

DISCUSSION PAPER SERIES

IZA DP No. 13557

**Rising Concentration and Wage
Inequality**

Guido Matias Cortes
Jeanne Tschopp

JULY 2020

DISCUSSION PAPER SERIES

IZA DP No. 13557

Rising Concentration and Wage Inequality

Guido Matias Cortes

York University and IZA

Jeanne Tschopp

University of Bern

JULY 2020

Any opinions expressed in this paper are those of the author(s) and not those of IZA. Research published in this series may include views on policy, but IZA takes no institutional policy positions. The IZA research network is committed to the IZA Guiding Principles of Research Integrity.

The IZA Institute of Labor Economics is an independent economic research institute that conducts research in labor economics and offers evidence-based policy advice on labor market issues. Supported by the Deutsche Post Foundation, IZA runs the world's largest network of economists, whose research aims to provide answers to the global labor market challenges of our time. Our key objective is to build bridges between academic research, policymakers and society.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Rising Concentration and Wage Inequality*

Wage inequality has risen in many countries over recent decades. At the same time, production has become increasingly concentrated in a small number of firms. In this paper, we show that these two phenomena are linked. Theoretically, we show that shocks that increase concentration will also lead to an increase in wage dispersion between firms. Empirically, we use industry-level data from 14 European countries over the period 1999-2016 and show robust evidence of a positive and statistically significant correlation between concentration and between-firm wage inequality, driven by increases in market shares and wages in high productivity firms.

JEL Classification: J31, L11, E24

Keywords: wage inequality, market power, heterogeneous firms, Europe

Corresponding author:

Guido Matias Cortes
Department of Economics
York University
4700 Keele Street
Toronto, ON M3J 1P3
Canada

E-mail: gmcortes@yorku.ca

* We thank David Autor, Anna Salomons, Max von Ehrlich, seminar participants at the St Louis Fed, and participants at the EALE Conference 2019 and SOLE Conference 2020 for valuable comments and suggestions. We acknowledge financial support from the Social Sciences and Humanities Research Council of Canada. Dennis Ko provided expert research assistance.

1 Introduction

In recent years, two important economic phenomena have received a large amount of attention from academics and policymakers. On the one hand, there has been a strong increase in wage inequality since the 1980s (Juhn et al., 1993; Katz and Autor, 1999; Acemoglu and Autor, 2011). A recent literature has shown that a large fraction of this increase in inequality is driven by increased wage dispersion between firms, rather than within firms (Card et al., 2013; Song et al., 2018; Barth et al., 2016). On the other hand, a separate literature has shown that product markets have become increasingly concentrated, with a smaller number of firms becoming increasingly dominant in many industries (De Loecker et al., 2020; Autor et al., 2017, 2020; Grullon et al., 2019; Kehrig and Vincent, 2018; Azar et al., 2020, 2018; Benmelech et al., 2018; Bajgar et al., 2019). So far, the rise in inequality and the rise in concentration have been studied in isolation.¹ In this paper, we show that the two phenomena are related.

From a theoretical perspective, we show that a shock that increases concentration, such as an increase in consumer price sensitivity (which in turn may be driven by factors such as greater economic integration and the availability of new web technologies), will also lead to increased wage dispersion between firms. Empirically, we use firm-level data aggregated to the 2-digit industry level for 14 European countries over the period 1999–2016 to show that there is a significant positive correlation between inequality and concentration, which is robust to controlling for unobservable factors in a variety of ways.

We motivate our analysis using the heterogeneous firm search and bargaining framework of Helpman et al. (2010). We analyze the implications of an increase in consumer price sensitivity – modeled as an increase in the elasticity of substitution between varieties in consumption, as in Autor et al. (2020).² The model predicts that this type of shock will lead to increased concentration of production in the most productive firms within industries, while at the same time increasing wage inequality between firms within industries.

Intuitively, an increase in price sensitivity shifts consumer demand towards more productive firms, which are able to produce goods at a lower price. These firms will increase their market share, while also increasing both the quantity and the quality of

¹To the best of our knowledge, the only exception is Rinz (2018), who empirically studies the correlation between concentration and inequality at the local labor market level in the U.S.

²The conceptual framework considered by Autor et al. (2020) features a competitive labor market, which does not allow for any type of wage inequality.

the workers that they hire. This will lead to an increase in their average wages (relative to less productive firms), thus increasing between-firm wage inequality.

We test the predictions of the model regarding the link between concentration and inequality using data from the Competitiveness Research Network (CompNet). The dataset provides information on concentration and between-firm wage inequality at the 2-digit industry level for 14 European countries over the period 1999-2016. The dataset also provides information on sales and average wages for firms at different deciles of the productivity distribution within each industry-country-year cell.

Our empirical strategy consists of regressing inequality on concentration as well as various combinations of industry, country, and year fixed effects. This strategy allows us to control for different types of confounding factors, and to exploit different sources of variation for identification in order to determine the robustness of the results. Our key finding is that there is a positive and statistically significant correlation between concentration and inequality, which is robust to various ways of restricting identification. This suggests that, as predicted by the model, the two phenomena are indeed linked to each other.

To further explore the mechanisms highlighted by the model, we explore how employment, sales and wages vary across firms with different productivity levels in industries that become more concentrated. In line with the model, we find that increases in concentration are associated with higher sales in the most productive firms in an industry. These firms also become larger in terms of their total employment, and pay higher average wages to their workers. This evidence provides further support for the plausibility of the idea that the increase in concentration in Europe is driven by an increase in competitive pressures rather than increased barriers to entry.

Our paper provides an important contribution to the literature by providing evidence of the link between the growth of concentration and the growth of wage inequality. Our results highlight the importance of considering common driving forces that can account for both of these patterns. We contribute to the literature that studies the drivers of increased wage inequality by proposing a mechanism which gives a relevant role to firms in the widening of the wage distribution. In spite of the empirical evidence regarding the importance of increased between-firm wage dispersion, the literature has mostly focused on the role of changing demand for skills and tasks without allowing for heterogeneous firms to play a relevant mediating role (see e.g. Acemoglu and Autor, 2011, for a review of this literature).

We also contribute to the literature on concentration by highlighting the importance

of considering the implications for wage inequality. Our findings speak to the debate about the drivers of concentration (see Covarrubias et al., 2019) by providing supportive evidence for the idea that increased competitive pressures have led to the increased dominance of highly productive firms in Europe. This type of concentration is evidently less concerning in terms of efficiency and welfare than if it were driven by unproductive firms becoming entrenched and imposing barriers to entry. In such a case, one might expect that the firms that are becoming more dominant might exploit their market power in order to reduce worker rents, leading to a (relative) reduction in the wages at top firms (see Stansbury and Summers, 2020) and a potential compression of the sectoral wage distribution – a pattern which is at odds with our empirical evidence for Europe.

2 Theoretical Motivation

In order to motivate our analysis of the relationship between concentration and inequality, we illustrate the theoretical link between these two variables within the closed-economy framework of Helpman et al. (2010). Their model introduces Diamond–Mortensen–Pissarides (Diamond, 1982a,b; Mortensen and Pissarides, 1994) search and matching frictions into a Melitz (2003) model with heterogeneous firms. The model is able to generate wage differences between firms through the combination of: (i) search frictions and wage bargaining, and (ii) heterogeneous match-specific ability and the availability of a screening technology.³ We refer the reader to the Helpman et al. (2010) paper for full details on their model. Here, we briefly highlight the key features of the model, and we focus on the implications of their equilibrium conditions for concentration and between-firm wage inequality. In particular, we analyze the impact of an increase in consumer price sensitivity, modeled as an increase in the price elasticity of demand, as in Autor et al. (2020). Autor et al. (2020) discuss how consumers may have become more price-sensitive due to greater product market competition (e.g., through globalization) or new technologies (e.g., due to greater availability of price compar-

³Note that match-specific ability and screening are crucial ingredients to generate wage variation across firms in their model; search frictions and wage bargaining alone are not sufficient (see for instance Felbermayr et al., 2011). Intuitively, in a model without match-specific ability and screening, firms face a common search cost; hence, the additional value created by the marginal worker will be identical across firms in equilibrium. This implies that in a standard search and bargaining model with heterogeneous firms, more productive firms would be larger, but wages would be equated across firms with different productivity levels.

isons on the Internet). While Autor et al. (2020) consider the implications of this type of shock within the setting of a competitive labor market (with no wage inequality), we extend their analysis in order to consider the implications within the context of a framework that allows for wage heterogeneity between firms.

2.1 Key Features of the Helpman et al. (2010) Model

As in Melitz (2003), each sector features a continuum of horizontally differentiated varieties, with total consumption Q being given by a constant elasticity of substitution (CES) aggregate:

$$Q = \left[\int_{j \in J} q(j)^\beta dj \right]^{1/\beta},$$

where j indexes varieties, J is the set of varieties within the sector, $q(j)$ denotes consumption of variety j and $\beta \in (0, 1)$ is a function of the elasticity of substitution between varieties, σ , namely $\beta \equiv (\sigma - 1)/\sigma$.

The product market is characterized by a continuum of monopolistically competitive firms, each producing a unique variety and facing a fixed cost of production. Firm output is given by:

$$y = \theta h^\gamma \bar{a}, \quad 0 < \gamma < 1$$

where θ is the firm's idiosyncratic productivity draw, h is the measure of workers hired and \bar{a} denotes the average match-specific ability of these workers. The productivity distribution, $G(\theta)$, is assumed to be Pareto with shape parameter z .⁴

Workers are ex-ante identical but differ in terms of their match-specific ability, which is not transferable across firms.⁵ Workers' match-specific ability is drawn from a Pareto distribution, $G_a(a) = 1 - (a_{min}/a)^k$. Ability is not directly observable when a firm and a worker match, but firms have access to a screening technology. In particular, by paying a screening cost of ca_c^δ/δ , a firm can identify whether workers are above or below an (endogenously chosen) ability threshold a_c . Neither the firm nor the worker know the match-specific ability of any individual worker, so bargaining occurs under conditions

⁴The assumption of a Pareto distribution is common in the literature on heterogeneous firms. Corcos et al. (2012) find empirical support for this assumption; Axtell (2001) shows that the observed distribution of firm sizes follows a Pareto distribution.

⁵In the Appendix, we discuss an extension of Helpman et al. (2010) to two types of workers that allows us to relate increased concentration and wage inequality to changes in worker sorting on observables. We discuss this extension in further detail below.

of symmetric information.

In equilibrium, more productive firms have incentives to screen more intensively, due to the complementarity between workers' abilities and firm productivity. Hence, more productive firms will hire workers with higher match-specific abilities and will pay higher wages. Intuitively, firms are able to adjust their bargained wage down to the replacement cost of a worker. Since more productive firms hire workers of higher average ability, their workers are costlier to replace, and hence are paid a higher wage. From the perspective of the worker, the expected wage conditional on being sampled is the same across all firms. Under the assumption that $\delta > k$, more productive firms will also be larger.⁶

Equilibrium firm-level revenues, employment and wages are given by:

$$\begin{aligned}
 r(\theta) &= r_d \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}}, & r_d &\equiv \frac{1 + \beta\gamma}{\Gamma} f_d \\
 h(\theta) &= h_d \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}(1-k/\delta)}, & h_d &\equiv \frac{\beta\gamma f_d}{\Gamma b} \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{min}^\delta} \right]^{-k/\delta} \\
 w(\theta) &= w_d \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta k}{\delta\Gamma}}, & w_d &\equiv b \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{min}^\delta} \right]^{k/\delta}
 \end{aligned}$$

where θ_d is the equilibrium productivity threshold below which firms would choose not to operate, b is the search cost, f_d is the fixed cost of production, and $\Gamma \equiv 1 - \beta\gamma - \frac{\beta}{\delta}(1 - \gamma k)$. As in Helpman et al. (2010), it is assumed that $0 < \gamma k < 1$ so that firms have an incentive to screen.

2.2 Concentration and Wage Inequality

In order to measure concentration, consider the set of firms in the top $\mu\%$ of the productivity distribution (among operating firms).⁷ The equilibrium relationships described above imply that the share of sectoral revenues accruing to these firms, and the share

⁶This is in line with the well-studied empirical relationship between firm size and wages (e.g. Oi and Idson, 1999).

⁷Derivation details of the concentration measures and wage distribution can be found in Appendix A.

of employment concentrated in these firms is given, respectively, by:

$$C_r = \mu^{1-\frac{\beta}{\Gamma z}} \quad C_\ell = \mu^{1-\frac{\beta}{\Gamma z}(1-k/\delta)}. \quad (1)$$

Regarding inequality, we are interested in the distribution of wages across firms. The equation for equilibrium wages implies that this distribution is given by:

$$G_f(w) = 1 - \left(\frac{w_d}{w}\right)^{\frac{\delta\Gamma z}{\beta k}} \quad (2)$$

This is a Pareto distribution with scale parameter w_d and shape parameter $\frac{\delta\Gamma z}{\beta k}$. Scale-invariant measures of inequality, such as the coefficient of variation, the Gini coefficient, or the Theil index, are decreasing in the shape parameter and are independent of the scale parameter.

2.3 Effects of an Increase in Consumer Price Sensitivity

As mentioned, we consider the impact of an increase in consumer price sensitivity, modeled as an increase in the elasticity of substitution between varieties, σ (as in Autor et al., 2020). Our two key predictions are the following:

Prediction 1: An increase in consumer price sensitivity increases concentration in terms of revenues and in terms of employment.

Proof: Given the definitions of β and Γ and since $\gamma k \in (0, 1)$, we have that:

$$\frac{\partial\beta}{\partial\sigma} = \frac{1}{\sigma^2} > 0 \quad \text{and} \quad \frac{\partial\Gamma}{\partial\sigma} = -\frac{\partial\beta}{\partial\sigma} \left[\gamma + \frac{1}{\delta}(1 - \gamma k) \right] < 0.$$

It is then straightforward to show that:

$$\frac{\partial C_r}{\partial\sigma} > 0 \quad \text{and} \quad \frac{\partial C_\ell}{\partial\sigma} > 0.$$

Prediction 2: An increase in consumer price sensitivity increases inequality in firm-level wages.

Proof: Recall that the shape parameter of the distribution of firm-level wages is $s \equiv \frac{\delta\Gamma z}{\beta k}$.

Given the definitions of β and Γ , it is straightforward to show that:

$$\frac{\partial s}{\partial \sigma} < 0.$$

A decrease in the shape parameter will unambiguously increase any scale-invariant measure of inequality.⁸

2.4 Intuition

A shock that increases consumers' price-sensitivity will shift consumer demand towards the lower cost varieties produced by higher-productivity firms. To meet increased demand, high-productivity firms increase their output by hiring more workers and screening more intensively, which leads to an increase in both employment and wages relative to less productive firms. As a result, sectoral concentration and wage inequality increase.

Note that the shift in demand will also have implications in terms of the types of firms that remain operative. As demand for the higher cost varieties produced by low productivity firms falls, they are no longer able to operate profitably and must exit. This leads to an increase in the productivity threshold θ_d . Although the increase in the threshold reduces the range of firm types that operate, under the assumption that productivity is Pareto-distributed, this will actually *increase* the variance of productivity among operating firms.⁹ Since employment and wages are proportional to productivity, the exit of unproductive firms will further compound the increase in concentration and inequality arising from changes in employment and wage choices conditional on firm type.

The implications of an increase in consumer price sensitivity for concentration and wage inequality carry over to other heterogeneous firm settings that feature a mechanism that links worker wages to firm rents. For example, in the fair-wage framework of Egger and Kreckemeier (2012), workers adjust their effort according to whether they perceive

⁸It is worth noting that the wage distribution in Equation 2 is obtained when assigning equal weight to all firms, regardless of their size. This is in line with the variable used in our empirical analysis below. The fact that employment becomes more concentrated in high-productivity/high-wage firms implies that employment-weighted measures of between-firm wage inequality would increase even more strongly in response to an increase in consumer price sensitivity.

⁹Intuitively, this occurs because the increase in the threshold is associated with the exit of a mass of firms that are relatively homogenous (at the bottom end of the distribution, where the mass is large given the Pareto assumption), and a relative increase in the mass of firms towards the tail. Detailed derivations of the change in θ_d can be found in Appendix B.

the wage that they receive to be fair. The “fair-wage” is anchored by a firm-external point of reference (workers should consider their wage to be fair relative to the wage of employees at other firms), as well as a firm-internal point of reference (workers should consider their wage to be fair given their own firm’s performance). In equilibrium, it is in the firms’ best interest to pay workers the fair wage in order to elicit the optimal amount of effort. In this framework, an increase in consumer price sensitivity also leads to a relative increase in the demand for the varieties produced by the most productive firms. As in the Helpman et al. (2010) model, this will lead to an increase in the relative size of the most productive firms in terms of employment and sales (increased concentration). The increased profitability of the most productive firms translates into higher relative wages for workers in these firms, due to the fair-wage considerations. Hence, in this model, increases in concentration and increases in (between-firm) wage inequality are also linked.

2.5 Accounting for Sorting on Observables

The empirical literature has shown that there is an important role for changes in worker sorting in accounting for the increase in between-firm wage inequality in several countries (Card et al., 2013; Song et al., 2018).¹⁰ There is also evidence that increased outsourcing opportunities have led to increased establishment specialization and worker segregation (Cortes and Salvatori, 2019).

The model discussed so far only predicts increases in wage dispersion among ex-ante homogeneous workers, without allowing for these empirically-relevant changes in sorting patterns. In Appendix C, we consider the Helpman et al. (2010) extension that features two types of workers (skilled and unskilled). We show that, in this setting, an increase in consumer price sensitivity also leads to increases in both concentration and between-firm wage inequality. The increase in between-firm wage inequality in this case is driven both by: (i) increased sorting of skilled workers to the most productive firms, and (ii) increases in between-firm wage inequality conditional on (observable) worker skill type. These predictions are consistent with the observed increases in between-firm wage inequality documented in the empirical literature. In our empirical analysis below, we only observe overall average firm-level wages, and cannot disentangle the importance of changes in sorting from changes in wages conditional on observable skills. However, regardless of the relative importance of these two channels, the key conclusion from the

¹⁰See also Akerman (2019).

theoretical framework is that a common shock (increased consumer price sensitivity) would lead to simultaneous increases in both concentration and between-firm wage inequality at the industry level.

3 Data

In order to test the prediction of the model about the link between concentration and inequality, we use data from the Competitiveness Research Network (CompNet). This dataset draws on various administrative and public sources, and compiles information for non-financial corporations with at least one employee in various European countries. The latest edition of the data (6th vintage) provides information for 18 countries over the period 1999–2016, though not all years are available for all countries. We focus on the 14 countries which have representative data for the full universe of firms.¹¹ The data is made available at various levels of aggregation. We work with the finest level of aggregation available in the data, which is the industry-country-year level, where industries are coded at the 2-digit NACE Revision 2 level.

For roughly 40% of the overall sample, the CompNet dataset also provides information on various outcomes for firms at different deciles of the productivity distribution within each industry-country-year cell. Productivity is measured by a firm’s total factor productivity (TFP), computed from the estimation of a production function using a weighted two-step instrumental variable regression.¹² This information allows us to directly analyze how various outcomes evolve for firms with different productivity levels. In what follows, we refer to firms in the top decile of the productivity distribution within an industry-country-year as “superstar” firms. Appendix Figure A.1 confirms that, as implied by the model, more productive firms within an industry-country-year

¹¹The 14 countries are Belgium, Croatia, Denmark, Finland, France, Hungary, Italy, Lithuania, Netherlands, Portugal, Romania, Slovenia, Spain and Sweden. Due to missing data on some variables, Denmark, Netherlands and Romania are not included in all specifications. Representativeness is achieved through a reweighting procedure as detailed in the CompNet User Guide available at https://www.comp-net.org/fileadmin/_compnet/user_upload/Documents/User_Guide_6th_Vintage.pdf. Although data for the Czech Republic is available, its use is not recommended due to the very low coverage rate of small firms.

¹²Specifically, TFP is estimated by pooling all firms operating in a given sector and assuming a Cobb Douglas production function expressed in logs. Firm output is measured as real gross output (real revenues), and is regressed on the firm real book value of net capital and firm total employment. To deal with endogeneity issues, the estimation is performed using a control function approach, following the methodology developed by Olley and Pakes (1996) and Levinsohn and Petrin (2003). More details on the estimation of the TFP measure can be found in the user guide for the 6th vintage of the CompNet dataset.

cell are larger in terms of both sales and employment, and pay higher average wages.

The CompNet dataset provides two concentration measures: the share of sales (turnover) in the top 10 firms in each industry-country-year cell, and the Herfindahl-Hirschman index of market concentration. Our measures of sectoral wage inequality are also constructed from the CompNet data. Information is available on the distribution of labor costs per employee across firms. This allows us to measure dispersion in average firm-level wages by computing the log 90-10 ratio of labor costs per worker. It is worth noting that this measure of between-firm wage dispersion is not employment weighted, in line with the distribution of firm-level wages obtained from the theoretical framework (see also the discussion in footnote 8). It is therefore not mechanically affected by changes in the distribution of firm size within industries. Note also that the wage differentials that we compute from the data do not account for heterogeneity in worker characteristics and therefore capture both pure firm wage premia (i.e. wage differences for otherwise identical workers), as well as sorting of workers to firms based on observable or unobservable characteristics (see also the discussion in Section 2.5).

Panel A of Table 1 presents summary statistics for our measures of concentration and between-firm wage inequality. We show summary statistics separately for our full sample, and for the restricted sample where information across deciles of the productivity distribution is available. All summary statistics are weighted using each industry's time-averaged share of total value added in each country.¹³ Both in the full and in the restricted sample, the average log 90-10 ratio of firm-level wages is around 1.5 (implying nearly a fivefold gap in average firm wages), with quite a bit of heterogeneity across industry-country-year cells. The concentration measures are somewhat lower in the restricted sample, but even for this sample, more than 15% of overall sales in each cell are on average concentrated in just 10 firms.

At the bottom of Panel A we present two alternative measures of concentration which, instead of ranking firms according to their total sales (as would be done for the computation of the top 10 and HHI concentration measures), focus on the share of sales or employment in the *most productive* firms in an industry (namely firms in the top productivity decile). Consistent with the fact that more productive firms are larger, we find that firms in the top decile of productivity account for more than 40% of sales and more than 20% of employment.

Panel B analyzes the correlation between these alternative measures of concentra-

¹³Using time-averaged industry shares will allow us to rule out any effects driven by changes in the industry structure within countries in our empirical analysis.

tion based on productivity rankings and the more traditional measures based on the top 10 firms or the HHI. We find that all correlations are positive and statistically significant. This means that in industry-country-year cells where concentration is higher based on the share of sales in the top 10 firms or the HHI index, we also observe a larger concentration of both employment and sales in the most productive firms in the industry.

4 Findings: Concentration and Inequality

In order to explore the empirical link between concentration and between-firm wage inequality, we exploit variation across industry-country-year cells in the CompNet data. Our equation of interest is:

$$INEQ_{ict} = \alpha CONC_{ict} + \gamma_i + \delta_c + \tau_t + u_{ict} \quad (3)$$

The dependent variable is a measure of inequality in firm-level wages in industry i in country c at time t . The key independent variable is a measure of concentration in industry i in country c at time t . In order to exploit different sources of variation for identification, and to control for different sources of shocks, we experiment with different combinations of industry, country and time fixed effects, as discussed below.¹⁴

Table 2 presents our main set of results. Different panels use different measures of concentration, and each column considers a specification with a different set of fixed effects, as detailed at the bottom of the table. All regressions are weighted using each industry's time-averaged share of total value added in each country. Using time-averaged industry shares allows us to rule out any effects driven by changes in the industry structure within countries, while giving equal total weight to each country-year cell.

The top panels of Table 2 show results for each of the two standard concentration measures using our full sample. Column (1) presents a specification which includes industry, country and year fixed effects. We find that there is a positive and statistically significant correlation between concentration and between-firm wage inequality. The remaining columns of Table 2 consider different combinations of fixed effects in order to exploit different sources of identification. Column (2) includes country-year and

¹⁴The inclusion of these different sets of fixed effects also alleviates potential concerns about the cross-country comparability of the data sources underlying CompNet.

industry fixed effects. This would control for any country-specific policy changes that affect outcomes across industries. Identification is achieved from differential variation across industries within country-year cells. Column (3) includes industry-year and country fixed effects. This would account for any industry-specific variation in outcomes over time. Here, identification is achieved from differential changes over time within industries across countries. Column (4) is our most restrictive specification, which includes both industry-year and country-year fixed effects. Regardless of the source of variation used for identification, the correlation between concentration and inequality remains positive and strongly statistically significant.

The remaining panels of Table 2 show results for our restricted sample, where we also consider the concentration measures based on the share of sales or employment in the most productive firms in an industry. In all specifications, the estimated coefficient is positive, and statistically significant at the 1% level, confirming the positive empirical correlation between concentration and inequality.

To get a sense of the magnitude of the correlation, consider the estimates for the full sample in Column (4). Conditioning on country-year and industry-year fixed effects, a one standard deviation increase in concentration (which as shown in Table 1 is around 0.1 if measured through the HHI and around 0.3 if measured through the share of sales in the top 10 firms) would be associated with an increase in inequality of around 0.05. This is slightly less than 10% of one standard deviation in the sample.

Appendix Figure A.2 explores the extent to which the correlation between concentration and inequality is observed within each of the countries and each of the industries in our sample. The panels on the left show estimates from separate regressions of inequality on concentration for each country, including industry and time fixed effects, while the panels on the right show estimates from analogous regressions for each broad sector, including country and time fixed effects. The figure plots the estimated coefficient on concentration for each country/sector, along with the corresponding 95% confidence interval (where concentration is measured using the HHI in the top panel, and the share of sales in the top 10 firms in the bottom panel).

The correlation between concentration and inequality (conditional on industry and time fixed effects) is positive for the majority of the countries in our sample, with the only exceptions being Finland, France and Lithuania. This confirms that the results are widespread across European countries and do not seem to be particularly related to country-specific institutions. The correlation between concentration and inequality (conditional on country and time fixed effects) is also positive and statistically signif-

icant in most sectors. The main exception is wholesale and retail trade – which is excluded from the graph for visual clarity – where the estimated coefficient is substantially negative. Negative point estimates are also obtained for the administrative sector when measuring concentration based on the HHI and for the real estate sector when measuring concentration based on the top 10 firms; however, neither of these negative point estimates are statistically significant. Overall, we conclude that the positive association between concentration and inequality is widespread across different sectors of the economy.

Figure 1 provides more details about the link between concentration and inequality, by considering the correlations along the entire distribution of firm-level wages. In particular, we regress firm-level wages at percentile p in an industry-country-year cell on concentration in that cell (based on the HHI in the top panel and based on the share of sales in the top 10 firms in the bottom panel), as well as industry-year and country-year fixed effects. The figure plots the estimated coefficients and 95% confidence intervals obtained from these regressions at different percentile levels p , ranging from the fifth to the 99th percentile. Consistent with our finding regarding the positive correlation between concentration and inequality, we find that concentration is associated with a widening of the distribution of average firm-level wages. Depending on the measure of concentration used, we find that wages at the bottom of the distribution in more concentrated industries are either lower or only slightly higher than in less concentrated industries, whereas wages at the top of the distribution are much higher in more concentrated industries, with coefficients of 0.2 to 0.4 above the 90th percentile.

5 Changes along the Productivity Distribution

The theoretical framework presented in Section 2 features an increase in concentration that is driven by the increased dominance of the most productive firms within an industry. In practice, concentration increases may be associated with relatively less productive firms becoming entrenched and exploiting their market power. In Table 3 we analyze the correlation between changes over time in overall sales concentration and changes over time in concentration of sales or employment in the *most productive* firms in an industry. We do this by regressing the shares of sales (or employment) in superstar (top TFP decile) firms on standard concentration measures, plus a full set of industry-country fixed effects, and year fixed effects. We thus identify the correlation

between the two measures exploiting only differential changes over time within industry-country cells. We find that these correlations are positive and statistically significant at the 1% level. Hence, in line with the theoretical framework, we find that increases in concentration in Europe are associated with increasing concentration in the most productive firms within industries.

Figure 2 further analyzes the link between firm productivity, concentration and inequality. The left-hand panels consider how sales for firms at different deciles of the TFP distribution within an industry-country-year cell evolve as concentration increases. We do this by regressing average sales at each productivity decile on concentration (using the HHI in the top panel and the share of sales in the top 10 firms in the bottom panel), as well as a full set of country-year and industry-year fixed effects. In line with the evidence in Table 3, we see that increases in concentration are associated with nearly monotonic changes in sales along the TFP distribution, with low productivity firms experiencing reductions in their average volume of sales, and higher productivity firms experiencing increases.

The middle panels show the associated wage changes, by running an analogous set of regressions, but with average firm-level wages at each productivity decile as the outcome of interest. Increases in concentration are associated with wage declines in the less productive firms in an industry, and wage increases for more productive firms. In line with the prediction of the model, as highly productive firms become larger in terms of their sales, they also increase their average wages.

Finally, the panels on the right of Figure 2 show the results when considering employment as an outcome. Once again we see a (nearly) monotonic pattern: rises in concentration are associated with differential increases in employment of more productive firms. As high productivity firms become more dominant in terms of sales, they also become more dominant in terms of employment. This has an additional important implication for wage inequality. Although our measures of inequality give equal weight to all firms, the fact that these more productive, high wage firms not only increase their average wage but also become larger in terms of employment implies that if we were to construct *employment-weighted* measures of between-firm inequality, they would increase even more strongly as concentration increases, given this reallocation of employment.

Overall, the patterns observed along the productivity distribution are in line with the mechanisms highlighted by the model, thus supporting the plausibility of the type of shock that we analyze in terms of an increase in competitive pressure. The results show

that increases in concentration are associated with the best firms in the sector becoming more dominant, rather than unproductive firms being able to keep competitors out due to barriers to entry. In spite of their increased dominance, these highly productive firms pay higher average wages, such that certain groups of workers benefit from this increase in market concentration. This is consistent with increased competitive pressures being an underlying driver of both concentration and inequality.

6 Conclusions

We document a theoretical and an empirical link between rising concentration and rising between-firm wage inequality. Conceptually, a shock that favors the most productive firms in an industry (e.g. an increase in consumer price sensitivity, as in Autor et al. (2020)), will increase the concentration of employment and revenues in those firms. In a setting that links firm demand to worker wages (e.g. the search and bargaining framework of Helpman et al. (2010) or the fair wage framework of Egger and Kreickemeier (2012)), the expansion of production in the most productive firms will be accompanied by an increase in the relative wages of workers in those firms, hence increasing between-firm wage inequality.

We confirm the empirical relevance of this conceptual link using data on concentration and between-firm wage inequality for 14 European countries over the period 1999-2016. We indeed find evidence of a statistically significant positive correlation between concentration and inequality at the industry-country-year level, which is robust to allowing for a variety of different combinations of industry, country and year fixed effects. These changes are associated with increased sales, employment, and average wages in the most productive firms in the industry, suggesting that increased competitive pressures are a plausible mechanism underlying changes in both concentration and inequality in Europe.¹⁵

Further understanding how and why competitive pressures have changed, as well as the underlying micro-level adjustments occurring at the firm level using detailed micro-data would be promising avenues for future research.

¹⁵See also the evidence in Gutiérrez and Philippon (2018) regarding the role of supra-national institutions in the European Union in driving the increased enforcement of competition policies within the region.

References

- Acemoglu, Daron and David Autor (2011), “Skills, tasks and technologies: Implications for employment and earnings.” *Handbook of Labor Economics*, 4, 1043–1171.
- Akerman, Anders (2019), “The relative skill demand of superstar firms and aggregate implications.” *Working Paper*.
- Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen (2017), “Concentrating on the fall of the labor share.” *American Economic Review: Papers & Proceedings*, 107, 180–185.
- Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John van Reenen (2020), “The fall of the labor share and the rise of superstar firms.” *Quarterly Journal of Economics*, 135, 645–709.
- Axtell, Robert L. (2001), “Zipf distribution of U.S. firm sizes.” *Science*, 293, 1818–1820.
- Azar, José, Ioana Marinescu, and Marshall I. Steinbaum (2020), “Labor market concentration.” *Journal of Human Resources*, Forthcoming.
- Azar, José, Ioana E. Marinescu, Marshall Steinbaum, and Bledi Taska (2018), “Concentration in US labor markets: Evidence from online vacancy data.” *NBER Working Paper No. 24395*.
- Bajgar, Matej, Giuseppe Berlingieri, Sara Calligaris, Chiara Criscuolo, and Jonathan Timmis (2019), “Industry concentration in Europe and North America.” *OECD Productivity Working Papers, No. 18*.
- Barth, Erling, Alex Bryson, James C. Davis, and Richard Freeman (2016), “It’s where you work: Increases in the dispersion of earnings across establishments and individuals in the United States.” *Journal of Labor Economics*, 34, S67–S97.
- Benmelech, Efraim, Nittai Bergman, and Hyunseob Kim (2018), “Strong employers and weak employees: How does employer concentration affect wages?” *NBER Working Paper No. 24307*.
- Card, David, Jörg Heining, and Patrick Kline (2013), “Workplace heterogeneity and the rise of West German wage inequality.” *The Quarterly Journal of Economics*, 128, 967–1015.

- Corcos, Gregory, Massimo Del Gatto, Giordano Mion, and Gianmarco I.P. Ottaviano (2012), “Productivity and firm selection: Quantifying the ‘new’ gains from trade.” *The Economic Journal*, 122, 754–798.
- Cortes, Guido Matias and Andrea Salvatori (2019), “Delving into the demand side: Changes in workplace specialization and job polarization.” *Labour Economics*, 57, 164–176.
- Covarrubias, Matias, Germán Gutiérrez, and Thomas Philippon (2019), “From good to bad concentration? u.s. industries over the past 30 years.” In *NBER Macroeconomics Annual 2019, volume 34*, University of Chicago Press.
- De Loecker, Jan, Jan Eeckhout, and Gabriel Unger (2020), “The rise of market power and the macroeconomic implications.” *The Quarterly Journal of Economics*, 135, 561–644.
- Diamond, Peter A (1982a), “Aggregate demand management in search equilibrium.” *Journal of Political Economy*, 90, 881–894.
- Diamond, Peter A (1982b), “Wage determination and efficiency in search equilibrium.” *The Review of Economic Studies*, 49, 217–227.
- Egger, Hartmut and Udo Kreickemeier (2012), “Fairness, trade, and inequality.” *Journal of International Economics*, 86, 184–196.
- Felbermayr, Gabriel, Julien Prat, and Hans-Jörg Schmerer (2011), “Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity.” *Journal of Economic Theory*, 146, 39–73.
- Grullon, Gustavo, Yelena Larkin, and Roni Michaely (2019), “Are US industries becoming more concentrated?” *Review of Finance*, 23, 697–743.
- Gutiérrez, Germán and Thomas Philippon (2018), “How EU markets became more competitive than US markets: A study of institutional drift.” *NBER Working Paper No. 24700*.
- Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding (2010), “Inequality and unemployment in a global economy.” *Econometrica*, 78, 1239–1283.

- Juhn, Chinhui, Kevin M. Murphy, and Brooks Pierce (1993), “Wage inequality and the rise in returns to skill.” *Journal of Political Economy*, 410–442.
- Katz, L.F. and D.H. Autor (1999), “Changes in the wage structure and earnings inequality.” *Handbooks of Labor Economics*, 3, 1463–1558.
- Kehrig, Matthias and Nicolas Vincent (2018), “The micro-level anatomy of the aggregate labor share decline.” *NBER Working Paper No. 25275*.
- Levinsohn, James and Amil Petrin (2003), “Estimating production functions using inputs to control for unobservables.” *The Review of Economic Studies*, 70(2), 317–341.
- Melitz, Marc J (2003), “The impact of trade on intra-industry reallocations and aggregate industry productivity.” *Econometrica*, 71, 1695–1725.
- Mortensen, Dale T and Christopher A Pissarides (1994), “Job creation and job destruction in the theory of unemployment.” *The Review of Economic Studies*, 61, 397–415.
- Oi, Walter Y. and Todd L. Idson (1999), “Firm size and wages.” In *Handbook of Labor Economics*, volume 3, 2165 – 2214, Elsevier.
- Olley, Steven and Ariel Pakes (1996), “The dynamics of productivity in the telecommunications equipment industry.” *Econometrica*, 64(6), 1263–1297.
- Rinz, Kevin (2018), “Labor market concentration, earnings inequality, and earnings mobility.” *CARRA Working Paper 2018-10*.
- Song, Jae, David J Price, Fatih Guvenen, Nicholas Bloom, and Till von Wachter (2018), “Firming up inequality.” *The Quarterly Journal of Economics*, 134, 1–50.
- Stansbury, Anna and Lawrence H Summers (2020), “The declining worker power hypothesis: An explanation for the recent evolution of the American economy.” *NBER Working Paper No. 27193*.

Table 1: Summary statistics and correlations of concentration measures

Panel A: Descriptive statistics					
	Mean	Std. Dev.	P10	P90	N
<i>Full Sample</i>					
Log 90-10 ratio of wage bill per worker	1.617	0.653	0.914	2.479	8820
Concentration (HHI)	0.034	0.098	0.0003	0.087	9809
Concentration (top 10)	0.226	0.271	0.015	0.714	9118
<i>Restricted Sample</i>					
Log 90-10 ratio of wage bill per worker	1.502	0.559	0.894	2.236	3660
Concentration (HHI)	0.012	0.031	0.0002	0.025	4185
Concentration (top 10)	0.156	0.199	0.010	0.433	4025
Share of sales in superstar firms	0.416	0.175	0.179	0.628	3262
Share of employment in superstar firms	0.223	0.142	0.072	0.397	3154
Panel B: Correlations					
	HHI	Top 10	Share of sales		
Top 10	0.826*** <i>4025</i>				
Share of sales in superstar firms	0.287*** <i>3262</i>	0.331*** <i>3162</i>			
Share of employment in superstar firms	0.305*** <i>3154</i>	0.383*** <i>3067</i>	0.840*** <i>3026</i>		

Note: The restricted sample refers to industry-country-year cells for which all deciles of the TFP distribution are available for the computation of the share of sales and employment in superstar (top TFP decile) firms. Summary statistics are weighted using each industry's time-averaged share of total value added in each country. The number of observations for each correlation in Panel B is shown in italics. ***p<0.01, **p<0.05, *p<0.1

Table 2: Concentration and wage inequality

	<i>Dep var: log 90-10 ratio of wage bill per worker</i>			
	(1)	(2)	(3)	(4)
Full sample:				
Concentration (HHI)	0.183 (0.072)**	0.231 (0.059)***	0.452 (0.081)***	0.503 (0.065)***
Obs.	8820	8813	8806	8798
R^2	0.683	0.798	0.726	0.833
Concentration (top 10)	0.237 (0.036)***	0.197 (0.029)***	0.225 (0.039)***	0.176 (0.03)***
Obs.	8175	8168	8166	8158
R^2	0.689	0.814	0.727	0.842
Restricted sample:				
Concentration (HHI)	0.653 (0.163)***	0.588 (0.144)***	0.668 (0.192)***	0.520 (0.169)***
Obs.	3660	3657	3577	3574
R^2	0.881	0.912	0.895	0.922
Concentration (top 10)	0.155 (0.038)***	0.143 (0.034)***	0.118 (0.043)***	0.101 (0.038)***
Obs.	3506	3503	3402	3399
R^2	0.883	0.913	0.897	0.923
Share of sales in superstar firms	0.124 (0.031)***	0.134 (0.027)***	0.134 (0.035)***	0.122 (0.031)***
Obs.	2819	2812	2680	2672
R^2	0.895	0.926	0.907	0.934
Share of employment in superstar firms	0.212 (0.038)***	0.219 (0.034)***	0.229 (0.043)***	0.208 (0.038)***
Obs.	2743	2738	2595	2589
R^2	0.9	0.928	0.911	0.935
Industry FE	Yes	Yes		
Country FE	Yes		Yes	
Year FE	Yes			
Country x Year FE		Yes		Yes
Industry x Year FE			Yes	Yes

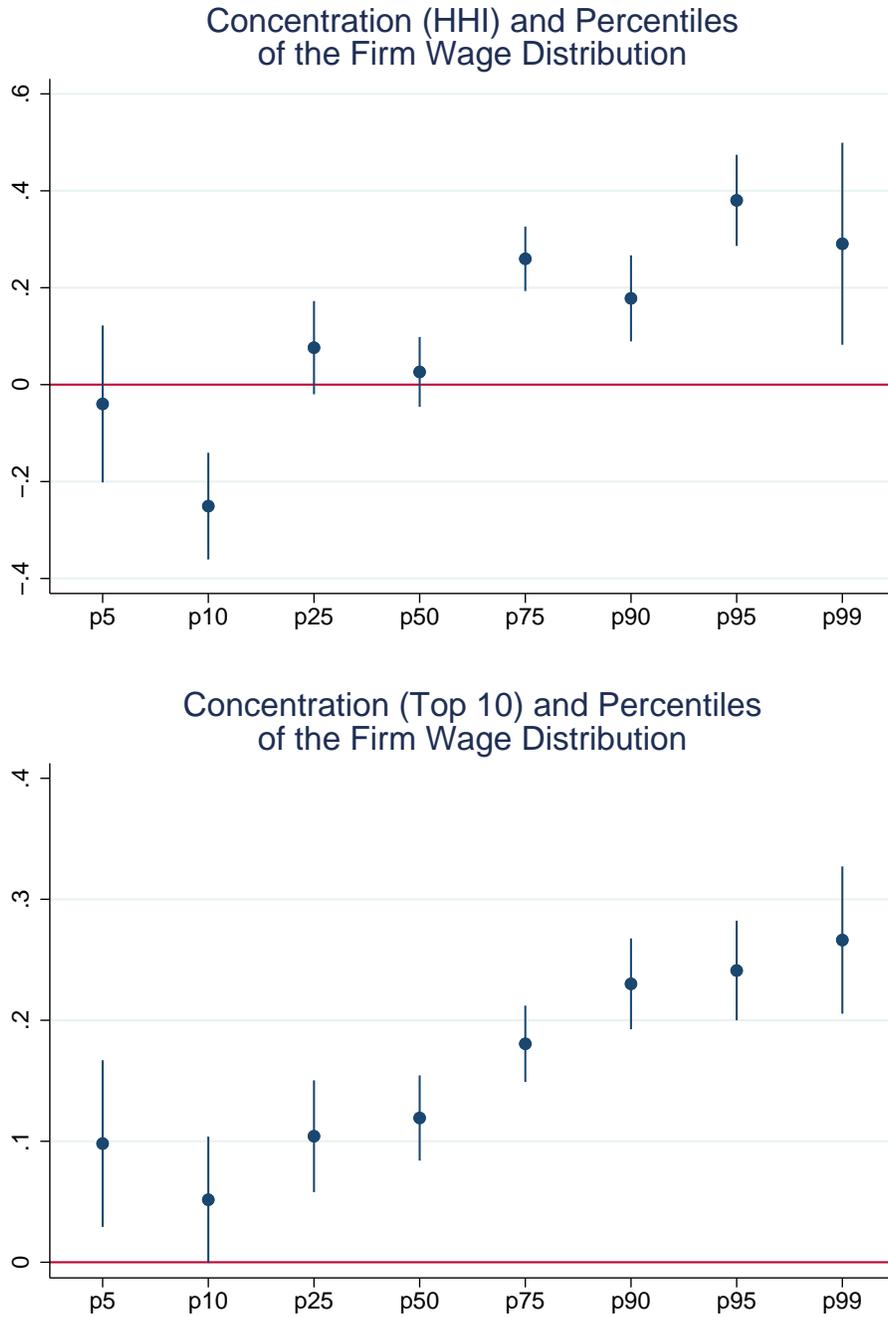
Note: Observations are at the country-industry-year level. All regressions are weighted using each industry's time-averaged share of total value added in each country.

Table 3: Concentration and the share of sales and employment in superstar firms

	Share of sales in superstar firms		Share of employment in superstar firms	
	(1)	(2)	(3)	(4)
Concentration (HHI)	0.638 (0.088)***		0.433 (0.081)***	
Concentration (top 10)		0.183 (0.023)***		0.116 (0.021)***
Obs.	3252	3156	3144	3060
R^2	0.964	0.964	0.958	0.958
Industry x Country FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

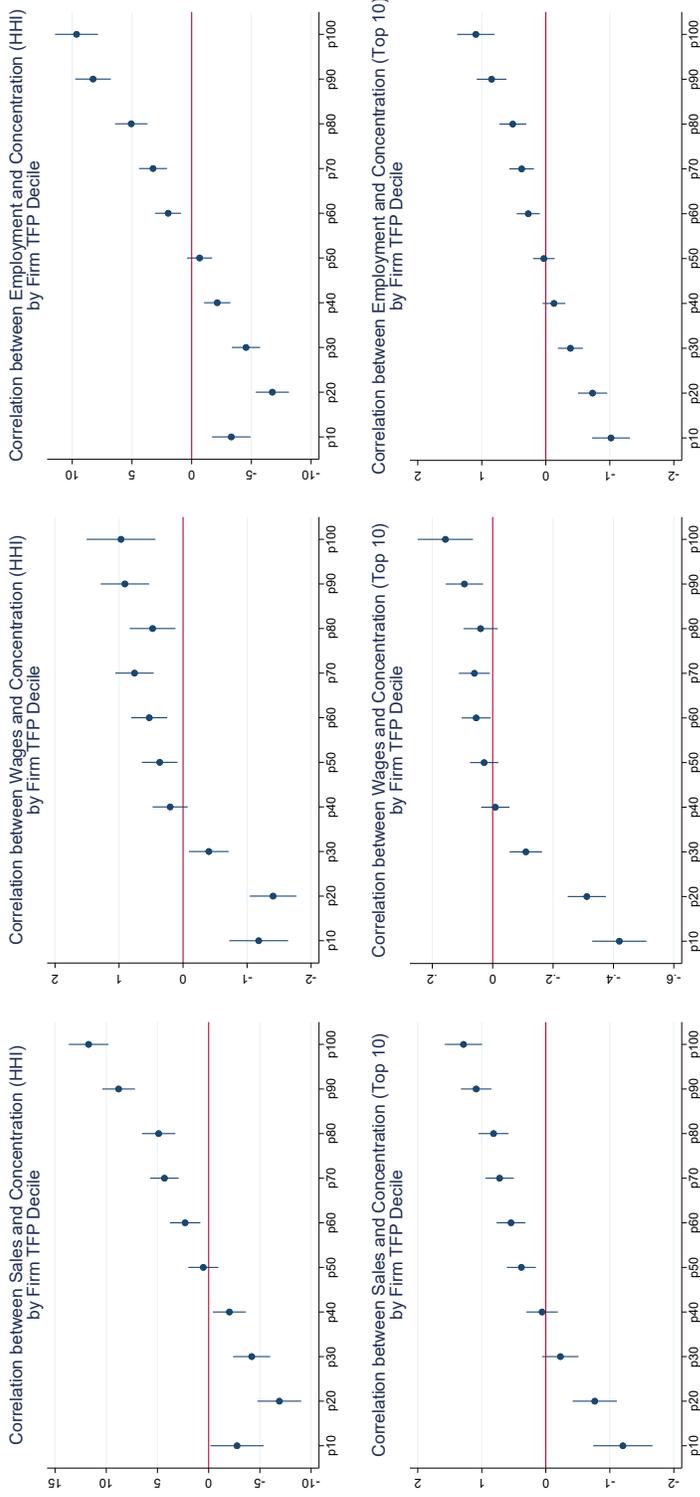
Note: Superstar firms are those in the top decile of the TFP distribution within their industry-country-year cell.

Figure 1: Results across the distribution of firm-level wages



Note: The figure plots the estimated coefficients and 95% confidence intervals obtained from regressions of wages at different percentiles of the firm wage distribution within an industry-country-year cell on concentration in that cell, controlling for industry-year and country-year fixed effects. Regressions are weighted using each industry's time-averaged share of total value added in each country.

Figure 2: Differential changes in sales, wages and employment along the productivity distribution



Notes: The figure shows the estimated coefficients and 95% confidence intervals obtained from regressions of industry-country-year mean firm sales (wages, employment) at each decile of the TFP distribution on industry-country-year concentration. Each regression includes country-year and industry-year fixed effects, and is weighted using each industry's time-averaged share of total value added in each country.

Online Appendix for: Rising Concentration and Wage Inequality

Guido Matias Cortes (York University and IZA)
Jeanne Tschopp (University of Bern)

Appendix A Deriving Concentration Measures and the Wage Distribution

Concentration Measures Let $\bar{\theta}$ denote the productivity level corresponding to the $(100 - \mu)$ th percentile of the productivity distribution. The share of sectoral revenues accruing to firms in the top $\mu\%$ of the productivity distribution is given by:

$$\begin{aligned}
 C_r &= 1 - \frac{\int_{\theta_d}^{\bar{\theta}} r(\theta) dG_{\theta}(\theta)}{\int_{\theta_d}^{\infty} r(\theta) dG_{\theta}(\theta)} \\
 &= 1 - \frac{\int_{\theta_d}^{\bar{\theta}} \theta^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta)}{\int_{\theta_d}^{\infty} \theta^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta)} \\
 &= \left(\frac{\bar{\theta}}{\theta_d} \right)^{\frac{\beta}{\Gamma} - z}.
 \end{aligned} \tag{A.1}$$

where the second equality is obtained using the equation of equilibrium firm-level revenues $r(\theta) = r_d \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}}$, and the last equality uses the fact that with a Pareto distribution $g(\theta) = z\theta^{-(z+1)}$ and $1 - G_{\theta}(\theta_d) = \theta_d^{-z}$.

Finally, to express C_r as a function of μ , it is useful to relate $\frac{\bar{\theta}}{\theta_d}$ to the share of firms with $\theta \geq \bar{\theta}$:

$$\begin{aligned}
 \mu &= 1 - \int_{\theta_d}^{\bar{\theta}} g(\theta \mid \theta \geq \theta_d) d\theta \\
 &= 1 - \frac{1}{\theta_d^{-z}} \int_{\theta_d}^{\bar{\theta}} z\theta^{-(z+1)} d\theta \\
 &= \left(\frac{\bar{\theta}}{\theta_d} \right)^{-z}.
 \end{aligned}$$

It follows that $\frac{\bar{\theta}}{\theta_d} = \mu^{-\frac{1}{z}}$. Replacing the latter equation in (A.1) we obtain an expression for the share of revenues accruing to firms in the top $\mu\%$ of the productivity distribution:

$$C_r = \mu^{1 - \frac{\beta}{\Gamma z}}. \quad (\text{A.2})$$

The concentration measure of employment is obtained in a similar way by computing the share of sectoral employment concentrated in the firms in the top $\mu\%$ of the productivity distribution.

Wage Distribution Let θ_w denote the productivity level associated with $w(\theta_w) = w$. The wage distribution is given by:

$$\begin{aligned} G_f(w) &= Pr [w(\theta) \leq w] \\ &= Pr [\theta \leq \theta_w \mid \theta \geq \theta_d] \\ &= 1 - \left(\frac{\theta_d}{\theta_w} \right)^z, \end{aligned} \quad (\text{A.3})$$

where the last equality uses the fact that productivity follows a Pareto distribution with shape parameter z . Finally, using the fact that $w(\theta_w) = w_d \left(\frac{\theta_w}{\theta_d} \right)^{\frac{\beta k}{\delta \Gamma}}$, we have that $\frac{\theta_d}{\theta_w} = \left(\frac{w_d}{w} \right)^{\frac{\delta \Gamma}{\beta k}}$ and we can rewrite (A.3) as follows:

$$G_f(w) = 1 - \left(\frac{w_d}{w} \right)^{\frac{\delta \Gamma z}{\beta k}}. \quad (\text{A.4})$$

Hence, firm-level wages are Pareto distributed with scale parameter w_d and shape parameter $\frac{\delta \Gamma z}{\beta k}$.

Appendix B Productivity Threshold for Production and Selection

In the closed economy version of the Helpman et al. (2010) model, the equilibrium productivity cutoff for production is given by:

$$\theta_d = \left(\frac{\beta}{z\Gamma - \beta} \right)^{1/z} \left(\frac{f_d}{f_e} \right)^{1/z} \theta_{min}. \quad (\text{A.5})$$

From (A.5) it is straightforward to see that

$$\frac{\partial \theta_d}{\partial \beta} = \left(\frac{f_d}{f_e}\right)^{1/z} \left(\frac{\beta}{z\Gamma - \beta}\right)^{1/z-1} \frac{\theta_{min}}{(z\Gamma - \beta)^2} > 0. \quad (\text{A.6})$$

Hence, an increase in the elasticity of substitution increases the productivity threshold for production and leads to a reduction in the range of firm types.

This, however, does not imply that the variance of productivity among operating firms is reduced. The productivity distribution among operating firms is a truncated version of the Pareto distribution $G(\theta)$, with the same shape parameter z , but with scale parameter θ_d . The variance of productivity among operating firms is given by

$$\frac{z\theta_d^2}{(z-1)^2(z-2)},$$

which is increasing in θ_d . Intuitively, this is due to the fact that an increase in θ_d implies that firms with relatively homogeneous firm types at the bottom of the productivity distribution exit, and hence there is a relative increase in the mass of firms towards the tail. The increase in the productivity threshold induced by an increase in consumer price sensitivity will therefore lead to an increase in the variance of productivity among operating firms, further compounding the increase in inequality due to changes in employment and wages within continuing firm types. Scale-invariant measures of the dispersion of productivity among operating firms would not be affected by the change in θ_d .

Appendix C Concentration, Wage Inequality and Worker Sorting

Helpman et al. (2010, Section 5.1) present an extension to their model that allows for worker heterogeneity in observable characteristics. This makes it possible to also think about the impacts operating through the increased sorting of good workers to good firms in terms of observables. Here we illustrate the key features of this extension to the model, and derive the key predictions of interest for our purposes regarding concentration and between-firm wage inequality.

Consider an economy with two types of workers, $\ell = H, L$, with H denoting skilled workers and L unskilled workers. The production function is given by:

$$y = \theta (\bar{a}_H h_H^{\gamma_H})^{\lambda_H} (\bar{a}_L h_L^{\gamma_L})^{\lambda_L}, \quad \lambda_H + \lambda_L = 1 \quad (\text{A.7})$$

The match-specific ability of each group has a Pareto distribution with shape parameter k_ℓ and lower bound $a_{min,\ell}$. Search and matching for skilled and unskilled workers occur in separate markets, so search costs b_ℓ are allowed to differ by type.

Helpman et al. (2010) show that firm-level employment and wages for workers of type ℓ are given by:

$$h_\ell(\theta) = h_{d,\ell} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}(1-k_\ell/\delta)}, \quad h_{d,\ell} \equiv \frac{\lambda_\ell \beta \gamma_\ell f_d}{\Gamma b_\ell} \left[\frac{\lambda_\ell \beta (1 - \gamma_\ell k_\ell)}{\Gamma} \frac{f_d}{c a_{min,\ell}^\delta} \right]^{-k_\ell/\delta}$$

$$w_\ell(\theta) = w_{d,\ell} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta k_\ell}{\delta \Gamma}}, \quad w_{d,\ell} \equiv b_\ell \left[\frac{\lambda_\ell \beta (1 - \gamma_\ell k_\ell)}{\Gamma} \frac{f_d}{c a_{min,\ell}^\delta} \right]^{k_\ell/\delta}$$

where now:

$$\Gamma = 1 - \beta(\lambda_H \gamma_H + \lambda_L \gamma_L) - \frac{\beta}{\delta} [1 - (\lambda_H \gamma_H k_H + \lambda_L \gamma_L k_L)]$$

The relative employment of skilled workers within a firm with productivity θ is given by:

$$\frac{h_H(\theta)}{h_L(\theta)} = \frac{h_{d,H}}{h_{d,L}} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\delta \Gamma}(k_L - k_H)}$$

And the relative wage of skilled workers is given by:

$$\frac{w_H(\theta)}{w_L(\theta)} = \frac{w_{d,H}}{w_{d,L}} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\delta \Gamma}(k_H - k_L)}$$

For sufficiently high values of $\frac{w_{d,H}}{w_{d,L}}$, we have that in all firms, skilled workers are paid more than unskilled workers, i.e. $\frac{w_H(\theta)}{w_L(\theta)} > 1 \forall \theta$.

Assuming that $k_H < k_L$, i.e. that the match-specific ability distribution is more dispersed among skilled workers than among unskilled workers, we have that the relative employment of skilled workers is increasing in firm productivity.

Average firm wages will be higher in more productive firms because they: (i) employ a larger proportion of skilled workers, and (ii) pay higher wages to both worker types.

Hence, wages differ across firms both because of the composition/sorting of workers, and because of firm premia conditional on worker type.

The concentration of type ℓ workers in the top $\mu\%$ of firms is given by:

$$C_{h,\ell} = \mu^{1 - \frac{\beta}{\Gamma z} (1 - k_\ell / \delta)} \quad (\text{A.8})$$

Given that $k_H < k_L$, we have that $C_{H,h} > C_{L,h}$.

We have the following prediction:

Prediction: An increase in the elasticity of substitution, σ , increases concentration of employment in the most productive firms, particularly so for skilled workers:

$$\frac{\partial C_{h,H}}{\partial \sigma} > \frac{\partial C_{h,L}}{\partial \sigma} > 0$$

Corollary: The disproportionate increase in employment concentration for skilled workers implies stronger sorting of skilled workers to high productivity firms. This increased sorting and the implied changes in the composition of workers across firm types will increase between-firm inequality in average firm-level wages.

The distribution of wages across firms for workers of type ℓ is given by:

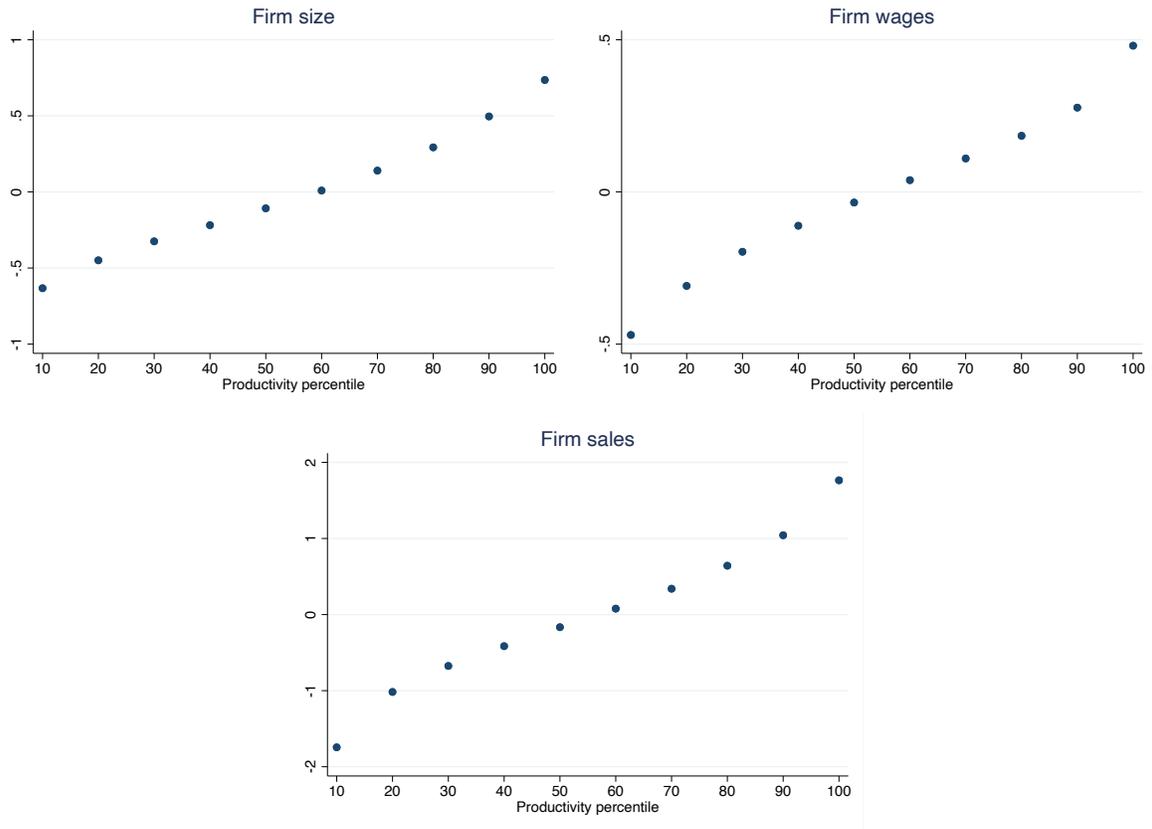
$$G_f(w_\ell) = 1 - \left(\frac{w_{d,\ell}}{w_\ell} \right)^{\frac{\delta \Gamma z}{\beta k_\ell}}$$

This is a Pareto distribution with scale parameter $w_{d,\ell}$ and shape parameter $\frac{\delta \Gamma z}{\beta k_\ell}$. Inequality, as measured by any scale-invariant measure, will be a function of the shape parameter only.

Prediction: An increase in the elasticity of substitution, σ , increases within-group, between-firm wage inequality for both worker types.

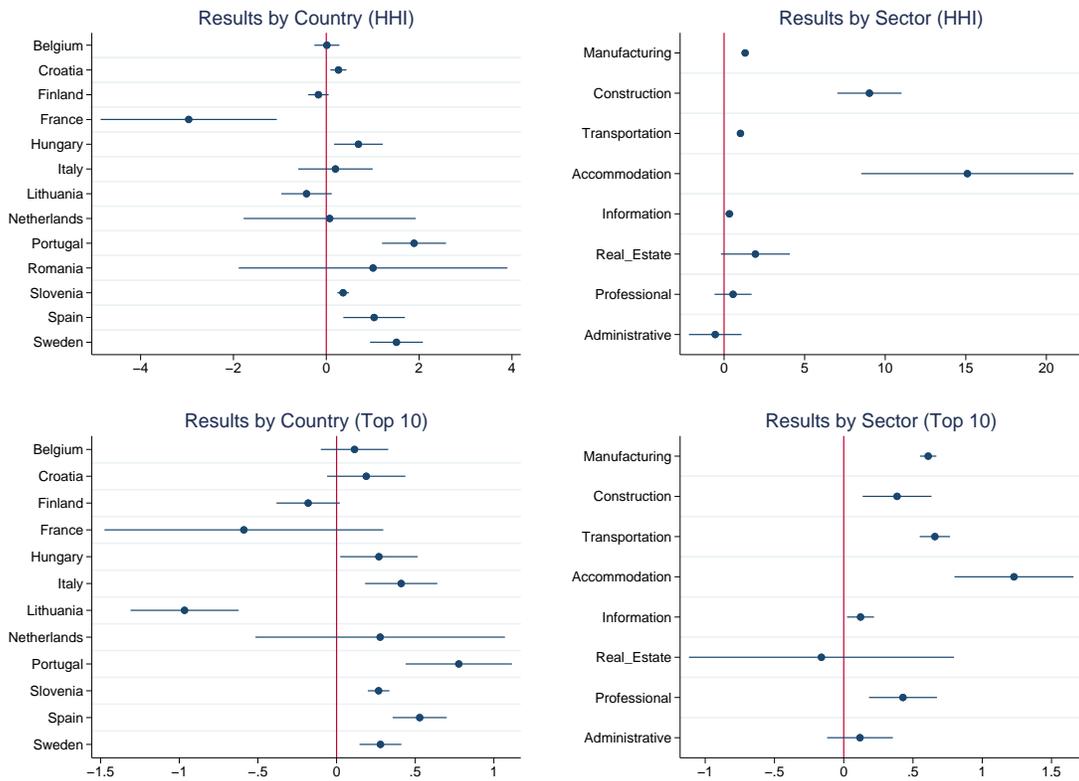
Corollary: An increase in the elasticity of substitution, σ , increases inequality in average firm wages both because of (i) increased worker sorting and (ii) increased dispersion in firm premia conditional on worker types.

Figure A.1: Firm size, wages and sales (turnover) along the productivity distribution



Note: The left (right, bottom) panel shows the relationship between firm size (wages, sales) and firm productivity across percentiles of the TFP distribution. Firm size (wages, sales) is the average of residuals from an unweighted regression of log firm size (log wages, log sales) on a full set of industry-country-year fixed effects, averaged across firms in a given TFP percentile bin.

Figure A.2: Results by Country and by Broad Sector



Note: The figure plots the estimated coefficients and 95% confidence intervals obtained from regressions of inequality on concentration. The top panels use the HHI index as the concentration measure, while the bottom panels use the share of sales in the top 10 firms. In the left panels, regressions are run separately for each country, controlling for industry and time fixed effects. Note that we drop Denmark as we only have 18 observations (industry-year cells). In the right panels, regressions are run separately for each broad sector, controlling for country and time fixed effects. Wholesale and retail trade is excluded from the figure for visual clarity, but it is included in the baseline regressions.