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ISSN: 2365-9793

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ABSTRACT

Motivational Goal Bracketing with Non-rational Goals*

We provide a tractable model of motivational goal bracketing by a present-biased individual, extending previous work to show that the main insights from models with rational goals carry over to a setting with non-rational goals. Goals motivate because they serve as reference points that make substandard performance psychologically painful. A broad goal allows high performance in one task to compensate for low performance in the other. This partially insures against the risk of falling short of ones' goal(s), but creates incentives to shirk in one of the tasks. Narrow goals have a stronger motivational force and thus can be optimal, providing an explanation for observed instances of narrow bracketing. In particular, if one task outcome becomes known before working on the second task, narrow bracketing is always optimal.

JEL Classification: A12, C70, D91

Keywords: non-rational goals, multiple tasks, motivational bracketing, self-control

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* Declarations of interest: none.

1 Introduction

People often set narrow rather than broad goals. For example, a student might set himself the narrow goals to "achieve a score of 60% in each exam this term", rather than the broad goal to "get an average score of 60% over all exams this term". When viewed through the lens of standard economic preferences, such narrow goals are at odds with maximizing behavior (e.g., Read et al., 1999). For individuals with time inconsistent preferences, however, narrow goals can help to overcome self-control problems – a process for which Read et al. (1999) coined the term *motivational bracketing*.

In previous work, we have given a theoretical and experimental foundation for the motivational bracketing hypothesis, by showing that narrow goals can outperform broad goals when individuals face uncertainty and work on multiple tasks simultaneously (Koch and Nafziger, 2016) or sequentially (Koch and Nafziger, 2020). In both studies, we assume that goals are rational in the sense that individuals set effort goals that correspond to the effort that the individual actually expects to exert given these goals. The goals induce rational expectations about the benefits (such as task output) and costs (such as effort costs) that then serve as reference points in the utility evaluation of the realized outcomes.

Whether goals are rational or not is an open issue (see, e.g., Allen et al., 2017). In our previous empirical work, we indeed find that a large fraction of participants do not reach their goals and that this cannot be fully explained by naiveté (Koch and Nafziger, 2020).[1] In the present paper, we demonstrate that the key intuitions and effects from these studies carry over to a model with non-rational goals. That is, the goals that the individual takes as a reference standard need not correspond to the outcomes that the individual expects. A further aim of the paper is to provide an easily applicable model of goal bracketing.

Our model is built around a self-control problem that arises from a present bias (Laibson, 1997; O'Donoghue and Rabin, 1999). For instance, an individual may judge that going for morning runs is in his long-run interest. But come the start of a day, the distant health benefits suddenly do not seem worth the effort. The preference reversal occurs because the costs become more salient when they are immediate and loom larger than the gains which only accrue in the future.

Personal goals play an important role in helping to overcome such motivational problems. Bandura, for example, writes that "the regulation of motivation by goal setting is a remarkably robust phenomenon" (cf. p.xii of his foreword to Locke and Latham, 1990). Goals serve as reference points against which actual task outcomes are measured, and people display loss aversion regarding goal achievement (for a foundation of this idea in the psychology literature see Heath et al., 1999; Locke and Latham, 2002). Falling short of a goal hence causes a

[1]In that paper we apply the idea of "perception-perfect" strategies O'Donoghue and Rabin (1999). A (partially) naïve individual forms expectations that are consistent with the goals. But expectations are not rational because they are based on a biased belief about the present bias of the individual.

loss that weighs more heavily than the gain from achieving the goal. Consequently, setting a higher goal today raises the motivation of one's future self to work hard. Yet, a higher goal also increases the chances of falling short of the performance standard because of bad luck and thereby suffering a loss. So goals are painful self-disciplining devices whenever there is uncertainty whether they will be achieved; and this limits their attraction for self-regulation (see Koch and Nafziger, 2011).

The key question that we address is how an individual uses goals when facing multiple tasks. Does an individual prefer to set narrow goals (i.e., to measure each task outcome against a separate performance standard) or a broad goal (i.e., to evaluate the combined task outcomes against a global performance standard)? We do so in a model with discrete effort where an individual set non-rational output goals, complementing our previous work addressing this question in the context of rational goals (Koch and Nafziger, 2016, 2020).

Our analysis starts by looking at the case where an individual works sequentially on two independent tasks, but does not learn about the outcome of the first task before deciding how much effort to put into the second task (the *no-information scenario*). We then turn to the case where the individual receives intermediate information (the *information scenario*). Intuitively, in the no-information scenario, a broad goal seems appealing because it pools outcomes across tasks and thus provides a partial hedge against the risk of falling short of the goal despite best efforts: A high outcome in one task can compensate for a low outcome in the other task. With narrow goals, in contrast, the individual suffers a loss in a task as soon as the respective task outcome is low. Due to this *risk-pooling effect*, the expected cost of using a broad goal is lower than that for corresponding narrow goals.

The possibility of compensating for low performance in one task with high performance in the other task however also makes it more attractive to "lean back": If the individual shirks in one of the tasks, he can still achieve the broad goal and avoid a loss. But the fear of facing such a loss when shirking is where the incentive effect of a goal stems from.[2] So a broad goal provides lower motivation for working hard in both tasks than corresponding narrow goals. Our model shows that the *leaning-back effect* alone is not sufficient for narrow goals to always be optimal. There still are cases where the risk-pooling effect dominates, so that a broad goal performs better than narrow goals.[3]

An additional effect arises if the individual receives intermediate information between the tasks. Again, a broad goal allows for low performance in one task to be offset by high

[2]In line with this, Heath et al. (1999) observe that workers are twice as willing to provide effort to just meet a given goal than they are willing to work toward surpassing the goal.

[3]A similar effect arises also in the model with rational goals by Koch and Nafziger (2016). Yet, here broad goals may also have a positive effect on incentives, which is due to way goals create stochastic reference points that then are incorporated into the expectations formed in a personal equilibrium. This positive incentive effect may dominate the negative one for a low probability of success. Still it is shown that narrow goals can be optimal.

performance in the other task, giving rise to the leaning-back effect. But, because the individual can observe the outcome of the first task before working on the second task, a second effect arises that further undermines the motivating force of a broad goal. If the individual observes that he was successful in the first task and has already achieved his goal, he will "rest on his laurels" and put in low effort in the second task. In contrast, if he observes a failure in the first task, he will work hard in the second task to still meet the goal. This asymmetric response, in turn, exacerbates the temptation to shirk in the first task as the present-biased individual rather wants to work hard tomorrow than feel the effort costs today – a process referred to as *effort substitution* in Koch and Nafziger (2020). To overcome this temptation, the broad goal has to be much higher than what would be needed if the individual received no intermediate information. As we show, the effort substitution effect and its adverse impact on first-period incentives make the broad goal perform worse than narrow goals: in the information scenario, narrow bracketing is always optimal in our model.

The paper is organized as follows. Next, we discuss the related literature. Section 2 introduces our model of goal setting in an environment with multiple tasks, which is based on the single-task model in Koch and Nafziger (2011). This section also prepares the analysis of the no-information and information scenarios, which follow in Sections 3 and 4, respectively. In Section 5 we address the robustness of our key insights to incorporating into the model diminishing sensitivity or continuous output and effort. Section 6 concludes the paper.

Related Literature. We theoretically analyze how individuals choose and design goals for themselves, thereby contributing to the literature in economics on goal setting in single tasks when individuals face self-control problems (Koch and Nafziger, 2011; Jain, 2009; Suvorov and van de Ven, 2008; Hsiaw, 2013). Specifically, we extend our model of non-rational goals (Koch and Nafziger, 2011) to multiple tasks to study whether broad or narrow goals are better self-regulation tools. In doing so, we relate to Shefrin and Thaler (1988) and Fudenberg and Levine (2006) who study narrow bracketing as a self-regulation tool.[4] Next to the aforementioned studies by Koch and Nafziger (2016) and Koch and Nafziger (2020), most closely related to our study are Jain (2009) and Hsiaw (2018). The latter extends her single-task model (Hsiaw, 2013) to study goal bracketing with two sequential continuous-time optimal stopping problems. In contrast to our model, costs and benefits from a project accrue at the same time, and it is the tension between the option value of waiting versus stopping today at a known project value that gives rise to a self-control problem. Goals help counter the tendency of the present-biased individual to stop projects too early. So the intuition is quite different from the one developed in our model. First, narrow goals allow better regulation of stopping times relative to a broad goal, but cause disutility from frequent

[4]Another, complementary, explanation for narrow mental accounts in consumption decisions is limited attention (cf. Kőszegi and Matějka, 2020).

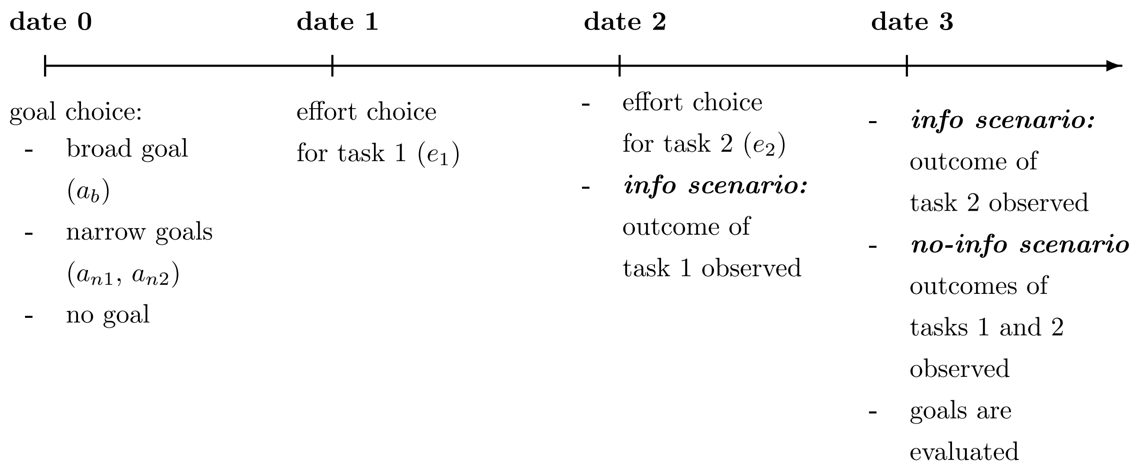

Figure 1: Timing

goal evaluation. Second, as Hsiaw (2018) assumes that the agent receives no intermediate information between the periods, the effort substitution effect that arises in our model plays no role in her results. Jain (2009) provides a two-period model sequential task model with deterministic output, where the individual sets an overall effort goals for both tasks. As output is deterministic, the risk pooling and leaning back effects cannot arise. Further, Jain does not compare narrow and broad goals.

2 The model

The tasks. The individual faces two symmetric tasks $i \in \{1, 2\}$. Each task leads to an outcome $y_i \in \{\underline{y}, \bar{y}\}$, where $\bar{y} > \underline{y}$. We normalize $\underline{y} \equiv 0$. For each task, the individual can choose whether to work hard ($\bar{e}_i = 1$) or shirk ($\underline{e}_i = 0$). Effort causes an instantaneous utility cost $c \cdot e_i$. The outcome of a task depends on both effort and luck: effort in task i leads to a successful outcome $y_i = \bar{y}$ with probability $p \cdot e_i$, where $p \in (0, 1)$. The outcome realization for each task is independent of that in the other task.

Goal setting. Before working on the tasks, the individual can choose a goal which then becomes anchored in his head as a standard for the future outcome to be achieved: he can either specify a separate reference standard for each task $i \in \{1, 2\}$, i.e., a *narrow goal* $a_{n_i} \in [0, \infty)$, or an overall reference standard for the sum of the outcomes in both tasks, i.e., a *broad goal* $a_b \in [0, \infty)$. Note that the individual can choose a reference standard that will be met for sure, i.e., $a_{n_i} = 0$ or $a_b = 0$. We will refer to this as *no goal* henceforth, to distinguish it from meaningful goals.

Timing and utilities. Figure 1 summarizes the timing. At date 0, the individual chooses the goal bracket and level(s): a broad goal (a_b), two narrow goals (a_{n1}, a_{n2}), or no goal. No payoff-relevant events occur, so $u_0 = 0$. Date-1 utility reflects the immediate cost of effort exerted in the first task: $u_1 = -c(e_1)$. At date 2, the individual works on the second task, so $u_2 = -c(e_2)$. In the *information scenario*, he observes the outcome of task 1 before he provides effort at date 2. In the *no-information scenario*, he does not observe the outcome in task 1.[5] At date 3, the individual experiences gain-loss utility related to the realized task outcomes and the goal(s). Namely, the individual experiences a loss if an outcome falls short of the respective goal and a gain if the outcome exceeds it. Losses loom larger than gains of equal size.

We capture these features with a value function in the spirit of Kahneman and Tversky (1979). Depending on the difference between the outcome realization y and the goal a, the individual experiences a gain $\mu^+(y-a)$ from satisfying the goal if $y \geq a$, and a loss $\mu^-(a-y)$ from falling short of the goal if $y < a$.[6] As in many applications, we assume for tractability that the individual has a piecewise-linear value function: $\mu^+(y-a) = \max\{y-a, 0\}$ and $\mu^-(y-a) = \mu \cdot \max\{a-y, 0\}$, with $\mu > 1$. With narrow goals a_{n_i} for $i = 1, 2$, gain-loss utility relates to individual task performance y_i, providing date-3 utility

$$u_3 = \sum_{i=1}^{2} \left(\max\{y_i - a_{n_i}, 0\} - \mu \max\{a_{n_i} - y_i, 0\}\right).$$

With a broad goal a_b, the overall performance $y_1 + y_2$ is measured against the goal, such that

$$u_3 = \max\{(y_1 + y_2) - a_b, 0\} - \mu \max\{a_b - (y_1 + y_2), 0\}.$$

In the absence of a goal, simply set $a_b = 0$ (or $a_{ni} = 0$).

Our setup is a simplified version of the single-task goal setting model in Koch and Nafziger (2011). There, both the goal level and the outcome have the same continuous support, i.e., a goal always corresponds to a possible outcome level. The key property however is that varying the goal level shifts the expected gain from exceeding the goal relative to the expected loss from falling short of it. Our setting preserves this essential feature, while making the model sufficiently tractable for analyzing multi-task problems. In Section 5, we outline the challenges of incorporating continuous output in this setting and discuss the robustness of our main effects when we incorporate continuous output and effort. Further, we address robustness of our main effects when we incorporate diminishing sensitivity in the utility function.

[5]The no-information scenario yields the same results as a scenario where the individual works simultaneously on both tasks, as we show in Appendix B.

[6]Kőszegi and Rabin (2006) distinguish gain-loss utility (the value function) and outcome-based consumption utility. Adding consumption utility (say, $v(y) = y$), does not change our qualitative results. With $v(\bar{y}) > v(\underline{y}) \geq 0$, a high outcome is more valuable relative to our setting. So the parameter range where a conflict of interest between self 0 and self 1 arises (Assumption 1) becomes smaller, but still is non-degenerate. Moreover, self 0 is willing to specify a 'painful' goal for a larger range of parameter values.

Present bias. The individual has (β, δ)-*preferences* (Laibson, 1997; O'Donoghue and Rabin, 1999). The first parameter, δ, corresponds to the standard exponential discount factor (without loss of generality, we assume $\delta = 1$). The second parameter, $\beta \in [0, 1)$, captures the extent of the present bias, and is the parameter of interest in our model as it causes time inconsistency in preferences. The utility of the individual at date $t \in \{0, 1, 2, 3\}$ is given by:

$$U_t \quad = \quad u_t + \beta \left[\sum_{\tau=t+1}^{3} u_\tau \right], \tag{1}$$

where u_t is the *instantaneous* utility at date t. For instance, the date-0 incarnation of the individual (*self 0*) weighs future utilities u_1, u_2 and u_3 equally; but the date-1 incarnation of the individual (*self 1*) puts a larger relative weight on u_1 by discounting u_2 and u_3 with $\beta < 1$, reflecting his present bias. We assume that self 0 knows about the present-biased preferences of his future selves, i.e., is *sophisticated* in the sense of O'Donoghue and Rabin (1999).[7]

Self-control problem. For self 0, the choice of effort still lies in the future, and he therefore weighs equally the effort costs and the future benefits related to each task outcome. High effort in a task is therefore optimal from the date-0 perspective if the expected utility it yields ($\beta\ \{p\,\bar{y} - c\}$) exceeds that from low effort ($\beta\,\underline{y} = 0$). That is, self 0 wants his future self to work hard in the tasks if and only if

$$\beta\,\{\,p\,\bar{y} - c\} \geq 0. \tag{2}$$

Only in the case with no present bias ($\beta = 1$) does self 1 (or self 2) choose exactly as self 0 would. But because $\beta < 1$, the individual overemphasizes immediate costs relative to the distant benefits. As a result, self 1 (self 2) may prefer to shirk, while self 0 wants his later incarnation(s) to work hard. To see this formally, note that in the absence of a goal it is optimal to provide low effort from the perspective of self 1 (self 2) if and only if

$$\beta\,p\,\bar{y} - c < 0. \tag{3}$$

Thus, whenever both (2) and (3) hold, the individual has a self-control problem. We assume that a conflict of interest between self 0 and later-date selves arises in the absence of a positive goal level:

Assumption 1 (Self-control problem) *Self 0 prefers high effort in each task, but in the absence of a goal, the self making the effort decision on a task will choose low effort, i.e.,* $p\,\bar{y} \geq c$ *and*

$$\beta_{sc} \equiv \frac{c}{p\,\bar{y}} > \beta. \tag{4}$$

[7] In our related work Koch and Nafziger (2020), we show that the main insights carry over to (partially) naïve individuals.

3 The no-information scenario

We start our analysis of the two-task setting with the *no-information scenario.* Here the individual does not learn about the outcome in the first task before working on the second task. For example, think of studying for two exams in unrelated subjects that take place in close sequence. Before even taking the first exam, you need to already study for the second exam. So all you know is how much (or little) effort you put into preparing for the first exam.

3.1 Self-regulation with narrow goals

If the individual sets narrow goals, he subsequently evaluates task outcomes separately. For example, "Did I reach the target grade in the first exam? Did I in the second?" Given the symmetry of tasks, the problem for each task mirrors the one in the single-task setting and hence the analysis follows Koch and Nafziger (2011).
When designing his task goal, self 0 anticipates what it takes to motivate self 1 to provide effort. We therefore solve by backward induction. Starting with the effort choice at date 1, we ask how self 1 responds to a given goal. Then we determine what is the optimal goal from the perspective of self 0.

3.2 The incentive effect of a given goal

For a given goal, self 1 provides high effort in the task for which the goal was set if and only if the expected utility of self 1 when working hard exceeds that from shirking: $U_1(a;\bar{e}) \geq U_1(a;\underline{e})$. For $a \leq \bar{y}$ this incentive constraint is given by (see Appendix A.1):

$$a\,\beta\,(\mu-1)\,[\Pr(\text{loss}|\underline{e}) - \Pr(\text{loss}|\bar{e})] \geq c - \beta\,p\,\bar{y}, \tag{5}$$

where $\Pr(\text{loss}|e)$ gives the probability that the individual falls short of the goal (thereby suffering a loss) when providing effort level e. The incentive constraint shows how and why a goal motivates an individual to work hard. Think of studying for an exam. Cramming yields self 1 utility $\beta\,p\,\bar{y} - c$. Note that the utility is negative (by Assumption 1). Because of the individual's present bias, the benefit from an expected good grade does not compensate for the cost of studying hard. Hence, self 1 would refrain from working hard in the absence of a goal.
Motivating himself to study for an exam may work if he has a target grade, as he then fears suffering a loss from not reaching his goal. If self 1 shirks, the probability of failing the goal is larger (he would fail it with probability 1) than if he works hard (he would fail it with probability $1-p$) – so the left-hand side of the incentive constraint is positive. Hence, with an appropriately chosen goal, the loss avoided by exerting effort offsets the net cost from

hard work. More precisely, the goal level, where (5) just holds, is given by

$$\hat{a}_n = \frac{c - \beta\, p\, \bar{y}}{\beta\, p\, (\mu - 1)}. \tag{6}$$

The more severe the present bias (i.e., the lower β), the more difficult it is to incentivize self 1 and thus the higher the required goal level $\hat{a}$. Self-regulation becomes impossible if the present bias is so strong that $\hat{a}$ exceeds $\bar{y}$. The reason is that in this case the incentive constraint (5) needs to be modified: with such a goal the individual experiences a loss no matter whether he works hard or shirks. As loss aversion no longer kicks in, the goal simply cancels out from the incentive constraint.[8] In sum, self-regulation with a goal is *feasible* if and only if $\hat{a} \leq \bar{y}$, which is equivalent to $\frac{1}{\mu}\,\beta_{sc} \leq \beta$ (β_{sc} is defined in Assumption 1).

3.3 What goal does self 0 choose?

Holding the effort level e fixed, a higher goal increases the chance of falling short of the aspired outcome a. But this matters more than the chance of exceeding a, because losses loom larger than gains of equal size. So a higher goal level is painful, because it reduces the utility that self 0 expects from a given effort level e: $\frac{d}{d\,a}\,U_0(a; e) < 0$.
Consequently, the only purpose of setting $a > 0$ is to discipline self 1 to put in more effort than he would exert in the absence of a positive goal. Thus, the choice of self 0 is a relatively simple one: either avoid the pain from an ambitious goal by abstaining from goal setting and live with low effort; or pick the goal level that just suffices to motivate self 1 to work hard (if this is feasible). In other words, self 0 will specify a positive goal if his expected utility with goal $\hat{a}$ and high effort in the future exceeds the utility from $a = 0$ and low effort in the future: $U_0(\hat{a}; \bar{e}) \geq U_0(0; \underline{e}) = 0$. Lemma 1 below characterizes this in terms of β (it is the analogue to Proposition 1 in Koch and Nafziger, 2016).

Lemma 1

(i) Self-regulation with narrow goals is feasible if and only if $\frac{1}{\mu}\,\beta_{sc} \leq \beta$.

(ii) There exists a cutoff β_n*, satisfying* $\frac{1}{\mu}\,\beta_{sc} < \beta_n < \beta_{sc}$*, such that for* $\beta \in [\beta_n, \beta_{sc})$ *self 0 sets the narrow goal* $\hat{a}_n$ *from (6) for each task rather than no goal, and selves 1 and 2 provide high effort.*

Lemma 1 shows that there exists a range for the present bias parameter β, for which self 0 would find it worthwhile to engage in self-regulation with adequately chosen task-specific goals ($a_{ni} = \hat{a}_n$). What happens outside of this range? If there was no intra-personal conflict of interest – i.e., if β exceeded β_{sc} – no goal would be needed as self 1 would exert high effort anyway. Assumption 1 rules out this case. In contrast, if β falls in between $\frac{1}{\mu}\,\beta_{sc}$ and β_n, it

[8] $-\beta\,\mu\,\{p\,(a - \bar{y}) + (1 - p)\,a\} - c \geq -\beta\,\mu\,a. \Leftrightarrow \beta\,p\,\bar{y} \geq c$, which does not hold because of Assumption 1.

would be feasible to overcome the self-control problem with narrow goals, yet the required goal levels are too painful for self 0. So he rather gives up on self-regulation. If the self-control problem is very severe, $\beta < \frac{1}{\mu}\beta_{sc}$, no feasible narrow goal can help overcome it (but by the previous argument, it would be too painful to do so anyhow).

3.4 Self-regulation with a broad goal

The alternative to narrow task-specific goals is to set a broad goal for the overall outcome from both tasks (e.g., reach some average grade score in the upcoming exams). One needs to distinguish between two cases. Is it enough to set a broad goal $a_b \leq \bar{y}$ that is "easy" in the sense that it can be achieved with one task outcome alone? Or does the goal have to be a "difficult" broad goal $\bar{y} < a_b \leq 2\,\bar{y}$? The difficult goal is more painful, and therefore only makes sense if an easy broad goal does not do the job of motivating selves 1 and 2 to work hard.

We now explore the intuition for when an easy broad goal makes sense, relegating formal details to Appendix A.3. Because the individual evaluates the outcomes of the two tasks jointly, the (previous or expected future) effort choice of the other self influences the incentives of the current self. That is, we have to consider two cases: Does the other self work hard, or does he shirk? If there will be shirking in one task, say by self 1, the broad goal necessary to motivate self 2 just equals the narrow goal for that task, i.e., $a_b = \hat{a}_n$ from equation (6). Yet if high effort in this task is worth the cost of the goal, then effort in the other task is also worth it. So in this case, narrow goals $(\hat{a}_n, \hat{a}_n)$ dominate the broad goal.

Now consider what it takes to get both selves 1 and 2 to put in effort. If self 2 knows that his previous self worked hard at date 1, a broad goal $a_b \leq \bar{y}$ provides incentives for him to work hard in the second task if:

$$a_b\,\beta\,(\mu-1)\,[\Pr(\text{loss}|\underline{e}_2,\bar{e}_1) - \Pr(\text{loss}|\bar{e}_2,\bar{e}_1)] \geq c - \beta\,p\,\bar{y}. \tag{7}$$

The interpretation is similar to that of the incentive constraint (5) for the single-task setting/narrow goals: while shirking saves $c - \beta\,p\,\bar{y}$ (this is the net-utility cost of effort of self 2 in the task; note that the effort cost of self 1 is already sunk), it exposes the individual to the risk of falling short of the goal. Shirking now however does not for sure lead to a loss (as it does under narrow goals), but only with probability $1-p$: A high outcome in the first task, where self 1 already put in effort, can compensate for the low outcome that shirking causes in the second task. By working hard, self 2 can reduce the probability of a loss to $(1-p)^2$. So $\Pr(\text{loss}|\underline{e}_2,\bar{e}_1) - \Pr(\text{loss}|\bar{e}_2,\bar{e}_1) = p\,(1-p)$, and the broad goal necessary to motivate self 2 is given by

$$\hat{a}_b = \frac{c - \beta\,p\,\bar{y}}{\beta\,p\,(1-p)\,(\mu-1)}. \tag{8}$$

The goal $\hat{a}_b$ also motivates self 1 to work hard if he expects self 2 to put in effort. The reason is that the incentive constraint of self 1 simply reduces to the one in (7): The anticipated

effort cost of self 2 appears on both sides of the incentive constraint, and thus cancels out without influencing the decision of self 1 whether to work hard or shirk.
The following result provides the condition under which the "easy" broad goal $\hat{a}_b \leq \bar{y}$ is better than abstaining from setting a positive goal altogether (i.e., when $U_0(\hat{a}_b; \bar{e}_1, \bar{e}_2) \geq U_0(0; \underline{e}_1, \underline{e}_2)$).

Lemma 2

(i) Self-regulation for both tasks with an "easy" broad goal ($\bar{y} \geq a_b > 0$) is feasible if and only if $\frac{1}{(1-p)\,\mu+p}\,\beta_{sc} \leq \beta$.

(ii) There exists a cutoff β_b *satisfying* $\frac{1}{(1-p)\,\mu+p}\,\beta_{sc} < \beta_b < \beta_{sc}$*, such that for* $\beta \in [\beta_b, \beta_{sc})$ *self 0 sets the "easy" broad goal* $\hat{a}_b$ *from (8) rather than no goal, and selves 1 and 2 provide high effort.*

Even if a broad goal that would help overcome the self-control problem is feasible, this does not mean that the individual is willing to use it – similar to the case of narrow goals. For $\frac{1}{(1-p)\,\mu+p}\,\beta_{sc} < \beta < \beta_b$, it is better to set no goal than to ensure effort at the cost of a painful broad goal.
This also implies that a difficult broad goal ($\bar{y} < a_b \leq 2\,\bar{y}$) can never be optimal. Self 0 will consider such a goal only if it is not possible to solve the motivation problem with an easy broad goal. In the parameter range where this is true, $\beta < \frac{1}{(1-p)\,\mu+p}\,\beta_{sc}$, the benefit of higher effort however does not even outweigh the cost of setting an easy broad goal. So, a fortiori, a difficult broad goal would yield negative utility for self 0. First, there is a larger loss of falling short of the goal, as it is larger (by construction) than any easy broad goal. Second, a high outcome in one task can no longer fully compensate for a low outcome in the other task, so the individual also suffers a loss more often than with an easy broad goal.

Lemma 3 *Self 0 will never adopt a "difficult" broad goal ($\bar{y} < a_b \leq 2\,\bar{y}$).*

As the easy broad goal $\hat{a}_b$ is the only candidate that can beat narrowly bracketed goals, we will henceforth refer to the easy broad goal simply as the broad goal.

3.5 Motivational bracketing: Broad vs narrow goals

Is broad or narrow goal bracketing better suited for self-regulation? To answer this question, suppose first that those goals that motivate the individual to work hard – be they broad or narrow – dominate setting no goal (i.e., $\beta \geq \max\{\beta_n, \beta_b\}$).

3.5.1 The risk-pooling effect

Broad bracketing would be superior if we were able to abstract from incentive considerations (say, because self 1 always works hard) and simply compared the utility of self 0 in the broad and narrow bracketing scenarios for the *same* goal level (say, $a = a_b = a_n$).[9] Under both scenarios, the individual experiences disutility $2\,c$ from working hard in the tasks, and he looks forward to a future expected utility flow from the task outcomes of $2\,p\,\bar{y}$. In addition, the individual has negative expected gain-loss utility, which represents the cost of goal setting. This cost under broad bracketing differs from the one under narrow bracketing because of a *risk-pooling effect*.

Specifically, a high outcome in one task can compensate for a low outcome in the other task (the broad goal is lower than $\bar{y}$ according to Lemma 3). The individual therefore experiences a loss only if the outcomes in both tasks are low, so the expected loss is $(1-p)^2\,\mu\,a$. In contrast, narrow goals provide no scope for risk pooling and the individual experiences a loss as soon as the outcome is low in *any* of the two tasks. Hence, the probability of a loss under broad bracketing ($(1-p)^2$) is lower than the probability of a loss under narrow bracketing ($2\,(1-p)$). With the respective complementary probabilities, the individual achieves his goal(s) and experiences a gain (i.e., the goal then is not weighed by the loss aversion parameter μ). Combining the terms we obtain

$$\text{the gain-loss utility cost of a broad goal } a_b = a: \quad \underbrace{[2\,p - p^2 + (1-p)^2\,\mu]}_{\equiv[broad]} \times a, \tag{9}$$

and

$$\text{the gain-loss utility cost of narrow goals } a_{n1} = a_{n2} = a: \quad \underbrace{2\,[p + (1-p)\,\mu]}_{\equiv[narrow]} \times a. \tag{10}$$

As losses loom larger than gains ($\mu > 1$), the overall cost under a broad goal is lower than that under corresponding narrow goals:

$$\text{gain-loss utility cost(broad goals)} < \text{gain-loss utility cost(narrow goals)}. \tag{11}$$

3.5.2 The leaning-back effect

Yet the above comparison is incomplete because it neglects the incentive side. The optimal broad goal ($\hat{a}_b \leq \bar{y}$ by Lemma 3) provides a buffer against a loss also if the individual shirks in one of the tasks; and this buffer creates an incentive to 'lean back' in one of the tasks.

To understand this *leaning-back effect*, take as given the goal level under the narrow and broad bracketing scenarios. Now ask where the individual has a greater motivation to work hard? If selves 1 and 2 both exert effort, the probability of falling short of the broad goal

[9] This corresponds to Proposition 1 in Koch and Nafziger (2011).

is $(1-p)^2$. Shirking in a single task by one of the selves raises this probability, but only to $1-p$. The reason is that a high outcome in the other task – where effort is put in – can compensate for the low outcome that shirking will cause in the task at hand. In contrast, under a narrow goal, shirking in a single task leads to a loss with probability one. Effort lowers this probability to $1-p$ for each task. So the difference that it makes to provide effort in both tasks relative to shirking in one task and only providing effort in the other task is smaller with the broad goal than with the corresponding narrow goals: a drop in the probability of a loss of $p(1-p)$ versus one of p. As a result, the individual needs to set a more ambitious broad goal to motivate effort than the required goal level under narrow bracketing.[10]

Lemma 4 *Motivating self 1 requires a larger goal level under broad bracketing than under narrow bracketing:*

$$\hat{a}_b = \frac{\hat{a}_n}{1-p} > \hat{a}_n.$$

3.5.3 When is narrow bracketing optimal?

What is the overall impact on the utility of self 0? Comparing the expected utility of self 0 from setting two narrow goals $\hat{a}_n$ with that from setting the broad goal $\hat{a}_b$, the next result shows that narrow bracketing can indeed be optimal.

Proposition 1 (No-information scenario) *Suppose that setting some type of goal is optimal, i.e., $\beta \geq \min\{\beta_n, \beta_b\}$. Then the individual brackets goals narrowly rather than broadly if and only if*

$$p^2 - (1-p)^2\mu \geq 0,$$

or equivalently, if and only if $\beta_b \geq \beta_n$.

The formula shows that for large p narrow bracketing is optimal, while for $p \leq 1/2$ the broad goal is optimal.[11] A broad goal $\hat{a}_b$ allows the individual to hedge against the risk of suffering a loss. While the direct impact of the risk-pooling effect on the utility of self 0 is positive, it decreases the motivational power of a given goal. The leaning-back effect dominates for a high success probability p. The reason is that the incentive power (the impact of effort in terms of lowering the probability of suffering a loss) increases with p under narrow bracketing, whereas for a broad goal it is strongest at $p = 1/2$. Proposition 1 is the analogue to Proposition 2 in Koch and Nafziger (2016), where the model considers only rational effort goals .

[10] The effects described here are robust to correlation in task outcomes. Unless tasks are perfectly positively correlated, the risk-pooling effect arises (i.e., broad goals have a lower gain-loss utility cost than narrow goals). And unless tasks are perfectly negatively correlated, the leaning-back effect arises.

[11] To see the latter, note that the broad goal is better than the narrow goals if it involves a lower cost, i.e., if $\frac{\hat{a}_n}{1-p} \times [broad] \leq 2\,\hat{a}_n \times [narrow]$. For $p \leq \frac{1}{2}$, we have that $\frac{\hat{a}_n}{1-p} \leq 2\,\hat{a}_n$. And from (9) and (10) we know that $[broad] < [narrow]$. So broad bracketing is for sure better if $p \leq 1/2$.

4 The information scenario

So far we assumed that the individual does not receive intermediate information between the tasks. In many settings though, people work in tasks sequentially and receive information about task outcomes in between tasks. For instance, a student taking a series of exams over the course of an academic year often learns about how well he did in one exam before studying for the next exam. So we consider now the information scenario, where the individual gets to know the outcome of task 1 before providing effort at date 2 (see Figure 1).[12]

4.1 Narrow goals are invariant to information

For self-regulation with narrow goals it is irrelevant whether the individual receives intermediate information or not, because task outcomes are evaluated separately. A past success provides no excuse to slack off today, and a past failure does not provide extra motivation to make up for it. So our results from Section 3.1 carry over.

4.2 The effort substitution effect under a broad goal

Again we have to distinguish between an "easy" and a "difficult" broad goal. It turns out that a difficult broad goal ($\bar{y} < a_b \leq 2\,\bar{y}$) can never be optimal, essentially for the same reasons as in Section 3.4. We show this in Appendix A.9, and simply refer to an easy broad goal ($a_b \leq \bar{y}$) as the broad goal in the following. We now consider the incentives of self 2 to work hard if he observes a first-period success and if he observes a first-period failure.

In the case of a first-period success, the risk-pooling effect of the broad goal undermines incentives because self 2 knows that he has already exceeded the goal and will not risk a loss even if he shirks on the second task. So the fear of a loss – which we know is needed to motivate self 2 to exert effort – is gone. Hence, self 2 will "rest on his laurels" and shirk on the second task – even though effort would be optimal from the perspective of self 0.

In contrast, if self 2 observes a first-period failure, he knows that he can achieve the broad goal by working hard on the next task. In this case, the incentive problem for self 2 mirrors the one under narrow bracketing: The focus is on the single task ahead, because the effort cost of self 1 is sunk and the first-period outcome cannot be influenced – no matter whether self 2 works hard or not. So to motivate self 2 to work hard (after a low first-task outcome)

[12]The choice bracketing literature suggests that people evaluate differently events that are spread over time, depending on whether they bracket narrowly or broadly (e.g., Read *et al.* (1999), Thaler (1999)). Without changing our results, we could assume that under narrow bracketing the goal for the first task is evaluated at date 2 (instead of date 3): With narrow goals, mental accounts can be settled as soon as a goal-related outcome realizes. A broad goal, in contrast, leaves the account open until the final goal-related outcome becomes known. That is, goal evaluation only takes place at date 3, after all goal-related events have realized.

it takes a broad goal level of at least $\hat{a}_n$ from equation (6).

The asymmetric response of self 2 to the outcome of the first task changes the incentive constraint for self 1 as well. Self 1 anticipates correctly that self 2 will rest on his laurels if he succeeds today. While self 1 thinks that shirking today is the optimal thing to do in the absence of a sufficiently ambitious goal (on account of his present bias), he still prefers self 2 to work hard tomorrow rather than to shirk. And self 1 can, by shirking today, always push self 2 to work hard (effort substitution): The first task outcome then will for sure be low and hence self 2 will for sure work to make up for this – resulting in expected utility $\beta\,(p\,\bar{y} - c) > 0$ for self 1. In contrast, if self 1 works hard he reduces the probability that self 2 will work hard to $1 - p$ (the probability with which the first task outcome will be low). Thus, the individual has lower incentives to work hard on the task at date 1 compared to a situation were the individual receives no intermediate information.

It therefore is quite intuitive that the broad goal required to motivate self 1 in the information scenario is larger compared to the no-information scenario. This, in turn, means that the goal also is larger than the narrow goal $\hat{a}_n$ (which would suffice to motivate self 2 to work hard after a first-period failure). Specifically, the broad goal required is given by (see Appendix A.7):

$$\hat{a}_b^{info} = \underbrace{\frac{\hat{a}_n}{1-p}}_{=\hat{a}_b} + \frac{p\,\bar{y} - c}{(1-p)\,(\mu - 1)}. \tag{12}$$

Note that the leaning-back effect described in Section 3.5 still kicks in and shows up in the first component (which is the same as for the no-information scenario). In addition, the goal has to counter the effort substitution effect, as captured by the second component. The following lemma summarizes our observations:

Lemma 5

(i) If the outcome on task 1 is high, no "easy" broad goal ($a_b \leq \bar{y}$) can motivate self 2 to provide high effort on task 2.

(ii) If the outcome on task 1 is low, any broad goal that weakly exceeds $\hat{a}_n$ leads to high effort on task 2.

(iii) The broad goal $\hat{a}_b^{info}$ from (12), that motivates self 1 to work hard (and self 2 if the outcome on task 1 is low), is larger than the narrow goal $\hat{a}_n$ from (6), and larger than the broad goal in the no-information scenario $\hat{a}_b$ from (8).

Summarizing, the negative incentive effects of a broad goal are more pronounced if the individual receives intermediate information than if no such information is received. Firstly, the goal level increases. Secondly, the individual rests on his laurels after observing a high outcome in the first period (resulting in a utility loss of $\beta\, p\, (p\, \bar{y} - c)$ for self 0).[13] In contrast, the risk-pooling effect described in Section 3.5 is not affected by the intermediate information. The gain-loss utility cost of the broad goal still behaves as in (9). Overall, moving from the information to the no-information scenario decreases the utility of self 0 under broad bracketing, but leaves his utility under narrow bracketing unaffected. As a consequence, broad goals must perform relatively worse if the individual receives intermediate information than if he does not.

4.3 Optimality of narrow goals

Can a broad goal still be optimal? The answer is no:

Proposition 2 (Information scenario) *It is never optimal for self 0 to motivate future selves with a broad goal. Narrow goals are optimal if and only if $\beta \geq \beta_n$.*

To gain some intuition for the result, note that for relatively low values of p – where the individual does not succeed often – the effort substitution effect plays a minor role. However, for low values of p it also becomes quite difficult in general to motivate future selves to work hard as the expected gains from effort are not much larger than the ones from shirking – precisely because the individual does not succeed very often. That is, the broad goal needs to be very large and, consequently, very painful for self 0. Goal setting thus is generally unattractive for low values of p, and self 0 will rather let self 1 shirk. Indeed, the proof of Proposition 2 shows that whenever the negative effects of broad bracketing are weak and a broad goal stands a chance to do better than narrow bracketing (for low values of p), the broad goal that motivates effort by self 1 yields negative utility.
Thus, with intermediate information on the first task outcome, self 0 will either specify two narrow goals or no goal. A broad goal is never optimal even though it provides a hedge against the risk of falling short of the goal.

5 Discussion and extensions

The model that we outlined in Section 2 is highly tractable, but it makes some restrictive assumptions. First, to obtain closed form solutions for the goal, we assumed that the gain-loss utility related to the goal is piece-wise linear. Yet, the psychology literature that proposes that goals serves as reference points (cf. Heath et al., 1999; Markle et al., 2018) emphasizes

[13]Of course, the individual could choose not to fight against the consequences of the effort substitution effect. A lower broad goal will lead to shirking on the first task, but $a_b = \hat{a}_n$ will still get self 2 to work hard on the second task. But setting two narrow goals $\hat{a}_n$ will motivate effort also on the first task for the same gain-loss utility cost as the broad bracketing strategy incurs related to just the second task. So narrow bracketing clearly dominates such a strategy.

not only loss aversion and reference dependence as plausible properties of the goal related gain-loss utility, but also diminishing sensitivity. While we have incorporated the two former properties, we have thus far sidestepped the third property for tractability.
Further, while the individual can choose any non-negative number as goal, we assumed that output is discrete to obtain closed form solutions for the goals. A model with continuous output requires a characterization of the distribution function of the sum of two independent continuous random variables relative to the distribution for a single random variable. The kink that loss aversion introduces into the value function further complicates the analysis. In the following, we outline the robustness of our main effects with respect to diminishing sensitivity and continuous output and effort. We also incorporate consumption utility next to gain-loss utility (cf. footnote 6).

5.1 Diminishing sensitivity

To incorporate diminishing sensitivity into our basic model with discrete effort, let the gain-loss function be $\mu(x) = \mu^+(x)$ for $x \geq 0$ and $\mu(x) = \mu^-(-x)$ for $x < 0$. The functions μ^+ and μ^- are both strictly increasing. We assume that $\mu^{+''}(x) < 0$ and $\mu^{-''}(x) > 0$ (diminishing sensitivity) and $v(y) + v(-y) > v(x) + v(-x)$ for $y > x \geq 0$ (loss aversion), as well as $\mu^+(0) = \mu^-(0) = 0$.

Risk-pooling. Holding the goal level fixed, the expected gain-loss utility under broad goals differs from that under narrow goals by

$$p^2 \left[\mu^+(2\,\bar{y} - a) - 2\,\mu^+(\bar{y} - a)\right] - (1 - p^2)\mu^-(-a). \tag{13}$$

Under linear gain-loss utility, the first term in (13) boils down to $p^2\,a$ and the second to $(1 - p^2)\,\mu\,a$. With diminishing sensitivity, we however have that the first term is negative because concavity of the value function for gains implies that $2\,\mu^+(\bar{y} - a) > \mu^+(2\,(\bar{y} - a))$. That is, feeling a gain twice gives a higher utility to the individual than the utility from twice the gain. Compared to our main model, there hence is a countervailing force to the risk pooling effect captured by the second term in (13) .[14]
While the second term $-(1 - p^2)\mu^-(-a) > 0$, it will be small if p close to 1 or if a is small. Thus, the expression in (C.26) can become negative, so that a broad goal is dominated by narrow goals (because the incentive effects discussed next works against a broad goal).[15]

[14]In a similar vain, Thaler (1985) argued that seggregating gains and integrating losses would be optimal for maximizing utility. Compared to our model, the individual needs to wait for the outcomes to realize before deciding on how to evaluate them. Thaler however later rejected such hedonic editing based on whether outcomes are gains or losses as a good descriptive model of how people actually evaluate joint outcomes (Thaler, 1999).

[15]If we assumed $\underline{y} > 0$, the second term would be $(1 - p)^2\,\mu^-(2\,\underline{y} - a) - 2\,(1 - p)\,\mu^-(\underline{y} - a)$ instead of $-(1 - p^2)\mu^-(-a)$. As $2\,\mu^-(\underline{y} - a) < \mu^-(2\,(\underline{y} - a)) < \mu^-(2\,\underline{y} - a)$, this effect works in favor of a broad goal.

Incentive effects. In Appendix C, we state the incentive constraints for the information and no-information scenarios. We observe that the incentive effects outlined in Lemmas 4 and 5 continue to hold under diminishing sensitivity. That is, in the no-information scenario, the incentive constraint is more difficult to satisfy under a broad than under a narrow goal. Further, for a broad goal that satisfies $a < \bar{y}$, self 2 will shirk in task 2 after observing a high output in task 1 and will work hard after observing a low output. Finally, it is more difficult to satisfy the incentive constraint for self 1 under a broad goal in the information scenario compared to the no-information scenario. The latter insight stems to some extent from the assumption that output and effort are discrete. We next extend our model to incorporate continuous output and effort.

5.2 Continuous output and effort

To illustrate the robustness of our main effects, we first consider a piece-wise linear gain-loss function. We then incorporate diminishing sensitivity in the information scenario to show that an additional incentive effect can arise here.

Suppose that output in task 2 is given by $y_2 = e + \epsilon_2$, where $e \in I\!R_0^+$ and the shock ϵ_2 is distributed according to the function $F(\epsilon_2)$ with associated density $f(\epsilon_2)$. Effort costs are $c(e)$, with $c'(e) > 0$ and $c''(e) > 0$. For simplicity, we assume that output in task 1 is $y_1 = \epsilon_1$, where ϵ_1 is distributed according to the function $F(\epsilon_1)$ with density $f(\epsilon_1)$ and ϵ_1 and ϵ_2 being independent. That is, task 1 constitutes 'background risk' and broad bracketing amounts to integrating the background risk with the outcome in task 2. Let $H(\cdot)$ be the distribution of the sum of the two shocks. The associated density $h(\cdot)$ is obtained by the convolution of the individual densities. We assume that all functions are continuously differentiable.

No-information scenario. As we show in Appendix D, the optimal effort under narrow goals is characterized by:

$$\beta\,[1 + (\mu - 1)\,F(a_n - e_n)] = c'(e_n). \tag{14}$$

Under a broad goal it is characterized by:

$$\beta\,[1 + (\mu - 1)\,H(a_b - e_b)] = c'(e_b). \tag{15}$$

For a uniform distribution, the distribution of a single random variable, $F(\cdot)$, is first order stochastically dominated by the distribution of the sum of the two random variables, $H(\cdot)$, i.e., $H(y) \leq F(y)\,\forall\, y$. Thus, for $a_n = a_b$, the individual provides less effort under a broad than under a narrow goal. As in the discrete case, incentives are lower with a broad goal. Yet, the property that the distribution of a single random variable is first order stochastically dominated by the distribution of the sum of two random variables is not a general property. An examination of the extent to which the insights here generalize is beyond the scope of this paper.

Information scenario. Suppose now that the individual observes the realization of the background risk before providing effort. The first order conditions for the optimal effort then are given by:

$$\beta\left[1+(\mu-1)\, F(a_b-e_b-\epsilon_1)\right]=c'(e_b). \tag{16}$$

Comparing equation (16) to the first order conditions for a narrow goal, equation (14) shows that incentives are weaker under the broad goal than under narrow goals. The reason is that under the broad goal a higher realization of ϵ_1 substitutes for a higher effort: The closer the individual is to reaching the goal, the weaker are the incentives to work hard. Intuitively, if the realization of ϵ_1 is large, the probability of falling short of the goal is small. Yet, it is the fear of falling short of the goal that motivates the individual. Thus, the individual is less motivated to provide effort – in line with our previous analysis.

Adding diminishing sensitivity adds two additional terms to the incentive constraint, as we we illustrate in Appendix D. The first term is decreasing in ϵ_1 and captures the intuition described by Heath et al. (1999) that "People should be willing to exert less effort as they move away from their goal." The second term is increasing in ϵ_1 so that being closer to the goal increases incentives. This captures the intuition described by Heath et al. (1999) that "[...] [people] should be willing to exert more effort as they approach their goal." Overall, depending on which of the effects dominates, we might either observe that being closer to the goal decreases or increases effort so that a broad goal can be more or less motivating than narrow goals. In the appendix, we employ a specific parametric value function in conjunction with a uniform distribution to show, in accordance with our previous analysis, that in this case the negative effect dominates and a broad goal is less motivating.

Exploring further when the positive and when the negative effect dominates is an interesting avenue for future (empirical) work and may help to shed light on why the literature thus far presents no clear findings. Namely, Campion and Lord (1982) predict and observe that people who are further from their goal provide more effort. In contrast, Heath et al. (1999) predict and observe the opposite. The two effects to which we point may explain this difference. For example, Campion and Lord (1982) study grade goals, where goal achievement also depends on random components, while Heath et al. (1999) consider goals for sit-ups, where uncertainty is likely to play less of a role.

Finally, adding effort of self 1 to task 1, so that $y_1=e_1+\epsilon_1$ implies similar effects as discussed before for background risk. In the absence of diminishing sensitivity, the effort of self 2 is decreasing in the effort of self 1, which in turn undermines the incentives of self 1 as outlined above. If the effects of diminishing sensitivity are strong, then the effort of self 2 may be increasing in the effort of self 1 so that self 1 is more motivated to work hard with a broad goal.

6 Conclusion

An important reason why we often see people setting goals is that this allows them to overcome self-control problems. With our model of goal setting by a present-biased individual we provide a tractable framework for studying the trade-offs between broad and narrow goals that extends the previous theoretical foundation for the motivational bracketing hypothesis (Koch and Nafziger, 2016, 2020) to a setting with non-rational goals.

Goals serve as reference points that make substandard performance psychologically painful. And the fear of suffering a loss from falling short of a goal is what can provide the impetus for effort, that an individual with a present bias may otherwise lack. But because there is a risk of falling short of a goal even if the individual works hard, goal setting is costly. Setting broad goals – i.e., evaluating jointly the performance in two tasks – reduces this risk through a *risk-pooling effect*, because high performance in one task can compensate for low performance in another task. However, the possibility of offsetting outcomes also makes it less painful to shirk in one of the tasks (the leaning-back effect). Furthermore, if the individual learns about the outcome of the first task before facing the next task, incentives for effort on the second task are lower following a good outcome than following a bad outcome – diminishing first period incentives (the effort substitution effect). The latter two effects do not arise if the individual sets narrow task goals. We show that the key effects driving the results in our main model are robust to adding diminishing sensitivity to the value function or having a model with continuous output and effort.

Acknowledgements

We thank Johannes Abeler, Hideo Owan, Heiner Schumacher, Yoram Weiss, and George Wu for helpful comments.

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Appendix

A Proofs

A.1 Derivation of the incentive constraint (5)

Written out, the incentive constraint for self 1 is:

$$\beta \{p\,(\bar{y}-a)-(1-p)\,\mu\,a\}-c \geq -\beta\,\mu\,a.$$

Working hard, the goal is surpassed with probability p (resulting in a gain $\bar{y}-a$); and with probability $1-p$ the task outcome falls short of the goal (resulting in a loss $\mu\,a$). Shirking results in a substandard outcome with associated loss $\mu\,a$. Rewritten in terms of the respective probabilities of obtaining a gain or a loss ($\Pr(\text{gain}|\bar{e})=p$, $\Pr(\text{loss}|\bar{e})=1-p$, $\Pr(\text{gain}|\underline{e})=0$, and $\Pr(\text{loss}|\underline{e})=1$), the incentive constraint becomes:

$$\beta \{\Pr(\text{gain}|\bar{e})\,(\bar{y}-a)-\Pr(\text{loss}|\bar{e})\,\mu\,a\}-c \geq \beta \{\Pr(\text{gain}|\underline{e})\,(\bar{y}-a)-\Pr(\text{loss}|\underline{e})\,\mu\,a\}.$$

Rearranging:

$$\beta\,a\,[\{\Pr(\text{gain}|\bar{e})-\Pr(\text{gain}|\underline{e})\}+\mu\,\{\Pr(\text{loss}|\underline{e})-\Pr(\text{loss}|\bar{e})\}] \geq c-\beta\,p\,\bar{y}.$$

Using $\Pr(\text{gain}|e)=1-\Pr(\text{loss}|e)$ yields (5).

A.2 Proof of Lemma 1

With narrow goals, the utility is separable across tasks. So we can consider each task on its own, using the single-task incentive constraint (5).

(i) To motivate self 1 (self 2), a narrow goal of at least $\hat{a}$ is required. And this goal has to be feasible, i.e., $\hat{a} \leq \bar{y} \Leftrightarrow \frac{1}{\mu}\,\beta_{sc} \leq \beta$ (using the definition of β_{sc} in Assumption 1).

(ii) By Assumption 1, for the utility component of self 0 related to task $i=1,2$ ($U_{0i}(a_{ni};e)$) we have $U_{0i}(0;\bar{e})-U_{0i}(0;\underline{e})>0$. Moreover, $U_{0i}(\bar{y};\bar{e})-U_{0i}(0;\underline{e})=\beta\,[-(1-p)\,\mu\,\bar{y}-c]<0$. Note that $U_{0i}(a;\bar{e})-U_{0i}(0;\underline{e})$ is a continuous and strictly decreasing function in a, because $\mu>1$:

$$\frac{\partial}{\partial\,a}\,U_{0i}(a;\bar{e})=-\beta\,[\mu-p\,(\mu-1)]<0.$$

Applying the intermediate value theorem, there exists a unique value $\tilde{a}\in(0,\bar{y})$ such that $U_{0i}(\tilde{a};\bar{e})-U_{0i}(0;\underline{e})=0$. This is the maximum narrow goal level that self 0 would ever choose to get a future self to exert high effort in a task.

As tasks are symmetric, consider without loss of generality the effort choice of self 1 in task 1. The next argument shows that there exists a $\beta_n\in(\underline{\beta},\beta_{sc})$ such that the single-task incentive constraint (5) holds at $\tilde{a}$, where $\underline{\beta} \equiv \frac{1}{\mu}\,\beta_{sc}$. Because utility is separable across tasks, we can write the incentive constraint in terms of a function Φ that captures the utility difference related to effort by self 1 on task 1, and that is indexed by the present bias parameter β:

$$\Phi(a,\beta) \equiv \underbrace{[\beta\,\{p\,(\bar{y}-a)-(1-p)\,\mu\,a\}-c]}_{=U_1(a;\bar{e})} - \underbrace{[-\beta\,\mu\,a]}_{=U_1(a;\underline{e})}.$$

From Assumption 1 we know that for $a=0$ the incentive constraint binds at $\beta=\beta_{sc}$: $\Phi(0,\beta_{sc})=0$. Note that $\frac{\partial}{\partial a}\,\Phi(\cdot)>0$. So, $\Phi(\tilde{a},\beta_{sc})>0$. Similarly, $\Phi(\bar{y},\underline{\beta})=0$, which implies that $\Phi(\tilde{a},\underline{\beta})<0$. Using $\frac{\partial}{\partial \beta}\,\Phi>0$, by the intermediate value theorem, there exists a unique $\beta_n \in (\underline{\beta},\beta_{sc})$ such that $\Phi(\tilde{a},\beta_n)=0$. Thus, β_n is the *minimum* value of the present-bias parameter β for which self 0 would be willing to set a narrow goal to incentivize self 1. While the precise formula for β_n will be of little importance for our analysis, we state it here for completeness:

$$\beta_n = \frac{[(1-p)\,\mu+p]\;c}{p\;[\mu\,\bar{y}-(\mu-1)\,c]}.$$

A.3 Proof of Lemma 2

i) The relevant "easy" broad goal is $\hat{a}_b$ from equation (8).

For an easy broad goal level $a_b \leq \bar{y}$, consider the incentive constraint of self 2 after he observes that self 1 worked hard:

$$\beta\,\{p^2\,(2\,\bar{y}-a_b)+2\,p\,(1-p)\,(\bar{y}-a_b)-(1-p)^2\,\mu\,a_b\}-c \geq \beta\,\{p\,(\bar{y}-a_b)-(1-p)\,\mu\,a_b\}. \quad \text{(A.17)}$$

Rewritten in terms of the respective probabilities of obtaining a gain or a loss[16] and rearranging, the incentive constraint becomes:

$$a_b\,[\Pr(\text{gain}|\underline{e}_2,\bar{e}_1)-\Pr(\text{gain}|\bar{e}_2,\bar{e}_1)-\mu\,\{\Pr(\text{loss}|\bar{e}_2,\bar{e}_1)-\Pr(\text{loss}|\bar{e}_2,\bar{e}_1)\}] \geq c-\beta\,p\,\bar{y}.$$

Using $\Pr(\text{gain}|e_2,\bar{e}_1)=1-\Pr(\text{loss}|e_2,\bar{e}_1)$ yields equation (7). Solving for a_b and using $\Pr(\text{loss}|\underline{e}_2,\bar{e}_1)-\Pr(\text{loss}|\bar{e}_2,\bar{e}_1)=p\,(1-p)$ yields $\hat{a}_b$ in equation (8).

Now consider the incentive constraint for self 1 (given that he expects self 2 to work hard):

$$\beta\,\{p^2\,(2\,\bar{y}-a_b)+2\,p\,(1-p)\,(\bar{y}-a_b)-(1-p)^2\,\mu\,a_b-c\}-c \geq \beta\,\{p\,(\bar{y}-a_b)-(1-p)\,\mu\,a_b-c\}. \quad \text{(A.18)}$$

Note that $-\beta\,c$ (the effort cost of self 2) appears on both sided of the incentive constraint and cancels out. Hence, the incentive constraints (A.17) and (A.18) coincide, so the broad goal $\hat{a}_b$ given in equation (8) also motivates self 1.

[16] $\Pr(\text{gain}|\underline{e}_2,\bar{e}_1)=p$, $\Pr(\text{loss}|\underline{e}_2,\bar{e}_1)=1-p$, $\Pr(\text{gain}|\bar{e}_2,\bar{e}_1)=p^2+2\,p\,(1-p)$, and $\Pr(\text{loss}|\bar{e}_2,\bar{e}_1)=(1-p)^2$.

Similarly, the incentive constraint of self 2 after he observes that self 1 shirked is:

$$a_b\,\beta\,(\mu-1)\,\underbrace{[\Pr(\text{loss}|\underline{e}_2,\underline{e}_1)-\Pr(\text{loss}|\bar{e}_2,\underline{e}_1)]}_{1-(1-p)=p}\geq c-\beta\,p\,\bar{y}. \tag{A.19}$$

Again this coincides with the incentive constraint for self 1, if self 1 expects self 2 to shirk. Note also that the incentive constraint (A.19) coincides with the single-task/narrow-goals incentive constraint (5), so it binds at $a_b = \hat{a}_n$. As $\hat{a}_n < \hat{a}_b$, self $t \in \{1, 2\}$ will work hard under $\hat{a}_b$, even if the self responsible for the other task does not.

ii) Part (i) of Lemma 2.

The upper bound on an "easy" broad goal, $a_b \leq \bar{y}$, implies that $\hat{a}_b$ is feasible if and only if

$$c \leq \beta\,p\,\bar{y}\,[(1-p)\,\mu+p] \quad \Leftrightarrow \quad \frac{1}{(1-p)\,\mu+p}\,\beta_{sc}\leq\beta.$$

iii) Part (ii) of Lemma 2.

The proof of Part (ii) is analogous to that of Lemma 1 and therefore omitted. Again, the precise formula for the cutoff value is of little importance, but we state it here for completeness:

$$\beta_b = \frac{[\mu\,(1-p)^2+p\,(2-p)]\;c}{p\;[\mu\;((1-p^2)\,\mu+p^2)\;\bar{y}-2\,(\mu-1)(1-p)\,c]}.$$

A.4 Proof of Lemma 3

See Section 3.4.

A.5 Proof of Lemma 4

Comparing the narrow goal $\hat{a}_n = \frac{c-\beta\,p\,\bar{y}}{\beta\,p\,(\mu-1)}$ with the broad goal $\hat{a}_b = \frac{c-\beta\,p\,\bar{y}}{\beta\,p\,(1-p)\,(\mu-1)}$, it is straightforward that $\hat{a}_b = \hat{a}_n/(1-p)$.

A.6 Proof of Proposition 1

For narrow goals the (normalized) expected utility of self 0 is:

$$U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2)/\beta \;=\; 2\,(p\,\bar{y}-c)-\hat{a}_n\,[2\,p+2\,(1-p)\,\mu].$$

For the broad goal it is:

$$U_0(\hat{a}_b; \bar{e}_1, \bar{e}_2)/\beta \;=\; 2\,(p\,\bar{y}-c)-\hat{a}_b\,[2\,p-p^2+(1-p)^2\,\mu].$$

So the expected utility of self 0 under narrow bracketing is larger than the one from broad bracketing if and only if the expected cost of goal setting with narrow bracketing, $\hat{a}_n\,[2\,p + 2\,(1-p)\,\mu]$, is lower than the one for broad bracketing, $\hat{a}_b\,[2\,p - p^2 + (1-p)^2\,\mu]$. Using $\hat{a}_b = \frac{\hat{a}_n}{1-p}$ and rearranging:

$$\frac{1}{1-p} \geq \frac{2\,p + 2\,(1-p)\,\mu}{2\,p - p^2 + (1-p)^2\,\mu}.$$

Rearranging again then shows that

$$U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) \geq U_0(\hat{a}_b; \bar{e}_1, \bar{e}_2) \quad \Leftrightarrow \quad p^2 - (1-p)^2\,\mu \geq 0.$$

Exploiting the monotonicity of the utility functions in β (which follows from $\frac{\partial}{\partial\beta}\,\hat{a}_b < 0$, and $\frac{\partial}{\partial\beta}\,\hat{a}_n < 0$, respectively), we obtain another equivalence relation in terms of the respective minimum level of the present bias parameter for which an "easy" broad goal or narrow goals are feasible:

$$U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) \geq U_0(\hat{a}_b; \bar{e}_1, \bar{e}_2) \quad \Leftrightarrow \quad p^2 - (1-p)^2\,\mu \geq 0 \quad \Leftrightarrow \quad \beta_b \geq \beta_n. \tag{A.20}$$

A.7 Derivation of $\hat{a}_b^{info}$ [Equation (12)]

There is an asymmetric effort response of self 2, depending on the outcome in the first task: $\bar{e}_2(\underline{y}_1)$ and $\underline{e}_2(\bar{y}_1)$. So the incentive constraint of self 1 is:

$$\begin{aligned} &\beta\,a_b\,(\mu-1)\,\underbrace{[\Pr(\text{loss}|\underline{e}_1, \bar{e}_2(\underline{y}_1), \underline{e}_2(\bar{y}_1)) - \Pr(\text{loss}|\bar{e}_1, \bar{e}_2(\underline{y}_1), \underline{e}_2(\bar{y}_1))]}_{=1-p-(1-p)^2=p\,(1-p)} \\ \geq \quad & c - \beta\,p\,\bar{y} + [\beta\,(p\,\bar{y} - c) - \beta\,(1-p)\,(p\,\bar{y} - c)]. \end{aligned}$$

The goal that makes this incentive constraint just bind is:

$$\hat{a}_b^{info} = \frac{c - \beta\,p\,\bar{y} + \beta\,p\,(p\,\bar{y} - c)}{\beta\,p\,(1-p)\,(\mu-1)}.$$

Subtracting $\hat{a}_b$ from $\hat{a}_b^{info}$

$$\hat{a}_b^{info} - \hat{a}_b = \frac{p\,\bar{y} - c}{(1-p)\,(\mu-1)},$$

and using $\hat{a}_b = \frac{\hat{a}_n}{1-p}$ gives (12).

A.8 Proof of Lemma 5

See Section 4.2.

A.9 Proof of Proposition 2

i) A "difficult" broad goal $\bar{y} < a_b \leq 2\,\bar{y}$ is never optimal

The utility for self 0 from setting narrow goals is the same for the no-information and information scenario. Abstaining from goal setting yields self 0 utility of zero. To obtain an upper bound on the (normalized) utility of self 0 with a "difficult" broad goal, set $a_b^d = \bar{y}$ (recall that $\frac{\partial}{\partial a} U_0(a; e_1, e_2) < 0$ for fixed e_1, e_2), and suppose that selves 1 and 2 provide high effort (if they do not, the utility of self 0 can only be lower):

$$U_0(\bar{y}; \bar{e}_1, \bar{e}_2)/\beta = \bar{y}\,[p^2 - (1-p)^2\,\mu] - 2\,c.$$

Hence, a necessary condition for a difficult broad goal to do strictly better than no goal is that

$$p^2 - (1-p)^2\,\mu > 0. \tag{A.21}$$

Given that, for fixed effort levels, the utility is decreasing in the goal level, we also know that $U_0(\bar{y}; \bar{e}_1, \bar{e}_2) \leq U_0(\hat{a}_b; \bar{e}_1, \bar{e}_2)$. Therefore, the equivalence relation from (A.20) tells us that a necessary condition for *any* type of broad goal to do strictly better than narrow goals is that

$$p^2 - (1-p)^2\,\mu < 0. \tag{A.22}$$

But the two necessary conditions for a difficult broad goal to be optimal conflict with each other. For example, if Condition (A.22) holds, Condition (A.21) is violated and tells us that the utility from the goal is negative; this means that setting no goal is preferred over setting a difficult broad goal. So whenever narrow goals are feasible (i.e., if $\frac{1}{\mu}\,\beta_{sc} \leq \beta$), either narrow goals dominate a difficult broad goal, or setting no goal is optimal. As argued in the proofs of Lemma 1 and 3, at $\beta = \frac{1}{\mu}\,\beta_{sc}$ we have $U_0(\bar{y}; \bar{e}_1, \bar{e}_2) < 0$. This is the upper bound on the profit of a difficult broad goal for $\beta < \frac{1}{\mu}\,\beta_{sc}$ as well, because for any type of broad goal $\frac{\partial}{\partial \beta} U_0(a; \bar{e}_1, \bar{e}_2) > 0$. Taken together this shows that a difficult broad goal can never be optimal.

ii) An "easy" broad goal $a_b \leq \bar{y}$ is never optimal

The (normalized) expected utility of self 0 from setting narrow goals is:

$$U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2)/\beta \;\; = \;\; 2\,(p\,\bar{y} - c) - \hat{a}_n\,[2\,p + 2\,(1-p)\,\mu].$$

For the "easy" broad goal the utility can be written as:

$$\begin{aligned} U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2)/\beta \;\; &= \;\; (2-p)\,(p\bar{y} - c) - \hat{a}_b^{info}\,[2\,p - p^2 + (1-p)^2\,\mu] \\ &= \;\; (2-p)\,(p\bar{y} - c) - \left[\hat{a}_b + \frac{p\,\bar{y} - c}{(1-p)\,(\mu - 1)}\right]\,[2\,p - p^2 + (1-p)^2\,\mu] \\ &= \;\; (2-p-\kappa)\,(p\,\bar{y} - c) - \hat{a}_b\,[2\,p - p^2 + (1-p)^2\,\mu], \end{aligned} \tag{A.23}$$

where $\kappa \equiv \frac{2\,p-p^2+(1-p)^2\,\mu}{(1-p)\,(\mu-1)}$, and we use from (12) that $\hat{a}_b^{info} = \hat{a}_b + \frac{p\,\bar{y}-c}{(1-p)\,(\mu-1)}$. Substituting $\hat{a}_n = (1-p)\,\hat{a}_b$ we obtain:

$$\begin{aligned} & U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) - U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2) \geq 0 \\ \Leftrightarrow \quad & (p+\kappa)\,(p\,\bar{y}-c) - \hat{a}_b\,\left[(1-p)^2\,\mu - p^2\right] \geq 0. \end{aligned} \tag{A.24}$$

Thus, if $p^2-(1-p)^2\,\mu \geq 0$ we know that $U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) - U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2)$ is strictly positive. This is exactly the condition from Proposition 1: so whenever narrow bracketing is optimal in the no-information scenario, it is also optimal in the information scenario.
Now suppose $p^2 - (1-p)^2\,\mu < 0$. We will now see that $U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2) \geq 0$ implies $U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) - U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2) \geq 0$, i.e., whenever a broad goal would yield a positive utility, it is dominated by narrow bracketing. For this we show that

$$U_0(\hat{a}_n, \hat{a}_n; \bar{e}_1, \bar{e}_2) - U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2) \geq U_0(\hat{a}_b^{info}; \bar{e}_1, \bar{e}_2). \tag{A.25}$$

Comparing (A.23) and (A.24), note first that $\hat{a}_b\,[(1-p)^2\,\mu - p^2] \leq \hat{a}_b\,[2\,p - p^2 + (1-p)^2\,\mu]$. Second, note that $(p+\kappa)\,(p\,\bar{y}-c) \geq (2-p-\kappa)\,(p\,\bar{y}-c) \Leftrightarrow \kappa \geq 1-p$. The result in (A.25) follows because

$$\kappa \equiv \frac{2\,p - p^2 + (1-p)^2\,\mu}{(1-p)\,(\mu-1)} \geq 1-p \quad \Leftrightarrow \quad 2\,p - p^2 + (1-p)^2\,\mu \geq (1-p)^2\,(\mu-1),$$

where the latter inequality clearly holds.

B The simultaneous-tasks scenario

In Section 3, a broad goal $a_b \leq \bar{y}$ gives rise to a leaning-back effect, according to which self 1 can substitute effort on the first task with (for self 1 less painful) effort by self 2 on the second task. What if the individual works on the two tasks simultaneously? The difference with the *no-information scenario* is that both effort costs are felt at the same time, and we therefore have two incentive constraints. Providing effort in both tasks must be better than providing effort in just one task:

$$\begin{aligned} & a_b\,\beta\,(\mu-1)\,[\underbrace{\Pr(\text{loss}|\underline{e}_j, \bar{e}_i) - \Pr(\text{loss}|\bar{e}_j, \bar{e}_i)}_{p\,(1-p)}] \geq c - \beta\,p\,\bar{y} \\ \Leftrightarrow \qquad & a_b \geq \frac{c-\beta\,p\,\bar{y}}{\beta\,p\,(\mu-1)\,(1-p)} \equiv \hat{a}_b, \end{aligned}$$

because this corresponds exactly to the relevant incentive constraint in the *no-information scenario* [Equation (8)]. And providing effort in both tasks must also be better than shirking in both tasks:

$$\begin{aligned} & \beta\,a_b\,(\mu-1)\,[\underbrace{\Pr(\text{loss}|\underline{e}_j, \underline{e}_i) - \Pr(\text{loss}|\bar{e}_j, \bar{e}_i)}_{p\,(2-p)}] \geq 2\,(c - \beta\,p\,\bar{y}) \\ \Leftrightarrow \qquad & a_b \geq \frac{c-\beta\,p\,\bar{y}}{\beta\,p\,(\mu-1)\,(1-\frac{p}{2})} \equiv \breve{a}. \end{aligned}$$

To satisfy both incentive constraints, $a_b \geq \max\{\hat{a}_b, \breve{a}\} = \hat{a}_b$. So $\hat{a}_b^{sim} = \hat{a}_b$ from equation (8), and the results from the *no-information scenario* carry over.

To gain some intuition for why the the "prevent-shirking-in-one-task" incentive constraint is the binding one, consider first the effort cost side (net of the present-bias discounted expected benefit from the task outcome). If, instead of working hard in both tasks, self 1 were to shirk in one task he would gain $c - \beta\, p\, \bar{y}$; and if he were to shirk in both tasks he would gain twice that amount, $2\,(c - \beta\, p\, \bar{y})$. But shirking exposes him to the risk of falling short of the goal. Consider this side now. Intuitively, by not working at all, the individual would definitely fall short of the goal and suffer a loss. In contrast, putting in effort in one of the two tasks already goes some way toward avoiding this: with probability p the task outcome is high, and this is enough to satisfy the easy broad goal. So the difference from working hard in both tasks is not that big anymore. More specifically, shirking only in one task would increase the probability of a loss by $\Pr(\text{loss}|\underline{e}, \bar{e}) - \Pr(\text{loss}|\bar{e}, \bar{e}) = p\,(1-p)$, whereas shirking in both tasks would increase by more than twice that amount the probability of a loss: $\Pr(\text{loss}|\underline{e}, \underline{e}) - \Pr(\text{loss}|\bar{e}, \bar{e}) = p\,(2-p) > 2\,p\,(1-p)$.

Putting both sides together, we thus see that the risk of falling short of the goal is proportionately larger relative to the net cost of effort in the case of shirking in both tasks. As a result, the "prevent-shirking-in-one-task" incentive constraint is the binding one.

C Diminishing sensitivity

C.1 No information scenario

The incentive constraint for self 2 under a narrow goal is:

$$\begin{aligned} & \beta\,\{p\,(\bar{y} + \mu^{+}(\bar{y} - a) + (1-p)\,\mu^{-}(-a)\} - c \geq \beta\,\mu^{-}(-a) \\ \Leftrightarrow \quad & \beta\,p\{\mu^{+}(\bar{y} - a) - \mu^{-}(-a)\} \geq c - \beta\,p\,\bar{y}. \end{aligned}$$

The incentive constraint for self 2 under a broad goal satisfying $\bar{y} > a$ is:

$$\begin{aligned} & \beta\,\{p\,\bar{y} + p^2\,\mu^{+}(2\,\bar{y} - a) + 2\,p\,(1-p)\,\mu^{+}(\bar{y} - a) + (1-p)^2\,\mu^{-}(-a)\} - c \geq \beta\,\{p\,\mu^{+}(\bar{y} - a) + (1-p)\,\mu^{-}(-a)\} \\ \Leftrightarrow \quad & \beta\,p\,\{p\,\mu^{+}(2\,\bar{y} - a) + (1-2\,p)\,\mu^{+}(\bar{y} - a) - (1-p)\,\mu^{-}(-a)\} \geq c - \beta\,p\,\bar{y} \\ \Leftrightarrow \quad & \beta\,p\,\{\mu^{+}(\bar{y} - a) - \mu^{-}(-a) + p\,[\mu^{+}(2\,\bar{y} - a) - 2\,\mu^{+}(\bar{y} - a) + \mu^{-}(-a)]\} \geq c - \beta\,p\,\bar{y}. \end{aligned}$$

Comparing the incentive constraints for narrow and broad goals for the same goal a, we see that the left-hand side under a broad goal differs from the one under narrow goals by

$$p\,[\mu^{+}(2\,\bar{y} - a) - 2\,\mu^{+}(\bar{y} - a) + \mu^{-}(-a)].$$

By diminishing sensitivity, $2\,\mu^{+}(\bar{y} - a) \geq \mu^{+}(2\,\bar{y} - 2\,a)$, and by loss aversion, $\mu^{-}(-a) < -\mu^{+}(a)$. Hence,

$$\mu^{+}(2\,\bar{y} - a) - 2\,\mu^{+}(\bar{y} - a) + \mu^{-}(-a) \leq \mu^{+}(2\,\bar{y} - a) - \mu^{+}(2\,\bar{y} - 2\,a) - \mu^{+}(a). \qquad \text{(C.26)}$$

By the concavity of $\mu^+(\cdot)$ and $\mu^+(0) = 0$, when we make the individual $2\,\bar{y} - 2\,a$ units richer the increase in gain-loss utility from getting a is smaller:

$$\underbrace{\mu^+(2\,\bar{y} - a)}_{=\mu^+(2\,\bar{y} - 2\,a + a)} - \mu^+(2\,\bar{y} - 2\,a) \leq \mu^+(a) - \mu^+(0). \tag{C.27}$$

Inequalities (C.26) and (C.27) imply that

$$\mu^+(2\,\bar{y} - a) - 2\,\mu^+(\bar{y} - a) + \mu^-(-a) \leq 0$$

Thus, for the same goal a, the incentive constraint under a broad goal is less likely to be satisfied than the incentive constraint under a narrow goal. To restore incentives, the individual hence needs to set a higher broad goal compared to the narrow goals.

C.2 Information scenario

Suppose the broad goal is such that $\bar{y} > a$ holds. As in the main model, the individual shirk if he observes that first period output is $\bar{y}$ because the goal already is fulfilled; and he works hard if he observes that first period output is $\underline{y}$. So the first period incentive constraint is:

$$\begin{aligned} & \beta\,\{p\,[\bar{y} + \mu^+(\bar{y} - a)] + (1-p)\,[p\,\bar{y} - c + p\,\mu^+(\bar{y} - a) + (1-p)\,\mu^+(-a)]\} - c \\ \geq\;& \beta\,\{p\,\bar{y} + p\,\mu^+(\bar{y} - a) + (1-p)\,\mu^-(-a) - c\}. \end{aligned}$$

Rearranging, we get

$$\beta\,(1-p)\,p\,\{\mu^+(\bar{y} - a) - \mu^-(-a)\} \geq c - \beta\,p\,\bar{y} + \beta\,p\,[p\,\bar{y} - c].$$

The left-hand sides of the incentive constraints under a broad goal in the no-information vs. the information scenario differ by:

$$\begin{aligned} & \beta\,p^2\,\{[\mu^+(\bar{y} - a) - \mu^-(-a)] + [\mu^+(2\,\bar{y} - a) - 2\,\mu^+(\bar{y} - a) + \mu^-(-a)]\} \\ =\;& \beta\,p^2\,\{\mu^+(2\,\bar{y} - a) - \mu^+(\bar{y} - a)\} > 0. \end{aligned}$$

The right-hand side of the incentive constraints differ by $-\beta\,(1-p)\,[p\,\bar{y} - c] < 0$.
Thus, for the same goal a, the incentive constraint under a broad goal in the information scenario is less likely to be satisfied than the incentive constraint under a broad goal in the no information scenario. As in the main model, this implies that the individual hence needs to set a higher broad goal compared to the required narrow goals.

D Continuous output and effort

D.1 No-information scenario

The utility of self 1 for task 2 under a narrow goal is given by:

$$U(e) \;=\; \beta\left\{\int_{\underline{y}}^{\bar{y}} (e + \epsilon_2)\,f(\epsilon_2)\,d\,\epsilon_2 + \int_{\underline{y}}^{\bar{y}} (e + \epsilon_2 - a)\mathcal{I}_{e+\epsilon_2 \geq a}\,f(\epsilon_2)\;\epsilon_2\} + \mu\int_{\underline{y}}^{\bar{y}} (e + \epsilon_2 - a)\mathcal{I}_{e+\epsilon_2 < a}\,f(\epsilon_2)\right\} - c(e)$$

$$
\begin{aligned}
&= \beta \left\{ e + E(\epsilon_2) + (e-a)\,(1-F(a-e)) + \mu\,(e-a)\,F(a-e) + \int_{a-e}^{\bar{y}} \epsilon_2\, f(\epsilon_2)\, d\,\epsilon_2 + \mu \int_{\underline{y}}^{a-e} \epsilon_2\, f(\epsilon_2)\, d\,\epsilon_2 \right\} \\
&- \quad c(e).
\end{aligned}
$$

Differentiating gives the first order condition stated in the text. Replacing ϵ_2 by $\epsilon = \epsilon_1 + \epsilon_2$ and the density f and distribution F by h and H, respectively, gives the utility under a broad goal.

D.2 Diminishing sensitivity

The utility of self 2 after observing the realization of the background risk, ϵ_1, is:

$$
\begin{aligned}
U(e) &= \beta \left\{ e + E(\epsilon_1) + \int_{a-e-\epsilon_1}^{\bar{y}} \mu^{+}(e+\epsilon_1+\epsilon_2-a)\, f(\epsilon_2)\, d\,\epsilon_2 + \mu \int_{\underline{y}}^{a-e-\epsilon_1} \mu^{-}(e+\epsilon_1+\epsilon_2-a)\, f(\epsilon_2)\, d\,\epsilon_2 \right\} \\
&- \quad c(e).
\end{aligned}
$$

The first order conditions are given by:

$$
\beta\,\{1 + \int_{a-e-\epsilon_1}^{\bar{y}} \mu^{+'}(e+\epsilon_1+\epsilon_2-a)\, f(\epsilon_2)\, d\,\epsilon_2 + \int_{\underline{y}}^{a-e-\epsilon_1} \mu^{-'}(e+\epsilon_1+\epsilon_2-a)\, f(\epsilon_2)\, d\,\epsilon_2\} = c'(e).
$$

Applying integration by parts to the gain-loss utility on the left hand side, we obtain:

$$
\begin{aligned}
&\underbrace{[\mu^{-'}(0) - \mu^{+'}(0)]\, F(a-e-\epsilon_1)}_{A} + \underbrace{\mu^{+'}(e+\bar{y}+\epsilon_1-a)}_{B} \\
&\underbrace{- \left\{ \int_{a-e-\epsilon_1}^{\bar{y}} \mu^{+''}(e+\epsilon_1+\epsilon_2-a)\, F(\epsilon_2)\, d\,\epsilon_2 + \int_{\underline{y}}^{a-e-\epsilon_1} \mu^{-''}(e+\epsilon_1+\epsilon_2-a)\, F(\epsilon_2)\, d\,\epsilon_2 \right\}}_{C}.
\end{aligned}
$$

Term A is decreasing in ϵ_1 just as in the case without diminishing sensitivity. Because of the concavity of μ^{+}, term B is also decreasing in ϵ_1. It captures the second effect described by Heath et al. (1999) that “People [are] willing to exert less effort as they move away from their goal.” The final term also arises because of diminishing sensitivity; its sign can be positive or negative.

To calculate the sign of term C and the overall gain-loss utility for a specific example, consider the following 2-nd-order Taylor approximation of a standard value function (cf. Iantchev, 2009): $x - \frac{1}{2}\,\gamma\, x^2$ for $x \geq 0$ and $\mu\, x + \frac{1}{2}\,\mu\,\gamma\, x^2$ for $x < 0$, where $\mu > 1$ and $\gamma \in (0,1)$. This gives

$$
C = \gamma \left[\int_{a-e-\epsilon_1}^{\bar{y}} F(\epsilon_2)\, d\,\epsilon_2 - \mu \int_{\underline{y}}^{a-e-\epsilon_1} F(\epsilon_2)\, d\,\epsilon_1 \right].
$$

Term C is increasing in ϵ_1 because the parameter of loss aversion $\mu > 1$. The total gain-loss utility is given by:

$$
\gamma\,(\mu-1)\, F(a-e-\epsilon_1) + 1 - \gamma\,(e+\bar{y}+\epsilon_1-a) + \gamma \left[\int_{a-e-\epsilon_1}^{\bar{y}} F(\epsilon_2)\, d\,\epsilon_2 - \mu \int_{\underline{y}}^{a-e-\epsilon_1} F(\epsilon_2)\, d\,\epsilon_1 \right].
$$

Differentiating, we obtain

$$\gamma\,[(\mu-1)\,\{F(a-e-\epsilon_1)-f(a-e-\epsilon_1)\}-1].$$

Effort is decreasing in ϵ_1 if, for example, $F-f<0$, which holds for the uniform distribution. Effort is decreasing in ϵ_1 for μ not too large, because $F(a-e-\epsilon_1)\leq 1$.