of a surprise inclusion on debt issuance $\mathcal{B}_{t+1}^j - b_{t+1}^{j,1}$ in (24) and dividends $\mathcal{D}_t^j - d_t^{j,1}$ in (25), respectively.[16] The sign of the regression coefficients in all three specifications are collected in table 3. We compare the results from the model-implied regression to coefficients reported in several empirical papers.[17]

Table 3: Cross-Sectional Regression Results

	Data			Model		
Control	$r_t^{j,0} - r_t^j$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,1}$	$\mathcal{D}_t^j - d_t^{j,1}$	$r_t^{j,0} - r_t^j$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,1}$	$\mathcal{D}_t^j - d_t^{j,1}$
Eligibility	+	+	+	+	+	+
Leverage × Eligibility	-	-	-	-	-	-

For the yield reaction, the coefficient on eligibility naturally equals +11bp in both cases,

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[16]The equilibrium dividend in period t is given by

$$\mathcal{D}(b_t^j, \mu_t^j) = e^{\mu_t^j} - (\pi + \kappa)b_t^j + q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j)\left(\mathcal{B}(b_t^j, \mu_t^j) - (1-\pi)b_t^j\right)$$

while the dividend of an in-eligible, but otherwise identical firm can be written as

$$d^1(b_t^j, \mu_t^j) = e^{\mu_t^j} - (\pi + \kappa)b_t^j + q(b^1(b_t^j, \mu_t^j), \mu_t^j)\left(b^1(b_t^j, \mu_t^j) - (1-\pi)b_t^j\right) \,.$$

[17]In all three cases, we assume that standard errors are i.i.d. across firms. Note that we indexed outcome and control variables by $t+1$ and t, respectively. However, the regressions are still performed on the simple cross-section and not on a panel. Since we sample from a parsimonious structural model, all coefficients are highly significant.

since this a targeted moment of our calibration. The negative coefficient on the interaction of eligibility with leverage is in line with Mota (2021), who showed that the 'safety premium' on US corporate bonds declines in their default risk. The positive impact of eligibility on debt issuance is consistent with findings by Pelizzon et al. (2020), while the positive effect of eligibility on dividends has been described in Todorov (2020). Crucially, debt issuance and dividend payouts respond more strongly for less risky firms, as the negative coefficients on the interaction term of eligibility and beginning-of-period leverage demonstrate.[18] The negative coefficient can therefore be interpreted as evidence for state-dependent risk-taking, which has been documented by Mota (2021). Firms with high leverage respond only weakly to eligibility and, in some cases, even decrease their risk-taking. The negative coefficient on the interaction term is also in line with the results of Grosse-Rueschkamp et al. (2019), who report that in particular firms rated A or high increased their leverage ratio by 1.8 p.p. in response to CSPP-eligibility, as opposed to eligible BBB-rated firms, which only weakly increase leverage by 0.8 percentage points. Taken together, our model is able to capture the impact of eligibility requirements on multiple firm outcome variables documented in the data.

5.2 Firm Distribution

Next, we show the effects of collateral easing on the firm distribution. Specifically, we compare the bond spread distribution for the baseline calibration to lenient (BBB) eligibility requirements, and to lenient eligibility requirements with constant firm behavior, which we mark in purple. Differentiating between the full equilibrium response and the share of eligible bonds with constant firm behavior allows us to decompose the total collateral supply response into *mechanical effects* and firm responses.

We divide firms into different spread buckets and show the resulting histogram in figure 5. The colored part of each bar represents *eligible firms*, while the shaded gray area represents the mass of *all additional, non-eligible firms* in the respective bucket. For example, about half firms in the lowest four risk buckets are eligible in the baseline calibration. This share increases substantially in the case of collateral easing with constant debt choices: almost all firms in the lowest five risk buckets are eligible, while the (unconditional) distribution of firms does not change by definition. Allowing firms to change their debt choice markedly increases the share of firms in the left tail of the distribution. This follows from the high likelihood of satisfying the minimum rating requirement in future periods, which are associated with low bond spreads. The quartiles of the spread distribution decrease from 30bp, 58bp, and 80bp to 0bp, 39bp, and

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[18]Since leverage is the main driver of default in our model, higher levered firms can be classified as more risky.

70bp, respectively.

Figure 5: Firm Distribution over Bond Spreads
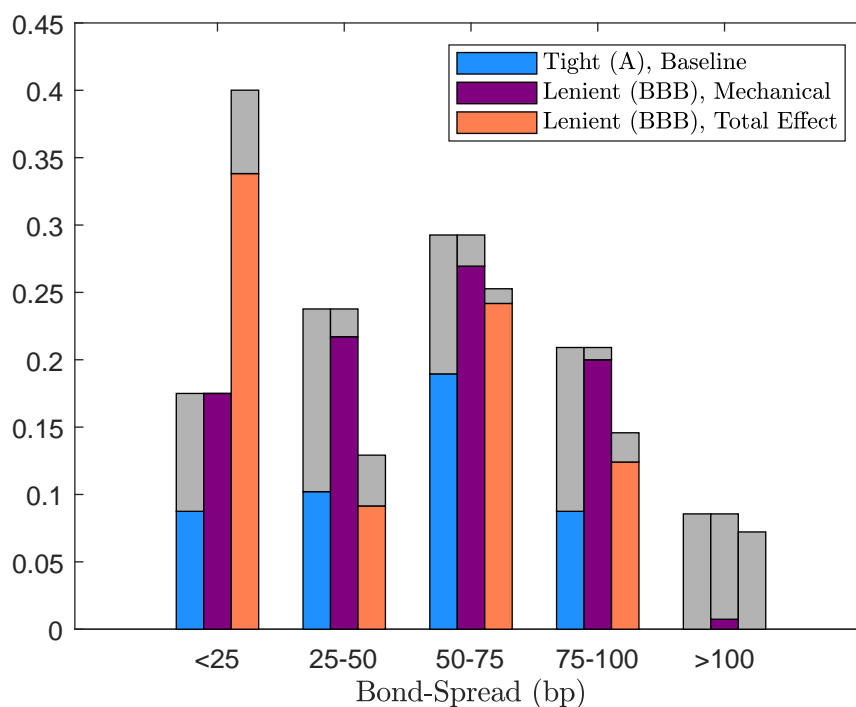
Since bond spreads contain both a fundamental and a collateral service component, we also inspect the (annualized) one-period ahead default probabilities, which is a direct measure of corporate default risk. The right shift of the unconditional distribution over default probabilities in figure 6 reveals that collateral easing is associated with an increase in firm default risk. Nevertheless, all firms in the lower default risk buckets are eligible after collateral easing, while only firms in the lowest two risk buckets were eligible in the baseline.

Figure 6: Firm Distribution over Default Probabilities
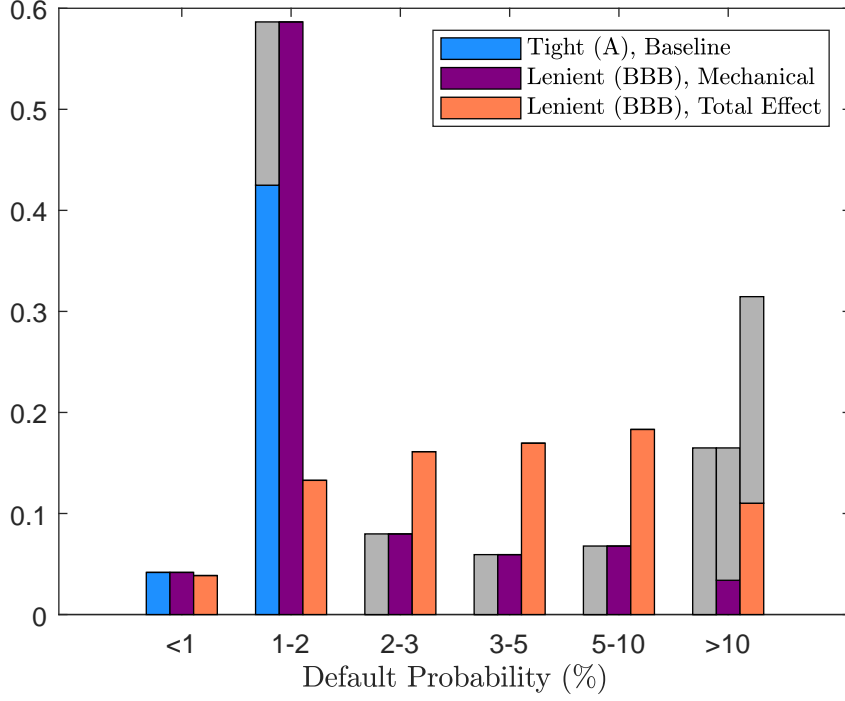
Having studied the changes to the cross-sectional firm distribution induced by collateral eas-ing, we now turn to aggregate effects. Our discussion is based on a decomposition of collateral supply into a *mechanical effect*, the change in *collateral quality* and *collateral quantity*. In the full model, this decomposition obtains from expanding the total effect as follows:

$$\Delta(\overline{B}) \equiv \int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^A < \overline{F}^A\} q^A b^A dG^A(\mu, b)$$

$$= \underbrace{\int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^A < \overline{F}^{BBB}\} q^A b^{BBB} dG^A(\mu, b)}_{\Delta \text{Collateral Quality}}$$

$$+ \underbrace{\int \mathbb{1}\{F^A < \overline{F}^{BBB}\} q^A b^{BBB} dG^A(\mu, b) - \int \mathbb{1}\{F^A < \overline{F}^{BBB}\} q^A b^A dG^A(\mu, b)}_{\Delta \text{Collateral Quantity}}$$

$$+ \underbrace{\int \mathbb{1}\{F^A < \overline{F}^{BBB}\} q^A b^A dG^A(\mu, b) - \int \mathbb{1}\{F^A < \overline{F}^A\} q^A b^A dG^A(\mu, b)}_{\text{Mechanical Effect}} .$$

The first line captures the change in collateral quality, which is the difference in collateral

supply due to rating and spread changes, represented by the default probability F and bond prices q, fixing debt issuance, the firm distribution, and the eligibility requirement $\overline{F}$ at the new equilibrium. The collateral quantity effect in the second line takes into account the overall change in debt issuance *conditional on eligibility*, represented by the mass of firms below the BBB-threshold, all else being equal. Finally, the mechanical effect keeps firm behavior and the cross-sectional distribution at the baseline calibration, varying only the eligibility requirement.

Applying this decomposition to our calibrated model, it stands out that the mechanical effect exceeds the total effect, as we show in table 4. We also report the effect found in two robustness checks. The details on these robustness checks are deferred to appendix E.1. Across all specifications, firm responses *dampen* the impact of eligibility requirements on collateral supply, which we can almost fully attribute to collateral quality. Consequently, there are adverse effects on the corporate bond market as measured by default costs.[19] Risk-taking effects turn out to be pivotal, when comparing the negative collateral quality effect to sign and magnitude of the bond quantity effect. Quantity expands by 1 to 2.5%, depending on the specification, which amplifies the impact of eligibility requirements. However, this is dominated by the collateral quality channel, which exceeds the quantity channel by a factor of 5 to 10 and implies that endogenous firm responses decrease collateral supply by up to 10%. This is quantitatively relevant, given a mechanical effect size of 68 to 85%.

This result is directly related to the persistence of revenue shocks and the stickiness of leverage: high-revenue firms issuing bonds below their eligible debt capacity find it optimal to increase their debt issuance and increase current dividends. If revenues are sufficiently persistent and firm managers are sufficiently impatient, this only leads to a modest increase in default risk in the current period. Ultimately however, firms will receive adverse revenue shocks and - due to the inherent stickiness of leverage as in Gomes et al. (2016) - find themselves in a debt overhang, which makes default more likely.[20]

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[19]There are no mechanical effects on aggregate default costs by construction.

[20]This feature is not present in our simplified setting with i.i.d. shocks and one-period bonds. In such a setting, it is not optimal for firms to increase debt issuance to a point that it decreased the market value of bonds outstanding.

Table 4: Macroeconomic Effects of Collateral Easing

Baseline	Total Effect	Collateral Quantity	Collateral Quality	Mechanical Effect
Collateral Supply $\overline{B}$	+68.2%	+1.9%	-10.7%	+77.1%
Default Costs $\mathcal{M}$	+13.9%	-	-	-
Low Collateral Premium L = 0.0025				
Collateral Supply $\overline{B}$	+68.4%	+1.1%	-8.3%	+75.6%
Default Costs $\mathcal{M}$	+10.5%	-	-	-
High Fundamental Risk $\sigma_\mu = 0.0475$				
Collateral Supply $\overline{B}$	+65.6%	+2.3%	-12.6%	+75.9%
Default Costs $\mathcal{M}$	+13.1%	-	-	-

The relative importance of collateral quality across all specifications suggests that firm responses are detrimental to the conduct of collateral policy in two dimensions. First, they induce adverse side effects on the corporate bond market. In practice, higher prevalence of default risk can directly increase restructuring costs or inefficient liquidation of firms and indirectly make the financial system fragile, for example due to counterparty default risk. Second, the overall dampening requires the central bank to relax eligibility requirements more aggressively to achieve a specific increase in collateral supply.

5.3 Eligibility Covenants

Finally, we propose eligibility covenants as a potential central bank instrument to mitigate adverse quality effects while at the same maintaining a positive quantity effect. These covenants restrict the eligible debt capacity of firms beyond the maximum default probability $\overline{F}$. Since the persistence of idiosyncratic states is key to adverse collateral quality effects, it comes naturally to make the covenant state-dependent as well. In addition, the covenant should satisfy an implementability condition in the sense that it conditions period t-eligibility on observable firm characteristics at time t. Suitable and implementable covenants can be either based on beginning-of-period leverage b_t^j or revenues μ_t^j in our setting.

We consider leverage-based covenants in the following, since leverage is common knowledge for firms that are sufficiently large to issue marketable debt securities. This still leaves us with all functions mapping from the debt state space into the binary eligibility indicator

$\Psi \in \{0, 1\}$. Therefore, we focus on the exponential class, parametrized by $f_b > 0$, such that the eligible debt capacity is decreasing in leverage:

$$\widetilde{b}^j_{t+1} = \exp\{-f_b b^j_t\} \cdot \frac{\exp\{\sigma_\mu \Phi^{-1}(\overline{F}) + \rho_\mu \mu^j_t\}}{\pi + \kappa} . \tag{27}$$

Note that this is equivalent to making the eligibility threshold negatively dependent on leverage $\frac{\partial \overline{F}^j_t}{\partial b^j_t} < 0$. Because current collateral frameworks are typically based on ratings or related measures of default risk, this is a relevant alternative to implement eligibility covenants. The debt choice under the (optimal) debt covenant is shown in figure 7, where we again fixed revenues at the median level and consider the case of lenient eligibility requirements. Intuitively, introducing an eligibility covenant reduces the maximum amount of debt that can be issued without losing eligibility, if firms enter the period with a large legacy debt stock. The downward sloping shape of the eligible debt capacity implies a larger disciplining effect, since the optimal debt choice $\mathcal{B}$ is located below b^1 for a larger range of legacy debt stocks. This is particularly pronounced in comparison to the debt choice without covenants in figure 4.

Figure 7: Firm Policy with (Optimal) Covenant
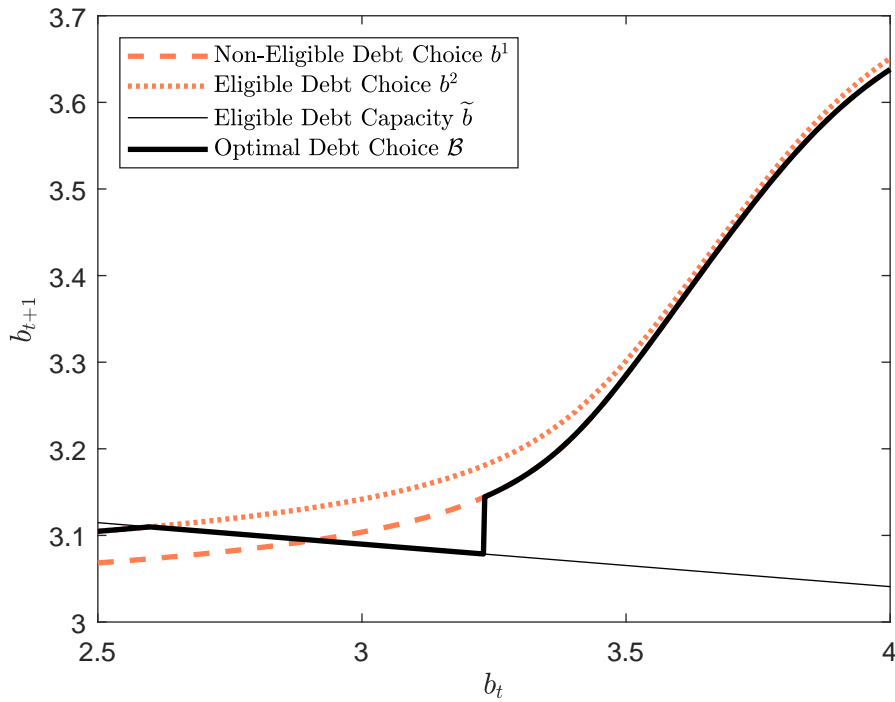
Optimal Covenants The policy trade-off regarding the covenant is to set f_b to a value that moderately punishes debt issuance of firms facing adverse revenue shocks. Setting an overly harsh covenant (a large f_b) reduces collateral supply, since it dis-incentives firms from issuing bonds altogether. By contrast, an overly lenient covenant (a small f_b) fails to limit the risk-taking by eligible firms. We maximize collateral supply $\overline{B}$ by varying the covenant parameter f_b over a large interval and show the results in figure 8. This procedure reveals the existence of a "collateral Laffer curve". The case without a covenant is given by the left bound $f_b = 0$, while the optimal covenant parameter is indicated by the dashed vertical line.

Figure 8: Collateral Laffer Curve
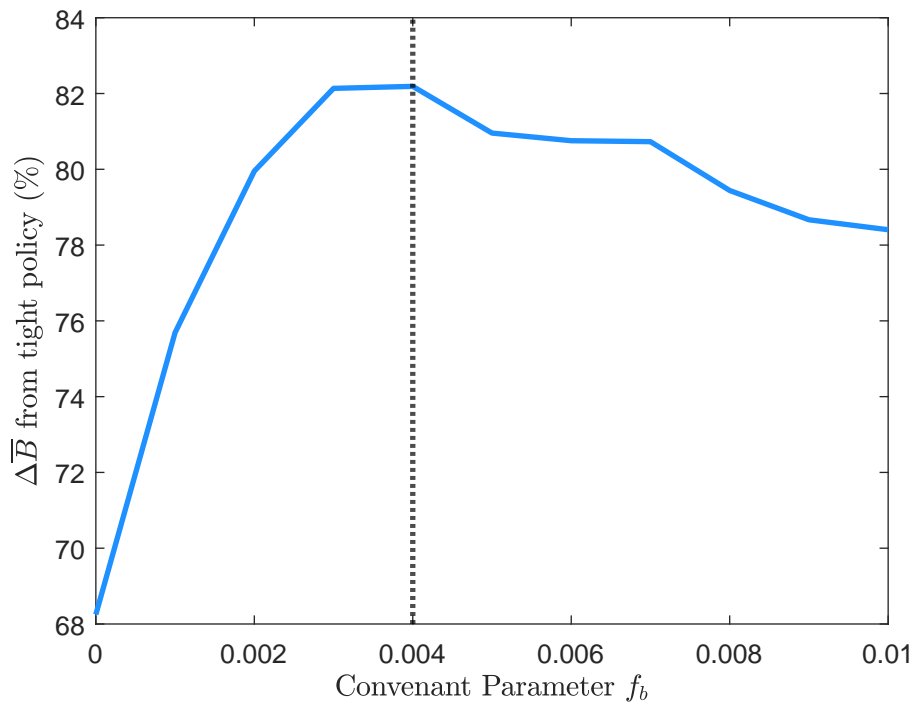
We decompose the collateral supply effect of the optimal covenant in table 5. In comparison to macroeconomic aggregates without covenants (table 4), we observe a substantial increase in collateral quality, while at the same time there is a small negative on collateral quantity. This result is robust to using a lower collateral premium and higher fundamental risk.

Table 5: Macroeconomic Effects of Collateral Easing with Optimal Covenant

Baseline	Total Effect	Collateral Quantity	Collateral Quality	Mechanical Effect
Collateral Supply $\overline{B}$	+82.2%	-0.7%	+5.8%	+77.1%
Default Costs $\mathcal{M}$	-3.2%	-	-	-
Low Collateral Premium $L = 0.0025$				
Collateral Supply $\overline{B}$	+79.3%	-1.5%	+5.3%	+75.6%
Default Costs $\mathcal{M}$	-12.2%	-	-	-
High Fundamental Risk $\sigma_\mu = 0.03$				
Collateral Supply $\overline{B}$	+80.7%	-0.3%	+5.2%	+75.9%
Default Costs $\mathcal{M}$	+0.0%	-	-	-

Macroeconomic Relevance   To illustrate the macroeconomic relevance of our results, we plot aggregate collateral supply by the private sector over time in figure 9. The major relaxation of eligibility requirements in 2008 is marked by the vertical line. While there is an upward trend in collateral supply by the corporate sector between 2004-2007, this has been attributed to the secular trend in market-based financing in Europe (Darmouni and Papoutsi, 2021), global trends towards securitization (Acharya et al., 2013), maturing capital markets following the introduction of the currency union in 1999, and the large increase in credit to the private sector prior to the Great Financial Crisis. Since data at a higher observation frequency are not available before 2012 and the policy relaxation occurred mid-way through 2008, we focus on the difference between 2007 and 2009. Interpreting total collateral supply of EUR 1876 billion in 2007 as the baseline in our model, the increase to EUR 3065 billion in 2009 amounts to a total effect of 68%, which we targeted when calibrating $\overline{F}^{BBB}$.

Figure 9: Aggregate Collateral Supply by Corporate Sector over Time
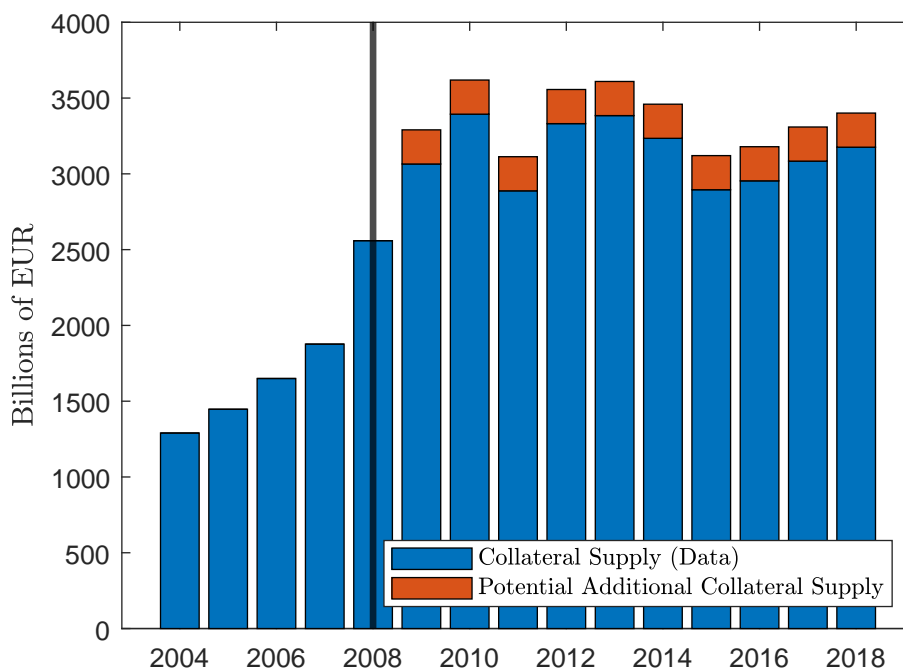
The orange bars represent endogenous firm responses implied in our decomposition of total collateral supply into a mechanical effect and endogenous firm responses. We obtain the counterfactual by computing the difference in collateral supply between the equilibrium with and without covenant, which amounts to 12%. Since the tight policy reference point is identical in both cases, we apply this increase of 12% to the total collateral supply in 2007. The potential additional collateral supply under the assumption of constant firm behavior is then given by EUR 225 billion, which we add to the total collateral supply in 2009 and all years thereafter. This exercise should only be interpreted as illustration, since our quantitative analysis abstracts from aggregate risk and alternative sources of collateral, such as government bonds, which was subject to considerable variation in 2011 and 2012.

Implementing eligibility covenants in practice requires information both about current profitability and current indebtedness of firms. Moreover, the debt repayment schedules of large firms are often difficult to determine, particularly if firms have multiple subsidiaries or parent

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companies. Due to the large amount of information necessary to evaluate the credit risk of bonds, central banks typically base their collateral framework on ratings determined by external rating agencies. In line with this practice, one natural way to implement covenants is to condition eligibility on the rating outlook, which is akin to making the eligibility threshold negatively dependent on leverage. Firms rated A, but with negative outlook can for example be interpreted as being on a financially unsustainable path and could therefore be subjected to a tighter eligibility requirement than a firm rated BB+, but with a positive outlook.[21] Fine-tuning the minimum rating requirements would require a richer model with an explicit representation of ratings, which is beyond the scope of this paper.

6 Conclusion

This paper evaluates the effects of central bank eligibility requirements on the debt and default decision of firms, i.e. the collateral supply side. Adding collateral premia and eligibility requirements to a heterogeneous firm model with default risk reveals that firms can be affected in different ways: low-risk firms increase their debt issuance and risk-taking, whereas high-risk firms are be disciplined by the prospect of benefiting from collateral premia. Both effects increase aggregate collateral quantity, while they have opposing effects on collateral quality. Which of these two effects is the dominating force is therefore a numerical question. Consistent with empirical evidence at the firm level, our numerical findings suggest that risk-taking is the dominating force in the aggregate. Endogenous firm responses are quantitatively relevant and substantially dampen the impact of collateral easing on collateral supply. Eligibility covenants are suitable instruments to alleviate adverse quality effects and therefore help maintaining a high level of collateral supply.

Our work can be extended along multiple dimensions. Interacting endogenous collateral supply with frictions on the collateral demand side, such as aggregate liquidity risk, can potentially generate interesting interactions with non-trivial implications for the conduct of collateral policy. It should also be stressed that we take investment opportunities as exogenous. A model with endogenous investment allows to study real effects of eligibility requirements using a richer trade-off between distributing cashflows as dividends and investment. We also do not account for bank loans as alternative source of financing, which is also a margin affected by eligibility requirements. All extensions add additional layers of complexity to our present framework, and we leave them to future research.

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[21]An alternative way would be a conditioning on CDS-spreads.

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A   Proofs

This section contains the proofs of Proposition 1 and Lemma 1.

A.1 Proof of Proposition 1

The partitioning of firms into different groups (unconstrained eligible, constrained eligible, and ineligible) uses the fact that there are three *potentially* optimal debt choices for every s. The first possibility is to issue bonds $\widetilde{b}_{t+1}(s)$ to be exactly at the eligibility threshold. By the strict monotonicity of $F(b_{t+1}|s)$ in b_{t+1}, there is a unique $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ where the corporate bond is just eligible. Second, there is a debt level b^1_{t+1} satisfying the first-order condition (7) for the case of ineligibility. Third, the level b^2_{t+1} solves (8), the first-order condition in the eligibility case. Under the monotonicity assumption on $h(b_{t+1}) \cdot b_{t+1}$, both conditions are satisfied by a unique b^1_{t+1} and b^2_{t+1}, respectively. Moreover, since $\frac{1-\beta+L}{1+L} > 1 - \beta$ for every $L > 0$ and $0 < 1 - \beta < 1$, it holds that $b^1_{t+1} < b^2_{t+1}$, which reflects that the outward shift of the bond price schedule due to eligibility incentivizes the firm to choose a higher leverage. The remainder of the proof characterizes which of these three debt levels is optimal, given the type parameter s.

Existence of Type Space Partitions There is a positive mass of unproductive firms, such that $\widetilde{b}_{t+1}(s) = 0 < b^1_{t+1}(s) < b^2_{t+1}(s)$, which holds at least for $s = s^-$ by assumption. These firms are not able to issue any bonds without exceeding the minimum quality requirement $\overline{F}$, i.e. their eligible debt capacity is zero. On the other hand, there are firms with positive eligible debt capacity. This can be shown by finding values s_1 and s_2 such that $b^1_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ and $b^2_{t+1}(s_2) = \widetilde{b}_{t+1}(s_2)$, i.e. firms are able to issue debt according to (7) and (8) without losing eligibility. We then show that the cut-off values satisfy $s^- < s_1 < s_2$. From the mass-shifting property of s, we can express the eligible debt capacity as

$$\widetilde{b}_{t+1}(s) = F^{-1}(\overline{F}) + s. \tag{A.1}$$

Plugging this into the first-order conditions (5) and (6), we get

$$\frac{\partial V_1(s)}{\partial b}\bigg|_{\widetilde{b}_{t+1}(s)} = (1-\beta)(1-\overline{F}) - \left(F^{-1}(\overline{F})+s\right) f(F^{-1}(\overline{F})) \tag{A.2}$$

$$\frac{\partial V_2(s)}{\partial b}\bigg|_{\widetilde{b}_{t+1}(s)} = \frac{1-\beta+L}{1+L}(1-\overline{F}) - \left(F^{-1}(\overline{F})+s\right) f(F^{-1}(\overline{F})). \tag{A.3}$$

For a sufficiently profitable firm, i.e. firms with a large s, eligible debt capacity $\widetilde{b}_{t+1}(s)$ lies on the downward sloping part of the objective function. Since the objective is concave by the monotone hazard rate assumption, $\widetilde{b}_{t+1}(s)$ is not optimal and such a firm voluntarily issues less debt than in could without losing eligibility. From $1-\beta < \frac{1-\beta+L}{1+L}$, it follows that $s_1 < s_2$. We can

exploit the monotonicity of the first-order conditions in s and monotonicity of the eligible debt capacity $\frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} = 1$. Implicitly differentiating (5) and (6) with respect to s

$$\frac{\partial b^1_{t+1}(s)}{\partial s} = \frac{(1 - F(b^1_{t+1}|s)) f'(b^1_{t+1}|s) b^1_{t+1} + f(b^1_{t+1}|s)^2 b^1_{t+1}}{(1 - F(b^1_{t+1}|s)) [f'(b^1_{t+1}|s) b^1_{t+1} + f(b^1_{t+1}|s)] + f(b^1_{t+1}|s)^2 b^1_{t+1}}$$
$$\frac{\partial b^2_{t+1}(s)}{\partial s} = \frac{(1 - F(b^2_{t+1}|s)) f'(b^2_{t+1}|s) b^2_{t+1} + f(b^2_{t+1}|s)^2 b^2_{t+1}}{(1 - F(b^2_{t+1}|s)) [f'(b^2_{t+1}|s) b^2_{t+1} + f(b^2_{t+1}|s)] + f(b^2_{t+1}|s)^2 b^2_{t+1}} \ .$$

The partial derivatives $\frac{\partial b^1_{t+1}(s)}{\partial s}$ and $\frac{\partial b^2_{t+1}(s)}{\partial s}$ are strictly smaller than one, since the first-order conditions (5) and (6) imply that firms are risky, i.e. $f(b^1_{t+1}|s) > 0$ and $f(b^2_{t+1}|s) > 0$. Since by assumption $\widetilde{b}_{t+1}(s^-) = 0$ and $b^1_{t+1}(s^-) > 0$, $s_1 > s^-$ follows immediately. Furthermore, the cut-off values s_1 and s_2 are unique.

Characterizing Debt Choices   For every $s > s_2$, firms issue less debt than it could issue without losing eligibility. All firms with $s > s_2$ choose leverage according to their first-order condition and are called *unconstrained eligible*. Consider next firms which cannot choose their optimal borrowing without losing eligibility, i.e. firms with $s < s_2$. Define the hypothetical value functions for a never eligible firm $V^1(b_{t+1}|s)$ and an always eligible firm as $V^2(b_{t+1}|s)$. All firms between s_1 and s_2 choose to be just eligible and lever up until $\widetilde{b}_{t+1}(s)$, since for them $V^2(b^2_{t+1}(s)|s)$ is not feasible and $V^1(b^1_{t+1}(s)) < V^2(b^1_{t+1}(s)) < V^2(\widetilde{b}_{t+1}(s))$. The first inequality follows from $V^2(b_{t+1}|s) > V^1(b_{t+1}|s)$ for all b_{t+1}, holding s constant. The second inequality follows from the fact that V_2 is increasing between $b^1_{t+1}(s)$ and $\widetilde{b}_{t+1}(s)$. Finally, there is a threshold $s_0 < s_1$, below which firms choose $b^1_{t+1}(s)$ and are not eligible. All firms between s_0 and s_1 also choose $\widetilde{b}_{t+1}(s)$. The value s_0 is implicitly defined through the indifference condition $V^2(\widetilde{b}_{t+1}|s_0) = V^1(b^1_{t+1}|s_0)$. The assumptions on the revenue distribution will imply the existence of exactly one s_0 by the intermediate value theorem. To see this, consider their difference

$$\Delta(s) \equiv V^2(\widetilde{b}_{t+1}(s)|s) - V^1(b^1_{t+1}(s)|s) \ . \tag{A.4}$$

Obviously $\Delta(s_1) > 0$, because $b^1_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ and $V^2(\widetilde{b}_{t+1}(s_1)|s_1) > V^1(\widetilde{b}_{t+1}(s_1)|s_1)$. In addition, there exists a level s^- where $F(0|s^-) > \overline{F}$ by assumption. At this level $V^2(\widetilde{b}_{t+1}(s^-)|s^-) - V^1(b^1_{t+1}(s^-)|s^-) < 0$, because $\widetilde{b}_{t+1}(s^-) = 0$ and $V^2(\widetilde{b}_{t+1}(s^-)|s^-)$ is the value of the unlevered firm. Choosing $b_{t+1} = 0$ however violates (7) and therefore $V^1(b^1_{t+1}(s)|s)$ exceeds the value of an unlevered firm for every s. Together with continuity of s, this already implies existence of at least one s_0 by the intermediate value theorem. To establish uniqueness, we differentiate $\Delta(s)$ with

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respect to s. The first part of $\Delta(s)$ can be written explicitly as

$$V^2(\widetilde{b}_{t+1}(s)|s) = (1-\overline{F})(1+L)\widetilde{b}_{t+1} + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - \widetilde{b}_{t+1}(s)dF(\mu_{t+1}|s),$$

and its total derivative is given by

$$\frac{\partial V^2(\widetilde{b}_{t+1}(s)|s)}{\partial s} = \frac{\partial V^2(\widetilde{b}_{t+1}(s)|s)}{\partial \widetilde{b}_{t+1}}\frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} + \frac{\partial V^2(\widetilde{b}_{t+1}(s)|s)}{\partial s}\Bigg|_{\widetilde{b}_{t+1} const}$$

$$= \left((1-\overline{F})(1+L) + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} (-1)dF(\mu_{t+1}|s)\right)\frac{\partial \widetilde{b}_{t+1}}{\partial s} + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1}-\widetilde{b}_{t+1}(s))df(\mu_{t+1}|s)$$

$$= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) - \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1}df(\mu_{t+1}|s) + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \widetilde{b}_{t+1}(s)df(\mu_{t+1}|s)$$

$$= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) - \beta\left(f(\overline{\mu})\overline{\mu} - f(\widetilde{b}_{t+1}(s)|s)\widetilde{b}_{t+1}(s) - (1-F(\widetilde{b}_{t+1}(s)|s))\right)$$

$$\quad + \beta\widetilde{b}_{t+1}(s)\left(f(\overline{\mu}) - f(\widetilde{b}_{t+1}(s)|s)\right)$$

$$= (1-\overline{F})(1+L). \tag{A.5}$$

Here we used again that $\frac{\partial \widetilde{b}_{t+1}}{\partial s} = 1$. The second part of (A.4) is given by $\frac{\partial V^1(b_{t+1}(s),s)}{\partial s}$, since $\frac{\partial V^1(b_{t+1}(s),s)}{\partial b_{t+1}} = 0$ by the principle of optimality, when totally differentiating $V^1(b_{t+1}(s)|s)$ with respect to s. Specifically,

$$\frac{\partial V^1(b_{t+1}^1(s)|s)}{\partial s} = f(b_{t+1}^1(s) - s) \cdot b_{t+1}^1(s) + \beta \int_{b_{t+1}^1(s)}^{\overline{\mu}} -(\mu_{t+1} - b_{t+1}^1(s))f'(\mu_{t+1}|s)d\mu_{t+1}$$

$$= (1-\beta)\left(1 - F(b_{t+1}^1(s)|s)\right) + \beta\left(1 - F(b_{t+1}^1(s)|s)\right)$$

$$= 1 - F(b_{t+1}^1(s)|s). \tag{A.6}$$

In the second line, we directly used the first-order condition (7). Putting both parts together

$$\frac{\partial \Delta(s)}{\partial s} = \frac{\partial V^2(\widetilde{b}_{t+1}(s)|s)}{\partial s} - \frac{\partial V^1(b_{t+1}^1(s)|s)}{\partial s} = (1-\overline{F})(1+L) - \left(1 - F(b_{t+1}^1(s)|s)\right) > 0.$$

The sign follows from the fact that $\widetilde{b}_{t+1}(s) < b_{t+1}^1(s)$ holds in the region of interest. This implies that the default probability at $b_{t+1}^1(s)$ exceeds the eligibility threshold, i.e. $F(b_{t+1}^1(s)|s) > \overline{F}$. The inequality follows from $(1 - F(b_{t+1}^1(s)|s)) < 1 - \overline{F}$ and $L > 0$. Since $\Delta(s)$ is continuous and mono-

tonically increasing, there exists an unique s_0 where the firm is indifferent between constrained eligibility and non-eligibility by the intermediate value theorem. All firms between s_0 and s_2 are called *constrained eligible*, firms below s_0 are *non-eligible*. $\square$

A.2 Proof of Lemma 1

To see that $\frac{\partial s_0}{\partial \overline{F}} < 0$, consider the indifference condition (A.4). The value of being constrained eligible $V^2(\widetilde{b}_{t+1}(s)|s)$ increases in $\overline{F}$. Differentiating eligible debt capacity $\widetilde{b}_{t+1}(s)$ with respect to the eligibility threshold yields

$$\frac{\partial \widetilde{b}_{t+1}(s)}{\partial \overline{F}} = \frac{\partial F^{-1}\left(\overline{F}|s\right)}{\partial \overline{F}} = \frac{1}{f\left(F^{-1}(\overline{F}|s)\right)} \, , \tag{A.7}$$

where the last step follows from the inverse function theorem. Relaxing eligibility requirements increases the eligible debt capacity and a constrained firm will always be better off after a relaxation of eligibility requirements $V^2(\widetilde{b}_{t+1}^A|s^{0,A}) < V^2(\widetilde{b}_{t+1}^{BBB}|s^{0,A})$. Furthermore, we showed in (A.5) that the value of a constrained eligible firm is increasing in the shifting parameter. Then, denoting eligible debt capacity and cut-off values before the policy change by $(s^{0,A}, \widetilde{b}_{t+1}^{0,A})$, the effect on the value of being constrained eligible is unambiguous

$$V^2(\widetilde{b}_{t+1}^A|s^{0,A}) < V^2(\widetilde{b}_{t+1}^{BBB}|s^{0,A}) < V^2(\widetilde{b}_{t+1}^{BBB}|s^{0,BBB}) \, .$$

Since the value of being ineligible $V^1(b_{t+1}^1(s)|s)$ does not depend on the eligibility threshold, the indifference point s^{BBB} satisfies $V^2(\widetilde{b}_{t+1}^{BBB}(s)|s) - V^1(b_{t+1}^1(s)|s)$ shifts to the left, i.e. $s^{0,BBB} < s^{0,A}$. To see the effect of eligibility thresholds on s_2, it suffices to note that $V^2(b_{t+1}^2|s)$ is independent of $\overline{F}$ and restrict attention to the condition pinning down the eligible debt capacity $F(\widetilde{b}_{t+1} - s) = \overline{F}$. Rearranging for s and differentiating yields $-\frac{\partial s_2}{\partial \overline{F}} = -\frac{1}{f(\overline{F})} < 0$ $\square$

B Corporate Bond Eligibility in Collateral Frameworks

This section reviews the eligibility of corporate bonds in ECB operations and on the interbank market. As we show in table B.1, eligibility of corporate bonds as collateral in central bank operations varies across countries and over time.

Table B.2 gives an overview of changes in the ECB collateral framework since 2007. Corporate bonds were eligible prior to the 2008-09 crises at a minimum rating requirement of A. In response to the financial crises, the minimum requirements were reduced from A to BBB,

Table B.1: Non-Financial Corporate Bonds in Various Collateral Frameworks

Central Bank	Pre GFC (Min. Rating)	Post GFC (Min. Rating)	Post Covid-19 (Min. Rating)
Australia	No	Yes (AAA)	Yes (BBB)
Eurosystem	Yes (A)	Yes (BBB)	Yes (BB)*
Japan	Yes (A)	Yes (BBB)†	Yes (BBB)
Switzerland	Yes (AA)	Yes (AA)	Yes (AA)
United Kingdom	No	Yes (A)	Yes (A)
United States††	Yes (AAA)	Yes (AAA)	Yes (BBB)

Notes: †: Multiple changes after Financial Crisis; *: For the duration of PEPP; ††: Only allowed in the discount window. Source: Bank for International Settlements (2013) & national CBs.

which substantially extended the amount of eligible assets and thereby broadened financial intermediaries' access to central bank liquidity. The smaller changes in 2011 and 2013 suggest that some fine-tuning was necessary after the initial relaxation. Nevertheless, the reduction of the minimum rating requirement was by far the largest adjustment, which motivates our choice of modelling collateral policy as a step function.

Table B.2: Corporate Bonds in the ECB Collateral Framework

Timespan	Regime	Haircut: A- or higher	Haircut: BBB- to BBB+
01 Jan 2007 - 24 Oct 2008	Fitch, S&P and Moody's are accepted ECAI, minimum requirement A-	4.5 %	100 %
25 Oct 2008 - 31 Dec 2010	DBRS legally and practically accepted as ECAI, minimum requirement BBB-	4.5 %	9.5 %
01 Jan 2011 - 30 Sep 2013	Tightening of haircuts	5 %	25.5 %
01 Oct 2013 - 01 Dec 2019	Relaxation of haircuts	3 %	22.5 %

Notes: Corporate bond with fixed coupon and maturity of 3 to 5 years; DBRS: Dominion Bond Rating Service, ECAI: external credit assessment institutions.

C Data

Corporate Bond Data We merge monthly data on the corporate bond universe in Europe from the iBoxx High Yield and Investment Grade Index families, provided by *IHS Markit*. We apply the following inclusion criteria:

1. Bond issuers are head-quartered in Euro Area member countries.

2. Issuers are non-financial firms.

3. The bond is denominated in Euro, senior, not callable, uncollateralized, and fixed coupon.

4. The issuer is part of the constituent list for at least 48 months.

Bond issuers are provided by *Markit* and we consider only the parent company level, since it can be reasonably assumed that dedicated financial management subsidiaries are identical from an economic perspective to the respective parent company.

Company Data Next, we match company names to their unique *Compustat* identifier (gvkey) and drop all companies which are not represented in the *Compustat Global* database. For the remaining firms we query *Compustat* for long-term liabilities (gvkey) in the firmq database and EBIT (gvkey) in the firma database.

D Computational Algorithm

We solve the individual firm problem using policy function iteration over a discrete set of collocation points using piecewise linear interpolation. The revenue shock is discretized using the method of Tauchen on an equispaced grid with $n_\mu = 25$ points over the interval $[-3\hat{\sigma}_\mu, +3\hat{\sigma}_\mu]$ with $\hat{\sigma}_\mu = \frac{\sigma_\mu}{1-\rho_\mu^2}$ denoting the unconditional variance of the revenue process. We denote the corresponding transition matrix Π_μ. Debt is discretized on an equispaced grid with $n_b = 21$ points over the interval $[5.5, 15.5]$.

To overcome the typical convergence issues in models with long-term debt and default, we use taste shocks when computing the debt choice (20), as proposed by Gordon (2018). The mass shifter for endogenous states follows immediately from the debt choice and is denoted Π_b. This matrix maps the current idiosyncratic state (μ_t^j, b_t^j), into next periods endogenous state b_{t+1}^j, i.e. has dimension $n_\mu \cdot n_b \times n_\mu \cdot n_b$.

Together with the transition matrix of idiosyncratic revenues and the aggregate state, the combined mass shifter $\Pi_g = \Pi_b \otimes \Pi_\mu$. Π_g implicitly defines the firm distribution G via $G' = G'\Pi_g$,

where G denotes the firm distribution. Extracting the distribution thus boils down to computing the right Eigenvalue to Π_g.

Starting with a guess for firm policies and bond prices, each iteration ι consists of four different steps.

(i) solve the firm problem taken as given the bond price schedule and value function from the previous iteration.

(ii) compute the eligible debt capacity (19), the associated values of the objective function, and determine the debt choice according to (20).

(iii) obtain the ensuing mass shifter Π_g from the policy functions and the transition matrix for revenue shock Π_μ and update the distribution G by iterating on $G = G'\Pi_g$.

(iv) Update bond price schedules and value functions.

We then iterate on the policy functions until convergence, i.e. $||\mathcal{B}^\iota(b_t^j, \mu_t^j) - \mathcal{B}^{\iota-1}(b_t^j, \mu_t^j)||_\infty < 10^{-5}$. The standard derivation of the taste shock is set to 0.01 to ensure convergence. This is typically achieved within 200 iterations.

E Additional Numerical Results

This section contains supplementary numerical results to our quantitative policy analysis. Appendix E.1 provides additional information to our robustness checks reported in table 4 and table 5.

E.1 Sensitivity to Calibrated Parameters

Low Collateral Premium   First, we show the results of lowering the collateral valuation parameter to $L = 0.0025$. This value implies a collateral premium of 7bp as reported in Mésonnier et al. (2021). Reducing L is associated with higher bond spreads ceteris paribus. Therefore, we decrease the revenue shock variance to $\sigma_\mu = 0.0325$ to ensure that spreads are consistent with the data. The recalibrated eligibility thresholds are given by $\overline{F}^A = 1.3\%$ and $\overline{F}^{BBB} = 16.4\%$, respectively. The overall model fit is of similar quality as the baseline calibration and, as table C.1 shows, the effect of relaxing eligibility requirements on spreads is negative across the entire firm distribution as well.

Table C.1: Targeted Moments, Low Collateral Valuation

Moment	Data	Model	Lenient	Covenant
Collateral premium $r - r_0$	7	7	7	7
Debt/EBIT $Q_{0.50}(b/\mu|\overline{F}^A)$	4.2	3.16	3.19	3.13
Bond spread $Q_{0.25}(x|\overline{F}^A)$	31	37	17	-16
Bond spread $Q_{0.50}(x|\overline{F}^A)$	51	54	42	-1
Bond spread $Q_{0.75}(x|\overline{F}^A)$	81	72	68	28
Eligible bond share $\overline{B}/(QB)|\overline{F}^A$	50%	52%	-	-
Eligible bond share $\overline{B}/(QB)|\overline{F}^{BBB}$	86%	-	86%	92%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

High Fundamental Risk Second, we consider the case of higher fundamental risk and target the spread level over a sample encompassing the financial crisis of 2008/09. To match the elevated level of spreads, we set $\sigma_\mu = 0.03$ and reduce $\rho_\mu = 0.92$ to match the increased cross-sectional dispersion. Similarly, we use $\overline{F}^A = 1.7\%$ and $\overline{F}^{BBB} = 18.5\%$ to recover the share of eligible bonds before and after relaxing eligibility requirements. Again, collateral easing has a similar impact on the cross-section of spreads as in the baseline calibration.

Table C.2: Targeted Moments, Higher Fundamental Risk

Moment	Data	Model	Lenient	Covenant
Collateral premium $r - r_0$	11	11	11	11
Debt/EBIT $Q_{0.50}(b/\mu|\overline{F}^A)$	4.2	3.06	3.11	3.06
Bond spread $Q_{0.25}(x|\overline{F}^A)$	44	58	30	-14
Bond spread $Q_{0.50}(x|\overline{F}^A)$	72	92	81	16
Bond spread $Q_{0.75}(x|\overline{F}^A)$	117	118	115	43
Eligible bond share $\overline{B}/(QB)|\overline{F}^A$	50%	50%	-	-
Eligible bond share $\overline{B}/(QB)|\overline{F}^{BBB}$	86%	-	81%	87%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

E.2 Endogenous Size of Collateral Premia

This section presents a robustness check of our results by endogenizing the *size* of collateral premia. While these have been fixed to a constant L in the baseline, we make them dependent on aggregate collateral supply. In this case, collateral premia decline after collateral easing, which reduces both risk-taking incentives for eligible firms and disciplining effects for firms slightly below the eligibility requirement. Whether and how this affect macroeconomic effects of collateral easing and optimal eligibility covenants can therefore only be assessed numerically. Assume that banks directly draw utility from holding collateral. For numerical and analytical tractability, we impose a CARA-functional form

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1\overline{B}\right\} \tag{F.1}$$

The collateral premium in this case is given by $L = l_0 \exp\{-l_1\overline{B}\}$. While we calibrate l_0 to match the collateral premium, the CARA-parameter l_1 governs the curvature of (F.1) and will be normalized to $l_1 = 1$. In table C.3 we show the model fit corresponding to a parameter choice of $\sigma_\mu = 0.029$ and $l_0 = 6$. The optimal eligibility covenant in this case is characterized by the parameter $f_b = 0.011$.

Table C.3: Targeted Moments, Robustness

Moment	Data	Model	Lenient	Covenant
Collateral premium $r - r_0$	11	12	1	1
Debt/EBIT $Q_{0.50}(b/\mu|\overline{F}^A)$	4.2	3.18	3.18	3.13
Bond spread $Q_{0.25}(x|\overline{F}^A)$	31	36	122	79
Bond spread $Q_{0.50}(x|\overline{F}^A)$	51	59	144	92
Bond spread $Q_{0.75}(x|\overline{F}^A)$	81	83	165	123
Eligible bond share $\overline{B}/(QB)|\overline{F}^A$	50%	50%	-	-
Eligible bond share $\overline{B}/(QB)|\overline{F}^{BBB}$	86%	-	85%	90%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

Different from the baseline model with constant collateral premia, the large increase in collateral supply induces a drastic decline of the collateral premium to $L \approx 1$bp. The spread levels are therefore substantially higher under lenient eligibility requirements compared to a tight

policy. This underscores the relevance of risk-taking effects in our model. Even though collateral quality effects are still negative, default costs experience a slight *decline*: firms take on more risk and are less likely to be eligible, but they default less often. Nevertheless, eligibility covenants have a positive effect, both on total collateral supply and on aggregate default costs.

Table C.4: Macroeconomic Effects, Robustness

No Covenant	Total Effect	Collateral Quantity	Collateral Quality	Mechanical Effect
Collateral Supply $\overline{B}$	+64.1%	-0.2%	-7.7%	+72.0%
Default Costs $\mathcal{M}$	-1.7%	-	-	-
Optimal Covenant				
Collateral Supply $\overline{B}$	+72.9%	-2.7%	+3.6%	+72.0%
Default Costs $\mathcal{M}$	-24.2%	-	-	-