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IZA DP No. 12811

Inequality Comparisons with Ordinal Data

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ISSN: 2365-9793

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ABSTRACT

Inequality Comparisons with Ordinal Data*

Non-intersection of appropriately-defined Generalized Lorenz (GL) curves is equivalent to a unanimous ranking of distributions of ordinal data by all Cowell and Flachaire (*Economica* 2017) indices of inequality and by a new index based on GL curve areas. Comparisons of life satisfaction distributions for six countries reveal a substantial number of unanimous inequality rankings.

JEL Classification: D31, D63, I31

Keywords: inequality, ordinal data, subjective well-being, life satisfaction, World Values Survey

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* The research was part-supported by core funding of the Research Centre on Micro-Social Change at the Institute for Social and Economic Research by the University of Essex and the UK Economic and Social Research Council (award ES/L009153/1). I gratefully acknowledge the hospitality of the School of Economics, University of Queensland, where I was based when drafting this paper. The estimates were derived using my Stata module ineqord, to be released on SSC shortly.

1. Introduction

Cowell and Flachaire (2017) provide a new approach to measuring inequality of ordinal data such as life satisfaction, happiness, and self-assessed health status that differs significantly from the approach taken in most recent research. This paper builds on Cowell and Flachaire's work by adding dominance results and a new inequality index.

Since the critique by Allison and Foster (2004), most economists have accepted that it is inappropriate to assess ordinal data inequality using the tools developed to assess the inequality of cardinal data on income and wealth. The latter methods associate greater inequality with greater dispersion about the mean, but the mean is an improper benchmark for an ordinal variable. For ordinal variables, Allison and Foster (2004) propose instead that greater inequality means greater spread about the median and they demonstrate that, for distributions with the same median, a unanimous ordering by all indices incorporating this concept is equivalent to '*S*-dominance' – a particular configuration of cumulative distribution functions.[1] Allison and Foster (2004) and other researchers, including Abul Naga and Yalcin (2008) and Apouey (2007), have developed inequality indices consistent with *S*-dominance. A distinguishing feature of the Allison-Foster approach is that it measures inequality in terms of polarization: 'inequality' is maximized when half the population has the lowest value on the ordinal scale and half the population has the largest value. Cowell and Flachaire's (2017) inequality indices are different – and hence complementary to the median-based ones – because greater inequality reflects greater spread in a sense other than greater polarization. However, no dominance results currently exist for Cowell-Flachaire indices.

I show that non-intersection of appropriately-defined Generalized Lorenz (GL) curves is equivalent to a unanimous ranking of distributions by all Cowell and Flachaire (2017) indices of inequality and by a new index based on areas below GL curves. The results are not restricted to distributions with the same median. Comparisons of life satisfaction distributions for six countries show that the new dominance results reveal a substantial number of unanimous inequality rankings. Supplementary materials are reported in the Appendix.

I use Cowell and Flachaire's 'peer-inclusive downward-looking' definition of individual status (explained below) as this definition is consistent with the focus in the median-related inequality measurement literature. (Cumulative distribution functions are the building blocks in common.) There are analogous results for Cowell and Flachaire's 'peer-

[1] See also Kobus (2015) for characterization results.

inclusive upward-looking' status definition but, for brevity, I summarize these in the Appendix.[2] My dominance results differ from the dual *H*-dominance results of Gravel et al. (2015) which are based on the concept of Hammond transfers. Gravel et al. do not relate their findings to any specific inequality indices; and it remains an open question whether Cowell-Flachaire (2017) inequality indices are Hammond-transfer-consistent.

2. Cowell-Flachaire inequality indices for ordinal data

The well-being of each of N individuals is measured on an ordinal scale characterized by a set of numerical labels (l_1, l_2, …, l_K), with $-\infty < l_1 < l_2 < \ldots < l_K < \infty$, and $K \geq 3$. Thus, the distribution of well-being is summarized by an ordered categorical variable. The proportion of individuals in the k^{th} category is denoted f_k with $0 \leq f_k \leq 1$ and $\sum_{k=1}^{K} f_k = 1$. The proportion of individuals in the k^{th} category or lower is F_k, with $F_k = \sum_{j=1}^{k} f_j$ and $F_K = 1$.

Cowell and Flachaire (2017) propose a two-step approach to inequality measurement for ordinal data. First, decide how to summarize 'status', s_i, for each individual $i = 1, 2, 3, \ldots, N$. The 'peer-inclusive downward-looking' status of an individual with scale level k is F_k, and hence does not depend on the particular values attached to (l_1, l_2, …, l_K). That is, the measure is scale-independent.

Second, define inequality as an aggregate summary of the 'distances' between each person's status and an appropriate reference value (distance encapsulates dispersion). Cowell and Flachaire argue persuasively that the reference status value for a peer-inclusive status measure should be the maximum value, i.e. 1 (the maximum of F_k). With some auxiliary axioms including a requirement that the minimum value of the inequality index is 0 (when all individuals have the same scale value), Cowell and Flachaire (2017) characterize a one-parameter family of inequality indices, $I(\alpha)$, with $0 \leq \alpha < 1$:

$$I(\alpha) = \frac{1}{\alpha(\alpha - 1)}\left[\frac{1}{N}\sum_{i=1}^{N} s_i^{\alpha} - 1\right], \qquad 0 < \alpha < 1;$$

$$I(0) = -\frac{1}{N}\sum_{i=1}^{N} \log(s_i). \tag{1}$$

[2] No researchers have used Cowell and Flachaire's (2017) 'peer-exclusive' definitions in applied work, including the authors themselves.

The smaller that α is, the greater the weight that is put on small status values relative to high status values. Cowell and Flachaire also cite a closely-related class of 'Atkinson-like' indices, $A(\alpha)$:

$$A(\alpha) = 1 - \left(\frac{1}{N}\sum_{i=1}^{N} s_i^{\alpha}\right)^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1;$$
$$A(0) = 1 - \left(\prod_{i=1}^{N} s_i\right)^{\frac{1}{N}}. \tag{2}$$

$A(\alpha)$ equals one minus the generalized mean of order α, and is a monotonically increasing transformation of the corresponding $I(\alpha)$ index.

Let $D(\boldsymbol{s})$ denote an inequality index that is a decreasing convex function of individual status s_i and $\mathcal{D}$ denote the set of all such functions. Clearly, every $I(\alpha)$ and $A(\alpha)$ index belong to $\mathcal{D}$. Also consider the class of indices $W(\boldsymbol{s}) = W(D(\boldsymbol{s}))$ where $W(.)$ is a monotonically decreasing function and hence of a similar form to the social welfare functions commonly used in income distribution analysis: by construction, $W(.)$ is an increasing concave function of individual status. The greater is W, the greater is equality. Let $\mathcal{W}$ denote the set of increasing concave social welfare functions. Equality indices $E(\alpha) = 1 - A(\alpha)$, $0 \le \alpha < 1$, are examples of members of $\mathcal{W}$.

The next section presents a tractable method based on Generalized Lorenz curve comparisons for assessing whether one distribution of status is unambiguously more (un)equal than another regardless of the differences in social judgements encapsulated in different inequality indices.

3. Generalized Lorenz curves for distributions of (peer-inclusive) status, and an inequality dominance result

I define the Generalized Lorenz (GL) curve for the distribution of status, $GL(\boldsymbol{s}, p)$ given $0 \le p \le 1$, following Shorrocks (1983) closely. With the distribution of status placed in ascending order, i.e. $s_1 \le s_2 \le s_3 \le \ldots \le s_N$, we have:

$$GL\left(\boldsymbol{s}, \frac{m}{n}\right) = \frac{1}{N}\sum_{i=1}^{m} s_i, \qquad m = 1, \ldots, N, \text{and } GL(s, 0) = 0. \tag{3}$$

The GL curve is drawn using straight lines to connect adjacent points of the form $\{m/n, GL(\boldsymbol{s}, m/n)\}$. The vertices of the curve are at $\{p_0 = 0, 0\}$ and $\{p_k, \sum_{j=1}^{k} f_j F_j\}$ for each $k = 1, \ldots, K$ with $p_k = F_k$. The GL ordinate at $p = 1$ is the arithmetic mean of the status distribution. Figure 1 provides an illustrative example for the case $K = 4$.

<Figure 1 near here>

The 45° ray from (0,0) to (1,1) is the GL curve representing complete equality – when all individuals have the same scale value and hence the same status. With inequality, the GL curve lies below the 45° ray and, intuitively, the further below the ray the curve is, the greater is inequality.

Indeed, one can formally demonstrate that a unanimous ranking of a pair of distributions in terms of their equality (or inequality) is equivalent to the non-crossing of their GL curves:

Result 1: For two status distributions $\boldsymbol{s}$ and $\boldsymbol{s'}$, $W(\boldsymbol{s}) \geq W(\boldsymbol{s'})$ for all $W(.) \in \mathscr{W}$ iff $GL(\boldsymbol{s}, p) \geq GL(\boldsymbol{s'}, p)$ for all p.

Result 2: $D(\boldsymbol{s}) \leq D(\boldsymbol{s'})$ for all $D(.) \in \mathscr{D}$ iff $GL(\boldsymbol{s}, p) \geq GL(\boldsymbol{s'}, p)$ for all p.

The proof of Result 1 follows directly from Shorrocks (1983, Theorem 2). Result 2 follows from Result 1 and the relationship between $D(.)$ and $W(.)$.

The connection between GL curve location and inequality suggests a new index of inequality for ordinal data, J. With reference to Figure 1, J is the ratio of area A to area $A + B$; equivalently, J equals 1 minus twice area B. It is Generalized Lorenz-consistent because a ranking of a pair of distributions by J is the same as the ranking by all $D(.) \in \mathscr{D}$ when the two GL curves do not cross. Using the expression for the vertices of the GL curve, and applying the Trapezium Formula, one can show that:

$$J = 1 - \sum_{j=0}^{K-1} (p_{j+1} - p_j)(GL_j + GL_{j+1}) = 1 - \sum_{j=0}^{K-1} f_{j+1}(GL_j + GL_{j+1}). \quad (4)$$

The minimum value of J is 0, achieved when there is perfect inequality.

The inequality dominance results relate to GL curves, not to Lorenz curves as some readers might expect. The reason is that the mean is an inappropriate reference point with ordinal data (Allison and Foster 2004), and hence also shares of the total are not a suitable building-block for inequality measurement in this context. Differences between maximum and observed status are what matters for Cowell-Flachaire indices. The situation considered here has analogies with the measurement of poverty. Non-intersection of two Three Is of

Poverty (TIP) curves is equivalent to a unanimous ranking according to all 'generalized poverty gap' poverty indices (Jenkins and Lambert 1997). But a TIP curve shows, at each p, the vertical distance between two GL curves, one for the distribution of income censored above at the poverty line and the other for the distribution in which every income equals the poverty line (a distribution with perfect equality). Index J, based on the area between two GL curves, is analogous to the Shorrocks (1995) modified-Sen poverty index (twice the area beneath a TIP curve, i.e. twice the area between two GL curves).

Polarized distributions and uniform distributions provide potential maximum-inequality benchmarks for ordinal data. As mentioned earlier, inequality indices in the Allison-Foster (2004) tradition reach their maximum if the distribution is polarized. But do Cowell-Flachaire indices reach a maximum in this case, and what if the distribution is uniform?

Results 1 and 2 imply that inequality is greater for a uniform distribution than for a polarized distribution according to J and all $D(.) \in \mathcal{D}$.[3] With a polarized distribution, $N/2$ individuals have the minimum scale value (status $F_1 = 0.5$) and $N/2$ have the maximum value ($F_K = 1$) for all possible K. The corresponding GL curve has two segments connecting points {(0,0), (0.5, 0.25), (1, 0.75)}. By contrast, with a uniform distribution, $f_k = 1/K$, all $k = 1, \ldots, K$, and the GL curve has vertices at $\{k/K, k(k+1)/(2K^2)\}$ for each k. Exploiting the expression for a straight line between two points, one can show that the GL curve for the polarized distribution lies above the curve for a uniform distribution at all p and regardless of the value of K.[4] For example, to three d.p., $J = 0.375$ for a polarized distribution and for a uniform distribution, $J = 0.481$ if $K = 3$, 0.531 if $K = 4$, and 0.615 if $K = 10$. $I(0) = 0.347$ for a polarized distribution and, for a uniform distribution, $I(0) = 0.501$ if $K = 3$, 0.592 if $K = 4$, and 0.807 if $K = 10$.

Results 1 and 2 are also informative about whether J and all $D(.) \in \mathcal{D}$ reach their maximum values in the case of a uniform distribution. It is straightforward to show that if one starts from a uniform distribution, a small shift in the proportion of individuals from one scale level to next level up (or down) must lead to a crossing of the pre- and post-shift GL curves.

[3] The results cited in this paragraph require that N is sufficiently large so that any difference in the number of individuals in each category is negligible, with attention restricted to the two populated categories in the case of a polarized distribution.
[4] Similarly, one can also show that the GL curve for a uniform distribution over $K+1$ levels lies everywhere on or below the GL curve for a uniform distribution over K levels. Illustrating these results, Appendix Figure A1 shows Generalized Lorenz curves for a polarized distribution and uniform distributions with $K = 3$, 4, 5, and 10.

Hence, according to GL-consistent indices, inequality in a non-uniform distribution may be larger or smaller than in the uniform distribution.

4. Empirical illustration: ranking countries by life satisfaction inequality

To illustrate the inequality dominance results, I use data about life satisfaction from the mid-2000s for six countries (Australia, Canada, Great Britain, New Zealand, Australia, USA, South Africa), drawn from the fifth wave of the World Values Survey (WVS), the latest available when this research was undertaken. The six countries are 'white settler' economies plus their colonial mother country (Great Britain).[5] Life satisfaction is measured using a 10-point integer-valued scale ranging from 1 (completely dissatisfied) to 10 (completely satisfied). The data are the same as those employed by Cowell and Flachaire (2017), though I analyse fewer countries. For more details about the data, see their paper and the WVS documentation (Inglehart et al. 2014). All my estimates use the WVS-supplied sample weights. Country-specific relative frequency distributions are shown in Appendix Figure A2.

Figure 2 shows the GL curves for two of the 15 possible pairwise cross-national comparisons. Panel (a) provides an example of inequality dominance: the GL curve for Britain lies every on or above the GL curve for South Africa, and hence life satisfaction inequality is lower in Britain than South Africa according to all indices J and $D \in \mathcal{D}$. In contrast, panel (b) shows that there is no unambiguous ranking of Australia and New Zealand. Their GL curves intersect four times. To assess whether inequality is higher in one or other of these two countries requires use of indices and the ordering derived may depend on the index used.

<Figure 2 near here>

Table 1 summarizes the results of all 15 pairwise GL curve comparisons (entries below the main diagonal) as well as for checks for first-order stochastic dominance ('F-dominance') based on comparisons of cumulative distribution functions (entries above the main diagonal).[6]

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[5] Ireland could be included in this description but there are no Irish data in the WVS. For additional dominance and inequality index comparisons based on WVS data, see Jenkins (2019).

[6] See Appendix Figures A3 and A4 for charts showing all the pairwise GL curve and cumulative distribution function comparisons.

GL curves do not cross in 8 out of 15 pairwise comparisons, demonstrating that the inequality dominance result has empirical usefulness. The most striking finding concerns South Africa: its life satisfaction inequality is unambiguously greater than in each of the other 5 countries. No country stands out as being unambiguously more equal than every other country, but Great Britain is more equal than its comparator countries in three of its five comparisons.

<Table 1 near here>

There is *F*-dominance in 8 of the 15 pairwise comparisons. For example, average life satisfaction in the USA is lower than in Canada, or Great Britain, or New Zealand, regardless of the life satisfaction scale that is used. However, the *S*-dominance criterion has little discriminatory power by comparison with the GL dominance criterion. There is only one case of *S*-dominance: there is greater spread away from the median in New Zealand than in Canada. *S*-dominance is rare partly because of the prevalence of *F*-dominance – if there is *F*-dominance, there cannot also be *S*-dominance (Allison and Foster 2004) – and partly because median life satisfaction is lower in South Africa than in the other countries (7 rather than 8). *S*-dominance applies only to distributions with a common median – a restriction that does not apply to GL-dominance.

To derive a complete inequality ordering of the 6 countries an inequality index must be used. However, different indices incorporate different social judgements about how to assess differences in different parts of the life satisfaction distribution. It is therefore important to use a portfolio of indices to check the robustness of rankings. I report estimates for 6 GL-consistent indices in Figure 3: $I(\alpha)$ for $\alpha = 0$, 0.25, 0.5, 0.75, and 0.9, plus J. For comparison, I also include 3 *S*-dominance-consistent indices from the Abul Naga and Yalcin (2007) class. *ANY*(1, 1) weights observations in categories above and below the median equally; *ANY*(4, 1) is more sensitive to above-median spread than below-median spread; and *ANY*(1, 4) is the opposite (i.e. relatively bottom-sensitive). Figure 3 shows point estimates and 95% confidence intervals. I estimate standard errors using a repeated half-sample bootstrap approach in order to appropriately account for the sample weights (Saigo et al. 2001; Van Kerm 2013) with 500 bootstrap replications.

<Figure 3 near here>

Consider the $I(\alpha)$ estimates. Figure 3 confirms that inequality is distinctly greater in South Africa than in every other country. For example, according to $I(0)$, South Africa's inequality is 4% larger than NZ's (with the null hypothesis of no difference decisively rejected: test statistic = 5.2).[7] According to $I(0.9)$, the difference is 6% (test statistic = 4.8). Canada and Great Britain appear to have the lowest inequality according to all five $I(\alpha)$ estimates. (Although the $I(0)$ point estimate appears slightly smaller for Canada, the CA-GB difference is not statistically different from zero.) The countries ranked second, third, and fourth by $I(\alpha)$ are NZ, the USA, and Australia, but differences between the three estimates are not statistically significant. (Differences between NZ on the one hand and Great Britain and Canada on the other hand are significantly different, however.) Rankings by J are very similar to those by $I(\alpha)$.

The bottom row of Figure 3 shows that median-based indices can yield different conclusions about inequality orderings by comparison with GL-consistent indices. Although the country ranking by $ANY(1,1)$ mimics those by $I(0)$ and J, the top- and bottom-sensitive indices $ANY(4,1)$ and $ANY(1,4)$ provide different patterns. For example, according to these two indices New Zealand is the second-ranked country by life satisfaction inequality after South Africa. For top-sensitive index $ANY(4,1)$, Australia moves down the ranking by comparison with the rankings from the other indices. As well, the precision of the estimates of $ANY(4,1)$ and $ANY(1,4)$ seems to be lower than for the other indices: look at the width of the confidence intervals for South Africa and the USA in particular.

4. Summary and conclusions

Cowell and Flachaire's (2017) innovative approach to inequality measurement with ordinal data complements the predominant approach to date that conceptualizes greater inequality as greater spread around the median. This paper builds on Cowell and Flachaire's work by adding dominance results and a new inequality index. I show that non-intersection of appropriately-defined GL curves is equivalent to a unanimous ordering of distributions according to all Cowell-Flachaire (2017) inequality indices and the new index based on GL curve areas. In contrast with *S*-dominance, the results presented here can be applied when distributions do not have a common median. Cross-national inequality comparisons based on WVS data show that the GL curve-based results have useful empirical content in the sense of revealing a substantial number of unanimous inequality rankings.

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[7] The narrower confidence bands for South Africa's estimates partly reflect that country's distinctly larger WVS sample size (Jenkins 2019).

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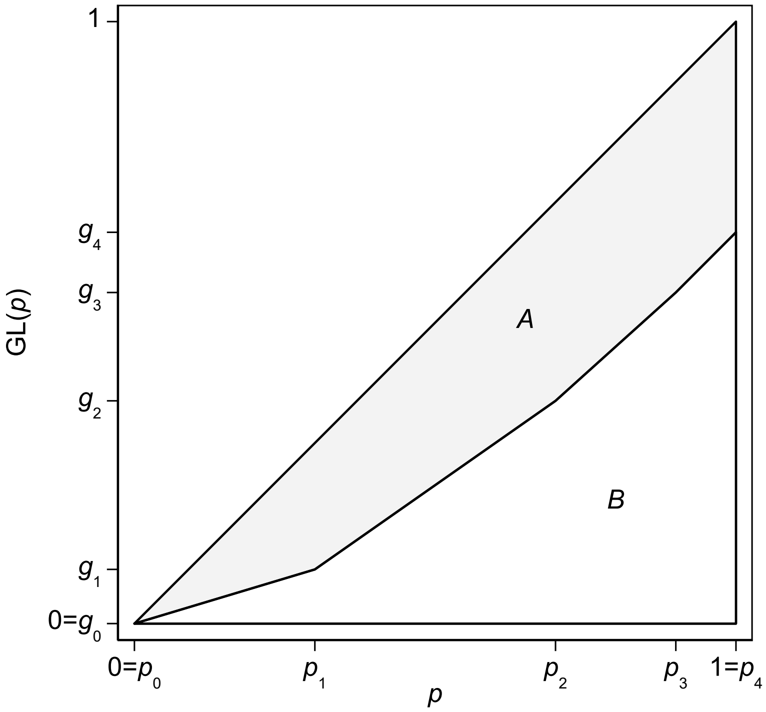
Table 1. Cross-national comparisons of life satisfaction distributions: summary of dominance checks

		Country *y*					
		AU	CA	GB	NZ	US	ZA
	AU		<	<	<	–	>
	CA	–		–	–*	>	–
Country	GB	>	–		–	>	–
x	NZ	–	<	<		>	>
	US	–	–	–	–		–
	ZA	<	<	<	<	<	

Notes Entries above the diagonal summarize checks for *F*-dominance (first-order dominance): ‘>’, *x* *F*-dominates *y*; ‘<’, *y* *F*-dominates *x*; ‘–’, no dominance. *: NZ *S*-dominates CA. Entries below the diagonal summarize GL dominance checks: ‘>’, *x* is more equal than *y*; ‘<’, *y* is more equal than *x*; ‘–’, no dominance.

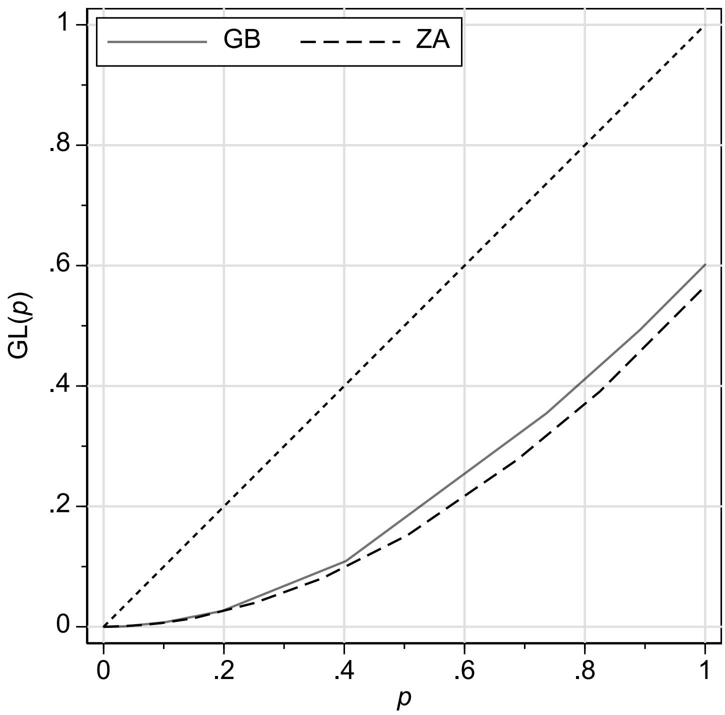
Figure 1. A Generalized Lorenz (GL) curve for the distribution of status (K = 4)



Note Area-based inequality index $J = A/(A + B) = 1 - 2B$.

Figure 2. GL curve comparisons of life satisfaction distributions: selected examples

(a) Dominance



(b) Non-dominance

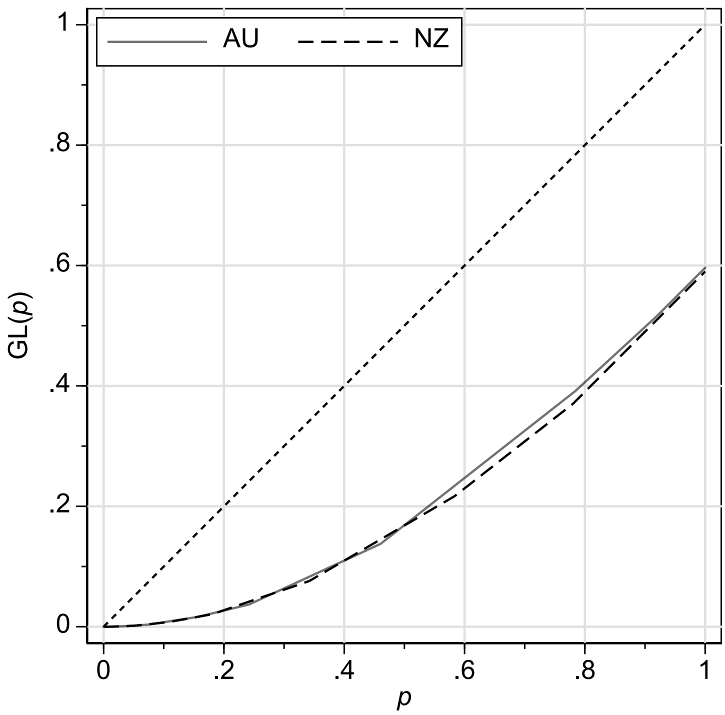
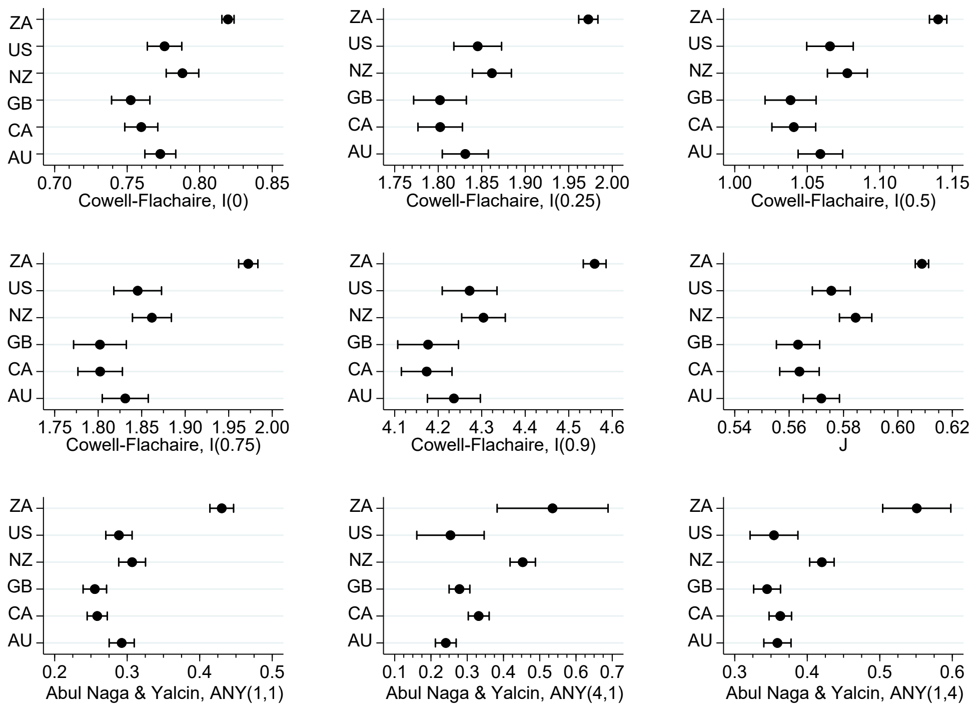


Figure 3. Estimates of life satisfaction inequality across countries: 9 indices



Notes. The figure shows point estimates of inequality indices and their associated 95% confidence intervals.

APPENDIX

(Supplementary Material)

for

Inequality comparisons with ordinal data

Stephen P. Jenkins

Analysis in the case of peer-inclusive upward-looking status

The main text focuses on Cowell and Flachaire's (2017) peer-inclusive downward-looking status. This note explains how the analysis needs to be modified if Cowell and Flachaire's peer-inclusive upward-looking status definition is used instead.

According to Cowell and Flachaire (2017), your peer-inclusive downward-looking status is the fraction of individuals with the same status as you or lower, with higher values corresponding to higher status. Your peer-inclusive upward-looking status is measured by the proportion of individuals with the same status as you or higher, with lower values corresponding to higher status. Let $\boldsymbol{u}$ represent this distribution of status.

Distribution $\boldsymbol{u}$ can be represented by the survivor function – the proportions of individuals in each category k or higher, $S_k = \sum_{j=k}^{K} f_j$, $k = 1, \ldots, K$, with $S_1 = 1$ and $S_K = f_K$.

The GL curve is defined as in (3) except that $\boldsymbol{u}$ replaces $\boldsymbol{s}$ and individuals are ranked in ascending order of upward-looking status (the reverse of the ranking by downward-looking status). The vertices of this revised GL curve are at $\{p_0 = 0, 0\}$ and $\{p_{K+1-k}, \sum_{j=k}^{K} f_k S_k\}$ for each $k = 1, \ldots, K$, with $p_{K+1-k} = S_{K+1-k}$. The labelling reflects the fact that the first segment of the revised GL curve refers to the individuals with the highest level of the ordinal variable and the last segment refers to those with the lowest level.

Results 1 and 2 are as stated earlier except that $\boldsymbol{u}$ replaces $\boldsymbol{s}$ and one uses the revised definition of the GL curve.

The area-based inequality index for the peer-inclusive upward-looking status case is

$$1 - \left[f_K GL_K + \sum_{j=1}^{K-1} f_k \left(GL_j + GL_{j+1} \right) \right],$$

which is calculated using the revised definition of the GL curve.

The statements in the main text about the relative inequality of polarized and uniform distributions are unaffected by the change in status definition because polarized and uniform distributions are symmetric and, for symmetric distributions, the distributions of peer-inclusive downward-looking and upward-looking status are identical (Cowell and Flachaire 2017).

Abul Naga and Yalcin (2008) and Apouey (2007) indices are defined using values of F_k. One could develop analogous indices using values of S_k.

Figure A1. Generalized Lorenz curves for a polarized distribution and uniform distributions (N = 500; K= 3, 4, 5, 10)

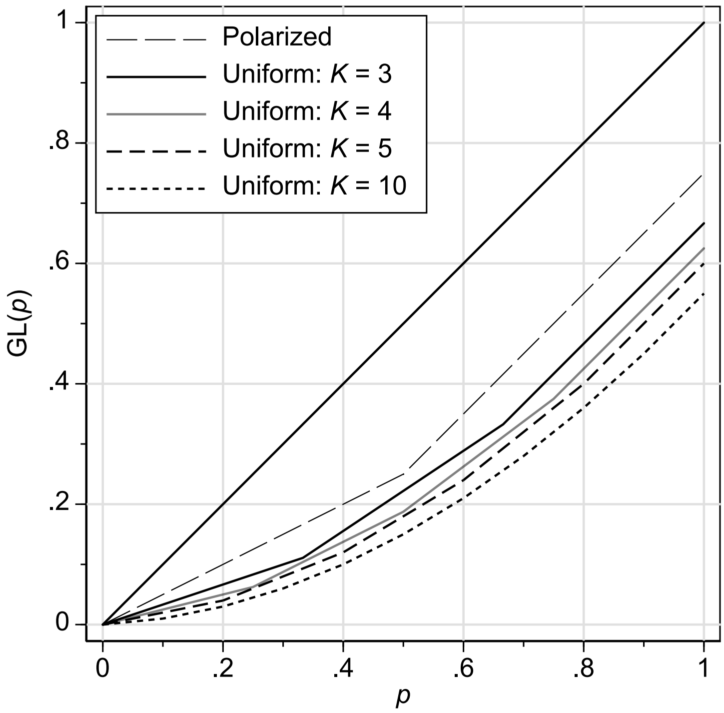


Figure A2. Life satisfaction relative frequency distributions, by country

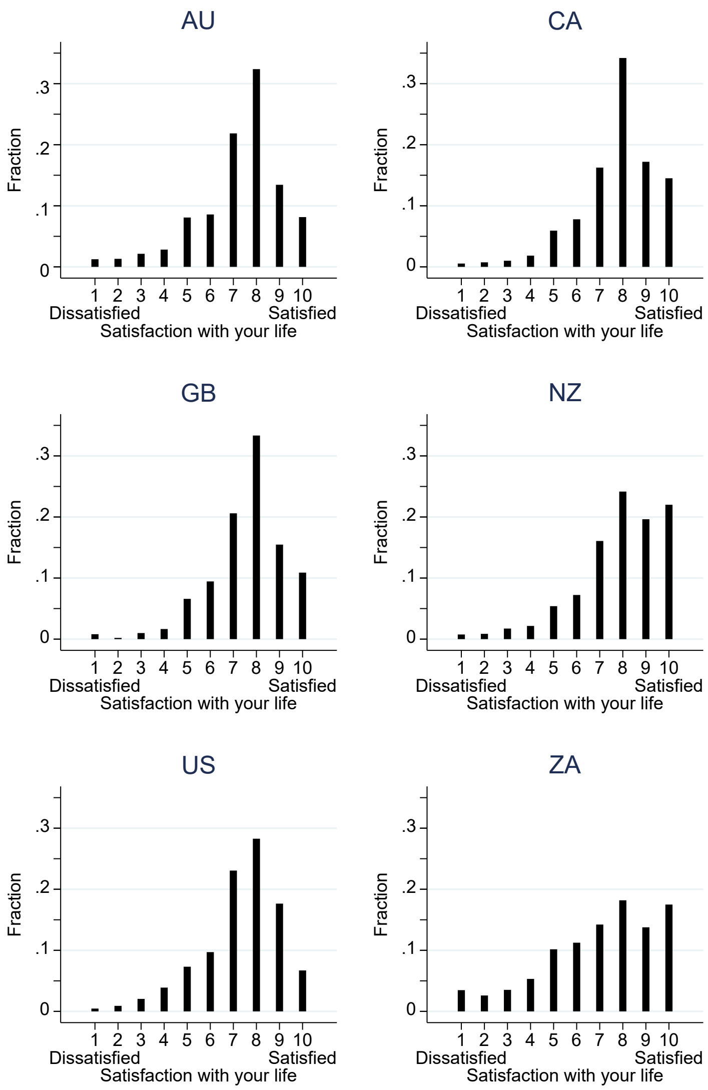


Figure A3. Life satisfaction distributions: GL dominance checks

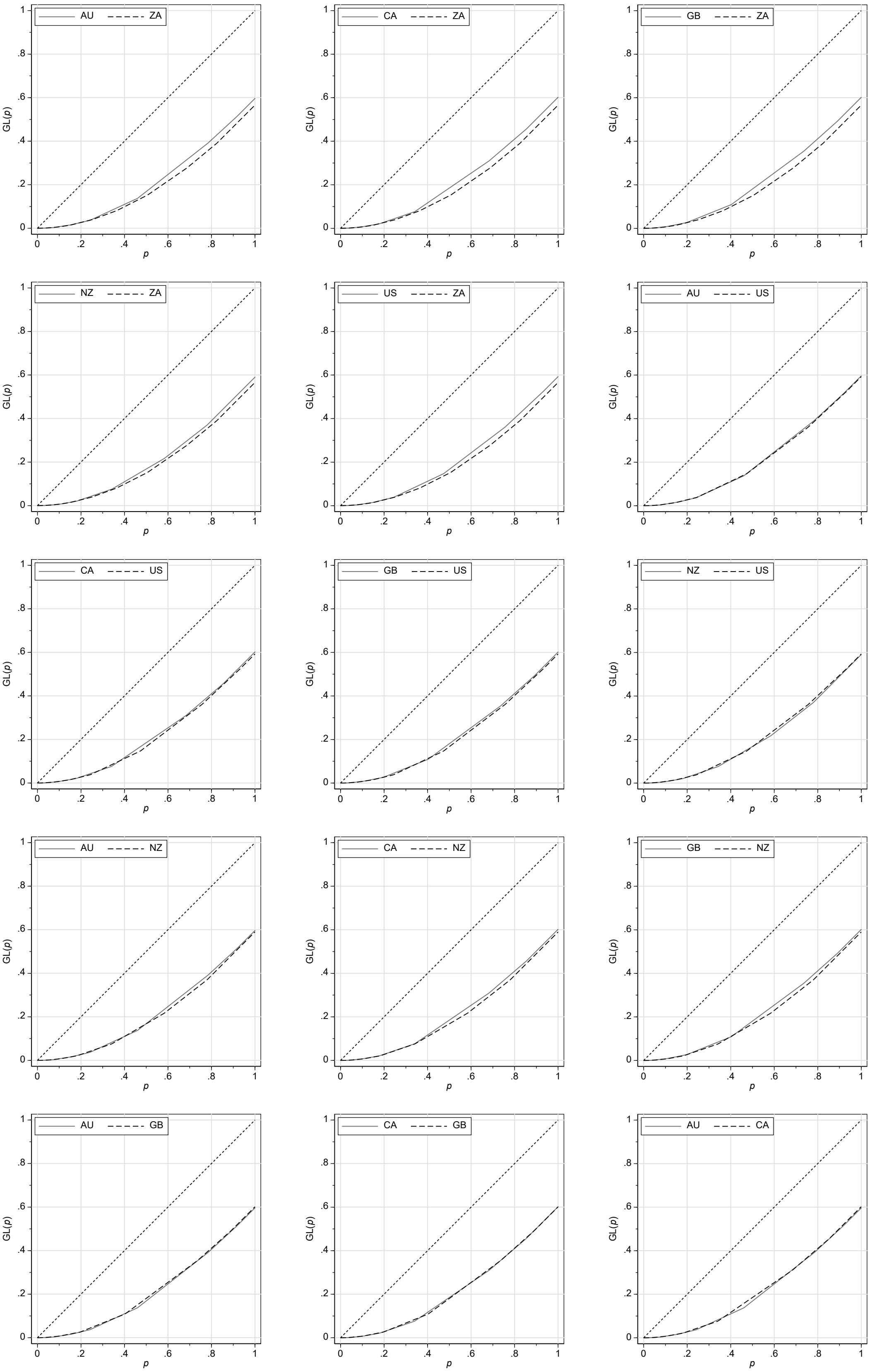


Figure A4. Life satisfaction distributions: first-order dominance (*F*-dominance) checks