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ABSTRACT

Decomposing US Income Inequality à La Shapley: Race Matters, but Gender Too*

This paper is an application of a new Shapley income decomposition methodology, in which we isolate two subjective factors in income differences - race and gender - that contribute to income inequality within the population of blacks and whites in the United States over the period 2005-2017. We show that the purely racial contribution to income inequality as defined by the Gini index varies from 1% to 4% depending on the geographical administrative divisions used. Race tends to contribute more to inequality in the Western and Southern part of the country. Whatever the division, the share of income inequality associated with gender exceeds greatly that of race. While gender income inequality falls over time, income inequality associated with race tends to increase.

JEL Classification: C71, D63, J15, J71

Keywords: income inequality, decomposition, Shapley value, racial discrimination, gender discrimination

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1 Introduction

Income inequality, especially involving race and gender, is still a hot topic in the United States. A nationwide poll conducted in 2015 by CNN and the Kaiser Family Foundation found that 49% of US citizens think racism is "a big problem" in society today, while being "only" 28% in 2011. Title VII of the Civil Rights Act of 1964 prohibits employment discrimination based on race, color, religion, sex and national origin. In 1965, President Johnson signed an executive order promoting "affirmative action" that would ensure that all applicants and employees will be considered with no distinction of race, creed, color, or national origin. Since then these measures have been strongly criticized by conservatives, and several states opted to take action against positive discrimination policies in public institutions. The importance of such issues in public debates underlines the need to have convenient indicators that are easy to use by public policy makers.

The existing income decomposition literature most often focuses on decomposing the evolution of income inequality over time or the difference in inequality between two groups (see Fortin *et al.* 2011 for a literature review or Montes-Rojas *et al.* 2017 for a recent application). Our contribution rather proposes a practical Shapley decomposition tool that is able to decompose the inequality of an income distribution by individual characteristics (hereafter referring to as attributes). Our methodology has the advantage of not requiring to choose a modality of reference for categorical variables (eg. level of diploma, gender, race) known to affect the results (see Oaxaca and Ransom 1999 for a discussion). Moreover, contrarily to the distribution decomposition by income sources proposed by Lerman and Yitzhaki (1985), our methodology is suitable for a decomposition by attributes and it is applicable to a large variety of inequality index: Gini, Theil, Atkinson, etc. For the best of our knowledge, there is no other decomposition instrument in the literature to carry out inequality index decomposition by attributes that presently meet all of these advantages. Our contribution is an application of this new Shapley income decomposition methodology, in which we concentrate on the discriminatory part of observed inequalities through two subjective factors in income differences - race and gender - that contribute to income inequality as defined by the Gini index within the population of blacks and whites taken together for each of the geographical administrative divisions of the United States over the period 2005-2017.

Literature on racial and gender inequality often concentrate on the income gap between groups, *ie* comparing different ethnical groups or women versus men. Following this perspective, Sites and Parks (2011), and Couch and Daly (2002), show that racial income inequalities in the United States diminished significantly following the passage of the Civil Rights Act of 1964 and other measures aimed at reducing labor market discrimination, but have not changed significantly since 1974; the black-white wages gap remaining around 30% to the later 1980s, when a new convergence was observed in the 1990s. As for recent years, the median annual income of a family in 2014 was $76,658 for whites and $43,151 for blacks, the second-largest ethnic minority after Hispanics (Economic Report of the President, 2016). As a matter of allocation, decades of sociological research showed that black-white inequality in local areas is greater where the black population is relatively large (see for instance Huffman and Cohen (2004)). Using the decomposition techniques of Juhn *et al.* (1991), Couch and Daly (2002) found that greater occupational diversity partly explain the reduction of the racial income gap during the 90s. Convergence is partly due to equalization in the attainment of education and experience and also to the distribution of employment across industries and occupations, rather than being a purely ethnic matter. Sections of the literature explain part of the observed income inequality by the reduction in unionization rates (see for instance Koniger *et al.* 2007 and Rosenfeld 2014). Some authors find a significant impact of unionization rates on the male black-white gap, resulting in a wage premium for blacks (Jones and Shmitt 2014). As for gender income inequality, others indicate that unionization would be beneficial to women (Rosenfeld and Kleykamp 2012). Blau and Kahn (2017) note that the gender wage gap has been intensively investigated for decades, but still remains an area of active research. While the long-term trend shows a significant reduction in the wage gap, convergence has been slower and uneven since the 90s. Income decomposition methods show that the gender repartition with respect to occupation and industry are the main factors explaining this gap nowadays. Still, gender differences in workforce interruptions and working time are significant sources of income gap.

Our contribution differs from the previous literature by concentrating on racial or gender inequality as defined by the contribution of race or gender to the overall inequality. In doing so, applying decomposition methods to measures of income inequality seem to provide an attractive framework for the appraisal of inequality associated to these two characteristics. Among income decomposition methods, those inspired by the Shapley value seem particularly interesting, since they allow explanation of income inequality by determining the contribution of the various income sources, or the contribution of different sub-populations to overall inequality.[2] However, the Shapley decomposition methods developed so far do not permit the estimation of the share of overall wage inequality due to an attribute such as race or even gender. In fact, if the two sub-populations are made up of blacks and whites respectively, the results of the decomposition will give the contribution of blacks to the overall income inequality on the one hand and the contribution of whites to the overall inequality on the other hand, that is inequality observed within each sub-population. But this contribution does not reflect inequality between sub-populations, that is between blacks and whites. Hence, this decomposition framework does not allow the determination of the contribution of race to income inequality. In order to resolve this drawback of the "classical" Shapley decomposition rule, Chantreuil and Lebon (2015) extended this framework to a third dimension, namely the decomposition of income inequality by attributes. Defining the wage received by an individual as the sum of several elements, each element representing the part of income resulting from each individual's attribute, the Shapley decomposition rule then offers a simple way to

[2] See Chantreuil and Trannoy (2011, 2013) and Shorrocks (2013) for the definition of the Shapley decomposition rules determining the contribution of different income sources or different sub-populations to overall income inequality.

determine the contribution of race as well as all other individuals' attributes, and especially gender, to overall income inequality. Here, the order of attributes matters. Our contribution adds to this latter literature as we do not pre-assume attributes' ordering.

The Shapley decomposition method enables us do distinguish inequality arising from several attributes, some which can be regarded as objective such as age and education, from inequality arising from subjective factors, which can be regarded as discriminative such as race and gender. Focusing on the latter, we find that share of inequality attributable to racial affiliation is about 1% to 4% depending on the 9 designated divisions of the United States Census Bureau, while the contribution of gender to the total observed inequality in the blacks and white population taken together is much larger and amount for 9% to 13%. Time comparison shows that the contribution of race to income inequality has tended to increase in all administrative divisions over the past decade, whereas that of gender tends to have been lower recently.

The paper is organized as follows: Section 2 presents empirical evidences on racial income inequality in the United States over the period 2005-2017. Section 3 outlines the Shapley decomposition methodology according to conditional decomposition. Section 4 analyses the results, whose robustness is verified in section 5. Finally, section 6 contains some concluding comments.

2 Empirical evidence of racial inequality

We use annual earnings, defined as wages plus self-employment incomes in the American Community Surveys (ACS-PUMS, about 1% of the total United States population) from 2005 to 2017[3]. To be able to draw unambiguous conclusions, highlighting the impact of racial factors that can be observed, we limit our income inequality study to the comparison of pure backs and pure whites; we exclude from our analysis other ethnic groups, as well as those black and white persons that declared themselves as Hispanic.

Individuals are described according to several characteristics: the geographical administrative division to which they belong (see appendix A for a list of states per division); their gender; their level of education (lower than high-school certification, high-school graduation plus some college but without a degree-level qualification, undergraduate degree, and graduate degree or more); and cohort (one group every 4 years from 25-29 years old up to 60-64 years old).[4] Individuals over 65 years old, or individuals with a declared annual income below one hour of the federal minimum wage for each year, are dropped from the data set. Finally, individual US states are grouped into 9 regional sub-sets according to the United States Census Bureau's designated divisions. The total sample over

[3]Using the IPUMS-CPS from University of Minnesota with weekly earnings from 1989 to 2015, gives similar results over the period 2005-2015, but it open to criticism arising from a much weaker sample size.

[4]We exclude from our analysis individuals between 20-24 since the sample contains few observations with a graduate degree or more.

the period 2005-2017 is more than 12.3 millions observations, among which 11.87% are defined as pure blacks in our white/black sample.

Table 1 shows the distribution of black and white population per administrative division together with the average yearly income over time.

Table 1: Descriptive statistics of the 2005-2017 sample

Division	**black share*** (%)	**annual earnings** (constant 2017 USD)		**B/W earnings ratio** (%)
		whites	**blacks**	
1.New England	5.32	68851.35	46093.96	67
2.Middle Atlantic	11.31	65734.75	46519.41	71
3.East North Central	8.14	53581.88	38825.35	72
4.West North Central	4.20	50498.77	36213.30	72
5.South Atlantic	21.26	61673.36	40753.78	66
6.East South Central	17.40	50792.49	33486.56	66
7.West South Central	15.99	61172.86	38048.12	62
8.Mountain	3.95	57862.42	42547.05	74
9.Pacific	7.61	71149.73	50411.06	71
FEDERAL	11.87	60312.74	40888.43	68

* in the black and white population, excluding other ethnical groups.

In line with Huffman and Cohen (2004), Table 1 shows that the black population earns on average much less than the white population in most administrative divisions where the black share of (black and white) population is important. This is the case for East South Centrale, West South Central and South Atlantic. The smallest gap is observed in the Mountain division, where the black share is only 3.95%. New England and Middle Atlantic seem to present different patterns, with New England having a low share of black (5.32%) and an earning gap higher than 30% and Middle Atlantic having a somewhat significant share of blacks (11.31%) and a relatively low earnings gap of less than 30%. Even if this distribution hides large disparities, since administrative divisions cover many different states (see Appendix A for a list of states per division), racial differences consistently disadvantage the black population.

3 The income decomposition framework

The decomposition of income inequality into appropriate component contributions usually follows two main paths. The first studies situations in which different sources of total income are examined,[5] while the second considers the influence of population sub-

[5]See Fei, Ranis and Kuo (1978), Shorrocks (1982), and Lerman and Yitzhaki (1985).

groups.[6] For both types of decomposition, the Shapley value has been proved useful in many applications;[7] nevertheless, the use of the Shapley decomposition rule by population subgroups such as race or gender does not lead to a clear-cut answer looking to the question of the "real" contribution of such individual characteristics. Chantreuil and Lebon (2015) resolved this problem by proposing a solution "*assimilating the different dimensions of the status of individuals to a particular wage source in order to assess the contribution of each status*". We call this proposed framework "income decomposition by attributes". Chantreuil *et al.* (2019) offers a complete methodological note on this method of decomposition (cf. Appendix B for a short presentation).[8]

This method does however not deal with a question which remained unresolved in the conditional decomposition: that of the incidence of the ordering of attributes. To properly understand the manner in which distributions associated with each attributes are built, let us consider a brief example. Let us assume 8 individuals ($i = A, B, ..., H$) for whom we know the income w_i as well as two characteristics (also referred to as attributes) x and y having each two possible modalities ($x = x_1, x_2; y = y_1, y_2$). The income of an individual i can be decomposed in two ways, as follows:

$$w_i = \overline{w}_{x_i} + (\overline{w}_{x_i,y_i} - \overline{w}_{x_i}) + (w_i - \overline{w}_{x_i,y_i}) \tag{1}$$

or

$$w_i = \overline{w}_{y_i} + (\overline{w}_{x_i,y_i} - \overline{w}_{y_i}) + (w_i - \overline{w}_{x_i,y_i}) \tag{2}$$

In equation (1) (respectively eq. (2)), the individual income w_i is written as the sum of three terms:

- the income share associated with observed attribute x (resp. y): the average income of individuals for which attribute x takes the modality x_i (resp. y_i),
- the income share associated with observed attribute y (resp. x): the difference between the average income of individuals for which both attribute x (resp. y) takes the modality x_i (resp. y_i) and attribute y (resp. x) takes the modality y_i (resp. x_i),
- the income share associated with unobserved attributes: the difference between individual i's income and the average income of individuals presenting the same modalities as the individual i for attributes x and y.

Table 2 presents an example with virtual incomes. The first part of the table displays characteristics associated with each of the 8 individuals, the second part presents the

[6]See Bourguignon (1979), Cowell (1980), Shorrocks (1980, 1984), Foster and Shorrocks (1988), or Cowell and Jenkins (1995).

[7]An incomplete list of applications of the Shapley value to inequality decomposition includes Sastre and Trannoy (2002), Israeli (2007), Bargain and Callan (2010), Devicienti (2010) and Charpentier and Mussard (2011).

[8]We use conditional decomposition (see appendix B) as opposed to pair-wise decomposition. See Ramik and PetrKorviny (2010) and Brunelli (2011) for a mathematical explanation of the inconsistency of a pair-wise comparison matrix, and why it should not be used.

income figures associated with equation (1), the third part presents the incomes figures associated with equation (2).

Table 2: Example of income disaggregation by attribute

i	w_i	x_i	y_i	$\overline{w}_{x_i}$	$\overline{w}_{x_i,y_i} - \overline{w}_{x_i}$	$w_i - \overline{w}_{x_i,y_i}$	$\overline{w}_{y_i}$	$\overline{w}_{x_i,y_i} - \overline{w}_{y_i}$	$w_i - \overline{w}_{x_i,y_i}$
A	1000	x_1	y_1	1225	-200	-25	1575	-550	-25
B	1050	x_1	y_1	1225	-200	25	1575	-550	25
C	1350	x_1	y_2	1225	200	-75	2250	-825	-75
D	1500	x_1	y_2	1225	200	75	2250	-825	75
E	2200	x_2	y_1	2600	-475	75	1575	550	75
F	2050	x_2	y_1	2600	-475	-75	1575	550	-75
G	2900	x_2	y_2	2600	475	-175	2250	825	-175
H	3250	x_2	y_2	2600	475	175	2250	825	175

This table shows that figures associated with attribute x and with attribute y differ according to the order in which attribute appears in the income decomposition. The share of income inequality (as defined in what follows by the Gini index) associated with each attribute therefore differ slightly depending on the ordering of attributes, as shown in the following results' Table 3.

Table 3: Relative contributions and Gini index

	Relative contrib. x	Relative contrib. y	Relative contrib. unobserved	Gini index
eq. (1)	60.48%	32.67%	6.85%	0.22
eq. (2)	64.67%	28.48%	6.85%	0.22
mean	62.58%	30.58%	6.85%	0.22

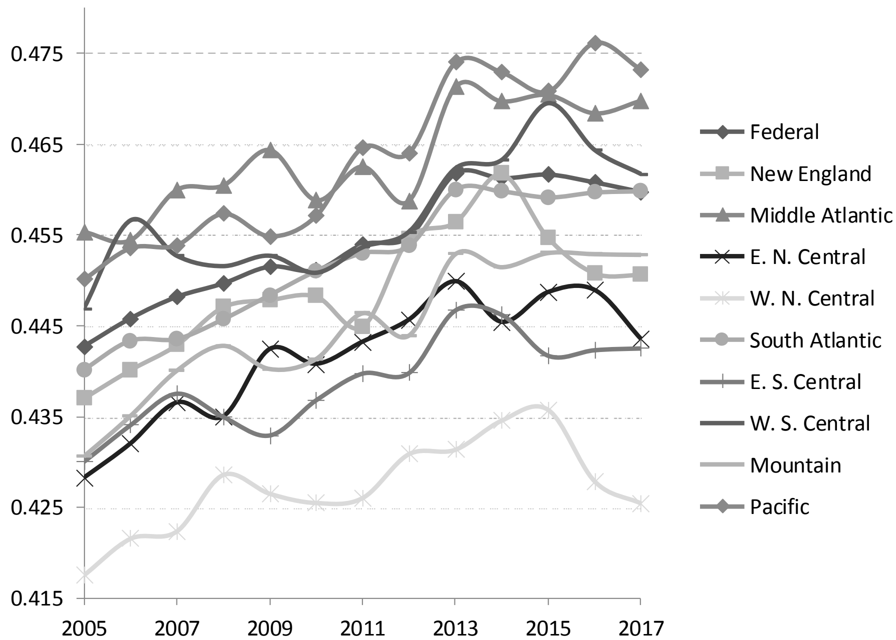
In Table 3 the Gini index is 0.22 and the relative contribution of unobserved attributes to income inequality is 6.85%, *i.e.* 6.85% of the Gini value remains associated with unobserved characteristics. The share of the observed inequality associated to attribute x is 60.48% (resp. 32.67% for attribute y) in the first decomposition and 64.67% (resp. 28.48% for attribute y) in the second decomposition. Because there is no *a priori* reason to choose one decomposition over an other, in the following we choose to evaluate the share of the observed inequality associated with each attribute as the mean of the share obtained in each decomposition ordering as shown in Table 3. We empirically discuss the incidence of the ordering of attributes in section 5.1.

4 Contributions of attributes to observed inequality

We now apply the methodology outlined above to US observations, determining the average contribution to income inequality of several individual characteristics - among which are gender and race - and their development over time. For each year between 2005 and 2017, and each geographical division, the income of each individual is treated as explicable in terms of four attributes capable of generating income differences either objectively (*i.e.* based on individual productivity), or subjectively: by age, education, gender, or race.[9] These four attributes represent 24 (=4!) permutations of the ordering of attributes that we must consider in order to obtain the mean contribution of each attribute to income inequality. Overall results are presented in appendix C.

Table 6 of appendix C reports the mean contribution of each attribute at the federal level and the geographical division level between 2005 and 2017 as a percentage of the overall observed income inequality as defined by the Gini index. As shown in Figure 1, this total income inequality measured by the Gini index varies over the period at federal level and for each administrative division.[10]

Figure 1: Change in the Gini index from 2005 to 2017



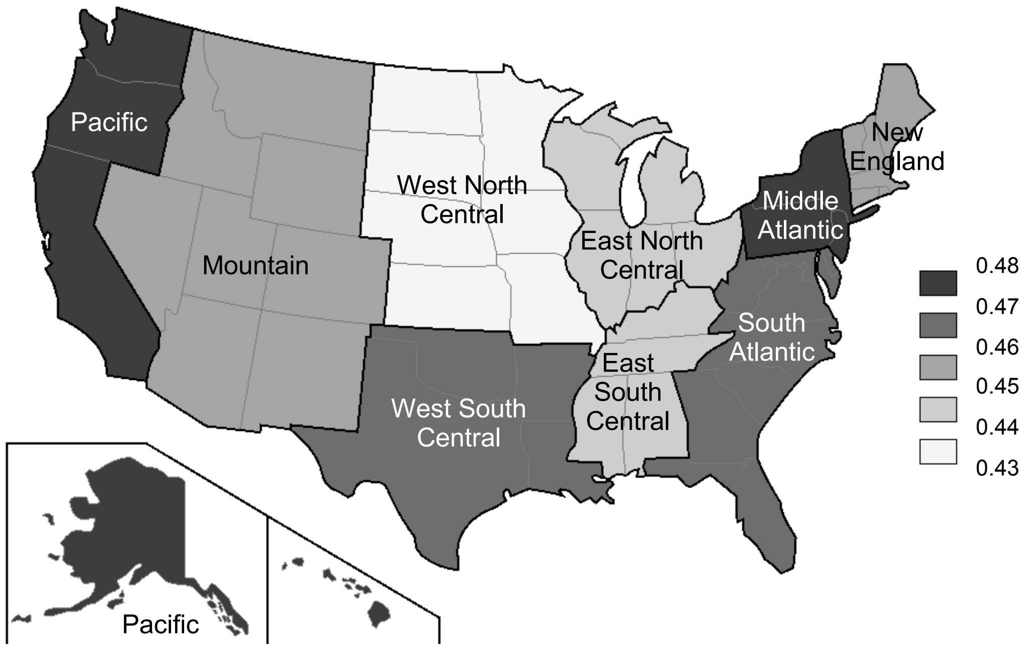
The Gini index tends to increase uniformly from 2005 to 2013-2014, irrespective of division. At the end of the period studied, inequality is tending towards a slowly reduction. This observation can be associated with the return of economic growth after the subprime crisis of 2009. Figure 1 also shows that the relative value of regional income inequality is somewhat stable over time. Figure 2 shows the regional map for 2017. It can be noted

[9]We concede that income also depends on occupation. This attribute is however excluded from the analysis - implying that the impact of occupation on income inequality is captured by the residual - due to the limited size of each subset of the sample.

[10]Reported Gini indices are larger than those observed in reality. The World Bank estimate is 0.411 for 2007 and 0.417 for 2016, while our federal index gives respectively 0.471 and 0.484. This different is mostly due to the data considered, given that we excluded from our analysis all individuals who were not "pure" blacks and whites, thus displaying larger inequalities.

that seaboard regions (which are among the most populous) are more inegalitarian, since they have a higher Gini index. West, North and East North Central as well as Mountain have the lowest Gini index.

Figure 2: Division map of Gini's index (year 2017)



What is the contribution of each attribute to this income inequality? Not surprisingly, education accounts for most of the income inequality among the four selected attributes, with a relative contribution varying from about 15% to 20% of income inequality. Then comes gender (about 9% to 14%), age (about 6% to 9%) and finally race (about 1% to 4%). Such a result shows that pure racial discrimination with respect to incomes exists, but that the observed inequality between blacks and whites is mostly linked to non-racial characteristics. Note that this pure and direct racial discrimination in terms of income has to be treated with caution, since another indirect discriminatory aspect of income difference is possibly linked to the difficulty that blacks have, for example, in securing prestigious jobs.

Figure 3 shows the time variation of each of the four attributes at the federal level, while figures 5 and 4 respectively concentrate on the change in gender and racial contribution by geographical division over time.

Figure 3: Change in the averaged relative contribution of attributes to income inequality as percentage at federal level

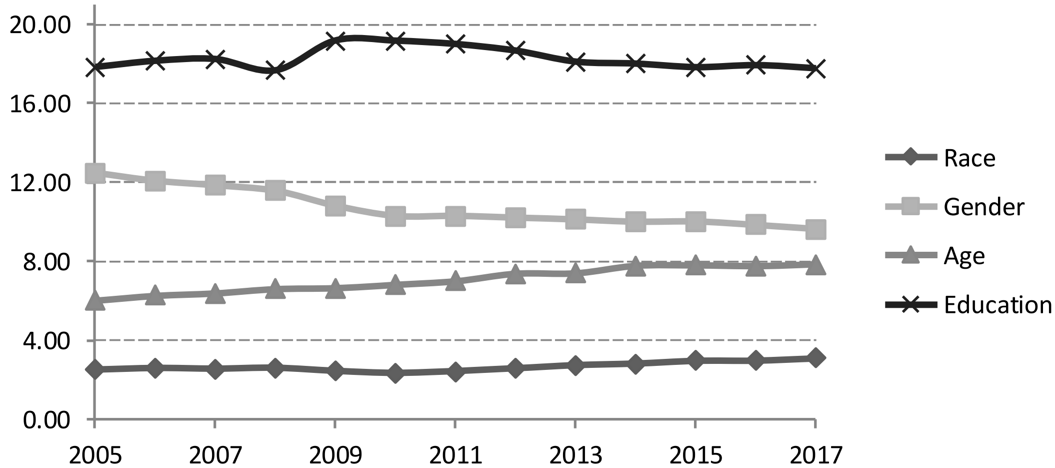


Figure 3 shows a reduction in relative gender inequality over the entire period. This trend is found in all administrative divisions (see Figure 5), with a share of inequality associated with gender diminishing by 2 to 3% (from about 12% to about 9%) between 2005 and 2017. However, from 2010 onward this reduction seems to slow at the Federal level and in most divisions. The contribution of education to income inequality is approximately the same in 2005 and in 2017 (about 18%), but is not stable over time. The subprime crisis might have induced a significant increase in income inequality associated with education, since educated workers were mostly less affected than those with lower educational qualification. This effect diminishes with the economic recovery. By contrast, the contribution of age to income inequality tends to increase, except during the three more recent years. As for race, its contribution is stable from 2005 to 2008, then less for two years before consistently increasing up to 2017. An increase in the most recent years can be observed in almost all geographical divisions (see Figure 4). It might be supposed that the rise of income inequality between blacks and whites also originates in the financial and economic crisis. However, this phenomenon did not reverse during the recovery, and even seems to be getting worse.

To sum up, the share of income inequality associated with observed attributes is mainly captured by education and age, two individual characteristics which legitimately affect earnings since they relate to productivity. Higher certification is indeed associated with higher skills, and greater age to greater experience. By contrast, gender and race are not objective criteria when dealing with productivity. These two characteristics nevertheless account for more than 12% of income inequality in a context where total income inequality of the black and white populations taken together is tending to increase. While gender income inequality falls, income inequality associated with race tends to increase.

Figure 4: Change in the averaged relative contribution of race to income inequality as percentage

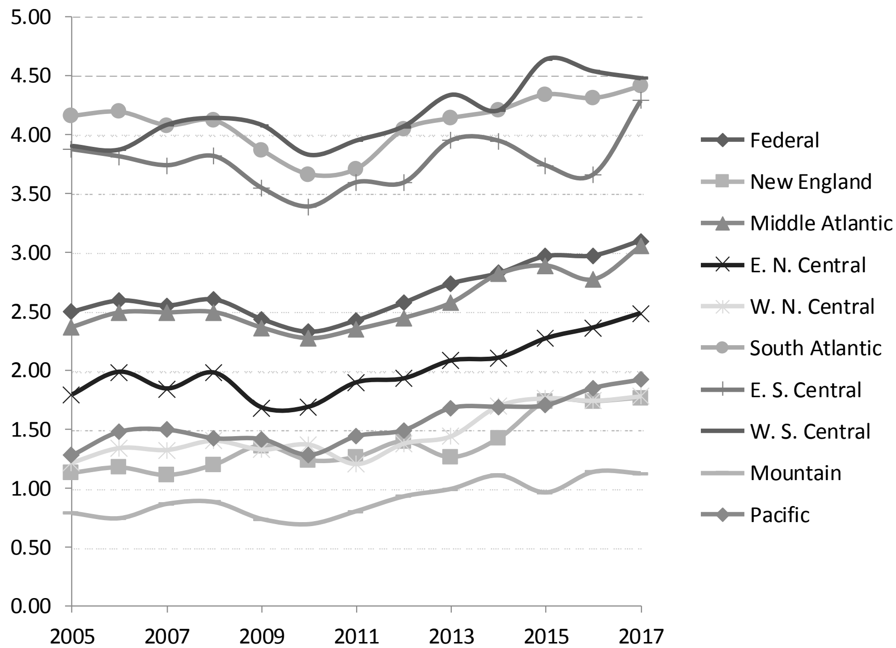
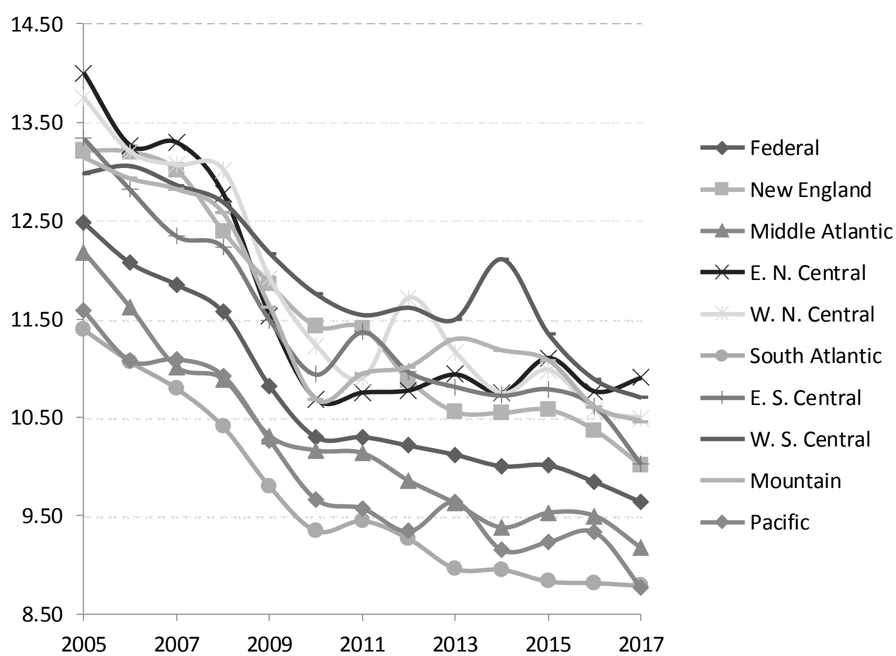


Figure 5: Change in the averaged relative contribution of gender to income inequality as a percentage

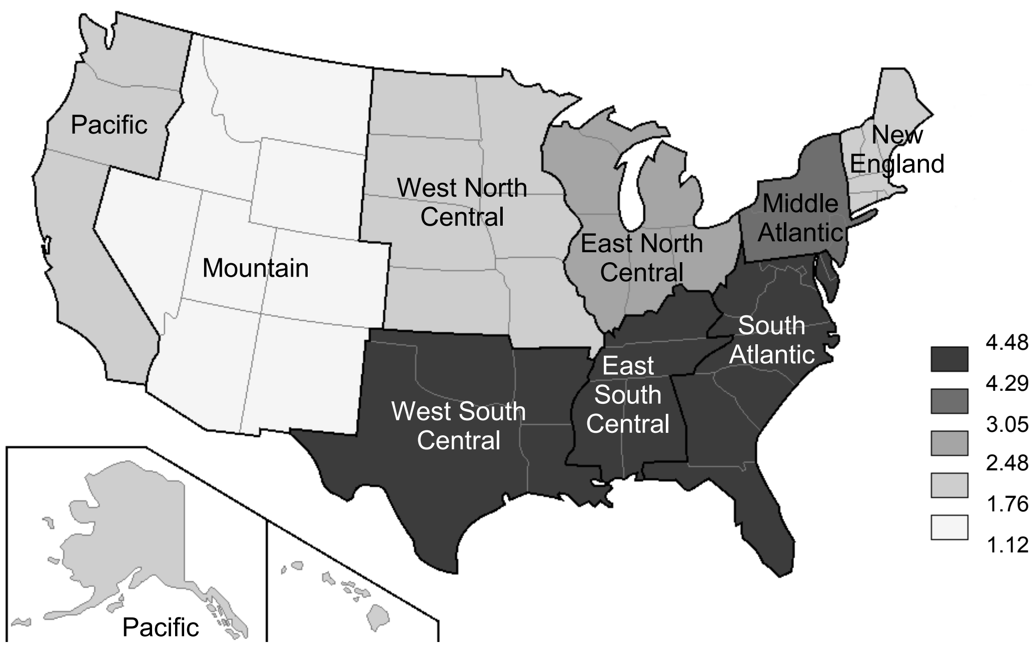


Let us concentrate on race and gender. Figures 4 and 5 allow us to observe the income inequality associated with these two attributes between 2005 and 2017. Three divisions (West South Central, East South Central and South Atlantic) show a particularly high share of income inequality associated with race, moving from about 4% in 2005 to 4.5% in

2017. By contrast, four divisions register a low contribution of race to income inequality (in 2017 less than 2% in Pacific, West North Central and New England), the lowest being Mountain with a contribution of "only" about 1%.[11] As for the gender contribution to income inequality, this falls by at least 2% in every division during the period studied. Middle Atlantic, South Atlantic and Pacific, which corresponds to most of the East and West coast, are the least affected by inequality associated with gender (lower than 10% in 2017). Other divisions display a gender share of from 10.5% to 11.5% in 2017.

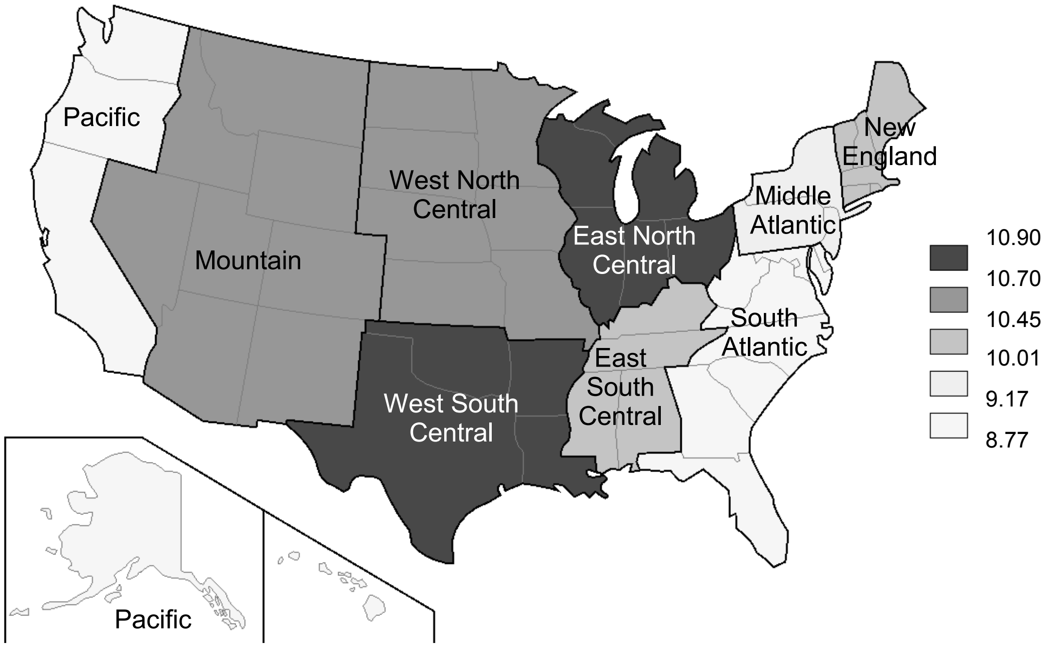
Comparing figure 2 of the Gini index map, the contribution of race (fig. 6) and that of gender (fig. 7) highlight some additional points. Coastal divisions, which are the most heavily affected by income inequality, are also the least affected by gender inequality. West South Central is the only division which has to deal with both a high level of income inequality and a large share of income inequality associated with gender. It is also one of the divisions where the contribution of race to income inequality is high.

Figure 6: Division map of contribution of race to income inequality (year 2017)



[11]Results for the Mountain division are to be treated with caution since this division has a very low share of blacks in the black and white population (3.95%, see Table 1). Studying sub-sample size shows that the Mountain division shows several sub-samples with less than 5 observations by subgroup (accounting for 223 of the 573 observations belonging to a subgroup with less than 5 observations), especially in the black population with least educational qualifications and in those with the highest.

Figure 7: Division map of contribution of gender to income inequality (year 2017)



Let us now look at the absolute contribution to income inequality as measured by the Gini index. The relative contribution is the percentage of the Gini index explained by each of the attributes. The absolute contribution is directly the "level" of inequality explained by each of the attributes. Figures 8 and 9 display for each US administrative division in 2017 the absolute contribution of race (or gender) with respect to the Gini index.

Figure 8: Absolute contribution of race (Y axis) with respect to the Gini index (X axis) (year 2017)

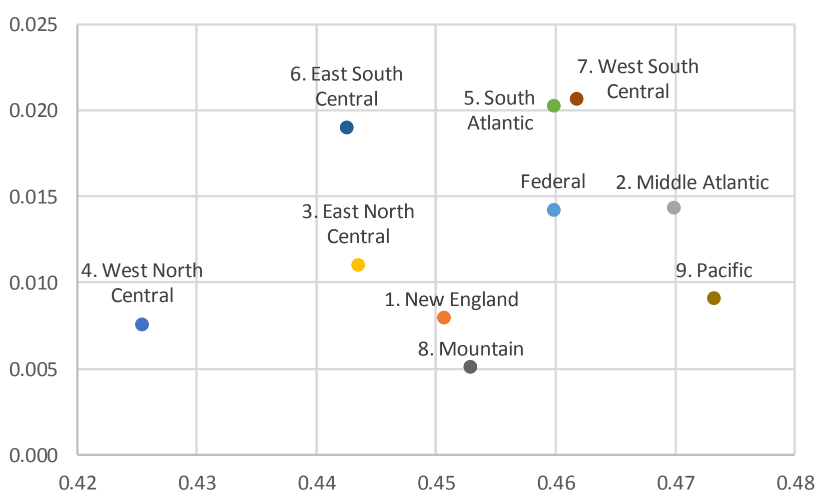
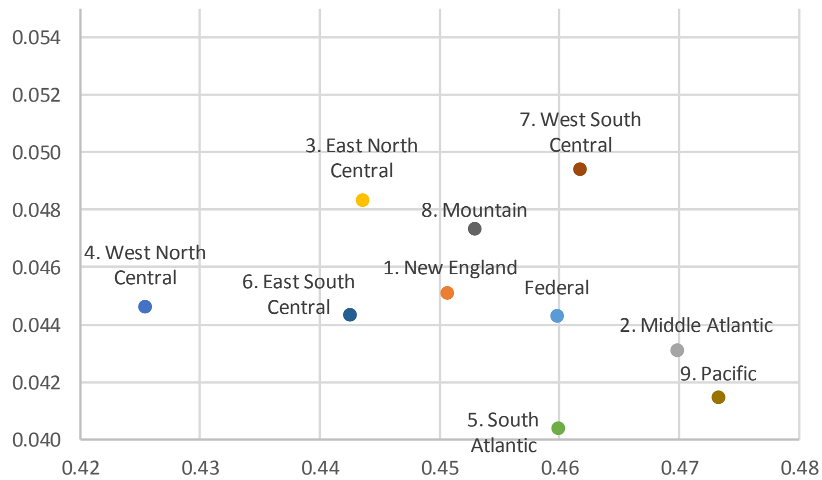


Figure 9: Absolute contribution of gender (Y axis) with respect to the Gini index (X axis) (year 2017)



These two figures highlight specific features of several divisions. West North Central is distinguished by the lowest level of inequality and also by a racial contribution that is among the lowest, with a limited gender contribution. Symmetrically, the figures from division West South Central show that a high level of inequality tends to go along with a high absolute contribution of both race and gender. Middle Atlantic and Pacific, which display the highest income inequality level, do not however conform to this trend: the absolute racial contribution to inequality is moderate, and the gender contribution is among the lowest. Finally, South Atlantic and Mountain display both a moderate level of inequality but with contrasting pattern with regard to the racial and gender contribution. In the South Atlantic division, the absolute contribution of gender is among the lowest and that of race among the highest, while in the Mountain division gender has the lowest contribution and race has a contribution among the highest.

5 Robustness

In order to evaluate the validity of our results, we first discuss the impact of the ordering of attributes for both race and gender. We note that the order does not significantly reduce the contribution of one attribute or the other. We then perform a robustness check of the results obtained in section 4 using the bootstrap percentile method.

5.1 Incidence of the ordering of attributes

As mentioned in section 3, the value associated with an attribute appears in the income disaggregation. The first attribute takes the mean value of one of its different modalities, while the following attributes are only successively associated with mean differences. As a consequence, an attribute located in the first position logically captures a higher proportion of income than the following attributes. One might think that it also captures more of the total inequality than when it has a lower position, or at the extreme. This is the case which appears in the example presented in section 3; however the results presented in table 4 show that this effect is not systemic.

Table 4 presents on the left-hand side the racial contribution when race (R) is the first attribute to be considered (then comes education (E), age (A) and gender (G) in that order) and when race is the last attribute to be considered (following education, age and gender in that order); and on the right-hand side is gender contribution when gender (G) is the first attribute and when gender is the last attribute.

Table 4: Incidence of attributes' ordering on the relative contribution of race and gender to income inequality at teh federal level, as a percentage of income inequality as defined by the Gini index

	Race		Gender	
	REAG	EAGR	GREA	REAG
2005	2.87	2.11	10.80	13.83
2006	2.98	2.18	10.19	13.58
2007	2.93	2.13	9.90	13.40
2008	2.94	2.20	9.61	13.15
2009	2.83	1.99	8.76	12.46
2010	2.73	1.87	8.29	11.89
2011	2.80	1.98	8.22	11.97
2012	2.94	2.14	8.10	11.94
2013	3.12	2.25	7.99	11.86
2014	3.23	2.32	7.85	11.80
2015	3.37	2.47	7.84	11.83
2016	3.40	2.44	7.62	11.72
2017	3.52	2.55	7.38	11.54

The results in Table 4 confirm some effect of the ordering of attributes on the relative contribution to income inequality obtained by the Shapley decomposition method. The impact of race is reduced when considered in the last position as compared with the first position. This reduction (of about 1% in 2017) does not however challenge our main finding: the income spread between blacks and whites has an impact on total inequality that is not insignificant.

Regarding gender, its position in the ordering of attributes also slightly modifies the explained part of total inequality, the magnitude of the impact remaining similar. Notice that the relative contribution of gender is higher when it is placed in the final position. This observation calls into question our previous supposition, that the first position does not automatically correspond to the maximum contribution of an attribute.

Such a result can easily be explained. Income associated with the gender attribute is spread over many more modalities when placed in the final position rather than in the first position. In fact, income is decomposed into 160 modalities (different age classes, education levels, two races, and two genders) rather than 2 modalities (male/female). If all other attributes are first taken into account, and if the gap between income values associated with men and with women is important, the gender contribution to income inequality might increase when placed in the final position.

At this stage we observe for all orders proposed in Table 4, and for those resulting from the set of all possible permutations, race and gender discriminating factors explain a significant part of the overall income inequality. Bootstrap testing will allow us to confirm the significance of these attributes.

5.2 Confidence interval

The contributions obtained via the Shapley decomposition method do not *a priori* follow a normal distribution. We therefore perform a bootstrap sampling for calculating a simple percentile confidence interval. For 2017 at the Federal level, we perform 1000 resamples with replacement of the initial database and calculate for each sample the (absolute) contribution as defined in section 4. Bootstrap Confidence Intervals are reported in Table 5.[12]

Table 5: Absolute contribution and robustness Bootstrap test, Federal 2017

	Initial	95% Bootstrap Confidence Intervals			
		lower	mean	upper	std.dev
Race	0.014235	0.014227	0.014237	0.014247	0.000165
Gender	0.044305	0.044297	0.044315	0.044332	0.000276

The 2017 absolute contribution is found to be within 95% of the CI, thus demonstrating the robustness of our results regrading the existence of the two discriminational characteristics (race and gender) when decomposing income inequalities.

6 Conclusion

Income inequality decomposition *à la* Shapley (1953) enables us to derive the contribution of an individual characteristic to total observed income inequality in the US over the period 2005-2017. An important share of this inequality is explained by determinants of productivity, *i.e.* the level of skills and the duration of professional experience that we capture via education and age. Other significant factors are less objective. We concentrate on the discriminational part of observed inequalities, that is inequalities associated with pure ethnic concerns or gender issues.

[12]Note that bootstrapping on the ACS database requires a large amount of memory and calculation time. Such a calculation would not have been possible without the use of the ATOS BULL Myria computer available at CRIANN, Normandy, France.

In a context in which income inequalities have increased at the federal level over the period 2005-2017, and for each administrative division, we show that the share of inequality explained by race has also increased. In 2017 we find that the racial contribution to income inequality is about 1% to 4% for all of the nine United States Census Bureau designated divisions. Our results confirm those of Huffman and Cohen (2004): the contribution of race to income inequalities is much higher in areas densely populated with blacks. But race is not the most important source of income discrimination. Gender indeed explains a much higher share of income inequality (about 10%) whatever the geographical division. However, unlike race, inequalities associated with gender tend to be noticeably lower between 2005 and 2017.

There seems to have been a general stability to developments in income inequality during the period of study with regard to the contribution of race and gender to this inequality; there is a general stability from one geographical division to another. This means that the relative situation of divisions persists over time. We identified geographical areas lastingly affected by strong income inequalities, with a marked impact of gender and race, such as in West South Central. By contrast, we identified a low level of inequalities, with the weak impact of the two attributes in, for instance, West North Central.

Our analysis does not account for disparities between state. Work at the state level is compromised because of the size of sub-samples. We do not expect any different results for Middle Atlantic, since the states (New York, New Jersey, Pennsylvania) are similar, but we might expect different results for instance for South Atlantic states, which include both former Confederate states (South Virginia, North and South Carolina, Georgia and Florida) and Union states (DC, Delaware, Maryland, West Virginia). One way to bypass the sub-sample size issue would be to re-aggregate states according to their economic and historical characteristics. This will not present thinner results, but will result in a smaller standard error at the division level.

Apart from our caution regarding the geographical constraint, our results have several consequences public policy. While it does exist, the direct impact of race on income remains moderate. This means that the observed income gap between blacks and whites also find its origins in characteristics with which ethnic minorities are associated (low education, poverty, etc.), rather than being a matter of race in itself. In terms of public policy, such a result places in question affirmative action in public institutions in general, but would make them more important in education. Our work also highlights the significant contribution of gender to income inequality, justifying public action in favor of gender equality.

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A United States Census Bureau designated divisions

1. **New England**
   (ME) Maine
   (NH) New Hampshire
   (VT) Vermont
   (MA) Massachusetts
   (RI) Rhode Island
   (CT) Connecticut

2. **Middle Atlantic**
   (NY) New York
   (NJ) New Jersey
   (PA) Pennsylvania

3. **East North Central**
   (OH) Ohio
   (IN) Indiana
   (IL) Illinois
   (MI) Michigan
   (WI) Wisconsin

4. **West North Central**
   (MN) Minnesota
   (IA) Iowa
   (MO) Missouri
   (ND) North Dakota
   (SD) South Dakota
   (NE) Nebraska
   (KS) Kansas

5. **South Atlantic**
   (DE) Delaware
   (MD) Maryland
   (DC) District of Columbia
   (VA) Virginia
   (WV) West Virginia
   (NC) North Carolina
   (SC) South Carolina
   (GA) Georgia
   (FL) Florida

6. **East South Central**
   (KY) Kentucky
   (TN) Tennessee
   (AL) Alabama
   (MS) Mississippi

7. **West South Central**
   (AR) Arkansas
   (LA) Louisiana
   (OK) Oklahoma
   (TX) Texas

8. **Mountain**
   (MT) Montana
   (ID) Idaho
   (WY) Wyoming
   (CO) Colorado
   (NM) New Mexico
   (AZ) Arizona
   (UT) Utah
   (NV) Nevada

9. **Pacific**
   (WA) Washington
   (OR) Oregon
   (CA) California
   (AK) Alaska
   (HI) Hawaii

B Methodological appendix

Let us consider an income distribution $X = (x_1, x_2, ..., x_n)$ among a set of individuals $N = \{1, ..., i, ..., n\}$ and a set of attributes $A = \{1, ..., j, ..., a\}$ such as age, level of education or race. If the overall income inequality is measured by an inequality index I, such that the value of zero is assigned to an equal income distribution, the contribution of attribute j to the overall inequality $I(X)$ computed with the Shapley decomposition rule is defined by the following formula:

$$Sh_j(X, A, I) = \sum_{S \subseteq A, j \in S} \frac{(s-1)!(a-s)!}{a!} \Big[I\big(Y(S)\big) - I\big(Y(S - \{j\})\big) \Big] \tag{3}$$

with s the cardinality of S, a the cardinality of A and $S \in 2^A$ a subset of the set of attributes A.

By convention, for $S = \emptyset$, $Y(S) = 0$ and for all $S \in 2^A$, $S \neq \emptyset$, $Y(S)$ is defined as follows:

$$Y(S) = \left(\sum_{j \in S} y_1^j + \sum_{\substack{h \notin S \\ h \in A}} \frac{\sum_{i=1}^n y_i^h}{n}, ..., \sum_{j \in S} y_i^j + \sum_{\substack{h \notin S \\ h \in A}} \frac{\sum_{i=1}^n y_i^h}{n}, ..., \sum_{j \in S} y_n^j + \sum_{\substack{h \notin S \\ h \in A}} \frac{\sum_{i=1}^n y_i^h}{n} \right) \tag{4}$$

$Y(S)$ is thus the income distribution obtained from the income distribution $Y(A)$ when the shares of income related to the attributes $h \notin S$ are equally distributed among individuals.

Equation 4 refers to what Chantreuil and Trannoy (2011) defines as an equalized inequality game. Sastre and Trannoy (2002), as part of the analysis of income inequality in the US and UK, show that this framework is the one to be implemented in decomposing income inequality by sources.

The distribution $Y(A)$ according to the set of attributes A can be derived from the distribution of income X using the conditional decomposition by attributes. Such a method is a generalization of the framework proposed by Chantreuil and Lebon (2015) for the case in which more than two attributes have to be accounted for.

In order to present formally the two approaches, let us introduce the following notations. We consider that each attribute $j \in A$ has $m(j)$ modalities, such that $1 \leq k_j \leq m(j)$, where k_j is the k-th modality of the attribute j.

The number of individuals who have the k-th modality of the attribute j is denoted by n_{k_j} such that

$$\sum_{k_j=1}^{m(j)} n_{k_j} = n \tag{5}$$

The number of individuals who have the k-th modality of the attribute 1 and the k-th modality of the attribute 2 is denoted by n_{k_1,k_2} such that

$$\sum_{k_2=1}^{m(2)} n_{k_1,k_2} = n_{k_1} \tag{6}$$

The number of individuals who have the k-th modality of all attribute 1 to a is denoted by $n_{k_1,...,k_j,...,k_a}$ such that

$$\sum_{k_a=1}^{m(a)} n_{k_1,...,k_j,...,k_a} = n_{k_1,...,k_j,...,k_{a-1}} \tag{7}$$

The income of individuals $i \in N$ who have the k-th modality of the attribute j is denoted by $x_i^{k_j}$.

The income of individuals $i \in N$ who have the k-th modality of the attribute 1 and the k-th modality of attribute 2 is denoted by $x_i^{k_1,k_2}$.

The income of an individual $i \in N$ who has the k-th modality of all attributes 1 to a is denoted $x_i^{k_1,...,k_j,...,k_a}$.

The income distribution $Y(A)$ is based on the assumption that the set of attributes is ranked by order of importance from 1 to a. Given this ranking of the individuals' attributes, the share of income of an individual i coming from the attribute 1 is defined as the average income of individuals who have the same attribute 1's modality and the share of income of an individual coming from attribute j is defined as the average income of individuals who have the same sequence of modalities for all attributes from 1 to j.

Thus the distribution $Y(A) = (y_1, ..., y_i, ..., y_n)$ is such that for all $i \in N$

$$y_i = \sum_{j=1}^{a} y_i^j + \left[y_i - \sum_{j=1}^{a} y_i^j \right] \tag{8}$$

where

$$y_i^1 = \left[\frac{\sum_{k_1=1}^{m(1)} x_i^{k_1}}{n_{k_1}} \right] \tag{9}$$

$$y_i^2 = \left[\frac{\sum_{k_2=1}^{m(2)} x_i^{k_1,k_2}}{n_{k_1,k_2}} - \frac{\sum_{k_1=1}^{m(1)} x_i^{k_1}}{n_{k_1}} \right] \tag{10}$$

$$y_i^j = \left[\frac{\sum_{k_j=1}^{m(j)} x_i^{k_1,...,k_j}}{n_{k_1,...,k_j}} - \frac{\sum_{k_{j-1}=1}^{m(j-1)} x_i^{k_1,...,k_{j-1}}}{n_{k_1,...,k_{j-1}}} \right] \tag{11}$$

and

$$y_i^a = \left[\frac{\sum_{k_a=1}^{m(a)} x_i^{k_1,...,k_a}}{n_{k_1,...,k_a}} - \frac{\sum_{k_{a-1}=1}^{m(a-1)} x_i^{k_1,...,k_{a-1}}}{n_{k_1,...,k_{a-1}}} \right] \tag{12}$$

From the previous equations, for all $i \in N$ we thus have

$$\begin{aligned} y_i \;=\; & \left[\frac{\sum_{k_1=1}^{m(1)} x_i^{k_1}}{n_{k_1}} \right] + ... + \left[\frac{\sum_{k_j=1}^{m(j)} x_i^{k_1,...,k_j}}{n_{k_1,...,k_j}} - \frac{\sum_{k_{j-1}=1}^{m(j-1)} x_i^{k_1,...,k_{j-1}}}{n_{k_1,...,k_{j-1}}} \right] \\ & +... + \left[\frac{\sum_{k_a=1}^{m(a)} x_i^{k_1,...,k_a}}{n_{k_1,...,k_a}} - \frac{\sum_{k_{a-1}=1}^{m(a-1)} x_i^{k_1,...,k_{a-1}}}{n_{k_1,...,k_{a-1}}} \right] + \left[y_i - \frac{\sum_{k_a=1}^{m(a)} x_i^{k_1,...,k_a}}{n_{k_1,...,k_a}} \right] \end{aligned} \tag{13}$$

C Averaged relative contribution of attributes

Table 6: Averaged relative contribution of attributes to income inequality as percentage and Gini index

Geog., year	Race	Gender	Age	Education	Unobserved	GINI index
Federal, 2005	2.50	12.47	6.03	17.84	61.16	0.4427
Federal, 2006	2.59	12.07	6.28	18.15	60.91	0.4458
Federal, 2007	2.55	11.85	6.40	18.24	60.97	0.4482
Federal, 2008	2.60	11.57	6.61	17.67	61.55	0.4496
Federal, 2009	2.44	10.82	6.66	19.18	60.92	0.4515
Federal, 2010	2.33	10.30	6.82	19.16	61.38	0.4511
Federal, 2011	2.43	10.30	7.00	18.99	61.28	0.4539
Federal, 2012	2.57	10.21	7.37	18.66	61.18	0.4548
Federal, 2013	2.74	10.12	7.40	18.11	61.64	0.4618
Federal, 2014	2.82	10.00	7.77	18.01	61.40	0.4613
Federal, 2015	2.97	10.01	7.79	17.83	61.39	0.4617
Federal, 2016	2.97	9.84	7.76	17.93	61.50	0.4608
Federal, 2017	3.10	9.64	7.85	17.77	61.65	0.4598
Division 1, 2005	1.13	13.21	6.09	18.45	61.12	0.4370
Division 1, 2006	1.18	13.20	6.14	18.30	61.19	0.4401
Division 1, 2007	1.11	13.01	6.73	18.47	60.68	0.4429
Division 1, 2008	1.20	12.39	6.92	18.09	61.40	0.4471
Division 1, 2009	1.36	11.85	7.13	19.56	60.10	0.4479
Division 1, 2010	1.24	11.42	7.20	19.40	60.74	0.4483
Division 1, 2011	1.26	11.41	7.37	19.36	60.60	0.4449
Division 1, 2012	1.40	10.88	7.80	19.23	60.70	0.4546
Division 1, 2013	1.26	10.55	8.04	18.90	61.24	0.4564
Division 1, 2014	1.42	10.55	8.50	18.62	60.92	0.4617
Division 1, 2015	1.73	10.57	8.27	18.44	60.99	0.4547
Division 1, 2016	1.73	10.37	8.54	18.11	61.25	0.4508
Division 1, 2017	1.76	10.01	8.43	18.70	61.10	0.4506
Division 2, 2005	2.37	12.18	5.48	18.92	61.05	0.4553
Division 2, 2006	2.49	11.62	5.55	19.30	61.04	0.4544
Division 2, 2007	2.49	11.01	5.62	19.34	61.55	0.4600
Division 2, 2008	2.49	10.89	6.07	18.52	62.03	0.4605
Division 2, 2009	2.36	10.31	6.33	19.96	61.04	0.4644
Division 2, 2010	2.27	10.16	6.15	19.72	61.70	0.4588
Division 2, 2011	2.35	10.14	6.50	19.42	61.61	0.4625
Division 2, 2012	2.44	9.85	6.71	19.24	61.75	0.4588
Division 2, 2013	2.57	9.63	7.13	18.85	61.83	0.4713
Division 2, 2014	2.81	9.38	7.35	18.59	61.87	0.4697
Division 2, 2015	2.89	9.53	7.54	18.73	61.31	0.4704
Division 2, 2016	2.77	9.50	7.48	18.94	61.32	0.4684
Division 2, 2017	3.05	9.17	7.47	18.39	61.92	0.4698
Division 3, 2005	1.79	14.00	6.18	17.37	60.65	0.4282
Division 3, 2006	1.98	13.26	6.74	17.89	60.13	0.4320
Division 3, 2007	1.84	13.30	6.64	18.42	59.81	0.4365
Division 3, 2008	1.98	12.78	6.60	17.58	61.07	0.4350
Division 3, 2009	1.68	11.53	6.63	19.52	60.64	0.4424
Division 3, 2010	1.69	10.68	6.95	19.60	61.08	0.4408
Division 3, 2011	1.89	10.75	7.39	19.48	60.49	0.4432
Division 3, 2012	1.93	10.78	7.78	18.73	60.79	0.4456
Division 3, 2013	2.08	10.94	7.48	18.53	60.98	0.4499
Division 3, 2014	2.10	10.75	7.84	18.34	60.97	0.4454
Division 3, 2015	2.27	11.10	7.72	18.10	60.80	0.4487
Division 3, 2016	2.36	10.76	7.54	18.26	61.07	0.4489
Division 3, 2017	2.48	10.90	7.72	18.17	60.72	0.4435
Division 4, 2005	1.21	13.75	6.44	16.33	62.27	0.4175
Division 4, 2006	1.34	13.19	6.32	16.88	62.27	0.4215
Division 4, 2007	1.32	13.06	6.66	17.01	61.95	0.4223
Division 4, 2008	1.40	13.02	6.47	16.70	62.42	0.4286
Division 4, 2009	1.33	11.91	7.15	18.01	61.61	0.4265
Division 4, 2010	1.37	11.22	7.12	18.32	61.97	0.4255
Division 4, 2011	1.20	10.90	7.45	17.70	62.75	0.4260
Division 4, 2012	1.38	11.71	7.22	17.44	62.25	0.4308
Division 4, 2013	1.44	11.15	7.51	16.84	63.05	0.4313

Division 4, 2014	1.70	10.74	7.82	16.86	62.89	0.4345
Division 4, 2015	1.76	10.98	7.47	17.27	62.52	0.4357
Division 4, 2016	1.74	10.57	7.55	17.12	63.02	0.4279
Division 4, 2017	1.78	10.49	7.22	17.35	63.16	0.4254
Division 5, 2005	4.16	11.39	5.88	18.84	59.74	0.4401
Division 5, 2006	4.19	11.06	6.15	18.90	59.70	0.4433
Division 5, 2007	4.08	10.79	6.21	19.22	59.71	0.4435
Division 5, 2008	4.12	10.41	6.36	18.76	60.36	0.4457
Division 5, 2009	3.86	9.79	6.31	20.49	59.54	0.4484
Division 5, 2010	3.66	9.35	6.56	20.10	60.33	0.4510
Division 5, 2011	3.71	9.45	6.60	20.26	59.99	0.4530
Division 5, 2012	4.04	9.26	7.31	19.75	59.64	0.4538
Division 5, 2013	4.14	8.96	7.41	19.31	60.18	0.4600
Division 5, 2014	4.21	8.95	7.95	19.23	59.66	0.4598
Division 5, 2015	4.34	8.83	8.01	18.78	60.04	0.4591
Division 5, 2016	4.31	8.82	8.05	18.58	60.25	0.4597
Division 5, 2017	4.41	8.79	8.27	18.30	60.23	0.4598
Division 6, 2005	3.87	13.34	7.00	17.10	58.68	0.4300
Division 6, 2006	3.81	12.81	6.90	17.53	58.94	0.4340
Division 6, 2007	3.74	12.34	7.16	17.81	58.95	0.4375
Division 6, 2008	3.81	12.23	7.22	17.22	59.51	0.4350
Division 6, 2009	3.54	11.49	6.97	18.36	59.65	0.4329
Division 6, 2010	3.38	10.94	7.21	18.39	60.09	0.4368
Division 6, 2011	3.59	11.37	7.33	17.87	59.85	0.4397
Division 6, 2012	3.59	10.96	7.44	18.36	59.64	0.4398
Division 6, 2013	3.95	10.81	7.78	17.30	60.16	0.4467
Division 6, 2014	3.95	10.72	7.99	17.10	60.24	0.4462
Division 6, 2015	3.73	10.79	8.23	16.58	60.68	0.4417
Division 6, 2016	3.66	10.61	8.37	16.73	60.63	0.4423
Division 6, 2017	4.29	10.02	8.10	16.99	60.60	0.4425
Division 7, 2005	3.90	12.98	6.24	17.25	59.63	0.4469
Division 7, 2006	3.87	13.05	6.47	17.46	59.16	0.4566
Division 7, 2007	4.08	12.86	6.74	17.14	59.19	0.4527
Division 7, 2008	4.14	12.68	7.26	16.02	59.90	0.4516
Division 7, 2009	4.08	12.16	6.87	17.35	59.53	0.4527
Division 7, 2010	3.83	11.75	7.40	17.44	59.58	0.4508
Division 7, 2011	3.95	11.54	7.20	17.44	59.87	0.4536
Division 7, 2012	4.07	11.61	7.47	16.86	59.99	0.4553
Division 7, 2013	4.33	11.50	7.08	16.27	60.82	0.4623
Division 7, 2014	4.20	12.11	7.35	16.07	60.27	0.4633
Division 7, 2015	4.64	11.35	7.60	16.07	60.35	0.4695
Division 7, 2016	4.53	10.88	7.48	16.53	60.57	0.4644
Division 7, 2017	4.48	10.70	7.63	16.17	61.02	0.4617
Division 8, 2005	0.79	13.14	6.54	14.59	64.94	0.4306
Division 8, 2006	0.75	12.93	6.92	14.39	65.02	0.4351
Division 8, 2007	0.87	12.82	6.68	14.66	64.97	0.4401
Division 8, 2008	0.88	12.58	7.46	14.41	64.67	0.4428
Division 8, 2009	0.74	11.62	7.48	15.97	64.19	0.4403
Division 8, 2010	0.70	10.67	7.47	16.46	64.71	0.4414
Division 8, 2011	0.81	10.94	7.55	15.93	64.78	0.4464
Division 8, 2012	0.93	11.01	7.43	16.20	64.43	0.4440
Division 8, 2013	0.99	11.29	7.76	14.97	64.98	0.4529
Division 8, 2014	1.11	11.18	8.09	15.45	64.17	0.4515
Division 8, 2015	0.96	11.07	8.24	15.35	64.37	0.4531
Division 8, 2016	1.14	10.60	8.41	15.39	64.45	0.4529
Division 8, 2017	1.12	10.45	9.06	14.96	64.40	0.4529
Division 9, 2005	1.28	11.58	6.57	15.91	64.66	0.4501
Division 9, 2006	1.48	11.07	6.92	16.89	63.64	0.4535
Division 9, 2007	1.50	11.09	7.25	16.14	64.02	0.4539
Division 9, 2008	1.42	10.92	7.54	16.06	64.06	0.4574
Division 9, 2009	1.41	10.27	7.53	17.25	63.54	0.4549
Division 9, 2010	1.28	9.66	7.59	17.69	63.78	0.4571
Division 9, 2011	1.44	9.57	7.91	17.71	63.37	0.4646
Division 9, 2012	1.49	9.34	8.78	17.32	63.07	0.4640
Division 9, 2013	1.68	9.64	8.45	16.81	63.42	0.4740

Division 9, 2014	1.69	9.15	8.97	16.72	63.47	0.4729
Division 9, 2015	1.70	9.23	9.06	16.26	63.74	0.4708
Division 9, 2016	1.85	9.33	8.80	16.68	63.35	0.4762
Division 9, 2017	1.92	8.77	8.98	16.44	63.90	0.4732