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Optimal Corrective Policies under Financial Frictions[1]

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Abstract

This paper examines credit market policies under pecuniary externalities induced by collateral constraints. Pigouvian taxes/subsidies on debt or savings are derived as Ramsey-optimal policies. Firstly, prudential (ex-ante) debt taxes can restore constrained efficiency. Secondly, when policies are non-state and non-time contingent, debt subsidies can be superior to debt taxes. Thirdly, ex-ante saving subsidies are desirable when distributive effects dominate collateral effects. Fourthly, both effects can simultaneously be addressed by non-contingent saving subsidies. The analysis indicates that optimal policies can improve on constrained efficiency and that inefficiencies due to financial externalities can most effectively be addressed by interest rate reductions.

JEL classification: E44, G28, H23

Keywords: Pecuniary externalities, collateral constraint, incomplete markets, Pigouvian policies, inequality

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1 Introduction

Pecuniary externalities in financial markets can lead to inefficiencies and crises. The basic mechanism relies on price-dependent borrowing limits or margin constraints that tighten when asset prices fall, which can cause acceleration of adverse effects. Given that atomistic agents do not internalize the impact of their decisions on asset prices, corrective interventions in financial markets can potentially enhance efficiency. Prudential policies, like debt taxes and capital controls, have been established by Bianchi (2011), Benigno et al. (2016), Schmitt-Grohe and Uribe (2017), Korinek and Sandri (2016), Bianchi and Mendoza (2018), Korinek (2018), or Jeanne and Korinek (2019, 2020) to restore constrained efficiency, defined in the tradition of Stiglitz (1982), by addressing "overborrowing" of agents. These analyses are conducted within a class of models where interest rates are exogenously determined and agents take borrowing limits as given.[3]

This paper examines optimal Pigouvian credit market policies in a finite horizon incomplete markets model with domestic household debt and two types of agents. Limited commitment induces borrowing to be limited by borrowers' holdings of durables, serving as collateral. This leads to inefficiencies due to pecuniary externalities with respect to the durables price and the interest rate. Following the classification of Davila and Korinek (2018), the model features "collateral effects" as well as "distributive effects".[4] The former are responsible for the main mechanism in the above cited studies, while the latter are turned off therein. As the main novel contribution, we show that a prudential debt tax implementing the constrained efficient allocation can be outperformed by non-contingent corrective policies that stimulate borrowing.[5] Hence, the concept of constrained efficiency, by which pecuniary externalities can be identified, is not suited to guide optimal policy choices in general. Moreover, we find that borrowers' underconsumption, which is the primary failure due to externalities induced by collateral constraints,[6] can be most effectively addressed by corrective policies that reduce the costs of borrowing.

[3] An exception is Bianchi and Mendoza's (2018) model, where agents' stocks of physical capital serve as collateral. See Erten et al. (2021) for an overview.

[4] Yet, the model is held deliberately simple to enable the derivation of analytical results.

[5] We thereby abstract from apparently superior policies inducing borrowing constraints never to bind.

[6] This is supported by a supplementary analysis of Davila and Korinek's (2018) model (in Section 5).

	Ex-ante	Non-contingent
Debt	1. **tax**/subsidy	2. tax/**subsidy**
Savings	3. tax/**subsidy**	4. tax/**subsidy**

Table 1: Pigouvian policies (computation results in bold)

We apply the Ramsey approach to optimal policy under commitment and analyze Pigouvian taxes/subsidies on debt or savings (see rows of Table 1), which are non-equivalent under potentially binding borrowing constraints. We further compare prudential or ex-ante policies, which are exclusively imposed before borrowing constraints might become binding, to non-contingent policies, where the tax/subsidy rate is held constant regardless of the particular state/period (see columns of Table 1).[7] Thereby, we tie the policy maker's hands, in the sense that he cannot reduce market incompleteness by state-contingent policies.[8] Non-contingency is further motivated by the model property that inequality of income/endowments between agents rather than on aggregate income serves as a trigger for borrowing limits to get binding, such that state-contingency cannot simply be induced by cylicality of policy instruments.

The policy regimes under consideration highlight the role of collateral effects, which refer to uninternalized changes in the price of durables affecting the borrowing limit, and of distributive effects, which refer to interest rate changes and are based on marginal rates of substitutions that differ between agents: We *firstly* confirm that ex-ante taxes on debt enhance efficiency by addressing collateral effects. *Secondly*, we consider a non-contingent tax/subsidy on debt, which is constant and always imposed regardless whether the constraint is binding or not. We show that collateral effects are rather addressed by a debt subsidy than by a debt tax, if they are sufficiently small. *Thirdly*, we consider an ex-ante tax/subsidy imposed on lenders. A policy maker imposes an ex-ante subsidy (rather than a tax) on savings, which reduces the interest rate and thus the terms of borrowing for potentially constrained agents, when distributive effects dominate collat-

[7] Complementary to our analysis of optimal policy under commitment for different instruments, Benigno et al. (2020) study different allocations that can be implemented with the same set of instruments applying the Ramsey approach.

[8] See Benigno et al. (2013), Bianchi (2016) and Jeanne and Korinek (2020) for welfare-enhancing state-contingent policies that can be applied when borrowing constraints bind.

eral effects. *Fourthly*, we show that a non-contingent saving subsidy can simultaneously address distributive and collateral effects by stimulating borrowing and borrowers' consumption ex-ante and ex-post. For the optimal policy choices in the cases 2-4, the policy maker thus trades off collateral and distributive effects as well as the effects before and while borrowing constraints bind.[9]

The first policy regime, i.e. the optimal ex-ante debt tax, implements the constrained efficient allocation, which is defined as in Stiglitz (1982), Bianchi et al. (2011), Davila et al. (2012), or Davila and Korinek (2018). Concretely, a constrained efficient allocation is chosen by a social planer who determines borrowing and maximizes social welfare subject to budget and borrowing constraints, conditional on maintaining equilibrium price relations under laissez faire. Because an optimal ex-ante debt tax leaves the relevant equilibrium price relations under laissez faire unchanged, it implements the constrained efficient allocation.[10] In contrast, the other policy regimes alter the relevant price relations via two channels: Non-contingent credit market policies, which are imposed before and while borrowing constraints are binding, influence agents' willingness to pay for collateral via the valuation (i.e. the multiplier) of the borrowing constraint, and a savings tax/subsidy directly shifts the interest rate charged by lenders. Notably, the former channel does not exist when agents take borrowing limits as given and the latter channel when interest rates are exogenously determined. We show that the policy maker can exploit these price effects to address collateral and distributive effects in a more direct way than under an ex-ante debt tax, which operates indirectly via its influence on the endogenous state variable (debt). Specifically, the non-contingent debt tax/subsidy changes the equilibrium relation for the durables price, the ex-ante saving tax/subsidy changes the equilibrium relation for the interest rate, and the non-contingent savings tax/subsidy changes both equilibrium price relations compared to laissez faire.

To illustrate the effects of these four policy regimes, we provide a numerical analysis computing prices, the allocation, and social welfare. The four optimal policies are 1) an

[9] The latter latter trade-off apparently suggests that state-contingent policies would be able to enhance efficiency even further.

[10] See Benigno et al. (2016) or Davila and Korinek (2017) for corresponding results, and Bianchi and Mendoza (2018) for the case of time-consistent policies under discretion.

ex-ante debt tax, 2) a non-contingent debt subsidy, 3) an ex-ante saving subsidy, and 4) a non-contingent saving subsidy (see bold policies in Table 1). Except for the ex-ante debt tax, all policies tend to raise debt before the borrowing constraint binds,[11] and the non-contingent policies induce the largest increases in the collateral price. The ex-ante debt tax has the least impact on borrowers' consumption and leads to the smallest welfare gains relative to laissez faire, which are virtually negligible based on the distance to first best. In contrast, a non-contingent saving subsidy substantially (by more than 50%) reduces welfare losses under a competitive equilibrium compared to first best.

To demonstrate the robustness of our findings and to facilitate direct comparisons with related studies, we further examine ex-ante and non-contingent debt taxes/subsidies in a production economy. For this, we adopt the model from Davila and Korinek's (2018) application where "collateral externalities cause overborrowing". We confirm their result that a policy maker chooses a debt tax when it is imposed ex-ante, implementing the constrained efficient allocation.[12] If however the policy maker has a non-contingent debt tax/subsidy at his disposal, debt is subsidized. Thus, depending on the available policy instrument, the inefficiency seems to rely either on agents borrowing too much or too little. Under the ex-ante debt tax, addressing the collateral externality requires to raise consumption by reducing debt repayments. Under the non-contingent debt subsidy, borrowers tend to borrow and to consume more compared to laissez faire, raising the collateral value via their own willingness to pay. Given that borrowers consume to little under binding borrowing constraints and that the collateral price is positively related to their consumption, the analysis indicates that underconsumption – instead of overborrowing – is the primary failure due to externalities induced by collateral constraints.

Related literature This paper is related to several studies on policies that correct for pecuniary externalities based on financial frictions, like Bianchi (2011), Benigno et al. (2016), Korinek and Sandri (2016), Schmitt-Grohe and Uribe (2017), Bianchi and Mendoza (2018), Korinek (2018), or Jeanne and Korinek (2019). These studies focus on

[11] Relatedly, Bianchi and Mendoza (2018) find numerical results for higher debt compared to laissez faire when a social planer acts under commitment.

[12] Likewise, we replicate their result on over- and underinvestment depending on productivity of borrowers, applying the Ramsey approach to optimal policy.

constrained efficient allocations and prudential policies, like debt taxes or capital controls, that are imposed when borrowing constraints are not binding. Benigno et al. (2016) further examine optimal policies imposed on other markets, like for non-tradable goods. In contrast to these studies, we analyze non-contingent credit market policies that are also imposed when borrowing constraints bind and find that they can outperform prudential debt taxes. Benigno et al. (2013), Bianchi (2016) and Jeanne and Korinek (2020) consider less restrictive policy regimes and derive welfare gains from state-contingent (ex-post) policies. In contrast to our paper, none of these studies examines changes in the debt price (interest rate) as an outcome of financial market interventions or distributive effects. The latter term is adopted from the classification of Davila and Korinek (2018), who provide a comprehensive analysis of pecuniary externalities under financial frictions. Distributive effects under pecuniary externalities in financial markets are further studied in Lorenzoni (2008) and Davila et al. (2012). In contrast to the previous studies, which focus on the analysis of constrained efficient allocations, we examine Ramsey-optimal policies leading to potentially superior allocations.

The Ramsey approach is also used in Schmitt-Grohe and Uribe's (2017, 2021) analyses of state-contingent capital controls, for which Schmitt-Grohe and Uribe (2021) show the existence of multiplicity, giving rise to equilibria with underborrowing due to excessive precautionary savings. In this model, like in the model of Bianchi (2011), Benigno et al. (2016), Korinek and Sandri (2016), Korinek (2018), or Jeanne and Korinek (2019), agents take borrowing limits as given, since they solely depend on aggregate values. Therefore, a Ramsey-optimal debt tax does not affect the equilibrium relation for the collateral price, such that it implements a constrained efficiency allocation. Benigno et al. (2020) re-examine policy instruments used in Benigno et al. (2016), applying the Ramsey approach to optimal policy. Complementary to our analysis of different policy instruments, they show that a set of instruments that can implement a constrained efficient allocation can also be used to implement an allocation where borrowing constraints never bind.

Outline The remainder is structured as follows. Section 2 develops the model. Section 3 examines optimal policies. Section 4 presents numerical illustrations. Section 5 presents a robustness analysis for an economy with productive capital. Section 6 concludes.

2 An incomplete markets model with collateral constraints

The model has two main characteristics: Only non-state contingent bonds are available, such that financial markets are incomplete, and agents are not able to commit to debt repayment. The latter leads to a price-dependent borrowing constraint, which limits borrowers' intertemporal allocation possibilities and can drive a wedge between marginal rates of substitutions between borrowers and lenders. Agents do not internalize how their choices affect the collateral price and the price of debt, giving rise to collateral and distributive effects (see Davila and Korinek, 2017), when the borrowing constraint is binding with positive probability.

2.1 Details

There are two mass-one groups $\{b, l\}$ with infinitely many households, who live for three periods $t = 1, 2, 3$. In each period t, a household $i \in \{b, l\}$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good, $d_{i,t}$, as given by the function $u_{i,t} = u(c_{i,t}, d_{i,t})$. Agents maximize their expected lifetime utility, $E\sum_{t=1}^{3}\beta^{t-1}u(c_{i,t}, d_{i,t})$, where u is strictly increasing and concave, E denotes an expectations operator conditional on information in period 1, and $\beta \in (0, 1)$ is a discount factor. In each period, agents receive a potentially random endowment $y_{i,t}$ of non-durable goods and they exhibit an initial endowment of durables $d_{i,0}$. Agents can borrow and lend only in terms of non-state contingent one-period bonds $b_{i,t}$, which are issued at the price $1/r_t$. The budget constraint of a household i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + (1 - \tau_{i,t})b_{i,t}/r_t = b_{i,t-1} + y_{i,t} + T_{i,t}, \tag{1}$$

where $\tau_{i,t}$ denotes distortionary taxes/subsidies on debt/savings. Specifically, we consider Pigouvian-type fiscal interventions, where budgetary effects of taxes/subsidies are (ex-post) neutralized in a non-distortionary way:

$$T_{i,t} = -\tau_{i,t}b_{i,t}/r_t, \tag{2}$$

which is not internalized by households. There is no uncertainty in the periods 1 and 3, where total endowment with non-durables is equally distributed: $y_{b,1} = y_{l,1}$. Agents b

(l) start with negative (positive) initial net financial wealth $b_{b,0} < 0$ ($b_{l,0} > 0$) and will be called borrowers (lenders). In period 2, endowment is randomly determined and can either take the same values as in period 1 (state L) or can be unequally distributed (state H). Specifically, both states are equally likely and endowment of borrowers in state H (with Higher inequality) is $y_{b,2} = y/(1 + \delta_H)$, where $\delta_H > 1$.

We assume that agents cannot commit to repay debt and that debt can be renegotiated after issuance in the same period. Borrowers can make a take-it-or-leave-it offer to reduce the value of debt. If a lender rejects the offer, he can seize a fraction γ of the borrower's durable goods, which he can sell at the market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets. Without loss of generality, we assume that default and renegotiation never happen in equilibrium. When debt is issued, the amount of debt $-b_{i,t}$ is therefore constrained by

$$-b_{i,t} \leq \gamma q_t d_{i,t}. \tag{3}$$

According to (3), newly issued debt is constrained by the current market value of durables, which is consistent with empirical evidence (see Cloyne et al., 2019). The borrowing constraint (3) can generate a feedback from agents' demand for durables and the debt limit, which is not internalized in individual decisions. Moreover, the borrowing constraint can lead to unequal marginal rates of substitutions between states and agents, giving rise to distributive effects, as discussed in Davila and Korinek (2018).

The available stock of durables equals d and the total non-durable endowment equals y. Since there is no borrowing/lending in the final period, the borrowing constraint is irrelevant and there are also no taxes/subsidies on debt/savings in $t = 3$, i.e. $\tau_{i,3} = T_{i,3} = 0$. A competitive equilibrium is then given by an allocation of durables, non-durables, and debt $\{c_{i,1}, d_{i,1}, b_{i,1}, c_{i,2}(s), d_{i,2}(s), b_{i,2}(s), c_{i,3}(s), d_{i,3}(s)\}$ for $i \in \{b, l\}$ and $s \in \{L, H\}$, a set of prices $\{r_1, r_2(s), q_1, q_2(s), q_3(s)\}$ satisfying agents' maximization problem s.t. the budget constraints (1) and the collateral constraints (3), and the market clearing conditions, $d_{b,t} + d_{l,t} = d$ and $b_{b,t} + b_{l,t} = 0 \; \forall t$, given taxes/subsidies $\{\tau_{i,t}, T_{i,t}\}$ for $i \in \{b, l\}$ and $t \in \{1, 2\}$, and an initial distribution of debt and durables and sequences of non-durable endowments $\{y_{i,t}\}_{t=1}^{3}$ for $i \in \{b, l\}$.

2.2 Equilibrium properties

To further facilitate the analysis of optimal policies, we impose some simplifying assumptions on preferences and on the relevance of the borrowing constraint.

Assumption 1 *Agents preferences satisfy* $u_{l,t} = c_{l,t}$ for $t \in \{1, 2, 3\}$, $u_{b,t} = \log c_{b,t} + v(d_{b,t})$ for $t \in \{1, 2\}$, and $u_{b,3} = c_{b,3} + v(d_{b,3})$, *where* $v_d > 0$ *and* $v_{dd} \leq 0$.

Assumption 2 *Initial debt* ($b_{b,0}$) *is small enough that the borrowing constraint is slack in period 1 and income inequality* (δ_H) *in period 2 is large enough that the borrowing constraint is binding in state* H *under laissez faire. The fraction* γ *of seizable assets satisfies* $\gamma > (2\beta + \beta^2)^{-1}$.

The restrictions on agents' preferences in Assumption 1 facilitate the derivation of analytical results and allows isolating distinct effects of policy regimes. Specifically, as durables do not provide utility to lenders, which relates to studies on fire sales where borrowers have a superior use for assets (see also Davila and Korinek, 2018), the distribution of durables will be degenerate and only borrowers will hold durables in equilibrium.[13] Due to linear utility of lenders, the interest rate is constant under laissez faire and under debt taxes/subsidies, as in small open economy models (see Bianchi, 2011, or Benigno et al., 2016) or in most applications in Davila and Korinek (2018). Here, it can however be adjusted if a saving tax/subsidy is imposed on lenders. With these preferences, we switch off distributive effects with regard to durables and focus on collateral effects when debt taxes/subsidies are applied. In contrast, a saving tax/subsidy might also address distributive effects via changes in the real interest rate.

Assumption 2 ensures that the borrowing constraint is not binding in period 1, while there is a positive probability that it is binding in period 2. Policies that are exclusively imposed in period 1 are therefore summarized as ex-ante policies. We further examine (non-contingent) policies that are neither time nor state dependent and that apply equally in period 1 and 2. Notably, the parameter restriction in Assumption 2 is hardly restrictive under reasonable values for γ and β.

[13] Consistently, we restrict the initial endowment by $d_{b,0} = d$.

Laissez faire Before we discuss welfare-enhancing policies, we briefly describe the equilibrium under laissez faire, i.e. without fiscal interventions, which will serve as the main reference case. Under Assumptions 1 and 2, the borrowers' optimality conditions can be summarized as $c_{b,1}^{-1}q_1 = v_d(d_{b,1}) + \beta E q_2 c_{b,2}^{-1}$,

$$c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{4}$$

$$c_{b,2}^{-1}q_2 = v_d(d_{b,2}) + \beta q_3 + \mu_{b,2}\gamma q_2, \tag{5}$$

$$c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{6}$$

$$-b_{b,2} = \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} > 0 \text{ or } -b_{b,2} \leq \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} = 0, \tag{7}$$

and $q_3 = v_d(d_{b,3})$, where $\mu_{b,2}$ denotes the multiplier on the borrowing constraint (3). Notably, the borrowers' optimality conditions for debt and durables in period 2, (5) and (6), would differ from corresponding optimality conditions of lenders even under identical preferences, since lenders do not face a (potentially binding) borrowing constraint. Given that the borrowing constraint (3) depends on individual collateral holdings,[14] borrowers value durables also for their ability to raise the borrowing limit. This effect is captured by the multiplier $\mu_{b,2}$ entering the RHS of (5), implying that the borrowing decision (6) relates to the durables price via the tightness of the borrowing constraint. This channel will be particularly relevant when non-contingent policies taxes/subsidies are imposed while borrowing constraints bind (see below). In contrast to the optimal borrowing decision, optimal lending is apparently not affected by (3)

$$1/r_1 = 1/r_2 = \beta. \tag{8}$$

It can be shown in a straightforward way that lenders will not hold durables under Assumption 1. The laissez faire equilibrium is further characterized by the binding budget constraints, $c_{b,3} = b_{b,2} + y_{b,3}$, $c_{b,2} = b_{b,1} + y_{b,2} - b_{b,2}/r_2$, and $c_{b,1} = b_{b,0} + y_{b,1} - b_{b,1}/r_1$, as well as $c_{l,3} = b_{l,2} + y_{l,3}$, $c_{l,2} = b_{l,1} + y_{l,2} - b_{l,2}/r_2$, and $c_{l,1} = b_{l,0} + y_{l,1} - b_{l,1}/r_1$.

Combining (6) with (8) shows that the marginal utility of non-durables consumption of

[14] Notably, this is not the case in the small open economy models of Bianchi (2011), Benigno et al. (2016), Jeanne and Korinek (2019), or Schmitt-Grohe and Uribe (2021), where the borrowing limit depends on aggregate values.

borrowers in period 2 exceeds the marginal utility of non-durables consumption of lenders (=1) under a binding borrowing constraint: $(c_{b,2}^{-1} - 1)\beta = \mu_{b,2} > 0$. Under Assumption 2, the same property is implied by (4) for period 1:

$$(c_{b,1}^{-1} - 1) = E[c_{b,2}^{-1} - 1] > 0. \tag{9}$$

Apparently, agents' decisions are distorted by the borrowing constraint, when there is a positive probability that it is binding. Pecuniary externalities can then affect the allocation in an adverse way. Subsequently, we will summarize collateral effects and distributive effects of externalities following the classification of Davila and Korinek (2018). Concretely, collateral effects refer to uninternalized changes in the price of durables affecting the borrowing limit. Distributive effects refer to uninternalized changes in the debt/savings price, which are relevant under marginal rates of substitutions that differ between different agents.

As in Davila and Korinek's (2017) application for collateral externalities (see Section 5), the collateral price is determined by the borrowers' optimality conditions. Like in their model, (5) and (6) imply the collateral price to be an increasing function of borrowers' consumption in the same period:

$$q_2 = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1}(1-\beta\gamma) + \beta\gamma}, \tag{10}$$

where we used $q_3 = v_d(d_{b,3})$ and $d_{b,t} = d$. In state L, where $\mu_{b,2} = 0$ holds under Assumption 2, condition (10) simplifies to $q_2 = c_{b,2} v_d(d)(1+\beta)$. Hence, the impact of borrowers' non-durables consumption $c_{b,2}$ in period 2 on the durables price is enhanced under a binding borrowing constraint in state H. This effects stems from lower $c_{b,2}$ levels, which raise agents' valuation of relaxing the borrowing constraint $\mu_{b,2}$ (see 6); the latter exerting a positive effect on the price of collateral/durables (see 5). The effects of non-durables consumption on the durables price and thus on the collateral constraint are not internalized by the agents, though by the policy maker (see below).

Moreover, higher non-durables consumption of borrowers in period 1 relative to period 2 is associated with a lower interest (see 4). Yet, under laissez faire and under debt taxes/subsidies, lenders are only willing to lend at the constant rate $1/\beta$ (see 8). If, however a social planer imposes a tax/subsidy on savings, the interest rate can be altered, which can potentially enhance efficiency as borrowers' marginal rates of substitution are distorted by the borrowing constraint (see 9).

First best As a further reference case, we briefly describe the first best allocation in this economy. It can easily be shown that the allocation would be efficient (even though financial markets are incomplete), if borrowing were not constrained. The allocation would then be equivalent to the allocation a social planer would choose who maximizes utilitarian social welfare,

$$W = E\sum_{t=1}^{3}\beta^{t-1}\left(u_{b,t}+u_{l,t}\right), \tag{11}$$

subject to the resource constraints. It can be shown in a straightforward way that the first best allocation, which maximizes (11), is characterized by $d_{b,t}=d$,

$$c_{b,1}^{fb}(s)=c_{b,2}^{fb}(s)=1, \quad c_{l,1}^{fb}(s)=c_{l,2}^{fb}(s)=y-1, \tag{12}$$

$c_{b,3}^{fb}(s)=((b_{b,0}+y_{b,1}-c_{b,1}^{fb})\beta^{-1}+y_{b,2}(s)-c_{b,2}^{fb}(s))\beta^{-1}+y_{b,3}$ and $c_{l,3}^{fb}(s)=y-c_{b,3}^{fb}(s)$. Even though individual endowment with non-durables is random in period 2 and markets are incomplete, borrowers' consumption of non-durables is identical in the periods 1 and 2 under first best. In contrast, period-3-consumption of non-durables is state-dependent.

Under a positive probability that the borrowing constraint is binding (see Assumption 2), the first best allocation, in particular (12), cannot be realized under laissez faire, where $c_{b,2}$ is state dependent (see 6) and $c_{b,1}<1$ holds (see 9).

3 Corrective debt market policies

Given that the laissez faire allocation is inefficient, social welfare can be enhanced by addressing externalities via corrective polices. We thereby disregard welfare improvements by state-contingent policies that can reduce market incompleteness. Specifically, we examine four different types of policy interventions in the credit market (see Table 1): 1) an ex-ante tax/subsidy on debt, 2) a non-contingent tax/subsidy on debt, 3) an ex-ante tax/subsidy on savings, and 4) a non-contingent tax/subsidy on savings. For this, we apply the Ramsey approach to optimal policy, where the policy maker acts under commitment and internalizes equilibrium price relations.[15] Under 1), the allocation is identical to a constrained efficient allocation, where the social planer respects budget and borrowing constraints as well as equilibrium price relations that are unchanged compared to laissez faire (see Bianchi, 2011, or Davila et al., 2012). In contrast, the equilibrium relation for the durables/collateral price is altered under 2) and for the debt price under 3), while the equilibrium relations for both prices are simultaneously altered under 4). The allocation under optimal choices of these instruments will therefore differ and the policy maker can more directly address the pecuniary externalities under regimes 2-4 than under regime 1.

3.1 An ex-ante Pigouvian tax on debt

We first consider the case, where a tax/subsidy on debt might be introduced in period 1, whereas no policy instrument is applied in period 2. This policy regime has already been examined in several related studies (see Davila and Korinek, 2018, for an overview), establishing that it can implement a constrained efficient allocation. Under an ex-ante debt tax, borrowers' optimality condition (4) changes to

$$(1-\tau_{b,1})c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}]. \tag{13}$$

In equilibrium, this condition – combined with the optimal lending choice $1/r_1 = \beta$ – implies $(1-\tau^b) = c_{b,1}E[c_{b,2}^{-1}]$. By taxing debt in period 1, agents can be induced to borrow less, which tends to raise non-durables consumption $c_{b,2}$ and the durables price q_2 (see 10).[16] Given that borrowers do not internalize the adverse effect of period-1-borrowing on the durables/collateral price and thus the borrowing limit in period 2, a policy maker can enhance efficiency by addressing this collateral effect with an ex-ante debt tax. This principle is well-established in the literature on prudential regulation and capital controls, and is typically summarized by the term "overborrowing" (see e.g. Lorenzoni, 2008, or Bianchi, 2011).

[15] Thus, we disregard the issue of time-consistency. See Bianchi and Mendoza (2018) for an analysis of constrained efficient allocations chosen by a social planer acting under discretion.

[16] Notably, this positive effect of higher net worth of borrowers on the durables/collateral price corresponds to the effect in Davila and Korinek (2018) imposed by their condition 1.

Proposition 1 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on debt before the borrowing constraint might be binding. Then, the optimal allocation is constrained efficient and associated with a tax on debt, satisfying*

$$\tau_{b,1} = c_{b,1}\gamma dE\left[\mu_2^{tb1}\frac{\partial q_2}{\partial c_{b,2}}\right] > 0, \tag{14}$$

where $\mu_2^{tb1} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

Proof. See Appendix. ■

The optimal ex-ante debt tax described in Proposition 1 implements a constrained efficient allocation, which is chosen by a social planer respecting budget and borrowing constraints and allowing markets for durables and non-durables to clear in a competitive way (see Stiglitz, 1982, Bianchi, 2011, Davila et al., 2012, or Davila and Korinek, 2018). Concretely, a constrained efficient allocation is chosen by a social planer who determines borrowing and maximizes social welfare W subject to budget and borrowing constraints, while taking the competitive equilibrium relations for interest rates (8) and the durables price (10) under laissez faire into account. The Ramsey optimal ex-ante debt tax leads to the same outcome, since the it leaves the equilibrium price relations (8) and (10) unaffected.[17] In contrast, we will examine polices in the subsequent sections that alter (8) and (10), such that the prices q_2, $1/r_1$, and $1/r_2$ can directly be adjusted by policy choices. These direct effects are taken into account by applying the Ramsey approach to optimal policy. Under alternative credit market polices, competitive equilibrium allocations can thereby be implemented that are superior to the constrained efficient allocation.

3.2 A non-contingent tax/subsidy on debt

In this framework, state contingency cannot simply be induced by cyclicality of policy instruments. We therefore consider that the debt tax/subsidy can neither be made contingent on specific periods nor on the state of the economy, i.e. on the distribution of agents' endowment, such that the debt tax/subsidy would be constant and equally imposed in the periods $t = 1$ and $t = 2$. In this case, the tax/subsidy has ex-ante and

[17] Benigno et al. (2016) and Davila and Korinek (2017) also show that this approach can be equivalent to a Ramsey optimal policy where the policy maker chooses taxes ex-ante or on first period allocations.

ex-post effects with regard to the state of the economy where the borrowing constraint is binding. The borrowers' optimality conditions (4) and (6) then change to

$$(1-\tau_b)c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{15}$$

$$(1-\tau_b)c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{16}$$

where $\tau_{b,1} = \tau_{b,2} = \tau_b$ and $1/r_1 = 1/r_2 = \beta$ hold. Combining (15) with (16), shows that the multiplier $\mu_{b,2}$ satisfies the relation $\mu_{b,2} = \beta(c_{b,2}^{-1}c_{b,1}^{-1}E[c_{b,2}^{-1}] - 1)$, which differs from its laissez faire version ($\mu_{b,2} = (c_{b,2}^{-1} - 1)\beta$) and can be used together with (5) to get the following equilibrium relation for the durables price:

$$q_2 = \frac{v_d(d)\,(1+\beta)}{c_{b,2}^{-1}(1-\beta\gamma c_{b,1}E[c_{b,2}^{-1}]) + \beta\gamma}, \tag{17}$$

while the durables price in state L simplifies to $q_2 = c_{b,2}v_d(d)(1+\beta)$. Notably, the durables price q_2 tends to be higher under larger values for the multiplier $\mu_{b,2}$ (see 5), while a non-contingent debt tax $\tau_b > 0$ tends to reduce the multiplier (see 16). Due to this multiplier effect on q_2 and the negative effect of the debt tax on non-durables consumption $c_{b,1}$ relative to $c_{b,2}$ (see 15), the durables price q_2 is here characterized by a positive relation to $c_{b,1}$ in equilibrium (see 17).

A non-contingent debt tax tends to induce agents to borrow and to consume less in period 1 (as in the case of the ex-ante tax), but also tends to reduce borrowing and consumption in period 2 when the borrowing constraint can be binding. A debt tax can thereby induce a reduction in the durables price and in the borrowing limit in period 2, which can reinforce adverse effects of the borrowing constraint. It might therefore be preferable to apply a subsidy rather than a tax on debt. The following proposition summarizes properties of an optimal non-contingent tax/subsidy on debt:[18]

Proposition 2 *Suppose that the policy maker can apply a non-contingent Pigouvian tax/subsidy on debt in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on debt satisfying*

$$\tau_b = c_{b,1}\gamma d E\left[\mu_2^{tb}\left(\frac{\partial\phi_2^d}{\partial c_{b,2}} - \beta\frac{\partial\phi_2^d}{\partial c_{b,1}}\right)\right], \tag{18}$$

[18] Notably, a policy maker under commitment fully accounts for agents conditioning their expectations, for example on the RHS of (17), on policy choices.

where $\partial \phi_2^d / \partial c_{b,1} > 0$, $\partial \phi_2^d / \partial c_{b,2} \gtreqless 0$, $\mu_2^{tb} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem, and the RHS is positive iff $\tau_b > (\beta\gamma)^{-1} [\beta\gamma (2+\beta) - 1] > 0$.

Proof. See Appendix. ■

As revealed by Proposition 2, the collateral effects given on the RHS of (18) imply that an optimal non-contingent debt policy can either be a tax ($\tau_b > 0$) or a subsidy ($\tau_b < 0$). The condition stated in the last part of the proposition, implies that the optimal allocation is associated with a subsidy on debt, if the collateral effects and the required intervention are sufficiently small, i.e. if the tax/subsidy rate is less than $(\beta\gamma)^{-1} [\beta\gamma (2+\beta) - 1] > 0$. To understand this, recall that a debt tax τ_b tends to reduce non-durables consumption $c_{b,1}$ relative to $c_{b,2}$ (see 15). If the collateral effect and the magnitude of the policy intervention are large, so is the policy induced shift between $c_{b,1}$ and $c_{b,2}$. Given that the impact of $c_{b,2}$ on the durables price q_2 can be positive for small values of $c_{b,1}$ (see 17), an increase in the durables price q_2 can be induced by a debt tax that reduces $c_{b,1}$ and raises $c_{b,2}$. If the collateral effects are however not too large, i.e. $\gamma d E[\mu_2^{sp} \partial \phi_2^d / \partial c_{b,1}] < 1/(2c_{b,1})$, a corrective rise in q_2 can be induced by a simultaneous increase in $c_{b,1}$ and $c_{b,2}$, which requires a debt subsidy.

3.3 An ex-ante Pigouvian tax/subsidy on savings

We now consider a tax/subsidy on savings as a closely related policy instrument, which is however imposed on lenders rather than borrowers. Given that borrowers and lenders structurally differ with regard to preferences and constraints, the impact of a tax/subsidy on savings on the competitive equilibrium will in general not be equivalent to the impact of a tax/subsidy on debt. Specifically, the analysis will reveal that distributive effects might play an important role for the policy maker's choice under a savings policy. As long as only borrowers were taxed, the interest rate has been constant due to the linear utility function of lenders (see Sections 3.1 and 3.2), which relates to the specification of foreign lenders in small open economy models.

Under an ex-ante tax/subsidy on savings, the interest rate in period 1 can directly be altered as shown by the lenders' optimal savings decision

$$(1 - \tau_{l,1})/r_1 = \beta, \tag{19}$$

implying that the interest rate is reduced by a saving subsidy $\tau_{l,1} > 0$. Combining (19) with the borrowers' optimality condition (4), gives

$$1/(1 - \tau_{l,1}) = c_{b,1} E[c_{b,2}^{-1}], \tag{20}$$

implying that borrowers's period-1 non-durables consumption $c_{b,1}$ tends to increase relative to $c_{b,2}$ with a saving subsidy (whereas it tends to decrease with a debt tax). The durables/collateral price satisfies the laissez faire relation (10) as under the ex-ante debt tax. An ex-ante tax/subsidy on savings can indirectly alter the borrowing limit via the effect of $c_{b,2}$ on the collateral price similar to the ex-ante debt tax (see 10), while it can additionally affect the price of debt in a direct way via (19). The social planer can utilize the latter effect and lower the interest rate to address distributive effects induced by the borrowing constraint. In fact, the distributive effects call for a subsidy on savings and the collateral effects for a tax on savings. The sign of the tax/subsidy rate therefore depends on the relative magnitudes of both effects.

Proposition 3 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on savings before the borrowing constraint might be binding. Then, the optimal allocation is associated with a tax/subsidy rate on savings, satisfying*

$$\tau_{l,1} = \underbrace{\left\{ -b_{b,1} r_1 E\left[\mu_2^{tl1}\right] \cdot \left(E\left[\frac{\partial \phi_1^b}{\partial c_{b,1}}\right] - r_1 E\left[\frac{\partial \phi_1^b}{\partial c_{b,2}}\right] \right) \right\}}_{>0} + \underbrace{\left\{ -r_1 \beta \gamma d E\left[\mu_2^{tl1} \frac{\partial q_2}{\partial c_{b,2}}\right] \right\}}_{<0}, \tag{21}$$

where $\partial \phi_1^b / \partial c_{b,1} > 0$, $\partial \phi_1^b / \partial c_{b,2} < 0$, *and* $\mu_2^{tl1} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

Proof. See Appendix. ■

The condition for the optimal tax/subsidy rate (21) in Proposition 3 reveals that the sign of the tax/subsidy rate depends on two opposing effects: The first term on the RHS (in curly brackets) is strictly positive and summarizes the distributive effects induced by the borrowing constraint that is binding with a positive probability (see Assumption

2).[19] These effects call for a saving subsidy, inducing a lower interest rate. Due to the higher debt price $1/r_1$, borrowers can increase their period-1 non-durables consumption above the laissez faire level (see 9). The second term on the RHS (in curly brackets) is strictly negative and summarizes the collateral effects, which can be addressed by reducing borrowing via a savings tax, $\tau_{l,1} < 0$, that tends to reduce the supply of debt (like an ex-ante debt tax tends to reduce the demand for debt, see Proposition 1). Evidently, the policy makers applies a saving subsidy, $\tau_{l,1} > 0$, when collateral effects are dominated by distributive effects.

3.4 A non-contingent tax/subsidy on savings

Now suppose that the tax/subsidy on savings can neither be made contingent on particular periods nor on the state of the economy, such that the tax/subsidy rate is equally imposed in the periods $t = 1$ and $t = 2$. Notably, this policy regime would even be non-equivalent to a non-contingent tax/subsidy on debt if all agents were ex-ante identical. The reason is the asymmetry of agents' problems in period 2 induced by the borrowing constraint, which is apparently irrelevant for saving decisions. Here, the lenders' optimality conditions are characterized by

$$(1 - \tau_l)/r = \beta, \text{ where } r_1 = r_2 = r, \tag{22}$$

instead of (8), implying that the non-contingent saving tax/subsidy reduces the interest rate in both periods, 1 and 2. These interest rate effects of the non-contingent saving tax/subsidy further affect the borrowing decisions in period 1 and 2,

$$c_{b,1}^{-1}/(1 - \tau_l) = E[c_{b,2}^{-1}], \tag{23}$$

$$c_{b,2}^{-1}\beta/(1 - \tau_l) = \beta + \mu_{b,2}, \tag{24}$$

where $\tau_{l,1} = \tau_{l,2} = \tau_l$. The conditions (23) and (24) indicate that a saving subsidy $\tau_l > 0$ tends to raise borrowers' non-durable consumption in period 1 and 2. Simultaneously, it alters the valuation of the borrowing constraint, measured by the multiplier on the

[19] Notably, the multipler μ_2^{tl1} includes the difference between the marginal utilities of borrowers and lenders (see proof of Proposition 3), such that the distributive effects in (14) include the same componenets (sufficient statistics) as summarized in Proposition 2 in Davila and Korinek (2017).

borrowing constraint, $\mu_{b,2}$. Combining (23) and (24) gives $\mu_{b,2} = c_{b,2}^{-1} c_{b,1} \beta E[c_{b,2}^{-1}] - \beta$, which can be used to substitute out the multiplier $\mu_{b,2}$ in (5). Then, the durables price relation differs from the laissez faire version (10) and again satisfies (17). By these changes in the price relations, the policy maker can use a non-contingent tax/subsidy on savings to address collateral effects via the durables price as well as distributive effects via the interest rate.

Proposition 4 *Suppose that the policy maker can apply a non-contingent Pigouvian tax/subsidy on savings in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on savings satisfying*

$$\tau_l = \beta \overbrace{\left\{ -b_{b,1} r \beta E\left[\frac{\mu_2^{tl}}{\phi^b}\right] \left(E\left[\frac{\partial \phi^b}{\partial c_{b,1}}\right] - rE\left[\frac{\partial \phi^b}{\partial c_{b,2}}\right] \right) - E\left[b_{b,2} \frac{\mu_2^{tl}}{\phi^b} \left(\frac{\partial \phi^b}{\partial c_{b,1}} - r \frac{\partial \phi^b}{\partial c_{b,2}} \right) \right] \right\}}^{>0} + \Psi$$

$$\textit{with } \ \Psi = \beta \gamma d E\left[\mu_2^{tl} \left(\frac{\partial \phi_2^d}{\partial c_{b,1}} - r \frac{\partial \phi_2^d}{\partial c_{b,2}} \right) \right] \gtreqless 0, \tag{25}$$

where $\partial \phi_1^b / \partial c_{b,1} > 0$, $\partial \phi_1^b / \partial c_{b,2} < 0$, $\partial \phi_2^d / \partial c_{b,1} > 0$, $\partial \phi_2^d / \partial c_{b,2} \gtreqless 0$, *and* $\mu_2^{tl} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem. The term* Ψ *is strictly negative iff* $\tau_l < -[\gamma\beta(2+\beta) - 1] < 0$.

Proof. See Appendix. ■

According to Proposition 4, the distributive effects, which are given by the terms in the curly brackets in (25), can be addressed by a non-contingent subsidy on savings, which relates to the findings in Proposition 3. In contrast to an ex-ante saving subsidy, a non-contingent saving subsidy also reduces the interest rate in period 2, where the borrowing constraint might be binding. This additional effect is captured by the last term in the square brackets. In contrast to the terms referring to the distributive effect, the sign of the term Ψ, which summarizes the collateral effects, is ambiguous and, particularly, depends on the relative values for borrowers' consumption in the period 1 and 2. This corresponds to the ambiguity of the policy responses to the collateral effects under a non-contingent debt tax/subsidy (see Proposition 2). As stated in Proposition 4, Ψ can in fact only be negative if τ_l takes a sufficiently large negative value, which implies a tax on savings. This, however requires that the collateral effects are large enough to dominate the distributive effects. Otherwise, the optimal policy is a saving subsidy $\tau_l > 0$.

4 Prices, allocation and welfare

In this Section, we aim at demonstrating the impact of corrective policies and the possibility to improve on the constrained efficient allocation. To illustrate optimal policy choices and their effects on prices, the allocation and social welfare, we introduce a functional form for $v(d_{b,t})$: $v(d_{b,t}) = \varkappa \log d_{b,t}$. We further assign the following reasonable (non-calibrated) values for the model parameters: $y = 2$, $d/y = 1.5$, $b_{b,0}/y = -0.25$, $\varkappa = 0.1$, and $\beta = 0.9$. The benchmark values for the inequality measure δ_H and for the share of seizable collateral γ are 1.1 and 0.8, respectively. We then examine sensitivity of the effects by altering the tightness of the borrowing constraint γ and income inequality δ_H. The solutions for the equilibrium objects under the four policy regimes summarized in Table 1 and under laissez faire are presented in the Figures 1-3. The first row in all Figures refers to a variation in γ, where an increase in γ (from 0.5 to 1) reduces the tightness of the borrowing constraint and thereby the strength of the financial friction. The second row in all Figures refers to a variation in δ_H, where an increase in δ_H (from 1.05 to 1.55) increases the inequality of agents' non-durables endowment in period 2 and thereby the relevance/valuation of the financial friction.

The first column of Figure 1 shows the tax and subsidy rates under all five regimes. The laissez faire case (black dotted lines) provides reference values with zero tax/subsidy rates. The first policy regime (solid black lines with crosses) is the optimal ex-ante tax on debt $\tau_{b,1} > 0$ (see Proposition 1), which decreases with γ and increases with δ_H. The second policy regime (red dashed lines with crosses) is the optimal non-contingent subsidy on debt, $\tau_b < 0$, with a subsidy rate that changes with γ in a non-monotonic way, consistent with the two effects summarized in Proposition 2. The third (blue solid lines with circles) and the fourth regime (green dashed lines with circles) are the optimal ex-ante and the optimal non-contingent optimal savings subsidies, $\tau_{l,1} > 0$ and $\tau_l > 0$, as characterized in Propositions 3 and 4. The second column shows the durables price in period 2, which is slightly increased compared to laissez faire under the ex-ante debt tax. The non-contingent debt subsidy, which tends to raise borrowing in both periods 1 and 2 (see Figure 2), also leads to higher durables prices (except for the lowest γ values). In contrast, the ex-ante saving subsidy, which raises debt in period 1 and reduces non-

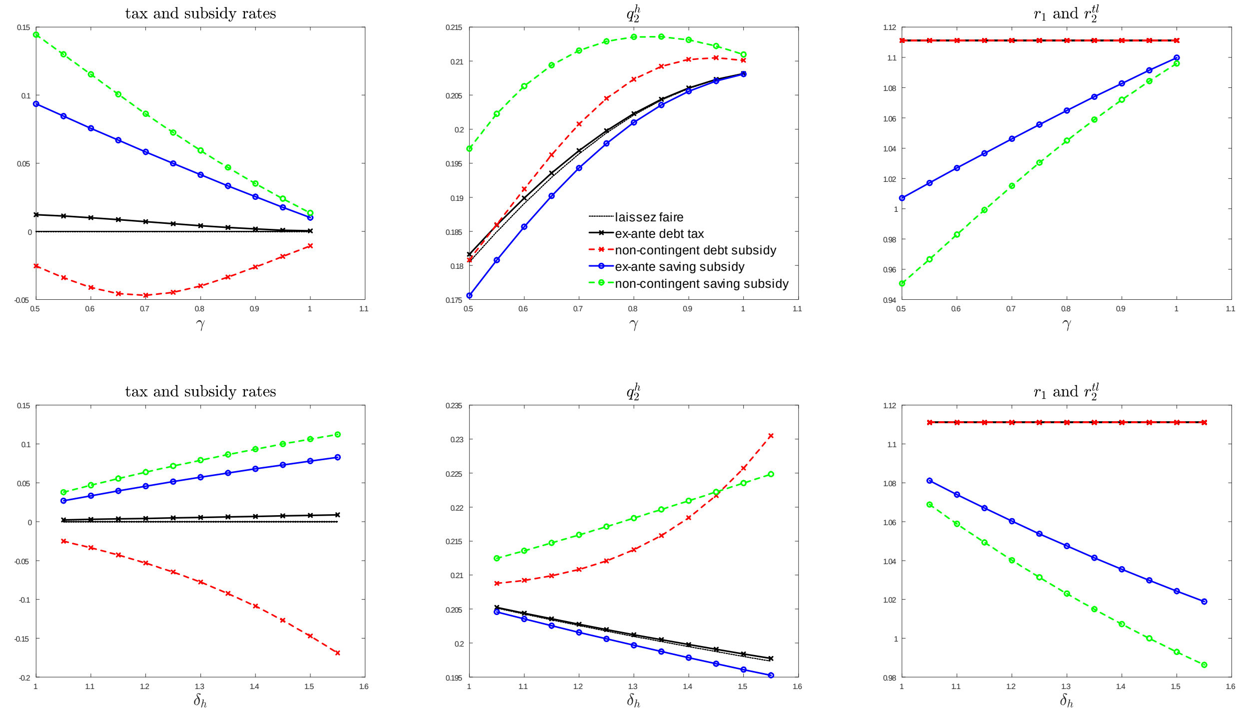

Figure 1: Instruments and prices (benchmark values $\gamma = 0.8$ and $\delta_h = 1.1$)

durables consumption $c_{b,2}$ in period 2 (see Figure 2), leads to lower durables prices q_2. Simultaneously, it reduces the interest rate in period 1 below its laissez faire value (see third column), such that borrowing funds requires issuance of less debt $b_{b,1}$. The non-contingent saving subsidy leads to the most pronounced increase in the durables price q_2. It further leads to a reduction in the interest rate r_1 in period 1 that is larger than under the ex-ante saving subsidy and it reduces the interest rate r_2 in period 2. Figure 2 further shows that all three subsidies raise debt, suggesting that agents tend to borrow too little under laissez faire, and lead to higher levels of non-durables consumption in period 1, closing the gap to first best (see 12), which in contrast increases under the ex-ante debt tax. Yet, the non-contingent debt subsidy and the ex-ante saving subsidy reduce consumption $c_{b,2}$ due to a higher debt burden in period 2. The opposite outcome is induced by the ex-ante debt tax (as discussed in Section 3.1) and by the non-contingent saving subsidy, which lowers borrowing costs in both periods 1 and 2.

Figure 3 presents the welfare effects of the policy regimes. Social welfare W (see 11) is measured in terms of equivalents of borrowers' non-durables consumption in period 1.

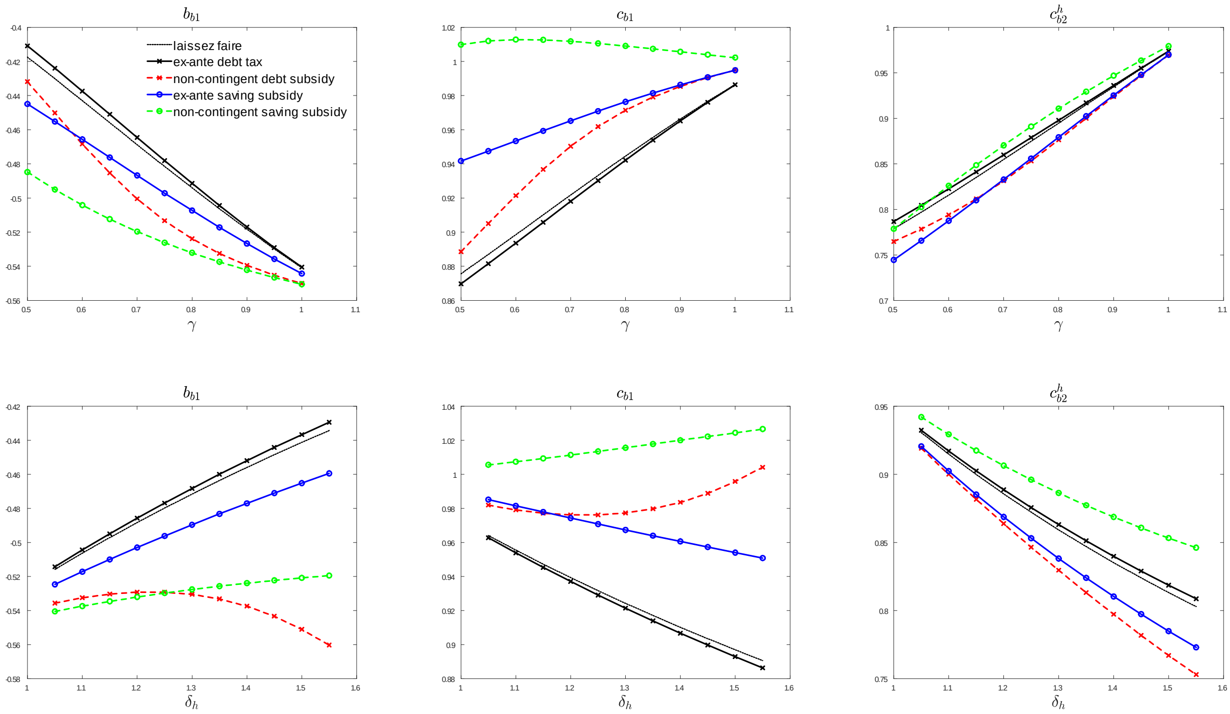

Figure 2: Debt and consumption (benchmark values $\gamma = 0.8$ and $\delta_h = 1.1$)

The first column shows welfare effects of the four policy regimes relative to the laissez faire case. Apparently, the debt policies (ex-ante debt tax and non-contingent debt subsidy) lead to much smaller welfare gains than the saving subsidies. This results is simply due to the fact that the former policies can – by construction – not address distributive effects by changes in the interest rate, in contrast to the latter policies.[20] The second column of Figure 3 zooms in into the welfare effects of the debt policies, revealing that the ex-ante debt tax leads to the smallest welfare gains under the benchmark parameter values. Yet, it shows that the ex-ante debt tax can principally be superior to the non-contingent debt subsidy, particularly for tighter borrowing constraints (i.e. for lower γ values), which enhance the relevance of the collateral effect (see discussions in Section 3.2). The last column of Figure 3 presents welfare losses compared to first best. The values for laissez fare and the ex-ante debt tax can hardly be distinguished, indicating that the total welfare gains of an ex-ante debt tax are negligible relative to first best. In

[20]This possibility would in principle be possible under alternative specifications of lenders' utility function, for example, logarithmic utility, which is neglected here to keep the exposition transparent using polar cases.

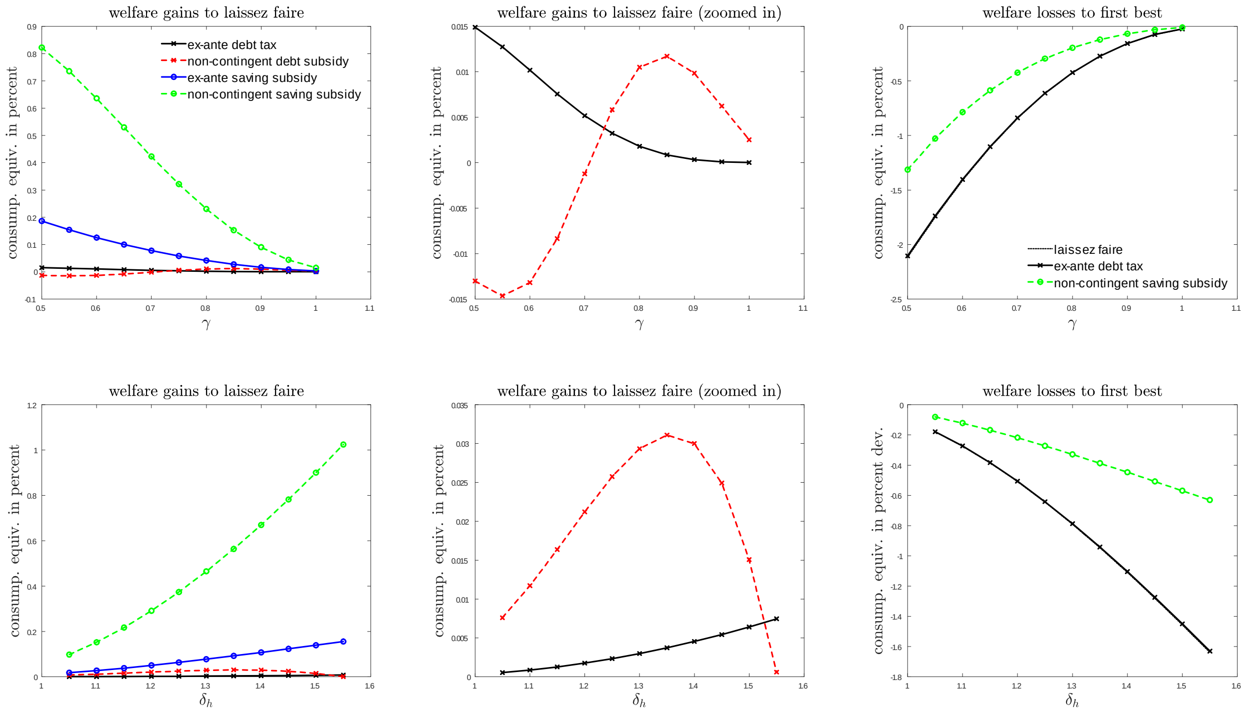

Figure 3: Social welfare (benchmark values $\gamma = 0.8$ and $\delta_h = 1.1$)

contrast, the non-contingent saving subsidy can substantially reduce the welfare loss in a competitive equilibrium compared to first best. For the benchmark values, the welfare loss is reduced by more than 50%.

5 Collateral externalities in a production economy

To assess the robustness of our findings with regard to collateral effects and to facilitate comparisons, we further adopt the model of Davila and Korinek (2018) with endogenous capital formation. The following model details are introduced to exactly replicate their (fourth) application "collateral externalities" and their results: Suppose that there is no uncertainty and that there are no durable consumption goods in this economy. Agents' lifetime utility satisfies $u_l = c_{l,1} + c_{l,2} + c_{l,3}$ and $u_b = \log c_{b,1} + \log c_{b,2} + c_{b,3}$, which accords to Assumption 1 without durables ($v = 0$) and implies no discounting ($\beta = 1$). Borrowers have access to an investment technology, by which capital $k_{b,2}$ can be installed under convex costs $\alpha k_{b,2}^2/2$ in the first period. Capital can be traded in the second period at the price q^k and remains constant until it fully depreciates at the end of the last period. In the periods 2 and 3, borrowers use their full stock of capital to produce according to the technology $A_t k_{b,t}$ with $t \in (2,3)$. Given that there is no uncertainty and no discounting, lenders' optimal saving decision pins down the price of debt at 1. The borrowing constraint for the second period is given by

$$-b_{b,2} \leq \phi q^k k_{b,2}, \tag{26}$$

with $\phi > 0$, while borrowing is (de facto) unconstrained in the first period. Since lenders have no use for capital, the entire stock of capital is held by borrowers: $k_{b,2} = k$. Under under laissez faire, the borrowers' first order conditions can be written as $1/c_{b,1} = 1/c_{b,2}$, $1/c_{b,2} = 1 + \kappa_{b,2}$, $\alpha k(1/c_{b,1}) = (1/c_{b,2})\left(A_1 + q^k\right)$, and $q^k(1/c_{b,2}) = A_2 + \kappa_{b,2}\phi q^k$, where $\kappa_{b,2} \geq 0$ denotes the multiplier on the borrowing constraint. Combining the last two conditions, gives the equilibrium relation for the price of capital

$$q^k = A_2 \left[\phi + (1-\phi)/c_{b,2}\right]^{-1}, \tag{27}$$

implying $\partial q^k / \partial c_{b,2} > 0$, while the stock of capital satisfies $k = \left(A_1 + q^k\right)/\alpha$. The unconstrained equilibrium allocation satisfies $c_{b,1} = c_{b,2} = 1$ and $k = (A_1 + A_2)/\alpha$, which is identical to the first best allocation maximizing utilitarian social welfare function (11) for $\beta = 1$. This first best allocation cannot be realized in a laissez faire equilibrium under a binding borrowing constraint, where $\kappa_{b,2} > 0 \Rightarrow c_{b,2} = c_{b,1} < 1$.

We consider the two policies in the first line of Table 1 with compensations satisfying (2), namely, an ex-ante debt tax/subsidy $\tau_{b,1}$ and a non-contingent debt tax/subsidy τ_b. Like in Davila and Korinek (2018), the policy maker has further access to an ex-ante tax/subsidy on capital investment $\tau_{k,1}$, which we assume for both cases. Notably, (27) is unaffected in the first case, but is replaced by a different price relation in the second case. This difference is induced by the policy intervention in the second period, causing the capital price q^k also to be increasing in $c_{b,1}$ in a competitive equilibrium. The following results are derived by applying the Ramsey approach to optimal policy.[21]

[21] Note that the result for the investment tax/subsidy is only reported for the first case to confirm Davila and Korinek's (2017) findings. For convenience, we do not report results for the investment tax/subsidy for the second case, since it does not relate to the main purpose of the analysis.

Proposition 5 *Consider the production economy with a binding borrowing constraint, and suppose that the policy maker can apply a Pigouvian ex-ante tax/subsidy on capital investment.*

1. *Suppose that the policy maker can further apply a Pigouvian tax/subsidy on debt in the first period. Then, the optimal allocation is constrained efficient and characterized by a tax on debt satisfying*

$$\tau_{b,1} = (1 - c_{b,1})\,\phi k\left(\partial q^k / \partial c_{b,2}\right) > 0, \tag{28}$$

   where $\partial q^k / \partial c_{b,2} > 0$, *and a subsidy on capital investment,* $\tau_{k,1} > 0$, *iff* $A_1 + \phi q^k > 0$.

2. *Suppose that the policy maker can further apply a Pigouvian non-contingent tax/subsidy on debt. Then, the optimal allocation is characterized by a subsidy on debt satisfying*

$$\tau_b = -\mu_2^{t1}[2\phi^2 k c_{b,1}^2 / c_{b,2}]\left(\partial q^k / \partial c_{b,1}\right) < 0, \tag{29}$$

   where $\partial q^k / \partial c_{b,1} > 0$ *and* $\mu_2^{t1} > 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

Proof. See Appendix. ■

In the first case, the policy maker applies an ex-ante tax on debt (see 28) and further imposes an investment subsidy for a sufficiently high productivity level, $A_1 + \phi q^k > 0$, which exactly replicates Davila and Korinek's (2018) results for the constrained efficient allocation. In the second case, the policy maker applies a non-contingent debt subsidy (see 29). Thus, depending on the disposable policy instrument, agents either seem to over- or to underborrow. The reason for this ambiguity is that the externality is addressed by an increase in the collateral price that does not directly depend on the level of debt. Capital trading implies the collateral price to satisfy $q^k = A_2/[(1/c_{b,2}) - \kappa_{b,2}\phi]$ under both regimes, such that it increases with consumption $c_{b,2}$ and with multiplier $\kappa_{b,2}$ on the borrowing constraint (26). Under an ex-ante policy, (27) holds and raising q^k requires reducing debt repayment to raise resources available for consumption in period 2. Under the non-contingent policy, the multiplier $\kappa_{b,2}$ and thereby the willingness to pay for capital/collateral increases when borrowing is stimulated. At the same time, first period consumption, which is inefficiently low under laissez faire ($c_{b,1} < 1$), is also stimulated. As a common feature, optimal policies thus address borrowers' underconsumption.

6 Conclusion

This paper derives optimal corrective policies in an incomplete market model with pecuniary externalities based on financial frictions. Applying the Ramsey approach to optimal policy, we confirm the well-established result that overborrowing can be addressed by a Pigouvian ex-ante tax on debt, implementing the constrained efficient allocation (as typically defined in the literature). We further examine alternative credit market policies that are non-contingent or imposed on lenders. By manipulating competitive equilibrium price relations, these policies address collateral effects and distributive effects in more direct ways than the ex-ante debt tax, leading to allocations that can be superior to the constrained efficient allocation. These results show that the concept of constrained efficiency, which has been established for the identification of pecuniary externalities, is less suited to guide optimal policy choices in general. We further examine optimal policy in a production economy and show that a policy maker either taxes or subsidizes debt depending on whether the policy is imposed ex-ante or in a non-contingent way. Overall, the results indicate that a common property of economies with collateral externalities is rather underconsumption than overborrowing; the latter being typically found in a class of models where interest rates are exogenously determined and agents take borrowing limits as given.

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8 Appendix

Proof of Proposition 1. Using $c_{l,3} = -b_{b,2} + y_{l,3}$, $c_{l,2} - b_{b,2}/r_2 = -b_{b,1} + y_{l,2}$, $c_{l,1} - b_{b,1}/r_1 = b_{l,0} + y_{l,1}$, $c_{b,3} = b_{b,2} + y_{b,3}$ and $d_{b,t} = d$, the objective (11) can by be written as

$$\begin{aligned} W = & \log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta \left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ & + \beta^2 \left[y_{b,3} + v(d) + y_{l,3}\right]. \end{aligned} \tag{30}$$

The primal problem of a policy maker who applies an ex-ante tax/subsidy on debt $\tau_{b.1}$ and a compensating lump-sum transfer/tax $T_{b,t} = -\tau_{b.1} b_{b,t}/r_t$ is identical to the problem of a social planer who determines period-1-borrowing, such that (4) does not apply, and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (8) and (10) under laissez faire into account, leading to a constrained efficient allocation. It can be summarized as

$$\begin{aligned} \max_{c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}} \quad & E\{\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta \left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ & + \beta^2 \left[y_{b,3} + v(d) + y_{l,3}\right]\} \\ \text{s.t.} \quad & 0 = b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta, \\ & 0 = b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta, \\ & 0 \leq \gamma q_2(c_{b,2}) d + b_{b,2}, \end{aligned} \tag{31}$$

where $q_2(c_{b,2})$ satisfies (10), leading to the optimality conditions

$$\lambda_{b,1}^{tb1} = 1/c_{b,1}, \tag{32}$$

$$\lambda_{b,2}^{tb1} = (1/c_{b,2}) + \mu_2^{tb1} \gamma d \partial q_2(c_{b,2})/\partial c_{b,2}, \tag{33}$$

$$\lambda_{b,1}^{tb1} = E \lambda_{b,2}^{tb1}, \tag{34}$$

$$\mu_2^{tb1} = \beta(\lambda_{b,2}^{tb1} - 1) \geq 0, \tag{35}$$

where $\lambda_{b,1}^{tb1}$, $\lambda_{b,2}^{tb1}$, and μ_2^{tb1} are the multipliers for the constraints in order of their appearance in (31). Applying expectations conditional on period-1-information and substituting out the multipliers $\lambda_{b,1}^{tb1}$ and $\lambda_{b,2}^{tb1}$ in (32)-(34) leads to

$$\frac{1}{c_{b,1}} = E\frac{1}{c_{b,2}} + E\left[\mu_2^{tb1}\gamma\frac{\partial q_2(c_{b,2})}{\partial c_{b,2}}d\right],$$

combining the latter with the optimality condition (13) and $1/r_1 = \beta$, gives the following condition for the tax rate on debt

$$\tau_{b,1} = c_{b,1}E\left[\mu_2^{tb1}\gamma\frac{\partial q_2(c_{b,2})}{\partial c_{b,2}}d\right] > 0,$$

where $\mu_2^{tb1} = \beta(c_{b,1}^{-1} - 1)$ further holds (see 32 and 35). ■

Proof of Proposition 2. For the formulation of the planer's primal problem under commitment in Lagrangian form we define $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1}-(c_{b,1}c_{b,2}^{-2}-1)\beta\gamma}$ and use (30):

$$\begin{aligned}
L = E\{&\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta\left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\
&+\beta^2\left[y_{b,3} + v(d) + y_{l,3}\right] \\
&+\lambda_{b,1}^{tb}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta\right] \\
&+\beta\lambda_{b,2}^{tb}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] \\
&+\beta\mu_2^{tb}[\gamma\phi_2^d(c_{b,1}, c_{b,2})d + b_{b,2}]\},
\end{aligned}$$

leading to the optimality conditions

$$\lambda_{b,1}^{tb} = c_{b,1}^{-1} + \beta E[\mu_2^{tb}\gamma d\cdot\partial\phi_2^d/\partial c_{b,1}], \tag{36}$$

$$\lambda_{b,1}^{sp} = E\lambda_{b,2}^{tb}, \tag{37}$$

$$\lambda_{b,2}^{tb} = c_{b,2}^{-1} + \mu_2^{tb}\gamma d\cdot\partial\phi_2^d/\partial c_{b,2}, \tag{38}$$

$$\mu_2^{tb} = \beta(\lambda_{b,2}^{tb} - 1) \geq 0. \tag{39}$$

Taking expectations and substituting out the multipliers $\lambda_{b,1}^{tb}$ and $\lambda_{b,2}^{tb}$ in (36)-(38) gives

$$\frac{1}{c_{b,1}} - E\frac{1}{c_{b,2}} = \gamma dE\left[\mu_2^{tb}\left(\frac{\partial\phi_2^d}{\partial c_{b,2}} - \beta\frac{\partial\phi_2^d}{\partial c_{b,1}}\right)\right].$$

Combining with $E\frac{1}{c_{b,2}} = (1-\tau_b)\frac{1}{c_{b,1}}$, which follows from (15) and $1/r_1 = \beta$, we get a condition for the optimal tax/subsidy rate

$$\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left(\frac{\partial\phi_2^d}{\partial c_{b,2}} - \beta\frac{\partial\phi_2^d}{\partial c_{b,1}}\right)\right], \tag{40}$$

where the multiplier μ_2^{tb} satisfies (see 38 and 39)

$$\mu_2^{tb} = \beta(c_{b,2}^{-1} - 1)/(1 - \beta\gamma d\partial\phi_2^d/\partial c_{b,2}) \geq 0.$$

Using $\partial\phi_2^d/\partial c_{b,1} = \phi_2^d\frac{c_{b,2}^{-2}\beta\gamma}{\left(c_{b,2}^{-1} - c_{b,1}c_{b,2}^{-2}\beta\gamma + \beta\gamma\right)} > 0$ and $\partial\phi_2^d/\partial c_{b,2} = \frac{c_{b,2} - 2c_{b,1}\beta\gamma}{c_{b,2}\beta\gamma}\frac{\partial\phi_2^d}{\partial c_{b,1}}$, condition (40) can be rewritten as

$$\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left(\frac{c_{b,2} - 2c_{b,1}\beta\gamma}{c_{b,2}\beta\gamma} - \beta\right)\frac{\partial\phi_2^d}{\partial c_{b,1}}\right]. \tag{41}$$

Applying (15), $1/r_1 = \beta$, and taking expectations, shows that the term in the round brackets in (41) and the RHS are strictly positive iff $\tau_b > (\beta\gamma)^{-1}\left[\beta\gamma\left(2+\beta\right) - 1\right] > 0$. ■

Proof of Proposition 3. For the formulation of the planer's primal problem under commitment in Lagrangian form we define $\phi_1^b(c_{b,1}, c_{b,2}) = \beta(c_{b,1}/c_{b,2})$ and use the goods market clearing conditions to rewrite the welfare function, for convenience:

$$\begin{aligned} L = & E\{\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\ & +\lambda_{b,1}^{tl1}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi_1^b(c_{b,1}, c_{b,2})\right] \\ & +\beta\lambda_{b,2}^{tl1}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] \\ & +\beta\mu_2^{tl1}[\gamma q_2(c_{b,2})d + b_{b,2}]\}, \end{aligned}$$

where $q_2(c_{b,2})$ satisfies (10), leading to the optimality conditions

$$\lambda_{b,1}^{tl1} = \left(c_{b,1}^{-1} - 1\right)/\left(1 + b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]\right), \tag{42}$$

$$\lambda_{b,1}^{tl1} = r_1\beta E\lambda_{b,2}^{tl1}, \tag{43}$$

$$\beta\lambda_{b,2}^{tl1} = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl1}b_{b,1}\frac{\partial\phi_1^b}{\partial c_{b,2}} + \beta\mu_2^{tl1}\gamma\frac{\partial q_2}{\partial c_{b,2}}d, \tag{44}$$

$$\mu_2^{tl1} = \beta\lambda_{b,2}^{tl1} \geq 0. \tag{45}$$

Applying expectations and substituting out the multipliers $\lambda_{b,1}^{tl1}$ and $\lambda_{b,2}^{tl1}$ in (42)-(44), gives

$$r_1 \frac{1 + b_{b,1} E\left[\partial \phi_1^b / \partial c_{b,1}\right]}{1 + r b_{b,1} E\left[\partial \phi_1^b / \partial c_{b,2}\right]} = \frac{c_{b,1}^{-1} - 1}{\beta(E c_{b,2}^{-1} - 1) + \beta\gamma d E\left[\mu_2^{tl1} \partial q_2 / \partial c_{b,2}\right]}.$$

Rewriting the latter with (19) and (20), leads to the following condition for the tax/subsidy rate on savings

$$\tau_{l,1} = -b_{b,1} r_1 E\left[\mu_2^{tl1}\right]\left(E\left[\partial \phi_1^b / \partial c_{b,1}\right] - r_1 E\left[\partial \phi_1^b / \partial c_{b,2}\right]\right) - r_1 \beta\gamma d E\left[\mu_2^{tl1} \partial q_2 / \partial c_{b,2}\right],$$

where $\partial \phi_1^b / \partial c_{b,1} = \beta / c_{b,2} > 0$, $\partial \phi_1^b / \partial c_{b,2} = -\beta(c_{b,1} c_{b,2}^{-2}) < 0$, $\partial q_2 / \partial c_{b,2} > 0$ (see 10). Combining (42), (44), and (45), shows that the multiplier μ_2^{tl1} further satisfies $E\mu_2^{tl1} = r_1^{-1}(c_{b,1}^{-1} - 1)/(1 + b_{b,1} E[\partial \phi_1^b / \partial c_{b,1}])$. ■

Proof of Proposition 4. For the formulation of the planer's primal problem under commitment in Lagrangian form we define $\phi^b(c_{b,1}, c_{b,2}) = \beta\left(c_{b,1}/c_{b,2}\right)$ and $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1} - (c_{b,1} c_{b,2}^{-2} - 1)\beta\gamma}$, and use the goods market clearing conditions to rewrite the welfare function, for convenience:

$$\begin{aligned} L = & E\{\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\ & + \lambda_{b,1}^{tl}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi^b(c_{b,1}, c_{b,2})\right] \\ & + \beta\lambda_{b,2}^{tl}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\phi^b(c_{b,1}, c_{b,2})\right] \\ & + \beta\mu_2^{tl}[\gamma\phi_2^d(c_{b,1}, c_{b,2}) d + b_{b,2}]\}, \end{aligned}$$

leading to the first order conditions

$$\begin{aligned} & \lambda_{b,1}^{tl}\left(1 + b_{b,1} E\left[\partial \phi^b / \partial c_{b,1}\right]\right) && (46) \\ & = \left(c_{b,1}^{-1} - 1\right) - \beta E\left[\lambda_{b,2}^{tl} b_{b,2}\left(\partial \phi^b / \partial c_{b,1}\right)\right] + \beta E\left[\mu_2^{tl}\gamma d\left(\partial \phi_2^d / \partial c_{b,1}\right)\right], \\ & \beta\lambda_{b,2}^{tl}\left(1 - b_{b,2}\left(\partial \phi^b / \partial c_{b,2}\right)\right) && (47) \\ & = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl} b_{b,1}\left(\partial \phi^b / \partial c_{b,2}\right) + \beta\left[\mu_2^{tl}\gamma d\left(\partial \phi_2^d / \partial c_{b,2}\right)\right], \\ & \lambda_{b,1}^{tl} = r\beta E\lambda_{b,2}^{tl}, && (48) \\ & \mu_2^{tl} = \phi^b \lambda_{b,2}^{tl} \geq 0, && (49) \end{aligned}$$

where we used $E\phi^b(c_{b,1}, c_{b,2}) = 1/r$. Taking expectations and substituting out the multipliers $\lambda_{b,1}^{tl}$ and $\lambda_{b,2}^{tl}$ in (46)-(48), leads to

$$
\begin{aligned}
& E\left[\mu_2^{tl}/\phi^b\right]\left(1+b_{b,1}E\left[\partial\phi^b/\partial c_{b,1}\right]\right)-E\left[\mu_2^{tl}/\phi^b\right]\left(1+rb_{b,1}E\left[\partial\phi^b/\partial c_{b,2}\right]\right) \\
= & \frac{1}{r\beta}\left(c_{b,1}^{-1}-1\right)-E\left(c_{b,2}^{-1}-1\right)-\frac{1}{r}E\left[\left(\mu_2^{tl}/\phi^b\right)b_{b,2}\left(\partial\phi^b/\partial c_{b,1}\right)\right]+\frac{1}{r}\gamma dE\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,1}\right)\right] \\
& +E\left[\left(\mu_2^{tl}/\phi^b\right)b_{b,2}\left(\partial\phi^b/\partial c_{b,2}\right)\right]-\gamma dE\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,2}\right)\right],
\end{aligned}
$$

and by applying (22), to the following condition for the tax/subsidy rate

$$
\begin{aligned}
\tau_l = & \left(-b_{b,1}\right)r\beta E\left[\mu_2^{tl}/\phi^b\right]\left(E\left[\partial\phi^b/\partial c_{b,1}\right]-rE\left[\partial\phi^b/\partial c_{b,2}\right]\right) \\
& +\beta E\left[\left(-b_{b,2}\right)\left(\mu_2^{tl}/\phi^b\right)\left(\left(\partial\phi^b/\partial c_{b,1}\right)-r\left(\partial\phi^b/\partial c_{b,2}\right)\right)\right] \\
& +\beta\gamma dE\left[\mu_2^{tl}\left(\left(\partial\phi_2^d/\partial c_{b,1}\right)-r\left(\partial\phi_2^d/\partial c_{b,2}\right)\right)\right],
\end{aligned}
\tag{50}
$$

where $\partial\phi^b/\partial c_{b,1}=\beta\left(1/c_{b,2}\right)>0$, $\partial\phi^b/\partial c_{b,2}=-\beta\left(c_{b,1}c_{b,2}^{-2}\right)<0$. Combining (42), (44), and (45) further shows that the multiplier μ_2^{tl} satisfies

$$
r\beta E\left[\frac{\mu_2^{tl}}{\phi^b}\right]\left(1+b_{b,1}E\left[\frac{\partial\phi^b}{\partial c_{b,1}}\right]\right)+\beta E\left[\frac{\mu_2^{tl}}{\phi^b}b_{b,2}\frac{\partial\phi^b}{\partial c_{b,1}}\right]=\left(\frac{1}{c_{b,1}}-1\right)+\beta E\left[\mu_2^{tl}\gamma\frac{\partial q_2^d}{\partial c_{b,1}}d\right].
$$

The last summand on the RHS of (50), $\beta\gamma dE\left[\mu_2^{tl}\left(\left(\partial\phi_2^d/\partial c_{b,1}\right)-r\left(\partial\phi_2^d/\partial c_{b,2}\right)\right)\right]$, can by using $\partial\phi_2^d/\partial c_{b,2}=\frac{c_{b,2}-2c_{b,1}\beta\gamma}{c_{b,2}\beta\gamma}\frac{\partial\phi_2^d}{\partial c_{b,1}}$, be rewritten as

$$
r\beta\gamma dE\left[\mu_2^{tl}\left(\frac{1}{r}-\frac{c_{b,2}-2c_{b,1}\beta\gamma}{\beta\gamma c_{b,2}}\right)\frac{\partial\phi_2^d}{\partial c_{b,1}}\right],
\tag{51}
$$

where $\frac{\partial\phi_2^d}{\partial c_{b,1}}=\phi_2^d\frac{c_{b,2}^{-2}\beta\gamma}{\left(c_{b,2}^{-1}-c_{b,1}c_{b,2}^{-2}\beta\gamma+\beta\gamma\right)}>0$. Applying (22) and (23) and taking expectations, reveals that the term in the round bracket in (51) and is negative if and only if $\tau_l<-\left[\gamma\beta\left(2+\beta\right)-1\right]<0$. ■

Proof of Proposition 5. Consider the production economy. In equilibrium, where capital is entirely held by borrowers, the borrowers' budget constraints for the first two periods can be written as $c_{b,1}+\alpha k^2/2+b_{b,1}=y_{b,1}$ and $c_{b,2}+b_{b,2}=y_{b,2}+b_{b,1}+A_1k$. The remaining budgets constraint are further given by $c_{b,3}=y_{b,3}+b_{b,2}+A_2k$, $c_{l,1}+b_{l,1}=y_{l,1}$, $c_{l,2}+b_{l,2}=y_{l,2}+b_{l,1}$, and $c_{l,3}=y_{l,3}+b_{l,2}$. Using the latter constraints, the social welfare function (11) can for $\beta=1$ be rewritten as $W=\log c_{b,1}+y_{l,1}+\left[\log c_{b,2}+y_{l,2}\right]+\left[y_{b,3}+b_{b,2}+A_2k+y_{l,3}\right]$.

To establish the claims made in the first part of the proposition, consider that the policy maker introduces an ex-ante capital investment tax/subsidy $\tau_{k,1}$ and an ex-ante debt tax/subsidy $\tau_{b,1}$, which are fully compensated (ex-post) by lump-sum transfers (see 2). The borrowers' optimality conditions then satisfy

$$(1-\tau_{b,1}) = c_{b,1}/c_{b,2}, \tag{52}$$

$$(1-\tau_{k,1})\alpha k\,(1/c_{b,1}) = (1/c_{b,2})\left(A_1+q^k\right). \tag{53}$$

The primal policy problem of the policy maker is then identical to the problem of a social planer who determines period-1-borrowing as well as the capital investment decision and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (27) under laissez faire into account, leading to a constrained efficient allocation. The problem can be summarized as $\max W$ w.r.t. $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k subject to $c_{b,1}+\alpha k^2/2+b_{b,1} = y_{b,1}$, $c_{b,2}+b_{b,2} = y_{b,2}+b_{b,1}+A_1 k$, and $b_{b,2}+\phi q^k(c_{b,2})k \geq 0$, where $q^k(c_{b,2})$ satisfies (27) and thus $\partial q^k/\partial c_{b,2} > 0$, and its Lagrangian form can be written as

$$\begin{aligned}
L = & \log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_2 k + y_{l,3}] \\
& +\lambda_1^{t1}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] \\
& +\lambda_2^{t1}\left[y_{b,2} + b_{b,1} + A_1 k - c_{b,2} - b_{b,2}\right] \\
& +\mu_2^{t1}\left[b_{b,2} + \phi q^k(c_{b,2})k\right]\},
\end{aligned}$$

leading to the first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k

$$\begin{aligned}
& \lambda_1^{t1} = 1/c_{b,1}, \;\; \lambda_2^{t1} = (1/c_{b,2}) + \mu_2^{t1}\phi k \partial q^k/\partial c_{b,2}, \;\; \lambda_1^{t1} = \lambda_2^{t1}, \\
& \mu_2^{t1} = \lambda_2^{t1} - 1, \\
& \lambda_1^{t1}\alpha k = A_2 + \lambda_2^{t1} A_1 + \mu_2^{t1}\phi q^k(c_{b,2}).
\end{aligned} \tag{54}$$

Substituting out the multipliers λ_1^{t1} and λ_2^{t1} using the first three conditions in (54), gives $(1/c_{b,1}) = (1/c_{b,2}) + \mu_2^{t1}\phi k \partial q^k/\partial c_{b,2}$. Using (52) to substitute out $1/c_{b,2}$ in the latter, leads to the following condition for the debt tax/subsidy rate $\tau_{b,1}$:

$$\tau_{b,1} = \mu_2^{t1} \phi k \partial q^k / \partial c_{b,2},$$

where the RHS is strictly positive under a binding borrowing constraint, $\mu_2^{t1} > 0$, implying a tax on debt, $\tau_{b,1} > 0$, and $\mu_2^{t1} = \lambda_2^{t1} - 1 = 1/c_{b,1} - 1 \Rightarrow c_{b,1} < 1$. Further substituting out the multipliers with $\lambda_1^{t1} = \lambda_2^{t1} = 1/c_{b,1}$ and $\mu_2^{t1} = 1/c_{b,1} - 1$ in the condition in the third line of (54), gives $\alpha k \left(1/c_{b,1}\right) = A_2 + \left(1/c_{b,1}\right) A_1 + \left(1/c_{b,1} - 1\right) \phi q^k$. Comparing the latter with (53) after rewriting with the capital trading decision $q^k(1/c_{b,2}) = A_2 + \kappa_{b,2}\phi q^k$ as $(1 + \tau_{k,1})\alpha k \left(1/c_{b,1}\right) = \left(\left(1/c_{b,2}\right) A_1 + A_2 + \left(\left(1/c_{b,2}\right) - 1\right) \phi q^k\right)$, shows that the capital tax/subsidy rate satisfies

$$\tau_{k,1} = -\frac{\left\{\left(1/c_{b,1}\right) - \left(1/c_{b,2}\right)\right\} \left[A_1 + \phi q^k\right]}{A_2 + \left(1/c_{b,1}\right) A_1 + \mu_2^{t1} \phi q^k}.$$

Since $\left(1/c_{b,1}\right) = \left(1/c_{b,2}\right) + \mu_2^{t1} \phi k \partial q^k / \partial c_{b,2}$ implies $1/c_{b,1} > 1/c_{b,2}$, the policy maker subsidizes capital $\tau_{k,1} < 0$ iff $A_1 + \phi q^k > 0$. Otherwise, $A_1 + \phi q^k < 0$, capital is taxed $\tau_{k,1} > 0$. This establishes the claims made in the first part of the proposition.

For the second part of the proposition, we consider a non-contingent debt tax/subsidy and again an ex-ante capital investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers (see 2). Agents' borrowing conditions and the capital investment decision then satisfy

$$(1 - \tau_b)/c_{b,1} = 1/c_{b,2}, \tag{55}$$

$$(1 - \tau_b)/c_{b,2} = 1 + \kappa_{b,2}, \tag{56}$$

and (52). Substituting out the multiplier $\kappa_{b,2}$ in the borrowers' capital trading condition $q^k(1/c_{b,2}) = A_2 + \kappa_{b,2}\phi q^k$ with (56) and then the tax/subsidy rate τ_b with (55), gives the equilibrium price relation

$$q^k = \frac{A_2 c_{b,2}}{1 - \left(c_{b,1}/c_{b,2}\right) \phi + c_{b,2}\phi} \ , \tag{57}$$

which differs from the corresponding relation under laissez faire (27). The equilibrium price relation (57) implies the price q^k to relate to $c_{b,1}$ and $c_{b,2}$ by $\partial q^k / \partial c_{b,1} =$

$\phi A_2 c_1^2 \left(\phi c_{b,1} - c_{b,2} - \phi c_1^2\right)^{-2} > 0$ and $\partial q^k / \partial c_{b,2} = \left(1 - 2\phi c_{b,1}/c_{b,2}\right) \partial q^k / \partial c_{b,1}$. The Lagrangian of the planer's primal problem can be written as

$$
\begin{aligned}
L = & \log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_2 k + y_{l,3}] \\
& + \lambda_1^t \left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] \\
& + \lambda_2^t \left[y_{b,2} + b_{b,1} + A_1 k - c_{b,2} - b_{b,2}\right] \\
& + \mu_2^t [b_{b,2} + \phi q^k (c_{b,1}, c_{b,2}) k]\},
\end{aligned}
$$

where $q^k(c_{b,1}, c_{b,2})$ satisfies (57). The first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k are

$$
\begin{aligned}
\lambda_1^t &= 1/c_{b,1} + \mu_2^t \phi k \partial q^k / \partial c_{b,1}, \quad \lambda_2^t = 1/c_{b,2} + \mu_2^t \phi k \partial q^k / \partial c_{b,2}, \quad \lambda_1^t = \lambda_2^t, \\
\mu_2^t &= \lambda_2^t - 1, \\
\lambda_1^t \alpha k &= A_2 + \lambda_2^t A_1 + \mu_2^t \phi q^k (c_{b,1}, c_{b,2}).
\end{aligned} \tag{58}
$$

Substituting out the multipliers λ_1^{t1} and λ_2^{t1} using the first three conditions in (58), $(1/c_{b,1}) + \mu_2^t \phi k \partial q^k / \partial c_{b,1} = (1/c_{b,2}) + \mu_2^t \phi k \partial q^k / \partial c_{b,2}$, and substituting out $1/c_{b,2}$ with (55), gives the following condition for the debt tax/subsidy rate $\tau_b : \tau_b = \mu_2^t c_{b,1} \phi k [(\partial q^k / \partial c_{b,2}) - (\partial q^k / \partial c_{b,1})]$. Using that the capital price q^k satisfies $\partial q^k / \partial c_{b,2} = \left(1 - 2\phi c_{b,1}/c_{b,2}\right) \partial q^k / \partial c_{b,1}$, the latter can be rewritten as

$$
\tau_b = -\mu_2^t c_{b,1} \phi k \left(2\phi c_{b,1}/c_{b,2}\right) \left(\partial q^k / \partial c_{b,1}\right).
$$

Given that $\partial q^k / \partial c_{b,1} > 0$, debt is subsidized, $\tau_b < 0$, under a binding borrowing constraint, $\mu_2^t > 0$, establishing the claim made in the second part of the proposition. ■