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ABSTRACT

Modelling Artificial Intelligence in Economics

Economists' two main theoretical approaches to understanding Artificial Intelligence (AI) impacts have been the task-approach to labor markets and endogenous growth theory. Therefore, the recent integration of the task-approach into an endogenous growth model by Acemoglu and Restrepo (AR) is a useful advance. However, it is subject to the shortcoming that it does not explicitly model AI and its technological feasibility. The AR model focuses on tasks and skills but not on abilities, while abilities better characterize AI services' nature. This paper addresses this shortcoming by elaborating the task-approach with AI abilities for use within endogenous growth models. This more ability-sensitive specification of the task-approach allows for more nuanced and realistic impacts of progress in artificial intelligence (AI) on the economy to be captured.

JEL Classification: O47, O33, J24, E21, E25

Keywords: Artificial Intelligence, endogenous growth theory, labor economics, mathematical models

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1 Introduction

Until recently, there was relatively little research in economics on artificial intelligence (Agrawal et al., 2019). This situation is rapidly changing, however. Most progress so far has been on understanding the potential labor market implications of AI, in particular through the task-approach to labor markets (e.g., Autor (2013)). This approach has been used to evaluate fears that AI-automated job losses would cause mass unemployment, as for instance, raised by Frey and Osborne (2013).

Acemoglu and Restrepo (2018) incorporated the task-approach into an endogenous growth model, representing further progress in modelling AI in economics. The Acemoglu-Restrepo (AR) model, however, lacks adequate modelling of the nature of AI itself, as the task-approach is, in essence, naive about the nature of AI as automation technology. The AR-model does not explicitly capture AI and its technological feasibility, and focuses on tasks and skills but not on abilities, even though abilities may better characterize the nature of the services that AI provides.

This paper addresses this shortcoming by elaborating the task-approach with AI abilities for use within endogenous growth models. This more ability-sensitive specification of the task-approach allows for more nuanced and realistic impacts of AI progress on the economy to be captured.

The paper will proceed as follows. In section 2, we describe how automation is modelled in the task-approach to labor economics. In section 3, we present the core of the AR model which incorporates the task-approach into an endogenous growth setting. In section 4 go beyond the AR-model, providing a novel re-specification of the task-approach. Section 5 concludes.

2 The Task-Approach and Automation

The main conceptual approach used by economists to investigate the labor market impacts of automation has been the task-approach to labor economics, see e.g. Autor (2013) and Autor and Dorn (2013).

Defining a task as "a unit of work activity that produces output" (Autor and Dorn, 2013, p.186) a final good or services is produced according to a Constant Elasticity of Substitution (CES) production function, as in Autor and Dorn (2013, p.187):

$$Y = \left[\int_0^1 y(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \tag{1}$$

Where Y is the output of a final good, $y(i)$ the different tasks needed to produce the output Y, and η the elasticity of substitution between tasks. Because a task can be produced or performed by either low (L), medium (M), high-skilled (H) labor or capital (K), the production function for a task is, following Autor (2013):

$$y(i) = A_L \alpha_L l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i) \tag{2}$$

Where $l(i)$, $m(i)$, $h(i)$ are respectively the number of low, medium and high-skilled laborers doing task (i), and $k(i)$ the capital used for task (i). The productivity of labor and capital in a task (i) are expressed by α_L, α_M, α_H and α_K. The A represents a factor-augmenting technology in the carrying out of tasks.

According to Autor and Dorn (2013, pp.188-189) "the most important innovation offered by this task-based framework is that it can be used to investigate the implications of capital (embodied in machines) directly displacing workers from tasks that they previously performed."

If improvements in algorithms (AI) occur (reflected in A_K) the α_K would improve, not for all tasks, but for a specific range $(i) \subset [I', I]$ of tasks - maybe those than can be more easily codified, such as routine tasks. Then, if some of these tasks are performed by medium-skilled workers, some of the $m(i)$ will be replaced by machines, robots and computers - i.e. their tasks will be automated.

The replacement of workers is however not the only consequence of automation in the task-based approach. A further consequence is that it can lead to creation of new tasks and jobs, through what has been termed "reinstatement" effects. These are due to the positive supply-side effect of AI on productivity - which generates higher wages, profits and demand, and thus new jobs. Autor and Salomons (2018, pp.12-13) decompose these reinstatement effects into three categories, namely *Uber*, *Walmart*, and *Costco* effects.

A shortcoming of the task-approach is that the extent of the reinstatement effect is fundamentally uncertain. This is because it depends on (i) the extent of economic growth created by AI, and (ii) the extent to which economic growth stimulates the demand for labor, which in turn depends (iii) on growth in labor productivity, labor wages, and the income share of labor (Gries and Naudé, 2020).

Because the task-approach is not an economic growth model, it is unable to model these dynamic aspects. Standard endogenous growth models on the other hand, although capable of tracking economic dynamics, typically fail to make the distinction between jobs and tasks, and hence tend towards extreme and unrealistic outcomes, such as infinite (singularity) growth or a collapse in employment. The evident solution is to incorporate the task-approach, or something like the task-approach, into endogenous growth models. This is the contribution of Acemoglu and Restrepo (2018), which will be described in the next section.

3 The Acemoglu-Restrepo (AR) Growth Model

Acemoglu and Restrepo (2018) (henceforth the AR-model) proposed a production function (p.1494) similar to that in (1), where β is a constant and $y(i)$ a unit measure of tasks:

$$Y = \beta \left[\int_{N-1}^{N} y(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \tag{3}$$

Whereas Autor and Dorn (2013) specify the production factor of a task incorporating the automation technology A in (2) as a factor-augmenting technology, Acemoglu and Restrepo (2018) specify separate production functions for tasks that can be automated, and for tasks that cannot be automated but provided only with labor. This follows from their indexing (3) tasks ranging from $N-1$ to N so that there can be a point $I \in [N-1, N]$ with tasks $i \leq I$ that can be automated, and tasks $i > I$ that cannot be automated - the assumption is that labor has a comparative advantage in tasks high up in the index. For tasks $i > I$ they specify the following CES production function (p.1494):

$$y(i) = \beta(\zeta) \left[\eta^{\frac{1}{\zeta}} q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)^{\frac{1}{\zeta}} (\gamma(i) l(i)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{4}$$

And for tasks $i \leq I$ a similar specification is used, except with the inclusion now of capital (k), which is a perfect substitute for labor l with CES elasticity $\eta \in (0, 1)$:

$$y(i) = \beta(\zeta) \left[\eta^{\frac{1}{\zeta}} q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)^{\frac{1}{\zeta}} (k(i) + \gamma(i) l(i)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{5}$$

Where $\gamma(i)$ is the productivity of labor in task i, $\zeta \in (0, \infty)$ the elasticity of substitution between intermediate inputs (q) and labor inputs (l).

Where is artificial intelligence (AI) in this model?

AI is, as in the task-approach, not explicitly modelled; rather it is firstly contained in q, which the authors define as “a task-specific intermediate [...] which embodies the technology used either for automation or for production with labor.” Furthermore, technological progress (e.g. progress in AI) can be of two kinds: it can either make more tasks amendable to automation (reflected in a shift of I) or transform old tasks that could be automated into new tasks in which labor has a comparative advantage, reflected in an increase in $N - I$ and a reduction in $I - (N - 1)$. In a static version of the AR-model, k is fixed and technology (including AI) exogenous. As such, technological innovation changes the allocation of tasks between capital and labor, and this in turn will change relative factor prices - with consequences for employment and the wage share of labor.

With these production functions for tasks carried over into a dynamic setting, Acemoglu and Restrepo (2018) endogenize capital and technological progress, and tease out the long-run implications of automation on jobs and inequality. Now, the price of capital relative to the wage rate will determine the extent to which new tasks are created, and they show that a stable balanced growth path is possible if progress in automation and creation of new tasks are equal. Any deviations from this will set corrective market forces in operation. In other words, the reinstatement effect (creation of new tasks) of the static task-approach continues to hold. As they put it (Acemoglu and Restrepo, 2018, p.1491) “This stability result highlights a crucial new force: a wave of automation pushes down the effective cost of producing with labor, discouraging further efforts to automate additional tasks and encouraging the creation of new tasks.”

4 Human Service Production

The AR-model, despite being a welcome addition to the economic growth literature concerned with AI, has two shortcomings for present purposes. The first, and more serious shortcoming, is that it does not take into account the specific nature of AI and its technological feasibility. A second, but less serious shortcoming, is that it is a rather complex model. Acemoglu and Restrepo (2018) do not provide a definition of AI, and as was noticed in the previous section AI is not explicitly modelled. As such, the task-approach as it is incorporated into the AR-model cannot model some of the distinguishing features of AI as an automation technology, which is different from the way in which robotics for instance automate jobs. Moreover, the AR-model seems to be more consistent with pre-AI automation technologies for which the task-approach to labor markets was initially developed - before the advent of Machine Learning (ML)-based AI, which started only after 2006/2007 (Naudé, 2021).

The task-approach is useful, but - in the context of the nature of AI - cannot simply transformed 1:1 into an AI-model. It focuses on tasks and skills but not on *abilities*, while abilities better characterizes the nature of the services that AI provide (Hernández-Orallo, 2017). According to Tolan et al. (2020, p.6-7) abilities are "a better parameter to evaluate progress in AI" because ML provide abilities to do tasks, and not skills, which are a human attribute requiring experience, knowledge, and common sense. Skills are not an attribute of AI. This means that, with AI providing abilities, such as the ability to understand human language or recognize objects, it is necessary to go beyond skills and tasks when evaluating any labor market impacts of AI, because the adoption of AI will ultimately depend on its abilities *relative* to the abilities of human labor.

Some abilities may be more (or less) likely to be provided by AI which means that "AI may cause workplaces to transform the way a task is performed" (Tolan et al., 2020, p.6). In other words, the technological feasibility of AI in automation will depend on the extent that is changes the very nature of tasks. This however cannot be modelled adequately by the task approach to labor markets and its incorporation into the AR-model. In the remainder of this section we attempt to address this shortcoming by proposing a model of AI as providing *abilities* in line with Tolan et al. (2020).

4.1 Human service as intermediate good

If AI is essentially a technology that provides certain abilities, it will always need to be combined or used in tandem with skills, which are, as we pointed out above, distinct human attributes requiring experience, knowledge and common sense. We define this combination of AI and human skills as *human services*, H. To be precise, a human service is an intermediate service good that is generated by variously skilled human labor and AI. Human service $[H = H(Labor, AI)]$ is produced following the task-approach to labor markets specification; however it can be easily included in any conventional production function leading to a nested production process $Y = Y(H, K)$. Due to this nested structure, the *human service task-approach* that we propose here allows us to analyze and separately discuss effects specific to the task-approach, without much increase in model complexity. Thus, a shortcoming of the AR modeling - its high complexity - is (somewhat) addressed.

The human service production function can be written as $H = H(L_L, A_L, A_{IT}, B_{IT})$. Here L_L is the number of workers each providing given hours of work, A_L is an index of human skills (reflecting experience and human abilities), A_{IT} is the total number of ML abilities (e.g. algorithms) in the economy, and B_{IT} are the IT-business owners or experts providing and running AI services. Hence, our approach enriches and extends the simple task-approach by integrating human skills with AI abilities, as per the arguments of Hernández-Orallo (2017) and Tolan et al. (2020).

The production function for human services can be specified as (note the similarities as well as the differences with (3) and (4) and (5)):

$$H = \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where z denotes each task in a unit interval $[N-1, N]$, and $h(z)$ is the output of task z. As tasks range between $N-1$ and N, the total number of tasks is constant. Note that whereas Acemoglu and Restrepo (2018) define total production as result of N-1 to N tasks, we propose to define total production as the result of human service inputs and other inputs like capital or other intermediates, where human service inputs consists of the outputs of different tasks. Further, with L_L and B_{IT} we separate between labor and owners respectively providers of AI as a more or less disembodied productive technology.

Each task z can either be produced with labor, $l(z)$, or only with AI services provided by AI-businesses, $b_{IT}(z)$, if the task can be done by AI. Therefore, there are two sets of tasks. Tasks $z \in [N-1, N_{IT}]$ can be produced by both labor and AI services [described by process (a) in (7)], and tasks $z \in (N_{IT}, N]$ can only be produced by labor [process (b) in (7)]. These tasks can be the niche in which labor can continue to specialize in the presence of AI driven services or automation, as per Arntz et al. (2017). Thus, the output of a task can be stated in two ways

$$h(z) = \begin{cases} \gamma_L(z) l(z) A_L + \gamma_{IT}(z) b_{IT}(z) A_{IT} D & \text{process (a) if } z \in [N-1, N_{IT}] \\ \gamma_L(z) l(z) A_L & \text{process (b) if } z \in (N_{IT}, N] \end{cases} \tag{7}$$

Note that the production process (a) implies perfect substitution of human labor abilities by AI, as the human labor ability $(A_L \gamma_L(z) l(z))$ is not a necessary input for this task. To provide further justification for the specification in (7) we can note the following:

First, while $l(z)$ is the volume of hours employed in the specific task z, A_L is a description of generally available skills, which includes human *abilities* and *experiences.* So humans who are employed, irrespective which tasks they perform, are endowed with A_L. Humans can identify problems, understand social signals and social interactions, detect and handle positive and negative social externalities in groups, can use common sense, and can think ahead. These very human skills have emerged over hundreds of thousands of years of biological evolution interacting with the environment and culture, including education. As these human skills indexed by A_L are homogeneously related to all human labor L_L this endowment is potentially available in each task z without rivalry and similar to a public good, $A_L l(z)$. However, in some tasks these human skills are particular valuable while in others they are not really needed. This task specific productivity is indicated by $\gamma_L(z)$. Thus, in (7) total human contribution to a task is $\gamma_L(z) A_L l(z)$.

Second, as far as production with AI is concerned, A_{IT} denotes the total number and quality of ML algorithms or *machine abilities* in the economy that can provide a general AI service. The idea here is that an AI service contains two components. One is a general AI algorithm or code and the other is specific application of the algorithm based on particular data. For example, A_{IT} would include various generic Machine Learning (ML) models and techniques, from logistical regressions to Deep Learning (DL) and Convolutional Neural Networks(CNN). These algorithms are non-specific with respect to a particular domain of usage. As such they can be used without rivalry, and to the extent that they may be excludable through licensing may have the characteristics of a club good. Since ML algorithms are trained on data (training can be either supervised or unsupervised by a skilled human), data D is the raw material needed to produce an AI service. Hence, we can denote the complementarity between data and algorithms as $A_{IT} D$, which is the fundamental infrastructure for specific AI services. Since the use of data is non-rival, $A_{IT} D$ is a club good. Note however that $A_{IT} D$ is yet not an AI service. The AI service is obtained when $A_{IT} D$ is applied to a particular task - where it creates value. This application is facilitated by IT experts $b_{IT}(z)$ who tailor $A_{IT} D$

for a particular purpose or business opportunity, z, adding up in total to B_{IT} applications.

Finally, AI services that have been tailored to a particular task will be characterised by different levels of task-specific productivity, $\gamma_{IT}(z)$. In total therefore, AI services production for a particular task z can be described as $\gamma_{IT}(z)b_{IT}(z)A_{IT}D$ in (7a).

If a task z with price $p_h(z)$ is produced with pure labor $h(z) = A_L\gamma_L(z)l(z)$, and labor rewards are calculated according to marginal productivity, then $p_h(z)A_L\gamma_L(z) = w_L$. Symmetrically, the same task could be produced with an AI technology so that $p_h(z)A_{IT}\gamma_{IT}(z) = w_{IT}$, with w_{IT} as the reward for the AI supplying expert or business. Given these two conditions, and given wages in the market, for any particular task the firm will choose the kind of service composition (AI service/automation or not) that results in the lowest unit labor costs. Thus, if the following condition holds, the task will be provided by the AI service:

$$\frac{w_{IT}}{p_h(z)A_{IT}\gamma_{IT}(z)} < \frac{w_L}{p_h(z)A_L\gamma_L(z)}$$

This rule leads to condition (8) which identifies the switching point between automated (AI) tasks and labor tasks. If tasks are ordered in such a way that $\frac{A_L\gamma_L(z)}{A_{IT}\gamma_{IT}(z)}$ is increasing in z and the tasks with lower numbers $z \in [N-1, N_{IT}]$ are the automated tasks, task N_{IT} is the switching point from an automation task to a labor task. N_{IT} is the highest number in this order for which

$$\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} < \frac{w_L}{w_{IT}} \tag{8}$$

holds. Apart from these automated (AI) tasks $[N-1, N_{IT}]$, all other tasks $(N_{IT}, N]$ are produced with standard labor. Thus, the costs and respectively the price $p_h(z)$ for any task z is

$$p_h(z) = \begin{cases} \frac{w_{IT}}{A_{IT}\gamma_{IT}(z)} & \text{if } z \in [N-1, N_{IT}] \\ \frac{w_L}{A_L\gamma_L(z)} & \text{if } z \in (N_{IT}, N] \end{cases} \tag{9}$$

We can use this to calculate the endogenous optimal number of *tasks provided by AI* in an economy with an efficient supply of the human sevice:[1]

$$N_{IT} = N_{IT}\left(B_{IT}, L_L, A_{IT}, ...\right), \text{ with } \frac{dN_{IT}}{dB_{IT}} > 0, \quad \frac{dN_{IT}}{dL_L} < 0, \quad \frac{dN_{IT}}{dA_{IT}} > 0 \tag{10}$$

This result indicates that the number of automated/machine produced tasks crucially depends on the relative availability of various input factors which are important for AI technologies. The extent of implementation and diffusion of AI technologies and automation of human services will depend on the relative availability of the specific inputs of the human service production. In particular we have to look at the relative availability of *human skills* to *machine abilities* A_L/A_{IT}; the relative abundance of the volume of *labor* to *AI-supplying experts* L_L/B_{IT}; the relative task-specific productivity at the switch point $\gamma_L(N_{IT})/\gamma_{IT}(N_{IT})$; and the volume and veracity of data available to run all these AI services, D. Thus, our modelling of AI provides a level of detail of specification that is lacking in the AR-model.

4.2 Optimal human service supply

From the demands for the various tasks, total human service production can be derived. Aggregating automated tasks and labor, equation (6) leads to

$$H = \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}.$$

[1]See the appendix.

Using (13), (18) and 19), respectively, and re-arranging gives the expression for total production of human services as:[2]

$$H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_{IT} L_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz \right)^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

With the definitions $\Gamma_{IT}(N_{IT}, N) = \int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz$ and $\Gamma_L(N_{IT}, N) = \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz = \Gamma(N_{IT}, N)\Pi(N_{IT}, N)^{\sigma-1}$ we can rewrite the aggregate optimal human service production as

$$H = \left[(\Gamma_{IT}(N_{IT}, N))^{\frac{1}{\sigma}} (A_{IT} L_{IT})^{\frac{\sigma-1}{\sigma}} + \Gamma_L(N_{IT}, N)^{\frac{1}{\sigma}} (A_L L_L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

This expression is similar to the familiar Constant Elasticity of Supply (CES) production function.

5 Concluding Remarks

In this paper, we contributed to the modeling of Artificial Intelligence (AI) in economics by adapting the task-approach to labor markets to reflect the distinctiveness of AI not as a task or skill, but as an ability. Our ability-sensitive specification of the task-approach allowed us to model in a more detailed and more nuanced manner, the labor market consequences of AI progress. A critical insight from our reformulation is that an economy will broadly (large N_{IT}) utilize AI technologies if (i) the economy is relatively abundant in sophisticated programs and machine abilities compared to human skills; (ii) the economy hosts a relatively large number of AI-providing businesses and experts; and (iii) the task-specific productivity of AI services are relatively high compared to the task-specific productivity of general labor and labor skills. Further, as access to data is essential for task-specific AI in our model, its

[2] For details see the Appendix.

relative abundance will be another determinant the diffusion of AI.

From a country perspective, the comparative abundance of the factors identified in the previous paragraph will determine the composition of human service in that country. If, for instance, IT experts or business solutions are widely available, more tasks will be automated. Similarly, if IT knowledge and AI algorithms are readily available, relative wages $\frac{w_L}{w_{IT}}$ will increase, and human labor tasks will become relatively more expensive, furthering automation.

Thus, our modification of the task-approach to labor markets provides not only a more detailed, but also a more nuanced description of AI automation, by describing how progress in AI abilities can turn more tasks into tasks that are amenable to automation.

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Appendices

Efficient production of human services

Optimal allocation within the task approach: Human service firms

$$\max : \pi_H = p_H H - p_h(z)h(z) = p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} - p_h(z)h(z).$$

F.O.C.

$$\begin{aligned}
p_H \frac{\sigma}{\sigma-1} \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} h(z)^{\frac{\sigma-1}{\sigma}-1} - p_h(z) = 0 \\
p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}-1} h(z)^{\frac{\sigma-1}{\sigma}-1} = p_h(z) \\
p_H \left(\int_{N-1}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{1}{\sigma-1}} h(z)^{-\frac{1}{\sigma}} = p_h(z) \\
p_H H^{\frac{1}{\sigma}} h(z)^{-\frac{1}{\sigma}} = p_h(z)
\end{aligned}$$

arriving at

$$h(z) = \frac{H}{p_h(z)^{\sigma}} p_H^{\sigma}, \tag{12}$$

see (12).

Demand for task z : Using marginal production and productivity rules

$h(z_{IT}) = A_{IT}\gamma_{IT}(z)l_{IT}(z)$	production (7)	$h(z_L) = A_L\gamma_L(z)l_L(z)$
$p_h A_{IT}\gamma_{IT}(z)l_{IT}(z) = l_{IT}(z)w_{IT}$	marginal productivity and factor reward	$p_h A_L\gamma_L(z)l_L(z) = l_L(z)w_L$
$p_h(z_{IT}) = \frac{w_{IT}}{A_{IT}\gamma_{IT}(z_{IT})}$	price = unit labor costs	$p_h(z_L) = \frac{w_L}{A_L\gamma_L(z_L)}$

and plugging in gives (13) as being the optimal demand for $h(z)$,

$$\begin{aligned}
h(z) &= \frac{H}{\left(\frac{w_{IT}}{A_{IT}\gamma_{IT}(z)}\right)^{\sigma}} p_H^{\sigma}, & h(z) &= \frac{H}{\left(\frac{w_L}{A_L\gamma_L(z)}\right)^{\sigma}} p_H^{\sigma}, \\
h(z) &= p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_{IT}(z)^{\sigma}, & h(z) &= p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma} \gamma_L(z)^{\sigma}.
\end{aligned} \tag{13}$$

Demand for each kind of labor in task z : In order to determine the marginal productivity for each total labor input, the productivity for each kind of labor is derived from (13) and (7), and we can obtain the optimal demand for IT labor :

$$A_{IT}\gamma_{IT}(z)l_{IT}(z) = h(z) = p_H^\sigma H\left(\frac{A_{IT}}{w_{IT}}\right)^\sigma \gamma_{IT}(z)^\sigma,$$

$$l_{IT}(z) = \begin{cases} \left(\frac{p_H}{w_{IT}}\right)^\sigma H\left(A_{IT}\right)^{\sigma-1}\gamma_{IT}(z)^{\sigma-1} & \text{if } z \in [N-1, N_{IT}] \\ 0 & \text{if } z \in [N_{IT}, N] \end{cases} \tag{14}$$

and standard labor:

$$A_L\gamma_L(z)l_L(z) = h(z) = p_H^\sigma H\left(\frac{A_L}{w_L}\right)^\sigma \gamma_L(z)^\sigma$$

$$l_L(z) = \begin{cases} 0 & \text{if } z \in [N-1, N_{IT}] \\ \left(\frac{p_H}{w_L}\right)^\sigma H\left(A_L\right)^{\sigma-1}\gamma_L(z)^{\sigma-1} & \text{if } z \in (N_{IT}, N] \end{cases} \tag{15}$$

Total IT labor is fully employed and allocates to all tasks using IT labor. This holds for standard labor respectively

$$L_{IT} = \int_{N-1}^{N_{IT}} l_{IT}(z)dz, \text{ and} \tag{16}$$

$$L_L = \int_{N_{IT}}^{N} l_L(z)dz. \tag{17}$$

Income of IT expert w_{IT}: expert With the integral in (14) $[l_{IT}(z) = \frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}]$ we obtain

$$\begin{aligned}
\int_{N-1}^{N_{IT}} l_{IT}(z)dz &= \int_{N-1}^{N_{IT}} \frac{p_H^\sigma}{w_{IT}^\sigma}H\gamma_{IT}(z)^{\sigma-1}\left(A_{IT}\right)^{\sigma-1}dz \\
L_{IT} &= \frac{p_H^\sigma}{w_{IT}^\sigma}H\left(A_{IT}\right)^{\sigma-1}\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz \\
w_{IT}^\sigma &= p_H^\sigma\frac{H}{L_{IT}}\left(A_{IT}\right)^{\sigma-1}\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz
\end{aligned}$$

such that with full employed IT labor we can determine the wages of IT labor as

$$w_{IT} = p_H\left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}}\left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}}\left(\int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}, \tag{18}$$

Symmetrically for standard labor,

$$
\begin{aligned}
\int_{N_{IT}}^{N} l_L(z)dz &= \int_{N_{IT}}^{N} \frac{p_H^\sigma}{w_{IT}^\sigma} H\gamma_{IT}(z)^{\sigma-1} \left(A_{IT}\right)^{\sigma-1} dz \\
L_{IT} &= \frac{p_H^\sigma}{w_{IT}^\sigma} H \left(A_{IT}\right)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_{IT}(z)^{\sigma-1}dz \\
w_{IT}^\sigma &= p_H^\sigma \frac{H}{L_{IT}} \left(A_{IT}\right)^{\sigma-1} \int_{N_{IT}}^{N} \gamma_{IT}(z)^{\sigma-1}dz
\end{aligned}
$$

$$
w_L = p_H \left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}} \left(A_L\right)^{\frac{\sigma-1}{\sigma}} \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}. \tag{19}
$$

The resulting internal relative factor productivity for labor is:

$$
\begin{aligned}
\frac{w_L}{w_{IT}} &= \frac{\left(\frac{p_H H}{L_L}\right)^{\frac{1}{\sigma}} \left(A_L\right)^{\frac{\sigma-1}{\sigma}} \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}}{\left(\frac{p_H H}{L_{IT}}\right)^{\frac{1}{\sigma}} \left(A_{IT}\right)^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz\right)^{\frac{1}{\sigma}}} \\
\frac{w_L}{w_{IT}} &= \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}}
\end{aligned}
$$

Endogenous switch to AI/automated tasks N_{IT} : From the discussion of (8) it is known that tasks are ordered such that $\gamma(z) = \frac{\gamma_L(z)}{\gamma_{IT}(z)}$, and $\frac{\partial \gamma(z)}{\partial z} > 0$. If it is assumed that task N_{IT} is the task that exactly separates the production mode, and if tasks are continues, the condition (8) can be rewritten as follows:

$$
\begin{aligned}
\frac{A_L\gamma_L(N_{IT})}{A_{IT}\gamma_{IT}(N_{IT})} &< \frac{w_L}{w_{IT}} = \left(\frac{L_{IT}}{L_L}\right)^{\frac{1}{\sigma}} \left(\frac{A_L}{A_{IT}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \\
0 &= G = \gamma\left(N_{IT}\right) - \left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}} \left(\frac{\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \qquad (20)
\end{aligned}
$$

If $\frac{dG}{dN_{IT}} \neq 0$, G implicitly defines a function $N_{IT} = N_{IT}\left(L_{IT}, L_L, A_{IT}, ...\right)$.Thus, we need to calculate the respective interesting derivatives.

$$\frac{dG}{dN_{IT}} = \frac{\partial \gamma\left(N_{IT}\right)}{\partial N_{IT}} + \left[\begin{array}{c} \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz}{\int_{N-1}^{I}\gamma_{IT}(z)^{\sigma-1}dz}\right)^{\frac{1}{\sigma}} \\ \left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{\int_{N_{IT}}^{N}\gamma_L(N_{IT})^{\sigma-1}dz} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\int_{N-1}^{N_{IT}}\gamma_{IT}(N_{IT})^{\sigma-1}dz}\right] \end{array} \right] > 0$$

and defining $\Gamma_{IT}\left(N_{IT}\right) = \int_{N-1}^{N_{IT}}\gamma_{IT}(z)^{\sigma-1}dz$, $\frac{d\Gamma_{IT}}{dN_{IT}} = \gamma_{IT}(N_{IT})^{\sigma-1}$; and $\Gamma_L\left(N_{IT}\right) = \int_{N_{IT}}^{N}\gamma_L(z)^{\sigma-1}dz$, $\frac{d\Gamma_L}{dN_{IT}} = -\gamma_L(N_{IT})^{\sigma-1}$ we obtain

$$\begin{aligned} \frac{\partial G}{\partial N_{IT}} &= \frac{\partial \gamma\left(N_{IT}\right)}{\partial N_{IT}} + \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\Gamma_L\left(N_{IT}\right)}{\Gamma_{IT}\left(N_{IT}\right)}\right)^{\frac{1}{\sigma}}\left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{\Gamma_L\left(N_{IT}\right)} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\Gamma_{IT}\left(N_{IT}\right)}\right] > 0 \\ \frac{\partial G}{\partial A_{IT}} &= -\frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}-1}\left(\frac{\Gamma_L\left(N_{IT}\right)}{\Gamma_{IT}\left(N_{IT}\right)}\right)^{\frac{1}{\sigma}}\frac{L_{IT}}{A_L L_L} < 0 \end{aligned}$$

and the derivative of the implicit function $N_{IT} = N_{IT}\left(A_{IT}\right)$ is

$$\frac{dN_{IT}}{dA_{IT}} = -\frac{\frac{\partial G}{\partial A_{IT}}}{\frac{\partial G}{\partial N_{IT}}} > 0$$

More specific:

$$\begin{aligned} \frac{dN_{IT}}{dA_{IT}} &= \frac{\frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\Gamma_L(N_{IT})}{\Gamma_{IT}(N_{IT})}\right)^{\frac{1}{\sigma}}\frac{1}{A_{IT}}}{\frac{\partial\gamma(N_{IT})}{\partial N_{IT}} + \frac{1}{\sigma}\left(\frac{A_{IT}L_{IT}}{A_L L_L}\right)^{\frac{1}{\sigma}}\left(\frac{\Gamma_L(N_{IT})}{\Gamma_{IT}(N_{IT})}\right)^{\frac{1}{\sigma}}\left[\frac{\gamma_L(N_{IT})^{\sigma-1}}{\Gamma_L(N_{IT})} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\Gamma_{IT}(N_{IT})}\right]} \\ \eta_{N_{IT},A_{IT}} &= \frac{dN_{IT}}{dA_{IT}}\frac{A_{IT}}{N_{IT}} = \frac{1}{\sigma\frac{\partial\gamma(N_{IT})}{\partial N_{IT}}\left(\frac{A_L L_L}{A_{IT}L_{IT}}\frac{\Gamma_{IT}(N_{IT})}{\Gamma_L(N_{IT})}\right)^{\frac{1}{\sigma}} + \frac{\gamma_L(N_{IT})^{\sigma-1}}{\Gamma_L(N_{IT})} + \frac{\gamma_{IT}(N_{IT})^{\sigma-1}}{\Gamma_{IT}(N_{IT})}}\frac{1}{N_{IT}} \end{aligned}$$

Total supply of human service inputs

From (13) it is known that $h(z) = p_H^{\sigma}H\left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\gamma_{IT}(z)^{\sigma}$ for $z \in [N-1, N_{IT}.$, and $h(z) = p_H^{\sigma}H\left(\frac{A_L}{w_L}\right)^{\sigma}\gamma_L(z)^{\sigma}$ for $z \in [N_{IT}, N]$. Plugging this in (6) generates an expression for the total value of H:

$$
\begin{aligned}
H &= \left(\int_{N-1}^{N_{IT}} h(z)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} h(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_{N-1}^{N_{IT}} \left(p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma} \gamma_I(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz + \int_{N_{IT}}^{N} \left(p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma} \gamma_L(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}.
\end{aligned}
$$

Using (18) and (19) results in: $w_{IT} = p_H \left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}} (A_{IT})^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}$

$$
\begin{aligned}
H &= \left(\int_{N-1}^{N_{IT}} \left(\gamma_{IT}(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz \left(p_H^{\sigma} H \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} + \int_{N_{IT}}^{N} \left(\gamma_L(z)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} dz \left(p_H^{\sigma} H \left(\frac{A_L}{w_L}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad (21) \\
&= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{w_{IT}}\right)^{\sigma-1} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{w_L}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

$$
= \left(\begin{array}{l}
\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{IT}}{p_H \left(\frac{H}{L_{IT}}\right)^{\frac{1}{\sigma}} (A_{IT})^{\frac{\sigma-1}{\sigma}} \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \\
+ \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{A_L}{p_H \left(\frac{H}{L_L}\right)^{\frac{1}{\sigma}} (A_L)^{\frac{\sigma-1}{\sigma}} \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1}
\end{array}\right)^{\frac{\sigma}{\sigma-1}}
$$

$$
= \left(\begin{array}{l}
\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_{IT}^{\frac{1}{\sigma}} A_{IT}^{\frac{1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1} \\
+ \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \left(\frac{p_H^{-1} H^{-\frac{1}{\sigma}} L_L^{\frac{1}{\sigma}} A_L^{\frac{1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}}}\right)^{\sigma-1}
\end{array}\right)^{\frac{\sigma}{\sigma-1}}
$$

$$
= \left(\begin{array}{l}
\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} \\
+ \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz p_H^{\sigma-1} H^{\frac{\sigma-1}{\sigma}} \frac{p_H^{-(\sigma-1)} H^{-\frac{\sigma-1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}}
\end{array}\right)^{\frac{\sigma}{\sigma-1}}
$$

$$
= \left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz \frac{(L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}} + \int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} d \frac{(L_L A_L)^{\frac{\sigma-1}{\sigma}}}{\left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{\sigma}{\sigma-1}}
$$

$$
H = \left(\left(\int_{N-1}^{N_{IT}} \gamma_{IT}(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}} (L_{IT} A_{IT})^{\frac{\sigma-1}{\sigma}} + \left(\int_{N_{IT}}^{N} \gamma_L(z)^{\sigma-1} dz\right)^{\frac{1}{\sigma}} (L_L A_L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

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