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ABSTRACT

Low, High and Super Congestion of a Renewable Natural Resource under Autarky and Trade*

Numerous developing economies depend vitally on renewable natural-resource (*NR*)-based commodities. This study develops a general equilibrium model to examine the steady-state impact of changes in a small economy's NR congestion under open access and optimal regulation. This issue has often been examined under 'low' congestion (LC) – with $MC > AC$ and both upward sloping. Two more categories, 'high' (HC) and 'super' (SC) congestion – whose AC is backwardbending and $MC < 0$ – are identified, with regulation's impact under SC opposite to that under HC [e.g., a tax reduces (raises) price and raises (reduces) output under SC (HC)]. Findings include: i) Welfare and NR losses under open access are typically a multiple to one order (one to two orders) of magnitude greater for HC (SC) than for LC countries, with congestion determined by population (world price) level under autarky (trade); ii) Trade openness (and termsof- trade improvements) reduces an exporter's welfare and NR, and reduce both sectors' output under HC and SC, though it may prevent population growth to cause NR and society's collapse; iv) Welfare and NR open-access costs increase (decline) with population under autarky (trade); v) Though trading partners' shift from open access to optimal regulation is said to create a 'NR destruction haven' effect and reduce exporters' NR, the opposite is likely under SC; vi) Results are robust to various alternative functional forms and parameter values (e.g., low vs. middle-income countries' food expenditure shares). Policy implications are provided.

JEL Classification: D62, F18, Q22, Q27, Q56

Keywords: natural resource, low, high and super congestion, open access and optimal regulation, autarky and trade, society's collapse

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1. Introduction

Many developing countries obtain a share of their income from open-access renewable natural resources (*NR*) like fisheries, forests, arable land, grazing grounds, and water resources. Imperfect property rights for the *NR* results in negative externalities,[1] excessive use of variable inputs (e.g., labor), and *NR* depletion, at times dramatically so – e.g., the North American bison near extinction due to open access in the US combined with a tanning innovation in Europe (Taylor 2011), the population-growth-related massive deforestation in the Philippines (Bee 1987), and many others.

The problem has affected many developing countries and has led to a decline or collapse of some communities – due to rapid population growth, access to a wider market, and other – and/or to mass migration. For instance, Brander and Taylor (1998) shed light on the causes of Easter Island society's collapse. They show that open access to its forests eventually led to their disappearance and to a dramatic decline in living standards and population.

The classic case of *NR* depletion is fisheries, and early analyses focused on open-access and optimal regulation of the sector (Gordon 1954, Scott 1955). Some recent studies have extended the analysis, using general equilibrium models to examine the steady-state and transition paths of economies with open-access *NR* (e.g., Brander and Taylor 1997, 1998; López and Schiff 2013). This study develops a simple general equilibrium model, focusing on steady-state outcomes.

<u>Congestion categories</u>

The issue of production of renewable *NR*-based commodities and *NR* depletion in the case of open access has often been examined under 'low' congestion (LC) where marginal cost is above average

[1] For instance, López (1997, 1998) finds negative *NR* externalities from use of village-level open-access lands in Ghana and Côte d'Ivoire, with an internalized share that declines with village size.

cost, i.e., $MC > AC$, and both are upward sloping. This is depicted in Figure 1. I identify two additional congestion categories (also shown in Figure 1), 'high' (HC) and 'super' (SC) congestion whose AC is backward bending and $MC < 0$. The HC (SC) category prevails in the shorter (longer) section of the backward-bending AC curve which is located to the South-East (North-West) of the MC curve. HC and SC are the categories where the impact of congestion is the most severe and thus of greatest concern to those affected by it, and they should therefore be of special interest to policymakers.

Analyses have frequently assumed that the *NR* congestion externality and *NR* depletion are a function of the commodity's output. However, a given output can be produced with different combinations of inputs, e.g., a small *NR* stock and large labor force or, vice versa, a large *NR* and small labor force. Pressure on *NR* is clearly greater in the former case, as is the externality and *NR* depletion. Moreover, these two input combinations can only generate the same output if HC or SC prevails in the former case and LC in the latter, as shown in Appendix Section 3.1.1.[2]

The study presents, for both autarky and trade, *i*) solutions for welfare, *NR*, sectoral employment, output and price, under both open access and optimal regulation;[3] *ii*) a new graphical – and more intuitive – analysis and demonstration of the results; *iii*) quantitative measures of the steady-state impact of open access relative to optimal regulation for each congestion category. It also *iv*) reexamines some results in the literature, including the impact on exporters in the South of the North's optimal *NR* regulation; and *v*) examines the impact of trade on *NR* under population-induced increasing pressure on *NR* and risk of collapse.

[2] This paper extends to trade Schiff's (2020) analysis where a comparison is also provided of Chile and the Philippines' (de facto) open-access farm-fishing industry on the one hand, and of Norway's highly regulated one on the other.
[3] Provision of private property rights to the *NR*, if feasible, is assumed to be equivalent to its optimal regulation.

The remainder of the paper is organized as follows. Section 2 develops a two-sector general equilibrium model of an economy under autarky. Section 3 solves the model under both an open-access *NR* and an optimally regulated one, derives the welfare and *NR* costs of open access – and the employment, output and price impact – for countries with different population levels. Section 4 does the same in the case of openness to trade, examines the impact of regulatory reform in the North on exporters in the South and trade's impact on a society's potential collapse. Section 5 examines the robustness of the results. Section 6 draws policy implications and Section 7 concludes.

2. Model

Section 2.1 presents the general equilibrium model's supply side and Section 2.2 the demand side. Section 2.3 presents the open-access and optimal solutions. Population (or, equivalently, labor force) varies across countries and is assumed to be exogenous. Reasons for population change that are unrelated to *NR* or income are provided in Diamond (2011).[4]

2.1. Supply

Assume an economy producing two goods under perfect competition, a manufacturing one, M, and a commodity, Q. The labor endowment is denoted by $\mathbb{L}$, and employment in sector Q (M) is denoted by $L(l)$, with $L + l = \mathbb{L}$. Interior solutions are assumed unless otherwise specified.

Following Brander and Taylor (1998), M is produced with l under a constant-returns-to-scale technology, with units chosen such that marginal product $MP_l = 1$. Thus, $M = l = \mathbb{L} - L$. Good

[4] Diamond (2011, Ch. 10) provides various reasons for population increase not (directly) related to *NR* or income when discussing the rapid growth of Africa's population in the final decades of the 20th century, including preventive medicine, improved hygiene, vaccination, antibiotics, greater control of malaria and other endemic African diseases, political consolidation, and other.

M is chosen as the numéraire, with its price normalized to one. Thus, labor's wage rate is $w = VMP_l = 1$ for $M > 0$, which holds with a Cobb-Douglas utility function under autarky.

The steady-state *NR* level, N, declines with employment L. Assume $N = \alpha - \beta L$ $(\alpha, \beta > 0)$, where α is the *NR* endowment – defined as the *NR* level when it is unexploited ($L = 0$), and $\partial N/\partial L = -\beta < 0$ is labor's (marginal) negative externality. Production functions for Q and M are:

$$Q = LN = L(\alpha - \beta L),\ \ M = l = \mathbb{L} - L;\ \alpha, \beta > 0;\ \ 0 < L < \alpha/\beta, \alpha > 3\underline{N}\,. \tag{1}$$

$L < \alpha/\beta \Leftrightarrow N = \alpha - \beta L > 0$ ensures an interior solution. The set of values, $N < \underline{N}$ constitutes a low-*NR* trap – in the sense that $N > \underline{N}$ is not feasible – and which may result in society's collapse. The *NR* endowment is assumed to be over three times the low $\underline{N}$ level.

2.1.1. <u>Average and marginal product and cost</u>

Labor's average product $AP_L = \frac{Q}{L} = \alpha - \beta L$. With $w = 1$, average cost $AC = \frac{w}{AP_L} = \frac{1}{\alpha - \beta L}$. Labor's marginal product $MP_L = \alpha - 2\beta L \gtrless 0 \Leftrightarrow L \lessgtr \hat{L} = \frac{\alpha}{2\beta}$, and marginal cost $MC = \frac{1}{MP_L} = \frac{1}{\alpha - 2\beta L} >$ $(<)$ 0 on the upward-sloping (backward-bending) part of the AC curve where low (high and super) congestion – or LC (HC and SC) – prevails (see Figure 1).[5]

Note that $MP_L \gtrless 0$ for $L \lessgtr \hat{L}$ implies that MC converges to ∞ $(-\infty)$ as L approaches $\hat{L}$ and Q approaches $\hat{Q}$ (Q_{MAX} in Fig. 1) from $L < \hat{L}$ $(L > \hat{L})$, i.e., $\lim\limits_{L^- \to \hat{L}} MC = \infty$ and $\lim\limits_{L^+ \to \hat{L}} MC = -\infty$.

[5] There may be other variable inputs (e.g., pens or cages). For simplicity's sake and following Brander and Taylor (1998), I abstract from non-labor costs.

2.1.2. Inflection point and negative MC

Denoting the upward-sloping (backward-bending) segment of the AC curve by AC_1 (AC_2), the related MC curve by MC_1 (MC_2), and Q (L) on AC_2 that separates HC and SC by Q_I (L_I). The following results are derived in Appendix 1:

i) $L_I = \frac{2\alpha}{3\beta} > \hat{L}$, $Q_I = \frac{2\alpha^2}{9\beta} < \hat{Q}$;
ii) Output Q_I is also the inflection point on AC_2, i.e., where $AC'' \equiv \frac{\partial^2 AC}{\partial Q^2} = 0$;
iii) Q_I is also the intersection point of AC_2 and MC_1 (associated with AC_1); and
iv) $MC_2 < 0$ (associated with AC_2) is a mirror image of $MC_1 > 0$ (as shown Figure 1). [6] [7]

MC_2 is negative because an increase in output is obtained by reducing employment. Population (or labor force) $\mathbb{L}$ varies by country and is assumed to be a function of exogenous factors in this paper. This issue is briefly discussed in Section 4.2.

The distinction between HC and SC is important. Denote optimal (open-access) output by Q^* (Q). We have $Q^* > (<)\ Q$ under SC (HC), with opposite implications for optimal regulation's impact in both the South and the North (see Sections 3.3 and 4.2). Note that HC and SC share the fact that $MP_L < 0$ (with $MP_L > 0$ under LC), and HC and LC share the fact that $Q^* < Q$ (with $Q^* > Q$ under SC).[8]

[6] The range of values for L is the smallest under HC, is larger under SC and largest under LC (see Appendix 1.3). Assuming random drawings of L, the likelihood of a SC (HC) (LC) drawing is 1/3 (1/6) (1/2).
[7] A fishery's backward-bending supply curve was examined in early studies, most with problematic analysis. Copes' (1970) seminal paper includes a backward-bending AC curve. He focused on demand shocks' impact on equilibrium stability. His results depend on a less elastic demand curve than the AC_2 curve, an assumption that need not hold in general. In fact, the opposite obtains in this model (see Appendix 2). Clark (1990) refers to a discounted supply curve that might be backward bending for an optimally managed fishery. However, that cannot be optimal because labor's marginal product must be positive at the optimum ($MP_L > 0$, and so $MC > 0$), i.e., the optimum must be on the upward-sloping segment AC_1 of the supply curve. In the case of road congestion, Else (1981) shows a backward-bending positive MC segment, even though MC is upward-sloping in its positive segment.

[8] $Q^* < Q$ under HC because, though AC is backward bending, it is located to the right of MC (see Figure 1), with optimal output smaller than the equilibrium one. The opposite holds under SC where AC is to the left of MC.

2.2. Demand

Individuals are assumed to have Cobb-Douglas preferences, namely:

$$U = q^{\gamma} m^{1-\gamma}, m = M/\mathbb{L}, q = Q/\mathbb{L}, 0 < \gamma < 1. \tag{2}$$

3. Autarky

The autarky model is relevant for small and micro states that are isolated and whose trade costs are prohibitively high due to the low volume of trade and large distance to the world markets, for comparison with the case of international trade, and for analysis of small economies that for economic, cultural or other reasons did not trade historically with the rest of the world, some of which suffered a massive decline as population growth put excessive pressure on the *NR* base (as shown in Brander and Taylor's (1998) insightful study of Easter Island). The issue of trade and collapse of some of these societies is examined in Section 4.3.

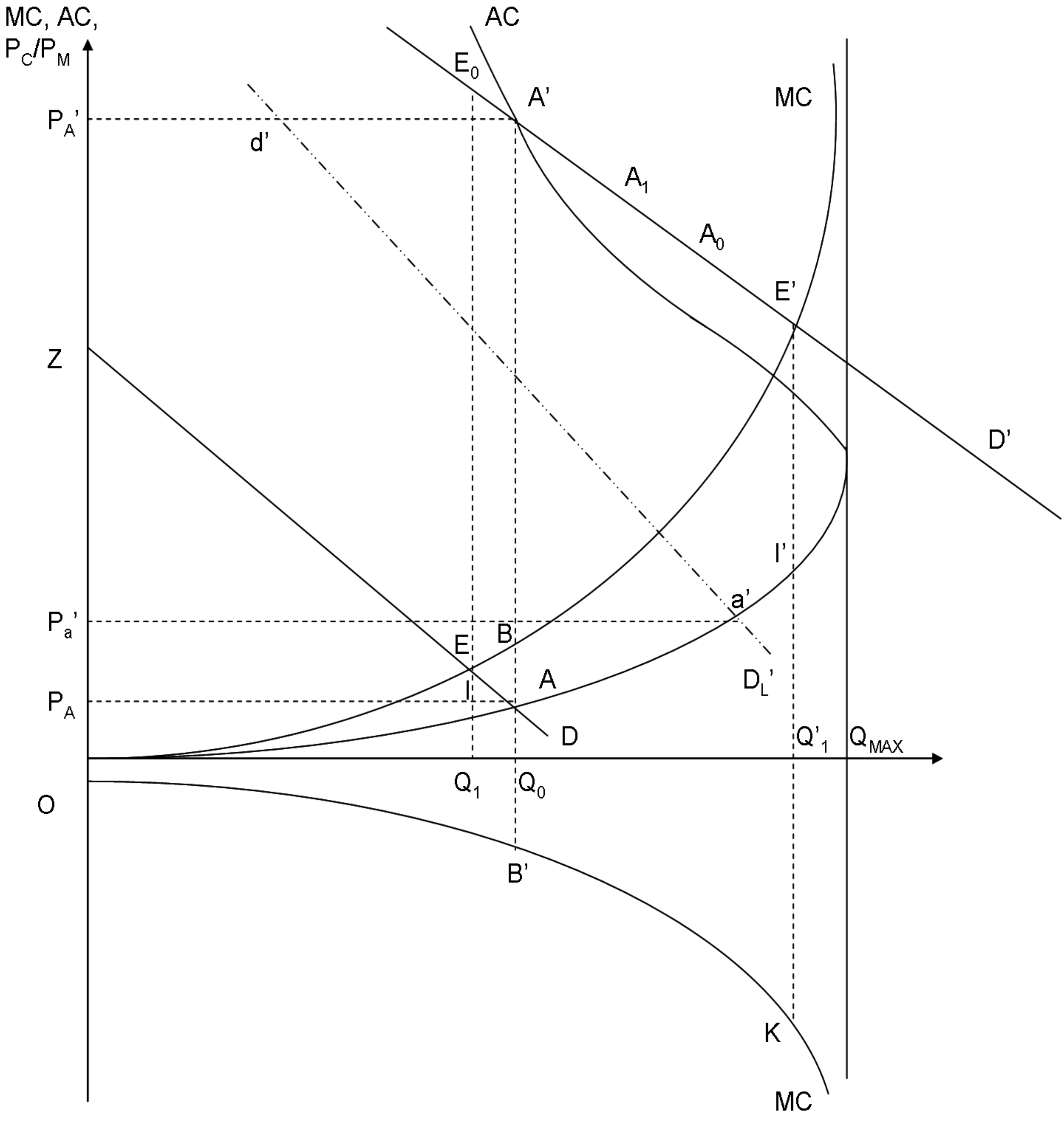
Section 3.1 presents a graphical analysis of the autarky case, Section 3.2 provides the model's solution, and Section 3.3 presents the simulations.

3.1. Graphical analysis

For clarity of exposition, demand curves are shown as straight lines in Figure l. Denote by AC_1 (AC_2) the upward-sloping (backward-bending) segment of the AC curve. Assume first that the demand curve is given by D on AC_1. Equilibrium is at point A, price $p = AC = P_A$, and output is $Q = Q_0$. The optimum is at point E where $p = MC =$ distance EQ_1 and output $Q = Q_1$. The welfare cost under open access is $\Delta W_{LC} = W_{LC}^* - W_{LC} = AEB$.

Assume now a country, C1, whose demand is given by D' (due to, say, a larger population or greater taste for Q; see Section 5.1). Open-access equilibrium is at A', on the SC segment of AC_2

Figure 1: Low, High and Super Congestion



where AC and D' intersect. For simplicity, assume D' is such that output is also Q_0 at A'. Thus, Q_0 – and any other output level – can be produced with a low (high) level of labor (*NR*), as in point A, or a high (low) level of labor (*NR*), as in point A'. Optimum is at E' where D' and MC intersect.

There are three ways to obtain the welfare cost, ΔW_{SC}, of open access:

1. *Higher cost (as measured by AC) and lower consumption*: The difference in the cost of producing Q_0 under demand D and D' is $P_A'A'AP_A = AA' * Q_0$. Moreover, the increase in output from Q_0 to Q_1' generates a welfare gain $A'E'B$. Hence, the welfare cost of open access is $\Delta W_{SC} = P_A'A'AP_A + A'E'B$.

2. *Zero producer surplus*: Under open access, the producer surplus is *nil* because $p = AC$. Thus, welfare is equal to the consumer surplus in the absence of intervention, and is equal to the consumer surplus plus the tax revenue under an optimal tax. At A', the consumer surplus is the area between the demand curve, the $P_A'A'$ line, the *y*-axis, and. At E', AC is given by point I', and ΔW_{SC} is the area $P_A'A'$, $A'E'$ on the demand curve, $E'I'$, and $P_I'I'$, the horizontal line at the I' level (P_I' is *not* shown), i.e., $P_A'A'E'I'P_I'$, or the sum of the consumer surplus and the optimal tax revenue $T^* = E'I' * Q_1'$.

3. *Higher cost (as measured by MC) and lower consumption*: As shown in footnote 9, the welfare cost is also given by $\Delta W_{SC} = A'B'KE' + E'\infty(-\infty)K$, where the second term is the area between the positive and negative segments of the MC curve for $Q_1' \leq Q \leq Q_{MAX}$.[9]

[9] Consumption is Q_0 rather than Q_1', with a loss $A'Q_0Q_1'E'$. The decline from Q_1' to Q_0 implies a higher cost, which consists of i) the cost of the output increase from Q_1' to Q_{MAX}, or the area below the MC curve, $E'Q_1'Q_{MAX}\infty$; ii) the cost of the decrease in output from Q_{MAX} to Q_1' (entailing an increase in L) on AC_2, i.e., $KQ_1'Q_{MAX}$(-∞); and iii) the cost of the decrease in output from Q_1' to Q_0, i.e., $B'Q_0Q_1'K$. Thus, the welfare cost under open access is $A'B'KE' + E'\infty(-\infty)K$. As the negative segment of the MC curve is the mirror image of its positive segment (see Appendix 1.2), the welfare cost is also $\Delta W_{SC} = A'B'KE' + 2(E'Q_1'Q_{MAX}\infty)$.

3.2. Solution

This section provides the solution to the model for both an unregulated or open-access *NR* and an optimally regulated one in country C1.

3.2.1. Open Access

Utility maximization implies the commodity's relative demand price, p_d, equals the ratio of marginal utilities. From (2), we have: $p_d = \frac{U_q}{U_m} = \frac{\gamma m}{(1-\gamma)q} = \frac{\gamma(\mathbb{L}-L)}{(1-\gamma)L(\alpha-\beta L)}$. The supply price is $p_s = AC = \frac{w}{AP_L}$. As $w = 1$, $p_s = AC = \frac{1}{\alpha-\beta L}$, and $p = p_d = p_s$ implies $L = \gamma\mathbb{L}$, $p = 1/(\alpha - \beta\gamma\mathbb{L})$, with $m = (1-\gamma)$, $q = \gamma(\alpha - \beta\gamma\mathbb{L}) = \gamma/p$, and $U = (\gamma/p)^\gamma(1-\gamma)^{1-\gamma}$.

Households in low-income and lower middle-income countries spend a large share of their budget on food. For instance, the share is between 40 and 50 percent in countries such as Myanmar, Kenya and Nigeria, and between 50 and 60 percent in countries such as Algeria, Azerbaijan, Cameroon, Guatemala, Pakistan and the Philippines. Thus, assume for simplicity that $\gamma = 1/2$, the value used in the first set of simulations.

Section 5.1 examines in detail the case of upper middle-income countries ($\gamma = 1/4$) and shows that the results on open-access welfare and *NR* costs for $\gamma = 1/2$ also hold for $\gamma = 1/4$. The case of $\gamma = 3/4$ has also been examined, with similar results.[10] The solution for $\gamma = \frac{1}{2}$ is:

$$L = M = \frac{\mathbb{L}}{2},\ m = \frac{1}{2},\ N = \alpha - \beta\frac{\mathbb{L}}{2},\ q = \frac{1}{2}\left(\alpha - \beta\frac{\mathbb{L}}{2}\right),\ U = \frac{1}{2}\left(\alpha - \beta\frac{\mathbb{L}}{2}\right)^{1/2} = \frac{1}{2p^{1/2}}. \qquad (3)$$

[10] These are available from the author upon request.

Welfare and *NR* increase with endowment α and decrease with externality β, population $\mathbb{L}$, and thus with price p. Denote the level of population where $\underline{N}$ is reached by $\overline{\mathbb{L}}$. From (3), $\mathbb{L} > \overline{\mathbb{L}} = \frac{2}{\beta}\left(\alpha - \underline{N}\right) \Leftrightarrow N < \underline{N}$, in which case C1 is caught in the low-*NR* trap.

By reducing $\mathbb{L}$ (before it reaches $\mathbb{L} > \overline{\mathbb{L}}$), emigration raises welfare and *NR*. The welfare and *NR* increases are especially large in the case where a SC equilibrium changes to a LC one, as with D_L' in Figure 1, where equilibrium moves from A' to a' and residents' welfare gain is $P_A' d' a' P_a'$.

3.2.2. Optimal Regulation

Under optimal regulation, $p_s = MC$, and $p_d = p_s$ imply $\frac{\mathbb{L}-L}{L(\alpha-\beta L)} = \frac{1}{\alpha-2\beta L}$, or $3\beta L^2 - 2(\alpha+\beta\mathbb{L})L + \alpha\mathbb{L} = 0$. With $m^* = 1 - \frac{L^*}{\mathbb{L}}$ and $N^* = \alpha - \beta L^*$, the solutions for L^*, N^* and U^* are:

$$L^* = \frac{1}{3\beta}\left(\alpha + \beta\mathbb{L} - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\right) < L, N^* = \frac{1}{3}\left(2\alpha - \beta\mathbb{L} + \sqrt{.}\right) > N,$$

$$U^* = \frac{1}{3\beta\mathbb{L}}\left[\alpha\beta\mathbb{L}(\alpha+\beta\mathbb{L}) - \frac{2}{3}(\alpha^3 + \beta^3\mathbb{L}^3) + \frac{2}{3}(\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L})^{3/2}\right]^{1/2} > U. \qquad (4)$$

The values of $\mathbb{L}$ for which $N^* < \underline{N}$ are given by $\mathbb{L} > \overline{\mathbb{L}} = \frac{(\alpha-\underline{N})(\alpha-3\underline{N})}{\beta(\alpha-2\underline{N})} < \frac{(\alpha-2\underline{N})}{\beta}$.[11]

The following results are derived in Appendix 2.

i) The open-access equilibrium is stable and unique.[12]

ii) From equations (3) and (4), $L^* < L = \frac{\mathbb{L}}{2}$, and thus $N^* > N$.

iii) The second solution, $L^* = \frac{1}{3\beta}\left(\alpha + \beta\mathbb{L} + \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\right)$, is not an optimum.

[11] $\mathbb{L} > \overline{\mathbb{L}}$ is the solution of inequality $N^* = \alpha - \beta L^* = \frac{1}{3}\left(2\alpha - \beta\mathbb{L} + \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\right) < \underline{N}$, with $\overline{\mathbb{L}} > 0$ since $\alpha > 3\underline{N}$ (see equation (1)).

[12] This matters only for equilibria on AC_2, the backward-bending part of the AC curve. Since $p = AC = 1/(\alpha - \beta L)$, it follows that $pQ = L$. Moving up the AC_2 curve, L increases, and so does $pQ = L$. Since Q declines on AC_2 as L increases, the proportional increase in p must be greater than the decline in Q, or $|d\log(Q)/d\log(p)| = d\log(Q)/d\log(AC)| < 1$, so the AC (or supply) curve is inelastic on its AC_2 segment. And as demand elasticity $|\eta| = 1$, excess demand prevails below the equilibrium and excess supply above it. Thus, the equilibrium is unique and stable. A formal solution is in Appendix 2.

3.3. Simulation

This section examines the relationship between $\Delta x \equiv \frac{x-x^*}{x^*}$ $(x = L, N, Q, U)$ and population $\mathbb{L}$. The values for α and β in the 'base case' are $\alpha = 10$ and $\beta = 1$, with $N = \alpha - \beta L = 10 - L = 10 - \frac{\mathbb{L}}{2} > 0$, with $\mathbb{L} < 20$. Subscripts used for all variables below refer to the level of the labor force, $\mathbb{L}$. Table 1 in Section A shows the Δx results for several 'individual' values of $1 \leq \mathbb{L} \leq 19$, and Table 2 in Section B does the same for central values of $\mathbb{L}$ in each congestion category.

Table 1. Open Access vs. Optimum under Low, High and Super Congestion

	Open Access (x)				Optimum (x^*)				Difference $\Delta x = \frac{x-x^*}{x^*}$ (%)			
$\mathbb{L}$	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	9.5	4.75	1.541	.49	9.51	4.63	1.542	2.7	-.11	1.06	-.065
4	2.0	8.0	16.0	1.414	1.8	8.2	14.5	1.424	15	-2.4	10.3	-.707
9	4.5	5.5	24.8	1.173	3.2	6.8	21.6	1.248	41	-19	14.6	-6.34
11	5.5	4.5	24.8	1.061	3.5	6.5	22.8	1.226	58	-31	9.0	-13.5
16	8.0	2.0	16.0	.7071	4.0	6.0	24.0	1.061	100	-67	-33	-33.3
19	9.5	.50	4.75	.3536	4.2	5.8	24.4	.991	127	-91	-81	-64.3

A. Individual $\mathbb{L}$ values

i) Welfare

Table 1 shows (in percent) for LC: $\Delta U_1 = -.065$; $\Delta U_4 = -.707$, and $\Delta U_9 = -6.34$; for HC: $\Delta U_{11} = -13.5$; for SC: $\Delta U_{16} = -33.3$, and $\Delta U_{19} = -64.3$. Thus, ΔU_{19} (ΔU_{16}) = 989 (513)ΔU_1, 90.9 (47.1)ΔU_4, 10.1 (5.3)ΔU_9, and 4.8 (2.5)ΔU_{11}. And $\Delta U_{11} = 207\Delta U_1$, 19.1$\Delta U_4$ and 2.1ΔU_9. Thus, the open-access welfare cost under SC (HC) is one order (a multiple) to two orders of magnitude greater than under LC. And the welfare cost under SC is a multiple of that under HC.

ii) Natural Resource

Under SC, i.e., for $\mathbb{L}$ =19 (16), $\Delta N_{19}(\Delta N_{16})$ is -91 (-67) percent or 827(608)ΔN_1 (LC), 38 (28) ΔN_4 (LC), 4.8 (3.5)ΔN_9 (LC), and 2.9 (2.2)ΔN_{11} (HC). For HC ($\mathbb{L} = 11$), $\Delta N_{11} = 282\Delta N_1$, $13\Delta N_4$ and $2.1\Delta N_9$. Thus, the *NR* depletion under SC and HC is between a multiple and two orders of magnitude greater than under LC. And the depletion under SC is 2.2 to 3 times that under HC. Footnote 12 below presents the output and employment results.[13]

B. Central $\mathbb{L}$ values

Table 2 presents welfare and *NR* results related to the central value $\mathbb{L} = 5.0$ (11.67) (16.67) for LC (HC) (SC). The welfare cost (in percent) $\Delta U_{SC} = -38.3$ or $29.9\Delta U_{LC}$, and $\Delta U_{HC} = -12.7$ or $9.9\Delta U_{LC}$. The *NR* cost $\Delta N_{SC} = -72$ or $15.3\Delta N_{LC}$, and $\Delta N_{HC} = -34$ or $7.2\Delta N_{LC}$. Thus, the difference between the impact of open access under SC and LC at the average $\mathbb{L}$-value remains large. The loss is an order of magnitude larger under SC than under LC for welfare (30 times) and *NR* (15 times). Under HC, the loss for welfare (*NR*) is 10 (7) times that under LC.

Table 2. Open Access vs. Optimum: Central $\mathbb{L}$ Values [a]

	Open Access (x)		Optimum (x^*)		Difference (%) $\Delta x = \frac{x-x^*}{x^*}$		Ratio $\Delta x/\Delta x_{LC}$	
$\mathbb{L}$	N	U	N^*	U^*	ΔN	ΔU	$\Delta N/\Delta N_{LC}$	$\Delta U/\Delta U_{LC}$
LC: 5.0	7.5	1.369	7.9	1.387	-4.7	-1.28	1	1
HC: 11.67	4.2	1.021	6.4	1.169	-34	-12.7	7.2	9.9
SC: 16.67	1.7	.6455	5.9	1.046	-72	-38.3	15.3	29.9

The results obtained in the case of autarky are collected in the following proposition.

[13] Optimal output is higher (lower) than open-access output under SC (LC and HC). In percent, ΔQ is 1.06 for $\mathbb{L} = 1$, 14.6 for $\mathbb{L} = 9$ (LC), 9.0 for $\mathbb{L} = 11$ (HC), and -81 for $\mathbb{L} = 19$ (SC). Employment $\Delta L_{19}(\Delta L_{16})(\Delta L_{11}) = 127$ (100) (58) = 47 (37) (21)ΔL_1, 8.5 (6.7) (3.9)ΔL_4, and 3.1 (2.4) (1.4)ΔL_9. Moreover, note that as $\mathbb{L}$ increases from 1 to 19, the decline in $U(N)$ is 3.3 (2.4) times the decline in $U^*(N^*)$. Thus, as $\mathbb{L}$ increases, optimal regulation – e.g., the increase in the optimal tax – dampens the decrease in welfare and *NR* relative to their decline under open access.

Proposition 1: Assume a developing country (South) that produces a commodity with labor L and a renewable natural resource *NR*, and a manufacturing good with L. Three congestion levels are identified: 'low' congestion (LC) where $MC > AC$ and both are upward sloping, and 'high' (HC) and 'super' (SC) congestion, whose AC is backward-bending and $MC < 0$. The main findings are: *i)* Open-access welfare is inversely related to the autarky price p, with $U = 1/2p^{1/2}$; *ii*) Relative to the optimum, *NR* and welfare losses under open access are generally a multiple to one order (one to two orders) of magnitude greater for HC (SC) than for LC countries, with average losses under HC (SC) a multiple of (an order of magnitude greater than) under LC; *iii*) Results under autarky are robust (see Section 5) to several alternative functional forms and parameter values (e.g., for both low and middle-income countries' food expenditure shares); *iv*) Both the open-access and optimal *NR* and welfare levels decrease with population, and the *NR* and welfare losses under open access increase with population; *v*) Optimal regulation – e.g., a tax – raises (reduces) output and reduces (raises) price under SC (HC and LC), one of the two main reasons for the larger gains under SC, the other being the greater reduction in cost.

4. International Trade

This section examines the case of a small open economy, Country 1 or C1, facing an exogenous world price, p_w. Section 4.1 presents the solution for welfare, *NR*, sectoral employment and output, under open access and optimal regulation, as well as simulations of the open-access welfare and *NR* costs. Section 4.2 examines the impact on C1 of a reform in the *NR* regulatory regime of the rest of the world, and obtains some standard results and some that differ from those in the literature. Section 4.3 examines the impact of trade on the likelihood of society's collapse.

4.1. Solution

4.1.1. Open Access

Assume C1's autarky price $p < p_w$, so that it exports commodity Q when it opens to trade. For instance, in Figure 1, assume $p_w = P_a'$ and $p = P_A$. Open-access producer surplus is nil since $AC = p_w$, so welfare equals the consumer surplus, which is smaller at P_a' than at P_A. The welfare loss is equal to the area between the demand curve D and the horizontal lines $P_a'a'$ and P_AA.

Denote supply (demand) (export) (import) by superscript $s(d)(X)(I)$ and trade by subscript T. For an exporter, $p_w = AC_T = \frac{1}{\alpha - \beta L_T} > p = \frac{1}{\alpha - \beta L}$, i.e., $L_T > L$, $M_T^s = \mathbb{L} - L_T < M = \mathbb{L} - L$, and $N_T = \alpha - \beta L_T = \frac{1}{p_w} < N = \alpha - \beta L = \frac{1}{p}$. As $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right)$, we have $\frac{\partial L_T}{\partial p_w} = \frac{1}{\beta p_w^2} > 0$, $\frac{\partial M_T}{\partial p_w} = \frac{-1}{\beta p_w^2}$, and $\frac{\partial N_T}{\partial p_w} = -\frac{1}{p_w^2} < 0$.

Note that $L_T > L$ implies $Q_T^s > (<)\ Q$ for LC (HC and SC) countries, which implies that output declines in *both* sectors in the case of HC and SC. Thus, the economy contracts in steady state when trade is liberalized in high- or super-congestion exporting countries.

As each individual owns one unit of labor and $w = 1$, open-access aggregate income $Y = \mathbb{L}$. Individual income $y = 1$, with budget constraint $y = m^d + p_w q^d = 1$. From utility function (2) and $\gamma = .5$, $m^d = \frac{1}{2}$, $q^d = \frac{1}{2p_w}$, and utility $U_T = (q^d m^d)^{1/2} = \frac{1}{2{p_w}^{1/2}}$. Given that the autarky price $p < p_w$, we have $U = \frac{1}{2p^{1/2}} > U_T$.

Thus, opening up to trade reduces an open-access commodity exporter's welfare and *NR*. The same holds for an improvement in the country's terms of trade, p_w (with $\frac{\partial U_T}{\partial p_w} = -\frac{1}{4{p_w}^{3/2}} < 0$).

As $\frac{\partial L_T}{\partial \mathbb{L}} = 0$, we have $\frac{\partial Q_T^s}{\partial \mathbb{L}} = \frac{\partial N_T}{\partial \mathbb{L}} = \frac{\partial U_T}{\partial \mathbb{L}} = 0$, and $\frac{\partial M_T^s}{\partial \mathbb{L}} = 1$, i.e., the manufacturing sector fully absorbs any population increase. Thus, M_T^s increases with $\mathbb{L}$ while L_T, Q_T^s and the *NR* remain unchanged. Demand for M is $M_T^d = \frac{\mathbb{L}}{2}$, supply is $M_T^s = \mathbb{L} - L_T$, and imports $M_T^I = M_T^d - M_T^s = L_T - \frac{\mathbb{L}}{2} = L_T - L > 0$ (since open-access $L = \mathbb{L}/2$ under autarky).

Thus, $M_T^I > 0$ once C1 opens up to trade, and declines as population increases. At some point, $\mathbb{L}$ reaches $2L_T$, in which case $M_T^I = 0$. Then, as $\mathbb{L}$ increases further, the pattern of trade is reversed, with C1 importing Q and exporting M.

While population growth reduces *NR* under autarky, it has no impact on *NR* under trade. However, opening up to trade itself reduces *NR* since $L_T > L$. The *NR* solution $N_T = 1/p_w$. The level of p_w, denoted by p_w^c, at which $N_T = \underline{N}$ is $p_w^c = 1/\underline{N}$. Thus, any $p_w \geq p_w^c$ implies a low-*NR* trap.

4.1.2. Optimal Regulation

In this case, $p_w = MC = \frac{1}{\alpha - 2\beta L_T^*}$. Under open access $p_w = \frac{1}{\alpha - \beta L_T}$, which implies that $L_T^* = \frac{L_T}{2}$ and $Q_T^{*S} = \frac{L_T}{2}\left(\alpha - \beta \frac{L_T}{2}\right)$. As $N_T^* = \alpha - \beta \frac{L_T}{2} = N_T + \beta \frac{L_T}{2} > N_T$, a higher world price, $p_w^{*c} > p_w^c$, is needed to reduce N_T^* to the $\underline{N}$ level, i.e., $p_w^{*c} = 1/\underline{N}$, and $p_w > p_w^{*c}$ implies a low-NR trap. [14]

The results below are derived in Appendix 3.2. The commodity sector rent $R = Q_T^{*S}(MC^* - AC^*)$ $= \frac{\beta L_T^2 p_w}{4}$. Aggregate income is $Y^* = \mathbb{L} + \frac{\beta L_T^2 p_w}{4}$, and individual income $y^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}}$. From equation (2) and $\gamma = .5$, we have $m_T^{*d} = p_w q_T^{*d} = \frac{y^*}{2} = \frac{1}{2} + \frac{\beta L_T^2 p_w}{8\mathbb{L}}$. As $q_T^{*d} = \frac{m_T^{*d}}{p_w}$, it follows that welfare $U_T^* = \left(q_T^{*d} m_T^{*d}\right)^{1/2} = \frac{m_T^{*d}}{p_w^{1/2}} = \frac{1}{p_w^{1/2}}\left(\frac{1}{2} + \frac{\beta L_T^2 p_w}{8\mathbb{L}}\right)$. As $U_T = \frac{1}{2p_w^{1/2}}$, we have:

$$U_T^* = U_T + \frac{\beta L_T^2 {p_w}^{1/2}}{8\mathbb{L}},\ \frac{\partial U_T^*}{\partial p_w} \gtrless 0;\ \Delta U_T = \frac{U_T - U_T^*}{U_T^*} = -\frac{\beta L_T^2 p_w^{1/2}/8\mathbb{L}}{U_T^*} < 0,\ \frac{\partial \Delta U_T}{\partial p_w} < 0;$$

$$N_T^* = \alpha - \beta\frac{L_T}{2} > N_T,\ \frac{\partial N_T^*}{\partial p_w} = -\frac{1}{2p_w^2} < 0;\ \Delta N_T = -\frac{\beta L_T/2}{\alpha - \beta L_T/2} < 0,\ \frac{\partial \Delta N_T}{\partial p_w} < 0. \qquad (6)$$

[14] Note that since $Q_T^{*S} = \frac{L_T}{2}\left(\alpha - \beta\frac{L_T}{2}\right)$, the difference between optimal and open-access output is $Q_T^{*S} - Q_T^S = \frac{3\beta L_T}{4}\left(L_T - \frac{2\alpha}{3\beta}\right) \gtrless 0 \Leftrightarrow L_T \gtrless L_I = \frac{2\alpha}{3\beta}$. This confirms that optimal output is larger (smaller) than open-access output under SC (LC and HC). Also, $M_T^{*S} = \mathbb{L} - \frac{L_T}{2} = M_T^S + \frac{L_T}{2} > M_T^S$.

Thus, *NR* declines with p_w. The impact on welfare, U_T^*, is ambiguous, which makes sense a priori. Since trade is free and optimal regulation corrects for the congestion externalities, labor and *NR* are allocated efficiently and welfare U_T^* is maximized. Hence, U_T^* increases (decreases) with p_w in the case where C1 is a commodity exporter (importer), i.e., $Q_T^{*x} = Q_T^{*s} - Q_T^{*d} \gtrless 0 \Leftrightarrow \frac{\partial U_T^*}{\partial p_w} \gtrless 0$. Trade balance occurs at $\mathbb{L} = L_T + \beta L_T^2 p_w/4$. [15]

As $L_T^* = \frac{L_T}{2}$ and $\frac{\partial L_T}{\partial \mathbb{L}} = 0$, we have $\frac{\partial L_T^*}{\partial \mathbb{L}} = 0$, $\frac{\partial M_T^*}{\partial \mathbb{L}} = 1$, i.e., any population increase is fully absorbed by the manufacturing sector and has no impact on the commodity sector's total rent, though it reduces the *per capita* rent and thus reduces U_T^*, with $\frac{\partial U_T^*}{\partial \mathbb{L}} = -\frac{\beta L_T^2 p_w^{1/2}}{8\mathbb{L}^2} < 0$. And $\frac{\partial U_T}{\partial \mathbb{L}} = 0$ implies that ΔU_T declines with $\mathbb{L}$ (in absolute value), i.e., $\frac{\partial \Delta U_T}{\partial \mathbb{L}} = \partial\left(\frac{-\beta L_T^2 p_w^{1/2}}{8\mathbb{L}U_T^*}\right)/\partial \mathbb{L} = \frac{\beta L_T^2}{16\mathbb{L}^2 (U_T^*)^2} > 0$. And $\frac{\partial \Delta N_T}{\partial \mathbb{L}} = 0$ since neither N_T nor N_T^* are functions of $\mathbb{L}$.

Both welfare and *NR* costs of open access *increase* with $\mathbb{L}$ under autarky. On the other hand, under trade, the welfare cost *decreases* with $\mathbb{L}$ and the *NR* cost is *unrelated* to $\mathbb{L}$. This reduces the incentive to regulate the *NR* as $\mathbb{L}$ increases compared to the autarky case.

4.1.3. Simulation

This section presents the welfare and *NR* costs of open access for different world prices and population levels, with $\alpha = 10$ and $\beta = 1$ as before. The results are derived in Appendix 3.3. As shown in the previous section, changes in $\mathbb{L}$ have no impact on L_T, N_T or U_T.

1. *Assume* $p_w = \frac{1}{2}$. So, $L_T = 8$, $N_T = 2$, $y_T = 1$, $m_T^D = \frac{1}{2}$, $q_T^D = \frac{m_T^D}{p_w} = 1$, $U_T = \frac{1}{2p_w^{1/2}} = .7071$.

[15] $M_T^{*s} = \mathbb{L} - \frac{L_T}{2}$, $M_T^{*D} = \frac{\mathbb{L}}{2} + \frac{\beta L_T^2 p_w}{8}$, so $M_T^{*I} = \frac{L_T}{2} - \frac{\mathbb{L}}{2} + \frac{\beta L_T^2 p_w}{8} = 0$, which implies $\mathbb{L} = \mathbb{L}_R^* = L_T + \frac{\beta L_T^2 p_w}{4}$.

1.A. $\mathbb{L} = 10$: $y_T^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}} = 1.8$, $U_T^* = 1.2728$, and $\Delta U_T = -44$ percent.

1.B. $\mathbb{L} = 20$: $y_T^* = 1.4$, $m_T^{D*} = .7$, $q_T^{D*} = 1.4$, $U_T^* = .9899$, so $\Delta U_T = -29$ percent.

1.C. $\mathbb{L} = 30$: $y_T^* = 1.267$, $m_T^{D*} = .633$, $q_T^{D*} = 1.267$, $U_T^* = .8957$, so $\Delta U_T = -21$ percent.

2. *Assume* $p_w = \frac{1}{8}$. So, $L_T = 2$, $N_T = 8$, and $y_T = 1$, $m_T^D = \frac{1}{2}$, $q_T^D = \frac{m_T^D}{p_w} = 4$, $U_T = 1.4142$.

2.A. $\mathbb{L} = 10$: $y_T^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}} = 1 + \frac{4/8}{40} = 1.0125$, $U_T^* = 1.4319$, and $\Delta U_T = -1.2$ percent.

2.B. $\mathbb{L} = 20$: $y_T^* = 1.00625$, $U_T^* = 1.4151$, and $\Delta U_T = -.06$ percent.

2.C. $\mathbb{L} = 30$: $y_T^* = 1.00417$, $U_T^* = 1.4148$, and $\Delta U_T = -.04$ percent.

The results above indicate that the welfare cost of open access declines with population $\mathbb{L}$, from 44 to 21 percent for $p_w = \frac{1}{2}$, and from 1.2 to .04 percent for $p_w = \frac{1}{8}$. And as $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right)$, L_T and N_T are fully determined by p_w. Thus, with $\alpha = 10$ and $\beta = 1$, $p_w = \frac{1}{2}$ implies $L_T = 8 > L_I = \frac{2\alpha}{3\beta} = 6.67$, which is the level of L separating HC and SC, i.e., super congestion (SC) prevails. And $p_w = \frac{1}{8}$ implies $L_T = 2 < \hat{L} = \frac{\alpha}{2\beta} = 5$, the level of L separating LC and HC, so low congestion (LC) prevails. Note that, as was the case under autarky, the welfare cost under SC or high-p_w is one to two orders of magnitude greater than under LC or low-p_w case, while the *NR* cost under SC is a multiple of that under LC.[16]

Finally, note that output $Q_T^S = L_T N_T = 16$ in both cases, with output under LC obtained with one fourth the employment ($L_{LC} = 2, L_{SC} = 8$) and *NR* level four times that under SC ($N_{LC} = 8, N_{SC} = 2$). In other words, the open-access input combination under SC entails a huge waste of resources.

[16] $N_{SC} = 2$, $N_{SC}^* = 10 - \frac{L_{SC}}{2} = 6$, $\Delta N_{SC} = -66.7$ percent; $N_{LC} = 8$, $N_{LC}^* = 10 - \frac{L_{LC}}{2} = 9$, $\Delta N_{LC} = -11.1$ percent. Thus, $\Delta N_{SC} = 6\Delta N_{LC}$.

4.2. The ‘*NR*-Destruction Haven’ hypothesis

An ongoing debate in the literature is whether a reduction in trade barriers contributes to ‘pollution havens,’ i.e., whether liberalizing trade leads to a shift of polluting or dirty industries from countries with stringent environmental regulation to countries with lax regulation.[17]

This section examines a similar issue, namely how stricter *NR* regulation in North affects *NR* and welfare in open-access C1. Two variants are examined: 1) C1 and North have identical open-access autarky prices, so that no trade occurs between C1 and North before the latter changes its *NR* policy; and 2) C1 has a lower open-access autarky price than North, so that C1 and North trade before North's policy change takes place. Thus, trade follows the change in *NR* regulation in Case 1, while initial trade precedes the change in *NR* regulation in Case 2.

Case 1: The impact of optimal regulation – or, equivalently, competitive private property rights to *NR* in North – was examined in Chichilnisky (1994) who assumed that North and South are identical except for open access in South and optimal regulation in North, and further assumed rising AC and MC curves, or low congestion (LC). Then, *NR* regulation in North, e.g., an optimal tax, raises the production cost (and raises its *NR* stock by reducing its output). This raises the world price, p_w, above South's autarky price, p. Hence, once it opens to trade, South raises its output and exports part of it, and its *NR* and welfare decline, as shown in Section 4.1.1.

Thus, North's tighter *NR* regulation results in a North-South shift in production of the *NR*-based commodity, raising *NR* in North and reducing it in South, i.e., North's *NR* regulation generates an

[17] For instance, Li and Zhou (2017) find that trade has a pollution-haven impact, and among studies reviewed in Taylor (2004). Antweiler et al. (2001) find trade to be good for the environment, and most find that the impact is small (e.g., Cole 2004, Birdsall and Wheeler 1993, and Copeland and Taylor's (2004) review).

'*NR*-destruction haven' effect. Assuming identical North and South except for *NR* policy in the North isolates the impact of the policy, providing an important insight about it. However, countries do exhibit many differences in reality.

Case 2: Assume a smaller population in C1 than in North, so that $p < p_w$.[18] Hence, C1 exports the commodity to North before North's policy reform occurs. Assume first that as North changes *NR* policy to optimal regulation, its commodity output declines and p_w increases, which holds under LC as well as under HC. Then, North's optimal regulation reduces C1's welfare and *NR*, i.e., it generates an '*NR*-destruction haven' effect in this case as well.

On the other hand, assume that super congestion (SC) prevails in North. Then, regulating the *NR* in North results in an increase in output and a decline in p_w (e.g., compare points A' and E' in Figure 1). Thus, C1's welfare and *NR* increase with North's regulatory reform in this case.

Thus, a developing commodity-exporting country, C1, would benefit from North's policy change from open access to optimal *NR* regulation under SC in North and would lose under LC and HC.

4.3. Collapse

In a significant contribution to our understanding of Easter Island's history, Brander and Taylor (1998) show how open access to its forests led to their total depletion and a catastrophic outcome for its people. The absence of trade due to the island's remoteness may have exacerbated the problem. Though opening up to trade results in a welfare and *NR* decline for an open-access commodity exporter, it may have a positive impact over time in the case of population growth.

[18] Other assumptions that result in a lower open-access price $p = w/(\alpha - \beta\gamma\mathbb{L})$ are: a larger *NR* endowment, α, a smaller externality, β, a smaller degree of preference for the commodity, γ, or higher productivity in the manufacturing sector ($w > 1$).

Thus, Easter Island did not suffer from export-related pressure on its *NR*, but neither did it benefit from trade's (arguably more important) positive impact as the situation worsened.

Assume the autarky price in the small open economy, C1, is below the world price ($p < p_w$) and it exports the *NR*-based commodity. Assume further that C1's population increases over time for reasons that are exogenous, as detailed in Diamond (2011) – see footnote 4. As shown in Section 4.1.1, $M_T^I = L_T - \frac{\mathbb{L}}{2}$ and trade ceases when population is $\mathbb{L} = \mathbb{L}_R = 2L_T$. With population $\mathbb{L} > \mathbb{L}_R$, C1 imports the commodity and exports the manufacturing product. Under optimal regulation, the critical value of $\mathbb{L}$ where trade reversal takes place is $\mathbb{L}_R^* = L_T + \frac{\beta L_T^2 p_w}{4} \neq \mathbb{L}_R$ (see footnote 13).

Under autarky, C1's price rises and *NR* falls continuously as $\mathbb{L}$ increases. As $\mathbb{L}$ reaches $\overline{\mathbb{L}}$ which, under open access, is $\overline{\mathbb{L}} = \frac{2}{\beta}\left(\alpha - \underline{N}\right)$, C1's *NR* falls to $N \leq \underline{N}$, i.e., it finds itself in a low-*NR* trap, a situation that might lead to society's collapse. On the other hand, in the case of trade, C1's commodity price cannot rise above p_w, which is the price at which all trade takes place. This means that further increases in population are entirely absorbed by the manufacturing sector and do not affect the commodity's output. Thus, the population increase results in an increase in commodity imports and manufacturing exports rather than in greater pressure on the *NR*.

In other words, though opening up to trade reduces welfare and *NR*, as long as $\mathbb{L} < \overline{\mathbb{L}}$ and $N > \underline{N}$ when doing so, increases in population are unlikely to affect *NR* and result in a low-*NR* trap and a possible collapse of society. For countries experiencing population growth and *NR* depletion, the short-run *NR* loss from opening up to trade is likely to be less important than its long-term benefit. Then, as population continues to grow, they will eventually experience a trade pattern reversal.[19]

Note that such a trade pattern reversal is more likely in larger, more diversified economies than in small remote ones whose exports often consist of a few *NR*-based products. Based on detailed empirical analysis of business costs for a large number of small and micro states, Winters and Martins (2004) find the smallness and remoteness-related high transactions may result in such low export prices that alternative products that can compete on the world market may not exist.[20] This provides a further argument for economies that depend heavily on *NR*-based products and whose population increases rapidly to broaden the range of products (or services) that can be exported.[21]

The results obtained are collected in the following proposition.

Proposition 2: Assume a developing country, C1, produces commodity Q and manufactures M with open-access *NR* and labor L, has autarky price $p < p_w$ (world price) and thus exports Q. Then: *i*) Under open access, $U_T = 1/2p_w^{1/2} < U = 1/2p^{1/2}$, and $N_T = 1/p_w < N = 1/p$, i.e., opening up to trade and a terms-of-trade improvement reduce C1's welfare and *NR*; *ii*) Trade raises sector Q's employment level ($L_T > L$), reduces manufacturing labor and output, and reduces both sectors' output under HC and SC, generating an economic contraction; *iii*) Increases in population $\mathbb{L}$ are fully absorbed by the manufacturing (M) sector under trade as, contrary to autarky's result, L_T is independent of population size $\mathbb{L}$, and so are N_T, Q_T and U_T; *iv*) While welfare and *NR* costs increase with $\mathbb{L}$ under autarky, the welfare cost under trade *decreases* with $\mathbb{L}$ and the *NR* cost is unrelated to $\mathbb{L}$, dampening the incentive to regulate the *NR*; *v*) Though North's change from open access to optimal *NR* regulation is said to reduce C1's *NR* and welfare, the opposite is likely to hold under SC in North; and *vi*) Though welfare and *NR* decline by opening up to trade, increases in $\mathbb{L}$ have no impact on *NR* and are therefore less likely to result in a low-*NR* trap and a possible collapse of society than under autarky.

[19] A question is the extent to which the reversal in the pattern of food trade prevails. Ng and Aksoy (2008) report, among others, that the largest reversal occurred in the 51 (non-oil and non-civil-conflict) 'Other middle-income countries,' where net raw-food exports of $9.1 billion in 1980/81 turned into net imports of $16.7 billion by 2004/5. Regarding Africa, Rakotoarisoa et al. (2011) report that its food trade surplus changed to a deficit in the early 1980s, and that it has been growing ever since. They provide several reasons for the change, a major one being rapid population growth.

[20] And even if such products do exist, they may generate excessively low incomes. Winters and Martins examine a number of other options regarding their capacity to help improve these countries' economic prospects.

[21] Gelb (2010) provides a useful review of issues related to economic diversification in resource-rich countries.

5. Robustness

This section examines the impact on the welfare and *NR* results of using alternative parameter values and functional forms for production and preference functions in the case of autarky. The results are derived in Appendix 4. Similar results obtain under trade.[22]

5.1. Alternative parameter values

5.1.1. *NR* and externality

I compare ΔU and ΔN under SC and LC for $(\alpha, \beta) = (6, 1)$ and $(\alpha, \beta) = (4, 1)$. Results, shown in Table 1A, Appendix 4.1.1, are similar to those in Table 1 for $(\alpha, \beta) = (10, 1)$. For instance, for $(\alpha, \beta) = (6, 1)$, $\Delta U_{11}/\Delta U_1 = 162$ and $\Delta N_{11}/\Delta N_1 = 210$. For $(\alpha, \beta) = (4, 1)$, $\Delta U_7/\Delta U_1 = 181$ and $\Delta N_7/\Delta N_1 = 79$.

5.2.2. Food expenditure share

The USDA reports a share of food in 2014 household expenditures of 20 to 30 percent in a number of upper middle-income countries (e.g., China, Mexico, South Africa) and high-income Russia. I select $\gamma = .25$ for this group of countries, and $\alpha = 10$ and $\beta = 1$ as before. Results for average values are presented in Appendix 4.1.2. They are similar to those for $\gamma = .5$ in Table 2, Section 3.3 above). Similar results are also obtained for $\gamma = .75$ (available from the author upon request).

5.2. Alternative functional forms

5.2.1 Utility functions

The first one is the constant-relative-risk-aversion function $U(x) = x^{1-\mu}/(1-\mu)$, $\mu \neq 1$. Under separability and $\mu = 1/2$, $U(m, q) = U(m) + U(q) = \frac{m^{1/2}}{1/2} + \frac{q^{1/2}}{1/2}$. Solutions and simulations are in Appendix 4.2.1. Panels A and B (Table A2) show, for $(\alpha, \beta) = (6, 1)$ and $(\alpha, \beta) = (4, 1)$,

[22] These can be obtained from the author upon request.

respectively, that ΔN_{SC} (ΔU_{SC}) is larger than ΔN_{LC} (ΔU_{LC}) by a multiple to one order (two orders) of magnitude. For $\mathbb{L}$'s central values (not shown), ΔU_{SC} (ΔN_{SC}) is an order of magnitude larger than ΔU_{LC} (ΔN_{LC}). The other utility function is $U = \left(m - \frac{m^2}{2}\right) + \left(q - \frac{q^2}{2}\right)$, with similar results (Appendix 4.2.2).

5.2.2. Production functions

The first production function is $Q = L[\alpha - \beta(\log L)]$, $L > 1$. Appendix 4.3.1 shows for each congestion category's central value of $\mathbb{L}$, that ΔU_{SC} (ΔU_{HC}) is an order of magnitude larger than (a multiple of) ΔU_{LC}. The second production function is $Q_{\varepsilon} = \varepsilon L(\alpha - \beta L) = \varepsilon Q$, $\varepsilon > 1$ (ε is TFP). Appendix 4.3.2 shows Δx ($x = L, N, Q, U$) is identical to the Δx obtained with equation (1).

Thus, the findings in Section 3, that welfare and *NR* costs of open access are greater under HC (SC) by a multiple to one order (two orders) of magnitude than under LC, holds under the alternative parameter values and alternative production and preference functions examined.

6. Policy Implications

Given the significantly larger *NR* and welfare cost of open access under high (HC) and especially super congestion (SC) in the case of autarky, it follows that in countries where it prevails, regulating the use of the *NR* – e.g., through an optimal producer tax – would generate massively larger gains over time than analyses of low congestion cases would suggest. The authorities of countries where HC and SC prevail should thus be more willing to regulate the *NR* and pay the short-term political cost compared to LC countries.

An optimal tax raises the consumer price in LC and HC countries under autarky, thereby hurting consumers in the short term.[23] On the other hand, the tax reduces the consumer price in SC countries and might therefore be politically easier to implement in this case.

Both welfare and *NR* costs of open access *increase* with population, $\mathbb{L}$, under autarky while the welfare cost under trade *decreases* with $\mathbb{L}$ and the *NR* cost is unrelated to $\mathbb{L}$. Thus, the incentive to regulate the *NR* as $\mathbb{L}$ increases might be lower under trade than under autarky.

A producer tax or other regulation may be harder to implement successfully in cases where the sector consists of a large number of small producers or if they are located in remote areas because of greater information and enforcement difficulties. This issue is likely to matter especially in the poorer developing countries. The problem would be greatly simplified under trade, as an export tax could be levied at the export points (e.g., at ports).

For a country whose autarky price is below the world price, opening up to trade raises employment in the commodity sector, reduces it in the manufacturing sector, reduces that sector's output, and reduces both sectors' output under HC or SC. Thus, opening up to trade results in an economy-wide contraction in countries where HC or SC prevails, which is more likely to occur in poorer countries where *NR* management is often weaker and where open access, de facto even if not de jure, is more frequent. This suggests that obtaining accurate information on *NR* stocks, their evolution, the extent of congestion and externality costs, would be especially important for these countries in order to evaluate the impact of alternative trade policies.

[23] The net-of-tax-producer price $p' = p(1-t)$ declines but does not affect producers' welfare as price always equals average cost under open access.

Countries exporting *NR*-based commodities have tended to favor terms-of-trade improvements because of their favorable impact on GDP, on the value and volume of exports, and thus on inflows of foreign exchange. However, the impact on *NR* is not always incorporated in analyses of the macroeconomic impact of changes in terms of trade. Including changes in the stock of *NR* in the Net National Product and Net Investment measures should help governments take them into account and help improve policy. The World Bank has incorporated the issue in its advice and programs, and has constructed a large annual database on key environmental and *NR* variables covering some 200 countries (e.g., World Bank, various years; Lange et al. 2018).[24]

7. Concluding Comments

This paper examined the case of an industry whose output is based on the exploitation of an open-access renewable natural resource, *NR*. I identified three congestion categories, each with distinct characteristics. Results are provided in the two propositions, at the end of Section 3 for the autarky case and at the end of Section 4 for the trade case. The main results are:

- Open-access welfare and *NR* losses are typically a multiple to an order (one to two orders) of magnitude greater for HC (SC) than for LC countries.
- Population growth does not affect commodity output or *NR* under trade, which may help prevent a *NR* and society collapse.
- Surprisingly, though open-access welfare and *NR* losses increase with population pressure in the case of autarky, welfare losses *decrease* under trade openness and the *NR* is unaffected.

[24] The issue of natural capital is especially important for poorer countries where it tends to be managed less efficiently and amounts to a larger share of national wealth, namely 47 (27) (17) percent in low- (lower-middle-) (upper-middle) income countries, and just three percent in high-income OECD countries (Lange et al. 2018).

- Opening up to trade reduces output in both sectors under HC and SC, generating an economic contraction.

Incorporating market power (large country), trade costs, diminishing returns in the manufacturing sector, and international migration is on my research agenda.

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Appendix

1. Properties of the AC and MC curves

1.1. L_I is the inflection point, with $AC_2 = MC_1$ at $Q_I = L_I(\alpha - \beta L_I)$

1.1.1. $AC_2 = MC_1$

Denote AC's backward-bending part by AC_2. The SC (HC) region consists of the upper (lower) part of AC_2 where optimal output $Q^* > (<)\ Q$, open-access output. Hence, $Q = Q^*$ at the border point between SC and HC, where AC_2 intersects MC_1 (i.e., MC's positive segment) and $L = L_I$. Thus, L_I is the level of L where $AC_2 = MC_1$, or $\frac{1}{\alpha - \beta L_2} = \frac{1}{\alpha - 2\beta L_1}$, i.e., $L_2 = 2L_1$ at output Q_I, and $N_2 = N_1/2$. In fact, any output Q can be produced with a high (low) L and a low (high) *NR* level.

Thus, $Q = L_1(\alpha - \beta L_1) = L_2(\alpha - \beta L_2)$, or $\beta L_2^2 - \alpha L_2 + (\alpha L_1 - \beta L_1^2) = 0$. The solution is $L_2 = \frac{1}{2\beta}\left[\alpha \pm \sqrt{\alpha^2 - 4\beta(\alpha L_1 - \beta L_1^2)}\right] = \frac{1}{2\beta}[\alpha \pm (\alpha - 2\beta L_1)]$. Thus, one solution is $L_2 = \frac{1}{2\beta}[\alpha + (\alpha - 2\beta L_1)] = \frac{\alpha}{\beta} - L_1$, or $L_1 + L_2 = \frac{\alpha}{\beta}$. The second solution is $L_2 = \frac{1}{2\beta}[\alpha - (\alpha - 2\beta L_1)] = L_1$. Thus, $L_2 = L_1 = \frac{\alpha}{2\beta} = \hat{L}$, the value of L where $Q = \hat{Q}$, the maximum output, and AC_1 switches to AC_2.

Since $L_2 = 2L_1$ at Q_I, we have $L_2 = \frac{2\alpha}{3\beta}$, $L_1 = \frac{\alpha}{3\beta}$ (and $N_2 = \frac{\alpha}{3}$, $N_1 = \frac{2\alpha}{3} = 2N_2$). As L_I is on AC_2, it follows that $L_I > \hat{L} = \frac{\alpha}{2\beta}$ or $L_I = L_2 = \frac{2\alpha}{3\beta}$, with $N_I = N_2 = \frac{\alpha}{3}$, $Q_I = \frac{2\alpha^2}{9\beta}$. In other words, the same output could be obtained with half the labor ($L_1 = L_2/2$) and twice the *NR* ($N_1 = 2N_2$), implying an enormous waste of human and natural resources.

For different levels of output Q, $L_1 \neq L_2$. Assume $L_2 > L_1$. The fact that $L_1 + L_2 = \frac{\alpha}{\beta} = 2\hat{L}$ implies that $L_2 = \frac{\alpha}{2\beta} + z = \hat{L} + z$, $L_1 = \hat{L} - z$, $z > 0$, i.e., HC or SC prevails in the case of L_2 and LC prevails in the case of L_1.

1.1.2. <u>Inflection point</u>

L_{I} which separates HC and SC, is also the inflection point where $AC'' = \partial^2 AC/\partial Q^2 = 0$, switching from $AC'' > 0$ (under SC) to $AC'' < 0$ (under HC). As $AC = \frac{1}{\alpha - \beta L}$, $AC' \equiv \frac{\partial AC}{\partial Q} = \frac{\partial AC}{\partial L} / \frac{\partial Q}{\partial L} = \frac{\beta}{(\alpha-\beta L)^2(\alpha - 2\beta L)}$. Thus, $AC' \gtreqless 0 \Leftrightarrow L \lesseqgtr \hat{L} = \frac{\alpha}{2\beta}$ ($L > \hat{L}$ implies L must fall for Q to rise). And $AC'' \equiv \frac{\partial^2 AC}{\partial Q^2} = \frac{\partial AC'}{\partial L} / \frac{\partial Q}{\partial L} = \frac{2\beta(2\alpha - 3\beta L)}{(\alpha-\beta L)^3(\alpha-2\beta L)^2}$, with $AC'' = 0 \Leftrightarrow L = L_{\mathrm{I}} = \frac{2\alpha}{3\beta}$. QED.

1.2. <u>At any given output Q, $MC_2 = -MC_1$</u>

$MC_1 = \frac{1}{\alpha - 2\beta L_1} > 0$, $L_1 < \hat{L}$. At $L_2 > \hat{L}$, $MC_2 = \frac{1}{\alpha - 2\beta L_2} < 0$. As shown in Section 2.1, $L_2 = \frac{\alpha}{\beta} - L_1$ for any Q. Thus, $\alpha - 2\beta L_2 = \alpha - 2\beta\left(\frac{\alpha}{\beta} - L_1\right) = -\alpha + 2\beta L_1 = -(\alpha - 2\beta L_1)$. Thus, $MC_2 = -\frac{1}{\alpha - 2\beta L_1} = -MC_1$, i.e., MC_2 is the mirror image of MC_1. QED.

1.3. <u>Range of L values under LC, HC and SC</u>

The range of L-values under SC is $L_I < L < \mathbb{L}$, or $\frac{\alpha}{\beta} - \frac{2\alpha}{3\beta} = \frac{\alpha}{3\beta}$, under HC it is $\hat{L} < L < L_I$, or $\frac{2\alpha}{3\beta} - \frac{\alpha}{2\beta} = \frac{\alpha}{6\beta}$, and under LC, it is $0 < L < \hat{L}$, or $\frac{\alpha}{2\beta}$. Assuming a random drawing of L, it follows that the LC (SC) (HC) category comprises $\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)$ of the L-values.

2. <u>Proofs and solutions for: 2.1. Stability and uniqueness of equilibrium; 2.2. $L > L^*$; 2.3. Sign in L^* is negative</u>

2.1. <u>Stability and uniqueness of equilibrium</u>: As the demand curve is downward sloping, stability and uniqueness is not an issue for equilibria on the upward-sloping part of AC, though it might be on the backward-bending part of AC where the elasticity $\varepsilon_{Q.AC} < 0$. These conditions require excess-demand to prevail below the equilibrium price and excess-supply above it, i.e., demand must be more elastic than supply.

Demand elasticity, η, under a Cobb-Douglas utility function is $\eta = -1$. Supply elasticity is $\varepsilon_{Q.AC} = \frac{\partial Q}{\partial AC} * \frac{AC}{Q}$, where $\frac{\partial Q}{\partial AC} = \frac{\partial Q/\partial L}{\partial AC/\partial L}$. As $\frac{\partial Q}{\partial L} = \alpha - 2\beta L$, and $AC = \frac{1}{\alpha - \beta L}$ implies $\frac{\partial AC}{\partial L} = \frac{\beta}{(\alpha - \beta L)^2}$, we have $\frac{\partial Q}{\partial AC} = \frac{(\alpha - 2\beta L)(\alpha - \beta L)^2}{\beta}$. With $\frac{AC}{Q} = \frac{1}{L(\alpha - \beta L)^2}$, we have $\varepsilon_{Q.AC} = \frac{\alpha - 2\beta L}{\beta L}$. Since $L > \hat{L} = \frac{\alpha}{2\beta}$ is located on AC_2 and $MC_2 = \alpha - 2\beta L < 0$, we have $\varepsilon_{Q.AC} < 0$. And as $AP_L = \alpha - \beta L > 0$, we have

$\alpha - 2\beta L > -\beta L$, or $\varepsilon_{Q.AC} = (\alpha - 2\beta L)/\beta L > -1$. Thus, $-1 < \varepsilon_{Q.AC} < 0$, and $\eta = -1 < \varepsilon_{Q.AC}$, i.e., the demand curve is more elastic than the AC_2 (or open-access supply) curve. QED.

2.2. $\underline{L > L^*}$: Say $L < L^*$ instead. Thus, $L = \frac{\mathbb{L}}{2} < L^* = \frac{1}{3\beta}\left[\alpha + \beta\mathbb{L} - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\,\right]$, or $\frac{3\beta\mathbb{L}}{2} < \alpha + \beta\mathbb{L} - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}$, i.e., $\sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}} < \alpha - \frac{\beta\mathbb{L}}{2}$, or $\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L} < \alpha^2 + \frac{\beta^2\mathbb{L}^2}{4} - \alpha\beta\mathbb{L}$, or $\frac{3\beta^2\mathbb{L}^2}{4} < 0$, which is false. Thus, $L > L^*$. QED.

2.3 Sign in L^* is negative: $L^* = \frac{1}{3\beta}\left[(\alpha + \beta\mathbb{L}) - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\right]$ has a negative sign before the square root. Assume it is positive. Since $\sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}} = \sqrt{(\alpha - \beta\mathbb{L})^2 + \alpha\beta\mathbb{L}} > \alpha - \beta\mathbb{L}$, we have $L^* > \frac{1}{3\beta}(\alpha + \beta\mathbb{L} + \sqrt{(\alpha - \beta\mathbb{L})^2} = \frac{2\alpha}{3\beta} = L_I$, which belongs to the SC category and cannot be an optimum. Hence, the sign before the square root must be negative. QED.

3. International Trade

Assume a small commodity-exporting country, referred to as C1. Solutions are provided for the open-access and the optimal *NR* regulation case. The world price, p_w, is given exogenously to C1.

3.1. Open Access

Denote supply, demand, export, import and trade by, respectively, s, d, X, I and T. Under trade, $p_w = AC = \frac{1}{\alpha - \beta L_T}$, or $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right)$, with $L_T > L = \frac{1}{\beta}\left(\alpha - \frac{1}{p}\right)$ since $p_w > p$. And $N_T = \alpha - \beta L_T < N = \alpha - \beta L$, with $\frac{\partial L_T}{\partial p_w} = \frac{1}{\beta p_w^2} > 0$. Thus, trade and better terms of trade raise employment L_T in the commodity sector, with $M_T^s < M$ and $Q_T^s > (<)\ Q$ under LC (HC and SC).

As each individual owns one unit of labor and $w = 1$, aggregate income $Y = \mathbb{L}$ under open access, and $y = w = 1$. The budget constraint is $m^d + p_w q^d = 1$. From utility function (2) and $\gamma = .5$, income is spent equally on Q and M, i.e., $m^d = \frac{1}{2}$, $q^d = \frac{1}{2p_w}$, and utility is $U_T = (q^d m^d)^{1/2} = \frac{1}{2{p_w}^{1/2}}$.[25] Under autarky, $U = q^{1/2}m^{1/2} = \frac{1}{2p^{1/2}}$, which implies that $U_T < U$ since $p_w > p$. Thus, opening up to trade under open access has a negative impact on a commodity-exporting country's welfare and *NR*. Moreover, it reduces output in *both* sectors under HC and under SC.

Given that $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right)$ and $\frac{\partial L_T}{\partial p_w} = \frac{1}{\beta p_w^2}$, we have $\frac{\partial N_T}{\partial p_w} = \frac{\partial(\alpha - \beta L_T)}{\partial p_w} = -\frac{1}{p_w^2} < 0$; and $\frac{\partial U_T}{\partial p_w} = \frac{\partial(1/2{p_w}^{1/2})}{\partial p_w} = -\frac{1}{4{p_w}^{3/2}} < 0$. Thus, terms of trade improvements reduce C1's welfare and *NR* under open access.

Note that $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right)$ implies $\frac{\partial L_T}{\partial \mathbb{L}} = \frac{\partial N_T}{\partial \mathbb{L}} = \frac{\partial Q_T^S}{\partial \mathbb{L}} = 0$, so that $\frac{\partial M_T^S}{\partial \mathbb{L}} = 1$. Thus, any population increase results in an equal increase in the manufacturing sector's employment, and since the rent in the commodity sector is nil under open access, $\frac{\partial U_T}{\partial \mathbb{L}} = 0$.

Also, given that $M_T^D = \frac{Y_T}{2} = \frac{\mathbb{L}}{2}$, we have $\frac{\partial M_T^D}{\partial \mathbb{L}} = \frac{1}{2}$. Since $\frac{\partial M_T^S}{\partial \mathbb{L}} = 1$, we have $\frac{\partial M_T^I}{\partial \mathbb{L}} = \frac{\partial M_T^D}{\partial \mathbb{L}} - \frac{\partial M_T^S}{\partial \mathbb{L}} = -\frac{1}{2}$, i.e., trade declines with $\mathbb{L}$. As $\mathbb{L}$ increases, M_T^I eventually reaches zero – at which point the autarky price p equals the world price p_w – and then becomes negative, i.e., the trade pattern is reversed, with country C1 importing commodity Q and exporting manufactures M.

[25] Manufacturing output $M^s = \mathbb{L} - L_T < M$, and import $M^I = M^d - M^s = \frac{\mathbb{L}}{2} - (\mathbb{L} - L_T) = L_T - \frac{\mathbb{L}}{2}$. Commodity output $Q^s = L_T N_T = L_T(\alpha - \beta L_T) = \frac{L_T}{p_w}$, with $N_T = \frac{1}{p_w} < N = \frac{1}{p}$. Also, $p_w Q^d = \frac{\mathbb{L}}{2}$, $p_w Q^s = L_T$, and $p_w Q^X = p_w(Q^s - Q^d) = L_T - \frac{\mathbb{L}}{2}$. Thus, $M^I = p_w Q^X$ and trade is balanced.

3.2. Optimal Regulation

In this case, $p_w = MC = \frac{1}{\alpha - 2\beta L_T^*}$. Under open-access, $p_w = \frac{1}{\alpha - \beta L_T}$. Thus, $L_T^* = \frac{L_T}{2}$, and $Q_T^{*S} = L_T^*(\alpha - \beta L_T^*) = \frac{L_T}{2}\left(\alpha - \beta \frac{L_T}{2}\right)$,[26] $q_T^{*S} = \frac{L_T}{2\mathbb{L}}\left(\alpha - \beta \frac{L_T}{2}\right)$, $M_T^{*S} = \mathbb{L} - \frac{L_T}{2}$, and $m_T^{*S} = 1 - \frac{L_T}{2\mathbb{L}}$. Denote the rent obtained in the commodity sector by R, i.e., $R = (MC^* - AC^*)Q_T^{*S}$. With $L_T^* = \frac{L_T}{2}$, we have $AC^* = \frac{1}{\alpha - \beta\frac{L_T}{2}} < AC = \frac{1}{\alpha - \beta L_T} = p_w$, $MC^* = \frac{1}{\alpha - 2\beta L_T^*} = \frac{1}{\alpha - \beta L_T}$ – i.e., $AC^* = MC^* = p_w$, and thus $MC^* - AC^* = \frac{1}{\alpha - \beta L_T} - \frac{1}{\alpha - \beta\frac{L_T}{2}} = \frac{\beta L_T/2}{(\alpha - \beta L_T)\left(\alpha - \beta\frac{L_T}{2}\right)}$, and $R = \frac{\beta L_T/2}{(\alpha - \beta L_T)\left(\alpha - \beta\frac{L_T}{2}\right)} * \frac{L_T}{2}\left(\alpha - \beta \frac{L_T}{2}\right) = \frac{\beta L_T^2}{4(\alpha - \beta L_T)} = \frac{\beta L_T^2 p_w}{4}$. Thus, aggregate income is $Y^* = \mathbb{L} + \frac{\beta L_T^2 p_w}{4}$, and individual income is $y^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}}$. From (2) and $\gamma = .5$, we have $p_w q_T^{*d} = m_T^{*d} = \frac{1}{2} + \frac{\beta L_T^2 p_w}{8\mathbb{L}}$.

Utility is $U_T^* = \left(q_T^{*d} m_T^{*d}\right)^{1/2} = \frac{m_T^{*d}}{p_w{}^{1/2}} = \frac{1}{p_w{}^{1/2}}\left(\frac{1}{2} + \frac{\beta L_T^2 p_w}{8\mathbb{L}}\right) = U_T + \frac{\beta L_T^2 p_w{}^{1/2}}{8\mathbb{L}} > U_T$, with $\frac{\partial U_T^*}{\partial p_w} \gtrless 0$ (see Section 3.2.1 below). As $N_T^* = \alpha - \beta\frac{L_T}{2}$, $\frac{\partial N_T^*}{\partial p_w} = -\frac{1}{2p_w^2} < 0$, with $N_T^* = N_T + \beta\frac{L_T}{2}$. Thus, optimal regulation raises welfare and *NR* relative to open access under free trade, and an increase in the terms of trade raises reduces *NR* and has an ambiguous impact on welfare.

From U_T^* and U_T, we have $\Delta U_T \equiv \frac{U_T - U_T^*}{U_T^*} = \frac{-\beta L_T^2 p_w{}^{1/2}/8\mathbb{L}}{\frac{1}{2p_w{}^{1/2}} + \frac{\beta L_T^2 p_w{}^{1/2}}{8\mathbb{L}}}$. Multiplying numerator and denominator by $8\mathbb{L}p_w{}^{1/2}$, we have $\Delta U_T = \frac{-\beta L_T^2 p_w}{4\mathbb{L} + \beta L_T^2 p_w}$, with $\frac{\partial \Delta U_T}{\partial p_w} = -\frac{4\mathbb{L}L_T(\beta L_T + 2/p_w)}{\left(4\mathbb{L} + \beta L_T^2 p_w\right)^2} < 0$. Thus, the open-access welfare cost and *NR* cost under trade increase with the country's terms of trade.

[26] $Q_T^{*S} - Q_T^S = \frac{L_T}{2}\left(\alpha - \beta\frac{L_T}{2}\right) - L_T(\alpha - \beta L_T) = \frac{3\beta L_T}{4}\left(L_T - \frac{2\alpha}{3\beta}\right) = \frac{3\beta L_T}{4}(L_T - L_I) \gtrless 0 \Leftrightarrow L_T \gtrless L_I$, where L_I is associated with output Q_I, the inflection point, which separates HC and SC, and where $MC_1 = AC_2$ (see Fig.1). This confirms that optimal output is larger (smaller) than open-access output in the case of SC (HC and LC).

3.2.1. $\underline{\partial U_T^*/\partial p_w \gtrless 0}$

The welfare impact of p_w under regulation is ambiguous. Since trade is free and optimal regulation corrects for the congestion externalities (a first-best situation), labor and *NR* are allocated efficiently and welfare U_T^* is maximized. The reason $\frac{\partial U_T^*}{\partial p_w} \gtrless 0$ is that U_T^* rises (declines) with an increase in p_w if the country exports (imports) the commodity. Thus, $Q_T^{*X} \gtrless 0 \Leftrightarrow \frac{\partial U_T^*}{\partial p_w} \gtrless 0$.

3.3. Simulation

Results for the welfare and *NR* costs of open access (relative to the optimum), for different world prices and population levels, using values $\alpha = 10$ and $\beta = 1$ as before, are presented below.

1. *Assume* $p_w = 1/8$, with $L_T = \frac{1}{\beta}\left(\alpha - \frac{1}{p_w}\right) = 2$, i.e., $L_T < \hat{L} = \frac{\alpha}{2\beta} = 5$ (where $MP_L = 0$ and output is maximized). Thus, we are in a low-congestion (LC) case. As we saw before, $m_T^D = \frac{1}{2}$ and $q_T^D = \frac{m_T^D}{p_w} = 8m_T^D = 4$, and $U_T = (m_T^D q_T^D)^{1/2} = 1.4142$.

1.A. $\mathbb{L} = 10$: Optimal income is $y_T^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}} = 1 + \frac{4/8}{40} = 1.0125$. Thus, $m_T^{D*} = .50625$, $q_T^{D*} = 4.05$, and $U_T^* = 1.4319$, so that $\Delta U_T = -1.24$ percent.

1.B. $\mathbb{L} = 20$: As noted earlier, U_T is unchanged. Optimal income $y_T^* = 1.00625$. Thus, $m_T^{D*} = .5003125$, $q_T^{D*} = 4.0025$, and $U_T^* = 1.4151$, so that $\Delta U_T = -.064$ percent.

1.C. Assume $\mathbb{L} = 30$. $y_T^* = 1.00417$. Thus, $m_T^{D*} = .5003125$, $q_T^{D*} = 4.0025$, and $U_T^* = 1.4148$, so that $\Delta U_T = -.042$ percent.

2. *Assume* now $p_w = \frac{1}{2}$. Then, $L_T = 8 > L_I = 6.67$, i.e., we are in a super-congestion (SC) case. And $m_T^D = \frac{1}{2}$, $q_T^D = \frac{m_T^D}{p_w} = 1$, $U_T = .7071$.

2.A. $\mathbb{L} = 10$: Optimal income is $y_T^* = 1 + \frac{\beta L_T^2 p_w}{4\mathbb{L}} = 1.8$. Thus, $m_T^{D*} = .9$, $q_T^{D*} = 1.8$, and $U_T^* = 1.2728$, so $\Delta U_T = -44.4$ percent.

2.B. $\mathbb{L} = 20$: $y_T^* = 1.4$, $m_T^{D*} = .7$, $q_T^{D*} = 1.4$, $U_T^* = .9899$, and $\Delta U_T = -28.6$ percent.

2.C. $\mathbb{L} = 30$: $y_T^* = 1.267$, $m_T^{D*} = .633$, $q_T^{D*} = 1.267$, $U_T^* = .8957$, and $\Delta U_T = -21.1$ percent.

4. Robustness

This section provides derivations and simulations of the results in Section 5, where the robustness of results (in Section 3.3) is examined under different parameter values and functional forms.

4.1. Different parameter values

4.1.1 Two sets of values for α and β parameters

The case of $(\alpha, \beta) = (6, 1)$ is examined in Table 1A, Panel A, and that of $(\alpha, \beta) = (4, 1)$ in Panel B. Panel A parameter values $(\alpha, \beta) = (6, 1)$ imply that $\mathbb{L} < 12$ and LC's upper limit for L is $\hat{L} = \frac{\alpha}{2\beta} = 3$ (and $\mathbb{L} = 6$) and SC's lower limit is $L_I = \frac{2\alpha}{3\beta} = 4$ (and $\mathbb{L} = 8$). We have $\Delta U_{11}(\Delta N_{11}) = -55\ (-86)$ percent or $162\Delta U_1$ $(210\Delta N_1)$, $52.7\Delta U_3$ $(17.6\Delta N_3)$ and $11.2\Delta U_5$ $(5.4\Delta N_5)$. Also, ΔU_9 $(\Delta N_9) = 88\Delta U_1$ $(144\Delta N_1)$, $25.9\Delta U_3$ $(12\Delta N_3)$ and $5.5\Delta U_5$ $(3.7\Delta N_5)$.

In Panel B, $(\alpha, \beta) = (4, 1)$, $\hat{L} = 2$ (and $\mathbb{L} = 4$), $L_I = 2\frac{2}{3}$ (and $\mathbb{L} = 5\frac{1}{3}$), and $\mathbb{L} < 8$. We have $\Delta U_7(\Delta N_7) = -45.7(-78.8)$ percent, or $180.5\Delta U_1(78.8\Delta N_1)$, and $12.4\Delta U_3(6.1\Delta N_3)$. For $\mathbb{L} = 6$, we have $\Delta U_6(\Delta N_6) = 106.8\Delta U_1$ $(58.9\Delta N_1)$ and $7.3\Delta U_3(4.6\Delta N_3)$. As for HC, $\Delta U_5(\Delta N_5) = 61.3\Delta U_1(40.7\Delta N_1)$ and $4.2\Delta U_3(3.2\Delta N_3)$.

Thus, both ΔU and ΔN under SC are between a multiple and two orders of magnitude larger than under LC. And ΔU and ΔN under HC are between a multiple and one order of magnitude larger than under LC. These results are similar to those obtained with $(\alpha, \beta) = (10, 1)$ in the main text.

4.1.2. Smaller share spent on food

The share of food in household expenditure in a number of middle-income countries ranges from .2 to .3, hence I select $\gamma = .25$ in this case, with $\alpha = 10$ and $\beta = 1$ as before. Results for individual (central) $\mathbb{L}$ values are shown in Table 2A (3A).

Table 1A. Open Access vs. Optimum

A: $\alpha = 6, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	5.5	2.8	1.173	.49	5.5	2.6	1.177	2.7	-.41	2.5	-.339
3	1.5	4.5	6.8	1.061	1.3	4.7	6.0	1.072	19	-4.9	13.3	-1.04
5	2.5	3.5	8.8	.935	1.8	4.2	7.6	.984	39	-16	15.1.	-4.91
6	3.0	3.0	9.0	.866	2.0	4.0	8.0	.928	50	-25	12.5	-6.65
9	4.5	1.5	6.8	.612	2.4	3.6	8.6	.839	91	-59	-21.4	-27.0
11	5.5	.50	2.8	.350	2.5	3.5	8.8	.784	112	-86	-69.0	-54.9

B: $\alpha = 4, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	3.5	1.75	.9354	.46	3.53	1.64	.9378	7.6	-1.0	6.5	-.253
3	1.5	2.5	3.75	.7906	1.13	2.87	3.25	.8209	33	-12.9	12.9	-3.69
5	2.5	1.5	3.75	.6124	1.47	2.53	3.72	.7240	70	-40.7	.76	-15.5
6	3.0	1.0	3.0	.50	1.57	2.43	3.81	.6850	91	-58.9	-21.4	-27.0
7	3.5	0.5	1.75.	.3536	1.64	2.36.	3.87	.6510	114	-78.8	-54.8	-45.7

Table 2A shows that ΔN_{39} (ΔU_{39}) is three orders of magnitude larger than ΔN_1 (ΔU_1), one order of magnitude larger than ΔN_8 (ΔU_8), and a multiple of (an order of magnitude larger than) ΔN_{18} (ΔU_{18}). In the case of $\gamma = .5$, $\mathbb{L} < 20$ (see Table 1, Section 3.3). Taking the value of $\mathbb{L} = 18$ in Table 2A for comparison purpose, ΔN_{18} (ΔU_{18}) is two orders of magnitude larger than ΔN_1 (ΔU_1).

Table 2A. Open Access vs. Optimum for $\gamma = .25$ [a]

$\mathbb{L}$	Open Access (x) N	U	Optimum (x^*) N^*	U^*	Difference (%) $\Delta x = \frac{x-x^*}{x^*}$ ΔN	ΔU	Ratio $\frac{\Delta N}{\Delta N_1}$	$\frac{\Delta U}{\Delta U_1}$
1	9.75	1.007	9.755	1.0072	-. 051	- .02	1	1
8	8.0	.9584	8.320	.9634	- 3.8	- .52	74.1	26.0
18	5.5	.8727	7.064	.9065	- 22.1	- 3.7	431	185
32	2.0	.6777	6.245	.8425	- 68.0	- 19.6	1327	980
39	.25	.4030	6.026	.8167	- 95.9	- 50.6	1871	2530

a: γ is the share of food in household expenditure.

Table 3A. Open Access vs. Optimum: Central $\mathbb{L}$ Values ($\gamma = .25$)[a]

$\mathbb{L}$	Open Access (x) N	U	Optimum (x^*) N^*	U^*	Difference (%) $\Delta x = \frac{x-x^*}{x^*}$ ΔN	ΔU	Ratio $\Delta x/\Delta x_{LC}$ $\Delta N/\Delta N_{LC}$	$\Delta U/\Delta U_{LC}$
LC: 10.0	7.5	.9431	8.0	.9514	-6.25	-.872	1	1
HC: 23.3	4.175	.8146	6.67	.8802	-37.4	-7.45	6.0	8.5
SC: 33.3	1.675	.6483	6.2	.8375	-73.0	-22.6	11.7	25.9

a: Results are for the central value of $\mathbb{L}$ in each congestion category.

Table 3A shows that, for each congestion category's central value of $\mathbb{L}$, the welfare (*NR*) impact of open access under SC, ΔU_{SC} (ΔN_{SC}) is 25.9 (11.7) times ΔU_{LC} (ΔN_{LC}) or an order of magnitude larger. Also, ΔU_{HC} (ΔN_{HC}) is 8.6 (6.0) times ΔU_{LC} (ΔN_{LC}) or a multiple. And as for $\gamma = .5$, $\Delta U_{SC} \cong 3\Delta U_{HC}$, and $\Delta N_{SC} \cong 2\Delta N_{LC}$. Moreover, comparing LC and SC congestion levels, Table 2A shows that the decline in $U(N)$ under open access is 2.6 (3.45) times the decline in $U^*(N^*)$.

Thus, results are similar to those for $\gamma = .5$ (see Table 2 in Section 3.3), though somewhat smaller given the lower demand. Similar, though larger, results are obtained with $\gamma = .75$.[27]

4.2. Two alternative utility functions

4.2.1. Constant relative-risk-aversion utility function

The utility function is $U(x) = \frac{x^{1-\mu}}{1-\mu}$ $(\mu \neq 1)$. Assuming separability, $U(m,q) = U(m) + U(q) = \frac{m^{1-\mu}}{1-\mu} + \frac{q^{1-\mu}}{1-\mu}$. With $\mu = 1/2$, we have:

$$U = \frac{m^{1/2}}{1/2} + \frac{q^{1/2}}{1/2}. \tag{1A}$$

Maximizing utility implies that the ratio of marginal utilities equals the relative price, i.e., $p = \left(\frac{m}{q}\right)^{1/2} = \left(\frac{M}{Q}\right)^{1/2} = \left[\frac{\mathbb{L}-L}{L(\alpha-\beta L)}\right]^{1/2}$. Under open access, $p = AC = \frac{1}{\alpha-\beta L}$. The two equations imply $\beta L^2 - (1+\alpha+\beta\mathbb{L})L + \alpha\mathbb{L} = 0$. The solution is:

$$L = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L} - \sqrt{(1+\alpha+\beta\mathbb{L})^2 - 4\alpha\beta\mathbb{L}}\right).^{28} \tag{2A}$$

At the optimum, $p = \left[\frac{\mathbb{L}-L}{L(\alpha-\beta L)}\right]^{1/2} = MC = \frac{1}{\alpha-2\beta L}$, or $\frac{\mathbb{L}-L}{L(\alpha-\beta L)} = \frac{1}{(\alpha-2\beta L)^2}$, which is rewritten as:

$$4\beta^2L^3 - \beta(1+4\alpha+4\beta\mathbb{L})L^2 + \alpha(1+\alpha+4\beta\mathbb{L})L - \alpha^2\mathbb{L} = 0. \tag{3A}$$

[27] Results and are available from the author upon request.

[28] The solution with a positive sign before the square root is $L = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L} + \sqrt{(1+\alpha+\beta\mathbb{L})^2 - 4\alpha\beta\mathbb{L}}\right) = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L} + \sqrt{(1+\alpha-\beta\mathbb{L})^2 + 4\beta\mathbb{L}}\right)$. As $\sqrt{(1+\alpha-\beta\mathbb{L})^2 + 4\beta\mathbb{L}} > (1+\alpha-\beta\mathbb{L})$, we have $L > \frac{1}{\beta}(1+\alpha)$, i.e., $\alpha - \beta L < -1$, which is not possible as $N = \alpha - \beta L \geq 0$. Thus, the sign before the square root must be negative.

Simulation results are presented in Table 4A, Panel A (B) for $(\alpha, \beta) = (6, 1)$ $\big((4, 1)\big)$. In Panel A, LC prevails for $\mathbb{L} = 1$, 3 and 5, and SC for $\mathbb{L} = 10$ and 50. For $\mathbb{L} = 50$, $\Delta U_{50} = -35 = 184\Delta U_{L1}$ $= 10.6\Delta U_{L3} = 3.9\Delta U_{L5}$, while $\Delta N_{50} = -95.9 = 103\Delta N_{L1} = 6.4\Delta N_3 = 2.7\Delta 5$. For $\mathbb{L} = 10$, $\Delta U_{10} = -21.5 = 113\Delta U_{L1} = 6.5\Delta U_3$, and $\Delta N_{10} = -71.7 = 77\Delta N_1 = 4.8\Delta N_3$.

In panel B, $\alpha = 4$ and $\beta = 1$, LC (SC) prevails for $\mathbb{L} < 4$ ($\mathbb{L} > 5.33$), with $\Delta U_{50} = 54\Delta U_{L1} = 4.4\Delta U_3$; $\Delta N_{50} = 37\Delta N_1 = 3.8\Delta N_3$. And $\Delta U_{10} = 41\Delta U_1 = 4.5\Delta U_3$; $\Delta N_{10} = 30\Delta N_{L1} = 3.0\Delta N_{LC}$. Thus, as with utility function (2), welfare and *NR* losses under SC are a multiple or a greater order of magnitude than those under LC.

4.2.2. Quadratic utility function

$$U = \left(m - \frac{m^2}{2}\right) + \left(q - \frac{q^2}{2}\right). \tag{4A}$$

Utility maximization implies that $p = \frac{U_q}{U_m} = \frac{1-q}{1-m}$; $m, q \,\epsilon\, (0, 1)$. With $M = l = \mathbb{L} - L$, we have $m = 1 - \frac{L}{\mathbb{L}}$, and $1 - m = \frac{L}{\mathbb{L}}$. Thus, $p = \frac{1-q}{L/\mathbb{L}} = \frac{(\mathbb{L}-Q)}{L} = \frac{\mathbb{L}}{L} - (\alpha - \beta L)$.

Open Access:

As $p = AC = \frac{1}{\alpha - \beta L}$, we have $\frac{\mathbb{L}}{L} - (\alpha - \beta L) = \frac{1}{\alpha - \beta L}$, a cubic equation in L, namely:

$$\beta^2 L^3 - 2\alpha\beta L^2 + (1 + \alpha^2 + \beta\mathbb{L})L - \alpha\mathbb{L} = 0. \tag{5A}$$

Optimum:

At the optimum, price $p = MC$, i.e., $\frac{\mathbb{L}}{L} - (\alpha - \beta L) = \frac{1}{\alpha - 2\beta L}$, a cubic equation. Thus, we have:

$$2\beta^2 L^3 - 3\alpha\beta L^2 + (1 + \alpha^2 + 2\beta\mathbb{L})L - \alpha\mathbb{L} = 0. \tag{6A}$$

Under open access, for $\alpha = 2$ and $\beta = 1$, we have from (6A): $L^3 - 4L^2 + (5 + \mathbb{L})L - \mathbb{L} = 0$. For $\mathbb{L} = 1$, a LC case, the solution is $L_1 = .4563$, $N_1 = 1.544$, $m_1 = .544$, $m_1 - \frac{{m_1}^2}{2} = .3961$, $q_L = .704$ and $q_L - \frac{q_L^2}{2} = .4561$. Thus, $U_1 = .8522$.

Table 4A. Open Access vs. Optimum

Panel A: $\alpha = 6, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.84	5.2	4.3	5.0	.79	5.2	4.1	5.01	6.1	-.93	5.1	-.19
3	2.4	3.6	8.6	4.3	1.8	4.2	7.5	4.45	33	-15	15	-3.3
5	3.6	2.4	8.6	3.7	2.1	3.9	8.3	4.1	71	-36	-5.1	-9.0
10	5.0	1.0	5.0	2.8	2.5	3.5	8.7	3.6	103	-72	-43	-21.5
50	5.9	.13	.78	1.9	2.8	3.2	9.0	2.9	111	-96	-91	-35

Panel B: $\alpha = 4, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.76	3.2	2.5	4.1	.68	3.3	2.3	4.1	13	-2.6	9.8	-.46
3	2.0	2.0	4.0	3.5	1.3	2.7	3.5	3.7	55	-25.5	14	-5.6
10	3.5	.53	1.8	2.5	1.7	2.3	3.9	3.1	109	-77	-53	-19
50	3.9	.08	.33	2.1	1.9	2.1	4.0	2.8	112	-96	-92	-25

For the optimum, we have $2L^3 - 6L^2 + 7L - 2 = 0$, with $L_1^* = .410$, $N_1^* = 1.590$, $q_1^* = .652$, $q_1^* - \frac{(q_1^*)^2}{2} = .4393$; $M_1^* = m_1^* = .590$, $m_1^* - \frac{(m_1^*)^2}{2} = .4158$ and $U_{LC}^* = .8555$. Thus, the welfare impact of open access (in percent) is $\Delta U_1 = -.375$, and $\Delta N_1 = -2.90$.

For $\mathbb{L} = 5$, a SC case, under open access, $L_5 = 1.629$, $N_5 = .371$, $Q_5 = .604$, $q_5 = .121$, $M_5 = 3.371$, $m_5 = .674$, and $U_5 = .560$. At the optimum, $L_5^* = .356$, $N_5^* = 1.644$, $Q_5^* = .585$, $M_5^* = 4.644$, and $U_5^* = .608$, with (in percent) $\Delta U_5 = -7.77$ or $20.7\Delta U_1$, and $\Delta 5 = -77.5$ or $26.7\Delta N_1$. Thus, the welfare (*NR*) cost under $\mathbb{L} = 5$ is over 20 (26) times that under $\mathbb{L} = 1$.

For $\mathbb{L} = 10$, also a SC case, $L_{10} = 1.8$, $N_{10} = .2$, $U_{10} = .519$, $L_{10}^* = .95$, $N_{10}^* = 1.05$, $U_{10}^* = .590$ and, in percent, $\Delta U_{10} = -12.1 = 32.1\Delta U_1$, and $\Delta N_{10} = -81 = 28\Delta N_1$.

Thus, welfare and *NR* losses under SC are an order of magnitude greater than under LC.

4.3. Two alternative production functions

Robustness of results is examined here under two alternative production functions.

4.3.1. Externality as function of log*L*

Assume now that the production function is

$$Q = L[\alpha - \beta(\log L)],\ L > 1. \tag{7A}$$

Under open access, $L = \frac{\mathbb{L}}{2}$, with $U = \frac{1}{2}\left[\alpha - \beta\left(\log\frac{\mathbb{L}}{2}\right)\right]^{1/2}$. The optimal value of L is

$$L^* = \frac{\mathbb{L}}{2}\left[1 - \frac{\beta}{2\alpha - \beta(1 + 2\log L^*)}\right] < L.^{29} \tag{8A}$$

The welfare cost for the central value of $\mathbb{L}$ under LC, HC and SC is (in percent) $\Delta U_{LC} = -4.3$, $\Delta U_{HC} = -17.7$, and $\Delta U_{SC} = -47.8$, i.e., $\Delta U_{SC} = 11.1\Delta U_{LC}$ and $\Delta U_{HC} = 4.1\Delta U_{LC}$. Thus, the welfare cost for $\mathbb{L}$'s central value under SC (HC) is greater by an order of magnitude than (a multiple of) that under LC.

[29] L^* has no closed-form solution as L^* is a function of $\log L^*$. For each value of $\mathbb{L}$, L^* was obtained by 'guessing' its level (denoted by x), obtaining $\log L^*$, and plugging $\log L^*$ in (8A) to obtain a solution for L^* (denoted by y), and verifying whether $y = x$. If not, I used a value for L^* between x and y, repeating the exercise until x and y converged.

4.3.2. Higher total factor productivity (TFP)

The production function now includes a TFP parameter ε, i.e.:

$$Q_\varepsilon = \varepsilon L_\varepsilon(\alpha - \beta L_\varepsilon),\ \varepsilon > 1. \quad (9A)$$

In equilibrium, $p_\varepsilon = AC_\varepsilon$, or $\frac{\mathbb{L}-L_\varepsilon}{\varepsilon L_\varepsilon(\alpha-\beta L_\varepsilon)} = \frac{1}{\varepsilon(\alpha-\beta L_\varepsilon)}$, and ε cancels out. Thus, $L_\varepsilon = L = \frac{\mathbb{L}}{2}$, $M_\varepsilon = M = \frac{\mathbb{L}}{2}$, $N_\varepsilon = N = \alpha - \beta\frac{\mathbb{L}}{2}$, with $Q_\varepsilon = \varepsilon Q$, and $U_\varepsilon = \varepsilon^{1/2}U.$ The same holds for the optimal values as ε also cancels out from the equation for the optimum, $\frac{\mathbb{L}-L_\varepsilon}{\varepsilon L_\varepsilon(\alpha-\beta L_\varepsilon)} = \frac{1}{\varepsilon(\alpha-2\beta L_\varepsilon)}$, implying that $L^*_\varepsilon = L^*$, $M^*_\varepsilon = M^*$, $N^*_\varepsilon = N^*$, $Q^*_\varepsilon = \varepsilon Q^*$ and $U^*_\varepsilon = \varepsilon^{1/2}U^*$. Hence, $\Delta x = \Delta x_\varepsilon$ $(x = L, N, Q, U)$.

Thus, the impact of open access on L, N, Q and U obtained with equation (9A) is identical to that obtained with equation (1).