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ABSTRACT

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We offer an explanation for the inconclusive results of empirical studies into the relationship between the magnitude of the Gini coefficient of income distribution at origin and the intensity of migration. Bearing in mind the substantial literature that identifies relative deprivation as an important determinant of migration behavior, we study the relationship between aggregate or total relative deprivation, TRD, the Gini coefficient, G, and migration. We show that for a given change of incomes, TRD and G can behave differently. We present examples where, in the case of universal increases in incomes, TRD increases while G does not change; G decreases while TRD does not change; and G decreases while TRD increases. We generalize these examples into formal criteria, providing sufficient conditions on the initial and final income vectors under which incongruence between the directions of changes of G and of TRD occur. Our analysis leads us to infer that when the incentive to migrate increases with TRD, then this response can co-exist with no change of G or with a decrease of G.

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1. Introduction

The intense interest in the causes and consequences of income inequality did not miss the intriguing topic of how measures or dimensions of income inequality contribute to the incidence of migration. Comprehensive evidence for the positive relationship between income inequality at origin and the intensity of migration assembled by Liebig and Sousa-Poza (2004), led Stark (2006) to outline an analytical-behavioral foundation for the relationship. Stark showed that when the aggregate income of a population is held constant, income inequality within the population, as measured by the Gini coefficient, is in functional terms positively related to Total Relative Deprivation, *TRD*, in the population, and he inferred that the positive relationship between income inequality at origin and migration documented by Liebig and Sousa-Poza can easily be confused with the true relationship, which is between *TRD* at origin and migration.

It is worth mentioning that considerable empirical evidence finds that relative deprivation is a statistically significant explanatory variable of migration behavior. Stark and Taylor (1991) show that relative deprivation increases the probability that household members will migrate from rural Mexico to the US. More recently, Quinn (2006) reports that relative deprivation is a significant motivating factor in domestic migration decisions in Mexico. Czaika (2012) finds that in India relative deprivation is an important factor in deciding whether a household member should migrate, especially over a short distance. Basarir (2012) observes that people in Indonesia are willing to bear a loss of absolute wealth if there is a relative wealth gain from migration. Jagger et al. (2012) report that relative deprivation is a significant explanatory variable of circular migration in Uganda. Vernazza (2013) concludes that, even though interstate migration in the US confers substantial increases in absolute income, the trigger for migration is relative deprivation (low relative income), not low absolute income. Drawing on data from the 2000 US census, Flippen (2013) shows that both black and white people who migrate from the North to the South generally have average lower absolute incomes than their stationary northern peers, yet enjoy significantly lower relative deprivation, and that the relative deprivation gains for black people are substantially larger than those for white people. Hyll and Schneider (2014) use a data set collected in the German Democratic Republic in 1990 to show that aversion to relative deprivation enhanced the propensity to migrate to western Germany. Kafle et al. (2018) use comparable longitudinal data from integrated household and agriculture surveys from Tanzania,

Ethiopia, Malawi, Nigeria, and Uganda, and find that wealth relative deprivation is positively associated with migration.

More recent studies than the inquiry by Liebig and Sousa-Poza do not yield an unequivocal verdict regarding the sign of the relationship between income inequality at origin, as measured by the Gini coefficient, and migration. Using 2005 Eurobarometer data for 25 EU countries, Fouarge and Ester (2007) find that income inequality at origin has a positive effect on the intention to migrate among individuals with average education, whereas no such effect is observed among less educated and highly educated individuals. Using the same 2005 Eurobarometer data for a smaller sample of the ten newly admitted countries of the 2004 EU enlargement, Zaiceva and Zimmermann (2008) find that the impact of income inequality at origin on the intention to migrate is positive and highly significant. Stark et al. (2009) find that regional income inequality in Poland had a positive impact on international migration from Poland and on interregional migration within Poland between 1999 and 2005. Drawing on annual data from several sources for 1975-2005, Agbola and Acupan (2010) do not find conclusive evidence that income inequality in the Philippines affects the migration decisions of its population. Based on averaged data from 1990s for a large set of developed and developing countries, Czaika (2013) argues that income inequality at origin is, if at all, negatively correlated with total migration rates. Mihi-Ramirez et al. (2017) find a positive effect of income inequality at origin on net migration among the rich countries of the EU 28, but no effect whatsoever among the poor countries of the EU 28. The overall impression that emerges from these results appears to be that the sign of the relationship between income inequality at origin, as measured by the Gini coefficient, and migration behavior can be any.

In view of this incongruence, we argue that whereas *TRD* measures a real inequality-based incentive for migration, the Gini coefficient does not. The potential influence of the Gini coefficient on migration behavior arises from its relationship with *TRD*. To delineate the distinction between *TRD* and the Gini coefficient as measures of inequality, we take a step beyond the analysis of Stark (2006), relaxing the assumption of a constant total income at origin. To obtain sharp results, we allow all incomes at origin to increase and then, depending on the precise characterization of the increase in incomes, we draw a distinction between the effect of this increase on the Gini coefficient and on *TRD*. What we find is that a proportional increase in

all incomes will have no effect on the Gini coefficient while it increases *TRD*; that a uniform increase in all incomes will reduce the Gini coefficient while it has no effect on *TRD*; and that a mixed (partly proportional, partly uniform) increase in all incomes will reduce the Gini coefficient while it increases *TRD*. Consequently, if the incentive to migrate increases when *TRD* at origin rises, holding constant all the relevant characteristics of the migration destination, then the increase of the inclination to migrate can co-exist not only with an increase of the Gini coefficient at origin but also with no change of the Gini coefficient at origin or with a decrease of the Gini coefficient at origin.

In Section 2 we sketch briefly the relationship between the Gini coefficient and *TRD*. This formulation serves as a background to the assessment in Section 3 of the possible repercussions of an increase in total income for the Gini coefficient, and for *TRD*. Section 4 summarizes and concludes.

2. The relationship between the Gini coefficient and Total Relative Deprivation

Let $V^n \subset \mathbb{R}^n$ be a set of ordered vectors, namely

$$V^n = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : x_1 \le x_2 \le \ldots \le x_n\}.$$

By *TI* ("Total Income") we delineate the aggregate or total income of a given population, namely for any $x = (x_1, x_2, \ldots, x_n) \in V^n$:

$$TI(x) = x_1 + x_2 + \ldots + x_n.$$

Let the *RD* ("Relative Deprivation") of individual i, $i = 1, 2, \ldots, n-1$, whose income is x_i be

$$RD(x_i) = \frac{1}{n} \sum_{j=i+1}^{n} (x_j - x_i),$$

and for individual n let $RD(x_n) = 0$.[1] By *TRD* we delineate the sum of the levels of relative deprivation of the members of a population, namely for any $x = (x_1, x_2, \ldots, x_n) \in V^n$:

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[1] This formula for *RD* can be taken, for example, from Stark (2006).

$$TRD(x) = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i)}{n}.$$

For any $x = (x_1, x_2, \ldots, x_n) \in V^n$, the Gini coefficient of income distribution of a population, $G(x)$, is [2]

$$G(x) = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i)}{n \sum_{i=1}^{n} x_i} = \frac{TRD(x)}{TI(x)}.$$

As can be seen at once, for a given $TI(x)$, $G(x)$ and $TRD(x)$ are positively related. It is on this basis that Stark (2006) sought to explain the positive relationship between the Gini coefficient at origin and the intensity of migration. Conjecturing that the incentive to migrate in a population is positively related to TRD in the population, and establishing that the Gini coefficient and TRD go hand in hand when TI is held constant, Stark argued that the Gini coefficient and migration are positively related.

3. Changing total income: Repercussions for the Gini coefficient and for Total Relative Deprivation

Relaxing the assumption of a constant TI enables us to extend the analysis of Stark (2006) by studying the consequences of varying TI for the nature of the relationship between the Gini coefficient, G, and TRD. For illustrative purposes, we first consider three examples referring, for

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[2] According to Sen (1977), the Gini coefficient for $x \in \mathbb{R}^n$ can be expressed as $G(x) = \dfrac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{2n \sum_{i=1}^{n} x_i}$. For $x \in V^n$

this formula can be rewritten as $G(x) = \dfrac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i)}{n \sum_{i=1}^{n} x_i}$. Taking into account that $TI(x) = \sum_{i=1}^{n} x_i$, and that

$TRD(x) = \dfrac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i)}{n}$, we get that $G(x) = \dfrac{TRD(x)}{TI(x)}$.

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simplicity's sake, to the case of two individuals. In the examples, incomes increase, so we can consider the constellation of income gains as a growth experience. However, across the examples the increase of incomes is distributed differently.

Scenario 1.

Incomes increase, *G* remains unchanged, yet *TRD* increases. For example:

$(1, 3) \rightarrow (2, 6)$.

Here incomes increase; *G* remains the same at 1/4; and *TRD* increases from 1 to 2.

Scenario 2.

Incomes increase, *G* decreases, yet *TRD* remains the same. For example:

$(1, 3) \rightarrow (2, 4)$.

Here incomes increase; *G* declines from 1/4 to 1/6; and *TRD* is the same at 1.

Scenario 3.

Incomes increase, *G* decreases, yet *TRD* increases. For example:

$(1, 3) \rightarrow (2, 5)$.

Here incomes increase; *G* decreases from 1/4 to 3/14; and *TRD* increases from 1 to 3/2.[3]

 These three scenarios suggest that an increase in all incomes at origin may affect *G* but not *TRD*; it may affect *TRD* but not *G*; or it may affect both *G* and *TRD* such that they move in opposite directions. These results might appear puzzling because both *G* and *TRD* measure income inequality. The reason behind this conundrum is that whereas *TRD* is "locally linear" with respect to the ordered incomes, the Gini coefficient is not linear. Therefore, these two indices do not measure the same type of inequality. We may say that whereas *TRD* measures the "absolute inequality" in a population, the Gini coefficient measures the "relative inequality" with

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[3] When considered "in reverse," namely when incomes decrease rather than increase, the three scenarios yield three additional possibilities for the relationship between *TRD* and *G*. For example, a reversed third scenario, namely (2, 5) → (1, 3), shows that it is possible that at the same time *TRD* decreases and *G* increases. We do not consider these possibilities as separate scenarios because they are the "dual" of the presented scenarios.

respect to the aggregate income of the population. Thus, it is not surprising that the two indices "behave" differently.

In the form of a claim, we next provide a generalization of Scenarios 1, 2, and 3.

Claim 1.

Let $x, y \in V^n$. Consider a population with an ordered vector of incomes x that changes to an ordered vector of incomes y. We have that:

(a) A constant rate of income growth for every individual, namely the same proportional income growth for every individual.

If there exists $a \in \mathbb{R}_+, a > 1$ such that $y = ax$, then

$$TRD(y) > TRD(x); \ TI(y) > TI(x); \ G(y) = G(x).$$

(b) Every individual receives the same lump sum income transfer.

If there exists $b \in \mathbb{R}_+$ such that $y = x + (b, b, \ldots, b)$, then

$$TRD(y) = TRD(x); \ TI(y) > TI(x); \ G(y) < G(x).$$

(c) Every individual receives a mix of a proportional income growth and a lump sum income transfer.

If there exists $a, b \in \mathbb{R}_+, a > 1$ such that $y = ax + (b, b, \ldots, b)$, then

$$TRD(y) > TRD(x); \ TI(y) > TI(x); \ G(y) < G(x).$$

Proof. In the Appendix.

Naturally, Claim 1 (a) is a generalization of Scenario 1 (where $a = 2$), Claim 1 (b) is a generalization of Scenario 2 (where $b = 1$), and Claim 1 (c) is a generalization of Scenario 3 (where $a = 3/2$ and $b = 1/2$).

A sufficient condition for an increase in incomes of all the individuals to co-exist with an increase of total relative deprivation and a decrease of the Gini coefficient is provided in the next claim.

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Claim 2.

Let $x, y \in V^n$. Consider a population with a vector of incomes x that changes to a vector of incomes y. If $y - x \in V^n$ and $G(y - x) < G(x)$, then $TI(y) > TI(x)$; $TRD(y) > TRD(x)$; and $G(y) < G(x)$.

Proof. In the Appendix.

Claim 2 informs us that if $y - x$ is an ordered vector such that $(y_1 - x_1, y_2 - x_2, \ldots, y_n - x_n) \in \mathbb{R}^n : y_1 - x_1 \leq y_2 - x_2 \leq \ldots \leq y_n - x_n$, and if the Gini coefficient calculated for that vector is smaller than the Gini coefficient calculated for the vector x (namely when the increase in incomes from x to y is bigger for richer individuals in absolute terms, but bigger for poorer individuals in relative terms), then an increase in the incomes of a population from x to y results in an increase in *TRD* of the population, and in a reduction of the Gini coefficient in the population. In other words, when the additional income is distributed among the individuals in such a way that the richer individuals obtain a larger part of the extra income in absolute terms (as per assumption $y - x \in V^n$), but a smaller part in relative terms, namely the additional income is divided more equally than the initial income (as per assumption $G(y - x) < G(x)$), then the Gini coefficient decreases, while *TRD* increases.

When we look into the construction of *G* and *TRD*, we notice that *TRD* is the aggregate of the levels of relative deprivation, *RD*, of members $i = 1, 2, \ldots, n - 1$ of the population. The *RD* of individual i is lowered when the income of an individual positioned to the right of individual i in the income distribution is reduced, but is not affected when the income of an individual who is positioned to the left of individual i in the income distribution is reduced. However, *G* is sensitive to both these changes (it will be reduced in the first case, it will be increased in the second case). It is this asymmetry between the two indices that gives rise to a divergence between their predictions.

4. Summary and conclusion

This paper underscores that when the incentive to migrate is studied, a distinction needs to be drawn between two measures of income inequality: the Gini coefficient and total relative deprivation, *TRD*. Intuitively, when the income vector of a population changes, the Gini coefficient and *TRD* could be expected to indicate the same qualitative change in inequality: if one of them increases (or decreases), then the other should similarly change. We showed that such an intuition can be misleading. We constructed numerical examples and we formulated sufficient conditions under which the Gini coefficient and *TRD* "behave" differently: with varying distributions of the increase in total income at origin, we generated divergence between changes of the Gini coefficient and *TRD*. Extending a preceding reasoning by Stark (2006), we relaxed Stark's assumption of a constant total income, and we showed that the Gini coefficient and *TRD* need not be positively correlated.

The current paper stays in line with Stark's (2006) argument that *TRD*, instead of the Gini coefficient, is the true driver of migration behavior. Empirical observations listing the Gini coefficient at origin among the determinants of migration behavior can stem from a functional link between the Gini coefficient and *TRD*: the *TRD* of a population is a product of the Gini coefficient of the population and total income of the population. This link, already determined by Stark (2006), delivers an explanation for the divergence of results of empirical studies regarding the sign of the relationship between the Gini coefficient of income distribution at origin and the intensity of migration. According to Eurostat (2018) data for EU countries, the sign of the relationship between the Gini coefficient and *TRD* (calculated as a product of the Gini coefficient, the mean disposable income in a population, and the size of the working-age population) differs depending on the country. While in some countries the correlation between the Gini coefficient and *TRD* between 2007 and 2017 is positive and strong (0.95 in Sweden and 0.92 in Denmark), in other countries it is positive and weak (0.16 in Latvia), negative and weak (-0.18 in Romania), or even negative and strong (-0.89 in Poland). Thus, we maintain that the preceding framework in Stark (2006) is delicate, and can be applied to match a new set of empirical findings. The framework presented in the current paper is richer than Stark's (2006), and will be useful in future studies on inequality and migration.

That an increase in all incomes will coincide with an increase in *TRD* or with an increase of the Gini coefficient may not be all that surprising. However, our analysis gives rise to a possibility that the incentive to migrate increases because *TRD* at origin increases, in spite of all incomes at origin increasing, and the Gini coefficient at origin decreasing. Such a possibility was not acknowledged before. The examples (scenarios) and claims presented in this paper serve to elucidate a need to exercise caution when basing a prediction of migration behavior on standard "push" indicators.

Appendix: Proofs of Claims 1 and 2

To facilitate proof of the claims, we first state and prove a supportive lemma.

Lemma 1.

TI and *TRD* are "linear" on V^n, namely for any $a \in \mathbb{R}_+$ and any $x, y \in V^n$

(i) $TI(x + y) = TI(x) + TI(y); \ TRD(x + y) = TRD(x) + TRD(y)$;

(ii) $TI(ax) = aTI(x); \ TRD(ax) = aTRD(x)$.

Moreover, if $x = (b, b, \ldots, b)$, $b \in \mathbb{R}_+$ then

(iii) $TI(x) > 0; \ TRD(x) = 0$.

Proof.

Properties (i), (ii), and (iii) are immediate consequences of the formulae of *TI* and *TRD* (it suffices to substitute the formulas for *TI* and *TRD* (presented in Section 2) into (i), (ii), and (iii)). Q.E.D.

Proof of Claim 1.

(a) By Lemma 1, part (ii)

$$TRD(y) = TRD(ax) = aTRD(x) > TRD(x);$$

$$TI(y) = TI(ax) = aTI(x) > TI(x);$$

$$G(y) = \frac{TRD(y)}{TI(y)} = \frac{TRD(ax)}{TI(ax)} = \frac{aTRD(x)}{aTI(x)} = \frac{TRD(x)}{TI(x)} = G(x).$$

(b) By Lemma 1, parts (i) and (iii)

$$TRD(y) = TRD(x + (b, b, \ldots, b)) = TRD(x) + TRD(b, b, \ldots, b) = TRD(x);$$

$$TI(y) = TI(x + (b, b, \ldots, b)) = TI(x) + TI(b, b, \ldots, b) > TI(x);$$

$$G(y) = \frac{TRD(y)}{TI(y)} = \frac{TRD(x)}{TI(x) + TI(b, b, \ldots, b)} < \frac{TRD(x)}{TI(x)} = G(x).$$

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(c) By Lemma 1

$$TRD(y) = TRD(ax + (b,b,\ldots,b)) = TRD(ax) + TRD(b,b,\ldots,b) = TRD(ax) = aTRD(x) > TRD(x);$$

$$TI(y) = TI(ax + (b,b,\ldots,b)) = TI(ax) + TI(b,b,\ldots,b) = aTI(x) + TI(b,b,\ldots,b) > TI(x);$$

$$G(y) = \frac{TRD(y)}{TI(y)} = \frac{TRD(ax + (b,b,\ldots,b))}{TI(ax + (b,b,\ldots,b))} = \frac{aTRD(x)}{aTI(x) + TI(b,b,\ldots,b)}$$

$$< \frac{aTRD(x)}{aTI(x)} = \frac{TRD(x)}{TI(x)} = G(x).$$

Q.E.D.

Proof of Claim 2.

By Lemma 1, part (i)

$$TI(y) = TI(x + (y - x)) = TI(x) + TI(y - x) > TI(x).$$

$$TRD(y) = TRD(x + (y - x)) = TRD(x) + TRD(y - x) > TRD(x).$$

From the assumption that $\dfrac{TRD(y - x)}{TI(y - x)} = G(y - x) < G(x) = \dfrac{TRD(x)}{TI(x)}$, it follows that

$$TRD(y - x) < \frac{TRD(x)TI(y - x)}{TI(x)}.$$

Thus,

$$G(y) = \frac{TRD(y)}{TI(y)} = \frac{TRD(x) + TRD(y - x)}{TI(x) + TI(y - x)}$$

$$< \frac{TRD(x) + \dfrac{TRD(x)TI(y - x)}{TI(x)}}{TI(x) + TI(y - x)}$$

$$= \frac{TRD(x)[TI(x) + TI(y - x)]}{TI(x)[TI(x) + TI(y - x)]} = \frac{TRD(x)}{TI(x)} = G(x).$$

Q.E.D.

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