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ABSTRACT

Low, High and Super Congestion of an Open-Access Natural Resource: The Autarky Case*

Production of commodities based on open-access renewable natural resources (NR) has usually been examined under "low" congestion (LC) – where $MC > AC$ and both increase with output. I identify two additional congestion categories, "high" (HC) and "super" (SC) congestion – where *AC* is backward-bending and $MC < 0$. Using a general equilibrium model, I derive the open-access impact on steady-state welfare, *NR*, sectoral employment, output and price, relative to optimal regulation. The main findings are: i) Welfare and NR losses under SC (HC) are one or more orders-of-magnitude greater (between a multiple and one or more orders-of-magnitude greater) than under LC; ii) These results are robust to alternative parameter values and functional forms, raising confidence in them and thus in regulation's importance; iii) One such regulation, an optimal tax, raises (reduces) the commodity's output and reduces (raises) its price under SC (LC and HC), generating significantly larger gains under SC. A companion piece examines the issue under openness to trade, showing that, though an increase in population unambiguously worsens open-access *NR* and welfare under autarky, this need not be the case under trade.

JEL Classification: D62, F18, Q22, Q27, Q56

Keywords: open access, natural resource, low, high and super congestion, autarky and trade

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1. Introduction

Many developing countries obtain an important share of their income from the exploitation of open-access renewable natural resources (*NR*), including fisheries, forests, arable land, grazing grounds, and water resources. Imperfect or lack of property rights for the *NR* results in the partial or non-internalization of negative externalities, an excessive use of labor and other variable inputs, and *NR* degradation.[1] Open access' negative impact may take time or can occur very rapidly.[2]

The problem has affected many developing countries and has led to the decline or disappearance of communities due to rapid population growth, access to a wider market, or other. For instance, Brander and Taylor (1998) argue that open access to Easter Island's forests initially led to economic growth, followed by population overshooting and the disappearance of the *NR*, resulting in a dramatic decline in population and living standards over time.

The classic case of *NR* depletion is fisheries. The problem has affected many countries and early analyses focused on this issue (Gordon 1954, Scott 1955). Some recent studies have extended the analysis, using general equilibrium models to examine the steady state and transition paths of economies with open-access *NR* (e.g., Brander and Taylor 1997, 1998; López and Schiff 2013). This study develops a general equilibrium model, focusing on steady-state outcomes.

It identifies, for the first time (as far as I know), three economically-relevant congestion categories, namely low (LC), high (HC) and super (SC) congestion. These are explained in Section 2 and are described in Figure 1 in Section 3. The study *i*) derives solutions for welfare, *NR*, sectoral

[1] For instance, López (1997, 1998) finds that the share of negative *NR* externalities – from use of village-level open access lands in Ghana and Côte d'Ivoire – that is internalized is around 30 percent and declines with village size.

[2] Appendix 1 provides a brief description of negative externalities from Chile's and the Philippines' aquaculture industry, where open access has had dramatic output and welfare effects.

employment, output and prices, under open access and optimal regulation under both a closed and an open economy; *ii*) sets forth a new graphical analysis to provide a more intuitive presentation of the results; and *iii*) performs simulations to obtain quantitative measures of the impact of open access relative to optimal regulation for each of the congestion categories.

The remainder of the paper is organized as follows. Section 2 sets forth a two-sector general equilibrium model and preference function, and solves the model under both an open-access and an optimally regulated *NR*. Section 3 examines the impact of open access on steady-state welfare, *NR*, employment, output and price. Section 4 looks at the robustness of the results. Section 5 draws policy implications and Section 6 concludes.

2. Model

Section 2.1 presents the general equilibrium model's supply side and Section 2.2 does so for the demand side. The open-access and optimal solutions are presented in Section 2.3. The model does not distinguish between population and labor force.

2.1. Supply

Assume an economy whose private sector produces two goods under perfect competition, a manufacturing good, M, and a commodity, Q. The economy's labor endowment is denoted by $\mathbb{L}$, and the amount of labor employed in sector Q (M) is denoted by $L(l)$, with $L + l = \mathbb{L}$.

Following Brander and Taylor (1998), I assume M is produced with l under a constant-returns-to-scale technology, with units chosen such that the marginal product $MP_l = 1$. Thus, $M = l = \mathbb{L} - L$. Good M is chosen as the numéraire, with its price normalized to one. Thus, labor's wage rate is $w = VMP_l = 1$ for $M > 0$, which holds under a Cobb-Douglas utility function (see Section 2.2).

The NR, N, declines with employment, L. Assume $N = \alpha - \beta L$ ($\alpha, \beta > 0$), where α is the NR when it is unexploited (i.e., $L = 0$) or NR endowment, and β is the absolute value of labor's negative externality, $\partial N/\partial L = -\beta < 0$, on the NR. Production functions for Q and M are:

$$Q = LN = L(\alpha - \beta L),\ M = l = \mathbb{L} - L;\ \ \alpha, \beta > 0,\ L \epsilon \left(0, \frac{\alpha}{\beta}\right), \tag{1}$$

where $L < \alpha/\beta \Leftrightarrow N = \alpha - \beta L > 0$ ensures an interior solution.

2.1.1. Average and marginal product and cost

Labor's average product $AP_L = \frac{Q}{L} = \alpha - \beta L$. As $w = 1$, average cost $AC = \frac{1}{\alpha - \beta L}$.[3] Labor's marginal product $MP_L = \alpha - 2\beta L$, and marginal cost $MC = \frac{1}{\alpha - 2\beta L}$, with $MC > (<)\, 0$ on the upward-sloping (backward-bending) part of the AC curve where low (high and super) congestion (or LC (HC and SC)) prevails, with $MP_L \gtrless 0 \Leftrightarrow MC \gtrless 0 \Leftrightarrow L \lesseqgtr \hat{L} = \alpha/2\beta$ (see Figure 1).[4]

Thus, $MP_L \gtrless 0$ for $L \lesseqgtr \hat{L}$ implies (see Figure 1) that MC converges to ∞ ($-\infty$) as Q approaches $\hat{Q}$ (Q_{MAX} in Fig. 1) from $Q < \hat{Q}$ ($Q > \hat{Q}$). Formally, $\lim_{L^- \to \hat{L}} MC = \infty$ and $\lim_{L^+ \to \hat{L}} MC = -\infty$.

2.1.2. Inflection point

The first three results below are derived in Appendix 2 and the fourth is derived in Appendix 3.

[3] There may be other variable inputs. Assuming their cost relative to that of labor, γ, is given, they can easily be incorporated in the analysis. For simplicity, and following Brander and Taylor (1998), I abstract from non-labor costs.

[4] The backward-bending supply curve in the case of a fishery was examined in some early partial-equilibrium studies, though some with questionable analysis. Copes' (1970) seminal paper, which includes a backward-bending AC curve, focused on the impact of demand shocks on the stability of the equilibrium. His results depend on a demand curve that is less elastic than the backward-bending part of the AC curve, an assumption that need not hold in general. In fact, the opposite obtains in this model (proof is in Appendix 2). Clark (1990) refers to a discounted supply curve that might be backward bending under an optimally managed fishery. However, that solution cannot be optimal because variable inputs' marginal product must be positive at the optimum ($MP_L > 0$, and so $MC > 0$), i.e., the optimum must be on the upward-sloping segment of the supply curve. In the case of road congestion, Else (1981) shows a backward-bending positive MC segment, even though MC is exclusively backward-bending in its negative segment.

Denoting the backward-bending (upward-sloping) segment of the AC curve by AC_1 (AC_2), the related MC curve by MC_1 (MC_2), and the level of Q (L) on AC_1 that separates HC and SC by Q_I (L_I), we have:

i) $L_I = \frac{2\alpha}{3\beta} > \hat{L}$, $Q_I = \frac{2\alpha^2}{9\beta} < \hat{Q}$;
ii) Output Q_I is the inflection point on AC_1, i.e., the point where $AC'' \equiv \frac{\partial^2 AC}{\partial Q^2} = 0$;
iii) Q_I is also the intersection point of AC_1 and MC_2 (associated with AC_2); and
iv) $MC_1 < 0$ (associated with AC_1) is a mirror image of $MC_2 > 0$, as depicted in Figure 1. [5]

The distinction between HC and SC is important. Denoting optimal (open-access) output by Q^* (Q), we have $Q^* > (<)\ Q$ under SC (HC),[6] which has opposite implications for the impact of optimal regulation.

2.2. Demand

Individual preferences, U, are given by $U = m^{\gamma} q^{1-\gamma}$, $0 < \gamma < 1$. For simplicity, assume the same amount is spent on *NR*-based goods (Q) and other ones (M), i.e., $\gamma = 1/2$. The impact of using other values of γ is examined in Section 4.1. Thus:

$$U = m^{1/2} q^{1/2},\ m = M/\mathbb{L},\ q = Q/\mathbb{L}. \quad (2)$$

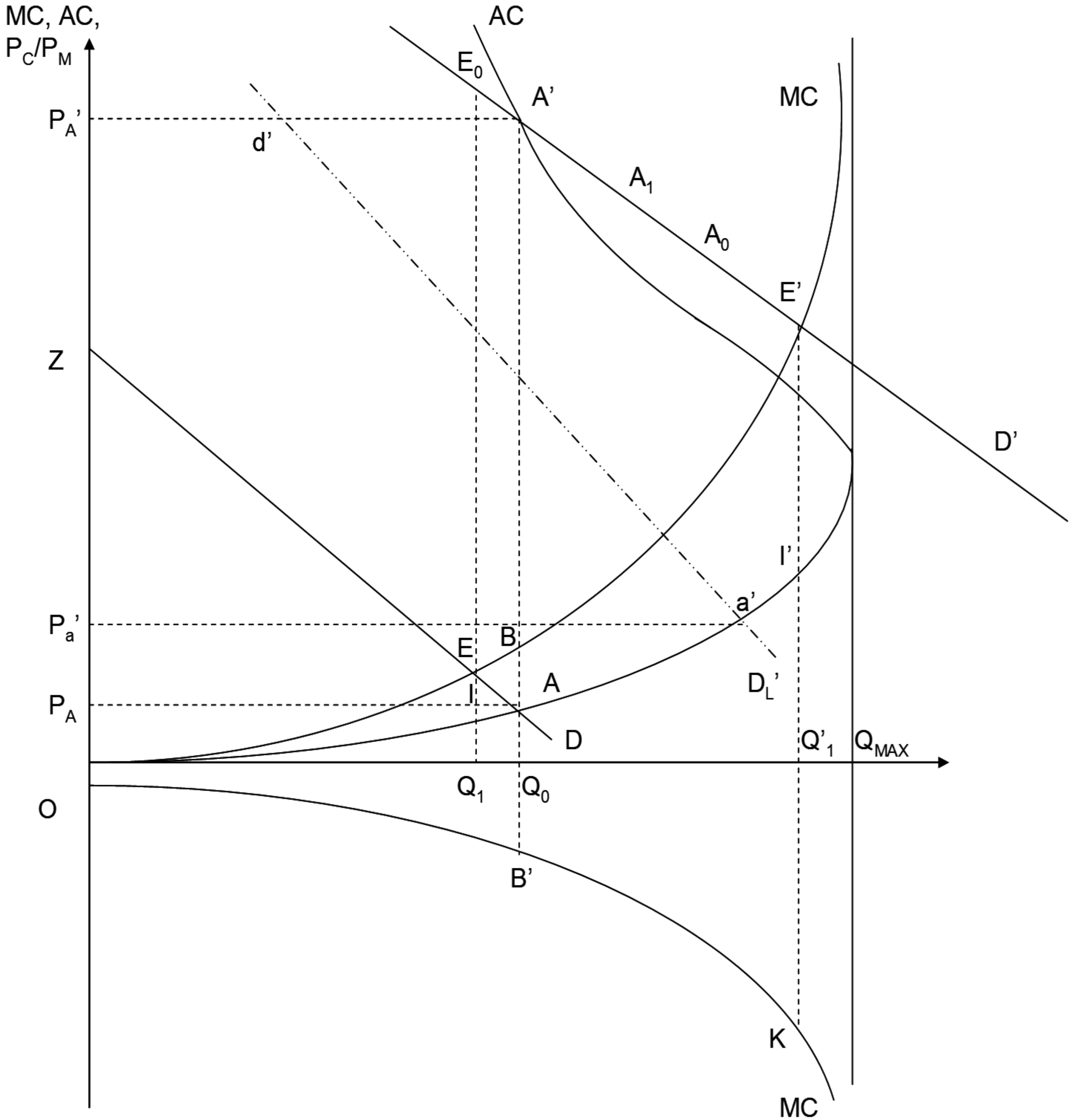
3. Solution and Simulation

Section 3.1 presents a graphical analysis, Section 3.2 provides the model's solution, and Section 3.3 presents various simulations. Note that welfare in Section 3.1 is aggregate welfare $W = \mathbb{L}U$, while welfare in Sections 3.2 and 3.3 is the representative individual's utility U, as given in (2).

[5] Note that, as shown in Appendix 4, the range of L-values is smallest under HC, larger under SC, and largest under LC. Assuming random drawings of L, the likelihood of a SC (HC) (LC) drawing is $\frac{1}{3}\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)$.

[6] The reason is that, though AC is backward bending under HC, it is located on the right of the MC curve (see Figure 1), with optimal output smaller than the equilibrium one. The opposite holds under SC where, by definition, the AC segment is to the left of the MC curve.

Figure 1: Low, High and Super Congestion



3.1. Graphical analysis

For clarity of exposition, demand curves are linear in Figure l. Assume first that the demand curve is given by D, with equilibrium at point A where price $p = AC = P_A$, and output is $Q = Q_0$. The optimum is at point E where $p = MC =$ distance EQ_1 and output is $Q = Q_1$. The welfare cost under open access is $\Delta W_{LC} = W_{LC}^* - W_{LC} = AEB$.

Assume now a country whose demand is represented by line D' because of, say, a larger population (or a greater preference for Q; see Section 4.1). Open-access equilibrium is at point A' where AC and D' intersect and which is located in the SC segment of the AC curve. For simplicity, assume D' is such that output is also Q_0 at A'. The optimum is at E' where D' and MC intersect.

There are three ways to obtain the welfare cost, ΔW_{SC}, of open access:

1. *Higher cost (as measured by AC) and lower consumption*: The difference in the cost of producing Q_0 under demand D and D' is $P_A'A'AP_A = AA' * Q_0$. Moreover, the increase in output from Q_0 to Q_1' generates a welfare gain $A'E'B$. Hence, the welfare cost of open access is $\Delta W_{SC} = P_A'A'AP_A + A'E'B$.

2. *Zero producer surplus*: Under open access, the producer surplus is *nil* because price equals average cost ($p = AC$). Thus, welfare is equal to the consumer surplus. At A', the consumer surplus is the area between the demand curve, the *y*-axis, and the $P_A'A'$ line. At E', AC is given by point I', and welfare is the area between the demand curve, the *y*-axis, and $P_I'I'$, the horizontal line at the I' level (P_I' is *not* shown).[7] Thus, ΔW_{SC} is equal to the area between the lines $P_A'A'$, $P_I'I'$ and the demand curve or $P_A'A'E'I'P_I'$.

[7] At the optimum, welfare consists of the sum of the consumer surplus and the tax revenue $T = (E'I') * Q_1'$.

3. Higher cost (as measured by MC) and lower consumption: As derived in footnote 8, the welfare cost is also given by $\Delta W_{SC} = A'B'KE' + E'\infty(-\infty)K$.[8]

3.2. Solution

This section provides the solution to the model for both an unregulated or open-access *NR* and an optimally regulated one. An interior solution prevails under a Cobb-Douglas utility function.

3.2.1. Open Access

Utility maximization implies the commodity's relative demand price, p, equals the ratio of marginal utilities: $p = \frac{U_q}{U_m} = \frac{m}{q} = \frac{M}{Q} = \frac{\mathbb{L}-L}{L(\alpha-\beta L)}$. Price $p = AC = \frac{1}{\alpha-\beta L}$ under open access. Hence, $\frac{\mathbb{L}-L}{L(\alpha-\beta L)} = \frac{1}{\alpha-\beta L}$, or $L = \frac{\mathbb{L}}{2}$, with $p = \frac{1}{\alpha - \frac{\beta\mathbb{L}}{2}}$. Recalling that $M = \mathbb{L} - L$, the solution is:

$$L = \frac{\mathbb{L}}{2}, M = \frac{\mathbb{L}}{2}, m = \frac{1}{2}, Q = \frac{\mathbb{L}}{2}\left(\alpha - \frac{\beta\mathbb{L}}{2}\right), q = \frac{1}{2}\left(\alpha - \frac{\beta\mathbb{L}}{2}\right), U = \frac{1}{2}\left(\alpha - \frac{\beta\mathbb{L}}{2}\right)^{1/2} = \frac{1}{2p^{1/2}}. \qquad (3)$$

Thus, utility increases with *NR* endowment α and declines with externality β and population $\mathbb{L}$, or declines with price p.

3.2.2. Optimum

Under optimal regulation, $p = MC$, or $\frac{\mathbb{L}-L}{L(\alpha-\beta L)} = \frac{1}{\alpha-2\beta L}$, which is a quadratic equation, namely $3\beta L^2 - 2(\alpha + \beta\mathbb{L})L + \alpha\mathbb{L} = 0$. The solution is:

[8] Consumption is Q_0 rather than Q_1', with a loss $A'Q_0Q_1'E'$. The decline from Q_1' to Q_0 implies a higher cost, which consists of i) the cost of the output increase from Q_1' to Q_{MAX}, or the area below the MC curve, $E'Q_1'Q_{MAX}\infty$; ii) the cost of the decrease in output from Q_{MAX} to Q_1' (entailing an increase in L) on the backward-bending part of the AC curve, $KQ_1'Q_{MAX}$(-∞); and iii) the cost of the decrease in output from Q_1' to Q_0, i.e., $B'Q_0Q_1'K$. Thus, the welfare cost under open access is $A'B'KE' + E'\infty(-\infty)K$. As the negative segment of the MC curve is the mirror image of its positive segment (see Appendix 3), the welfare cost is also $\Delta W_{SC} = A'B'KE' + 2(E'Q_1'Q_{MAX}\infty)$.

$$L^* = \frac{1}{3\beta}\left(\alpha + \beta\mathbb{L} - \sqrt{(\alpha+\beta\mathbb{L})^2 - 3\alpha\beta\mathbb{L}}\right),\ M^* = \mathbb{L} - L^*,^{9} \quad (4)$$

with $q^* = \frac{Q^*}{\mathbb{L}} = \frac{L^*(\alpha-\beta L^*)}{\mathbb{L}}$, $m^* = \frac{M^*}{\mathbb{L}} = 1 - \frac{L^*}{\mathbb{L}}$, $p^* = \frac{1}{\alpha - 2\beta L^*}$, and L^* is a function of α/β.[10]

The following results are derived in *Appendix 5*:

i) The equilibrium is stable, i.e., excess-demand (-supply) prevails below (above) the equilibrium, a potential issue on the backward-pending part of the AC curve.

ii) Equations (3) and (4) imply $L^* < L$, i.e., optimal regulation reduces the amount of labor employed in the *NR*-based sector. (The second solution of equation (4) is not an optimum).

iii) The gap between open-access and optimal solutions, as a share of the latter, $\Delta x \equiv \frac{x-x^*}{x^*}$ $(x = L, N, Q, U)$, depends not on α and β individually but on the ratio α/β, i.e., on the ratio of the *NR* endowment, $N = \alpha$ (where $L = Q = 0$) and the absolute value of the externality $\frac{\partial N}{\partial L} = -\beta$.

iv) The solution for optimal welfare, $U^* = (q^* m^*)^{1/2}$, is also presented.

3.3. Simulation

This section examines the relationship between $\Delta x \equiv \frac{x-x^*}{x^*}$ $(x = L,\ N,\ Q,\ U)$ and population parameter $\mathbb{L}$, for different values of the *NR* endowment α and externality parameter β. Robustness of results to different parameter values and functional forms is examined in Section 4.

The values for α and β selected in the 'base case' are $\alpha = 10$ and $\beta = 1$. Table 1 in Section A shows the Δx results for individual values of $\mathbb{L}$, and Table 2 in Section B does the same for central values of $\mathbb{L}$ in each congestion category.

[9] Appendix 5-*iii*) shows that the second solution for L^*, with a positive sign before the square root, is not an optimum.
[10] Note that national income Y is the sum of labor income $Y_L = w\mathbb{L}$ and income (or rent) $Y_N = (p - AC)Q$ from exploiting the natural resource, i.e., national income $Y = Y_L + Y_N = w\mathbb{L} + (p - AC)Q$. Since $p = AC$ under open access, $Y_N = 0$, and with $w = 1$, we have $Y = \mathbb{L}$.

A. <u>Individual $\mathbb{L}$ values</u>

i) <u>Welfare</u>

Subscripts used for all variables below refer to the level of the labor force, $\mathbb{L}$. Table 1 shows (in percent) that $\Delta U_1 = -.065$ (LC); $\Delta U_4 = -.707$ (LC), $\Delta U_9 = -6.34$ (LC); $\Delta U_{11} = -13.5$ (HC); $\Delta U_{16} = -33.3$ (SC) and $\Delta U_{19} = -64.3$ (SC).

Thus, for $\mathbb{L} = 19$ (16), i.e., under SC, we have ΔU_{19} (ΔU_{16}) $= 989$ (513)ΔU_1, 90.9 (47.1)ΔU_4, 10.1 (5.3)ΔU_9, and 4.8 (2.5)ΔU_{11}. For $\mathbb{L}$ =11, $\Delta U_{11} = 148\Delta U_1$, 19.1$\Delta U_4$ and 2.1ΔU_9. Thus, open-access welfare costs under SC and HC are both between *two orders of magnitude greater* and a *multiple* of that under LC.

ii) <u>Natural Resource</u>

For $\mathbb{L}$ =19 (SC case), ΔN_{19} is -91 percent or 827ΔN_1(LC), 38ΔN_4 (LC), 4.8ΔN_9 (LC), and 2.9ΔN_{11} (HC). For $\mathbb{L}$ =16 (SC case), $\Delta N_{16} = -67$ percent or 608ΔN_1, 28ΔN_4, 3.5ΔN_9 and 2.2ΔN_{11}. Thus, the *NR* depletion under SC is between *two orders of magnitude greater than* and *a multiple* of ΔN under LC, and is a multiple of ΔN under HC. And the same holds for $\mathbb{L} = 11$ (HC) relative to $\mathbb{L} = 1$ and $\mathbb{L} = 4$ (LC): $\Delta N_{11} = 282\Delta N_1$, 13$\Delta N_4$ and 2.1ΔN_9.

iii) <u>Employment</u>

ΔL_{19} $(\Delta L_{16})(\Delta L_{11})$ = 127 (100) (58) percent, or 47 (37) (21)ΔL_1, 8.5 (6.7) (3.9)ΔL_4, and 3.1 (2.4) (1.4)ΔL_9 (the three LC cases). Thus, employment under SC is between *an order of magnitude* larger and a *multiple* of that under LC, and similarly under HC for two of the three LC cases.

iv) <u>Output</u>

Open-access output is larger (smaller) than optimal output under LC and HC (SC), i.e., $\Delta Q >$ $(<)$ 0 under LC and HC (SC). Thus, at first, $\Delta Q > 0$ and increases with $\mathbb{L}$ (in percent) from 1.06 for $\mathbb{L} = 1$, 10.3 for $\mathbb{L} = 4$, to 14.6 for $\mathbb{L} = 9$, then declines to 9.0 for $\mathbb{L} = 11$, -33.0 for $\mathbb{L} = 16$ and

-81.0 for $\mathbb{L} = 19$. Thus, in the case of SC, equilibrium output is between one third and four fifths smaller than optimal output, in large part due to the *NR* depletion.

Table 1. Autarky: Open Access vs. Optimum [a]

	Open Access (x)				Optimum (x^*)				Difference $\Delta x = \frac{x-x^*}{x^*}$ (%)			
$\mathbb{L}$	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	9.5	4.75	1.541	.49	9.51	4.63	1.542	2.7	-.11	1.06	-.065
4	2.0	8.0	16.0	1.414	1.8	8.2	14.5	1.424	15	-2.4	10.3	-.707
9	4.5	5.5	24.8	1.173	3.2	6.8	21.6	1.248	41	-19	14.6	-6.34
11	5.5	4.5	24.8	1.061	3.5	6.5	22.8	1.226	58	-31	9.0	-13.5
16	8.0	2.0	16.0	.7071	4.0	6.0	24.0	1.061	100	-67	-33	-33.3
19	9.5	.50	4.75	.3536	4.2	5.8	24.4	.9910	127	-91	-81	-64.3

a: Figures have been rounded up, and in some cases, though Δx is correct, it may not appear so.

B. Central $\mathbb{L}$ values

Table 2 presents the welfare and *NR* results associated with the central value of $\mathbb{L}$ in each one of the three congestion categories, namely $\mathbb{L} = 5.0$ (11.67) 16.67) for LC (HC) (SC). The welfare cost $\Delta U_{SC} = -38.3$ percent or $29.7\Delta U_{LC}$, and $\Delta U_{HC} = 9.8\Delta U_{LC}$. The *NR* cost $\Delta N_{SC} = -72$ percent, or $14.7\Delta N_{LC}$, and $\Delta N_{HC} = 7.0\Delta N_{LC}$.

Table 2 confirms the main result that the negative impact of open access on both welfare and *NR* under SC (HC) are an order of magnitude greater than (a multiple of) that under LC, with the welfare cost under SC (HC) about 30 (10) times that under LC in this case, and a *NR* loss under SC (HC) about 15 (7) times that under LC.

Table 2. Open Access vs. Optimum: Central $\mathbb{L}$ Values [a]

	Open Access (x)		Optimum (x^*)		Difference (%) $\Delta x = \frac{x-x^*}{x^*}$		Ratio $\Delta x/\Delta x_{LC}$	
$\mathbb{L}$	N	U	N^*	U^*	ΔN [b]	ΔU	$\Delta N/\Delta N_{LC}$	$\Delta U/\Delta U_{LC}$
LC: 5.0	7.5	1.369	7.9	1.387	-4.9	-1.28	1	1
HC: 11.67	4.2	1.021	6.4	1.169	-34	-12.7	7.0	9.8
SC: 16.67	1.7	.6455	5.9	1.046	-72	-38.3	14.7	29.7

a: Results are for the central values of $\mathbb{L}$ in each congestion category; *b*: N and N^* figures have been rounded up, and ΔN results may not appear correct but are, e.g., N^* under LC is 7.87 rather than 7.9, so $\Delta N = -4.9\%$.

3.4. Optimal producer tax

The optimal tax, t^*, increases with $\mathbb{L}$, from (in percent), $t^* = 5.4$ for $\mathbb{L} = 1$ to $t^* = 100$ for $\mathbb{L} = 10$ to $t^* = 255$ for $\mathbb{L} = 19$. The fact that t^* rises with $\mathbb{L}$ helps dampen the increased pressure on the *NR* as well as the decline in welfare. This can be seen from Table 1 where the decline in N^* (U^*) as $\mathbb{L}$ increases from 1 to 19 is less than half the decline in $N(U)$. The solution for t^* and further simulation results are available upon request.

4. Robustness

I examine the results obtained by using alternative parameter values for production and utility functions, and alternative functional forms. All the results are derived in Appendix 6.

4.1. Alternative parameter values

First, I compare ΔU and ΔN under SC and LC for $(\alpha, \beta) = (6, 1)$ and $(\alpha, \beta) = (2, 1)$. Results shown in Appendix Table 2A (Section 6.1) are: ΔU_{SC} is greater than ΔU_{LC} by one to two orders of magnitude, and ΔN_{SC} is greater than ΔN_{LC} by a multiple to two orders of magnitude.

Second, a general form of equation (2) is $U = q^{\gamma}m^{1-\gamma}$, $\gamma \epsilon (0, 1)$. The USDA reports a share of food in 2014 household expenditures in countries like China, India, Mexico, Russia and South Africa of 20 to 30 percent. Hence, I select a value $\gamma = .25$, with (as before) $\alpha = 10$ and $\beta = 1$. I obtain similar results to those for $\gamma = .5$ in Section 3.3. For instance, I find that $\Delta U_{19} = 641\Delta U_1$ and $\Delta N_{19} = 218\Delta N_1$. Thus, welfare and *NR* costs of open access for $\mathbb{L} = 19$ are two orders of magnitude larger than for $\mathbb{L} = 1$ in this case.[11]

4.2. Alternative functional forms

Two alternative utility functions and production functions are examined below.

4.2.1 Utility functions

The first one is the constant-relative-risk-aversion function $U(x) = \frac{x^{1-\mu}}{1-\mu}$, $\mu \neq 1$. Under separability and $\mu = 1/2$, we have $U(m, q) = U(m) + U(q) = \frac{m^{1/2}}{1/2} + \frac{q^{1/2}}{1/2}$. Section 6.2.1 of Appendix 6 presents the solution and simulations.

I find for both $(\alpha, \beta) = (6, 1)$ and $(\alpha, \beta) = (4, 1)$ that ΔU_{SC} (ΔN_{SC}) is greater than ΔU_{LC} (ΔN_{LC}) by between a multiple and two (one) order/s of magnitude (see Panels A and B in Appendix Table A3). For $\mathbb{L}$'s central values (not shown), both ΔU_{SC} and ΔN_{SC} are an order of magnitude greater than under LC.

The second utility function is $U = \left(m - \frac{m^2}{2}\right) + \left(q - \frac{q^2}{2}\right)$. The results, which are shown in Section 6.2.2 in Appendix 6, are similar to those above.

[11] For $\gamma = .25$, $L = \mathbb{L}/4$, and $\mathbb{L} = 19$ means $L = 4.75 < \hat{L} = 5$, which is a LC equilibrium (see point a' on D_L' in Fig. 1). For $\gamma = .5$, $L = \frac{\mathbb{L}}{2}$ and $\mathbb{L} = 19$ means $L = 9.5 > L_I = \frac{2\alpha}{3\beta} = 6.67$ (see Table 1), a SC equilibrium (point A' in Fig.1). Thus, congestion increases from LC to SC as the income share spent on the *NR*-based commodity doubles.

4.2.2. Production functions

Assume $Q = L[\alpha - \beta(\log L)]$, $L > 1$. Section 6.3.1 in Appendix 6, shows that, for each congestion category's central value of $\mathbb{L}$, the relative welfare cost $\Delta U_{SC} = 11.1\Delta U_{LC}$ and $\Delta U_{HC} = 4.1\Delta U_{LC}$. Thus, ΔU_{SC} (ΔU_{HC}) is greater by an order of magnitude than (a multiple of) ΔU_{LC}.

The second production function is $Q_\varepsilon = \varepsilon L(\alpha - \beta L) = \varepsilon Q$, $\varepsilon > 1$, where ε is TFP. Section 6.3.1, Appendix 6 shows results for Δx ($x = L, N, Q, U$) are identical to those obtained with equation (1).

Thus, Sections 3 and 4 showed that welfare and *NR* costs of open access under SC (HC) are greater by between an order of magnitude (a multiple of) and two orders of magnitude than under LC and are robust under alternative parameter values and alternative production and preference functions.

5. Policy Implications

Given the significantly greater welfare cost of open access under super congestion (SC), it follows that in countries where SC prevails, regulating the use of the *NR* – e.g., through an optimal producer tax – would generate gains that are massively larger than found in analyses that deal with low-congestion (LC) cases. A similar, though somewhat weaker, conclusion holds for high congestion (HC).

Two issues arise in this context, namely political economy and implementation constraints.

1. Regarding political economy constraints, producers naturally favor an increase in the price of their natural-resource-based products – say due to a rise in population – while the government might also favor a higher price due to its positive impact on employment. Thus, it might not be inclined to impose an optimal tax or strict regulation on the sector. However, governments should be aware that the need for a sound, enforceable regulatory framework increases with price because it results in a decline in both the *NR* stock and welfare under open access, a decline that is

particularly large under HC and even more so under SC, compared to that under LC. Moreover, the tax-inclusive consumer price declines with the optimal tax under SC, so that the government might benefit politically for imposing a tax on basic food products in the case of super congestion.

2. A producer tax may be hard to levy because of administrative, logistical, enforcement and other problems, especially in the case of developing countries.[12] Other regulations – as found, for instance, in developed countries such as Norway and Scotland in the case of farm fishing [13] – designed to minimize these externalities are also likely to be needed, including the number and geographic distribution of licenses, selection of qualified applications in accordance with prioritization criteria, reporting requirements on the impact on the *NR*, etc.

Thus, an increase in demand associated with an increase in population size raises the importance of a sound, enforceable regulatory framework in order to ensure that the natural resource and welfare losses are limited.

6. Concluding Comments

This paper examined the potential impact on output, variable input use, natural resource (*NR*) and welfare in the case of an industry that is based on the exploitation of an open-access renewable *NR*. This issue is of great importance for a number of developing countries, particularly those

[12] This may be especially important in remote areas – e.g., in the case of farm fishing in remote villages with small fish farms, where it is difficult to ascertain the importance of these externalities and/or collect the tax.

[13] Before giving a license, Norwegian authorities must assess the risk of disease spread in an aquaculture facility and surrounding environment (such as distance to watercourses and other aquaculture facilities, type of species to be produced, farming system and production volume). And license proposals must be made public by local authorities in the municipality where the farm is to be located and must be published in two local papers, so the local population can react to the proposal. Moreover, an applicant must obtain a waste discharge permit to obtain a license and provide monthly reports on the farm's operation and on its impact. This is certainly not the case in, say, Chile or the Philippines, which suffer from a number of problems (see Appendix 1).

characterized by high congestion due to low *NR* endowment, high demand or both, and where the equilibrium is located on the backward-bending segment of the AC curve where SC and HC prevail.

The analysis compared outcomes in the case of an open-access resource and an optimally regulated one, and showed that:

- The welfare and *NR* cost of open access under high (HC) and super congestion (SC) is between a multiple and orders of magnitude larger than under low congestion (LC), is significantly larger under SC than under HC, and results in a massive waste of *NR* and labor.

- The optimal tax increases with the labor force or population size and with the negative externality, and decreases with the *NR* endowment level.

- The optimal tax raises price and reduces output under LC and HC. However, it reduces price and raises output under SC, generating much larger benefits.

- The results are robust to alternative parameter values and functional forms for production and preferences, raising confidence in the results and policy analysis.

Thus, the fact that countries' *NR* congestion might belong to the HC category, and especially to the SC one, raises the importance of sound regulation of sectors that are based on the exploitation of a renewable natural resource.

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Appendix

1. Negative Externalities: Aquaculture in Chile and the Philippines

Chile: Open access and lack of regulation led to excessively high density of salmon pens, high pollution levels and low productivity, and a disease outbreak in 2008 that spread rapidly across the industry destroyed two thirds of the sector's output through 2009 and 2010. This led to regulatory changes, though frequent problems have continued (Anderson 2012). Another issue is toxic algae bloom.[14] Chile's 2015 algae bloom reduced salmon output by 16 percent and 2016 production forecasts by 20 percent (Bajak 2016; Guardian, March 6, 2016), with the problem persisting several years later.

[14] High nutrient levels under high pen and fish stock density lead to high algae growth and exhausts nutrients. Decaying algae i) block the sun and deplete water's oxygen, suffocating the fish; and ii) is toxic and poisons the fish.

Philippines: High cage and stocking density in tilapia farms led to high feeding rates, very high ammonia and nitrogen levels and low oxygen level, mass fish mortality and chronic disease of surviving fish in Lake Taal (Yambot 2000). The same occurred in Luzon and Mindanao's coastal waters and milkfish in 2002 in Bolinao (Talaue-McManus 2006; San Diego-McGlone et al. 2008). Algae bloom badly affected tilapia and milkfish in Laguna de Bay, the Philippines' largest lake (ADB 1989).

2. L_I is the inflection point, where $AC_1 = MC_2$

2.1. $AC_1 = MC_2$

Denote the backward-bending segment of the AC curve by AC_1. The region SC (HC) is the upper (lower) part of AC_1 where optimal output $Q^* > (<)\ Q$, the open-access output. The border between SC and HC consists of a point where $Q = Q^*$, i.e., where AC_1 intersects MC_2, the positive segment of the MC curve, where $L = L_I$. Thus, L_I is the level of L where $AC_1 = MC_2$, or $\frac{1}{\alpha-\beta L_1} = \frac{1}{\alpha-2\beta L_2}$. Thus, $L_1 = 2L_2$ at output, Q_I, where $AC_1 = MC_2$.

Any output can be produced with a high level of labor and a low level of *NR*, or vice versa, with a low level of labor and a high level of *NR*. Thus, $Q = L_1(\alpha - \beta L_1) = L_2(\alpha - \beta L_2)$, which implies $\beta L_1^2 - \alpha L_1 + (\alpha L_2 - \beta L_2^2) = 0$. The solution is $L_1 = \frac{1}{2\beta}\left[\alpha \pm \sqrt{\alpha^2 - 4\beta(\alpha L_2 - \beta L_2^2)}\right] = \frac{1}{2\beta}[\alpha \pm (\alpha - 2\beta L_2)]$. Thus, under the solution $L_1 = \frac{1}{2\beta}[\alpha + (\alpha - 2\beta L_2)]$, we have $L_1 = \frac{\alpha}{\beta} - L_2$.[15] With $L_1 = 2L_2$ at Q_I, we have $L_2 = \frac{\alpha}{3\beta}$ and $L_1 = \frac{2\alpha}{3\beta}$. As L_I is on AC_1, it follows that $L_I > \hat{L} = \frac{\alpha}{2\beta}$. Thus, $L_I = L_1 = \frac{2\alpha}{3\beta}$, $N_I = \alpha - \beta L_I = \frac{\alpha}{3}$ and $Q_I = \frac{2\alpha^2}{9\beta}$, where L_I (Q_I) is the value of $L(Q)$ that separates

[15] The second solution, namely $L_1 = \frac{1}{2\beta}[\alpha - (\alpha - 2\beta L_2)]$, implies that $L_1 = L_2$, i.e., $L_1 = L_2 = \frac{\alpha}{2\beta} = \hat{L}$.

SC from HC. Note that Q_I can also be produced with half the employment and double the *NR* ($L_2 = \frac{L_1}{2} = \frac{\alpha}{3\beta}$, $N_2 = 2N_1 = \alpha - \beta L_2 = \frac{2\alpha}{3}$).

2.2. Inflection point

L_{I}, which separates HC and SC, is also the inflection point where the change in AC's slope, $AC' = \frac{\partial AC}{\partial Q}$, changes sign, i.e., where AC's second derivative, $AC'' = \frac{\partial^2 AC}{\partial Q^2}$, is zero, and switches from $AC'' > 0$ (under SC) to $AC'' < 0$ (under HC), with $AC'' = 0$ at L_{I}. As $AC = \frac{1}{\alpha - \beta L}$, $AC' \equiv \frac{\partial AC}{\partial Q} = \frac{\partial AC}{\partial L} / \frac{\partial Q}{\partial L} = \frac{\beta}{(\alpha - \beta L)^2(\alpha - 2\beta L)}$. Thus, $AC' \gtreqless 0 \Leftrightarrow L \lesseqgtr \hat{L} = \frac{\alpha}{2\beta}$ ($L > \hat{L}$ implies L must fall for Q to rise). The change in AC's slope, $AC'' \equiv \frac{\partial^2 AC}{\partial Q^2} = \frac{\partial AC'}{\partial L} / \frac{\partial Q}{\partial L} = \frac{2\beta^2(2\alpha - 3\beta L)}{(\alpha - \beta L)^3(\alpha - 2\beta L)^2}$. Thus, $AC'' \gtreqless 0 \Leftrightarrow L \lesseqgtr L_{\mathrm{I}} = \frac{2\alpha}{3\beta}$. QED.

3. At any given output Q, $MC_1 = -MC_2$

Optimal MC is $MC_2 = \frac{1}{\alpha - 2\beta L_2} > 0$, $L_2 < \hat{L}$. At $L_1 > \hat{L}$, $MC_1 = \frac{1}{\alpha - 2\beta L_1} < 0$. As shown in Section 2.1 of Appendix 2, $L_1 = \frac{\alpha}{\beta} - L_2$ for any Q. Thus, $\alpha - 2\beta L_1 = \alpha - 2\beta\left(\frac{\alpha}{\beta} - L_2\right) = -\alpha + 2\beta L_2 = -(\alpha - 2\beta L_2)$. Thus, $MC_1 = -\frac{1}{\alpha - 2\beta L_2} = -MC_2$, i.e., MC_1 is the mirror image of MC_2. QED.

4. Range of L values under LC, HC and SC

L under LC ranges from $L = 0$ to $\hat{L} = \frac{\alpha}{2\beta}$. Under HC, L ranges from L_I to $\hat{L}$, with $L_I - \hat{L} = \frac{2\alpha}{3\beta} - \frac{\alpha}{2\beta} = \frac{\alpha}{6\beta}$, i.e., the range of L-values under HC is one third that under LC. The range under SC is $\frac{\alpha}{\beta} - \frac{2\alpha}{3\beta} = \frac{\alpha}{3\beta}$, or twice that under HC and two thirds that of values under LC. Thus, one half (third) (sixth) of the L-values are in the LC (SC) (HC) region.

5. Proofs and solutions: *i*) Stability of equilibrium; *ii*) $L > L^*$; *iii*) Sign in L^* is negative ; *iii) ΔU* and ΔN depend on α/β; *iv*) Solution for U^*; v) Optimal producer tax

i) The open-access equilibrium is unique, so stability is both local *and* global. Stability is not an issue for equilibrium on the upward-sloping part of the AC curve. However, it might be an issue for equilibria on the backward-bending part of the AC curve, AC_1, where both the AC and demand curves are negatively sloped. It requires excess-demand to prevail below the equilibrium price and excess-supply above it, i.e., that demand be more elastic than supply.

The elasticity of demand, η, under a Cobb-Douglas utility function is $\eta = -1$. The elasticity of supply is $\varepsilon_{Q.AC} = \frac{\partial Q}{\partial AC} * \frac{AC}{Q}$, where $\frac{\partial Q}{\partial AC} = \frac{\partial Q/\partial L}{\partial AC/\partial L}$. With $\frac{\partial Q}{\partial L} = \alpha - 2\beta L$, and $AC = \frac{1}{\alpha - \beta L}$ implying $\frac{\partial AC}{\partial L} = \frac{\beta}{(\alpha-\beta L)^2}$, we have $\frac{\partial Q}{\partial AC} = \frac{(\alpha-2\beta L)(\alpha-\beta L)^2}{\beta}$. And with $\frac{AC}{Q} = \frac{1}{L(\alpha-\beta L)^2}$, we have $\varepsilon_{Q.AC} = \frac{\alpha-2\beta L}{\beta L}$, with $\varepsilon_{Q.AC} < 0 \Leftrightarrow L > \hat{L} = \frac{\alpha}{2\beta}$. Note that $\varepsilon_{Q.AC} = -1$ implies $L = \frac{\alpha}{\beta}$, $N = \alpha - \beta L = 0$ and $Q = LN = 0$, which cannot hold under a Cobb-Douglas utility function. Thus, employment $L < \frac{\alpha}{\beta}$ on AC_1. And with $-\frac{\partial \varepsilon_{Q.AC}}{\partial L} > 0$, we have $\eta < \varepsilon_{Q.AC} < 0$, i.e., demand is more elastic than supply, and the equilibrium is stable. QED.

ii) Assume instead that $L < L^*$. Thus, $L = \frac{\mathbb{L}}{2} < L^* = \frac{1}{3\beta}\left[\alpha + \beta\mathbb{L} - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}\,\right]$, or $\frac{3\beta\mathbb{L}}{2} < \alpha + \beta\mathbb{L} - \sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}}$, or $\sqrt{\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L}} < \alpha - \frac{\beta\mathbb{L}}{2} = N$, i.e., $\alpha^2 + \beta^2\mathbb{L}^2 - \alpha\beta\mathbb{L} < \alpha^2 + \frac{\beta^2\mathbb{L}^2}{4} - \alpha\beta\mathbb{L}$, or $\frac{3\beta^2\mathbb{L}^2}{4} < 0$, which is false. Thus, $L > L^*$. QED.

iii) Equation (4) is $L^* = \frac{1}{3\beta}\left(\alpha + \beta\mathbb{L} - \sqrt{(\alpha+\beta\mathbb{L})^2 - 3\alpha\beta\mathbb{L}}\,\right)$, and the sign in front of the square root must be negative. Assume the sign is positive. Since $\sqrt{(\alpha+\beta\mathbb{L})^2 - 3\alpha\beta\mathbb{L}} =$

$\sqrt{(\alpha-\beta\mathbb{L})^2+\alpha\beta\mathbb{L}} > \alpha-\beta\mathbb{L}$, we have $L^* > \frac{1}{3\beta}[(\alpha+\beta\mathbb{L})+(\alpha-\beta\mathbb{L})] = \frac{2\alpha}{3\beta} = L_{\mathrm{I}}$. However, this cannot be a solution as L^*, the optimal value of L, must be located in the LC segment of the AC curve where $MP_L > 0$, i.e., $L^* < \hat{L} = \frac{\alpha}{2\beta} < L_{\mathrm{I}} = \frac{2\alpha}{3\beta}$. Consequently, the sign in front of the square root must be negative. QED.

iv) Equation (4) can be rewritten as $L^* = \frac{1}{3}\left[1+\frac{\beta}{\alpha}\mathbb{L}-\sqrt{(1+\frac{\beta}{\alpha}\mathbb{L})^2-3\frac{\beta}{\alpha}\mathbb{L}}\right]$, i.e., L^* depends on α/β, and not on α and β individually, and so do ΔN and ΔU (as well as ΔL and ΔQ). Multiplying α and β by $\lambda > 0$ does not affect L^* or $M^* = \mathbb{L}-L^*$, or $m^* = M^*/\mathbb{L}$. And $Q_\lambda^* = L^*(\lambda\alpha-\lambda\beta L^*) = \lambda L^*(\alpha-\beta L^*) = \lambda Q^*$, with $q_\lambda^* = \lambda q^*$ and $U_\lambda^* = \lambda^{.5}U^*$. Also, as $L = \mathbb{L}/2$ is independent of α and β, so are M and m, with $q_\lambda = \lambda q$ and $U_\lambda = \lambda^{.5}U$. Thus, $\Delta U_\lambda = (U_\lambda - U_\lambda^*)/U_\lambda^* = \Delta U, \forall\,\lambda > 0$. Since $N_\lambda^* = \lambda N^*$ and $N_\lambda = \lambda N$, it follows that $\Delta N_\lambda = \Delta N$, $\forall\,\lambda > 0$. QED.

v) Substituting the solution for L^* in (4) in production and utility functions (1) and (2), the solution for optimal utility, U^*, is:

$$U^* = \frac{1}{3\beta\mathbb{L}}\left[\beta\mathbb{L}(\alpha^2+\alpha\beta\mathbb{L}+2\beta^2\mathbb{L}^2)+\frac{2}{3}(\alpha^2+\beta^2\mathbb{L}^2-\alpha\beta\mathbb{L})^{\frac{3}{2}}-\frac{2}{3}\alpha^3\right]^{1/2}. \qquad (1A)$$

6. Robustness simulations

This section provides derivations, simulations and detailed descriptions of the results provided in Section 3.4, which examines the robustness of results in Section 3.3 by using different values for parameters of the production and utility functions, as well as different functional forms for both.

6.1. Two sets of values for α and β parameters

The case of $(\alpha,\beta) = (6,1)$ is examined in Table 2A, Panel A, where $\mathbb{L} < 12$, and $(\alpha,\beta) = (2,1)$ in Panel B, where $\mathbb{L} < 4$. In Panel A, ΔU_{11} (ΔN_{11}) is equal to -55 (-86) percent or 162 (210) times $\Delta U_1(\Delta N_1)$, 107 (45) times ΔU_2 (ΔN_2), 53 (18) times ΔU_3 (ΔN_3), 8.3 (3.4) times ΔU_6 (ΔN_6) and 2.0 (1.5) times ΔU_9 (ΔN_9).

As $\mathbb{L} \leq 2$ are LC, and $\mathbb{L} \geq 5$ are SC, the welfare and *NR* losses are between two orders of magnitude and a multiple of t hose under LC. In Panel B, ΔU_3 (ΔN_3) is 21 (12) times ΔU_1 (ΔN_1), the losses of welfare and *NR* under SC are an order of magnitude larger than under LC.

Thus, the results are similar to those obtained in the main text: both ΔU and ΔN under SC and HC are between a multiple and two orders of magnitude larger than under LC.

Table 2A. Open Access vs. Optimum

A: $\alpha = 6, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	5.5	2.8	1.173	.49	5.5	2.6	1.177	2.7	-.41	2.5	-.339
2	1.0	5.0	5.0	1.118	.90	5.1	4.6	1.124	11	-1.9	8.7	-.516
3	1.5	4.5	6.8	1.061	1.3	4.7	6.0	1.072	19	-4.9	13.3	-1.04
5	2.5	3.5	8.8	.935	1.8	4.2	7.6	.984	39	-16	15.1.	-4.91
6	3.0	3.0	9.0	.866	2.0	4.0	8.0	.928	50	-25	12.5	-6.65
9	4.5	1.5	6.8	.612	2.4	3.6	8.6	.839	91	-59	-21.4	-27.0
11	5.5	.50	2.8	.350	2.5	3.5	8.8	.784	112	-86	-69.0	-54.9

B: $\alpha = 4, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.50	1.5	.75	.6124	.42	1.6	.67	.6204	18	-4.9	13	-1.3
3	1.5	.50	.75	.3536	.78	1.2	.95	.4845	91	-59	-21	-27

6.2. Two alternative utility functions

6.2.1. Constant relative-risk-aversion utility function

The utility function is $U(x) = \frac{x^{1-\mu}}{1-\mu}$ $(\mu \neq 1)$. Assuming separability, $U(m,q) = U(m) + U(q) = \frac{m^{1-\mu}}{1-\mu} + \frac{q^{1-\mu}}{1-\mu}$. With $\mu = 1/2$, we have:

$$U = \frac{m^{1/2}}{1/2} + \frac{q^{1/2}}{1/2}. \tag{3A}$$

Maximizing utility implies that the ratio of marginal utilities equals the relative price, i.e., $p = \left(\frac{m}{q}\right)^{1/2} = \left(\frac{M}{Q}\right)^{1/2} = \left[\frac{\mathbb{L}-L}{L(\alpha-\beta L)}\right]^{1/2}$. Under open access, $p = AC = \frac{1}{\alpha-\beta L}$. The two equations imply $\beta L^2 - (1+\alpha+\beta\mathbb{L})L + \alpha\mathbb{L} = 0$. The solution is:

$$L = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L} - \sqrt{(1+\alpha+\beta\mathbb{L})^2 - 4\alpha\beta\mathbb{L}}\right).^{16} \tag{4A}$$

At the optimum, $p = \left[\frac{\mathbb{L}-L}{L(\alpha-\beta L)}\right]^{1/2} = MC = \frac{1}{\alpha-2\beta L}$, or $\frac{\mathbb{L}-L}{L(\alpha-\beta L)} = \frac{1}{(\alpha-2\beta L)^2}$, which is rewritten as:

$$4\beta^2L^3 - \beta(1+4\alpha+4\beta\mathbb{L})L^2 + \alpha(1+\alpha+4\beta\mathbb{L})L - \alpha^2\mathbb{L} = 0. \tag{5A}$$

Simulation results are presented in Table 3A's Panel A (B) for $(\alpha,\beta) = (6,1)$ $\big((4,1)\big)$. In Panel A, $\alpha = 6$. LC prevails for $\mathbb{L} = 1$ (3) (5), and the welfare impact of open access (in percent) is $\Delta U_{L1} = -.19, \Delta U_{L2} = -3.4, \Delta U_{L3} = -9$. The *NR* impact under LC is $\Delta U_{N1} = -.93, \Delta U_{N2} = -14.9$, and $\Delta U_{N3} = -36.4$. SC prevails for $\mathbb{L} = 10$ and $\mathbb{L} = 50$. For $\mathbb{L} = 10$, $\Delta U_{S1} = -21.5 =$

[16] The solution with a positive sign before the square root is $L = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L}+\sqrt{(1+\alpha+\beta\mathbb{L})^2-4\alpha\beta\mathbb{L}}\right) = \frac{1}{2\beta}\left(1+\alpha+\beta\mathbb{L}+\sqrt{(1+\alpha-\beta\mathbb{L})^2+4\beta\mathbb{L}}\right)$. As $\sqrt{(1+\alpha-\beta\mathbb{L})^2+4\beta\mathbb{L}} > (1+\alpha-\beta\mathbb{L})$, we have $L > \frac{1}{\beta}(1+\alpha)$, i.e., $\alpha - \beta L < -1$, which is not possible as $N = \alpha - \beta L \geq 0$. Thus, the sign before the square root must be negative.

$113\Delta U_{L1} = 12.1\Delta U_{L2}$, and $\Delta N_{S1} = -71.7 = 77\Delta N_{L1} = 9.1\Delta N_{L2}$. For $\mathbb{L} = 50$, $\Delta U_{S2} = -23.7 = 125\Delta U_{L1} = 13.3\Delta U_{L2}$ and $\Delta N_{S2} = -95.9 = 103\Delta N_{L1} = 12.1\Delta N_{L2}$.

With $\mathbb{L} = 5$ (10) (50), L is 71 (103) (111) percent greater than the optimum, with an impact on output of $-5(-43)(-91)$ percent and an impact on *NR* of $-51(-77)(-96)$ percent, amounting to a massive waste of both human and natural resources.

Table 3A. Autarky: Open Access vs. Optimum

Panel A: $\alpha = 6, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.84	5.2	4.3	5.0	.79	5.2	4.1	5.01	6.1	-.93	5.1	-.19
3	2.4	3.6	8.6	4.3	1.8	4.2	7.5	4.45	33	-15	15	-3.4
5	3.6	2.4	8.6	3.7	2.1	3.9	8.3	4.1	71	-36	-5.1	-9.0
10	5.0	1.0	5.0	2.8	2.5	3.5	8.7	3.6	103	-71	-43	-21
50	5.9	.13	.78	2.1	2.8	3.2	9.0	2.8	111	-96	-91	-24

Panel B: $\alpha = 4, \beta = 1$

$\mathbb{L}$	Open Access				Optimum				Difference: $\frac{x-x^*}{x^*}$ (in %)			
	L	N	Q	U	L^*	N^*	Q^*	U^*	ΔL	ΔN	ΔQ	ΔU
1	.76	3.2	2.5	4.1	.68	3.3	2.3	4.1	13	-2.6	9.8	-.46
3	2.0	2.0	4.0	3.5	1.3	2.7	3.5	3.7	55	-26	14	-5.6
5	2.8	1.2	3.4	3.0	1.5	2.5	3.7	3.7	86	-51	-8.5	-12
10	3.5	.53	1.8	2.5	1.7	2.3	3.9	3.1	109	-77	-53	-19
50	3.9	.08	.33	2.1	1.9	2.1	4.0	2.6	112	-96	-92	-20

In panel B, $\alpha = 4$ and $\beta = 1$, LC (HC) (SC) prevail for $\mathbb{L} < 4$ $(4 < \mathbb{L} < 5.33)$ $(\mathbb{L} > 5.33)$. In percent, at $\mathbb{L} = 1$, $\Delta U_{L1} = -.46$ and $\Delta N_{L1} = -2.6$. At $\mathbb{L} = 3$, $\Delta U_{L2} = -5.6$ and $\Delta N_{L2} = -26.2$. The LC average is $\Delta U_{LC} = -3$ and $\Delta N_{LC} = -14.3$.

At $\mathbb{L} = 5$ (HC case), $\Delta U_{HC} = -12.3 = 25.5\Delta U_{L1} = 4.1\Delta U_{LC}$, $\Delta N_{HC} = -50.9 = 19.6\Delta N_{L1} = 3.6\Delta N_{LC}$. And for $\mathbb{L} = 10$ (SC), $\Delta U_{SC} = -19.3 = 42\Delta U_{L1} = 6.4\Delta U_{LC}$, and $\Delta N_{SC} = 78.0 = 30\Delta N_{L1} = 5.4\Delta N_{LC}$. At $\mathbb{L} = 50$, $\Delta U_{S2} = -20.2 = 44\Delta U_{L1} = 6.7\Delta U_{LC}$, and $\Delta N_{S2} = -96.8 = 37\Delta N_{L1} = 6.7\Delta N_{LC}$.

As with utility function (2), welfare and *NR* losses under SC are of a greater order of magnitude than, or a multiple of, those under LC.

6.2.2. <u>Quadratic utility function</u>

$$U = \left(m - \frac{m^2}{2}\right) + \left(q - \frac{q^2}{2}\right). \tag{6A}$$

Utility maximization implies that $p = \frac{U_q}{U_m} = \frac{1-q}{1-m}$; $m, q \in (0, 1)$. With $M = l = \mathbb{L} - L$, we have $m = 1 - \frac{L}{\mathbb{L}}$, and $1 - m = \frac{L}{\mathbb{L}}$. Thus, $p = \frac{1-q}{L/\mathbb{L}} = \frac{(\mathbb{L}-Q)}{L} = \frac{\mathbb{L}}{L} - (\alpha - \beta L)$.

<u>Open Access</u>:

As $p = AC = \frac{1}{\alpha-\beta L}$, we have $\frac{\mathbb{L}}{L} - (\alpha - \beta L) = \frac{1}{\alpha-\beta L}$, a cubic equation in L, namely:

$$\beta^2 L^3 - 2\alpha\beta L^2 + (1 + \alpha^2 + \beta\mathbb{L})L - \alpha\mathbb{L} = 0. \tag{7A}$$

<u>Optimum</u>:

At the optimum, price $p = MC$, i.e., $\frac{\mathbb{L}}{L} - (\alpha - \beta L) = \frac{1}{\alpha-2\beta L}$. Thus, we have:

$$2\beta^2 L^3 - 3\alpha\beta L^2 + (1 + \alpha^2 + 2\beta\mathbb{L})L - \alpha\mathbb{L} = 0. \tag{8A}$$

Under open access, for $\alpha = 2$ and $\beta = 1$, we have from (6A): $L^3 - 4L^2 + (5 + \mathbb{L})L - \mathbb{L} = 0$.

For $\mathbb{L} = 1$, a LC case, the solution is $L_L = .4563$, $N_L = 1.544$, $m_L = .544$, and $m_L - \frac{{m_L}^2}{2} = .3961$. Also, $q_L = .704$ and $q_L - \frac{q_L^2}{2} = .4561$. Thus, $U_{LC} = .8522$.

For the optimum, we have $2L^3 - 6L^2 + 7L - 2 = 0$, with $L_{LC}^* = .410$, $N_{LC}^* = 1.590$, $q_{LC}^* = .652$, $q_{LC}^* - \frac{(q_L^*)^2}{2} = .4393$; $M_{LC}^* = m_{LC}^* = .590$, $m_{LC}^* - \frac{(m_L^*)^2}{2} = .4158$ and $U_{LC}^* = .8555$. Thus, the welfare impact of open access (in percent) is $\Delta U_{LC} = -.375$, and $\Delta N_{LC} = -2.90$.

For $\mathbb{L} = 5$, a SC case, under open access, $L_{SC} = 1.629$, $N_{SC} = .371$, $Q_{SC} = .604$, $q_{SC} = .121$, $M_{SC} = 3.371$, $m_{SC} = .674$, and $U_{SC} = .560$. At the optimum, $L_{SC}^* = .356$, $N_{SC}^* = 1.644$, $Q_{SC}^* = .585$, $M_{SC}^* = 4.644$, and $U_{SC}^* = .608 = U_{SC}$, with (in percent) $\Delta U_{SC} = -7.77$ percent or $20.7\Delta U_{LC}$, and $\Delta N_{SC} = -77.5$ or $26.7\Delta N_{LC}$. Thus, the welfare (*NR*) cost under $\mathbb{L} = 5$ is over 20 (26) times that under $\mathbb{L} = 1$.

For $\mathbb{L} = 10$, also a SC case, $L_{S2} = 1.8$, $N_{S2} = .2$, $U_{S2} = .519$, $L_{S2}^* = .95$, $N_{S2}^* = 1.05$, $U_{S2}^* = .590$ and, in percent, $\Delta U_{SC2} = -12.1 = 32.1\Delta U_{LC}$, and $\Delta N_{SC2} = -81 = 28\Delta N_{LC}$.

Thus, welfare and *NR* losses under SC are an order of magnitude greater than under LC.

6.3. Two alternative production functions

Robustness of results is examined here under two alternative production functions.

6.3.1. Externality as function of log*L*

Assume now that the production function is

$$Q = L[\alpha - \beta(\log L)],\ L > 1. \tag{9A}$$

Under open access, $L = \mathbb{L}/2$, with $U = \frac{1}{2}\left[\alpha - \beta\left(\log\frac{\mathbb{L}}{2}\right)\right]^{1/2}$. The optimal value of L is

$$L^* = \frac{\mathbb{L}}{2}\left[1 - \frac{\beta}{2\alpha - \beta(1 + 2\log L^*)}\right].^{17} \tag{10A}$$

The welfare cost for the central value of $\mathbb{L}$ under LC, HC and SC is (in percent) $\Delta U_{LC} = -4.3$, $\Delta U_{HC} = -17.7$, and $\Delta U_{SC} = -47.8$, i.e., $\Delta U_{SC} = 11.1\Delta U_{LC}$ and $\Delta U_{HC} = 4.1\Delta U_{LC}$. Thus, the welfare cost for $\mathbb{L}$'s central value under SC (HC) is greater by an order of magnitude than (a multiple of) that under LC.

6.3.2. Higher productivity (TFP) level

The second production function is $Q_\varepsilon = \varepsilon L(\alpha - \beta L) = \varepsilon Q$, $\varepsilon > 1$ is TFP. Denote $\frac{p}{\varepsilon}$ by p_ε and $\frac{AC}{\varepsilon}$ by AC_ε. It is easy to see that L and N are independent of the value of ε as equilibria $p = AC = \frac{U_q}{U_m} = \frac{M}{Q}$ and $p_\varepsilon = AC_\varepsilon = \frac{U_{q\varepsilon}}{U_m} = \frac{M}{Q_\varepsilon}$ obtain for the same value of L, and thus the same value of $N = \alpha - \beta L$. The same holds for optimal values $p^* = MC = \frac{M}{Q}$ and $p_\varepsilon^* = MC_\varepsilon = \frac{M}{Q_\varepsilon}$. As $N = N_\varepsilon$ and $N^* = N_\varepsilon^*$, it follows that $\Delta N = \Delta N_\varepsilon$. And as $U_\varepsilon = \varepsilon^{1/2}U$ and $U_\varepsilon^* = \varepsilon^{1/2}U^*$, it follows that $\Delta U = \Delta U_\varepsilon$. In other words, all the results obtained in Section 3.3 also hold in this case.

[17] L^* has no closed-form solution (as L^* is a function of $\log L^*$). For each $\mathbb{L}$, L^* was obtained by 'guessing' a level of L^* (denoted by x), using it to obtain $\log L^*$, and using $\log L^*$ in equation (10A) to obtain a solution for L^* (denoted by y), and verifying whether $y = x$. If not, I used a value for L^* between x and y, and repeated the exercise until x and y converged.