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DISCUSSION PAPER SERIES

IZA DP No. 13666

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ISSN: 2365-9793

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ABSTRACT

Long Live the Vacancy*

We reassess the role of vacancies in a Diamond-Mortensen-Pissarides style search and matching model. In the absence of free entry long lived vacancies and endogenous separations give rise to a vacancy depletion channel which we identify via joint unemployment and vacancy dynamics. We show conditions for constrained efficiency and discuss important implications of vacancy longevity for modeling and calibration, in particular regarding match cyclicality and wages. When calibrated to the postwar US economy, the model explains not only standard deviations and autocorrelations of labor market variables, but also their dynamic correlations with only one shock.

JEL Classification: E24, E32, J63, J64

Keywords: Beveridge curve, business cycles, job destruction, random matching, separations, unemployment volatility, wage determination

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* The authors wish to thank Almut Balleer, Melvyn Coles, Russell Cooper, Wouter den Haan, Jason Faberman, Shigeru Fujita, Andreas Gulyas, Johanna Haefke, John Haltiwanger, Ayşe İmrohoroğlu, Leo Kaas, Philipp Kircher, Moritz Kuhn, Robert Kunst, John Leahy, Christian Merkl, Monika Merz, Jun Nie, Tamás Papp, Gilles St. Paul, Thijs van Rens, Etienne Wasmer, and participants of the 2017 Abu Dhabi Global Macro Workshop, 2017 Computing in Economics and Finance Conference, 2017 Labor Market Dynamics Workshop at the Board of Governors and the 11th Tsinghua Workshop in Macroeconomics 2018 for helpful discussions and valuable suggestions.

1 Introduction

In early 2017 mean vacancy duration ranged from fifteen days for small and medium enterprises to sixty five days for large enterprises[1]. Job search websites like indeed.com give guidelines on when and how to re-post jobs, resume-now.com provides strategies to job seekers who face reposted job ads. Clearly, vacancies have varying durations and are not necessarily destroyed at the end of a day/week/month if unfilled.

In the absence of free entry for firm startups, e.g. as in Diamond (1982) or Melitz (2003), this observation implies that an economy's stock of vacancies becomes a state variable and that in addition to the well studied job creation a second channel – vacancy[2] depletion – becomes relevant for labor market dynamics. Without explicitly labeling the channel, Coles and Moghaddasi Kelishomi (2018) illustrate the persistence and magnification of business cycle shocks that arise when a large number of newly unemployed job-seekers deplete the vacancy pool that then replenishes only gradually.

We propose a model with endogenous separations that encompasses the well known Hall Milgrom model (Hall and Milgrom, 2008) and the model of Coles and Moghaddasi Kelishomi (2018). We identify the key parameter that governs the relationship between vacancy creation and vacancy depletion and show how it relates to the lead/lag structure of unemployment and vacancies. We establish that vacancies lead unemployment for a set of OECD countries with long available vacancy series and the U.S. since 1951 (cf. Figure 1), which indicates an overall dominating vacancy creation channel. Nevertheless, the quantitative importance of the vacancy depletion channel is substantial and the impact of long-lived vacancies on the model is far-reaching theoretically and empirically, as has been anticipated by Elsby, Michaels and Ratner (2015) who wrote "...areas in which additional research seems especially fruitful [...] include the role of wage determination and entry into vacancy creation on the volatility and sluggishness of vacancy dynamics".

In addition to the important amplification of shocks generated by the depletion channel, the stock flow dynamics of vacancies generate highly persistent effects of temporary shocks (Fujita and Ramey, 2007) implying protracted deviations from steady state, in particular for unemployment. It is an empirical fact[3] that not every separation of an employment relationship leads to the destruction of the underlying job. Long lived vacancies naturally

[1] Based on data released by Steve Davis at `https://www.dice.com/indicators/contents/data/2017/08/2017-08-DHI-HIring-Indicators-FINAL-1.xlsx`. Mueller et al. (2020) report vacancy durations of comparable magnitude for Austrian linked employer-employee data.

[2] The analogous effect on the unemployment pool is widely acknowledged. Longer expansions typically lead to an exhaustion of the job-searcher pool, job-filling probabilities fall because it becomes increasingly harder to find qualified job applicants as expansions continue.

[3] On average the BLS reports 7.1 million destroyed jobs per quarter between 2001 and 2019 while an average of 13.7 million employment relationships were separated according to JOLTS.

allow the modeling of this fact, thus providing an important building block in modeling job and worker flows jointly.

In our model firms create jobs by paying a sunk cost to post a vacancy. This vacancy can either be filled or destroyed. Vacancies that are neither filled nor destroyed carry over as stock to the subsequent period. We call these vacancies long lived in contrast to the canonical approach of destroying unfilled vacancies with probability one at the end of a period. To our knowledge long lived vacancies (LLV) were first introduced to the literature by Fujita (2004) and Fujita and Ramey (2007) who documented the substantial improvement in the modelling of persistence. A key mechanism in their setup is the departure from free entry of vacancies towards a job creation process that Coles and Moghaddasi Kelishomi (2018) call Diamond entry after Diamond (1982). Effectively, entrepreneurs draw a one-time job/vacancy creation cost and will enter only if the cost is sufficiently low. It is the combination of Diamond entry and long lived vacancies that (i) makes vacancy stock an important state rather than jump variable in the model and (ii) generates a positive value for unfilled vacancies which will be important for wage determination and potentially separations. Coles and Moghaddasi Kelishomi (2018) extend Fujita and Ramey (2007) to allow for exogenous, time varying job destruction and document an important interaction effect. A large inflow of unemployed in response to a negative productivity shock will match and thus absorb an important fraction of the vacancy stock. In the absence of free entry a below steady state vacancy stock leads to substantial persistence in unemployment and vacancies, a substantial labor market tightness response and a pronounced Beveridge Curve relationship. Our model with fully endogenous separations maintains these desirable properties.

The original insight of Shimer (2005) and Costain and Reiter (2008) that Nash bargained wages fluctuate more than empirically observed is even more important in the absence of free entry when vacancies are long lived. Vacancy depletion substantially depresses wages in recessions. The wage that would decentralize the planner allocation varies substantially more than productivity. Over the last decade the bargaining protocol proposed by Hall and Milgrom (2008) has turned out to be a useful and convenient way to model wage determination in a Diamond Mortensen Pissarides (DMP) style search and matching model. As a generalization of Nash bargaining it offers a path to insulating wage negotiations from current unemployment and can thus generate empirically observed unemployment and wage fluctuations. When Hall and Milgrom introduced their bargaining protocol they focused on driving a wedge between workers' threat points and outside options. In our model their protocol applies both to the worker and the firm side.

Our model — when calibrated to US data for 1951–2003 — successfully matches a broad set of first and second moments including the Beveridge Curve, vacancy, unemployment and job finding variability as well as all relevant dynamic correlations. While the

Hall Milgrom benchmark does equally well in matching unemployment, we substantially improve for job finding probabilities as well as GDP persistence by allowing for labor market adjustments via both the hiring and separation margin through endogenous separations and the vacancy depletion channel. Finally, we fit our model to US labor market data from 1951–2018 and illustrate how the combination of features implied by long lived vacancies manages to capture the data surprisingly well with only one exogenous shock.

While our model is successful in replicating both average job destruction and separations, capturing a richer set of establishment level characteristics and worker characteristics as in Cooper, Haltiwanger and Willis (2007) would require firm heterogeneity. Then there would be room for job-to-job transitions (e.g. Menzio and Shi (2011)), giving rise to vacancy ladders, where each transition absorbs a vacancy and opens another one.

The entry mechanism based on noisy signals of productivity explored by Pries (2016) can be seen as an interesting complement to Diamond entry. The importance of the dynamics of vacancies and their values has also been pointed out by Shao and Silos (2013) in a model with Diamond entry, constant separations and long lived vacancies. Leduc and Liu (2020) estimate a two-shock model of endogenous search and recruiting intensity with exogenously varying match destruction in the spirit of Coles and Moghaddasi Kelishomi (2018) for 1967–2017 U.S. data. They show that both vacancy longevity and departure from free entry are necessary for recruiting intensity to depend on labor market conditions. Discount factor effects in models with long lived vacancies are particularly powerful because a vacancy can be considered capital. We focus on one shock and constant intensities to highlight the contribution of endogenous separations and microfounded wage bargaining. However, adding the discount factor shock and the variable intensity would help in further improving our model-fit in the post great recession period.

The remainder of this paper is organized as follows. In section 2 we summarize the data. Section 3 describes the model. Theoretical insights are presented in section 4, in particular how endogenous separations and the Beveridge Curve relationship inform estimates of the unemployment elasticity in the matching function. Section 5 presents the calibration strategy, main numerical results, extensions and robustness checks before section 6 concludes.

2 Data

To facilitate comparability we study the same period as Shimer (2005) for the major part of this paper and refer to his work for a comprehensive description of the data. Results remain qualitatively similar for alternative subperiods. Table 1 reports the usual business cycle statistics[4].

Table 1: Descriptive Statistics for Key Labor Market Variables, 1951–2003.

	u	v	M	ϕ^w	δ	GDP
Mean	$_{5.65}5.67_{5.70}$	—	$_{2.45}2.46_{2.47}$	$_{44.97}44.98_{45.00}$	$_{3.38}3.39_{3.40}$	—
StDev	$_{0.17}0.19_{0.21}$	$_{0.19}0.20_{0.22}$	$_{0.075}0.083_{0.093}$	$_{0.108}0.118_{0.126}$	$_{0.064}0.071_{0.080}$	$_{0.024}0.026_{0.029}$
RStDev	$_{6.79}7.28_{7.70}$	$_{7.10}7.77_{8.24}$	$_{2.88}3.16_{3.51}$	$_{4.18}4.53_{4.72}$	$_{2.45}2.70_{3.02}$	$_{1.00}1.00_{1.00}$
AC	$_{0.88}0.93_{0.98}$	$_{0.88}0.93_{0.98}$	$_{0.83}0.90_{0.97}$	$_{0.89}0.94_{0.99}$	$_{0.77}0.86_{0.95}$	$_{0.87}0.93_{0.98}$
u	1	$_{-.95}-.93_{-.92}$	$_{0.92}0.93_{0.95}$	$_{-.97}-.96_{-.95}$	$_{0.68}0.73_{0.80}$	$_{-.93}-.91_{-.89}$
v		1	$_{-.85}-.80_{-.76}$	$_{0.93}0.95_{0.96}$	$_{-.72}-.64_{-.58}$	$_{0.82}0.84_{0.88}$
M			1	$_{-.86}-.81_{-.78}$	$_{0.73}0.79_{0.85}$	$_{-.85}-.82_{-.75}$
ϕ^w				1	$_{-.72}-.63_{-.58}$	$_{0.88}0.90_{0.93}$
δ					1	$_{-.80}-.74_{-.68}$
GDP						1

Except for the means all statistics have been computed for detrended (HP, smoothing parameter 10e5) and seasonally adjusted logarithms of the respective quarterly time series. Unemployment, u, is the UNRATE series of the St. Louis Fred, vacancy, v, data has been provided by Regis Barnichon (Barnichon, 2010). Job finding, ϕ^w, and separation, δ, probabilities have been computed based on the methodology described in Shimer (2012). Matches, M, are computed by multiplying the unemployment rate (u) with the job finding probability (ϕ^w). GDP is real GDP as downloaded from FRED. 95% bootstrapped confidence bounds are reported in subscripts to the left and right of the respective statistic.

The average unemployment rate over the sample period, as published by the BLS based on CPS data, was 5.67%. Unemployment, u, fluctuates strongly around its trend with a standard deviation of 0.19, which is more than seven times the variability of real per capita GDP.

Regis Barnichon (2010) provides a methodology to combine data on the Help Wanted Index and the more recent JOLTS data. We use the vacancy series provided on his website[5], which combines the advantage of a long series from the Help Wanted Index with the accurate measurement of JOLTS since 2000. In Table 1 and Figure 2 we see that the cyclical component of vacancies, v, fluctuates approximately as much as unemployment and is highly persistent, consistent with the findings of Christiano, Eichenbaum and Trabandt (2016); Davis, Faberman and Haltiwanger (2013); Fujita and Ramey (2007) for the US and a number of other OECD countries (Amaral and Tasci, 2016). The contemporaneous correlation with unemployment is strongly negative with -0.91, however, the highest correlation between unemployment and vacancies obtains when vacancies are lagged one period, i.e. vacancies lead unemployment by one quarter. Similar behavior prevails for all European countries with at least 50 years of data availability in the OECD database as illustrated in Figure 1.

[4]When series are available at higher frequencies, they are averaged to quarterly series. For all series we study log-deviations from a slow moving HP trend with smoothing parameter 10^5. A detailed report of all data sources and transformations is provided in Appendix A together with extra figures.

[5]https://sites.google.com/site/regisbarnichon/cv/HWI_index.txt

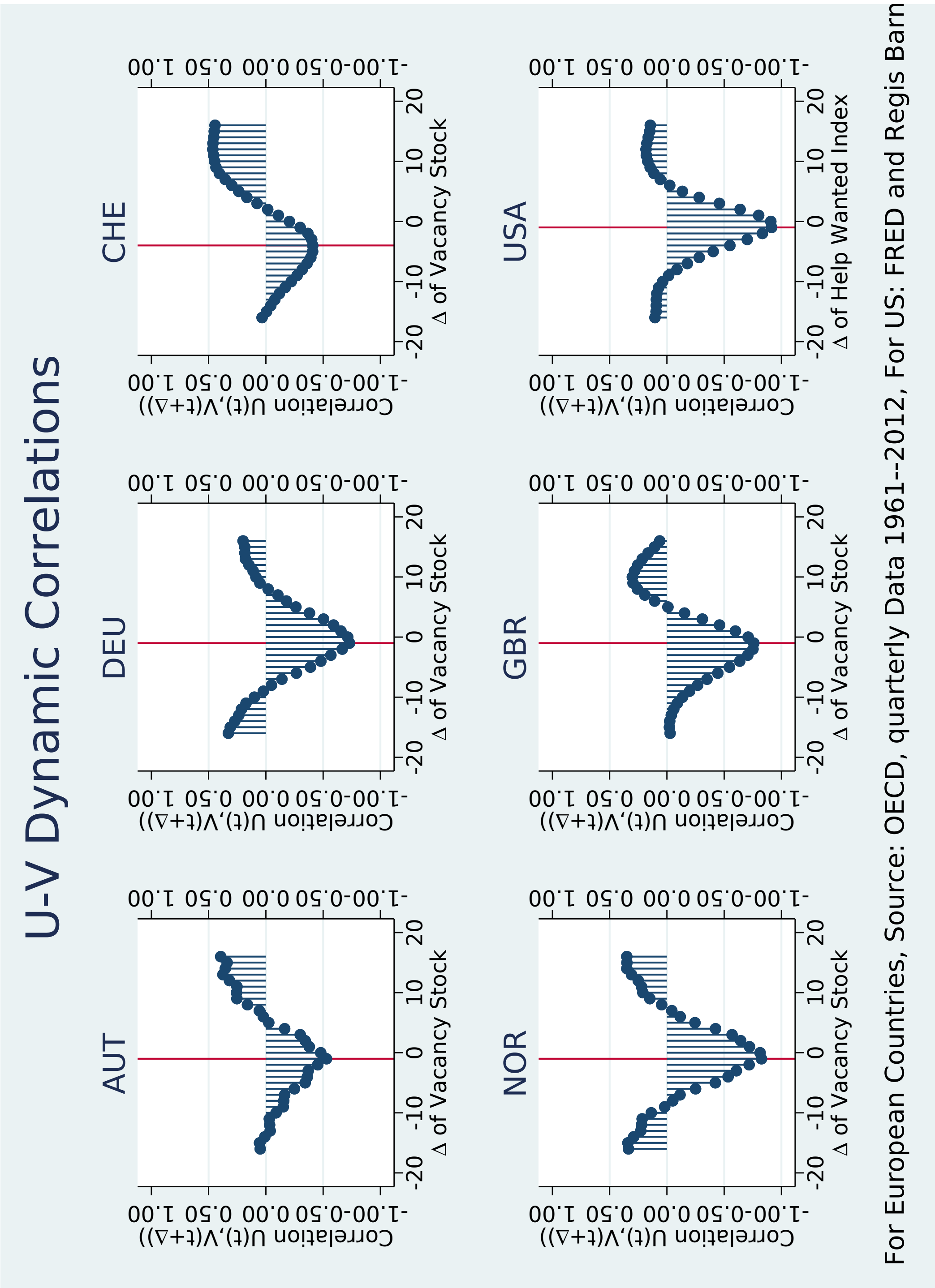

Figure 1: Dynamic Unemployment – Vacancy Correlations for USA and five European Countries.

US Data: FRED and Barnichon (2010), 1951–2003. European Data: OECD, 1961–2012. Δ denotes the time shift in the vacancy series when computing the dynamic correlations. The red line indicates the largest negative correlation. Vacancies lead by one quarter in all countries except Switzerland, where they lead by four.

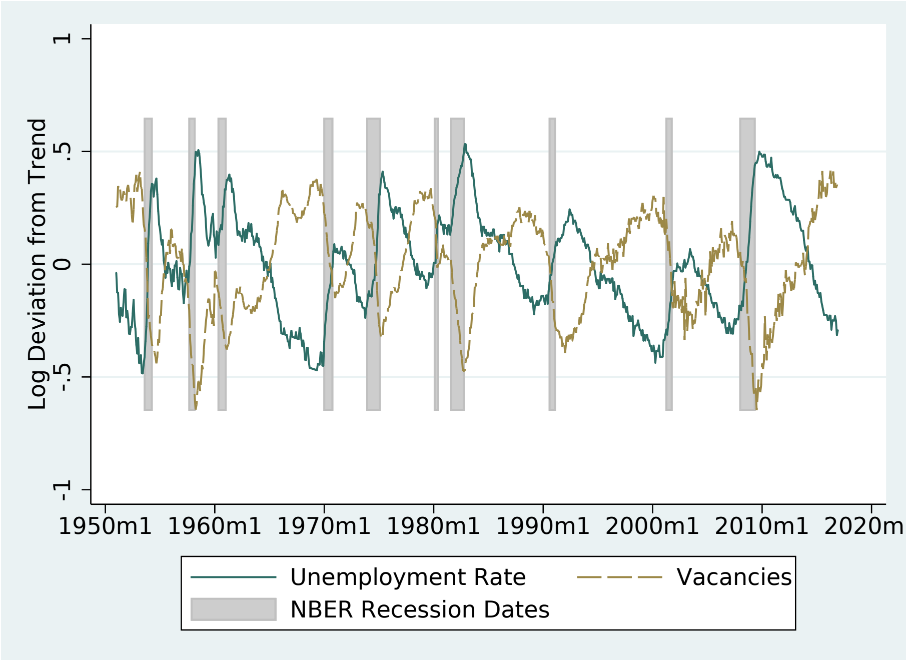

Figure 2: The Cyclical Component of Unemployment and Vacancies for the USA.

To compute the job finding probabilities, ϕ^w, we work with the series on short term unemployment as suggested by Shimer (2005). The average monthly[6] job-finding probability over the sample period is 44.4%. The cyclical component of the job finding probability is substantially less volatile than the unemployment rate and fluctuates approximately four and a half times as much as GDP. Job finding probabilities are highly negatively correlated with unemployment.

Following Shimer's assumption that all hires go through unemployment we compute the number of matches, M, as product of the unemployment rate and job finding probabilities. The cyclical component of new hires is substantially less volatile than unemployment and fluctuates approximately three times as much as GDP. As previously noted by Blanchard and Diamond (1990) and Mortensen (1994) flows from unemployment to employment are strongly positively correlated with unemployment.[7]

To compute separation probabilities, δ, we again follow the methodology suggested by Shimer (2005). The average monthly separation probability is 3.3% over the sample period. Separation probabilities fluctuate substantially less than unemployment and vacancies, approximately three times as much as GDP. Autocorrelation is substantially lower than for other reported labor market variables. Separation probabilities spike in recessions (see

[6]The literature typically reports quarterly averages of monthly probabilities, so we do the same. However, all other statistics are based on logarithms of quarterly averages.

[7]These observations are consistent with CPS micro-data on worker flows for the period 1984 onwards.

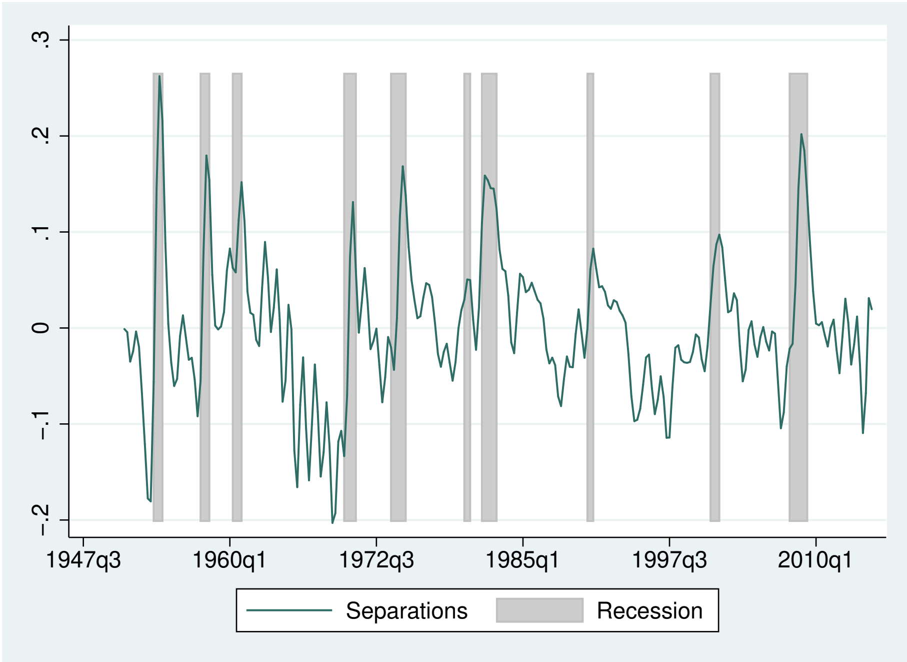

Figure 3: The Cyclical Behavior of the U.S. Separation Probability.
Separations are computed based on Shimer (2005). Series is in log-deviations from a low frequency HP trend.

Figure 3) which leads to a comparatively low autocorrelation of 0.8.

To obtain a wage series over the entire sample period from 1951–2003 we work with total compensation for private industries from the NIPA accounts which we divide by total hours worked and deflate using the GDP deflator. Wages exhibit a standard deviation relative to GDP of 0.72 with autocorrelation of 0.80 and a correlation with unemployment of -0.48. These numbers are qualitatively similar with other reports in the literature, e.g. Gertler and Trigari (2009) who use a lower smoothing parameter and shorter sample period.

3 The Model

Our model extends Hall and Milgrom (2008) in two dimensions. First, all separations are endogenous. Secondly, we allow for long lived vacancies that are potentially in finitely elastic supply as in Fujita and Ramey (2007) or Coles and Moghaddasi Kelishomi (2018).

There is one aggregate shock to labor productivity that generates business cycles,

$$\ln y_t = \rho_y \ln y_{t-1} + \sigma_y \varepsilon_t. \tag{1}$$

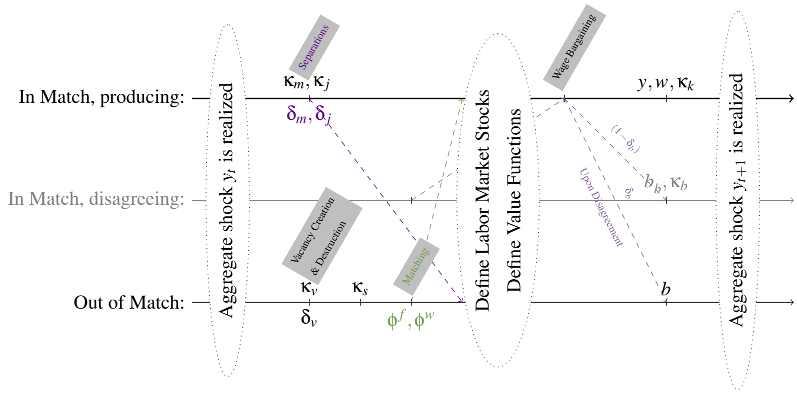

Figure 4: Model Timeline with Endogenous Separations and Hall Milgrom Wage Bargaining.

3.1 Firms and Vacancies

Every period there is a unit mass of potential vacancies. Analogously to Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2018) firms that would like to open such a vacancy pay a one-time stochastic vacancy posting cost κ_v drawn from a cost-distribution $\mathcal{V}$. Hence the vacancy posting decision will follow a threshold rule and firms with a draw $\kappa_v \leq \bar{\kappa}_{vt}$ pay, so that the flow of new vacancies in period t is given by:

$$n_t \quad = \quad \mathcal{V}(\bar{\kappa}_{vt}). \tag{2}$$

Free entry is the limiting case of (2) with degenerate cost distribution and a large mass of potential vacancies. For comparability with Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2018) it is useful to define[8] the elasticity ξ of the mass of new vacancies with respect to the vacancy posting cost threshold to be:

$$\xi \quad = \quad \frac{\mathrm{d}\mathcal{V}(\bar{\kappa}_v)}{\mathrm{d}\bar{\kappa}_v}\frac{\bar{\kappa}_v}{n}, \tag{3}$$

which goes to infinity in the limiting case of constant marginal vacancy posting cost, i.e.free entry.

A vacancy remains open until exogenously[9] destroyed with probability δ_v, or filled with filling probability ϕ_t^f. Matching and labor market flows are as described in Section 3.3. Denote the value of an unfilled vacancy at the end of period t by V_t^V and the value of being

[8]We solve the model by linearization. For these purposes the mean vacancy posting cost and the elasticity in steady state fully characterize the vacancy cost distribution.

[9]It would be straightforward to endogenize vacancy destruction. However, we will see from equation (10) that the lion share of vacancy outflow is attributable to new matches, which are at least one order of magnitude larger.

matched with a worker at the end of period t as V_t^J. The value of a newly created vacancy $V^{\check{V}}{}_t$ can then be written as

$$
\begin{aligned}
V^{\check{V}}{}_t &= -\kappa_s + \phi_t^f V_t^J + (1-\phi_t^f) V_t^V, && (4)\\
\text{and} & \\
V_t^V &= (1-\delta_v)\,\beta \mathbb{E}_t V^{\check{V}}{}_{t+1}, && (5)
\end{aligned}
$$

where $\mathbb{E}_t < \cdot >$ denotes expectations conditional on information available at time t. β denotes the discount factor that is common for firms and workers, and κ_s denotes a flow search cost that has to be paid whenever a firm is searching for a worker.

When matched, a worker-firm pair produces output with a linear technology so that firm level output in period t is simply labor productivity y_t. Every period that a firm produces, firms pay a wage w_t to workers and a capital cost κ_k.

3.2 Separations: Job and Match Destruction

Upon separation of an employment relationship, the job can either survive or be destroyed. We allow either of these to happen and call the two types of separation match destruction δ_{mt} and job destruction δ_{j_t}, respectively. Our benchmark model endogenizes both.

Firms are homogeneous unlike in Mortensen and Pissarides (1994) or den Haan, Ramey and Watson (2000) who introduced idiosyncratic productivity fluctuations. A job destruction event occurs with probability λ_j. Upon arrival of a job destruction event, firms draw a job maintenance cost κ_j from distribution $\mathcal{J}$. For all costs $\kappa_j \leq \bar{\kappa}_{j_t}$ firms will pay the cost and continue the relationship, otherwise the match dissolves and the job/vacancy is destroyed. Hence, a job is destroyed with probability $\delta_j = \lambda_j \left(1 - \mathcal{J}\left(\bar{\kappa}_{j_t}\right)\right)$. Similarly, a match destruction event occurs with probability[10] λ_m. Upon arrival of a match destruction event, firms draw a match maintenance cost κ_m from distribution $\mathcal{M}$. For all costs $\kappa_m \leq \bar{\kappa}_{mt}$ firms will pay the cost and continue the relationship, otherwise the match dissolves and the vacancy enters the existing stock of vacancies to be refilled. Hence, a match is destroyed with probability $\delta_m = (1-\lambda_j)\lambda_m \left(1 - \mathcal{M}\left(\bar{\kappa}_{mt}\right)\right)$. The overall separation rate is consequently given by $\delta_t = \delta_{mt} + \delta_{j_t}$.

3.3 Matching and Labor Market Stocks

The stock of unemployed, u_{t-1}, and the stock of vacancies, $\check{v}_t$, are matched using a constant returns to scale matching technology, where α denotes the elasticity of matches with respect to unemployment and A the constant matching productivity. Firms cannot immediately

[10]For simplicity we assume that a firm will only be hit by at most one destruction event. Hence, a match destruction event can only hit if the firm has not been hit by a job destruction event.

rematch upon match destruction as can be seen from the timeline in Figure 4. The worker's job finding probability ϕ_t^w and the firm's job filling probability ϕ_t^f follow:

$$\begin{aligned} M(u_{t-1}, \check{v}_t) &= A u_{t-1}^{\alpha} \check{v}_t^{1-\alpha}, && (6) \\ \phi_t^w &= \frac{M(u_{t-1}, \check{v}_t)}{u_{t-1}} = A\theta_t^{1-\alpha}, && (7) \\ \phi_t^f &= \frac{M(u_{t-1}, \check{v}_t)}{\check{v}_t} = A\theta_t^{-\alpha}, && (8) \end{aligned}$$

where labor market tightness in period t is defined as $\theta_t = \frac{\check{v}_t}{u_{t-1}}$.

Vacancies $\check{v}_t$ available for matching in period t are the surviving vacancies of the previous period plus newly formed vacancies, n_t,

$$\begin{aligned} \check{v}_t &= (1-\delta_v)v_{t-1} + n_t, && (9) \\ v_t &= \left(1-\phi_t^f\right)\check{v}_t + \delta_{mt} e_{t-1}. && (10) \end{aligned}$$

The vacancy stock v_t at the end of period t consists of unmatched vacancies and those employment relationships that separated by match destruction.

We assume that every worker is either employed or unemployed and normalize population to unity. Worker stocks are thus governed by:

$$u_t = \delta_t e_{t-1} + (1-\phi_t^w)\, u_{t-1}, \quad (11)$$

and

$$e_t = 1 - u_t. \quad (12)$$

3.4 Value Function of Filled Jobs

We can now state the firm's value of a filled job with wage w to be:

$$\begin{aligned} \tilde{V}_t^J(w) &= y_t - w - \kappa_k + (1-\lambda_j)(1-\lambda_m)\beta \mathbb{E}_t V_{t+1}^J \\ &+ \lambda_j \beta \mathbb{E}_t \left\langle -\int_{-\infty}^{\bar{\kappa}_{j_{t+1}}} \kappa_j \mathrm{d}\mathcal{J}(\kappa_j) + \mathcal{J}\left(\bar{\kappa}_{j_{t+1}}\right) V_{t+1}^J \right\rangle && (13) \\ &+ (1-\lambda_j)\lambda_m \beta \mathbb{E}_t \left\langle -\int_{-\infty}^{\bar{\kappa}_{mt+1}} \kappa_m \mathrm{d}\mathcal{M}(\kappa_m) + \mathcal{M}\left(\bar{\kappa}_{mt+1}\right) V_{t+1}^J + \left(1-\mathcal{M}\left(\bar{\kappa}_{mt+1}\right)\right) V_{t+1}^V \right\rangle. \end{aligned}$$

The relation between $\tilde{V}_t^J(w)$ and V_t^J as well as $\tilde{V}_t^E(w)$ and V_t^E will be described in section 3.6 on wage bargaining (Hall and Milgrom, 2008).

3.5 Workers

The worker side is completely standard. Every worker who is employed in period t receives wage w_t, every searcher receives b. Searchers find jobs with probability ϕ_t^w so that we can write the period t values for being employed at wage w ($\tilde{V}_t^E(w)$) and unemployed (V_t^U) as:

$$\tilde{V}_t^E(w_t) = w_t + \beta \mathbb{E}_t \left\langle V_{t+1}^E - \delta_{t+1}\left(V_{t+1}^E - V_{t+1}^U\right)\right\rangle, \tag{14}$$

$$V_t^U = b + \beta \mathbb{E}_t \left\langle V_{t+1}^U + \phi_{t+1}^w \left(V_{t+1}^E - V_{t+1}^U\right)\right\rangle. \tag{15}$$

3.6 Wages

Wages are bargained via an alternating offer protocol. We follow the credible bargaining (CB) assumption in Hall and Milgrom (2008) that the threatpoint in wage bargaining is not outright separation. Instead, when disagreeing, workers receive b_b, while firms have to pay the cost of idle capital κ_b[11]. With probability δ_b negotiations break down, the worker returns to the unemployment pool and the vacancy to the vacancy pool. If the worker makes the first offer, it will be $\tilde{w}_t^w$ which gives rise to the value $\tilde{V}_t^E(\tilde{w}_t^w)$ of being employed. If the firm makes the offer, it will offer $\tilde{w}_t^f$ which gives rise to the value $\tilde{V}_t^E(\tilde{w}_t^f)$ of being employed. Hence in equilibrium the firm needs to be indifferent between accepting $\tilde{w}_t^w$ and obtaining $\tilde{V}_t^J(\tilde{w}_t^w)$, or rejecting and offering $\tilde{w}_{t+1}^f$ in the subsequent period. Similarly, the worker has to be indifferent between accepting the firm's wage offer today or rejecting and making her own offer tomorrow. These considerations give rise to the following two indifference conditions that pin down $\tilde{w}_t^w$ and $\tilde{w}_t^f$:

$$\tilde{V}_t^J(\tilde{w}_t^w) = \delta_b V_t^V + (1-\delta_b)\left[-\kappa_b + \beta \mathbb{E}_t \tilde{V}_{t+1}^J(\tilde{w}_{t+1}^f)\right], \tag{16}$$

$$\tilde{V}_t^E(\tilde{w}_t^f) = \delta_b V_t^U + (1-\delta_b)\left[\ b_b + \beta \mathbb{E}_t \tilde{V}_{t+1}^E(\tilde{w}_{t+1}^w)\right]. \tag{17}$$

As suggested by Hall and Milgrom (2008) we assume that the worker gets to make the first offer with probability ω and the firm otherwise. As in Hall and Milgrom (2008) the solution to this bargaining protocol coincides with the Nash bargaining outcome for $\delta_b = 1$ and we define:

$$w_t = \omega \tilde{w}_t^w + (1-\omega)\tilde{w}_t^f, \tag{18}$$

$$V_t^E = \omega \tilde{V}_t^E(\tilde{w}_t^w) + (1-\omega)\tilde{V}_t^E(\tilde{w}_t^f), \tag{19}$$

$$V_t^J = \omega \tilde{V}_t^J(\tilde{w}_t^w) + (1-\omega)\tilde{V}_t^J(\tilde{w}_t^f). \tag{20}$$

3.7 Equilibrium

The aggregate state at the beginning of period t is summarized by the realization of aggregate labor productivity y_t, unemployed, u_{t-1}, and vacancies v_{t-1}: $\Omega_t = \{y_t, u_{t-1}, v_{t-1}\}$.

[11]Recent work by Chodorow-Reich and Karabarbounis (2016) has challenged the assumption of acyclical opportunity costs of employment. In section 5.5 we will allow b and b_b to be time-varying and cyclical. Results are robust.

Definition 1 (Equilibrium)**.** *A symmetric equilibrium for the model economy consists of a sequence of thresholds $\bar{\kappa}_{vt}, \bar{\kappa}_{j_t}, \bar{\kappa}_{mt}$; a sequence of labor market stocks e_t, u_t, v_t and flows n_t; and a sequence of wages $w_t, \tilde{w}_t^f, \tilde{w}_t^w$; such that for any time period t the following hold:*

1. *Vacancy Creation:*
   $\bar{\kappa}_{vt} = V^{\check{V}}{}_t$ *and Equation 2;*

2. *Endogenous Separations:*
   $\bar{\kappa}_{j_t} = V_t^J$ *and* $\bar{\kappa}_{mt} = V_t^J - V_t^V$*;*

3. *Labor Market Transitions:*
   Equations 10–12;

4. *Wage Bargaining:*
   The firm (Equation 16) and worker indifference (Equation 17) conditions; and wage aggregation by 18.

With value functions $V^{\check{V}}{}_t, V_t^V, \tilde{V}_t^J(w), V_t^J, V_t^U, \tilde{V}_t^E(w), V_t^E$ defined in Equations 4, 5, 13 – 15 and 19 – 20.

4 Analytical Results

In this section, we study some basic implications of long-lived vacancies. We first show that the well known conditions for constrained efficiency in Mortensen/Pissarides (MP) models continue to hold with long-lived vacancies. Then we investigate under which conditions long-lived vacancies make an important difference for labor market fluctuations. Even though these results only hold in a special case of the model with Nash bargaining and exogenous (but time-varying) separations, they yield important insights for the general case which is analyzed numerically in Section 5. We conclude section 4 by deriving some comparative steady state results, which are natural generalizations of the MP model and provide important guidance for the calibration.

4.1 Planner Solution and Decentralized Equilibrium

In the canonical labor market matching model, the decentralized equilibrium under Nash bargaining is constrained efficient if the worker's bargaining power equals the elasticity of matches w.r.t unemployment. Long-lived vacancies and Diamond entry seem to change the bargaining situation, because firms are left with a valuable vacancy if the bargain breaks down, while workers are unemployed. Nevertheless, the efficiency of the decentralized equilibrium is preserved. However, constrained efficiency holds only in the case of exogenous separations; the way we have modeled endogenous separations generates a hold-up problem that makes the equilibrium inefficient. Thus assume for this section that rather than being endogenously determined, match and job destruction follow a stochastic exogenous process.

The planner solves

$$\max_{\{n_t,\check{v}_t,v_t,u_t\}} \sum_{t=1}^{\infty} \beta^t \mathbb{E}_0 \left\langle (y_t - \kappa_k)(1-u_t) + bu_t - \kappa_s \check{v}_t - \mathcal{V}_k(n_t) \right\rangle \tag{21}$$

subject to vacancy and unemployment dynamics as given in (9 – 11) and the matching technology as represented in equation (6). Here $\mathcal{V}_k(n)$ denotes the total cost of creating n new vacancies, which is derived from the vacancy posting cost distribution as $\mathcal{V}_k(n) = \int_0^{\mathcal{V}^{-1}(n)} x d\mathcal{V}(x)$.

Denoting by $W^{\check{V}}{}_t$, W_t^V and W_t^S the Lagrange multipliers of constraints (9), (10) and (11), respectively, we can derive the optimality conditions:

$$W^{\check{V}}{}_t = \mathcal{V}_k'(n_t), \tag{22a}$$

$$W^{\check{V}}{}_t = -\kappa_s + W_t^V + \phi_t^f (1-\alpha)\left(W_t^S - W_t^V\right), \tag{22b}$$

$$W_t^V = \beta(1-\delta_v)\mathbb{E}_t W^{\check{V}}{}_{t+1}, \tag{22c}$$

$$W_t^S = y_t - \kappa_k - b + \mathbb{E}_t \left\langle (1-\delta_{t+1})\beta W_{t+1}^S + \delta_{mt+1}\beta W_{t+1}^V - \phi_{t+1}^w \alpha\beta\left(W_{t+1}^S - W_{t+1}^V\right)\right\rangle. \tag{22d}$$

Since firms unilaterally determine vacancies, it is necessary for optimality that the value of a vacancy coincides for the firm and the planner. It is also sufficient, because vacancy posting is the only decision in this version of the model. Therefore the planner's Lagrange multipliers W_t^V and $W^{\check{V}}{}_t$ must equal the vacancy values V_t^V and $V^{\check{V}}{}_t$ for firms. This is the case if the firm surplus $(V_t^J - V_t^V)$ equals $(1-\alpha)(W_t^S - W_t^V)$. Similarly, the planner's Lagrange multiplier of employment, W_t^S, equals total surplus $(V^J + V^E - V^U)$ in the decentralized economy if $(V^E - V^U) = \alpha(W^S - W^V)$. Both conditions are satisfied if wages are determined by generalized Nash bargaining with worker bargaining power $\omega = \alpha$, which extends the Hosios condition to the case of long lived vacancies with exogenously time varying separations. The details of the derivation are in Appendix B.1.

Result 1 (Constrained Efficiency)**.** *If wages are determined by Nash bargaining, and worker bargaining power* ω *equals the elasticity of matches with respect to unemployment* α*, then the Planner allocation (22) with exogenously time varying separations coincides with the solution to the decentralized economy.*

4.2 When does vacancy longevity matter?

Diamond entry and vacancy longevity are two important departures from the canonical model. Each on their own does not have a substantial impact, whereas their interaction does. It can be easily seen from (22) that short lived vacancies, i.e. $\delta_v = 1$, are equivalent to a standard model with time varying search costs.

Zero vacancy creation costs κ_v are a rather trivial special case where vacancy creation is determined by search costs κ_s. Equivalence with the canonical model and thus irrelevance of longevity then immediately holds because destroyed vacancies are worthless. More generally, with constant marginal entry costs, the impact of vacancy longevity is substantially diminished. First, by fixing the value of a vacancy, entry costs stabilize the capital cost related to vacancy destruction. The second role is probably more important: if vacancies can be created at a constant marginal cost, the vacancy stock adjusts to the value of a vacancy with infinite elasticity, eliminating the sluggishness of the stock-flow dynamics. The relationship between short and long lived vacancies is captured more formally in the following:

Result 2 (Equivalence Short and Long Lived Vacancies). *Assume that $\mathcal{V}_k'(n_t) = \kappa_v$ and the job destruction rate $\delta_{j_t} = \delta_t - \delta_{mt}$ are constant over time. We consider two economies that differ only in the vacancy destruction rate δ_v, vacancy creation cost κ_v, the search cost κ_s, and the capital cost κ_k. Index the two economies with S and L, so that economy S is characterized by the set of parameters $\{\delta_v^s, \kappa_v^s, \kappa_s^s, \kappa_k^s\}$ and economy L is characterized by the set of parameters $\{\delta_v^l, \kappa_v^l, \kappa_s^l, \kappa_k^l\}$. Given any $\{\delta_v^s, \kappa_v^s, \kappa_s^s, \kappa_k^s\}$ and $\{\delta_v^l, \kappa_v^l\}$, by setting*

$$\kappa_k^l = \kappa_k^s - [1-\beta(1-\delta_j)]\,\beta\left(1-\delta_v^l\right)\kappa_v^l + [1-\beta(1-\delta_j)]\,\beta(1-\delta_v^s)\,\kappa_v^s, \tag{23}$$

$$\kappa_s^l = \kappa_s^s + [1-\beta(1-\delta_v^s)]\,\kappa_v^s - \left[1-\beta\left(1-\delta_v^l\right)\right]\kappa_v^l, \tag{24}$$

the economies S and L have the same total surplus Σ_t, the same market tightness θ_t, and therefore also the same unemployment rate u_t. In the decentralized equilibrium with Nash bargaining, this also implies the same wage rate w_t and values of employment and unemployment.

To derive result 2, define total surplus Σ as $\Sigma_t = (W_t^S - W_t^V)$, and use (22c) to rewrite (22b) as

$$\kappa_s + W^{\check{V}}{}_t - \beta(1-\delta_v)\,\mathbb{E}W^{\check{V}}{}_{t+1} = (1-\alpha)\,\phi_t^f\,\Sigma_t. \tag{25}$$

Furthermore, rearrange (22d) to write the surplus as

$$\Sigma_t = y_t - \kappa_k - b + \beta\mathbb{E}_t\left\langle\left(1-\delta_{t+1}-\alpha\phi_{t+1}^w\right)\Sigma_{t+1} + \left[\beta W_{t+1}^V\left(1-\delta_{j_{t+1}}\right) - W_t^V\right]\right\rangle. \tag{26}$$

The left side of (25) can be interpreted as the user cost of an unfilled vacancy. Next to the flow cost κ_s, it captures the effect of discounting β, the depreciation rate δ_v and the expected capital gain $\mathbb{E}_t W^{\check{V}}{}_{t+1} - W^{\check{V}}{}_t$. In the special case where vacancy creation costs are constant ($\mathcal{V}_k'(n) = \kappa_v$), (22c) shows that the value of a vacancy is also a constant, given by $W_t^V = \beta(1-\delta_v)\kappa_v$, and the user cost of vacancies becomes $\kappa_h \equiv \kappa_s + (1-\beta(1-\delta_v))\,\kappa_v$.

Then (23) says that the user cost of unfilled vacancies must be identical across the two calibrations. The effect of vacancies on the surplus is given by the bracketed term on the right side of (26), which can be interpreted as the user cost of a filled vacancy. If, in addition, job destruction $\delta_{j_t} = \delta_t - \delta_{mt}$ is constant, this term is also constant and given by $-\beta\left[1-\beta\left(1-\delta_j\right)\right]\left(1-\delta_v\right)\kappa_v$. Then (24) says that the constant term in the surplus equation, given by $c = -\left[\kappa_k + b - \beta\left[1-\beta\left(1-\delta_j\right)\right]\left(1-\delta_v\right)\kappa_v\right]$ must be identical across the two calibrations. The details of the recalibration are in Appendix B.2.

Under these special assumptions, we get the following system of two dynamic equations, where the exogenous process y_t drives the dynamics of the surplus Σ_t and market tightness θ_t:

$$\Sigma_t = y_t - c + \beta\mathbb{E}\left\langle\left(1-\delta_t - \alpha\theta_{t+1}^{1-\alpha}\right)\Sigma_{t+1}\right\rangle \tag{27}$$

$$\kappa_h = (1-\alpha)\theta_t^{-\alpha}\Sigma_t. \tag{28}$$

In the general case, the user cost of a filled vacancy can be written as

$$\beta\mathbb{E}_t W_{t+1}^V\left(1-\delta_{j_t}\right) - W_t^V = \mathbb{E}_t\left\langle\left(W_{t+1}^V - W_t^V\right) - W_{t+1}^V(1-\beta) - \beta W_{t+1}^V\delta_{j_t}\right\rangle. \tag{29}$$

It includes three components: the capital gain $\mathbb{E}_t W_{t+1}^V - W_t^V$, the interest cost $(1-\beta)\mathbb{E}_t W_{t+1}^V$, and the expected loss through job destruction $\beta\mathbb{E}_t W_{t+1}^V\delta_{j_{t+1}}$. From this and the dynamics of vacancies, we see that LLV affect labor market dynamics in three ways. First, they affect the formation of new vacancies through the user cost of unfilled vacancies. Second, they affect the total surplus of a filled job through the user cost of filled vacancies. Third, they introduce stock-flow dynamics into vacancy creation. If vacancies are very persistent, the stock reacts sluggishly to the creation of new vacancies, which is underlying the mechanism in Coles and Moghaddasi Kelishomi (2018). Equation 25 can be interpreted as a generalized job creation condition. The share of the present value of the surplus going to the firm has to equal the user cost of an unfilled vacancy.

4.3 Steady State Responses

In this section we consider the deterministic steady state of the model with Nash bargaining. Throughout this section, variables without time index denote steady state values, hats denote log deviations from steady state, and η_y^x denotes the elasticity of any variable x with respect to exogenous productivity y. Similar to Shimer (2005), we find that the flow-equilibrium approximation is good and focus on it in this section. Hence any variable x can then be considered a function of productivity y only.

4.3.1 The Mechanics of the Beveridge Curve With Time-Varying Separations

We start by deriving a few relationships that just follow from the mechanics of unemployment and vacancy dynamics, and are independent of the economics of vacancy creation and job separations. In the standard MP model, it is well known that procyclical job destruction tends to make the correlation of unemployment and vacancies positive, From the steady state relationship $u = \frac{\delta}{\delta+\phi^w}$ and the matching relationships $\phi^w \propto \theta^{1-\alpha}$ and $\check{v} = \theta u$ we obtain

$$\hat{u} = -(1-\alpha)\frac{u\phi^w}{\delta}\hat{\theta} + (1-u)\hat{\delta} = (1-u)\left(\hat{\delta} - (1-\alpha)\hat{\theta}\right), \text{and} \tag{30}$$

$$\hat{\check{v}} = \left(1-(1-\alpha)\frac{u\phi^w}{\delta}\right)\hat{\theta} + (1-u)\hat{\delta} = \hat{\theta} + (1-u)\left(\hat{\delta} - (1-\alpha)\hat{\theta}\right). \tag{31}$$

From this we can infer that time varying separations, endogenous or exogenous, leave the Beveridge Curve (opposite sign of du and d$\check{v}$) intact as long as the tightness response is sufficiently strong. This is formalized in Result 3:

Result 3 (The Beveridge Curve)**.** *Let us assume that both tightness and the job destruction rate are a function of labor productivity. Across steady states, unemployment and vacancies go in opposite directions if*

$$-\eta_y^{\delta} < \left(\frac{1}{(1-u)} - (1-\alpha)\right)\eta_y^{\theta}. \tag{32}$$

In the data, the correlation between unemployment and vacancies is not just negative, but close to -1, and the fluctuations in the two variables are of about equal size, which implies $\frac{\hat{u}_t}{\hat{\check{v}}_t} \approx -1$. Using this in (30) and (31) yields the following

Result 4 (Calibration of α)**.**

$$\alpha = 1 - \frac{1}{2(1-u)} - \frac{\eta_y^{\delta}}{\eta_y^{\theta}}. \tag{33}$$

It follows from (30) and (31) that countercyclical separations (negative η_y^{δ}) tend to increase the fluctuations of unemployment relative to those of vacancies.

When the Beveridge Curve relationship is imposed, the elasticity of separations determines the cyclicality of matches. We have the following result about the log deviation from steady state of matches out of unemployment $\hat{M}$:

Result 5 (Cyclicality of Matches:)**.** *Since $\hat{M} = \alpha\hat{u} + (1-\alpha)\hat{\check{v}}$, we get from (30) and (31) that $\hat{M} = (1-\alpha)u\hat{\theta} + (1-u)\hat{\delta}$. Using (33), this gives*

$$\hat{M} = \left(\frac{u}{2(1-u)} + \frac{\eta_y^{\delta}}{\eta_y^{\theta}}\right)\hat{\theta}. \tag{34}$$

Since tightness is procyclical, so are matches in the model, if separations are constant (i.e. $\eta_y^{\delta} = 0$). Countercyclical matches, as observed in the data by Blanchard and Diamond (1990) or Mortensen (1994), are consistent with a strong Beveridge curve only if separations are sufficiently countercyclical ($\eta_y^{\delta} < 0$).

4.3.2 Tightness With Time-Varying Separations

We now analyze the effect of changes in labor productivity y and the total separation rate δ, which is considered here as an exogenous parameter. As in the standard MP model (Costain and Reiter, 2008; Hagedorn and Manovskii, 2008), the volatility of the labor market in this model is strongly affected by the size of the dynamic surplus in steady state, Σ, which we can derive from (26) as

$$\Sigma = \frac{y - \kappa_k - b + \left(\beta W^V (1-\delta_j) - W^V\right)}{1 - \beta\left(1 - \delta - \alpha\theta^{1-\alpha}\right)}. \tag{35}$$

It differs from the standard formula by the capital costs related to vacancies, the term in parentheses in the numerator, which reduces the surplus, everything else being equal. For brevity, write this term as $\bar{c} - \beta W^V \delta_j$ where $\bar{c} = \beta W^V - W^V - \kappa_k - b$. Use (35) in (28) and rearrange to obtain:

$$\frac{\kappa_h \theta^\alpha}{1-\alpha} = y + \bar{c} - \beta W^V \delta_j + \beta\left(1 - \delta - \alpha\theta^{1-\alpha}\right)\frac{\kappa_h \theta^\alpha}{1-\alpha}. \tag{36}$$

Total differentiation gives

$$[1 - \beta(1-\delta)]\,\alpha\Sigma\frac{d\theta}{\theta} = dy - \beta W^V d\delta_j - \beta\frac{\alpha\kappa_h}{1-\alpha}d\theta = dy - \beta W^V d\delta_j - \beta\Sigma\phi^w\frac{d\theta}{\theta}. \tag{37}$$

Since the steady state level of y is normalized to unity

$$\hat{\theta} = \frac{\hat{y} - \beta W^V \delta_j \hat{\delta}_j}{1 - \beta(1 - \delta - \phi^w)} \cdot \frac{1}{\alpha\Sigma}. \tag{38}$$

Naturally, higher labor productivity tends to decrease the job destruction rate, therefore $\hat{\delta}_j < 0$. From (38) we see that countercyclically varying separations allow to match a given magnitude of tightness fluctuations with either a larger surplus or lower exogenous productivity fluctuations.

4.3.3 Wages

It is straightforward to write the worker surplus in steady state as:

$$V^E - V^U = \frac{w_t - b}{1 - \beta[1 - \phi^w - \delta]}. \tag{39}$$

Since in a standard calibration the job finding rate is much higher than the separation and the discount rate, this can be approximated as

$$V^E - V^U \approx \frac{w_t - b}{\phi^w}. \tag{40}$$

This gives

$$w - b \approx \phi^w \Sigma = \theta\phi^f \Sigma \propto \theta \tag{41}$$

because $\phi^f\Sigma$ is a constant in the case of constant vacancy creation costs. In percentage deviations, this implies

$$\hat{w} = \hat{\theta}\frac{w-b}{w}. \tag{42}$$

In the data, tightness fluctuates about 13 times as much as output and almost 20 times as much as wages. If the model is suitable to explain the fluctuations in unemployment, and therefore in tightness, there are basically three alternatives:

1. average surplus $\frac{w-b}{w}$ is very small, as in Hagedorn and Manovskii (2008);
2. the link between tightness and the wage is weakened, e.g. by a different bargaining scheme as in Hall and Milgrom (2008);
3. wages fluctuate much more than output, which is what happens in Coles and Moghaddasi Kelishomi (2018).

Since the third alternative is clearly at odds with the data and we want to show that the model can explain unemployment fluctuations even with a big worker surplus, we assume a wage setting mechanism that dampens wage fluctuations. Nevertheless, our wages fluctuate about as much as labor productivity, and are not "rigid" by conventional criteria.

5 Numerical results

After discussing our baseline calibration in Section 5.1, we show in Section 5.2, that the model is very successful in matching the usual labor market statistics. As a benchmark, we compare our model to the well-established Hall Milgrom (HM) model. In Section 5.3 we investigate the main mechanisms in the model and explain the vacancy creation versus the vacancy depletion channel. As we have already discussed in Section 4.2, LLV matter little if new vacancies are extremely elastic with respect to the value of a new vacancy. Identifying this elasticity is therefore a key task in the empirical validation of the model, and we explain in Section 5.4 which aspects of the data can be used for identification. Section 5.5 contains a robustness check, showing that our model is able to explain labor market fluctuations even if workers' outside option is procyclical. Finally, in Section 5.6 we show that our model with only one shock goes a long way in explaining the whole postwar US labor market history.

5.1 Calibration

5.1.1 Calibration of the baseline model

We have chosen the time period as the 60th part of a quarter, which we refer to as a workday. We follow in this respect Christiano, Eichenbaum and Trabandt (2016), who subdivide their

Table 2: Calibrated Parameters: Values and Targets

Interpretation	Parameter	Value
Fixed Exogenously		
Discount factor	β	$0.96^{1/240}$
Utility of non-employment rtp	b	0.4544
Worker flow utility in disagreement rtp	b_b	0.4544
Worker Probability of making an offer	ω	0.6491
Firm Bargaining cost in disagreement rtp	κ_b	0.1698
Firm Flow Search Cost rtp	κ_s	0.1024
Arrival Prob of Job Dest. Event	λ_j	0.0023
Arrival Prob of Match Dest. Event	λ_m	0.0011
Arrival Prob of Vacancy Opportunity	λ_v	1.0000
For Averages		
Capital Cost	κ_k	0.0000
Mean of Vacancy Creation Cost	μ_v	2.4382
Mean of match destruction shock	μ_m	0.0631
Mean of job destruction shock	μ_j	2.4977
Efficiency of Matching Function	A	0.0403
For Second Moments		
Expected quarterly labor productivity		1
AR parameter for labor productivity	ρ_y	0.9987
Standard Deviation of labor productivity innovation	σ_y	0.0009
Elasticity of new vacancies	ξ	15.8780
Elasticity of matches wrt unemployment	α	0.6491
Breakdown of bargaining while disagreeing	δ_b	$4.93\bar{\delta}$
Std.Dev of Match destruction cost	σ_m	0.2003
Std.dev Job Destruction cost	σ_j	0.1758

rtp denotes relative to productivity.

quarterly period into 60 subperiods when computing the bargaining outcome. We set the discount factor β to the conventional value of 0.99 quarterly.

We think it is important that the model features a high worker surplus, that means a substantial difference between the wage and the unemployment benefit. In the benchmark, we set the unemployment benefit to 71 percent of the steady state wage, following Hall and Milgrom (2008). In a robustness check we also consider a replacement rate of 40 percent as in Shimer (2005).

Several parameters are chosen so as to match steady state values. The matching productivity $A = 0.0403$ and an average vacancy cost of $\mu_v = 2.4382$ are chosen so as to achieve a steady state unemployment rate of 5.67 percent, and a filling rate ϕ^f of one third per week. The latter value was taken from Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2018). We target a steady state wage of 64 percent of production, conforming to a 36 percent gross capital share. The wage is strongly influenced by the bargaining position of both sides, mainly the worker utility during disagreement, b_b, and the capital costs while in disagreement, κ_b. Since we do not have strong evidence on these parameters, we normalize $b_b = b$, and fix κ_b so as to obtain the 64 percent labor share. Out of the capital share, all the costs of maintaining the match must be covered. These include the capital costs κ_k, the expected job maintenance and match maintenance costs, as well as the expected costs of filling a vacancy. As we have explained above, κ_k should measure the cost of capital that is not tied to the specific job but can be hired flexibly. Since we do not have any direct observation of the split of capital into general and job-specific, we make the extreme assumption that all capital is job-specific and therefore set $\kappa_k = 0$, which leads to a very high average value of a vacancy. Following Silva and Toledo (2009), we set the search cost parameter κ_s so that the average cost per hire equals 4 percent of the quarterly wage.

We estimate the six parameters ρ_y, α, ξ, σ_j, σ_m, and δ_b, so as to match six second moments in the data, namely the quarterly autocorrelation of GDP and of vacancies, the variances of unemployment and of vacancies, and the variances of job destruction and of match destruction. Although the effects of all these parameters are inter-dependent, there is a clear interpretation of which parameter is driving which moment:

- We choose the autocorrelation coefficient of labor productivity so as to match the quarterly autocorrelation of GDP. This results in $\rho_y = 0.99893$, which conforms to a quarterly value of 0.938.

- We choose the elasticity of matches with respect to unemployment, α, so as to match the ratio of unemployment volatility to vacancy volatility. This gives $\alpha = 0.6491$, close to the value used in Coles and Moghaddasi Kelishomi (2018), and in-between the most common value in the literature, which is 0.5, and the value estimated in

Shimer (2005), which is 0.72. Motivated by the equivalence in Result 1 we impose the Hosios condition $\omega = \alpha$.

- The elasticity of vacancy creation ξ is the key parameter that determines the strength of the vacancy creation versus the vacancy depletion channel, and therefore the phase shift between vacancies and unemployment. We target a lead of vacancies to unemployment of 0.18 quarters (cf. Section 5.4). This gives a value of $\xi = 15.8780$ which substantially exceeds the estimate of $\xi = 1$ in Fujita and Ramey (2007), and the estimate of $\xi = 0.26$ in Coles and Moghaddasi Kelishomi (2018). One should notice that it is not easy to compare this value across models. Since it is an elasticity with respect to the value of a vacancy, it depends very much on the average value of a vacancy. Because of our choice $\kappa_k = 0$, we calibrate a high vacancy creation cost and therefore a high equilibrium value. Increasing κ_k and lowering the vacancy creation cost, the model would generate similar equilibrium dynamics with a substantially lower elasticity ξ.

- Mean μ_j and dispersion σ_j of the job maintenance cost drive mean and variance of job destruction; given job destruction, mean μ_m and dispersion σ_m of the match maintenance cost drive mean and variance of total separations. We observe an average monthly separation rate of 3.3 percent, and an average job destruction rate equalling 66.87 percent of all separations. It is difficult to obtain evidence on job-flows (rather than worker flows) for the entire sample period. However, based on the pioneering work of Davis and Haltiwanger (1992), quarterly aggregate numbers for total private sector establishment level job destruction are published by the BLS as the Business Dynamics Statistics of the US Census Bureau. For 1994–2014 the (imputed) average monthly probability of job destruction is 1.75%, i.e. approximately two thirds of total separations. The remaining separations are match destructions, each leaving an unfilled vacancy. We choose σ_j and σ_m so as to match the variance of the two destruction rates. Given any σ_j and σ_m, μ_j and μ_m can be set in steady state so as to obtain the right mean. This results in $\sigma_j = 0.1758$, $\sigma_m = 0.2003$, $\mu_j = 2.4977$ and $\mu_m = 0.0631$.

- The probability of break-up during disagreement, δ_b, determines how strongly the unemployment rate influences wages, and is therefore a key determinant of the variance of unemployment. Our calibrated value $\delta_b = 4.93\bar{\delta}$ is higher than in Hall and Milgrom (2008), who set $\delta_b = 4\bar{\delta}$. Endogenous separations make it easier for our model to match unemployment fluctuations.

This leaves three parameters undetermined. Two of them can basically be considered as normalizations, namely the arrival rate of job and match maintenance shocks. They determine the average cost of maintaining the match, but variations in those costs can be compensated by changes in other cost components, mainly vacancy creation costs and search costs. so that the average firm surplus in steady state is unchanged. We set the arrival rates such that on average, half of those shocks lead to a separation. Reasonable variations in these arrival rates have a minimal influence on the fluctuations in the model. Finally, for the rate of vacancy destruction, we are not aware of any direct evidence, so we follow Fujita and Ramey (2007) in setting this parameter equal to the job destruction rate.

5.1.2 Calibration of the Hall/Milgrom model

For a better comparison, we recalibrate the Hall/Milgrom model so that it matches the same targets as our baseline model, if possible. As in the original Hall/Milgrom calibration and our baseline we keep unemployment benefits $b_b = b$ at 71% of the steady state wage. We also maintain the same discount factor β. Unlike in the benchmark model, we cannot use ρ_y to match the autocorrelation of detrended GDP. The lack of persistence in vacancies affects GDP substantially, so that even with ρ_y very close to unity, GDP persistence falls short of what is observed in the data. We therefore keep ρ_y from the baseline. The job destruction and match destruction rates are constant in the model, both are set equal to the average destruction rates in the data.

Most importantly, we choose α and δ_b so as to match the volatility of vacancies and the unemployment rate. Because of a constant separation rate, the value of α differs from the baseline, as explained by Result 4.[12] As in the baseline, we apply the Hosios condition $\omega = \alpha$. The parameter values are summarized in Table 3.

5.2 Dynamics of the baseline model

Table 4 shows results for the second moments of unemployment (u), vacancies (v), the job finding rate (ϕ^w), the job separation rate (δ), the number of new matches (M), GDP (Y), labor productivity (y) and and the wage rate (w). All numbers are at quarterly frequency. The first section of the table shows the data, the second section the results of model simulations under our baseline calibration. As a benchmark, the third section shows the results for the HM model. Suitably calibrated, both models match the relative standard deviations of vacancies and of unemployment, generate a strong Beveridge curve (negative correlation

[12] The recalibration of α comes from the need to maintain the observed (u, v) relationship in the presence of endogenous separations. From Result 4, we get for constant separation rate ($\eta_y^\delta = 0$), $\frac{\delta}{\phi^w} = \frac{u}{e} \approx 0.06, \Rightarrow \alpha \approx 0.47$. Approximating $\frac{\eta_y^\delta}{\eta_y^\theta}$ by the relative volatility of δ and θ in the data, we get $\frac{\eta_y^\delta}{\eta_y^\theta} = -2.74/13 = -0.21$ so that $\alpha \approx 0.68$. These approximations are close to our calibrated values.

Table 3: Parameters Hall/Milgrom Model.

Parameter	target	our HM calibration	Hall and Milgrom (2008)
β	BL	$0.96^{1/240}$	$0.96^{1/240}$
ρ_y	BL	0.9987	0.9987
$b_b = b$	-	0.7100	0.7100
δ_b/δ	var(U)	4.0554	4.000
α	var(U)/var(V)	0.4599	0.5000
ω	Hosios	0.4624	0.5445
κ_s	mean(δ)	0.1611	0.4268
A	mean(U)	0.0488	0.0336

BL: we maintain the same parameter values as in our baseline calibration.

between vacancies and unemployment rate), and a strong amplification of GDP fluctuations relative to TFP.

In the baseline calibration our model improves over HM in several dimensions. First and most obviously, it explains the variability of the job separation rate, which the HM model fails to explain by construction. This has interesting consequences for other variables. In the HM model, the burden of generating unemployment fluctuations lies entirely on the job finding rate. This requires a much larger fluctuation of the finding rate in the model than in the data.

A further advantage of allowing for a variable job separation rate is the ability of the baseline model to explain the countercyclicality of new matches. In the data they are strongly positively correlated with unemployment, as was already emphasized by Blanchard and Diamond (1990) and Mortensen (1994).[13] As we have explained in Section 4, to get counter-cyclical matches requires a parameter $\alpha > 0.5$, consistent with the calibration of Shimer (2005). The baseline calibration has $\alpha = 0.6491$, and the HM model $\alpha = 0.4599$, again an almost mechanical consequence of constant separations, leading to slightly pro-cyclical matches.

Our model is also closer to the data in terms of the autocorrelation of some key variables. Sluggish vacancy creation generates more persistent fluctuations in the stock of vacancies, as was stressed by Fujita and Ramey (2007), which is realistic. The persistence of vacancies carries over to other variables such as GDP. We have calibrated the model so as to match the

[13]One should notice that matches are measured as flows from unemployment to employment. More comprehensive measures of new matches, including the flows from non-participation to employment as well as job-to-job transitions, are likely to be pro-cyclical. It remains for future work to explain these different types of matches, which requires a model with a household participation decision.

Table 4: Model Results: Second Moments.

	u	v	ϕ^w	δ	M	Y	y	w
Data, US 1951-2003								
StdevRel	7.28	7.77	4.53	2.70	3.16	1.0	0.77	0.70
Autocorr	0.94	0.94	0.95	0.87	0.91	0.94	0.88	0.76
CorrUnempl	1.0	-0.93	-0.96	0.74	0.93	-0.91	-0.40	-0.46
Model, baseline calibration								
StdevRel	7.28	7.77	5.28	2.70	2.02	1.00	0.58	0.71
Autocorr	0.95	0.95	0.95	0.88	0.95	0.93	0.89	0.92
CorrUnempl	1.00	-1.00	-1.00	0.91	0.99	-0.97	-0.91	-0.96
Hall/Milgrom model								
StdevRel	7.28	7.77	7.93	0.00	1.86	1.00	0.57	0.49
Autocorr	0.92	0.81	0.89	-	0.33	0.91	0.89	0.89
CorrUnempl	1.00	-0.90	-0.97	-	-0.24	-0.99	-0.97	-0.97
Nash bargaining, unchanged parameters								
StdevRel	2.11	2.23	1.52	0.85	0.59	1.00	0.88	1.26
ReplacementRatio = 0.4, recalibrated, $\delta_b = 3.67$								
StdevRel	7.28	7.77	5.28	2.70	2.02	1.00	0.58	0.71
Constant separation rates, unchanged parameters								
StdevRel	3.87	8.03	4.15	0.00	0.67	1.00	0.79	0.87

Notes: The log-linearized model was simulated 1000 times where each simulation is 104 years long. The first 50 years are discarded. The remaining 54 years are first aggregated from daily to quarterly frequency, then HP detrended with smoothing parameter 10^5. Reported numbers are averages over the 1000 simulations.

autocorrelation of GDP. This is not possible with the HM model; even when the autocorrelation of TFP equals 1, detrended GDP is less persistent in the HM model than in the data. The table therefore shows the HM model with the same TFP autocorrelation as the baseline calibration, generating a substantially lower autocorrelation of GDP.

Finally, the model is also consistent with the fact that job separations are much less autocorrelated than GDP, they are characterized by sharp spikes. Once employment goes down after an increase in separations, the value of employment goes back to normal relatively quickly, since the sluggish reaction of vacancies prevents the rapid generation of new jobs.

Necessary for the empirical success of the model is, next to endogenous separations and long-lived vacancies, the dampening of wage fluctuations through credible bargaining (CB). At a first glance, the model does not show any sign of "wage rigidity". In fact, wages fluctuate more than labor productivity. This is in contrast to the HM model, where wages fluctuate (slightly) less than productivity. Endogenous separations and LLV allow to explain fluctuations despite larger wage fluctuations. The effect of CB in both the HM model and our model is to shield wages from variations in the unemployment rate. This effect becomes apparent if we compare the baseline calibration to the version with conventional Nash bargaining (cf. last part of Table 4), i.e., the model where disagreement leads to separation rather than delay. In that case, wages fluctuate almost 50% more than productivity, which cuts relative unemployment fluctuations by a factor of more than three. One important conclusion from these results, already mentioned in Section 4, is that a unit elasticity with respect to productivity is not the right benchmark for assessing wage flexibility. Bringing the model to the data is tricky when it comes to wages, because it is not so clear what the relevant wage variable is. In models of exogenous separations, it is only the wages of new hires (Haefke, Sonntag and van Rens, 2013). Once separations are endogenous, wages of continuing matches matter, too. Above we report all wages, because data on wages of new hires are not available for our full sample (cf. Section 2).

The key parameter governing the volatility of wages is the probability of match separation in the (out-of-equilibrium) case of disagreement, δ_b, expressed relative to the total separation rate in equilibrium, δ. A higher δ_b increases the influence of the unemployment rate on the bargaining outcome and therefore makes wages more volatile. In Hall and Milgrom (2008) it was set to $\delta_b = 4\delta$. In our calibration of the Hall/Milgrom model we obtain $\delta_b = 4.05\delta$, In the baseline calibration we get $\delta_b = 4.96\delta$, which explains the high volatility of wages. If we increase the worker surplus by lowering the replacement rate from 0.71 as in Hall and Milgrom (2008) to 0.4 as in Shimer (2005), recalibration of the model leads to almost identical behavior of observable variables. Key to the recalibration is the reduction of δ_b to the value 3.67.

The last line in Table 4 shows that the volatility of unemployment is cut by almost half when separations are constant and all other parameters unchanged. We conclude that the endogenous separation margin is instrumental in generating pronounced labor market fluctuations.

5.3 Vacancy creation versus depletion

Figure 5 illustrates the dynamics of vacancies in response to a positive technology shock. The upper left panel of the figure shows dynamics for the *stock* of vacancies and unemployment that is very similar to what we know from a standard MP or HM model. Interesting is the behavior of *new* vacancies. The creation of new vacancies is above its steady state value for only two quarters. Afterwards, fewer new vacancies are posted than in steady state, but the stock of vacancies keeps going up for another quarter, and then recedes to steady state only slowly.

What is fueling this persistent increase in vacancies? The upper right panel of the figure provides a decomposition of the change in vacancies into four components: new postings of vacancies; the negative of job matches, the negative of vacancy destruction, and the repostings of vacancies, i.e. separations that leave an intact vacancy rather than destroying the job. so that they leave an intact vacancy. New vacancies capture the conventional vacancy creation channel. The remaining three comprise the vacancy depletion channel. All variables in this panel are expressed as percentages of the steady state vacancy stock, so that the dark blue line is in fact the sum of the other four lines. New vacancy postings, which are driven by changes in the profitability of new employment, only play an important role in the first quarter after the shock. Already in the second quarter, the increase in the vacancy stock is driven by the reduction in the number of new matches, so that fewer of the existing vacancies are absorbed. The number of matches is reduced because the percentage reduction in unemployment is as big as the percentage increase in vacancies, and unemployment has a bigger impact on matches since $\alpha = 0.6491$ in our baseline calibration.[14] The reduction in new matches is so strong that it more than compensates the reduction in re-postings, which is due to the decrease in match destruction. The last component to vacancy dynamics, namely the direct destruction of vacancies, is proportional to the stock of vacancies at rate δ_v, but only plays a minor role.

The mechanics of unemployment and vacancies can therefore be summarized as follows. The technology shock leads to an immediate reduction in the separation rate, so that unemployment starts falling. Due to higher profitability, new vacancy postings also

[14] Notice that Shimer (2005) estimates an even higher value of $\alpha = 0.72$, but the persistent effect on vacancies is not present in his model because vacancies are short-lived. Fujita and Ramey (2007) use the "conventional" value of $\alpha = 0.5$ and are therefore missing this effect.

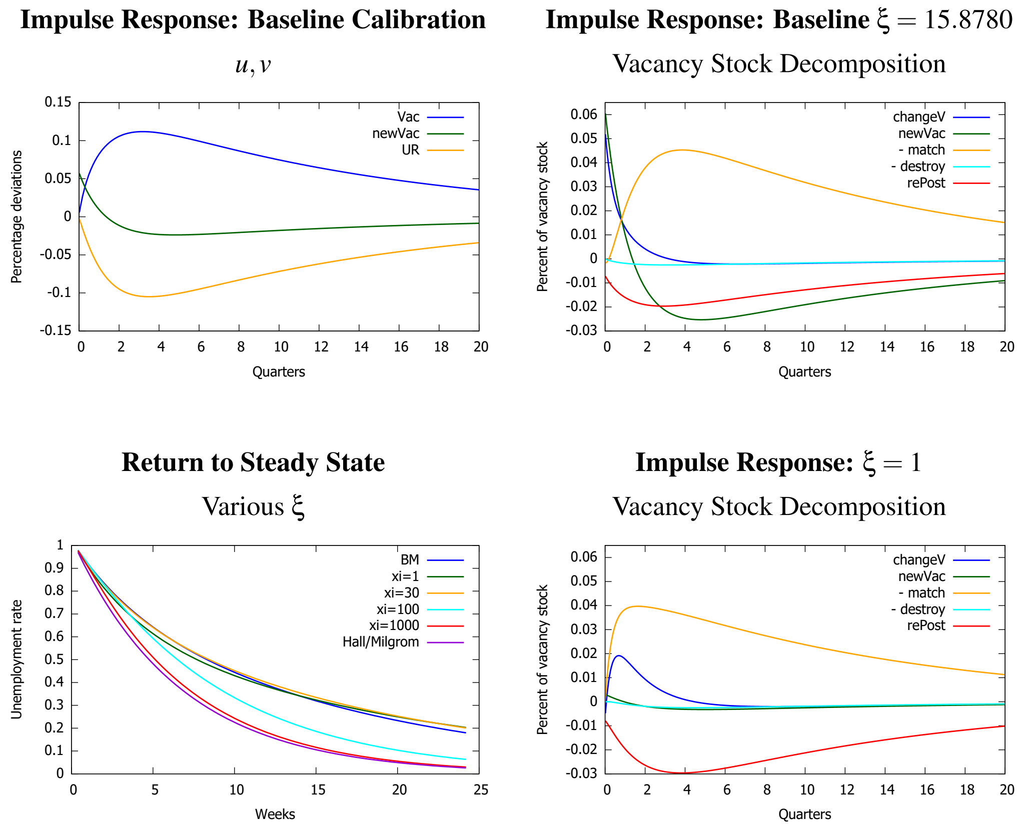

Figure 5: Key Model Dynamics.

Except for the bottom left panel the Impulse response functions depict responses to a 1% deviation in labor productivity at time 0. The bottom left panel depicts the return of unemployment to steady state when the model is initialized in steady state but with a one percentage point higher unemployment rate.

increase, so that the stock of vacancies start rising. Both effects lead to a gradual increase in the job finding probability. The reduction in unemployment leads to a reduction in the number of job matches, which in in turn leads to a further increase in the vacancy stock. The increase in vacancies then leads to a further reduction in unemployment, and so on. All this is the consequence of the interplay between endogenous vacancy destruction and LLV, which has been described very insightfully in Coles and Moghaddasi Kelishomi (2018). Notice that the peak in vacancies slightly precedes the peak in unemployment. This coincides with the finding that vacancies lead unemployment in the phase analysis, and is markedly different from the Coles-Kelishomi model with $\xi = 0.265$.

Figure 5 shows that vacancy dynamics depend critically on the value of ξ, the elasticity of new vacancies with respect to vacancy value. Our calibration identifies a very high elasticity, $\xi = 15.8780$. The bottom right panel displays the same decomposition, but now for the more conventional parameter value $\xi = 1$. In this case, the reduction in total matches becomes the driving force for vacancy dynamics almost immediately after the shock hits.

5.4 Identifying ξ

We have seen in Section 5.3 that a higher vacancy creation elasticity ξ strengthens the importance of vacancy creation versus vacancy depletion. Why does this matter? More elastic vacancy creation leads to a faster absorption of newly unemployed workers into employment. The lower left panel of Figure 5 confirms this with the following experiment. We start out of steady state, with an unemployment rate of 1 percentage point above steady state, while productivity is always at the steady state level. In the HM model, only 16 percent of the additional unemployment are left after 3 months, while 38 percent remain in our model. Roughly speaking, the return to normal is about twice as fast in the HM model. We perform the same experiment for a wide range of values of ξ. For very high values of ξ, our model converges to the HM model, but it makes little difference whether $\xi = 1$ or $\xi = 30$ in this respect. This is partly due to the fact that, for each value of ξ, we recalibrate the model so as to match the volatility of unemployment, vacancies, job destruction and match destruction. The difference to the case without recalibration is not big, however.

How is the value of ξ identified? There are two dimensions that are strongly affected by ξ: the phase relationship between vacancies and other variables, and the persistence of vacancies. As explained in the calibration section, we choose ξ so as to match the observed phase shift between unemployment and vacancies. Table 5 provides summary information on the phase relations between the four variables unemployment, vacancies, job finding and separation rates, for various values of ξ, again using the same recalibration strategy. A negative number means that the second variable is leading. For example, the number -0.18 in the first column means that unemployment (the first variable in u, v) has the strongest absolute correlation with the second variable, v, if v is lagged by 0.18 quarters.[15] The baseline calibration with $\xi = 15.8780$ does a very good job in matching the dynamic correlations. It gets the sign of all the leads right, and it matches the length of the lead within about half a quarter in all cases. As expected, the higher ξ, the stronger is the lead of vacancies relative to unemployment and other variables. No matter which phase shift one is targeting, the data always indicate a value of ξ between 5 and approximately 30. Although this is a huge numerical range, we have seen in Figure 5 that these values have about the same implications for the speed of absorption of unemployment.

The shape of the correlation functions is shown in Figure 6. In each panel, we show the observed correlations, bootstrapped 95% confidence bounds of the data, as well as the simulated correlations of the baseline model calibration. Since our one-shock model always generates stronger correlations than what we find in the data, all the model-generated correlation functions are shifted so as to have the same extreme value as the data. Then the shifted model correlations are within the confidence bounds almost always.

Table 6 presents autocorrelations implied by the model for various choices of ξ. As expected, the autocorrelation of vacancies (and most other variables) is higher the lower is ξ. Taking the more conventional value of $\xi = 1$ from Fujita and Ramey (2007) results in vacancies that are much more persistent than in the data. Matching the autocorrelation of vacancies would result in a somewhat higher value of ξ, around $\xi = 26$. All these values point to a range where the absorption of unemployment is much slower than in the HM model.

To sum up, the evidence relating vacancies, unemployment and job finding rates strongly points towards a high elasticity of vacancy creation, such that vacancies lead unemployment over the cycle. We have seen in the international data that this is a robust stylized fact. Although the elasticity is high, the model still predicts a speed of unemployment absorption that is much lower than in the standard MP and HM models.

5.5 Robustness check: cyclical employment opportunity cost

Chodorow-Reich and Karabarbounis (2016) provide empirical evidence that the opportunity costs of employment are procyclical, which makes the job surplus less procyclical and makes it harder for the model to explain unemployment fluctuations. In the HM model, there are two types of opportunity costs of employment: the utility of being unemployed, and the utility of disagreement in the bargaining process. It is mostly the latter which affects bargaining, and it is not clear to what extent the evidence in Chodorow-Reich and Karabarbounis (2016) applies to the disagreement situation. To make sure that cyclical opportunity

[15] Since we observe variables only at a quarterly frequency, we interpolate quarterly correlations by a cubic spline and report the phase shift at which the absolute values of this spline interpolation reach their maximum.

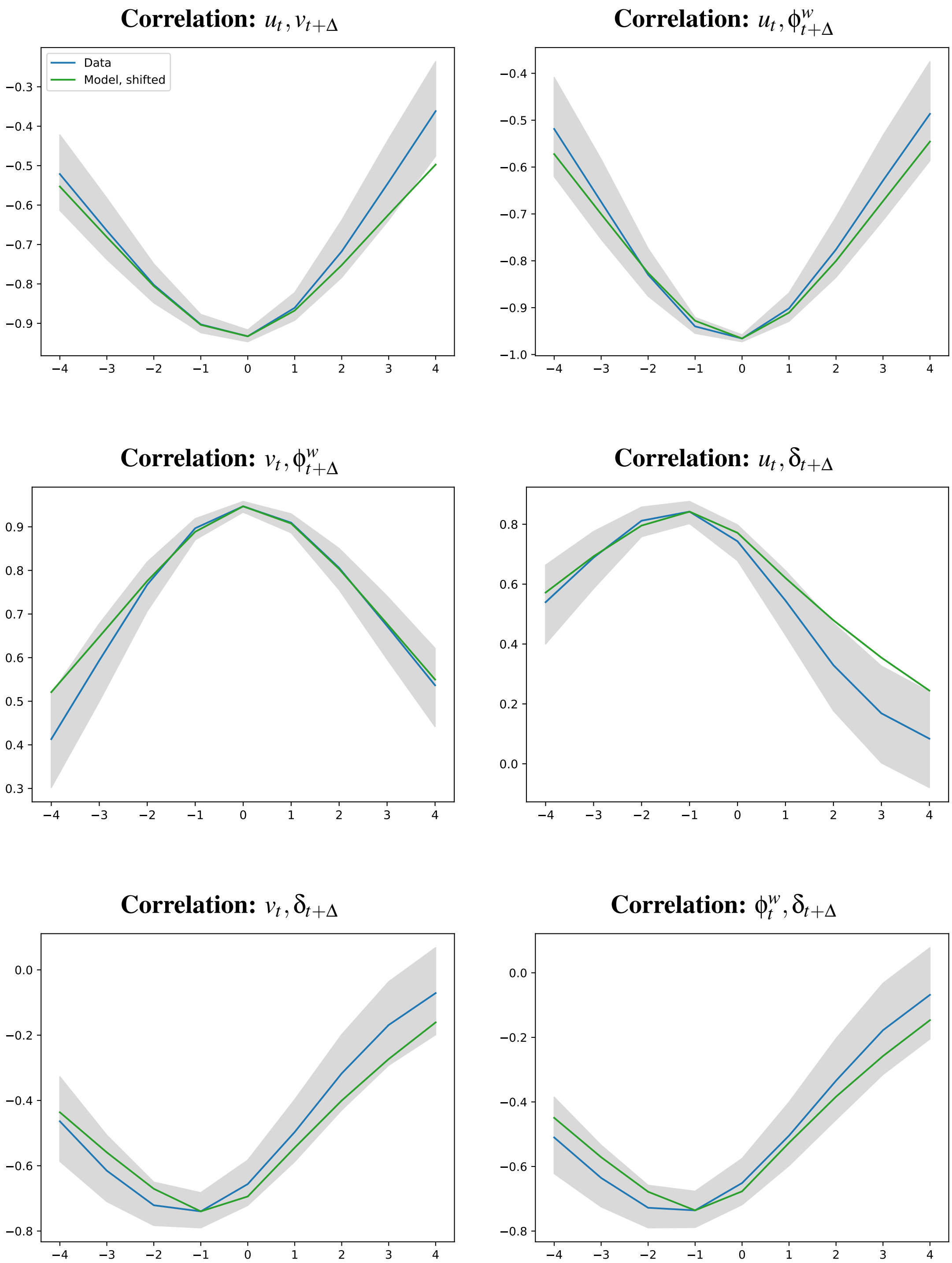

Figure 6: Dynamic Correlations.

The horizontal axis in each panel depicts the time-shift Δ, the vertical axis the correlation coefficient.

Table 5: Dynamic Correlations.

Model	u, v	u, ϕ^w	v, ϕ^w	u, δ	v, δ	ϕ^w, δ
Data	-0.18	-0.22	0.06	-1.25	-1.31	1.41
Model, $\xi = 1$	0.51	0.25	-0.24	-1.39	-2.00	1.71
Model, $\xi = 5$	0.27	0.14	-0.12	-1.30	-1.61	1.46
Model, $\xi = 10$	0.01	0.01	0.00	-1.17	-1.19	1.19
Baseline, $\xi = 15.8780$	-0.18	-0.09	0.09	-1.06	-0.86	0.97
Model, $\xi = 30$	-0.41	-0.21	0.17	-0.91	-0.41	0.65
Model, $\xi = 100$	-0.64	-0.34	0.21	-0.69	-0.02	0.24
Model, $\xi = 1000$	-0.76	-0.42	0.20	-0.50	0.14	0.04
Hall/Milgrom model	-0.80	-0.44	0.20	—	—	—

Quarterly correlations are interpolated by a cubic spline. We report the phase shift at which the absolute values of this spline interpolation reach their maximum. Thus -0.18 for (u, v) means unemployment needs to be lagged by 0.18 quarters ($0.18 \cdot 365/4 = 16.4$ days) in order to obtain the maximal correlation between unemployment and vacancies — i.e. vacancies lead unemployment.

Table 6: Autocorrelations.

	u	v	ϕ^w	δ	M	Y	y	w
Data	0.938	0.943	0.935	0.821	0.889	0.937	0.888	0.801
Model, $\xi = 1$	0.953	0.958	0.957	0.811	0.920	0.929	0.888	0.936
Model, $\xi = 5$	0.954	0.957	0.956	0.844	0.941	0.930	0.888	0.933
Model, $\xi = 10$	0.953	0.953	0.953	0.870	0.952	0.930	0.888	0.929
Baseline, $\xi = 15.8780$	0.950	0.945	0.948	0.883	0.951	0.929	0.888	0.924
Model, $\xi = 30$	0.944	0.927	0.938	0.892	0.934	0.927	0.888	0.916
Model, $\xi = 100$	0.931	0.876	0.914	0.893	0.873	0.920	0.888	0.900
Model, $\xi = 1000$	0.922	0.818	0.892	0.889	0.796	0.912	0.888	0.890
Hall/Milgrom model	0.921	0.805	0.889	-	0.330	0.910	0.888	0.889

Table 7: Cyclical Employment Opportunity Costs.

	u	v	ϕ^w	δ	M	Y	y	w
Data								
StdevRel	7.28	7.77	4.53	2.70	3.16	1.0	0.77	0.70
Baseline, $\delta_b = 4.93\bar{\bar{\delta}}$								
StdevRel	4.92	5.24	3.56	1.82	1.36	1.00	0.72	0.99
StdevRelZ	6.80	7.25	4.93	2.52	1.88	1.38	1.00	1.37
More rigid wages, $\delta_b = 2.20\bar{\bar{\delta}}$								
StdevRel	7.26	7.75	5.26	2.66	2.01	1.00	0.59	0.75
StdevRelZ	12.28	13.11	8.91	4.50	3.40	1.69	1.00	1.27

costs have an effect, we implement them by making both b and b_b proportional to TFP.

Does the model still succeed if we assume procyclical employment opportunity costs? In the part of Table 7 titled "Baseline, $\delta_b = 4.93\bar{\bar{\delta}}$", we have left all other parameters unchanged. The cyclical variability of unemployment is in fact reduced, to 4.92 from 7.28. A large part of this comes from a reduction in the variability of job separations: the incentive to separate in a recession is mitigated if outside options have deteriorated as well. To generate a realistic degree of unemployment variability, it appears necessary to shield the wage bargain more strongly from unemployment fluctuations. The results reported in the second part of Table 7 assume a breakup rate under disagreement of 2.20 times the total separation rate (all other parameters unchanged). Similarly to the case of low replacement ratio (cf. Table 4), a reduction in the breakup rate brings the model in line with the data, including the volatility of wages, which is higher than that of productivity. Again, there is no indication of "wage rigidity" when the usual criteria are applied.

5.6 The US labor market history 1951–2018

We have found that our model is quite successful in matching the dynamic properties of the data, even along some dimensions that are usually not considered, such as the dynamic correlations between different variables. The success may come as a surprise, given that our model has only one shock, namely a technology shock. This is in stark contrast to the recent DSGE literature, which assumes up to 10 different shocks, and usually finds that technology shocks contribute little to labor market dynamics.

To see whether the one-shock structure is an important constraint, we explore to what extent the model can fit the labor market history of the US in the postwar period. For this purpose, we construct the shock series such that the model matches the stock of vacancies perfectly in every quarter. Since the data are quarterly and the model period is daily, we assume that the data are constant over a quarter when we fit the model to the data. We initialize the state variables of the year 1951 by their expected values, according to our model, conditional on the observed values of unemployment, vacancies, the job finding rate and the total separation rate. With these starting values for 1951, and using the estimated series of shocks, we then simulate the US labor market variables for the whole time period 1951-2018. What we report are not one-period forecasts, but one long simulation, without resetting the states. Figure 7 shows the data and our simulated series. We show the total separation rate, not the components job separation and match separation separately, since there are no long enough data series for those.

The stock of vacancies is perfectly matched by construction. Perhaps surprisingly, both our model and the HM model match vacancies by using almost identical shock sizes. Conditional on matching vacancies, the fit for unemployment is very good for both models. This basically says that the matching function is a good approximation to reality. In terms of correlation, both models are very good at fitting the job finding probability, but HM over-predicts the amplitude. This results from the lower value of α, and ultimately comes from the fact that the HM model needs a more volatile job finding rate, because it has a constant separation rate (cf. our discussion in Section 5.2). Perhaps the biggest discrepancy between data and models is that both models predict a faster recovery of the unemployment rate in the great recession, given the observed vacancies. This would imply a reduction in matching efficiency in that period. Variations in advertisement intensity of vacancies, the mechanism stressed in Leduc and Liu (2020), might explain this behavior. For the separations, the correlation between model and data is 0.673, lower than for the other labor market variables, mostly due to a bad fit over the time period 1985-1995. Nevertheless, the model captures well the sharp spikes and lower autocorrelation of separations.

These findings suggest that the labor market variables are closely tied together through the mechanisms of the separation and the vacancy posting process, so that the dynamics can be explained by a model with only one shock. One can interpret this shock as the "marginal profitability of employment". In our model, the marginal profitability is closely linked to the average productivity of labor. This relationship can be tested in the data. The bottom right panel of Figure 7 displays our identified shock, together with measured labor productivity (productivity per person; productivity per hours would yield a similar picture), and an index of "financial tightness", namely Shiller's price-earnings-ratio (series "CAPE", detrended and aggregated to quarterly frequency). We can see that measured productivity follows our shock reasonably well until around 1985. After that, the series often run in opposite directions, so that the correlation over the full sample is only 0.085. This insight is not new; the "vanishing procyclicality" of productivity has been the object of intensive research over the last two decades. After 1985, there is a pretty good correlation between

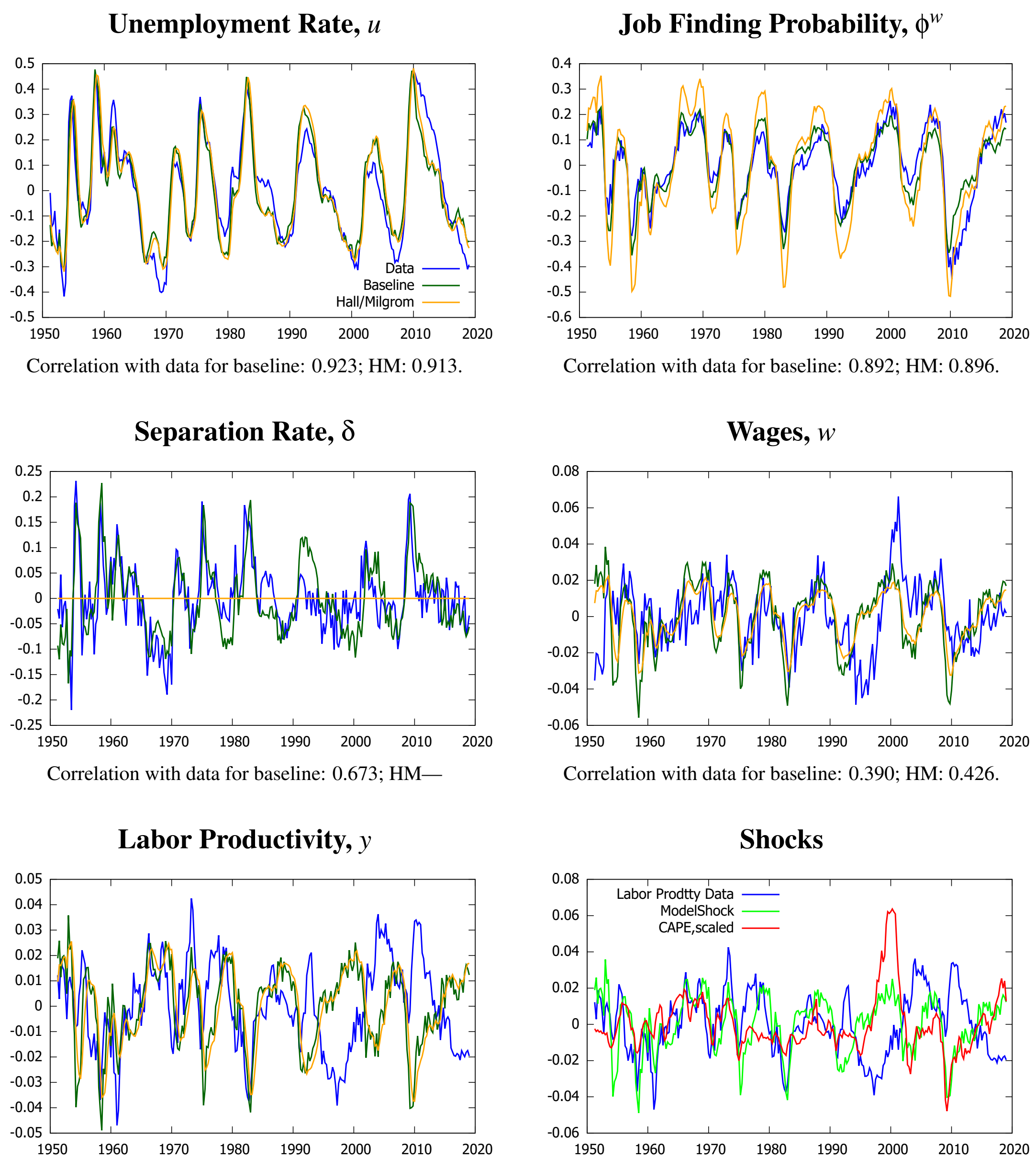

Figure 7: Key Labor Market Variables and Implied Productivity for Estimated Models.

our shock and the financial index. Although the underlying processes driving the incentives for job creation and separation are changing over time, the models still generate similar labor market dynamics. Identifying the structural shocks creating these incentives is still an open task.

6 Conclusions

We distinguish two channels generating labor market fluctuations: the vacancy creation channel, which is active in every textbook model of the Mortensen/Pissarides type, and the vacancy depletion channel, which results from the interplay of long-lived vacancies, less than perfectly elastic vacancy creation, and time-varying job separation rates. This mechanism was described, if not named, in Coles and Moghaddasi Kelishomi (2018). We develop a model that includes long-lived vacancies and endogenous job separations and use it to identify the importance of the two channels. The most important information for this identification are the phase relationships between labor market variables. While the vacancy creation channel is, overall, the dominating one, the depletion channel is quantitatively relevant. This is important for the question of how fast the unemployed can be absorbed into employment. Our estimates imply that the speed of absorption is about half of what is implied by the textbook labor market models which is important in the Covid crisis if we assume that many of the job losses are actually caused by job rather than just match destruction.

If the vacancy depletion channel is active, wages must fluctuate much more than productivity for the decentralized economy to achieve an efficient allocation of vacancies over the business cycle. Such a strong variation of wages is not observed in the data, not even in the wages for new hires. To bring the model in line with the data, we adopt the alternative-offer-bargaining framework of Hall and Milgrom (2008). Although this framework is usually seen as a form of "rigid wages", wages in our model still fluctuate more than productivity, which would probably not be considered rigid. Therefore, to assess whether wages react efficiently to changes in the economic situation, a unit elasticity with respect to productivity is not the appropriate benchmark.

If one accepts the view that vacancies are long lived, future work should focus on modeling job separations. We find that the model has no problem explaining the fluctuations in unemployment, if it is able to explain the fluctuations in job separations. The task is therefore to reconcile a sizeable match surplus, not only with large fluctuations in job finding rates (Ljungqvist and Sargent, 2017), but also with large fluctuations in job separation rates. As in den Haan, Haefke and Ramey (2005) a larger surplus increases the resilience of matches and reduces separations.

Finally, LLV create a link between the labor market and physical investment, because LLV are a form of capital, namely capital that is tied to a specific job. This changes the way that financial markets can affect the labor market. Preliminary results show that the effect of the interest rate on unemployment is larger in the model with long lived vacancies than with short lived vacancies by one order of magnitude or more, supporting recent work (Borovička and Borovičková, 2018; Vuillemey and Wasmer, 2020) on relating the labor market insights to stock market fluctuations.

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Appendix A Data

Detailed description of all data series used and whatever transformations we have made.

Unemployment (UNRATE), GDP (GDP) and deflator (GDPDEF) as well as working age population (CNP16OV) have been downloaded from FRED. Monthly unemployment numbers have been averaged to obtain quarterly data. We are working with real GDP per capita, which has obtained by dividing the GDP series by the deflator and the population.

To compute separation and job finding rates we follow Shimer (2005). We download short term unemployment (LNU03008396) and level of employment (LNU02000000) and unemployment (LNU03000000) from FRED. For data post December 1993 we increase both short and total unemployment numbers by 10% as advocated by Shimer[16]. Job Separation and job finding rates are then computed as in equations (1) and (2) of Shimer (2005). Let ϕ_t^w denote job finding probability, and δ_t separation probability. With u_t^s denoting short term unemployment in period t, u_t total unemployment in period t, and mp_t employment in t we have:

$$
\begin{aligned}
\phi_t^w &= 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}, && (43)\\
\delta_t &= \frac{u_{t+1}^s}{mp_t\left(1 - \frac{1}{2}\phi_t^w\right)}. && (44)
\end{aligned}
$$

All series are seasonally adjusted. Next average monthly probabilities to quarterly numbers and apply a bandpass filter to remove frequencies 2–4.

We compute total matches, M, by multiplying the unemployment rate with the job finding rate.

Data on vacancies for the US is downloaded from Regis Barnichon's website: https://sites.google.com/site/regisbarnichon/cv/HWI_index.txt?attredirects=0. Barnichon combines the earlier data on the Help Wanted Index with the JOLTS data on job openings.

JOLTS defines Job Openings as all positions that are open (not filled) on the last business day of the month. A job is "open" only if it meets all three of the following conditions:

A specific position exists and there is work available for that position. The position can be full-time or part-time, and it can be permanent, short-term, or seasonal, and The job could start within 30 days, whether or not the establishment finds a suitable candidate during that time, and There is active recruiting for workers from outside the establishment location that has the opening.

What is "active recruiting?" Active recruiting means the establishment is taking steps to fill a position. It may include advertising in newspapers, on television, or on radio; posting

[16] Results are quantitatively similar if we skip this adjustment.

Internet notices; posting "help wanted" signs; networking with colleagues or making "word of mouth" announcements; accepting applications; interviewing candidates; contacting employment agencies; or soliciting employees at job fairs, state or local employment offices, or similar sources.

Not included are: Positions open only to internal transfers, promotions or demotions, or recall from layoffs Openings for positions with start dates more than 30 days in the future Positions for which employees have been hired, but the employees have not yet reported for work Positions to be filled by employees of temporary help agencies, employee leasing companies, outside contractors, or consultants. A separate form is used to collect information from temporary help/employee leasing firms for these employees.

For European countries we download vacancies and unemployment from the OECD website https://stats.oecd.org/. The countries Austria, Germany, Switzerland, Norway, and United Kingdom are the only ones with both data on unemployment and vacancies available for an overlapping period of at least fifty years. For the European countries the data sample is 1961q2 – 2012 q2. Unemployed is data on registered unemployment. Vacancies are the total vacancy stock (i.e. public and private). We used the non-seasonally adjusted time series. Series are logged, then subject to a Bandpass filter and a regression on quarterly dummies to remove any season, and then HP-filtered with a smoothing parameter of 10^5.

A.1 Separations and Job Destruction

Finally, data on job destruction is downloaded from the BLS website of the Business Employment Dynamics, series BDS0000000000000000110004RQ5, which is the quarterly gross job loss rate, total private industries and is available at quarterly frequency starting 1994. We convert the series to a monthly probability to arrive at an average of 1.75% over the period 1994–2014. Over the same time the standard deviation of the log of this series relative to log real GDP is 2.43. Hence the average monthly probability of job destruction is approximately two thirds of the total separation rate as computed according to Equation 44 while the relative standard deviation is approximately the same.

A.2 Wages

There is no wage series that reaches back to the beginning of our sample. Instead, we use compensation of employees, wages and salaries, private industries (A132RC1Q027SBEA) and deflate it using the GDP deflator. To obtain compensation per hour we use annual hours worked by full-time and part-time employees (B4701C0A222NBEA) which is unfortunately available only at an annual frequency. We then divide total compensation (quarterly) by the corresponding annual hours observation. After detrending with the usual HP filter and smoothing parameter of 100000 we remove the deterministic seasonal pattern that arose from the mixing of annual and quarterly observations by regressing our resulting series on quarterly dummies and use the residual as our hourly wage series.

Appendix B Proofs

B.1 Proof of Result 1

Notice that $M_t = M(u_{t-1}, \check{v}_t) = \phi_t^w u_{t-1} = \phi_t^f \check{v}_t$. The planner solution is then characterized by the Lagrangian

$$
\begin{aligned}
\mathcal{L} = \sum_{t=1}^{\infty} \beta^t \mathbb{E}_0 \Big\langle & (y_t - \kappa_k)(1-u_t) + bu_t - \kappa_s \check{v}_t \\
& + W^{\check{V}}{}_t \left[-\check{v}_t + ((1-\delta_v) v_{t-1} + n_t) \right] \\
& + W_t^V \left[-v_t + \left(1 - \phi_t^f\right) \check{v}_t + \delta_{mt}(1-u_{t-1}) \right] \\
& - W_t^S \left[-u_t + \delta_t (1-u_{t-1}) + (1-\phi_t^w) u_{t-1} \right] - \mathcal{V}_k(n_t) \Big\rangle.
\end{aligned}
$$

Differentiating $\mathcal{L}$ with respect to n_t, $\check{v}_t$, v_t, and u_t, respectively, gives the optimality conditions:

$$
\begin{aligned}
W^{\check{V}}{}_t &= \mathcal{V}_k'(n_t), \\
W^{\check{V}}{}_t &= -\kappa_s + W_t^V + \phi_t^f (1-\alpha)\left(W_t^S - W_t^V\right), \\
W_t^V &= \beta(1-\delta_v)\mathbb{E}_t V^{\check{V}}{}_{t+1}, \\
W_t^S &= y_t - \kappa_k - b\mathbb{E}_t \left\langle (1-\delta_{t+1}) \beta W_{t+1}^S + \delta_{mt+1} \beta W_{t+1}^V - \phi_{t+1}^w \alpha \beta \left(W_{t+1}^S - W_{t+1}^V\right) \right\rangle.
\end{aligned}
$$

It is straightforward to derive the decentralized solution with Nash bargaining:

$$n_t = \mathcal{V}\left(V^{\check{V}}{}_t\right) \tag{46a}$$

$$V^{\check{V}}{}_t = -\kappa_s + \phi_t^f V_t^J + (1-\phi_t^f)(1-\delta_v)\beta \mathbb{E}_t V^{\check{V}}{}_{t+1} \tag{46b}$$

$$V_t^V = \beta(1-\delta_v)\mathbb{E}_t V^{\check{V}}{}_{t+1} \tag{46c}$$

$$V_t^J = y_t - w_t - \kappa_k + \beta \mathbb{E}_t \left\langle (1-\delta_{t+1}) V_{t+1}^J + \delta_{mt+1} \beta V_{t+1}^V \right\rangle \tag{46d}$$

$$V_t^E - V_t^U = w_t - b + \beta \mathbb{E}_t \left\langle \left[1 - \phi_{t+1}^w - \delta_{t+1}\right] \left(V_{t+1}^E - V_{t+1}^U\right) \right\rangle \tag{46e}$$

$$\omega\left(V_t^J - V_t^V\right) = (1-\omega)\left(V_t^E - V_t^U\right). \tag{46f}$$

The Nash bargaining condition (46f) reflects the fact that the firm is left with a vacancy of value V_t^V if the bargain breaks down.

To prove constrained efficiency we show that given the Hosios assumption of $\alpha = \omega$ $W_t^V = V_t^V$, $W^{\check{V}}{}_t = V^{\check{V}}{}_t$, and $W_t^S = V_t^J + V_t^E + V_t^U$ which then yields equivalence of first order conditions to the decentralized and the planner problem. Start out from 22c and 46c and equate $W_t^V = V_t^V$. Use 46c to replace $\mathbb{E}_t V_{t+1}^V$ in 46b and obtain:

$$V^{\check{V}}{}_t \quad = -\kappa_s + V_t^V + \phi_t^f \left(V_t^J - V_t^V\right).$$

Thus upon comparison with 22b it is immediately clear that

$$\left(V_t^J - V_t^V\right) = (1-\alpha)\left(W_t^S - W_t^V\right) \iff V^{\check{V}}{}_t = W^{\check{V}}{}_t.$$

Proceed by adding 46d to 46e:

$$\begin{aligned} V_t^J + V_t^E - V_t^U &= y_t - \kappa_k - b + \beta \mathbb{E}_t \left\langle (1-\delta_{t+1}) \left(V_{t+1}^J + V_{t+1}^E - V_{t+1}^U \right) \right\rangle + \beta \mathbb{E}_t \left\langle \delta_{mt+1} V_{t+1}^V \right\rangle \\ &- \beta \mathbb{E}_t \left\langle \phi_{t+1}^w \left(V_{t+1}^E - V_{t+1}^U \right) \right\rangle . \end{aligned}$$

Thus upon comparison with 22d it is immediately clear that for $V_t^V = W_t^V$:

$$\left(V_t^E - V_t^U \right) = \alpha \left(W_t^S - W_t^V \right) \iff V_t^J + V_t^E - V_t^U = W_t^S .$$

Use the Hosios assumption $\alpha = \omega$ in 46f to obtain

$$V_t^J - V_t^V = \frac{1-\alpha}{\alpha} \left(V_t^E - V_t^U \right) .$$

Then

$$V_t^J - V_t^V = (1-\alpha) \left(W_t^S - W_t^V \right)$$

as well as

$$V_t^E - V_t^U = \alpha \left(W_t^S - W_t^V \right)$$

immediately follow.

B.2 Proof of Result 2

Based on equations 27 and 28 the values for capital cost and search cost need to be derived so that the user cost of an unfilled vacancy κ_s and the surplus term remain constant. Pick κ_s^l such that κ_h remains constant, i.e.

$$\begin{aligned} \kappa_h &= \kappa_s^l + \left(1 - \beta \left(1 - \delta_v^l\right)\right) \kappa_v^l \\ &= \kappa_s^s + (1 - \beta (1 - \delta_v^s)) \kappa_v^s \\ \kappa_s^l &= \kappa_s^s - \left(1 - \beta \left(1 - \delta_v^l\right)\right) \kappa_v^l + (1 - \beta (1 - \delta_v^s)) \kappa_v^s . \end{aligned}$$

Pick κ_k^l such that c remains constant. Recall $W^V = \beta(1-\delta_v)\kappa_v$.

$$\begin{aligned} c + b &= [\beta (1 - \delta_j) - 1] \beta \left(1 - \delta_v^l\right) \kappa_v^l - \kappa_k^l \\ &= [\beta (1 - \delta_j) - 1] \beta (1 - \delta_v^s) \kappa_v^s - \kappa_k^s \\ \kappa_k^l &= \kappa_k^s - [1 - \beta (1 - \delta_j)] \beta \left(1 - \delta_v^l\right) \kappa_v^l + [1 - \beta (1 - \delta_j)] \beta (1 - \delta_v^s) \kappa_v^s . \end{aligned}$$