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DISCUSSION PAPER SERIES

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Maria Laura Di Tommaso
University of Torino, Collegio Carlo Alberto and Frisch Center for Economic Research

Dalit Contini
University of Torino

Dalila De Rosa
University of Torino and Ministry of Economy and Finance, Italy

Francesca Ferrara
University of Torino

Daniela Piazzalunga
University of Trento, CHILD and IZA

Ornella Robutti
University of Torino

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IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Tackling the Gender Gap in Mathematics with Active Learning Methodologies*

We implement a teaching methodology aimed at improving primary school children's mathematical skills. The methodology, grounded in active and cooperative learning, focuses on peer interaction, sharing of ideas, learning from mistakes, and problem solving. We evaluate the causal effect of the intervention on the gender gap in mathematics in Italy with a randomized controlled trial. The treatment significantly improves girls' math performance (0.14 s.d.), with no impact on boys, and reduces the math gender gap by more than 40%. The effect is stronger for girls with high pre-test scores.

JEL Classification: I21, I24, J16, C93

Keywords: gender gap, mathematics, school achievement, primary school, active learning, teaching methodologies, randomized controlled trial

Corresponding author:
Maria Laura Di Tommaso
University of Torino
Lungo Dora Siena 100
10153 Torino TO
Italy
E-mail: marialaura.ditommaso@unito.it

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1. Introduction

Over the past decades, the traditional female disadvantage in education has disappeared and turned into an advantage in most subjects. International learning assessments nonetheless indicate that girls still lag behind boys in mathematics in most countries (OECD 2019). According to the latest PISA survey with a specific focus on mathematics (PISA-2012), the OECD math competency at age 15 was on average 0.11 standard deviations greater for boys than for girls, albeit with considerable country variation ranging from -0.07 in Iceland to 0.24 in Austria.

The female disadvantage in math means that women are much less likely than men to choose STEM majors (science, technology, engineering and mathematics) at university (Turner and Bowen 1999, Card and Payne 2021). This gender imbalance in academic studies then translates into gender-based disparities in occupational choices and results in lower wages for women (Paglin and Rufolo 1990, Machin and Puhani 2003, Black et al. 2008, Piazzalunga 2018). Women are still underrepresented in the most productive sectors of the economy and in high-paying occupations, often in STEM fields, with long-term effects on gender differences in wages and wealth (Sierminska et al. 2019). Moreover, recent research underlines the importance of mathematical skills even for non-STEM degrees and occupations (Grinis 2019, Delaney and Devereux 2020).

A wide range of social and cultural factors contribute to the math gender gap (MGG), which is, in fact, narrower in countries with better gender equality (Guiso et al. 2008, Pope and Syndor 2010, Nollenberger et al. 2016, Lippman and Senik 2018, Gevrek et al. 2020). It is highly correlated with parents' and teachers' attitudes and stereotypes (Ertl et al. 2017, Alan et al. 2018, Carlana 2019, Dossi et al. 2019), and role models also exert an important influence (Dee 2007). Such forces can erode girls' sense of self-confidence and self-efficacy, and increase their anxiety about doing math (Ho et al. 2000, Gneezy et al. 2003, Niederle and Vesterlund 2010, OECD 2015, Di Tommaso et al. 2021).

A largely unexplored factor in the math gender gap is the way mathematics is taught to children. Qualitative research suggests that when the teaching methodology is problem-solving oriented and the students are engaged in discussions and investigative learning activities, the math gender gap narrows and can even disappear (Boaler and Greeno 2000, Boaler 2002a, Boaler 2002b, Zohar and Sela 2003, Boaler 2009, OECD 2016). Nonetheless, we are not aware of any quantitative studies conducted to determine the effectiveness of this approach in mitigating the gender gap in math.

Our paper fills this gap. This study therefore set out to implement and assess a mathematics teaching program based on active and cooperative learning aimed at improving children's mathematical skills in Italian primary school. We evaluate the program's impact with a randomized controlled trial (RCT). To the best of our knowledge, this is the first attempt to investigate the causal impact of a teaching methodology on the gender gap in mathematics. The intervention was found to significantly improve girls' math performance, reducing the gender gap by more than 40%.

Our approach to teaching mathematics is based on the "Mathematics Laboratory" ("*Laboratorio di matematica*"), a math education methodology developed in Italy in the early 2000s (Anichini et al. 2004). The basic building block of this approach is the active involvement of the children, who are engaged in individual and peer work in a collaborative and non-competitive environment. Children are encouraged to frame problems and to attempt to solve them by sharing and comparing ideas within small groups and in-class discussions. Mistakes are welcome and considered a crucial means to understanding. The central idea is that learning involves active participation on the part of the learner (Lave and Wenger 1991). In what follows, we refer to the intervention implemented for the purpose of this study as the "Math Active Learning" (MATL) program.

The MATL program consisted of 15 hours of laboratory activities delivered to third-grade pupils over five consecutive weeks in the spring of 2019. The reason for focusing on third graders is that students in Italy take their first standardized national achievement test at the end of second grade. That is when the math gender gap is first detected, and it typically increases throughout primary and secondary school (Contini et al. 2017). We wanted the intervention to take place when the gender gap had just emerged, before it had grown too large. Each school in the province of Torino was invited to choose at least two of its third-grade classes to apply for the program. We then randomly selected 25 of the schools that applied and randomly assigned one of each of those schools' classes to the treated group and the other to the control group. The final sample consisted of 1,044 children, with 519 children in the treatment group and 525 children in the control group. Tutors specially trained in the new teaching methodology delivered the intervention at the class level during the regular math time scheduled by the school. Thus, the intervention did not provide additional math instruction, but replaced the regular lessons with MATL activities. The regular math teachers remained in the classroom as observers. Children in the control classes followed the usual curriculum with their own teachers. To assess the impact of MATL on the children's performance, we administered math tests one month before the intervention (pre-test) and one month after the intervention (post-test). External supervisors involved in the design of the national assessment test (INVALSI) helped develop the tests, and the tests were scored blindly. This ensured that the tests had a conceptual framework and structure in line with the national achievement test.

Italy is of particular interest for two reasons. First, it had the highest gender gap among the 57 countries participating in TIMSS 4th grade test (Mullis et al. 2016) and the largest gender gap among OECD countries in the PISA test administered to 15-year-old students for the year 2018 (OECD 2019). Second, Italian teachers show the strongest preference for a teacher-centered approach over a student-centered approach (see the OECD teaching and learning international survey, TALIS-2008 (OECD 2009).

The findings from the impact evaluation of the MATL program are encouraging regarding the gender gap in math. The MATL program increased girls' math achievement by 0.14 standard deviations, without hampering boys' performance. Given that the intervention consisted in just 15 laboratory hours, this effect should be considered quite large in magnitude and policy relevant. Overall, the intervention led to an over 40% reduction in the math gender gap. We also evaluate how the impact of the MATL program varies with prior ability, as measured by the pre-test. We find that the treatment has no effect on boys, irrespective of their starting level, but that girls with above-average pre-test scores benefit most from the treatment. We also find heterogeneous effects by migratory background and parental education. Given prior ability, the treatment has a larger impact on migrant girls and girls with low educated parents than on girls from more advantaged family backgrounds.

The rest of the paper is organized as follows. In section 2, we provide an overview of the Italian institutional context and describe the intervention. Section 3 is devoted to the research design of the RCT, as well as to the data and estimation strategy. Results are presented in Section 4, while we explore potential mechanisms that might explain the results in Section 5. We discuss critical issues and problems in Section 6 and conclude in Section 7.

2. Institutional context and design of the program

2.1. Institutional context

In the Italian educational system, children enter formal schooling at age 6. Primary education lasts for five years until age 11. The system is largely composed of public institutions, with less than 7% of children attending private primary school. Families can choose between two schedules: a 40-hour school week, where children spend the whole day at school, or a more concentrated 27/30-hour week.[1] Curricula and learning targets are set at the national level and are the same for both schedules, but teachers are completely free to choose the teaching methods they feel are best. Each class typically has two or three generalist teachers who cover all the subjects between them (with the occasional exception of specialist teachers for foreign languages, gymnastics, and music). Didactic continuity is highly prized in the Italian school system. Children are assigned to a class that then remains the same for all five years of primary school and are normally taught by the same teachers. Primary school teachers receive training enabling them to teach all subjects,[2] although they often specialize in specific disciplines. However, once they have started teaching certain subjects to a class, they continue to teach those subjects to those students for the entire five-year cycle. The school year starts in early September and finishes in mid-June.

In primary school, math instruction covers the domains of numeracy, relations, data and predictions, space, and figures. National curricular guidelines recommend providing instruction in the different domains throughout the entire school year. In third grade, when the MATL intervention was delivered, math instruction is usually offered 6 to 8 hours a week.

2.2. The MATL intervention

Features of the MATL program

Educational research generally identifies two models in the teaching and learning paradigm: learner-centered and teacher-centered. The first conceives of teaching as a top-down activity and focuses on direct transmission of knowledge. In this view, the teacher's role is to "communicate knowledge in a clear and structured way, to explain correct solutions, to give students clear and resolvable problems, and to ensure calm and concentration in the classroom" (pg. 92, OECD 2009). The second, based on the constructivist approach, views students as active participants in the process of learning. More value is attached to the development of thinking and reasoning processes than to the acquisition of specific knowledge (Staub and Stern 2002). Students should become capable of developing solutions to problems on their own (Gutierrez and Boero 2006).

Our intervention consists in classroom-based activities aimed at improving children's mathematical understanding and is based on the theoretical framework of social

[1] The share of schools delivering a 40-hour schedule is much higher in the northern regions.
[2] Qualifying as a primary school teacher now requires a university degree in primary school education. Before 2001, a specific high school diploma (*Istituto magistrale*) was required.

constructivism. The main elements of social constructivism are: i) learning is inherently a social process because it is embedded within a social context as students and teachers work together to build knowledge; ii) knowledge cannot be directly imparted to students, so the goal of teaching is to provide experiences that facilitate the construction of knowledge. Rather than just passively take in information, as children accumulate experiences and reflect upon them, they build their own representations and incorporate new information into their pre-existing knowledge (Thompson 2014). Another conceptual pillar of the approach is the "growth mindset" paradigm, according to which ability is malleable, intelligence can be learned, and the brain can grow from exercise (Dweck 2006a, Boaler 2013), as there is evidence that students who acquire a growth mindset learn more effectively "displaying a desire for challenge and resilience in the face of failure" (Boaler 2013).

More specifically, the MATL intervention builds on the "*Laboratorio di matematica*", a math education methodology developed in Italy in the early 2000s and widely acknowledged in the international mathematics education community (Anichini et al. 2004, Arzarello and Robutti 2008, 2010, Arzarello, Ferrara and Robutti, 2012, Ferrara and Ferrari 2020).

The basic components of the MATL program can be summarized as follows:

(i) *Doing instead of Listening*. Focusing on problem framing and problem-solving as opposed to procedural work, the approach reverses the traditional teacher-centered instruction by putting children at the center of the learning process.

(ii) *Cooperative learning*. Students are engaged with individual and peer-group work, and are encouraged to enter into dialogue with the teacher, both individually and collectively.

(iii) *No pressure*. There is no demand for immediate answers or solutions at the individual level. Students are given suitable time to analyze the problem, explore different solutions, share and compare ideas, avoiding pressure and competition.

(iv) *Learning from mistakes*. Mistakes are seen as a crucial means to understanding. By giving positive attention to their own and others' mistakes, children explore their learning processes and develop a deeper understanding of the discipline.

(v) *Manipulative activities*. Children are engaged with materials (caps, straws, buttons of different size, boxes, cards…) that they manipulate with their hands and move around physically, as perceptual-motor learning has been proven to be effective in improving mathematics understanding (Antinucci 2001, Nemirovsky et al. 2004).

Each of these components aims at activating children's thinking and helping them construct mathematical meanings through self-reflection and interaction with the teacher and their peers. The different activities take place within a collaborative and non-competitive environment, where the teacher – the tutor, in our case – has the role of "orchestrating" the classroom activities.

MATL focuses on the subject area of "Numeracy", recognized as the most fundamental domain in the math field at this age and because we found that the MGG is highest in this domain.[3,4] In our experiment, the MATL program was implemented using two activities. In the first, named *Thousandville*, children must increase the size of a city without changing the proportions of the different components. The learning processes involved are counting, performing arithmetic operations, estimating the order of magnitude, and dealing with large numbers. The second activity, named *Forest Elves*, concerns a family of elves who must go to different places, at different speeds, and arriving at different times. The issues at stake are "who will arrive first in a given place?" and "when/where will they meet?". The learning processes involved are measuring quantities, comparing quantities, and discovering relations between quantities in terms of multiples and submultiples.[5]

Why should MATL contribute to reducing the gender gap in math?

Laboratory teaching practices are devised to help to develop a growth mindset. As shown by Dweck (2006a, 2006b) fixed mindset messages prevail among students across the entire achievement distribution, but high-achieving girls are especially damaged by fixed ability beliefs. Girls suffer most from the fixed ability concept that implies giving labels, like being or not being smart, or being good or not being good at math (Dweck 2006b).

The teaching practices embodied in the MATL intervention have the potential to reduce the gender gap in math for several reasons. First, the activities are meant to reduce pressure and competition. This should benefit girls, because girls are generally less competitive than boys; in competitive environments girls tend to develop more anxiety, and anxiety is detrimental to learning (Bohnet 2016). Second, the approach encourages a positive attitude to mistakes. Reframing mistakes as an opportunity to learn rather than as a sign of failure is particularly important for girls, because girls have been shown to be more risk-averse and

[3] The other subject areas in the primary school mathematics curriculum are relations, space and figures, data and predictions.
[4] For further details, see the final report of the MATHGAP project (Di Tommaso et al. 2020).
[5] Extracts from the methodological guidelines (English translation) are available in Appendix D. The full methodological guidelines are available in English (translation) or in Italian (original) upon request.

afraid of giving the wrong answer (Bohnet 2016). Moreover, girls might have a propensity for learning from mistakes through the development of constructive reasoning about their own cognitive processes because they are more thoughtful (Boaler 2016). MATL could also improve girls' test scores more than boys' test scores because it was specifically devised to embed mathematical activities within a narrative context and girls are typically better than boys at reading comprehension and languages. Another factor that might contribute to girls' activation and empowerment is the explicit support in the MATL guidelines for balanced participation in class discussions.

Delivery of the MATL intervention

The MATL program is delivered to children in grade 3, when they are about 8 years old. There were two reasons for this decision: (i) to tackle inequalities as early as possible and to contrast possible cumulative effects; (ii) to run the intervention at a point in time when the MGG already exists so we could observe gender differences before the intervention and analyze their (short-term) development.[6]

MATL was delivered between February and April 2019. The intervention took place at the class level during school-time and during the usual math time, and did not change the total amount of time devoted to math instruction. Each lab session lasted three hours, and the took place once a week for five consecutive weeks. The children were divided into small, heterogenous groups of mixed prior ability and gender. All the pupils in the treated classes took part in the activities, including children with disabilities, special education needs, or learning difficulties. In the meantime, children in the control group followed the usual curriculum with their class teacher. The intervention was conducted by four tutors with a background in mathematics education at the Master or Ph.D. level. The regular math teachers remained in the classroom as observers.

A pilot study aimed at evaluating the intervention format was conducted a few months before the beginning of the RCT, in two schools not taking part in the experiment. The treatment was then revised based on comments and suggestions from the tutors and the classroom teachers. This pilot also provided the opportunity to assess the length, difficulty, and discriminatory power of the items included in earlier versions of the pre- and post-tests. These tests were analyzed with item-response-theory (IRT) models and modified

[6] According to the literature, the MGG is often observed at a very young age and increases as children grow older; in Italy, the gap is already apparent at the end of second grade (Contini et al. 2017), when children take their first standardized national achievement test (INVALSI).

accordingly.[7]

3. Design, Data, and Estimation

3.1. Research Design

We evaluate the effectiveness of the intervention by exploiting a randomized controlled trial research design. The intervention was designed for delivery in public primary schools located in the province of Torino (Piedmont), in the north-west of Italy. There are 180 public primary schools in the province of Torino. We planned to enroll 25 schools and 50 classes, for a total of approximately 1000-1200 pupils.

The timeline of the implementation of the RCT is outlined in Figure 1.

Fig.1 Timeline of the intervention

Enrollment in the project was on a voluntary basis. In March 2018, all of the public primary school principals in the province of Torino received an official letter signed by the Regional Board of Education informing them of the project[8] and inviting them to a presentation about the project. To be eligible to participate in the project: (i) Schools had to apply with at least two classes, one to be randomized to the treatment group and the other to the control group. This served as a control for potential self-selection issues: parents have substantial leeway in choosing the children's school but cannot choose the specific class or teachers. Although random variability would ensure a fair allocation into the treated and control groups, due to the limited size of the sample of schools, some imbalances could occur. Including two classes per school eliminates school-specific effects related to school management, the socioeconomic composition of the student body, and school-level peer effects. In a broad sense, this procedure can be viewed as a matching method, set up to increase the comparability of the treated and control groups and to improve the accuracy of the estimates. (ii) Classes in the same school had to have different mathematics teachers, to limit the risk of spillover. (iii) Participating classes were not to be involved in other extra-curricular math projects in the same school year.

Thirty-one schools applied for the program. We excluded one school because it was already participating in another math-learning project and randomly selected 25 schools

[7] A full description of the pilot study and of the IRT analysis are available in the final report of the project (Di Tommaso et al. 2020).

[8] The Regional Board of Education is the highest authority of scholastic management at the regional level.

among those remaining. Since some schools applied with more than two classes, we also randomly selected the two participating classes (see Table A.1). We then randomly assigned one class from each school to the treatment group and the other to the control group.[9] The entire randomization process was public and took place at the University of Torino in June 2018.

All the children in the treatment and control classes attended the pre-test one month before the beginning of the MATL program (January 2019). The math laboratories were held between February and April 2019. The children attended the post-test approximately one month after the end of the intervention, between April and May 2019.

The trial and pre-analysis plan (PAP) were registered with the AEA RTC Registry on December 10, 2018, before the start of the intervention. This paper presents analyses on pre-specified outcomes, unless otherwise specified.

3.2. Outcome measures and additional data

Outcome measures

The tests assessing children's math competencies before and after the treatment, designed by experts in mathematics education, followed the same conceptual framework as the INVALSI national assessment for the domain of "Numeracy".[10] We could not use a pre-existing test because the INVALSI primary school assessments involve children in grades 2 and 5, and not children in grade 3. Each test consists of 20 items, to be completed in 40 minutes.[11] The tests cover different topics and mathematical dimensions (knowing, arguing, and problem-solving), and use both multiple choice-type answers and open answers.[12]

The tutors in charge of the laboratories administered the pre- and post-tests in the classrooms and later graded them blindly under the supervision of an external examiner.[13] Correct answers are assigned 1 point each and incorrect and missing answers 0 points, for a total possible of raw scores between 0 and 20 points. The individual raw score is then standardized to have zero mean and standard deviation 1.

[9] The sampling procedure was set before knowing how many schools and classes would apply for the project, and different rules were devised to deal with different numbers of applications. The details can be found in the pre-analysis plan registered with the AEA RCT Registry (Contini et al. 2018).
[10] For an overview of the INVALSI test see:
https://invalsi-areaprove.cineca.it/docs/2018/INVALSI_tests_according_to_INVALSI.pdf
[11] The results of the pre- and post-tests were analysed with an IRT model, available in the final report of the project (Di Tommaso et al. 2020).
[12] The English translation of the tests is available in Appendix C (C.1 and C.2).
[13] An expert in formulating and grading INVALSI tests.

The post-test is the main outcome variable for assessing the effectiveness of the intervention. The pre-test is used to evaluate the gender gap before the intervention and to assess the balance between treated and control classes, and it is included as a control variable to improve the accuracy of the estimates. Figure 2 shows the pre-test score distributions among girls and boys. On average, boys answered 11.23 items out of 20 correctly and girls 10.28; the difference is statistically significant and corresponds to 0.216 standard deviations (0.237 in the sample of children present both at the pre- and post-test). There is a gender gap in math across the entire distribution, confirming the findings from previous research (Contini et al. 2017). The gender gap measured by our test in grade 3 is close to the gap measured by INVALSI assessments in grade 2 in our experimental classes (0.241), but larger than the gap observed in the INVALSI tests in Piedmont (0.130) and Italy as a whole (0.099).[14]

Fig.2 Gender gap in the pre-test

We also collected information about children's attitudes towards math, as a second outcome variable, to explore possible mechanisms underlying the effect of the treatment on cognitive abilities. Attitudes were evaluated by means of a short questionnaire with five Likert-type questions, delivered immediately after the post-test. Details are provided in Section 5.2.

Additional data

A definition of all the variables used in the paper is available in the Appendix (Table A.2).

The schoolteachers provided information about children's special educational needs and disability (SEND), including any forms of learning difficulty, such as physical or mental disability, learning disorders, and attention disorders (ADHD).[15] The schools' administrative offices gave us information about parental education and migratory background. The tutors recorded absenteeism during the math labs for the children in the treated classes.

Data about the math teachers was collected via a brief questionnaire about gender, age, degree, experience overall and in the class, tenure, and type of contract. The tutors collected

[14] See also Section 6.2 on external validity.

[15] These data as all the other data collected in the project were treated with extreme confidentiality. They were collected following the code of ethics of the University of Torino and the Italian and European legislation for privacy.

information about the class, including class size and the schedule (40 hours per week ("full time"), or 27-30 hours per week ("normal").

INVALSI provided class-level data on math and language scores as well as socio-economic background from the national assessment following grade 2. This data was used for evaluating external validity, comparing average ability and social composition in the experimental classes with the corresponding statistics at the regional and national levels.

3.3. Sample

Table 1 shows the sample selection and Table A.3 in the Appendix provides additional details.

No school or class dropped out of the project, so 25 primary schools participated in the project with two third-grade classes each, for a total of 50 classes, and 1,044 children. Of the 1,044 children in the full sample (sample a), 933 pupils were present at the pre-test (sample b), 983 were present at the post-test (sample c), and 888 at both (sample d).[16] The sample used for the impact evaluation is sample d.

Tab.1 Sample selection

3.4. Balance, Attrition, and Compliance

Balance at baseline

Table 2 shows the balance between the treated and control groups at the baseline, i.e., before treatment, and descriptive statistics of the outcome variable (post-test). Panel A reports the mean values of the variables at the individual level, including pre-test scores of girls and boys, shares of girls and boys, native and migrant children, SEND and non-SEND children, parental education. Statistically significant differences can be seen in the maternal education variables. In the treatment group, we find a higher share of mothers with upper secondary education than in the control group, but the opposite occurs for tertiary education; considering the share with at least upper secondary education, the two groups appear perfectly balanced. The differences are slightly in favor of the control group, where mothers are more likely to have a tertiary degree. Panel B reports mean class size, schedule, mean class composition, and teacher characteristics. All variables are balanced. The only

[16] 4 children are excluded from the analysis because they were present at the post-test, but did not answer any of the test items (probably due to very serious disability).

exception is the number of years the math teacher has been teaching in the class,[17] in favor of the control group (2.79 years in control classes, 2.40 in treatment classes). It is worth noting that the number of statistically significant differences is similar to the figure expected due to random variability (i.e., close to 3, the expected number of times we would reject a correct null hypothesis using a level of significance of 0.10 in 30 independent tests).

The treated and control groups are well balanced for most characteristics, both at the overall level and by gender, indicating that the randomization was successful. In addition, we find that the two groups are very similar in terms of math performance, not only at the mean, but also across the entire distribution, as shown in Figure 3.

Tab.2 Baseline characteristics of treated and control children, full sample

Fig.3 Pre-test score distribution by treatment status

Attrition

In this study, there are two relevant sources of attrition: absences at the post-test and absences at the pre-test, which matters because our identification strategy relies on controlling for pre-test scores. We measure both overall attrition (share of units lost in the entire sample) and differential attrition (difference in units lost between treated and control groups) for all children, and separately for boys and girls. Attrition rates are reported in Table 3. The upper panel reports the attrition rates in the post-test compared to the full sample (1,044 children). Overall, 5.4% were absent at the post-test, with small differences between treated and control children and between girls and boys. The lower panel of Table 3 reports the share of children absent at either the pre- or the post-test (14.9%). More absences occurred at the pre-test, presumably because the test was administered during the winter of 2019, during the peak flu season. This attrition rate is significantly higher among treated than among control children (16.7% vs. 12.4%), with a larger gap among girls than among boys. The overall and the differential attrition rates are small enough not to raise concern about the validity of the estimates of the intervention effect.[18]

[17] As explained in the institutional context section, the Italian system values didactic continuity and the primary school teacher teaches the same group of students from grade 1 to grade 5.

[18] See the guidelines in WWC-What Works Clearinghouse (2013), which are based on an extensive simulation study.

Tab.3 Attrition at pre-test and post-test

We rerun balance checks for the sample of children who attended the post-test but not the pre-test (sample b - Tab A.4) and for the sample of children who were present at both tests (sample d - Tab A.5). The treatment and control groups still appear to be well balanced after attrition, and no substantial difference is found between the original and the analytical samples.

Comparison of the treated and control groups in sample (d) was further analyzed in a multivariate regression, by estimating a logit with the treatment status as the dependent variable and individual, teacher, and class characteristics as independent variables. The results are presented in Table A.6 and confirm the groups' comparability.

In the main empirical analyses, our preferred specification includes individual and class characteristics at the baseline as control variables, to account for the minor observed differences between the treated and control groups (despite the favorable results of the attrition analysis).

Compliance and spillover effects

In this experiment, none of the children assigned to the control group took part in the program. Children assigned to the treated classes, instead, were left untreated if they were absent on lab days. Noncompliance dilutes the treatment and yields underestimates of the average treatment effect (Bloom 2008).

In Table 4, we report statistics on MATL participation. No children missed all the lab sessions, 99.3% attended at least 50% of the time, and 73.8% attended all of the sessions, with a small difference in favor of boys (4 percentage points in the full participation). This may reduce the estimated impact on the MGG, yielding conservative estimates of the actual treatment effect. Given that full participation in the program was not reached, the impact evaluation estimates represent estimates of the Intention-to-Treat (ITT) effect.

Spillover effects are also not a matter of concern. First, it is highly unlikely that interactions between eight-year-old children in different classes would involve mathematics. Second, it is also unlikely that teachers in the control group learned sufficient details about MATL to modify their teaching practices in such a short space of time. The math teachers in the treated group were different from those in the control classes, and the intervention was delivered by external tutors with the treatment class teachers present as observers. Moreover, the methodological materials were released to teachers only a year after the project ended. If spillover did occur somehow, the treatment effect would be underestimated.

We cannot rule out the possibility that teachers of the treatment classes learned from observing the intervention. This is not problematic because our aim is to assess the total effect of the program, which consists of the direct effect of MATL on children's math achievement and the (potential) synergic indirect effect generated by the class teachers. Both channels are intended effects of the intervention.

Tab.4 Attendance of the laboratory sessions

3.5. Empirical strategy

Our goal is to assess the impact of participation in the math laboratories on pupils' math skills, and more specifically on boys' and girls' outcomes. The successful randomization into treated and control groups ensures that the two groups can be safely compared, without incurring selection bias. Nevertheless, to control for possible differences between the two groups generated by random variability, we do not simply compare the post-test scores of treated and control children but analyze these differences within a regression framework where we control for individual characteristics and pre-test scores. We estimate the effect of MATL using the following OLS specification, overall and separately for boys and girls:[19]

$$Y_{1iks} = \alpha + \beta T_{ks} + \gamma Y_{0iks} + \delta X_{iks} + \theta_s + \epsilon_{iks} \qquad (1)$$

where Y_{1iks} is the post-test score of individual i in class k of school s. T_{ks} is the binary treatment indicator, equal to one if the pupil is in a class randomly assigned to the treatment group and zero otherwise. Y_{0iks} is the outcome variable at baseline (pre-test score). X_{iks} is a vector of observable individual and class characteristics potentially predictive of the outcome (gender, special education needs or disability, migratory background, parental education, class size, and schedule). θ_s is a vector of school fixed effects (our randomization strata), and ϵ_{iks} are random errors normally distributed and clustered at the class level k. β is the coefficient of interest, capturing the intention-to-treat (ITT) effect of being offered the MATL program. β cannot be interpreted as the average treatment effect (ATE), because

[19] See the pre-analysis plan (Contini et al. 2018). Our empirical analysis is as close as possible to the pre-analysis plan. The analyses and outcomes investigated were pre-specified, unless otherwise indicated.

some pupils did not attend all the lab sessions. However, since most of the students did, we can expect ATE to be similar to the ITT in this case. We assess whether the treatment has a different impact on the two genders estimating equation (1) separately for boys and girls.

We then include an interaction effect between the pre-test score and the treatment dummy, for estimating heterogeneous effects by prior ability.

$$Y_{1iks} = \alpha + \beta T_{ks} + \gamma Y_{0iks} + \delta X_{iks} + \lambda T_{ks} * Y_{0iks} + \theta_s + \epsilon_{iks} \quad (2)$$

The coefficient λ captures the differential impact of the treatment according to the level of the pre-test.

We cannot simply compare gender gaps in the pre- and post-test scores to evaluate the effect of the treatment on the MGG, because the two tests are not equated. Although they were designed within the same conceptual framework, they do not have the same level of difficulty and are not measured on the same scale.[20] A better strategy consists in comparing the raw MGG in treated and control groups after treatment. Due to the successful randomization, we consider the post-test in the control group as a valid estimate of what would have happened to the children in the treated classes had they not been exposed to MATL (and vice versa). To account for the small differences in the pre-test, we estimate the counterfactual as the outcome of control group children had they been treated, using the coefficients estimates from (2) and setting value 1 to the treatment indicator. Similarly, we obtain a counterfactual outcome for treated children. Since there are two possible comparisons, we will obtain two distinct estimates of the magnitude of the change of the MGG due to treatment.

Explanatory variables

In addition to pre-test scores, we control for gender, special education needs or disability (dummy variable) (SEND), migratory background, parental education, class size, and time schedule, as well as school dummies, to account for school fixed effects. We also estimate simpler specifications where not all the control variables are included in the estimation.

Two different versions of the SEND variable are codified as dummy variables: a restricted version of the variable that assumes the value of 1 only for children with certified educational needs, and a broad version of the variable that assumes the value of 1 for all children

[20] See Di Tommaso et al. 2020.

reporting any kind of learning disorder/special needs, whether certified or merely demonstrated.

Family background variables included in models (1) and (2) above are defined in Table A.2. Parental education is denoted as "high education" if at least one parent has a tertiary degree, and 0 otherwise. The child's migratory background is coded as 3 dummy variables: native if the child and at least one parent were born in Italy, first-generation migrant if the child and both parents were born abroad, and second-generation migrant if the child was born in Italy and both parents were born abroad. To prevent the loss of numerous observations and to avoid self-selection issues, we include a dummy variable for each characteristic that is equal to 1 if the characteristic is missing.[21]

We use pre- and post-test scores in standardized version, thus the effect of the treatment reported in the results represents by how many standard deviations the test scores of the treated pupils differ on average from those of the control group.

Robustness checks

The main analytical sample includes only children who took both the pre- and the post-test. In a robustness check, we also include the children who were absent from the pre-test, identifying them with a dummy variable and assigning a zero value for the pre-test score. As for children absent from the post-test, we had scheduled a deferred session on a different date, as close as possible to the original one, and we use the resulting data in a second robustness check.[22]

In additional robustness checks, we exclude children with special education needs or disabilities. 15% of the pupils were reported by the teachers to have learning problems, with a slightly higher share among boys.[23] 8.1% are certified as children with special needs or disabilities. It is not uncommon for children with mild problems not to have obtained a

[21] We were able to collect information about the teachers' characteristics in 49 out of 50 classes (one teacher refused to provide consent for data processing). To avoid losing an entire (control) class, we do not include teachers' characteristics in the estimations at the class level. Teachers' characteristics are used in the balance tests.

[22] During regular sessions, the tutors administered the post-test within the classroom. In the deferred session, the post-test was administered by the class teacher while the other children were involved in normal classroom activities. These tests were then sent by mail to the research team. Of the 57 children absent from the post-test, 35 children took the deferred session. As it was impossible to have full control over this process, we chose not to include these children in the main analyses.

[23] Differences in the percentage of SEND between boys and girls are well-known and documented in the literature (e.g., Vogel 1990, Nass 1993) and can be partly ascribed to an existing gender bias against boys in referrals for special education (Anderson 1997, Wehmeyer and Schwartz 2001). This finding supports the decision to also include SEND children in the analysis.

certification by grade 3. The tests were designed for typically developing children, in line with the national assessments administered periodically at the national level by INVALSI. They may be not appropriate for children with severe learning problems. For this reason, in the pre-analysis plan we stated that we would exclude SEND children's results from the analysis. Because of problems identifying children with severe problems that we were not aware of before going into the field, we decided to deviate from the original plan. We include all SEND children in the main specification, leaving the estimations without them as robustness checks.

4. Results

To evaluate the ITT impact of the intervention on math performance, we compare the post-test results of the treated and control groups, overall and by gender, as described in the previous section. In section 4.1, we estimate the average impact on the entire group of participants, and on girls and on boys separately. In section 4.2 we analyze whether the treatment has heterogeneous effects according to prior achievement, parental education, and migratory background. In section 4.3, we describe the results of robustness checks.

4.1. Core results

Table 5 presents the main results. Considering all the children who sat the post-test (columns 1-3), we find that the intervention has significantly improved math performance (effect size 0.110 s.d.). Analysis by gender reveals that girls drive this effect (effect size 0.146 s.d.). The treatment did not influence boys' achievement instead. We then focus on our preferred sample, including the children who took both the pre- and the post-test; we present the results of specifications with different control variables, up to the preferred model, with school fixed effects, pre-test scores, individual and family background characteristics, class size, and schedule. The overall effect (0.083 s.d.) is entirely attributable to the positive impact of the treatment on girls' skills (0.142).[24,25] The table also shows the results if the treatment coefficients for boys and girls are significantly different.

Overall, the results on the treatment effect are quite stable across specifications: MATL increases girls' test scores by 0.131-152 standard deviations and has no effect on boys'

24 Complete results are presented in Table A.7 in the Appendix.

25 In Appendix B, we present the main and the heterogeneous results using the latent ability estimated with IRT models as a dependent variable rather than the standardized test-score. The results are confirmed and are similar in magnitude.

performance. For educational interventions, this effect is quite large in magnitude. By means of comparison, Bloom et al. (2008) report that the average annual gain in math tests between grade 2 and 3 of primary school is 0.89 standard deviations. Bloom (2008) shows that decreasing class size by 10 children (from 22-26 students) improves performance by 0.10-0.20 standard deviations. Slavin and Lake (2008) find that programs targeting teachers' practices lasting at least 12 weeks have a median effect size of 0.33 and Pellegrini et al. (2018) find a median effect size of 0.25 for similar programs.

A core question is how this impact translates into a raw reduction of the MGG. In the control group, the gender gap in math is 0.324, while in the treated group it is 0.221, implying a reduction of 31.7% in the treated group with respect to the control group.

To account for differences in the pre-test, we compute the reduction in MGG as follows. Firstly, we estimate counterfactual outcomes (of the control group children had they been treated, and of the treatment group had they not been treated) using the coefficient estimates from (2) and applying value 0 to the treatment indicator of the treated group children and value 1 to the treatment indicator of the control group children. Secondly, we compare each counterfactual MGG with the corresponding observed value. The actual MGG for the control group is 0.324, and the counterfactual MGG for this group had they been treated is 0.170, implying a reduction of 47.5%. The actual MGG for the treated group is 0.221, and the counterfactual MGG for this group had they not been treated is 0.369, implying a reduction of 40.1%.

Tab.5 Main results: effect of the treatment

4.2. Heterogeneity in treatment effects

Table 6 describes the estimates of a model with an interaction term between treatment and prior achievement. The intervention has no effect for boys, regardless of pre-test scores. Instead, we find that the treatment is more effective on well-performing girls. For each additional unit in standardized pre-test scores, the treatment effect increases by 0.127 post-test score units. We can appreciate how the treatment effect varies with pre-test scores and the corresponding confidence intervals by inspecting Figure 4. For instance, the point estimate of the treatment effect is close to zero for girls whose pre-test scores are 1 standard deviation below the average, while for girls who are 2 standard deviations above the average, the treatment effect is around 0.4 (=0.155+2·0.127). The effect is statistically significant for girls with pre-test scores exceeding -0.2, which is slightly below the girls' average pre-test score (-0.09).[26]

Tab.6 Heterogeneous effects of the treatment by prior achievement

Fig.4 Treatment effects by prior achievement levels

We then analyze how treatment affects children with different parental education and migratory backgrounds by including an interaction term between treatment and each of these two variables, for the overall sample and then separately for boys and girls.[27] The results are reported in Table 7.[28] The upper panel displays the estimates of the effects by parental education. Once again, we find no treatment effects for boys. Instead, we observe that in terms of point estimates, girls with low-educated parents benefit most from the treatment; however, the difference between girls with low and with high educated parents is not statistically significant. Results by migration background are shown in the bottom panel of Table 7. The benefit of attending the program is larger for migrant girls (0.399 s.d.) than it is for native girls (0.104 s.d.). The intervention has no effect on native boys, although there is evidence of a sizable effect, only mildly statistically significant, on migrant boys (-0.285 s.d.).

Summing up, we find evidence of the following heterogeneous effects: (i) other things being equal, well-performing girls benefit more than poor performing girls; (ii) other things being equal, migrant girls benefit more than native girls. We now analyze how prior performance affects the treatment impact on native and migrant girls, by estimating model (2) separately for the two subgroups. The results are shown in Figure 5. The effect of treatment increases with prior performance in particular for migrant girls, among whom even mid-low performers (prior test scores around -0.7) benefit from participation in the intervention (among native girls, only those better than the average are positively affected).

Overall, we observe that MATL labs improve the math skills of girls, and in particular, well-performing and migrant girls (and to some extent of girls with low educated parents). Instead, we find no effects on boys, or possibly even negative effects for migrant boys. These

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[26] As a robustness check, we replicated the analysis by interacting the treatment variable with pre-test quintiles instead of a continuous variable, allowing the treatment to be non-linearly related to pre-test score. The results are consistent with the described findings and indicate that the effect is approximately linear.

[27] We define as "low education" situations where neither parent has tertiary education qualifications and as "high education" situations where at least one parent has a tertiary degree.

[28] Full estimates are available from the authors upon request.

findings are not fully consistent with previous research. Two best-evidence review papers by Slavin and coauthors analyzing the effect of different active and cooperative math learning interventions (Slavin and Lake 2008, Pellegrini et al. 2018) indicate that students coming from different backgrounds benefit in a similar way and that low achievers benefit most by attending lengthy active learning math programs. MATL is a short-term program, and we speculate that the skills of boys from disadvantaged backgrounds might improve if the intervention were implemented over a longer period of time. Indeed, further investigation is needed to shed light on why the intervention in the present forms is not capable of improving the performance of boys and less performing children.

Tab.7 Heterogeneous effects of the treatment by migrant status and parents' education

Fig. 5 Treatment effect by prior achievement levels, migrant and native girls

4.3. Robustness checks

We replicate the main analyses on different samples. The results are reported in Table 8. First, we exclude from the analysis children with a certified special education need or disability (SEND, narrow definition). Second, we exclude children reporting special educational needs and disabilities even if not formally certified (SEND, broad definition). Third, we use the entire sample of children present at the post-test and we include a dummy variable for children absent from the pre-test. Fourth, we include the children who were absent from the post-test but were given a post-test on a deferred date.[29] In all models, we include pre-test scores, school fixed effects, and the usual additional controls.

The robustness checks largely confirm the results. The treatment has an impact on girls (effect size 0.12-0.17), but not on boys. The impact of the treatment is larger if we exclude children with any type of special educational needs and if we include all children. It is the smallest if we include children who took the test in the deferred session. Absences at the pre-test do not affect performance at the post-test, confirming our hypothesis that absences occurred randomly and that the peak observed in the pre-test was probably due to the flu season.

Tab.8 Robustness checks

[29] In the pre-analysis plan (PAP), we had decided to: exclude SEND children; include post-test taken in the deferred session; include children absent from the pre-test by labeling them with a missing dummy. Afterwards we decided to operate differently in the core analysis, but the choices specified in the PAP are presented here as Robustness checks.

5. Mechanisms

The MATL intervention has proven to be effective on girls. We now explore the potential channels through which the program might have improved girls' math skills. The program could improve abilities by increasing problem-solving competences, engagement and fun, reducing competitiveness, motivating discussion, and valuing the role of mistakes. MATL might act directly on children's competencies or/and indirectly via an effect on self-confidence and more generally on attitudes towards math.

Firstly, we investigate whether the intervention improves mathematical skills overall or only in some dimensions. The question is whether MATL works by enhancing the competencies in some dimensions but not others, or by improving children's skills in dealing with specific item formats. Secondly, we assess the role of attitudes towards math. We measure attitudes directly via a short questionnaire administered to children after the post-test and evaluate whether these measures vary according to whether the children underwent treatment or not. We also analyze if treated children are more likely than controls not to leave some items blank. Apart from the role of attitudes, these analyses were not specified in the pre-analysis plan, and should be considered exploratory.

We can anticipate that we find no evidence of the importance of these channels. The success of the intervention does not seem to be driven by improvement in specific cognitive dimensions or by raising the ability to answer specific types of questions, or by improving attitudes towards math, or by reducing the chances to leave questions unanswered. At the moment, this leads us to infer that MATL worked by directly improving girls' general math skills.

5.1. Type of question: item format, cognitive dimension, level of difficulty

We analyze whether the treatment has a differential impact by item format, cognitive dimension, or level of difficulty of the single test items. We classified the 20 items of the post-test by format, dimension, and difficulty. The item format can be open-response or multiple choice. The level of difficulty has been established with a one-parameter IRT analysis on the control group: we consider *easy* the items with difficulty below -0.5 (corresponding to 5 items), *difficult* those above or equal to 0.5 (5 items), and *medium* those in between (10 items). The cognitive dimension of the items – arguing, knowing, problem-solving – was assigned by experts in the field. The classification is shown in Table A.8 in the Appendix.

We calculate a new set of outcome scores, one for each category of items, by computing the share of correct answers within each category and standardizing the score. We have one post-test score constructed using only multiple-choice items, one constructed using only open-response items, one using only easy items, etc. We estimate the impact of the treatment on each one of the "new" outcome scores, applying a model similar to equation (1), but allowing for correlation among the error terms of the different equations for each group of outcomes (difficulty, format, dimension), by implementing a SUR (Seemingly Unrelated Regression) model.

The results are reported in Table 9. These models were estimated separately for boys and girls, controlling for pre-test scores and school fixed effects.[30] For each group of items, we tested the equality of the treatment coefficients across item categories.[31]

We find no significant effects for boys, so we concentrate on girls. The point estimate of the treatment effect on the multiple-choice score (0.163) is larger than the corresponding effect on the open-answer score (0.125), and both are significant at least at the 10% level. However, the difference between the effects is not significant. We find that the treatment effect is larger on the knowing dimension than on the other two scores (arguing and problem-solving), although the direction is the same and the magnitude is not very different. The treatment has no effect on the easy-items score, a substantial (but not highly significant) effect on the medium-items score, and a very large effect on the difficult-items score. This result is not surprising if we recall that high achieving girls are those who benefit the most.

These results suggest that the treatment enhances girls' math skills and is not driven by improvements in specific cognitive dimensions or in items with a specific format.

Tab.9 Treatment effect by type of item

[30] Since the test-scores in this section are based on the answers to just a few items, they are subject to larger measurement error (in the dependent variable). To simplify the model and avoid introducing many irrelevant variables, in these specifications we do not include all the controls included in the main specification. This should not be a problem, because all control variables are well balanced between treated and control groups (results with all control variables are similar and available from the authors upon request). To allow for appropriate comparisons, the estimate of the treatment effect from the comparable all-items model is reported in the first panel of Table 9.

[31] As reported in Table 9, the Breuch-Pagan test always rejects the null hypothesis of independent equations. As a comparison, we have also estimated single equation OLS models, with standard errors clustered at the level of the class. The results are very similar and available upon request.

5.2. Children's attitudes towards math

Girls generally display less positive attitudes towards math than boys and, in particular, lower interest and enjoyment, lower self-confidence in solving problems, lower beliefs in their own abilities, and higher levels of anxiety and stress (Mullis et al. 2008, Else-Quest et al. 2010, Hill et al. 2016, OECD 2016, Di Tommaso et al. 2021). Attitudes are a key factor to understanding performance in math: although the direction of causality is difficult to assess, there is empirical evidence of a strong relationship between attitudes and math achievement.

To explore whether MATL enhances children's attitudes towards math, we administered a short questionnaire on math self-beliefs and emotional response, right after the conclusion of the post-test.[32] The questionnaire consisted of 5 items with four-level Likert scale answers, ranging from 1 (more negative attitude) to 4 (more positive attitude). Our measure of attitudes is the raw sum of scores.

Consistent with the existing literature, we observe a sizable gender gap in attitudes in favor of boys (Table A.9 in the Appendix). We find a small negative effect of the treatment on the attitudes of both boys and girls, although the estimates are very imprecise and never statistically significant (Table A.10 in the Appendix).[33]

We may conclude that the success of MATL on girls' math skills was not mediated by a positive change in their attitudes towards math. This was a surprising finding. However, if the concept of what mathematics is, is grounded on traditional teaching practices and already heavily rooted in children's minds, it may be difficult to change. This would be especially true for a short intervention delivered by an external teacher rather than by the child's familiar classroom teacher. Longer programs may have more of an impact on pupils' attitudes.

5.3. Item non-response

The reduction of the gender gap in math observed for children exposed to treatment could be due to the tendency to leave questions unanswered. If girls in the treatment group experienced a strong reduction of non-response whereas boys did not, we could speculate

[32] The English translation of the full questionnaire is available in Appendix C (C.3).

[33] We also perform the analysis using the first component delivered by principal component analysis as a dependent variable and obtain very similar results.

that the effect of MATL on the gender gap in math test scores might be driven by a change in the propensity to give answers (even in the absence of a real improvement in math skills).

We use two models to estimate the effect of MATL on the tendency to leave items blank: an OLS linear model for the number of non-response items in the post-test, and a logit model for the probability to leave at least two items blank (see Table A.11). In addition to the treatment variable, we include the usual controls, school fixed effects, and the corresponding missing indicator in the pre-test. We find a negative and significant effect of the treatment on the number of non-response items. On average, the difference in the number of blank items in the post-test between treated and control children is approximately 0.14 and statistically significant. In terms of the probability of leaving at least 2 items blank, the average marginal effect of the treatment is -0.082. Hence there is evidence that MATL is effective in reducing non-response, although the effect is small.

When analyzing the probability to leave items blank separately by gender, we find similar results for girls and boys. We may conclude that there is no evidence that the decline in the MGG is related to differential changes in the propensity to leave items blank.

Finally, we may ask whether the observed improvement in test scores for girls could be largely driven by a decline in non-response. Back-of-the-envelope calculations show that this is not the case, because the change on item non-response is much too small to drive a substantial improvement in test scores.[34] Overall, these results do not support the hypothesis that MATL improves girls' performance by reducing the tendency to leave questions unanswered and suggests that the observed change is due to a real improvement in girls' math skills.

[34] If this were the case, the estimated improvement in test scores would have to be roughly the same as the number of questions that were previously left blank multiplied by the probability of getting the answer right by chance. This probability is difficult to establish, because some questions are open-answer, and the multiple-choice ones have a variable number of options. If the effect of treatment on the number of missing items for girls is -0.11 (meaning that treatment makes the number of blank items decrease by 0.11), even if the probability of giving the correct answer by chance was equal to 1 (obviously far from truth), we would end up with an increase of 0.11 correct answers (on a 20-item test). This value, still an upper bound of the true impact of treatment on the number of correct answers, is much smaller than the estimated impact of MATL for girls, amounting to 0.14 standard deviations in the post-test score variable and approximately equivalent to 0.6 questions. Employing a more reasonable figure for the probability to give the correct answer (say, 0.2-0.5), the distance would become even greater.

6. Limitations of the study

6.1. Threats to internal validity

We envisage two potential threats to internal validity. The first is related to the test design, and the second is related to awareness of the gender perspective of the mathematics laboratory.

Pre- and post-tests were designed by members of the research team, under the supervision of a member of the advisory board of the National Institute of Evaluation (INVALSI). There is some concern over the appropriateness of assessments made by developers of the program, as such measures have been found to overstate program impacts (Pellegrini et al. 2018). This feature could represent a weakness of the study. We believe that our results are still valid. First, the tests were standardized and scored blindly by the tutors (leaving no room for conscious or unconscious bias in grading, either in terms of gender or treatment class). Second, they were conceived as comprehensive measures of abilities with "Numeracy". Moreover, if a bias still existed, we would expect it to influence the results of both boys and girls; yet, this is not the case, as in our experiment the results of treated and controls differ only for girls.

A second issue of possible concern is related to the awareness of the ultimate goal of the intervention – reducing the gender gap in math – by the actors involved: the tutors conducting the laboratories and the schoolteachers. The crucial point is what exactly constitutes the intervention. Is it the teaching methodology or is there also an aspect of "gender awareness"?

The schools were informed that the aim of the project was an evaluation of the effects of the intervention on the gender gap in math because of transparency requirements set by the regional authorities. Teachers of both the treated and the control classes were aware of the gender perspective, and there are no major reasons to expect a difference between the two groups. Also keep in mind that the teachers were not actively involved in conducting the laboratories, but were merely observing. The teachers were also asked not to reveal the goal of the project to the children.

The tutors were also aware of the aim of their work. This was inevitable, as the inclusive participation of all children is a distinctive element of the program. The tutors were aware of the importance of conducting the activities in order to promote the active participation of the entire class. To some extent, the tutors' awareness of the gender perspective of the program might have contributed to improving the girls' performance more than the boys'.

In this light, we acknowledge that the program has two elements that cannot be disentangled. In future work aimed at evaluating a scale-up of the intervention, we should consider the implementation of two alternative programs: one like the current one, and the other with only the teaching component. This would be challenging to implement, however, because it would require a deliberate decision to provide incomplete information about the program to the school boards and regional authorities endorsing the project.

6.2. External validity

The study did not involve a representative sample of schools. Participation in the RCT was voluntary, so the principals and teaching staff of experimental units are likely to be positively selected in terms of interest in gender issues or in experimenting with new teaching methods.

To examine whether and how participating units differ from the regional and national levels, we exploit data from the second grade INVALSI standardized national achievement test held during the previous scholastic year 2017-18, and compare individual and family characteristics of the children in the experimental classes (treated and control) with the child population at large.[35] The results show that the children in the experimental classes perform substantially better on both the math and Italian INVALSI tests than children at the regional and national level (Table 10). It may be noticed that the gender gap in math is much larger in the participating classes: this is consistent with the common finding that girls lag behind boys in math test scores particularly among well performers. The educational level of the parents and the proportion of children who attended kindergarten are also higher in the experimental group.

Taken together, these results indicate that our study has limited external validity. Hence, further research is needed to evaluate ex-ante the potential effects of a scale-up of the intervention introducing the proposed teaching methodology in different contexts.

Tab.10 Comparison of experimental classes with Piedmont and Italy

[35] With the schools' consent, we obtained the experimental class averages of INVALSI test scores in math and Italian, oral marks in math and Italian, shares of pupils' childcare attendance, and mothers' and fathers' education levels. To analyze regional and national test scores, we analyzed the representative sample of classes where the test was administered under external supervision (to reduce cheating).

7. Discussion and conclusions

We implement a teaching methodology aimed at improving primary children's mathematical skills. The approach, grounded in active and cooperative learning practices, provided 15 hours of math laboratories focusing on peer interaction, the sharing of ideas, students' engagement, problem posing, and problem solving. We evaluate the methodology using a randomized controlled trial conducted in the province of Torino, involving 50 third grade classes in 25 schools, and 1,044 students.

The key finding of the paper is that active learning methodologies for teaching mathematics have the potential to reduce the gender gap in math. In our implementation of these methodologies, the treatment had a positive and statistically significant effect on girls' achievement (on average 0.14 standard deviations) without hampering boys' performance. In educational studies, an effect of this magnitude can be considered large and policy relevant. As a consequence, the intervention reduced the gender gap in mathematics by somewhere in the range of 40.1% to 47.5%. In addition, we found that girls with high pre-test scores, girls with low educated parents, and girls with migratory backgrounds benefit the most.

There are many studies on the gender gap in mathematics, but there are few or no rigorous evaluations of the impact of different teaching methodologies. This is the first study to establish that such a link does exist and thus provides a very important contribution to research on the causes of the gender gap in mathematics.

Given the concern and effort that many countries and the international community have shown on the issue of the gender gap in math and the career of women in STEM subjects, it is rather surprising that so little attention has been paid until now to the potential role of teaching methodologies in tackling these issues.

Our experiment introduces a fruitful area of research and can serve as the springboard for further work. The intervention could be scaled up and the class-based intervention be extended to a longer period (in this experiment it was only 15 hours) and delivered over several years. The sample could be increased to include different Italian regions and/or different countries. It would also be of interest to look at whether the intervention has a longer-term effect. In addition, the teachers themselves could implement the new teaching methodology and it could be included in a teachers' professional development program. If teachers instead of tutors delivered the intervention, its effect would likely be more lasting. Nevertheless, our results are encouraging and suggest that properly designed teaching methodologies may improve math performance among girls.

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TABLES

Tab. 1 Sample selection

Sample	Children	Treated	Controls
Full sample (a)	1,044	519	525
Present at the pre-test (b)	933	452	481
Present at the post-test (c)	983	490	493
Present at the pre-test and post-test (d)	888	431	457

Tab. 2 Baseline characteristics of treated and control children, and post-test, full sample

Panel A – Individual level	Control group	Treated group	P-value of the difference
Girl	0.500	0.514	0.436
SEND – broad definition	0.149	0.156	0.677
SEND – broad definition (F)	0.106	0.139	0.100
SEND – broad definition (M)	0.191	0.175	0.482
SEND – narrow definition	0.086	0.083	0.898
SEND – narrow definition (F)	0.046	0.064	0.270
SEND – narrow definition (M)	0.126	0.103	0.388
Native child	0.847	0.876	0.012
Migrant I generation	0.011	0.021	0.125
Migrant II generation	0.127	0.096	0.031
Migrant missing	0.013	0.005	0.321
Mother educ. (lower secondary)	0.219	0.229	0.322
Mother educ. (upper secondary)	0.280	0.354	0.001
Mother educ. (tertiary)	0.299	0.236	0.002
Mother educ. (missing)	0.201	0.179	0.137
Mother at least upper secondary	0.579	0.591	0.837
Father educ. (lower secondary)	0.224	0.254	0.126
Father educ. (upper secondary)	0.417	0.443	0.375
Father educ. (tertiary)	0.163	0.142	0.148
Father educ. (missing)	0.194	0.159	0.004
Father at least upper secondary	0.580	0.585	0.921
Observations	525	519	1,044
Raw pre-test score	10.786	10.703	0.680
Raw pre-test score (F)	10.394	10.152	0.489
Raw pre-test score (M)	11.179	11.274	0.902
Observations	481	452	933
Raw post-test score	9.842	10.335	0.032
Raw post-test score (F)	9.133	9.817	0.033
Raw post -test score (M)	10.566	10.924	0.332
Observations	493	490	983
Panel B – Class level			
Class size	21.000	20.760	0.818
Pre-test score (mean)	10.783	10.646	0.728
Pre-test score (s.d.)	4.310	4.219	0.621
Percent of female students	0.500	0.512	0.630
Percent of I gen. migrant students	0.011	0.018	0.422
Percent of II gen. migrant students	0.136	0.098	0.254
Percent of SEND (broad)	0.146	0.155	0.718
Percent of SEND (narrow)	0.083	0.082	0.954
Full time	0.800	0.720	0.517
Observations	25	25	50
Permanent contract teachers	1.000	0.920	0.164
Teaching experience (years)	21.375	22.560	0.720
Teaching exp. in math (years)	13.695	14.200	0.867
Teaching math in the class (years)	2.791	2.400	0.093
Teacher with a university degree	0.375	0.400	0.861
Teacher's age (years)	48.33	50.00	0.501
Observations	24	25	49

Notes: SEND stands for "special educational needs and disability". "SEND - broad definition" includes children with any form of special education needs or disability, "SEND - narrow definition" includes only children with a certified form of special education need or disability. Summary statistics refer to full sample (a). Summary statistics of pre-test refers to 933 observations (sample b), those of post-test refers to 983 observations (sample c). Teaching experience includes the year of the intervention, but some teachers started teaching in the second semester; thus, they reply that they have been teaching for less than one year, i.e., 0 years. P-value of the difference estimated including school fixed effects and standard errors clustered at class level.

Tab. 3 Attrition at pre-test and post-test

		Overall	Girls	Boys
Post-test [a]	Overall attrition	0.054	0.052	0.056
	Control	0.055	0.049	0.061
	Treated	0.054	0.056	0.051
	Difference (T-C)	-0.001	0.006	-0.009
		(0.141)	(0.194)	(0.020)
Pre- and post-test [b]	Overall attrition	0.149	0.153	0.138
	Control	0.124	0.125	0.123
	Treated	0.167	0.179	0.155
	Difference (T-C)	0.043**	0.053*	0.037
		(0.021)	(0.031)	(0.303)

Notes: Standard errors of the difference in parentheses. [a] Sample (c); [b] Sample (d).
*** p<0.01, ** p<0.05, * p<0.1

Tab. 4 Attendance of the laboratory sessions

Percent of labs. attended	% children	% boys	% girls
0%	0.00%	0.00%	0.00%
≥ 50%	99.30%	100%	98.63%
≥ 70%	95.82%	97.16%	94.52%
≥ 80%	94.19%	95.75%	92.69%
100%	73.78%	75.94%	71.68%
Observations	431	212	219

Notes: 100% of laboratories corresponds to 15 hours. Sample (d) (children present at pre- and post-test).

Tab. 5 Main results: effects of the treatment

	Post-test scores			Post-test scores			Post-test scores controlling for pre-test scores			Post-test scores controlling for pre-test, family background and class variables		
Variable	Overall (1)	Girls (2)	Boys (3)	Overall (4)	Girls (5)	Boys (6)	Overall (7)	Girls (8)	Boys (9)	Overall (10)	Girls (11)	Boys (12)
Treatment	0.110**	0.146**	0.065	0.084	0.131*	0.023	0.076**	0.152***	-0.028	0.083**	0.142**	-0.009
	(0.050)	(0.066)	(0.066)	(0.052)	(0.075)	(0.067)	(0.030)	(0.053)	(0.045)	(0.033)	(0.055)	(0.046)
Pre-test score							0.763***	0.744***	0.784***	0.739***	0.737***	0.748***
							(0.023)	(0.033)	(0.026)	(0.025)	(0.035)	(0.033)
Gender							-0.090*			-0.097**		
							(0.048)			(0.047)		
Constant	0.057	-0.160**	0.243*	0.156***	-0.031	0.308***	-0.001	-0.091**	0.008	0.163	-0.194	0.290
	(0.060)	(0.079)	(0.129)	(0.029)	(0.171)	(0.088)	(0.065)	(0.038)	(0.109)	(0.157)	(0.225)	(0.249)
Chi2 (girls = boys)		1.00			1.53			5.57**			4.14**	
Observations	983	501	482	888	448	440	888	448	440	888	448	440
R-squared	0.029	0.053	0.046	0.031	0.046	0.057	0.611	0.599	0.630	0.616	0.603	0.641
School FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Addit. controls										YES	YES	YES

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. Columns 1 to 3 use sample (c) (children present at the post-test); columns 4 to 12 use sample (d) (children present at the pre- and post-test). In columns 7 and 10, the control variable "Girl" is also included. Additional controls include SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (high-educated parents: at least one parent has a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results (columns 10-12) are available in Table A.7.

*** p<0.01, ** p<0.05, * p<0.1

Tab. 6 Heterogeneous effects of the treatment by prior achievement level

Variable	Overall (1)	Girls (2)	Boys (3)
Treatment	0.081**	0.155***	-0.013
	(0.033)	(0.053)	(0.048)
Pre-test score	0.719***	0.679***	0.735***
	(0.038)	(0.050)	(0.041)
Treatment* Pre-test score	0.062	0.127*	0.028
	(0.048)	(0.064)	(0.058)
Constant	0.139	-0.159	0.292
	(0.159)	(0.224)	(0.251)
Treatment: Chi2 (girls = boys)		5.05**	
Treatment*Pre-test score: Chi2 (girls = boys)		1.66	
Observations	888	448	440
R-squared	0.614	0.607	0.641
School FE	YES	YES	YES
Additional controls	YES	YES	YES

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. Sample (d). Additional controls include girl (in the Overall specification), SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (high-educated parents: at least one parent has a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results are available upon request.

*** p<0.01, ** p<0.05, * p<0.1

Tab. 7 Heterogeneous effects of the treatment by migrant status and parents' education

		Overall (1)	Girls (2)	Boys (3)
Effect of treatment by parents' level of education	Treatment	0.060	0.182**	-0.075
		(0.051)	(0.072)	(0.068)
	Treatment * high-educated parents	0.026	-0.099	0.119
		(0.096)	(0.133)	(0.148)
	Observations	*888*	*448*	*440*
	R-squared	*0.616*	*0.604*	*0.643*
Effect of treatment by migrant status	Treatment	0.092**	0.104*	0.032
		(0.041)	(0.062)	(0.062)
	Treatment * migrant	-0.071	0.295*	-0.317
		(0.117)	(0.154)	(0.204)
	Observations	*888*	*448*	*440*
	R-squared	*0.615*	*0.605*	*0.643*
	Pre-test scores	YES	YES	YES
	School FE	YES	YES	YES
	Additional controls	YES	YES	YES

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. Sample (d). Additional controls include girl (in the Overall specification), SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), class size and time schedule; in the first panel, migratory background (migrant I generation, II generation, information missing), and in the second panel parental education (high-educated parents: at least one parent with a tertiary degree; parents' education missing).
In the first panel, the interaction between treatment and parents' education missing is also controlled for. In the second panel, migrant includes first- and second- generation migrants and children with migratory background information missing. Full results are available upon request.
*** p<0.01, ** p<0.05, * p<0.1

Tab. 8 Robustness checks

	Post-test scores excluding children with certified special educational needs or disabilities			Post-test scores excluding children with any special educational needs or disabilities			Post-test scores including pre-test score missing dummy			Post-test score including children sitting the post-test deferred session		
	Overall	Girls	Boys	Overall	Girls	Boys	Overall	Girls	Boys	Overall	Girls	Boys
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Treatment	0.093**	0.144***	0.008	0.111***	0.159***	0.017	0.110***	0.165***	0.035	0.074**	0.118**	-0.002
	(0.035)	(0.053)	(0.051)	(0.037)	(0.053)	(0.054)	(0.037)	(0.056)	(0.047)	(0.032)	(0.050)	(0.046)
Pre-test scores	0.764***	0.740***	0.771***	0.769***	0.734***	0.786***	0.733***	0.716***	0.731***	0.744***	0.737***	0.739***
	(0.027)	(0.036)	(0.033)	(0.026)	(0.034)	(0.034)	(0.029)	(0.037)	(0.034)	(0.026)	(0.035)	(0.033)
Pre-test sc. missing							-0.069	-0.195	0.078			
							(0.097)	(0.128)	(0.151)			
Constant	0.032	-0.228	0.092	0.090	-0.034	0.152	-0.012	-0.419	0.262	0.153	-0.055	0.242
	(0.174)	(0.213)	(0.309)	(0.159)	(0.194)	(0.338)	(0.185)	(0.261)	(0.234)	(0.152)	(0.204)	(0.271)
Chi2 (girls = boys)		3.42*			3.96**			3.51*			2.84*	
Observations	818	425	393	757	396	361	983	501	482	916	462	454
R-squared	0.608	0.606	0.623	0.595	0.588	0.616	0.557	0.550	0.583	0.608	0.594	0.637
School FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Additional controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
SEND def.	Narrow version	Narrow version	Narrow version	Broad version	Broad version	Broad version						

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. Additional controls include girl (in the Overall specification), SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability) when appropriate (i.e., excluding models 4 to 6), parental education (high-educated parents: at least one parent with a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results are available upon request.
*** p<0.01, ** p<0.05, * p<0.1

Tab. 9 Treatment effect by type of item

		Girls		Boys	
All items	**Outcome**	**Treatm. Effect**	**S.E.**	**Treatm. Effect**	**S.E.**
	Post-test score	0.152**	0.059	-0.028	0.061
DIFFICULTY	**Outcome**	**Treatm. Effect**	**S.E.**	**Treatm. Effect**	**S.E.**
	Easy items score	0.014	0.077	0.032	0.073
	Medium items score	0.123*	0.067	-0.100	0.064
	Difficult items score	0.258***	0.071	0.080	0.078
		Chi2	**P-value**	**Chi2**	**P-value**
	Breusch-Pagan test	48.46	0.000	86.99	0.000
	Easy = Medium	1.392	0.238	2.445	0.118
	Easy = Difficult	5.586	0.018	0.238	0.626
	Medium = Difficult	2.627	0.105	4.660	0.031
FORMAT	**Outcome**	**Treatm. Effect**	**S.E.**	**Treatm. Effect**	**S.E.**
	Open Answers score	0.125*	0.065	-0.052	0.066
	Multiple Choice score	0.163**	0.067	0.013	0.066
		Chi2	**P-value**	**Chi2**	**P-value**
	Breusch-Pagan test	37.37	0.000	59.19	0.000
	Open Ans. = Multiple Choice	0.241	0.624	0.773	0.379
DIMENSION	**Outcome**	**Treatm. Effect**	**S.E.**	**Treatm. Effect**	**S.E.**
	Knowing score	0.162***	0.063	0.002	0.067
	Arguing score	0.108	0.080	-0.118	0.089
	Problem-solving score	0.101	0.069	-0.008	0.066
		Chi2	**P-value**	**Chi2**	**P-value**
	Breusch-Pagan test	75.53	0.000	79.62	0.000
	Knowing = Arguing	0.341	0.559	1.338	0.247
	Knowing = Problem-solving	0.615	0.433	0.018	0.893
	Arguing = Problem-solving	0.006	0.937	1.321	0.250
	Observations	448		440	
	School FE	YES		YES	
	Pre-test score	YES		YES	
	Additional controls	NO		NO	

Notes: Standardized test scores. Sample (d). The treatment effect is estimated with an OLS regression in the "All item" case. For each group of outcomes (difficulty, format, dimension) the treatment effects are estimated with a SUR (seemingly unrelated regression) model, in which the error terms are assumed to be correlated across equations. In all equations, school fixed effects and the pre-test score are included as controls. Below the SUR results, the results of the Breusch-Pagan test for independent equations and the tests of equivalence among the treatment coefficients of interest are reported, together with the corresponding p-values. Difficulty classifies the item's difficulty into three categories (easy, medium, high), using a one-parameter IRT model and (+/-) 0.5 as a threshold. Format classifies items by the type of answer (open answer vs. multiple choice). Dimension classifies the item according to the mathematical thinking behind a specific question (Knowing, Arguing, Problem-solving). The classification of single items can be seen in Table A.8.
*** p<0.01, ** p<0.05, * p<0.1

Tab. 10 Comparison of experimental classes with Piedmont and Italy

Variable	Experimental Classes (1)	Piedmont Classes (2)	P-value of the difference experimental vs. Piedmont classes (3)	Italian Classes (4)	P-value of the difference experimental vs. Italian classes (5)
INVALSI score in Italian	0.393	0.067	0.000	0.000	0.000
INVALSI score in Math	0.559	0.023	0.000	0.000	0.000
INVALSI score Italian Female	0.389	0.113	0.000	0.017	0.000
INVALSI score Italian Male	0.407	0.021	0.000	-0.044	0.000
INVALSI score Math Female	0.439	-0.052	0.000	-0.070	0.000
INVALSI score Math Male	0.681	0.086	0.000	0.029	0.000
Gender Gap Math	-0.241	-0.139	0.000	-0.099	0.000
School grade Italian	8.140	8.105	0.354	8.058	0.011
School grade Math	8.224	8.230	0.863	8.143	0.014
Kindergarten attendance	0.420	0.326	0.000	0.381	0.000
Girl	0.510	0.504	0.007	0.489	0.000
Mother’s education					
Lower secondary	0.258	0.339	0.000	0.331	0.000
Upper secondary	0.405	0.405	0.869	0.409	0.000
Tertiary	0.337	0.257	0.000	0.261	0.000
Father’s education					
Lower secondary	0.360	0.469	0.000	0.427	0.000
Upper secondary	0.405	0.353	0.000	0.391	0.000
Tertiary	0.235	0.178	0.000	0.183	0.000
Low-educated parents	0.697	0.743	0.012	0.754	0.000
High-educated parents	0.302	0.256	0.012	0.245	0.000
Parents’ educ. level missing	0.145	0.097	0.000	0.154	0.409
Max n. of obs.	1,044	1,391		26,142	

Notes: Maximum number observation reported. The number of observations varies depending on the variable and the missing values. Range of variation: Experimental classes 1,020 (min) – 1,044 (max); Piedmont classes 689 (min) – 1,391 (max); Italian classes 12,766 (min) – 26,142 (max). INVALSI scores are standardized (at the Italian level). The Gender Gap in Math is defined as the INVALSI score for Female minus the INVALSI score for boys. School grade refers to the grades given by the teacher, which varies between 1 and 10. Low-educated parents: no parent with a tertiary degree; high-educated parent: at least one parent with a tertiary degree.

FIGURES

Fig. 1 Timeline of the intervention

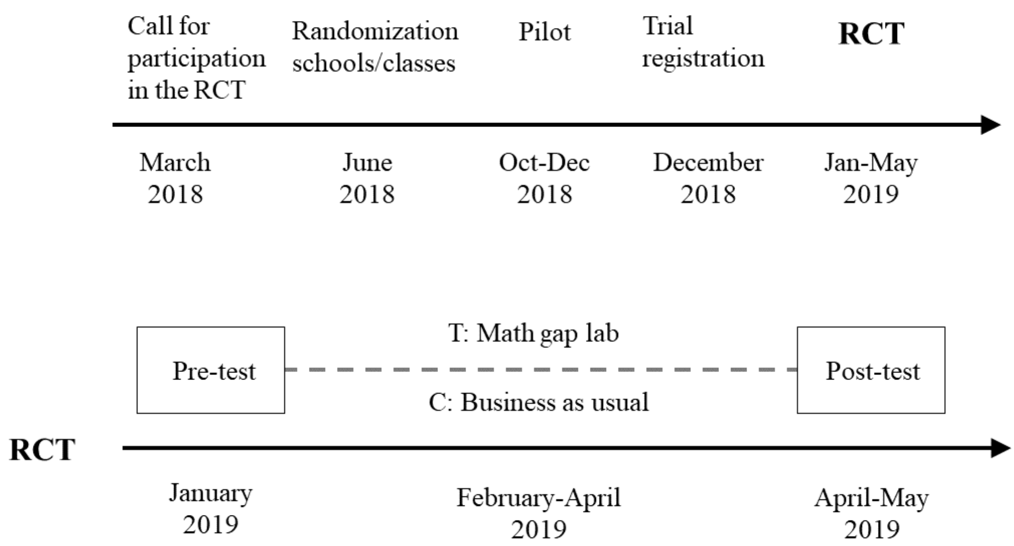
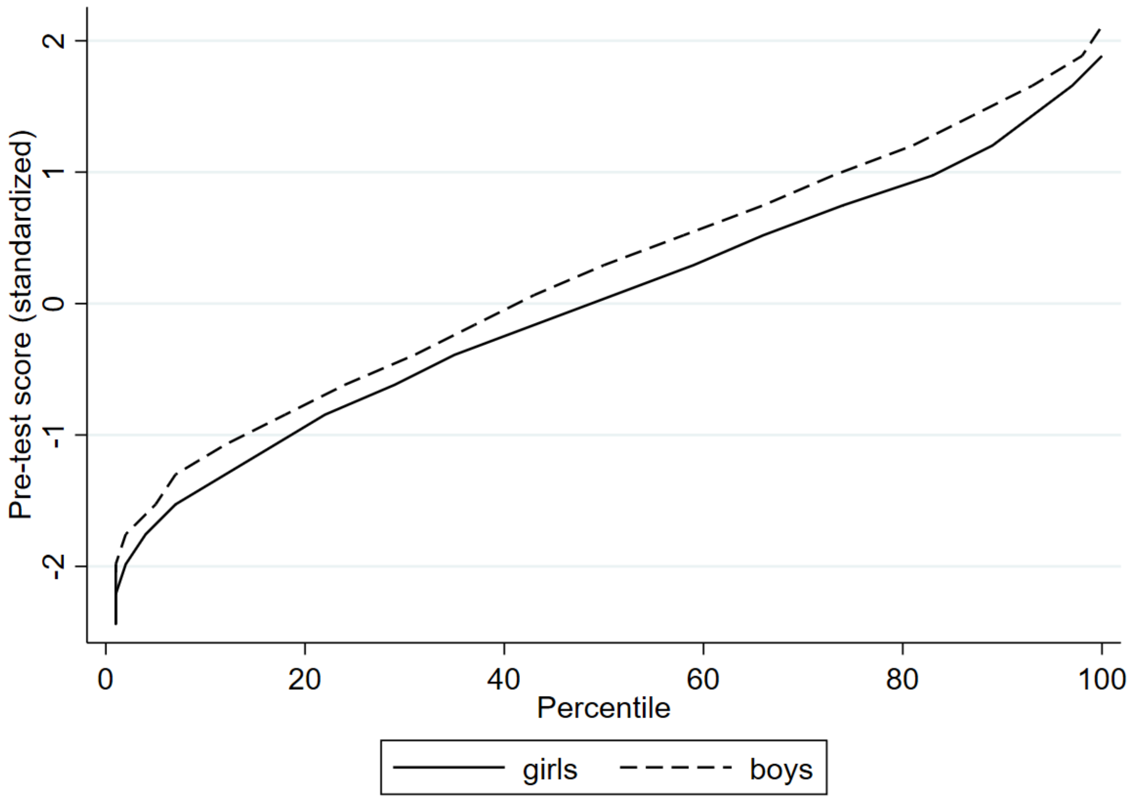
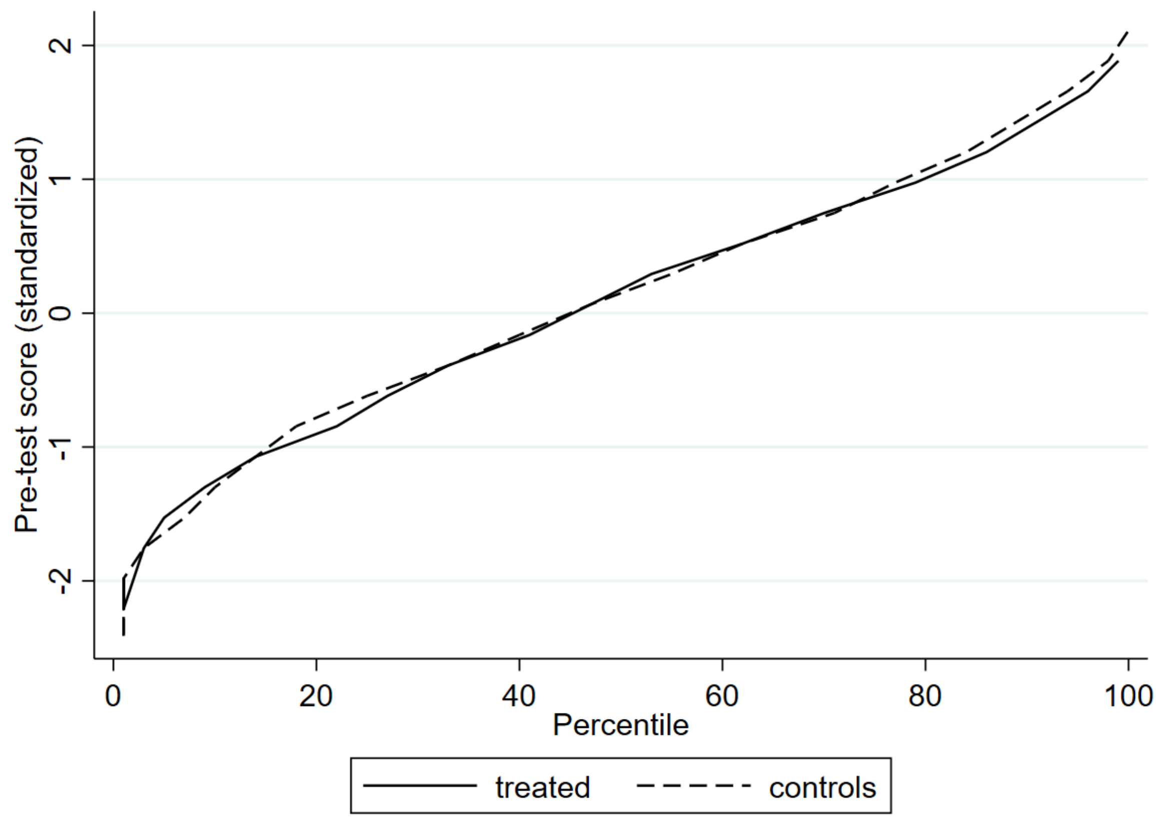


Fig. 2 Gender gap in the pre-test



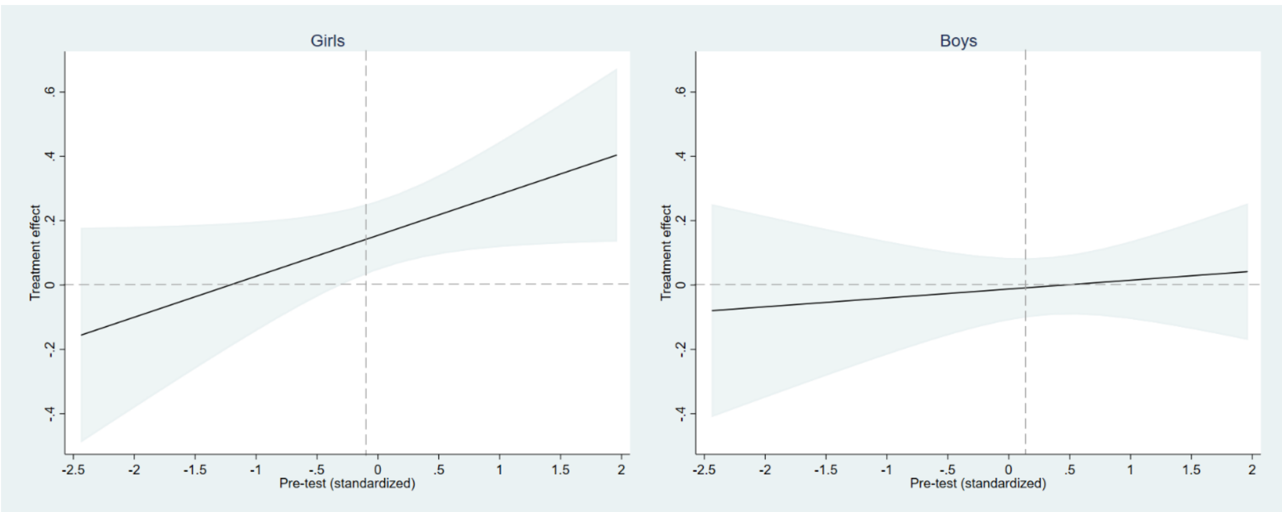
Note: Children present at the pre-test (sample b), 933 observations.

Fig. 3 Pre-test score distribution by treatment status



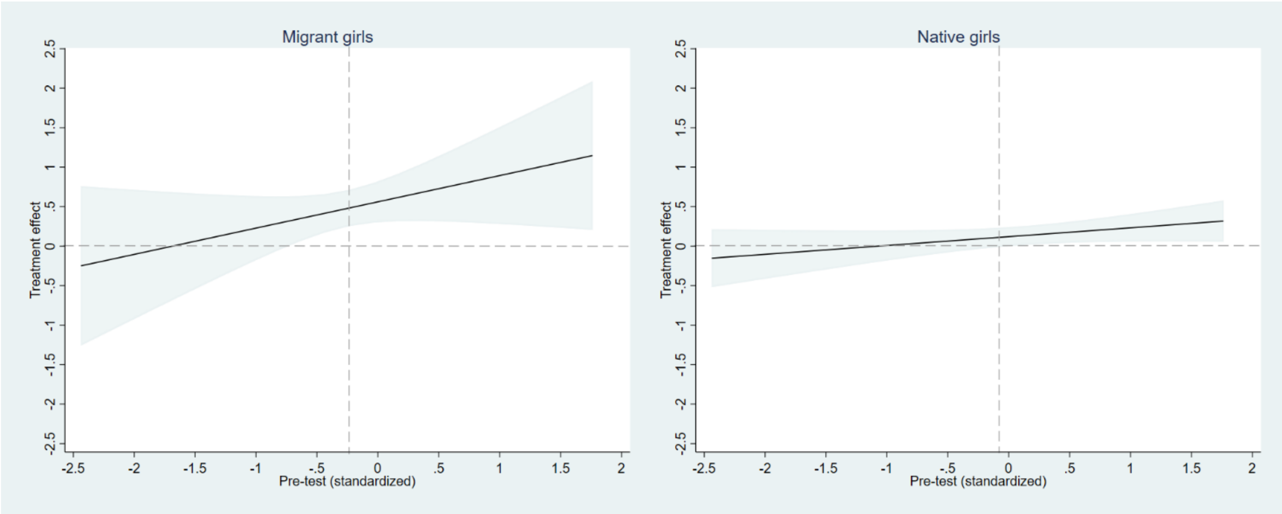
Note: Children present at the pre-test (sample b), 933 observations.

Fig. 4 Treatment effect by prior achievement levels



Notes: Effect of the treatment by pre-test scores for boys and girls (estimates from regression in Table 6). Sample (d), 888 observations. The dashed horizontal line represents a zero-treatment effect, whereas the dashed vertical line represents the pre-test score mean for girls and boys respectively.

Fig. 5 Treatment effect by prior achievement levels, migrant and native girls



Note: Effect of the treatment by pre-test scores for migrant and native girls (estimates available upon request). Sample (d), 888 observations. The dashed horizontal line represents a zero-treatment effect, whereas the dashed vertical line represents the pre-test score mean for each of the two groups respectively. Native girls include children born in Italy with at least one parents born in Italy, migrant girls include first- and second- generation migrants (i.e., those with both parents born abroad) and children with migratory background information missing.

APPENDIX

Appendix A – Additional tables

Tab. A.1 Primary schools in the province of Torino, application and participation into the program

	Schools	Classes
Population	180	-
Applicants	31	100
Eligible	30	82
Sampled	25	50

Tab. A.2 Variables' definition

Variable	Definition
Individual level	
Pre-test score	Pre-test score
Post-test score	Post-test score
Girl	1= girl; 0 = boy
SEND – broad definition	1= child with any form of special education needs or disability; 0 = otherwise
SEND – narrow definition	1= child with only certified special educ. needs or disability; 0 = otherwise
Native Child	1= child born in Italy with at least one parent born in Italy; 0 = otherwise
Migrant I generation	1= child born abroad with both parents born abroad; 0 = otherwise
Migrant II generation	1= child born in Italy with both parents born abroad; 0 = otherwise
Migrant missing	1= missing info on child and parents' birthplace; 0 = otherwise
Mother educ. (lower secondary)	1= mother level of education is lower secondary or less (including 3 years of professional education at high school); 0 = otherwise
Mother educ. (upper secondary)	1= mother level of education is upper secondary; 0 = otherwise
Mother educ. (tertiary)	1= mother level of education is tertiary or above; 0 = otherwise
Mother educ. (missing)	1= mother level of education is missing; 0 = otherwise
Mother at least upper secondary	1= mother level of education is at least upper secondary; 0 = otherwise
Father educ. (lower secondary)	1= father level of education is lower secondary or less (including 3 years of professional education at high school); 0 = otherwise
Father educ. (upper secondary)	1= father level of education is upper secondary; 0 = otherwise
Father educ. (tertiary)	1= father level of education is tertiary; 0 = otherwise
Father educ. (missing)	1= father level of education is missing; 0 = otherwise
Father at least upper secondary	1= father level of education is at least upper secondary; 0 = otherwise
Low-educated parents	1= no parent has tertiary degree; 0 = otherwise
High-educated parents	1= at least one parent has tertiary degree; 0 = otherwise
Parents' education missing	1= missing ingo on parental education; 0 = otherwise
Class level	
Class size	Number of children in each class
Full time	1= class with full time schedule (40 hours per week); 0 = otherwise (27/30)
Pre-test score (mean)	Mean of pre-test score at class level
Pre-test score (s.d.)	Standard deviation of pre-test score at class level
Percent of female students	Percent of female students in the class
Percent of I gen. migrant students	Percent of I generation migrants in the class
Percent of II gen. migrant students	Percent of II generation migrants in the class
Percent of SEND (broad)	Percent of children with any form of special educ. needs or disability in the class
Percent of SEND (narrow)	Percent of children with only certified special educ. needs or disability in the class
Permanent contract teachers	1= Teacher with a permanent contract; 0 = otherwise
Teaching experience (years)	Number of years teacher has been teaching
Teaching exp in math (years)	Number of years teacher has been teaching math
Teaching math in the class (years)	Number of years teacher has been teaching math in the class
Teacher with university degree	1= Teacher with a permanent contract; 0 = otherwise
Teacher's age (years)	Age of teacher

Tab. A.3 Sample selection, details

Sample	Children	Treated	Controls
Full sample (a)	1,044	519	525
Present at the pre-test (b)	933	452	481
Present at the post-test (c)	983	490	493
Present at the pre-test and post-test (d)	888	431	457
Provide background information (e)	759	385	374
Present at the pre-test and post-test and provide background information (f)	659	327	334
Number of pupils with all items missing (post-test)	4	1	3
Number of SEND narrow def. in the full sample	88	43	45
Number of SEND broad def. in the full sample	159	81	78
Post-test in the deferred session	35	20	15

Note: SEND stands for “special educational needs and disability”. “SEND - narrow definition” includes only children with a certified form of special education need or disability, “SEND - broad definition” includes children with any form of special education needs or disability.

Tab. A.4 Baseline characteristics of treated and control children, sample (c)

	Control group	Treated group	P-value of the difference
Girl	0.505	0.514	0.695
SEND – broad definition	0.139	0.148	0.699
SEND – broad definition (F)	0.100	0.126	0.136
SEND – broad definition (M)	0.180	0.172	0.597
SEND – narrow definition	0.079	0.077	0.922
SEND – narrow definition (F)	0.040	0.059	0.180
SEND – narrow definition (M)	0.118	0.094	0.313
Native Child	0.849	0.885	0.004
Migrant I generation	0.012	0.020	0.221
Migrant II generation	0.123	0.089	0.017
Migrant missing	0.014	0.004	0.208
Mother educ (lower secondary)	0.223	0.224	0.490
Mother educ (upper secondary)	0.290	0.348	0.003
Mother educ (tertiary)	0.290	0.246	0.025
Mother educ (missing)	0.196	0.179	0.071
Mother at least upper secondary	0.580	0.595	0.563
Father educ (lower secondary)	0.227	0.251	0.111
Father educ (upper secondary)	0.419	0.438	0.484
Father educ (tertiary)	0.164	0.144	0.166
Father educ (missing)	0.188	0.165	0.010
Father at least upper secondary	0.584	0.583	0.795
Observations	493	490	983
Pre-test score	10.772	10.856	0.879
Pre-test score (F)	10.358	10.232	0.740
Pre-test score (M)	11.188	11.500	0.397
Observations	457	431	888

Notes: SEND stands for "special educational needs and disability". "SEND - broad definition" includes children with any form of special education needs or disability, "SEND - narrow definition" includes only children with a certified form of special education need or disability. Summary statistics refer to children present at the post-test (sample c). Summary statistics of pre-test refers to 888 observations (sample d). P-value of the difference estimated including school fixed effects and standard errors clustered at class level.

Tab. A.5 Baseline characteristics of treated and control children, sample (d)

	Control group	Treatment group	P-value of the difference
Pre-test score	10.772	10.856	0.879
Pre-test score (F)	10.358	10.232	0.740
Pre-test score (M)	11.188	11.500	0.397
Girl	0.501	0.508	0.745
SEND – broad definition	0.144	0.150	0.612
SEND – broad definition (F)	0.104	0.127	0.093
SEND – broad definition (M)	0.184	0.174	0.758
SEND – narrow definition	0.080	0.076	0.871
SEND – narrow definition (F)	0.043	0.059	0.224
SEND – narrow definition (M)	0.118	0.094	0.350
Native Child	0.879	0.851	0.032
Migrant I generation	0.002	0.008	0.073
Migrant II generation	0.095	0.126	0.031
Migrant missing	0.004	0.013	0.218
Mother educ (lower secondary)	0.218	0.234	0.254
Mother educ (upper secondary)	0.293	0.364	0.003
Mother educ (tertiary)	0.295	0.225	0.003
Mother educ (missing)	0.192	0.176	0.029
Mother at least upper secondary	0.588	0.589	0.862
Father educ (lower secondary)	0.216	0.262	0.032
Father educ (upper secondary)	0.424	0.438	0.397
Father educ (tertiary)	0.168	0.127	0.020
Father educ (missing)	0.190	0.171	0.007
Father at least upper secondary	0.592	0.566	0.466
Observations	457	431	888

Notes: SEND stands for "special educational needs and disability". "SEND - broad definition" includes children with any form of special education needs or disability, "SEND - narrow definition" includes only children with a certified form of special education need or disability. Summary statistics refer to children present at the pre- and post-tests (sample d). P-value of the difference estimated including school fixed effects and standard errors clustered at class level.

Tab. A.6 Effect of baseline characteristics
on the probability of being treated

Variables	Treatment	Treatment
Pre-test score	0.100	0.098
	(0.080)	(0.074)
Girl	0.008	0.049
	(0.076)	(0.081)
SEND – broad definition	0.198	0.109
	(0.196)	(0.177)
Migrant I generation	1.074*	0.727
	(0.550)	(0.545)
Migrant II generation	-0.366**	-0.379**
	(0.164)	(0.162)
Migrant missing	-0.771	-1.039
	(0.790)	(0.850)
Parents high educated	-0.628***	-0.549***
	(0.164)	(0.139)
Parents education missing	-0.522***	-0.516***
	(0.163)	(0.165)
Class size	0.020	-0.062
	(0.316)	(0.208)
Full time	-3.898**	-1.343
	(1.902)	(1.357)
Teaching experience	-0.035	
	(0.059)	
Teacher’s university degree	1.187	
	(1.153)	
Teacher’s age	0.090	
	(0.087)	
Constant	-1.767	2.172
	(6.847)	(3.858)
Observations	845	888
Wald test of joint significance	82.17	49.98
	(0.000)	(0.000)
School FE	YES	YES

Notes: Standardized pre-test scores. Standard errors clustered at the class level in parentheses. Sample (d). Results of a logit model. Wald test performed on all variables, excluding schools’ dummies. “SEND - broad definition” includes children with any form of special education need or disability. Parents high educated: at least one parent with a tertiary degree. Reference categories are: boy, typically developed child, native child, parent’s low educated.
*** p<0.01, ** p<0.05, * p<0.1

Tab. A.7 Effect of the treatment controlling for individual and family background characteristics – full results

Variables	Overall (1)	Girls (2)	Boys (3)
Treatment	0.083**	0.142**	-0.009
	(0.033)	(0.055)	(0.046)
Pre-test score	0.739***	0.737***	0.748***
	(0.025)	(0.035)	(0.033)
Girl	-0.097**		
	(0.047)		
SEND broad definition	-0.106	0.034	-0.184*
	(0.067)	(0.129)	(0.101)
Migrant I generation	-0.061	-0.061	-0.059
	(0.156)	(0.237)	(0.146)
Migrant II generation	0.047	0.004	0.126
	(0.073)	(0.099)	(0.129)
Migrant missing	-0.152	-0.063	-0.484
	(0.122)	(0.244)	(0.351)
Parents high educated	0.121**	0.083	0.158*
	(0.055)	(0.081)	(0.085)
Parents education missing	-0.043	-0.159	0.145
	(0.095)	(0.121)	(0.110)
Class size	-0.012	0.006	-0.023*
	(0.008)	(0.013)	(0.012)
Full time	0.008	-0.076	0.057
	(0.051)	(0.065)	(0.074)
Constant	0.163	-0.194	0.290
	(0.157)	(0.225)	(0.249)
R-squared	0.616	0.603	0.641
Observations	888	448	440
School FE	YES	YES	YES

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. The Table corresponds to columns 10, 11, 12 of Table 5. SEND - broad definition" includes children with any form of special education need or disability. Parents high educated: at least one parent with a tertiary degree. Reference categories are: boy, typically developed child, native child, parents' low educated.
*** p<0.01, ** p<0.05, * p<0.1

Tab. A.8 Item classification, post-test

Question	Item	Difficulty score	Difficulty level	Format	Dimension
D1	1	1.244	Difficult	Open	Knowing
D2_a	2	-1.357	Easy	Open	Knowing
D2_b	3	1.323	Difficult	Open	Knowing
D3	4	-0.252	Medium	Multiple	Knowing
D4	5	0.207	Medium	Open	Knowing
D5_a	6	-0.991	Easy	Open	Problem-solving
D5_b	7	2.897	Difficult	Open	Problem-solving
D6	8	-0.272	Medium	Open	Problem-solving
D7_a	9	-1.466	Easy	Multiple	Knowing
D7_b	10	1.270	Difficult	Multiple	Arguing
D8_a	11	-0.242	Medium	Open	Knowing
D8_b	12	0.246	Medium	Open	Knowing
D9	13	-0.410	Medium	Open	Problem-solving
D10_a	14	-0.086	Medium	Multiple	Problem-solving
D10_b	15	0.838	Difficult	Multiple	Problem-solving
D11_a	16	0.276	Medium	Open	Arguing
D11_b	17	-0.164	Medium	Open	Arguing
D12	18	-0.802	Easy	Multiple	Knowing
D13_a	19	-0.696	Easy	Multiple	Problem-solving
D13_b	20	-0.500	Medium	Multiple	Problem-solving

Tab. A.9 Attitudes, summary statistics

	Obs.	Mean	S.D.	Min	Max
Overall	882	15.147	3.351	5	20
Boys	438	15.554	3.299	5	20
Girls	444	14.745	3.358	5	20
	Obs.	Difference	S.E.	P-value of the diff	
Mean diff. Boys vs. Girls	882	0.809	0.224	0.000	

Notes: The indexes for attitudes are constructed from five questions, with four possible Likert-type answers, coded from 1 (not at all) to 4 (a lot). Attitudes (sum) is an index built as the sum of these points.

Tab. A.10 Effect of the treatment on attitudes towards mathematics

Variable	Attitudes (1)	Attitudes (2)
Girls	-0.750*	-0.831**
	(0.388)	(0.375)
Treatment effect on boys	-0.474	-0.477
	(0.301)	(0.298)
Treatment effect on girls	-0.495	-0.486
	(0.358)	(0.350)
Constant	16.500***	16.094***
	(0.222)	(0.555)
Observations	882	882
R-squared	0.053	0.072
School FE	YES	YES
Additional controls	NO	YES

Notes: Standard errors clustered at the class level in parentheses. Sample (d). The indexes for attitudes are constructed from five questions, with four possible Likert-type answers, coded from 1 (not at all) to 4 (a lot). Attitudes is an index built as the sum of these points, Additional controls include SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (parents high educated: at least one parent has a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results available upon request.

*** p<0.01, ** p<0.05, * p<0.1

Tab. A.11 Treatment effect on blank items

	OLS			LOGISTIC		
	Overall	Boys	Girls	Overall	Boys	Girls
Variables	(1)	(2)	(3)	(4)	(5)	(6)
Treatment	-0.146**	-0.142*	-0.137*	0.284***	0.298***	0.223**
	(0.061)	(0.077)	(0.072)	(0.101)	(0.113)	(0.161)
Gender	0.008			0.799		
	(0.054)			(0.173)		
N. of blank items at pre-test	0.138***	0.146**	0.115***			
	(0.041)	(0.057)	(0.039)			
Pre-test score std.	-0.037	-0.028	-0.056	1.009	0.916	1.183
	(0.038)	(0.055)	(0.042)	(0.167)	(0.188)	(0.357)
At least 2 blank items pre-test				5.579***	3.955***	7.307***
				(1.650)	(1.741)	(4.749)
Constant	0.070	-0.260	0.441	0.043	0.257	0.000***
	(0.243)	(0.282)	(0.369)	(0.114)	(0.636)	(0.000)
Observations	888	448	440	888	440	448
R-squared	0.159	0.191	0.212			
School FE	YES	YES	YES	YES	YES	YES
Additional Controls	YES	YES	YES	YES	YES	YES
Dependent Variable	Num. of blank items at post-test	Num. of blank items at post-test	Num. of blank items at post-test	Dummy (at least 2 blank items at post-test)	Dummy (at least 2 blank items at post-test)	Dummy (at least 2 blank items at post-test)

Notes: Standardized test scores. Standard errors clustered at the class level in parentheses. In columns (1), (2), and (3) the dependent variable is the number of blank items at the post-test; in columns (4), (5) and (6) the dependent variable is a dummy variable equal to 1 if at least 2 items are left blank at the post-test, and a logistic model is estimated (coefficients reported in terms of Odd Ratio). Additional controls include SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (high-educated parents: at least one parent with a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results available upon request. *** p<0.01, ** p<0.05, * p<0.1.

Appendix B – IRT analysis

In this Appendix, we present the results from our preferred specification using as outcome variable Y_1 and as baseline control Y_0 the latent abilities estimated with IRT (Item Response Theory) models instead of pre- and post-test standardized results (Table B.1), and the heterogenous results by prior achievement (Table B.1). The first two columns present our main results to ease the comparison. More specifically, we have estimated three IRT models: (i) a one-parameter IRT logistic model, which accounts for the level of difficulty of the items; (ii) a two-parameter IRT logistic model, which accounts for the level of difficulty and the discriminatory power of the items; (iii) a two-parameter IRT logistic model estimated only on the control group, and predicted latent ability for both treated and control children, to reduce the risk that the treatment impacts the estimated latent ability.

All the results are confirmed and similar in magnitude to results using the standardized sum of correct answers as outcome, and thus we decided to keep the standardized test-scores in the main analysis, first so as to adhere as closely as possible to the pre-analysis plan, and second because the treatment itself could partially affect the estimated latent ability.

Tab. B.1 Main results with IRT scores as the dependent variable

Dependent var.	Post-test std.	Post-test std.	Ability from IRT 1 p.	Ability from IRT 1 p.	Ability from IRT 2 p.	Ability from IRT 2 p.	Ability from IRT 2 p. (controls)	Ability from IRT 2 p. (controls)
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Treatment	0.142**	-0.009	0.138***	-0.009	0.117***	-0.011	0.121***	-0.019
	(0.055)	(0.046)	(0.048)	(0.043)	(0.043)	(0.043)	(0.044)	(0.041)
Pre-test score std.	0.737***	0.748***						
	(0.035)	(0.033)						
Pre-test ability IRT 1p.			0.743***	0.732***				
			(0.038)	(0.035)				
Pre-test ability IRT 2p.					0.748***	0.759***	0.766***	0.778***
					(0.038)	(0.034)	(0.039)	(0.033)
Constant	-0.194	0.290	-0.150	0.230	-0.204	0.257	-0.084	0.442**
	(0.225)	(0.249)	(0.200)	(0.224)	(0.179)	(0.211)	(0.177)	(0.196)
Observations	448	440	448	440	448	440	448	440
R-squared	0.603	0.641	0.601	0.625	0.607	0.641	0.605	0.635
School FE	YES	YES	YES	YES	YES	YES	YES	YES
Additional controls	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Standard errors clustered at the class level in parentheses. Sample (d). Columns (1) and (2) report the results of our preferred specification and use standardized pre- and post-test scores (they correspond to columns (11) and (12) of Table 5). Columns (3) and (4) use as outcome and pre-test the latent abilities predicted with a one-parameter IRT (Item Response Theory) logistic model; columns (5) and (6) the latent abilities predicted with a two-parameter IRT model; columns (7) and (8) use as outcome the latent abilities predicted with a two-parameter IRT model estimated on the control group only (predicted abilities for both control and treated pupils). Additional controls include SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (high-educated parents: at least one parent with a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Full results available upon request.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Tab. B.2 Heterogeneous results by prior achievement with IRT scores as the dependent variable

Dependent var.	Post-test std.	Post-test std.	Ability from IRT 1 p.	Ability from IRT 1 p.	Ability from IRT 2 p.	Ability from IRT 2 p.	Ability from IRT 2 p. (controls)	Ability from IRT 2 p. (controls)
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Treatment	0.155***	-0.013	0.153***	-0.012	0.131***	-0.016	0.137***	-0.008
	(0.053)	(0.048)	(0.046)	(0.045)	(0.043)	(0.045)	(0.044)	(0.046)
Pre-test score	0.679***	0.735***	0.683***	0.722***	0.696***	0.741***	0.705***	0.753***
	(0.050)	(0.041)	(0.055)	(0.044)	(0.055)	(0.042)	(0.055)	(0.041)
Treatment* Pre-test score	0.127*	0.028	0.128*	0.021	0.114	0.039	0.115	0.031
	(0.064)	(0.058)	(0.069)	(0.063)	(0.068)	(0.061)	(0.069)	(0.062)
Constant	-0.159	0.292	-0.120	0.231	-0.170	0.261	-0.114	0.327
	(0.224)	(0.251)	(0.195)	(0.225)	(0.178)	(0.214)	(0.189)	(0.223)
Observations	448	440	440	440	448	440	448	440
R-squared	0.607	0.641	0.604	0.625	0.610	0.642	0.611	0.638
School FE	YES	YES	YES	YES	YES	YES	YES	YES
Additional controls	YES	YES	YES	YES	YES	YES	YES	YES

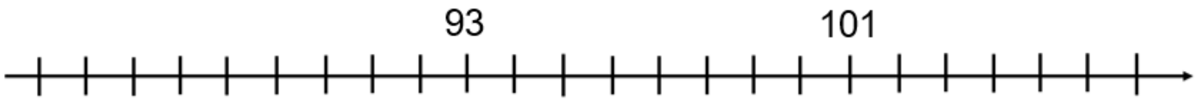
Notes: Standard errors clustered at the class level in parentheses. Sample (d). Columns (1) and (2) report the heterogeneous results of our preferred specification and use standardized pre- and post-test scores (they correspond to columns (2) and (3) of Tab.6). Columns (3) and (4) use as outcome and pre-test the latent abilities predicted with a one-parameter IRT (Item Response Theory) logistic model; columns (5) and (6) the latent abilities predicted with a two-parameter IRT model; columns (7) and (8) use as outcome the latent abilities predicted with a two-parameter IRT model estimated on the control group only (predicted abilities for both control and treated pupils). Additional controls include SEND (special education needs and disability) dummy broad definition (children with any form of special education needs or disability), parental education (high-educated parents: at least one parent with a tertiary degree; parents' education missing), migratory background (migrant I generation, II generation, information missing), class size, and time schedule. Pre-test scores are always the appropriate ones (e.g. standardized, IRT 1p., or IRT 2p.) depending on the outcome used. Full results available upon request.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Appendix C
Test: Pre- and post-test, and non-cognitive questionnaire

C1. Pre-test on math competences

NAME ……………………………………   **GOOD LUCK! ☺**

1) Look at the number line.



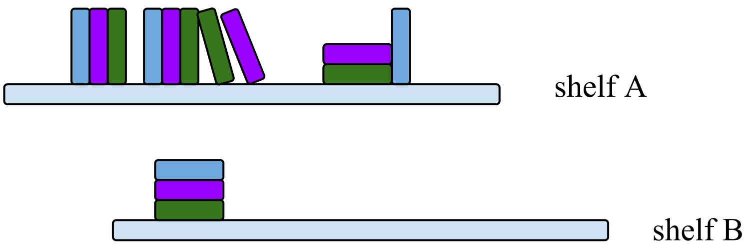
Add these numbers to the line: 89 and 97 and 105.

2) a. Which number has 3 units and 2 tens?

   A. 23
   B. 203
   C. 302

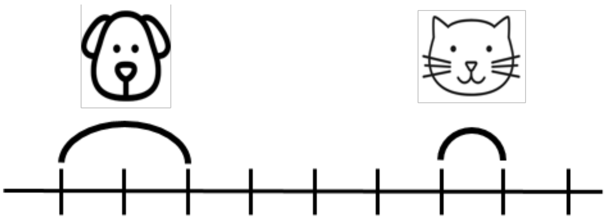
   b. Complete the sentence:
   There are ………… tens in the number 703.

3) Martha has to organize the bookshelves in her room:

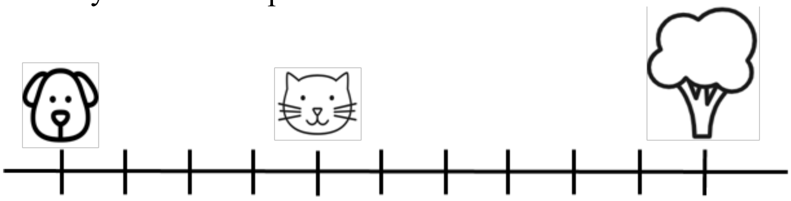


   Martha wants to have the same number of books on each shelf.
   How many books does she have to move from shelf A to shelf B?
   Answer: ………… books.

4) A dog and a cat are playing at chasing each other.
This is what the dog's step and the cat's step are like:



At a certain point they are in these positions:

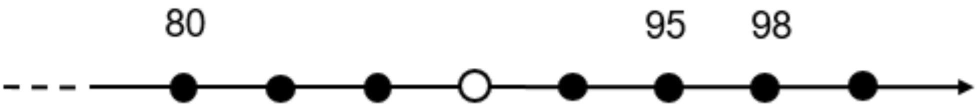


a. How many steps does the cat have to take to reach the tree?
Answer: ………… steps.

b. How many steps does the dog have to take to reach the tree?
Answer: ………… steps.

5) Add 7 units and 3 tens to the number two hundred and ten: what number do you get?
A. 283
B. 247
C. 220

6) Look at this figure:



What number can you put over the white circle?
Answer: …………

7) Today is Peter's birthday.
Peter has brought candy and cakes to celebrate with his friends.
This is how he distributes them:

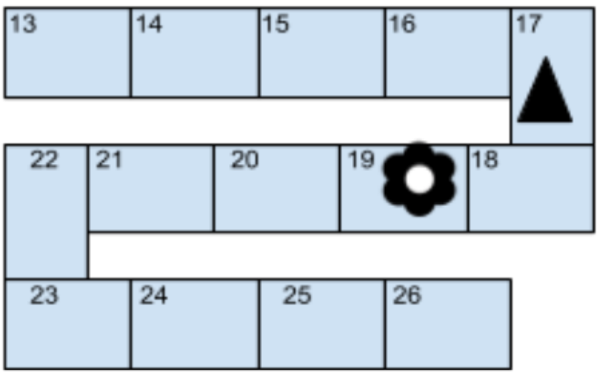
	1 cake for every 8 children
	3 pieces of candy for each child

There are 48 children at the birthday party.

a. How many cakes did Peter bring?
   A. 8
   B. 6
   C. 11

b. How many pieces of candy did Peter have in all?
   Answer: ………… pieces

8) Martina and Christian are playing Snakes and Ladders.
Martina's piece is shaped like a flower: she rolled a 6 and moved to the space shown in the figure.



a. What space was Martina's piece on before she rolled the dice?
Answer: On space …………

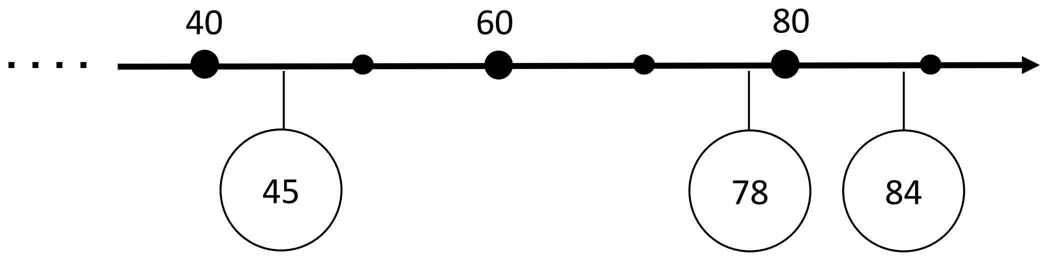
b. Christian's piece is shaped like a triangle and before he moved was on space 15.
What number did Christian roll last?
Answer: He rolled …………

9) Amanda is preparing a box of beads for a friend.
She bought 4 hundreds, 2 tens and 23 units.

a. Choose the operation to be used to count how many beads Amanda bought:
   A. 4 + 23 + 2
   B. 400 + 23 + 2
   C. 400 +20 + 23

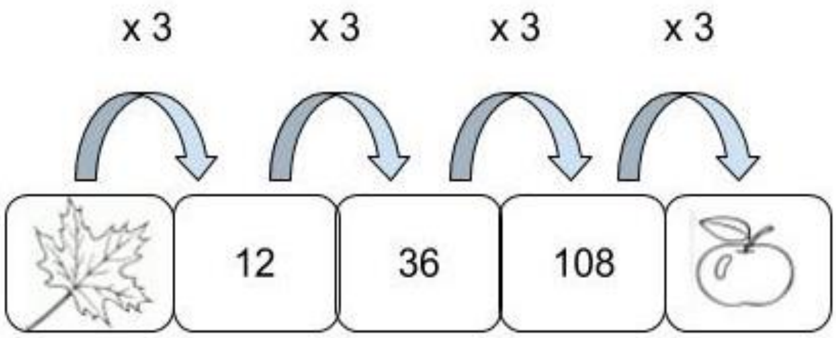
b. How many beads does Amanda have in all?
Answer: ………… beads

10) Look at the number line.



The number in one of the circles is wrong.
The wrong number is:
A. 45
B. 78
C. 84

11) Look at the sequence in the boxes and the operation indicated by the arrows.



The leaf and the apple cover two numbers.
a. What number is hidden behind the leaf?
Answer: …………

b. What number is hidden behind the apple?
Answer: …………

12) Frank’s birthday is February 22 and his brother Luke’s is 3 weeks earlier.

a. When is Luke’s birthday?
 A. February 1
 B. February 19
 C. January 31

b. Luke and Frank’s father celebrates his birthday on March 8.
Complete the sentence by writing a number on the dotted line:
The father’s birthday is exactly ………… weeks after Frank’s.

13) A t-shirt costs 8 euros and 70 cents.
Three friends have this much money:

Matt	
Mark	
Burt	

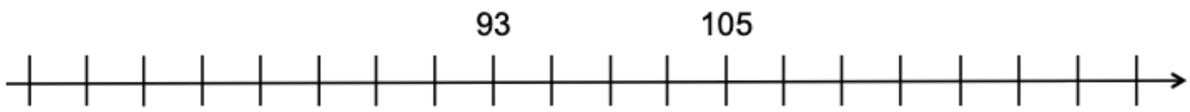
Who <u>can't</u> buy the t-shirt?

A. Matt
B. Burt
C. Mark

C2. Post-test on math competences

NAME …………………………………… **GOOD LUCK! ☺**

1) Look at the number line.

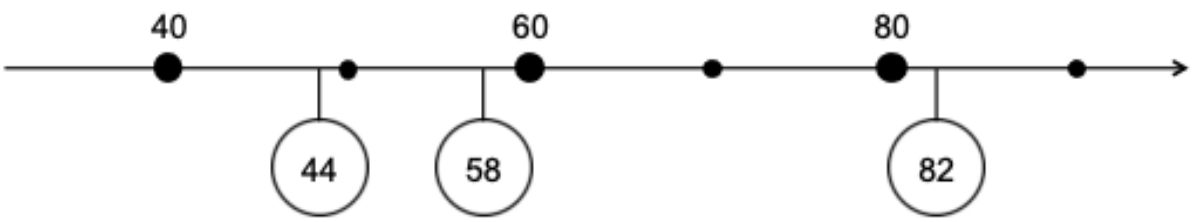


Write these numbers on the line: 90 and 99 and 114.

2) Think about the number 940.
a. What digit is in the tens place?
Answer: ……

b. How many tens make up the number 940?
Answer: …… tens

3) Look at the number line:

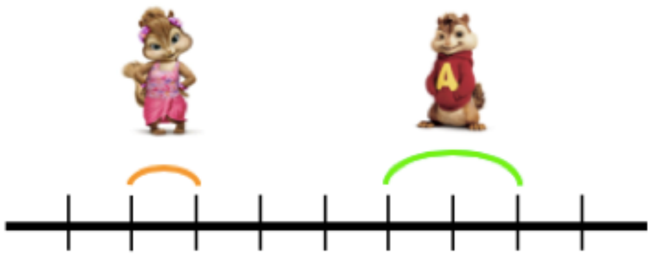


The number in one of the circles is wrong.
The wrong number is:
A. 44
B. 58
C. 82

4) Look at this figure:

What number can you put over the white circle?
Answer: …………

5) Chippie and Chip are racing to get an acorn.
Here is Chippie's step and Chip's step:



These are their positions at the beginning:

a. How many steps does Chippie have to take to arrive exactly at the acorn?
Answer: ………… steps

b. How many steps does Chip have to take?
Answer: ………… steps

6) Eliza has two bouquets:

BOUQUET A BOUQUET B

Eliza wants both bouquets to have the same number of flowers.
What does she have to do?
Complete the sentence:
Eliza moves ………… flowers from bouquet ………… to bouquet ………… .

7) Mr. Andrew, the teacher, prepares colored pencils for the class. He has 5 hundreds, 68 units and 3 tens.

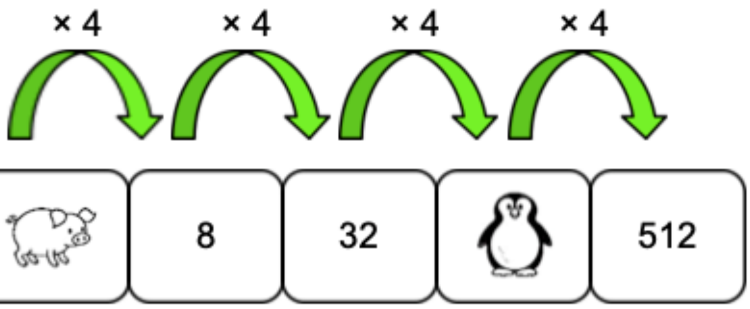
a. What operation does Mr. Andrew use to count how many pencils he has?

A. 50 + 3 + 68
B. 500 + 30 + 68
C. 68 + 3 + 500

b. Mr. Andrew takes only the red, blue and green pencils: he counts 120. He has 25 students in his class.
Can Mr. Andrew give 5 pencils of these colors to each student?

A. Yes, with 5 pencils left over.
B. No, he doesn't have enough pencils.
C. Yes, and he has no red, blue or green pencils left over.

8) Look at this picture:



a. What number is hidden behind the piglet?
Answer: …………

b. What number is hidden behind the penguin?
Answer: …………

9) A doll costs 7 euros and 90 cents.
Three friends have this much money:

Vittoria	
Lucrezia	
Sara	

Dipartimento ESt Economia e Statistica Cognetti de Martiis
DIPARTIMENTO DI MATEMATICA GIUSEPPE PEANO UNIVERSITÀ DI TORINO

Complete the sentence:
One of the three friends can’t buy the doll: it’s …………

10) Today the school cafeteria is serving pizza and French fries for lunch.
The cook made:

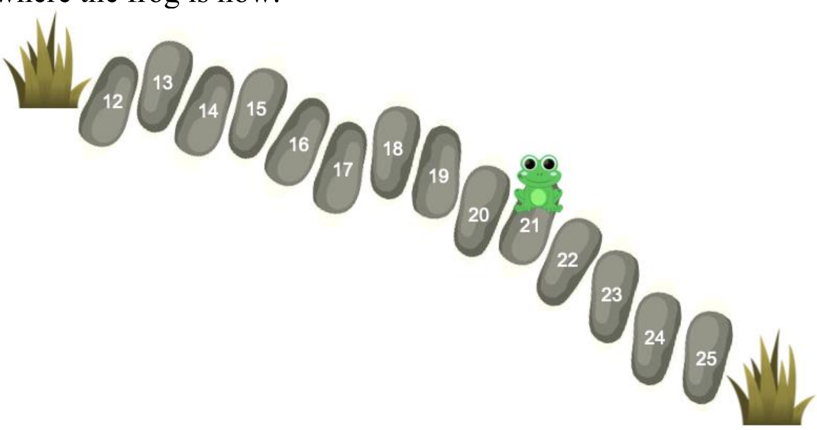
	1 pizza for every 7 children
	5 French fries for every child

There are 35 children in the cafeteria.

a. How many pizzas did the cook make?
- A. 12
- B. 5
- C. 7

b. How many French fries did the cook have to make?
- A. more than 170
- B. fewer than 150
- C. 165

11) A frog is hopping from stone to stone along a path.
Each stone is numbered as shown in the picture.
Look where the frog is now.



a. The frog hopped 7 times to get there.
What stone was she on before hopping 7 times?
Answer: she was on stone No. …………

b. Complete the sentence:
If the frog had been on stone No. 25, she would have had to hop ………… times to return to stone No. 13.

12) If you add 4 units and 2 tens to the number four hundred and thirty, you get:
A. 454
B. 472
C. 436

13) Julia’s birthday is January 29 and her friend Alexandra’s is exactly 1 week later.

Gennaio 2019

Lunedi	Martedì	Mercoledì	Giovedi	Venerdì	Sabato	Domenica
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

a. When is Alexandra’s birthday?
A. January 22
B. February 2
C. February 5

b. Alexandra’s sister celebrates her birthday exactly three weeks before Julia.
What day of the week did Alexandra’s sister’s birthday fall on in 2019?
A. Monday
B. Tuesday
C. Wednesday

C.3 Non-cognitive questionnaire

Name______________________________ Surname ____

1. Do you like math?
- ☐ not at all
- ☐ a little
- ☐ to some extent
- ☐ a lot

2. Are you good at math?
- ☐ not at all
- ☐ a little
- ☐ to some extent
- ☐ a lot

3. Are you worried to make a mistake when you do math?
- ☐ not at all
- ☐ a little
- ☐ to some extent
- ☐ a lot

4. Do you feel relaxed when doing math?
- ☐ not at all
- ☐ a little
- ☐ to some extent
- ☐ a lot

5. Are you worried not to finish the required tasks when you do math exercises in class?
- ☐ not at all
- ☐ a little
- ☐ to some extent
- ☐ a lot

Appendix D
Methodological guidelines of the activities for the teacher

D.1 Activity 1 - Thousandville: The City Gets Bigger

Lesson Plan (methodological guidelines for the teacher)

Thematic unit: Numeracy

Level: Primary school (3rd Grade)

Average time: 7 hours

Concepts
- Base-ten natural number system
- Writing natural numbers
- Place value in centesimal notation
- Comparing and ordering natural numbers
- Estimates and quantities

The lesson plan provides methodological guidelines for each stage of work.
The description of each stage is followed by the worksheet with the activities covered in it.

STAGE 0: Preliminaries and treasure hunt

Method: Group work

Time: A few minutes (around 10 minutes in all)

Materials needed:
- 120 bottle caps
- 500 drinking straws
- 100 small buttons, 50 medium buttons, 20 large buttons
- 1 container for each group of children

Classroom preparation
- Before starting the activity, the teacher hides piles of bottle caps, straws and buttons around the classroom. The teacher then divides the children into groups mixed by gender and aptitude level. The desks are arranged so that each group has a station, with a container for collecting the objects.

Description of activity
- The activity starts by reading the first part of the story:

Reesykle, the mayor of Thousandville, wants to make his city bigger. To do this, he has to make a model showing the plan for the new part of Thousandville. The model will be very large and will be made out of bottle caps, straws and buttons. Reesykle needs many helpers to make the model.

- Then proceed to the "treasure hunt": in 3 minutes, the children go around the classroom, collecting the required objects and putting them in the container assigned to their group. *Do not let the children count the collected objects* (if any of them try to count, the teacher should tell them not to).

STAGE 1: Narration, estimation and counting

Method: Group work, class discussion

Time: Around one hour (do not take too long over this stage)

Materials used:

- Objects collected in the container
- 2 additional containers for each group of children
- 3 colored cards (in three different colors) for each group

DESCRIPTION OF ACTIVITY

- The activity resumes with a reading of the second part of the **story** (in which Reesykle gives additional information to the children):

> *Reesykle has something else to tell us: "I'm such an airhead! I forgot to tell you how many objects we have to use! We need 100 bottle caps, 500 straws and 1000 buttons. Do you have everything we need for the model?"*
>
> *Are we sure we managed to collect all the material we need to make the model of Thousandville with Mayor Reesykle?*

- If necessary, the children should be told again *not to count the collected objects yet*. Before asking the children to count the objects on the desk, the teacher should ask the children a number of stimulus **questions** (it is not necessary to ask all of the questions listed below, which are provided only as suggestions):
  1. ***How can we figure out how many objects of each type we've collected?***
     OBJECTIVE: Give the groups two more containers and see how they divide the objects
     Some children will propose dividing the collected objects by type. At this point, the teacher will give them two more containers so that they can collect objects by type. The teacher should not suggest that the three materials be divided into the three boxes, but should wait for the children to do so themselves. If they do not, it will be necessary to lead them to this solution via class discussion.
  2. ***Which container do you think has the most objects in it?***
     OBJECTIVE: Quantity-dimension of the objects in the box […]
  3. ***Without counting them, how many straws do you think you've collected? How many bottle caps? And how many buttons?***
     OBJECTIVE: Rough estimates and concept of estimation […]
  4. ***What methods would you use to count the objects in a container quickly?***
     OBJECTIVE: Different counting strategies […]
  5. ***Now use your methods to count exactly how many objects of each kind you've collected.***
     OBJECTIVE: Counting […]

This stage involves working on the concepts of estimation and quantity. The first three questions are intended to stimulate the children's capacity to estimate/picture quantities. The last question provides the lead-in to the next stage, where the class identifies the best strategies for counting large quantities.

- After a suitable length of time, the students are asked to give the results of their count Each group will be given **3 cards** in different colors, one for each type of object. The children will write the number of objects they counted on each card (for example, as shown in the figure, the number of straws on the yellow card, the number of bottle caps on the red card, and the number of buttons on the green card).

Group A # straws	Group A # buttons	Group A # bottle caps

It is recommended that each group's cards be kept, so that the same situation as regards the materials can be replicated in the next session.

Guidelines for the class discussion:
The teacher:

- Starts the discussion by asking questions to draw attention to the differences between the students' choices, fostering balanced participation between girls and boys and children of different backgrounds;
- Asks more questions in order to discuss the choices;
- Reinforces good contributions by the children by means of approving looks, gestures, words, tone of voice and facial expression;
- Writes all of the answers on the blackboard or a poster;
- Encourages peer interaction and exchanges of views about different approaches, paying attention to the sensitive and multimodal aspects of understanding to promote the construction of mathematical meaning (e.g., by making use of sketches, turns of phrase introduced by the children, as well as their errors, silences, facial expressions and so forth);
- Shows willingness to listen to everyone in the class, aware of their diversity, without expressing judgements such as right/wrong, correct/incorrect;
- Stimulates the discussion to reach a consensus about the reasoning and strategies that can help answer the questions, paying particular attention to the strategies that are most effective in dealing with multiple-choice questions (for example, pointing out that not answering can be even more counterproductive than just guessing at an answer).

STAGE 2: Counting strategies

Method: Class discussion

Time: Around half an hour

Materials used:

- Objects collected in the containers
- Cards

The class discussion now focuses the children's attention on the number found by counting the objects in each container (written on the card) and on their initial estimates (written on the blackboard). […]

Figure 1. The teacher can write the estimates (shown in green in the photo) made by each group on the blackboard, and then the number which was counted (in white) in order to address the idea of estimation in the class discussion.

STAGE 3: Place value

Method: Group work, individual work

Time: Around 2 hours (approximately 1 hour for each worksheet, on separate occasions if necessary)

Materials needed:

- Worksheet 1A (group work)
- Worksheet 1B (individual work)
- Colored cards used in the previous stage

[…]

STAGE 4: Ordering on the number line

Method: Class discussion, group work

Time: Under two hours

Materials needed:

- String (3 pieces approximately 3 meters long each)
- Masking tape
- Colored cards marked with the numbers counted in the previous stage
- Flags to be placed on the target numbers
- Sufficient space (in the classroom, hall or gym)
- Worksheet 2

In this stage, the children must place their cards on lines marked on the floor. It is thus advisable to find a place that offers sufficient space for this activity. If the classroom is large enough, the desks can be moved to the sides and the three lines placed at a certain distance from them in the center of the room. Otherwise, the activity can be performed in the hall or gym. […]

STAGE 5: The value of 1000

Method: Class discussion, group work, individual work

Time: Around one hour

Materials:

- Worksheet 3 (group work)
- Worksheet 4 (individual work)
- Collected buttons
- New card or the new value of the buttons to be placed on the number line
- *Guidelines:* This stage is designed to make the children think about the difference between counting the number of objects and calculating their actual value (as is the case with coins, for example).

[…]

D.2 Activity 2 - Forest Elves

Lesson Plan (methodological guidelines for the teacher)

Thematic unit: Numeracy

Level: Primary school (3rd Grade)

Average time: 8 hours

Concepts:

- Number as measure
- Multiplicative reasoning
- Use of tables and the number line

The lesson plan provides methodological guidelines for each stage of work.
The description of each stage is followed by the worksheet with the activities covered in it.

STAGE 1: Narration and drawing

Method: Individual work, class discussion

Time: 1 ½ hours

Materials:
- Worksheet 1

DESCRIPTION OF ACTIVITY

- Hand out **worksheet 1** (individual).
- **Read** the worksheet (the worksheet should be read out loud, either by the teacher or a student)

> *Once upon a time, a family of forest elves lived in a house in the woods. The family was made up of Mummy Elf, Daddy Elf and their two children.*
>
> *It was autumn, time to start gathering provisions for the long cold winter ahead.*
>
> *The first to go out was Elf Girl. She left the house with her basket and went down the path. She took twenty steps towards the mountain and reached an apple tree. She filled her basket with apples and went back home.*
>
> *Then Elf Boy left the house, with his basket. He went down the path towards the mountains, took twenty steps and reached a chestnut tree. He gathered chestnuts until his basket was full and went home.*
>
> *A bit later, Mummy Elf came out of the house carrying an empty bucket. She went down the path towards the lake, took twenty steps and reached the pump. She filled*

- Before proceeding to individual work with the worksheet, it is advisable to ask the children to **repeat the content of the story** to make sure they have a firm grasp of its basic narrative (the elves walk along the path, and each takes 20 steps) and can thus picture it clearly to themselves. This will prevent them from representing steps that do not follow the path, which would prevent the exercise from reaching its goal.
  […]

Guidelines for the class discussion:
The teacher monitors the children's work, moving around the classroom to see what kinds of representation are being used and organize the class discussion. When all the students have finished (those who finish very early can color their drawings), the class discussion begins, directed by the teacher.
The teacher:

- Starts the discussion by asking questions to draw attention to the different possible choices (for example: *How did you draw the elves' routes? Where did you put the apple tree? And the chestnut tree?*), fostering balanced participation between girls and boys and children of different backgrounds;
- Asks more questions in order to discuss the choices;
- The end goal of the discussion is to reach a consensus about a representation that effectively captures the routes taken by the characters in the story and the points they reach. It can be helpful to draw this consensus representation on the blackboard.

Figure 1. On the blackboard, the teacher or one of the students can draw the different strategies used by the children to represent the routes, and the consensus representation chosen by the class at the end of the discussion.

STAGE 2: The length of the steps

Method: group work (groups mixed by gender and aptitude level), class discussion

Time: Around two hours

Materials:
- Reference worksheet
- Worksheet 2a
- Worksheet 2b

[…]

STAGE 3: New relationships and representations

Method: Group work (groups mixed by gender and aptitude level: the same as in the previous stage), class discussion, use of teaching aids

Time: Around two and a half hours

Materials:

- **Drinking straws** of different colors, cut into pieces whose length is proportional to that of the elves' steps, e.g.:
  - Four 12 cm pieces (Daddy Elf)
  - Six 8 cm pieces (Mummy Elf)
  - Eight 6 cm pieces (Elf Boy)
  - Twelve 4 cm pieces (Elf Girl)
- Worksheet 3a
- Worksheet 3b

DESCRIPTION OF ACTIVITY

- Hand out **worksheet 3a** (individual work).
  The worksheet requires the children to recognize a link between the steps taken by Mummy Elf and Daddy Elf, comparing different representations shown on the worksheet, which starts with a written description of this link. The teacher can direct a short class discussion of the children's answers, talking about the relationships shown by the different representations and then converging on the correct one.
- Afterwards, the teacher can draw a table on the blackboard showing how the number of steps taken by each character relates to the number taken by the others. In particular, the teacher can start by asking: "*If Daddy Elf reaches a place in 2 steps, how many steps will Mummy Elf have to take to reach the same place? And Elf Boy? Elf Girl?*"
- Hand out **worksheet 3b** (group work).
  For this activity, which is the most complex in the entire sequence:
  - Drinking straws cut into different lengths can be used to help represent the elves' steps during the group work.
  - It is also useful to employ concrete perceptual experiences (for example, reproducing the elves' steps by having two children and/or the teacher walk) to represent the paired relationships between the elves' steps that are to be compared on the worksheet.

Stage 4: Let's all go to Uncle and Aunt Elf's house!

Method: Class discussion

Time: Around 2 hours

Materials:
- Worksheet 4
- Large roll of graph paper
- Pictures of the characters and the points they reach

DESCRIPTION OF ACTIVITY

- Hand out **worksheet 4** "Let's all go to Uncle and Aunt Elf's house!", which contains a new piece of the story (may be read together)
- The teacher can give the children time to work individually on the worksheet, or proceed directly to producing a consensus representation using the roll of graph paper to make a poster.

Guidelines:

The poster shows only the elves' house, the path, the lake and the mountains (as on worksheet 1). The objective is to add all the other places reached by the elves: the apple tree, the chestnut tree, the pump, the market and Uncle and Aunt Elf's house. To do so, it is necessary to establish the length of the four elves' steps, knowing that (for example) Elf Girl's step is 1 square (1 cm) long. In this case, then, Elf Boy's step will be one and a half squares long, Mummy Elf's 2 squares long, and Daddy Elf's 3 squares long. Once the units have been found, the positions of the apple tree, the chestnut tree, the pump, the market and Uncle and Aunt Elf's house can be found. The poster can also be used to illustrate and understand the relationships between the different characters' steps, and the number of steps each character has to take to reach a given point on the map, as covered in the previous worksheets.

In this final stage, the class shares the discoveries made in following the story, and the children are normally highly involved in making the poster: they can color and paste on the characters and places mentioned in the story in the appropriate points.

Figure 2. An example of a poster made by a class, with the routes and the places mentioned in the story