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ABSTRACT

Accounting for Skewed or One-Sided Measurement Error in the Dependent Variable

While classical measurement error in the dependent variable in a linear regression framework results only in a loss of precision, non-classical measurement error can lead to estimates which are biased and inference which lacks power. Here, we consider a particular type of non-classical measurement error: skewed errors. Unfortunately, skewed measurement error is likely to be a relatively common feature of many outcomes of interest in political science research. This study highlights the bias that can result even from relatively "small" amounts of skewed measurement error, particularly if the measurement error is heteroskedastic. We also assess potential solutions to this problem, focusing on the stochastic frontier model and nonlinear least squares. Simulations and three replications highlight the importance of thinking carefully about skewed measurement error, as well as appropriate solutions.

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Introduction

Confronting measurement error in statistical research in political science and other disciplines has a lengthy history. Hausman (2001, p. 57) states, "The effect of mismeasured variables in statistical and econometric analysis is one of the oldest known problems." Asher (1974, p. 469) argues that "a more subtle problem of secondary analysis is that the investigator often has little feel for the quality of the data, for the extent and nature of the measurement error in the data." Nonetheless, the issue is often ignored by researchers. When it is not ignored, the focus is almost exclusively on classical measurement error. Here, "classical" refers to mean zero errors that are independent of covariates and the stochastic disturbance in the model; Asher (1974) refers to this as the case of "random" measurement error. Imai and Yamamoto (2010, p. 543) conclude that "existing research has either completely ignored the problem or exclusively focused on classical measurement error in linear regression models where the error is assumed to arise completely at random."

The focus on classical measurement error is particularly true in the case of mismeasurement of a continuous dependent variable in a linear regression framework. Here, classical measurement error reduces precision, but does not bias estimates. However, the assumption of classical measurement error is often invoked without any formal justification. In practice, a frequently encountered situation of nonclassical measurement error in the dependent variable occurs when the outcome is systematically over- or under-reported, resulting in measurement error that is skewed with non-zero mean; termed "nonrandom" in Asher (1974).

Skewed measurement errors, of which one-sided measurement errors are a special case, are likely to be a feature of many outcome variables used in political science research. Consider measures of the severity of civil conflict as an example. Measures of conflict intensity (e.g., casualties) are derived from either government reports, media accounts, or third-parties such

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as humanitarian groups. Each data source has its own unique challenges. Official government statistics face deficiencies related to insufficient resources and political manipulation. For example, Krueger and Laitin (2004, p. 8) state that "terrorism reports produced by the U.S. government do not have nearly as much credibility as its economic statistics, because there are no safeguards to ensure that the data are as accurate as possible and free from political manipulation." Media accounts risk missing data from remote areas (Wiedmann 2016). Third-parties face tension between providing accurate data and other objectives. For instance, Lacina and Gleditsch (2012, p. 1118) state that the PRIO battle deaths data may overstate the true death toll due to a "blurring between expertise and advocacy" as "experts may overestimate deaths because they seek to draw attention to ongoing conflicts or to underline the importance of the conflict on which they specialize."

Similarly, official death tolls from natural disasters may suffer from skewed measurement error. Consider the recent crisis in Puerto Rico arising from Hurricane Maria. Kishore et al. (2018) claim that the official death toll may severely understate the true loss of life. The authors state: "Accurate estimates of deaths, injuries, illness, and displacement in the aftermath of a disaster such as Hurricane Maria are critical...However, public health surveillance is extremely challenging when infrastructure and health systems are severely damaged. In early December 2017, the official death count in Puerto Rico stood at 64, but several independent investigations concluded that additional deaths attributable to the hurricane were in excess of 1,000 in the months of September and October." Kishore et al. (2018) estimate the death toll at 4,645.

Other examples of outcomes that may suffer from skewed measurement error abound. Katz and Katz (2010) discuss measurement error in self-reported voting behavior. Using validation data for elections over the period 1964 to 1990, the authors find that between 13% and 25% of self-reported voters did not vote; at most 4% of self-reported non-voters

did vote. The highly skewed errors at the individual level mean that aggregate measures of self-reported voter turnout will suffer from one-sided measurement error. Measurement of corruption, or criminal activity more generally, may also suffer from one-sided measurement error. Goel and Nelson (1998) and subsequent research on corruption in the United States utilize data on the number of public officials convicted for abuse of office. This measure only captures illegal activities that are discovered and prosecuted. Measurement of local pollution, necessary for analyses of environmental justice or other determinants of local environmental conditions, may also suffer from one-sided measurement error. For example, Daniels and Friedman (1999) and others rely on self-reported emissions by establishments in the United States obtained from the Toxic Release Inventory. The self-reported and public nature of the data make systematic under-reporting likely. Finally, survey data on actions or attitudes that may lead to social judgement may suffer from skewed measurement error, including data on political or charitable contributions, attitudes concerning racial or ethnic issues, etc.

With skewed measurement error in the dependent variable in a linear regression framework, the consequences extend beyond a loss in precision. Ordinary Least Squares (OLS) will no longer provide an unbiased estimate of the intercept (due to the non-zero mean of the composite error term) and may no longer provide unbiased estimates of the slope parameters if the skewed measurement error is heteroskedastic. Moreover, Instrumental Variable (IV) estimation, the traditional solution to measurement error when it is addressed econometrically, is not a viable solution as any potential instruments will necessarily be invalid. As a result, the typical response in political science and elsewhere is to ignore the issue. For example, Weidmann (2016, p. 206-7) states: "Thus, for scholars trying to explain the occurrence of political violence, this means that their dependent variable may be measured with error. This alone would not be a problem if this error was random; however, as is well known, systematic measurement error that is correlated with an independent variable can introduce

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statistical bias and lead to erroneous conclusions. Both conceptually and methodologically, the new wave of event-level analysis has not taken this issue serious enough."

However, solutions are available, but have rarely been used by researchers; never in political science to our knowledge. Our objective is to draw attention to these solutions and assess their performance via simulations and three replications.

The first solution entails directly modeling the measurement error assuming it stems from a particular parametric distribution.[1] Techniques to accomplish this are well-developed in the literature on efficiency modeling (Aigner et al. 1977; Parmeter and Kumbhakar 2014). Specifically, stochastic frontier analysis (SFA) allows for the composite error term to include a one-sided component (which can either be non-negative or non-positive) and a two-sided idiosyncratic component. While the one-sided error component is traditionally viewed as capturing inefficiency, one may instead interpret it as skewed measurement error; this stems from the statistics literature on closure properties related to many common skewed distributions. Moreover, heteroskedasticity in the skewed error may be accommodated in the estimation. SFA is now quite common; it is available in software packages such as *Stata*, *SAS*, and *R*. The second solution is similar to SFA, but involves Nonlinear Least Squares (NLLS) rather than maximum likelihood. NLLS replaces the distributional assumptions in SFA with an assumption referred to as the scaling property. Again, NLLS is available in software packages such as *Stata*, *SAS*, and *R*.

To our knowledge, the first paper to utilize SFA to address one-sided measurement error in the dependent variable is Hofler and List (2004). Subsequent research using SFA to address one-sided measurement error in the dependent variable has remained confined to the analysis of auction data where systematic over- or under-bidding may be common (e.g., Kumbhakar et al. 2012) or population and mortality data (Anthopolos and Becker 2010).

We make several contributions to this literature. First, we consider the case of skewed

measurement error in the outcome variable. Prior applications focus exclusively on one-sided measurement error. As illustrated in Katz and Katz (2010) with regards to self-reported voting behavior, often the outcome variable suffers from skewed, rather than strictly one-sided, measurement error. Second, whereas prior papers using SF models to address one-sided measurement error use the methodology for a specific application, we provide a rigorous analysis of the issue. This entails quantifying the impact on parameter estimates via simulations, as well as offering recommendations for researchers when confronting skewed measurement error. Finally, as the existing literature does not discuss the potential for NLLS under the scaling property to circumvent the need for distributional assumptions when confronting skewed or one-sided measurement error, we fill this gap.

Our results are striking. First, the simulations document that even a small amount of systematic, one-sided measurement error can lead to severe bias. Second, while correctly specified SFA and NLLS offer a solution, misspecification of the error components can have dire consequences. Third, the simulations confirm the superiority of SFA and particularly NLLS (in large samples) over ignoring measurement error when the errors are skewed instead of strictly one-sided. Finally, our three replications demonstrate that accounting for one-sided measurement error is important when evidence of heteroskedasticity is present.

Measurement Error in The Dependent Variable

To begin, we discuss one-sided measurement error as it illustrates the main empirical challenges. After, we consider the general case of skewed measurement error.

The Impact of One-Sided Measurement Error

Consider the following linear in parameters model

$$y_i^* = x_i\beta + v_i, \tag{1}$$

where y_i^* is the true measure of interest, such as the number of deaths during conflict i, x_i is the set of covariates, and v_i is a classical error term. In this setting, Ordinary Least Squares (OLS) will produce unbiased and consistent estimates of β. Moreover, $x_i\beta$ can be interpreted as the conditional expectation of y^* given x, $E[y^*|x]$.

Suppose one has access to a random sample, $\{y_i, x_i\}_{i=1}^N$, where y_i is the observed value of y^*. In the absence of measurement error, these measures coincide and $y_i^* = y_i$. However, with either over- or under-reporting, we have

$$y_i = y_i^* + su_i, \tag{2}$$

where $u_i \geq 0$ for all i and $s \in \{-1, 1\}$. When $s = -1$ ($s = 1$) values of y^* are systematically under- (over-)reported. The estimating equation is

$$y_i = x_i\beta + v_i + su_i = x_i\beta + \varepsilon_i. \tag{3}$$

The consequences of one-sided measurement error exceed those arising from classical measurement error. Whereas classical measurement error leads to a loss in precision, one-sided measurement error can potentially lead to substantial bias in the coefficient estimates. To see this, note that due to the one-sided nature of the measurement error, $E[u_i] \neq 0$. Thus, OLS applied to (3) will estimate the conditional mean $E[y|x] = E[y^*|x] + E[u|x]$.

For practitioners, the source of the bias does not matter, but it likely does have an impact

on whether there exists an expectation of bias. For instance, even a naïve researcher might realize that there are known issues with corruption data, but not necessarily with voter turnout data. While there is a difference in skewness arising from missingness due to case censoring (e.g., corruption data) versus respondent misreporting (e.g., voter turnout data), the issue may be handled directly by modeling the skewness in both settings.[2]

To assess the impact of this one-sided error, recall that OLS produces estimates that ensure the residuals are mean zero. Thus, the model that is estimated is

$$y_i = \beta_0 + x_i\beta + \varepsilon_i = (\beta_0 + sE[u]) + x_i\beta + (\varepsilon_i - sE[u]), \tag{4}$$

where we now explicitly include the intercept, β_0, and re-define x to include only the model covariates with associated parameter vector, β. Here, the main impact of one-sided measurement error is on the intercept. However, this assumes that the one-sided error is not systematic. If, alternatively, the level of over- or under-reporting depends on observable characteristics, the impact can be far more damaging.

Assume the level of over- or under-reporting depends on x. For example, in conflict research, the under-reporting of deaths may be a function of communication technology, and such technology may also be a direct determinant of conflict (Wiedmann 2016). In such a situation, the variance of the one-sided measurement error will depend on x. However, heteroskedasticity affects not only the variance of u_i, but also its conditional mean. Formally, with one-sided, heteroskedastic measurement error, $E[u|x] \neq E[u] \neq 0$.[3] Moreover, the nature of the problem ensures that any potential instrument (i.e., a variable $w \subsetneq x$ such that $Cov(w, x) \neq 0$) will be invalid. Finally, Wang and Schmidt (2002) show that the OLS estimates of β will also be biased if the variance of u depends on some other covariates z, where $z \subsetneq x$, unless z and x are independent.

In sum, in the presence of one-sided, heteroskedastic measurement error, the estimating equation becomes

$$y_i = \beta_0 + x_i\beta + sE[u|x] + (\varepsilon_i - sE[u|x]) \,. \tag{5}$$

Thus, estimation of the conditional mean results in an omitted variable problem that will yield biased coefficients except under specific assumptions that are unlikely to hold in practice. Moreover, this bias will not diminish even as the sample size increases.

Skewed Versus One-Sided Measurement Error

The assumption of one-sided measurement error may be too rigid in many applications. Instead, the assumption of measurement errors highly skewed in one direction may be more realistic. With skewed measurement error, it is likely that many of the issues that arise with one-sided measurement error continue to do so. Again consider the linear in parameters model in (1) and data that represent a random sample, $\{y_i, x_i\}_{i=1}^N$, where y_i is the observed value of y_i^*. However, now suppose that both over- and under-reporting simultaneously exist but in vastly different proportions. In this case,

$$y_i = y_i^* + u_i, \tag{6}$$

where u_i comes from a skewed distribution and can take both negative and positive values. As in the case of one-sided measurement error, skewed errors can potentially lead to substantial bias in the coefficient estimates.

To see this, consider a simple parametric example. The estimating equation is

$$y_i = x_i\beta + v_i + u_i = x_i\beta + \varepsilon_i. \tag{7}$$

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Suppose that ε is distributed as skew normal, which has pdf $f(\varepsilon) = \frac{2}{\omega}\phi(A)\Phi(\alpha A)$, where $A = \frac{\varepsilon - \xi}{\omega}$, ω is the variance of ε, α controls the skewness of ε, and ξ is the location (not to be conflated with the mean) of ε. The mean of ε is $\xi + \sqrt{\frac{2}{\pi}}\omega\delta$, where $\delta = \frac{\alpha}{\sqrt{1+\alpha^2}}$ (Azzalini 1985). Note, even if $\xi = 0$, the mean of ε is non-zero as long as there is skewness (i.e., $\alpha \neq 0$). Moreover, the conditional mean of ε depends on the variance parameter, ω. Thus, with skewed and heterokedastic measurement error, the conditional mean of ε is no longer constant. Thus, OLS applied to (7) estimates the conditional mean $E[y|x] = E[y^*|x] + E[\varepsilon|x] \neq E[y^*|x]$.

The use of the skew normal assumption is only for illustrative purposes. Nearly all members of the class of skewed distributions (skew normal, skew t, skew Laplace, etc.) possess these same features.[4] In addition, even with the assumption that v is distributed symmetrically, the distribution of $v + u$ will be asymmetric provided that u is distributed asymmetrically (whether u is one-sided or not). It is this asymmetry, and not one-sided measurement error, coupled with heteroskedasticity in the measurement error that leads to the bias and inconsistency of the OLS estimator as the conditional mean of the error term now depends on covariates, leading to the usual omitted variable bias. Consequently, skewness of the residuals provides a potential metric researchers can explore. Finally, note that with heteroskedastic but symmetric measurement error, the mismeasurement only affects efficiency and inference.

Confronting Skewed Measurement Error

Estimating models with one-sided errors has a long history in the analysis of productive efficiency (Kumbhakar and Lovell 2001; Parmeter and Kumbhakar 2014). This literature offers an array of methods to capture the one-sided nature of u_i. Thus, accounting for one-sided measurement error can be addressed with application of these methods.

The most common approach is SFA, which begins by making distributional assumptions on both errors in (3). Typically, $v_i \sim N\left(0, \sigma_v^2\right)$ and the one-sided error is assumed to be distributed half-normal, $u_i \sim N^+\left(0, \sigma_{u_i}^2\right)$.[5] If the observed outcome suffers from one-sided, heteroskedastic measurement error, a (correctly specified) SF model provides consistent estimates of the parameters.

It is noteworthy that while SFA typically uses the half-normal assumption for u, a nearly identical result holds if u is instead distributed as skew normal. The reason for this is that, as demonstrated in González-Farías et al. (2004), the sum of a normal random variable and a skew normal random variable is distributed as skew normal. More formally, the skew normal family is closed under addition; just as the sum of two normal random variables is distributed normal, the sum of two skew normal random variables is distributed skew normal (and a normal random variable is skew normal, with parameter $\alpha = 0$).

Maintaining the previous distributional assumptions on the errors and assuming that the errors are independent, the model is estimated via maximum likelihood with the following log-likelihood function

$$\ln L = \sum_{i=1}^{n}\left\{-\ln \sigma_\varepsilon + \ln \Phi\left[-\frac{s\varepsilon_i\left(\frac{\sigma_{ui}}{\sigma_v}\right)}{\sigma_{\varepsilon_i}}\right] - \frac{\varepsilon_i^2}{2\sigma_{\varepsilon_i}^2}\right\}, \tag{8}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\sigma_{\varepsilon_i}^2 = \sigma_v^2 + \sigma_{u_i}^2$ (Parmeter and Kumbhakar 2014). If u is heteroskedastic, the variance is commonly modeled as

$$\sigma_{u_i}^2 = \exp(z_i\gamma),$$

where typically $z_i = x_i$ (e.g., Caudill et al. 1995).

Alternative methods exist for researchers hesitant to make such distributional assumptions. One approach is to assume that u_i satisfies the scaling property (Simar et al. 1994;

Wang and Schmidt 2002). Under the scaling property, u_i has the following form

$$u_i \sim g(z_i; \delta)u_i^*, \tag{9}$$

where $g(\cdot) \geq 0$ is a function of exogenous variables while u_i^* is a random variable whose distribution does not depend on z_i.[6]

The scaling property captures the idea that the shape of the distribution of the asymmetric measurement error is the same for all observations. However, the scaling function collapses or expands the random variable so that the scale of u_i changes without changing the underlying shape (Parmeter and Kumbhakar 2014). In addition, the scaling property permits estimation without specifying a distribution for v or u^*. Combining (3), the scaling property, and placing structure on $g(\cdot)$ yields the following regression model

$$y_i = x_i\beta + v_i \pm \exp(z_i\delta)u_i^*, \tag{10}$$

where the conditional mean of y given x and z is

$$E[y|x, z] = x\beta \pm \exp(z\delta)\mu^* \tag{11}$$

with $\mu^* = E(u^*)$. We use the $\pm$ here to capture asymmetric measurement error that may either have a negative or positive mean. The regression model with mean zero error term is

$$y_i = x_i\beta \pm \exp(z_i\delta)\mu^* + v_i \mp \exp(z_i\delta)(u_i^* - \mu^*) = x_i\beta \pm \exp(z_i\delta)\mu^* + \varepsilon_i^*, \tag{12}$$

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which can be estimated using Nonlinear Least Squares (NLLS) as

$$\left(\widehat{\beta},\widehat{\delta},\widehat{\mu}^*\right) = \min_{\beta,\delta,\mu^*} n^{-1} \sum_{i=1}^{n} \left[y_i - x_i\beta \pm \mu^* \exp(z_i\delta)\right]^2. \tag{13}$$

The need for NLLS stems from the fact that the scaling function is positive. If one erroneously specifies it as linear, then two problems arise. First, the scaling function would not satisfy positivity everywhere, invalidating the function estimates. Second, the model would suffer from perfect multicollinearity if x and z overlap. In terms of implementation, note that the presence of μ^* implies that one cannot include a constant in z, as this leads to identification issues. Given that the error term from the model in (12) is heteroskedastic by definition, either a generalized NLLS algorithm is required[7] or heteroskedastic robust standard errors are needed for valid inference (White 1980).

SFA and NLLS methods have benefits and shortcomings. SFA will produce consistent and efficient estimates if the distributional assumptions and functional form for the skedastic function are correct. NLLS requires proper specification of the scaling function as well as the scaling property more generally. Other approaches are available that allow one to avoid parametric specification in general, but these typically require more advanced methods and also have shortcomings (Parmeter et al. 2017; Simar et al. 2017).

Assessing the (Likely) Presence of Skewed Measurement Error

While ignoring skewed, heteroskedastic measurement error is likely to cause significant estimation issues, applying the proposed solutions when such errors are not present is apt to be highly inefficient. Thus, testing for the likely presence of skewed, heteroskedastic measurement error seems wise. Fortunately, methods of testing for such properties of residuals are commonplace. In terms of testing skewness, several tests exist. For example, *Stata* im-

plements the test of D'Agostino et al. (1990) with the empirical correction developed in Royston (1991). For heteroskedasticity, again several tests are available. *Stata* implements tests developed in Pagan and Hall (1983), Bruesch and Pagan (1979), and others.

Monte Carlo Study

We undertake two small-scale Monte Carlo studies. The first considers the case of one-sided measurement error. The second examines skewed measurement error. The objectives are threefold. First, to assess the sensitivity of OLS to different degrees of one-sided or skewed measurement error in the dependent variable. Second, to assess the viability of correctly specified SFA and NLLS in such cases. Finally, to assess the sensitivity of the estimators to limited departures from the correct specification.

One-Sided Measurement Error

Design

Data are simulated from variants of the following DGP[8]:

$$
\begin{aligned}
y_i^* &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + v_i \\
y_i &= y_i^* - u_i \\
x_{1i}, x_{2i} &\overset{\text{iid}}{\sim} N(0,1) \\
v_i &\overset{\text{iid}}{\sim} N(0,1) \\
u_i &\sim \left| N(0, \sigma_{u_i}^2) \right| \\
\sigma_{u_i}^2 &= \exp(\gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}^2 + \gamma_4 x_{2i}^2 + \gamma_5 x_{1i} x_{2i})
\end{aligned}
$$

In all designs, we set $\beta_0 = \beta_1 = \beta_2 = 1$. In Design 1, we set $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0$. In Design 2, we set $\gamma_1 = 0.7$, $\gamma_2 = -0.5$, and $\gamma_3 = \gamma_4 = \gamma_5 = 0$. In Design 3, we set $\gamma_1 = 0.7$, $\gamma_2 = -0.5$, $\gamma_3 = 0.25$, $\gamma_4 = -0.25$, and $\gamma_5 = 0.5$. Thus, the one-sided measurement error, u, is homoskedastic in Design 1 and heteroskedastic in Designs 2 and 3. Finally, γ_0 is varied in order to assess the sensitivity of different estimators to the extent of the measurement error. Specifically, we vary γ_0 such that $E[\sigma_v^2]/E[\sigma_{u_i}^2] = \{1, 2, 5, 10\}$. Since $E[\sigma_v^2] = 1$, this is equivalent to $E[\sigma_{u_i}^2] = \{1, 0.5, 0.2, 0.1\}$.

For each experimental design, we conduct 1,000 simulations for two sample sizes, $N = 100$ and $N = 10,000$. We compare the bias, mean absolute error (MAE), mean squared error (MSE), and coverage rate of four estimators: OLS, homoskedastic SF, heteroskedastic SF, and NLLS. In the homoskedastic SF model, σ_{u_i} is assumed to be a constant for all i. In the heteroskedastic SF and NLLS models, $\sigma_{u_i}^2$ is parameterized as $\exp(\gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i})$.[9] As such, in Design 1 the skedastic function is over-specified. In Design 2 the skedastic function is correctly specified. In Design 3 the skedastic function is under-specified.

Results

The results are presented in Figures A1-A3 and Tables A1-A5 in the Supplemental Appendix. Tables A1-A4 report the bias, MAE, MSE, and coverage rate based on 95% confidence intervals. Each table is identical except for the value of γ_0 which then yields differences in $E[\sigma_v^2]/E[\sigma_{u_i}^2]$ (i.e., the relative importance of the idiosyncratic error). For ease of presentation, Figures A1 and A2 present the *median* squared error of our estimators relative to the *median* squared error of OLS for Designs 2 and 3. Figure A1 is based on the simulations with $N = 100$, while Figure A2 is based on the simulations with $N = 10,000$.

Several findings emerge. First, with one-sided, homoskedastic measurement error (Design 1), OLS performs well in terms of estimating the slope coefficients, but not the intercept.

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This is consistent with expectations as one-sided, homoskedastic measurement error only effects the intercept. In Design 1 the performance of the homoskedastic SF model in terms of estimating the slope parameters is nearly indistinguishable from OLS (except the coverage rates are a bit lower in small samples); estimation of the intercept and coverage rates across all parameters are very good in large samples. In Design 1 the heteroskedastic SF model is over-specified. As such, it is inefficient and the coverage rates are low, but the loss in precision is negligible in large samples if the measurement error is severe enough. Thus, with little measurement error, over-specifying the skedastic function leads to higher imprecision of all estimates, even in large samples, particularly for the intercept.

Second, with one-sided, heteroskedastic measurement error (Design 2 or 3), OLS performs poorly for all of the parameters of the model, even when the sample size is large and the measurement error is relatively small (Table A4). The homoskedastic SF model fares a bit better than OLS, but the improvement is marginal. Particularly salient is the fact that the coverage rates of the slope parameters for both OLS and the homoskedastic SF are less than 10% even when the expected variance of the measurement error is 10% of the variance of the idiosyncratic error (Table A4). Thus, even small heteroskedastic measurement error has important consequences; one should be wary of ignoring measurement error as a "minor" issue.

Third, the heteroskedastic SF model performs quite well when the heteroskedasticity function is correctly specified (Design 2), but the results are somewhat mixed when it is under-specified (Design 3). The performance of the heteroskedastic SF is increasing in the sample size and decreasing in the variance of the measurement error. However, even in Table A1 where the variance of the measurement and idiosyncratic errors are equal in expectation, the heteroskedastic SF performs very well with large samples and reasonably well in small samples. As such, a correctly specified SF model does offer a viable solution to researchers.

Fourth, the NLLS models performs quite poorly in small samples. Even when the model is correctly specified (Design 2), the median squared errors are higher than OLS and the correctly specified heteroskedastic SF model (Figure A1). With larger samples, the performance of the NLLS estimator does improve substantially, showing the consistency of the estimator. Nonetheless, the heteroskedastic SF model typically dominates the NLLS estimator when each is either correctly specified (Design 2) or over-specified (Design 1). Strikingly, though, the NLLS model appears to be more robust to misspecification. When the models are under-specified (Design 3), NLLS performs similarly to the heteroskedastic SF model when the measurement error is relatively small (Figure A2 and Tables A3 and A4) and exhibits some signs of superior performance when the measurement error is relatively large (Figure A2 and Tables A1 and A2).

The final set of results are presented in Table A5. In certain cases, one might be interested in the total number of occurrences of the dependent variable in the sample; formally, $Y^* \equiv \sum_{i=1}^{n} y_i^*$. For instance, with civil conflicts, one may be interested in the "true" death toll. The sample provides $Y \equiv \sum_{i=1}^{n} y_i$. Alternatively, estimates can be obtained based on different estimators of the model as $\widehat{Y}^* \equiv \sum_{i=1}^{n} \widehat{y}_i^* = \sum_{i=1}^{n} (\widehat{\beta}_0 + \widehat{\beta}_1 x_{1i} + \widehat{\beta}_2 x_{2i} + \widehat{u}_i)$, where $\widehat{u}_i$ is the estimated measurement error (equal to zero in the case of OLS). Note, OLS yields $\widehat{Y}^* = Y$. Utilizing the four estimators, the bias and mean absolute percentage error (MAPE) are reported in Table A5. The results indicate that OLS performs poorly across all experimental designs. The homoskedastic and heteroskedastic SF models, on the other hand, perform well as long the heteroskedastic function is correctly specified or over-specified. If the SF model is under-specified, it performs better than OLS, but perhaps still not well. The NLLS model continues to perform extremely poorly in small samples, and much worse than the heteroskedastic SF model in large samples if the skedastic function is correctly specified or over-specified. However, NLLS continues to be more robust to misspecification. Finally, Figure A3 plots

kernel density estimates of the distribution of y_i^* and $\widehat{y}_i^*$ for one simulated data set with $N = 10,000$ under Design 2. The results indicate that the observed density is shifted to the left, even in Panel D where the variance of the measurement error is low. The homoskedastic and heteroskedastic SF densities are fairly close to the truth, although the tails are a bit thinner. In contrast, the NLLS densities are over-corrected, lying to the right of the truth, when the measurement error is relatively small (Panels C and D). When the measurement error is relatively large (Panels A and B), the NLLS densities are fairly close to the truth, although the tails are a bit thicker.

Skewed Measurement Error

Design

Data are now simulated from variants of the following DGP:

$$
\begin{aligned}
y_i^* &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + v_i \\
y_i &= y_i^* - u_i \\
x_{1i}, x_{2i} &\overset{iid}{\sim} N(0,1) \\
v_i &\overset{iid}{\sim} N(0,1) \\
\widetilde{u}_i &\sim \left(\frac{\sigma_{u_i}}{20}\right) \chi_{20}^2 \\
u_i &= \widetilde{u}_i - 0.2 \\
\sigma_{u_i}^2 &= \exp(\gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}^2 + \gamma_4 x_{2i}^2 + \gamma_5 x_{1i} x_{2i})
\end{aligned}
$$

We set $\beta_0 = \beta_1 = \beta_2 = 1$ and consider three designs. In Design 1, we set $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0$. In Design 2, we set $\gamma_1 = 0.7$, $\gamma_2 = -0.5$, and $\gamma_3 = \gamma_4 = \gamma_5 = 0$. In Design 3, we set $\gamma_1 = 0.7$, $\gamma_2 = -0.5$, $\gamma_3 = 0.25$, $\gamma_4 = -0.25$, and $\gamma_5 = 0.5$. Thus, the skewed

measurement error, u, is homoskedastic in Design 1 and heteroskedastic in Designs 2 and 3. Finally, γ_0 is varied in order to assess the sensitivity of different estimators to the extent of the measurement error. Again, we vary γ_0 such that $E[\sigma_v^2]/E[\sigma_{u_i}^2] = \{1, 2, 5, 10\}$. Since $E[\sigma_v^2] = 1$, this is equivalent to $E[\sigma_{u_i}^2] = \{1, 0.5, 0.2, 0.1\}$.

To understand the properties of the skewed measurement error, u, define the rth central moments of u_i as

$$m_r = \frac{1}{N} \sum_{i=1}^{n} (u_i - \overline{u})^r .$$

The coefficient of skewness is $m_3 m_2^{-3/2}$. In Designs 1 and 2, the expected coefficients of skewness are approximately 0.6 and 1.8, respectively. In Design 3, it is at least 14. Finally, the expected fraction of observations with $u_i < 0$ varies across all designs parameter configurations. In Design 1, the expected fraction is less than 1%. In Designs 2 and 3, the expected fractions become as high as about 5% and 13%, respectively.

For each experimental design, we conduct 1,000 simulations with $N = 100$ and $N = 10,000$ and report the same metrics as in the prior section. However, there is one important distinction between this set of simulations and the prior set. In Design 3 we now specify the heteroskedasticity error term variance, $\sigma_{u_i}^2$, as $\exp(\gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}^2 + \gamma_4 x_{2i}^2 + \gamma_5 x_{1i} x_{2i})$ during estimation. Thus, the heteroskedastic variance is no longer under-specified during estimation in Design 3. As stated above, the degree of skewness is much higher in Design 3 than Design 2. By making this change, we are not confounding the effects of raising the degree of skewness and the effects of under-specifying the heteroskedastic variance.[10]

Results

The results are presented in Figures A4-A5 and Tables A6-A9 in the Supplemental Appendix. Several findings emerge. First, with skewed, homoskedastic measurement error

(Design 1), OLS performs well in terms of estimating the slope coefficients but not the intercept. Moreover, the performance of the homoskedastic SF model continues to be nearly indistinguishable from OLS, with the exception that the estimation of the intercept is much improved. This is noteworthy because, with skewed measurement error, the distributional assumption pertaining to the one-sided measurement error is not correct. In Design 1, the heteroskedastic SF and NLLS models are over-specified. As such, both are inefficient and, hence, less precise, particularly in small samples. Second, with skewed, heteroskedastic measurement error (Design 2 or 3), OLS and the homoskedastic SF model perform quite poorly. Furthermore, the coverage rates of the slope parameters for both OLS and the homoskedastic SF are close to zero even with relatively little measurement error.

Third, the heteroskedastic SF model performs quite well with skewed, heteroskedastic measurement error (Design 2 or 3) in terms of estimating the individual parameters. However, despite this performance, the coverage rates are not good, particularly with the larger sample size. This is not surprising. It is important to remember that the heteroskedastic SF model is misspecified, despite specifying the variance as a function of the correct covariates, since the distributional assumptions of the model are incompatible with those used to develop the likelihood function. However, despite this misspecification, the heteroskedastic SF model offers an improvement over ignoring the measurement error.

Finally, NLLS performs poorly in small samples. While the coverage rates are better than the heteroskedastic SF model, this reflects the imprecision in the estimates. With larger samples, NLLS performs substantially better, particularly in Design 3 with a high degree of skewness. Moreover, the coverage rates of NLLS in large samples continues to be much better than the heteroskedastic SF model. Thus, with large samples, NLLS can still be a bit imprecise, but the lack of distributional assumptions is a major advantage, particularly with a high degree of skewness.

Civil Conflict

To investigate the importance of addressing possible measurement error, we first revisit some of the analysis in Nepal et al. (2011).[11] The study investigates the role of inequality on conflict, analyzing killings by Nepalese Maoists in the People's War against their government. The conflict lasted from February 13, 1996 until the signing of the Comprehensive Peace Accord on November 21, 2006.

The main estimating equation in Nepal et al. (2011) is

$$\ln(y_{ij}^*) = x_{ij}\beta + \alpha_j + v_{ij} \tag{14}$$

where y_{ij}^* is the number of killings by Nepalese Maoists in village i in district j over the period 1996-2003, x_{ij} is a vector of village-level covariates, α_j are district fixed effects, and v_{ij} is a well-behaved error term. The observed variable, y_{ij}, comes from annual reports by the Informal Sector Services Center (INSEC). Because of the possibility of some violence being undocumented, the true number of killings is likely to exceed those reported. Thus, $y_{ij}^* \geq y_{ij}$ which implies that $s = 1$ in (2). The SF model is given by

$$\ln(y_{ij}) = x_{ij}\beta + \alpha_j - u_{ij} + v_{ij}. \tag{15}$$

Measurement error in the death toll during the Nepalese civil war is indicated by the variation in fatalities reported across sources. Do and Iyer (2010) put the death toll at over 13,000; ReliefWeb (2009) puts it at under 17,000. Reliance on INSEC's subnational data by researchers analyzing the Maoist conflict is common. Joshi and Pyakurel (2015, p. 604) state: "INSEC is highly respected among national and international human rights

communities for their monitoring of human rights issues in Nepal...For their commitment to human rights and to unbiased reporting, they are respected by the rank-and-file members of the Maoist party as well as by government officials." At the same time, the authors also note (p. 605) that "While it is possible that the INSEC data collection processes are biased for their focus on human rights issues, which could be used by domestic and international organizations as leverage to monitor the human rights situation in Nepal, the data are verifiable and correctable." Lastly, we note that the Nepalese government raised the official death toll in 2009 to 16,278 (BBC News 2009), higher than that contained in the INSEC database analysed in Joshi and Pyakurel (2015), which contains records for 15,021 deaths. Thus, the assumption that the INSEC suffers from strictly one-sided measurement error seems likely.

The set of village-level covariates include a measure of inequality (either the Gini coefficient or a polarization index), the percent below the poverty line, average years of education, mean months of employment in 2001, the percent of farmers, the percent speaking Nepali as their primary language, a rural dummy, and log population. The sample includes 3,857 villages across 75 districts. The variance of the error term is modeled as

$$\sigma^2_{u_{ij}} = \exp(z_{ij}\gamma), \tag{16}$$

where z_{ij} includes an intercept only (homoskedastic SF) or a linear or quadratic function of all variables in x_{ij} (heteroskedastic SF and NLLS).

Finally, the authors are concerned about the possible endogeneity of their primary regressor of interest, inequality. As such, they instrument for inequality using three village-level instruments: percent of households operating agricultural land, percent of households with female ownership of land, and the average number of big head livestock owned by women.

For the SF and NLLS models, we address endogeneity via a control function approach with bootstrap standard errors (see, e.g., Amsler et al. 2016). Specifically, (15) is augmented with the first-stage residual, becoming

$$\ln(y_{ij}) = x_{ij}\beta + \alpha_j + \widehat{\eta}_{ij} - u_{ij} + v_{ij}, \tag{17}$$

where $\widehat{\eta}_{ij}$ is the estimated residual from the first-stage model given by

$$x_{1ij} = w_{ij}\pi + \alpha_j + \eta_{ij}. \tag{18}$$

In (18), x_{1ij} represents the element of x corresponding to the inequality covariate and w_{ij} denotes the vector of excluded instruments and remaining (exogenous) elements of x.[12]

Coefficient estimates are presented in Tables 1 and 2.[13] The tables are identical except that the Gini coefficient is used to measure inequality in Table 1, whereas a polarization index is used in Table 2. The OLS and IV estimates (estimated using Two-Stage Least Squares) are identical to the extended model results in Tables 2 and 3 in Nepal et al. (2011).[14] Pagan and Hall (1983) tests of heteroskedasticity always reject the null of homoskedasticity ($p < 0.01$ in all cases) in the original models. Similarly, we easily reject the null that the error term in the original models are not skewed ($p < 0.01$ in all cases). While not shown, in our models, we find the variance of the one-sided error component depends on the poverty rate, population, share speaking Nepali as their primary language, and rural status. Moreover, the extent of measurement error is modest. Across the eight SF models, the ratio of the average variance of the idiosyncratic error to the one-sided error term varies from 4.3 to 4.7.

When inequality is treated as exogenous, the OLS estimates differ in three important ways from those addressing measurement error. First, poverty becomes a statistically significant determinant of violence in the various SF models. A one standard deviation increase

Table 1. Determinants of Killings by Nepalese Maoists: 1996-2003.

| Variable | OLS | IV | Heteroskedastic Stochastic Frontier | | | | NLLS | |
			Exogenous		Endogenous		Exogenous	Endogenous
Gini Coefficient	-0.2773	4.0288	-0.0394	-0.0129	3.9192	4.7353	0.8554	6.9770
	(0.281)	(1.582)	(0.335)	(0.359)	(1.656)	(1.742)	(0.933)	(5.973)
Poverty Rate (%)	-0.0055	0.4338	0.3624	0.2622	0.7741	0.7409	-0.1989	0.4665
	(0.105)	(0.191)	(0.115)	(0.118)	(0.194)	(0.189)	(0.282)	(1.266)
Ln(Population)	0.2118	0.0108	0.2264	0.2600	0.0408	0.0423	0.1993	-0.0902
	(0.023)	(0.076)	(0.027)	(0.029)	(0.083)	(0.086)	(0.053)	(0.751)
Education (in Years)	0.0130	0.0077	0.0159	0.0168	0.0117	0.0110	0.0338	0.0215
	(0.011)	(0.012)	(0.014)	(0.015)	(0.015)	(0.016)	(0.029)	(0.146)
Employment (Months)	-0.0041	-0.0048	-0.0045	-0.0070	-0.0052	-0.0077	0.0006	-0.0016
	(0.006)	(0.006)	(0.007)	(0.008)	(0.008)	(0.009)	(0.015)	(0.080)
Farmers (%)	-0.0761	0.1520	-0.0570	-0.0459	0.1657	0.2053	-0.4745	-0.2969
	(0.096)	(0.128)	(0.120)	(0.121)	(0.149)	(0.147)	(0.325)	(1.166)
Nelapi (%)	0.1089	0.0918	0.2765	0.2593	0.2576	0.2366	-0.1268	-0.0806
	(0.040)	(0.041)	(0.047)	(0.047)	(0.061)	(0.060)	(0.143)	(1.243)
Rural (1 = Yes)	-0.5696	-0.7166	-1.6501	-1.4743	-1.7876	-1.621	-1.2797	-1.4780
	(0.075)	(0.093)	(0.191)	(0.181)	(0.449)	(0.384)	(0.440)	(12.423)
Constant	-0.7908	-0.0984	0.0019	-0.3936	0.6396	0.3274	0.2996	1.2533
	(0.221)	(0.337)	(0.319)	(0.318)	(0.628)	(0.579)	(0.781)	(22.125)
Control Function					-4.2104	-4.9916		-6.2559
					(1.761)	(1.845)		(1.988)
Heteroskedasticity Function			Linear	Quadratic	Linear	Quadratic	Quadratic	Quadratic

Notes: OLS = Ordinary Least Squares. IV = Instrumental Variable. NLLS = Nonlinear Least Squares. Standard errors in parentheses; obtained from 500 bootstrap repetitions for the endogenous stochastic frontier and NLLS models. Number of observations = 3,857 villages. Killings is the number killed by Maoists from 1996-2003. 75 district fixed effects are also included. The Gini coefficient is instrumented for using the percent of households operating agricultural land, percent of households with women ownership of land, and average number of big head livestock owned by women. Heteroskedasticity refers to the variance of the one-sided error and depends on all covariates except the district fixed effects in the exogenous models and all covariates except the district fixed effects and inequality in the endogenous models.

in the village-level poverty rate is associated with a 4.7% increase in the expected number of killings (using the results from the quadratic specification). Second, the impact of the percent speaking Nepali as their primary language, while statistically significant at conventional levels in the OLS and SF models, more than doubles in magnitude in the SF models (roughly 0.11 in OLS models to 0.26 in the heteroskedastic SF models). Third, rural villages experience significantly fewer killings according to the SF and NLLS models. According to the OLS estimates, rural villages experienced 43% fewer killings in expectation. This increases to more than 60% according to the heteroskedastic SF and NLLS estimates. Finally, it is worth noting that the NLLS estimates tend to be much more imprecise.

When inequality is treated as endogenous, the NLLS estimates are extremely imprecise; consistent with the much larger MSEs found in the simulations when the sample size is not overly large. However, both the SF and NLLS models reject the null of exogeneity; the coefficients on the control function are statistically significant at conventional levels in

Table 2. Determinants of Killings by Nepalese Maoists: 1996-2003.

Variable	OLS	IV	Heteroskedastic Stochastic Frontier				NLLS	
			Exogenous		Endogenous		Exogenous	Endogenous
Polarization Index	1.202	12.9682	2.0526	2.3932	12.3418	15.7388	4.2649	21.6319
	(0.726)	(5.929)	(0.957)	(0.995)	(5.796)	(6.062)	(1.568)	(23.976)
Poverty Rate (%)	0.0246	0.0430	0.3673	0.259	0.3919	0.2863	0.0537	0.0580
	(0.101)	(0.104)	(0.111)	(0.112)	(0.147)	(0.141)	(0.120)	(0.779)
Ln(Population)	0.1949	0.1561	0.2188	0.2532	0.1823	0.2106	0.1949	0.1425
	(0.019)	(0.027)	(0.023)	(0.024)	(0.034)	(0.036)	(0.025)	(0.441)
Education (in Years)	0.0123	0.0090	0.0147	0.0149	0.0127	0.0123	0.0278	0.0284
	(0.011)	(0.012)	(0.014)	(0.015)	(0.016)	(0.016)	(0.014)	(0.120)
Employment (Months)	-0.0046	-0.0084	-0.0048	-0.0071	-0.0084	-0.0121	0.0011	-0.0057
	(0.006)	(0.006)	(0.007)	(0.008)	(0.008)	(0.009)	(0.007)	(0.033)
Farmers (%)	-0.0442	0.1234	-0.0200	-0.0075	0.1337	0.1792	0.0091	0.2328
	(0.096)	(0.129)	(0.121)	(0.121)	(0.156)	(0.152)	(0.112)	(0.550)
Nelapi (%)	0.1073	0.1022	0.2836	0.2658	0.2717	0.2517	0.0678	0.0461
	(0.040)	(0.041)	(0.047)	(0.047)	(0.059)	(0.060)	(0.051)	(0.655)
Rural (1 = Yes)	-0.5762	-0.5477	-1.6374	-1.4678	-1.6282	-1.4247	-1.0122	-1.0160
	(0.075)	(0.078)	(0.189)	(0.180)	(0.427)	(0.379)	(0.262)	(17.401)
Constant	-0.8974	-2.3773	-0.2713	-0.7087	-1.5337	-2.4064	-0.9893	-3.1420
	(0.235)	(0.778)	(0.343)	(0.337)	(0.953)	(0.964)	(0.438)	(19.805)
Control Function					-11.2448	-14.8098		-17.9706
					(5.781)	(6.038)		(6.598)
Heteroskedasticity Function			Linear	Quadratic	Linear	Quadratic	Quadratic	Quadratic

Notes: See Table 1.

all cases. Comparing the IV and SF estimates, two important differences arise. First, the positive effects of the poverty rate and share speaking Nepali as their primary language and the negative effects of being rural continue to be of much greater magnitude in the SF models. The effects of these covariates are significantly attenuated when one-sided, heteroskedastic measurement error is ignored. In fact, the effect of poverty is close to zero and not statistically significant according to traditional IV when polarization is used to measure inequality (Table 2). When the Gini coefficient is used to measure inequality (Table 1), a one standard deviation increase in the poverty rate is associated with a 7.9% increase in expected killings according to the IV estimates. The corresponding value is 13.8% according to the quadratic heteroskedastic SF model.

Second, while the IV results indicate a statistically significant impact of inequality on killings using both measures of inequality, the impact is also greater in magnitude in the heteroskedastic SF models. According to the traditional IV estimates, a one standard devia-

tion increase in the Gini coefficient (polarization index) leads to a 19% (14%) increase in the expected number of killings. The quadratic heteroskedastic SF model yields a corresponding estimate of 23% (17%). The NLLS estimates also suggest a larger effect of inequality, but the estimates are quite imprecise as noted above.

Table 3. Reported and Predicted Killings by Nepalese Maoists: 1996-2003.

| | Observed | Heteroskedastic Stochastic Frontier | | | | NLLS | |
		Exogenous		Endogenous		Exogenous	Endogenous
I. Measure of Inequality: Gini							
Aggregate	2629	7826	7782	7823	7780	9776	9062
Killings		(425)	(413)	(426)	(404)	(8e36)	(5e35)
II. Measure of Inequality: Polarization Index							
Aggregate	2629	7843	7796	7832	7785	7049	7179
Killings		(402)	(382)	(408)	(395)	(7e36)	(2e35)
Heteroskedasticity Function		Linear	Quadratic	Linear	Quadratic	Quadratic	Quadratic

Notes: NLLS = Nonlinear Least Squares. Standard errors in parentheses; obtained from 500 bootstrap repetitions for the endogenous stochastic frontier models. Number of observations = 3,857 villages. Aggregate killings is the number killed by Maoists from 1996-2003 across all villages. Results based on estimates in Tables 1 and 2. See text for further details.

Finally, Table 3 and Figure 1 compare the observed total number of killings – in aggregate (Table 3) or across villages and districts (Figure 1) – with estimates based on the heteroskedastic SF and NLLS models. Table 3 reveals that the observed number of killings is only about one-third the estimated number of killings obtained from the heteroskedastic SF models. The NLLS models point to even more killings, but the standard errors are enormous.[15] Figure 1 reveals that the modal number of reported killings experienced by village or district level is also about one-third to one-fourth the estimated number of killings.

In sum, while the qualitative findings in Nepal et al. (2011) remain after accounting for one-sided heteroskedastic measurement error using a SF model, the quantitative impor-
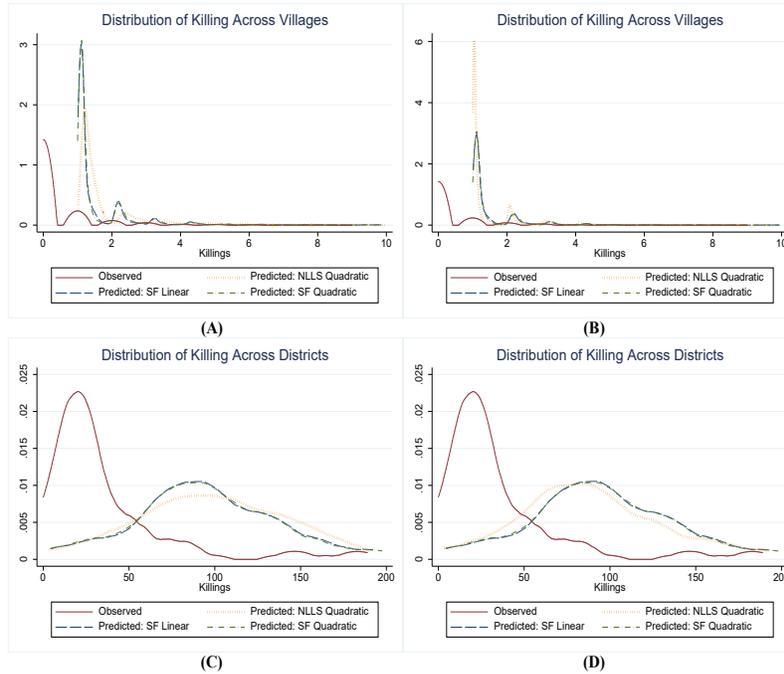
Figure 1. Distribution of Observed and Predicted Killings.
Notes: Predictions in (A) and (C) based on estimates in columns 5 and 6 in Table 6, with predicted killings aggregated at the village or district level. Predictions in (B) and (D) based on estimates in columns 5 and 6 in Table 7, with predicted killings aggregated at the village or district level. Kernel densities obtained using an Epanechnikov kernel and Silverman's (1986) rule-of-thumb bandwidth. See text for further details.

tance of economic variables – inequality and poverty – is found to be much larger. Thus, the authors conclusion that local economic conditions are salient determinants of internal conflict is enhanced. Moreover, our estimates reveal that the reported number of killings may significantly under count the actual death toll.

Criminal Activity

Next, we revisit some of the analysis in Galiani et al. (2011).[16] The study investigates the causal effect of peace time military service on subsequent criminal behavior using data from Argentina. The study is motivated by the numerous calls around the globe for conscription

as a tool to combat youth criminal activity. The main estimating equation in Galiani et al. (2011) is

$$y_{ij}^* = x_{ij}\beta + \alpha_j + v_{ij} \tag{19}$$

where y_{ij}^* is the crime rate of individuals in cohort i ($i = 1958, \ldots, 1962$) with a national identification number containing the last three digits j ($j = 000, 001, \ldots, 999$), x_{ij} is a vector of covariates, α_j are cohort fixed effects, and v_{ij} is a well-behaved error term. The observed variable, y_{ij}, comes from individual-level administrative records. Specifically, any individual ever prosecuted for or convicted of a crime is recorded in the data. The fraction of individuals in each cohort-identification number cell with a criminal record constitutes the observed crime rate. Because of the possibility of some criminal activity going undiscovered, the true crime rate is likely to exceed the observed crime rate. Thus, $y_{ij}^* \geq y_{ij}$ which implies that $s = 1$ in (2). The SF model is given by

$$y_{ij} = x_{ij}\beta + \alpha_j - u_{ij} + v_{ij}. \tag{20}$$

The set of covariates includes the fraction of each cohort-identification number cell that served in the military, the percent of Argentine-born indigenous individuals, the percent of naturalized citizens, and the percent from each of 24 districts. The sample size is 5000 (five cohorts times 1000 three digit endings on national identification numbers). The variance of the error term is modeled as

$$\sigma_{u_{ij}}^2 = \exp(z_{ij}\gamma), \tag{21}$$

where z_{ij} includes an intercept, the percent of Argentine-born indigenous individuals, the percent of naturalized citizens, and cohort fixed effects. The NLLS model is similar except that controls for district are omitted, as are the cohort fixed effects in the variance of the

error term. These omissions are needed as otherwise the NLLS model does not converge.

Finally, the authors are concerned about the possible endogeneity of their primary regressor of interest, conscription. As such, they instrument for military service using a single instrument: whether the individuals with a particular combination of the final three digits of their national identification number in a particular cohort were randomly chosen, via lottery, to be draft eligible. The authors also estimate reduced form specifications, where an indicator for being draft eligible replaces the conscription variable.

Again, we address endogeneity via a control function approach with bootstrap standard errors. Specifically, (20) is augmented with the first-stage residual, becoming

$$y_{ij} = x_{ij}\beta + \alpha_j + \widehat{\eta}_{ij} - u_{ij} + v_{ij}, \tag{22}$$

where $\widehat{\eta}_{ij}$ is the estimated residual from the first-stage model given by

$$x_{1ij} = w_{ij}\pi + \alpha_j + \eta_{ij}. \tag{23}$$

In (20), x_{1ij} represents the element of x corresponding to the conscription covariate and w_{ij} is a vector including the excluded instrument and remaining (exogenous) elements of x.

Coefficient estimates are presented in Table 4. The OLS and IV estimates are identical to columns 2 and 4 in Table 4 in Galiani et al. (2011). While not shown, the variance of the one-sided error component is found to be, at best, marginally related to the covariates included in (21). This is consistent with the results from Pagan and Hall (1983) tests of heteroskedasticity applied to the original models in Galiani et al. (2011); the null of homoskedasticity is never rejected ($p \approx 0.70$ in all cases). However, we easily reject the null that the error term in the original models are not skewed ($p < 0.01$ in all cases). Moreover, as shown in Galiani et al. (2011), the reduced form effect of being draft eligible or the

causal effect of conscription does not change when covariates are included in the model. Together, this suggests that we do not expect the reduced form effect of being draft eligible or the causal effect of conscription to change when allowing for the possibility of one-sided measurement error. Indeed, this is what we find. This is comforting in that allowing for the possibility of one-sided, heteroskedastic measurement error does not alter the findings when, in fact, heteroskedasticity is not present or at least not believed to be problematic.

Table 4. Determinants of Crime Rates in Argentina: 1958-1962 Cohorts.

Variable	OLS	IV	Heteroskedastic Stochastic Frontier		NLLS	
			Exogenous	Endogenous	Exogenous	Endogenous
Draft Eligible	0.0018		0.0018		0.0027	
	(0.0006)		(0.0006)		(0.0006)	
					(0.0004)	
Conscription		0.0027		0.0027		0.0027
		(0.0008)		(0.0008)		(0.0003)
Indigenous	-0.0594	-0.0588	-0.1110	-0.1203	-13.9970	-11.8457
	(0.1283)	(0.1284)	(0.1669)	(0.3367)	(9120)	(5.1464)
					(6.4731)	
Naturalized	0.1099	0.1114	0.1240	0.1146	37.2224	31.4631
	(0.1758)	(0.1760)	(0.1928)	(0.4110)	(24647)	(4.4234)
					(4.2287)	
Constant	0.0132	0.0131	0.0156	0.0165	9.8336	7.1076
	(0.0664)	(0.0664)	(0.0662)	(0.0668)	(13190)	(0.9882)
					(1.0243)	
Control Function				-0.0215		-0.0200
				(0.0057)		(0.0020)

Notes: OLS = Ordinary Least Squares. IV = Instrumental Variable. NLLS = Nonlinear Least Squares. All regressions also include controls for cohort. All regressions except for those estimated via NLLS also include controls for district. Robust standard errors in parentheses for the OLS, IV, and exogenous stochastic frontier models. Standard errors obtained from 500 bootstrap repetitions for the endogenous stochastic frontier and NLLS models. Robust standard errors computed as in Davidson and MacKinnon (2004) are in parentheses immediately under the point estimates for the exogenous NLLS model; standard errors from 500 bootstrap repetitions are provided beneath these for comparison. Number of observations = 5,000. Heteroskedasticity refers to the variance of the one-sided error and depends on cohort, indigenous, and naturalized in the stochastic frontier models; heteroskedasticity of the measurement error depends on indigenous and naturalized in the NLLS models.

Specifically, the SF estimates of the reduced form effect of being draft eligible and the causal effect of conscription are identical to those in Galiani et al. (2011). Coefficient estimates on the remaining controls are also qualitatively unchanged. The NLLS estimate of the causal effect of conscription is also identical to Galiani et al. (2011). However, the NLLS estimate of the reduced form effect of being draft eligible is larger (and identical to causal effect of conscription). Finally, the NLLS estimates of other covariates in the model are vastly different from the other models. As in the previous application, this results from the

more tenuous identification in the NLLS model relative to the SF model that incorporates distributional assumptions which aid identification.

Pollution

Lastly, we revisit some of the analysis in Kono (2017).[17] The study investigates the causal effect of country-level tariff reductions on per capita carbon dioxide (CO_2) emissions. The main estimating equation in Kono (2017) is

$$\ln(y_{it}^*) = x_{it}\beta + \alpha_i + v_{it} \tag{24}$$

where y_{it}^* captures per capita CO_2 emissions in country i in period t ($t = 1988, \ldots, 2013$), x_{ij} is a vector of covariates, α_i are country fixed effects, and v_{it} is a well-behaved error term. The observed variable, y_{ij}, comes from the World Banks's *World Development Indicators*. The World Bank states that the measure is derived from data on country-level fossil fuel consumption obtained from the United Nations Statistics Division's World Energy Data Set, along with data from the U.S. Department of Interior's Geological Survey on the global cement manufacturing.[18] In turn, the United Nations states that data on country-level fossil fuel consumption comes from annual questionnaires administered to "national statistical offices, ministries of energy or other authorities responsible for energy statistics in the country."[19] In addition to the potential for countries to under-report their fossil fuel consumption for political reasons, particularly for counties that are parties to the Kyoto Protocol, the World Bank notes that the measure of CO_2 emissions "excludes emissions from land use such as deforestation."[20] Thus, $y_{ij}^* \geq y_{ij}$ which implies that $s = 1$ in (2). The SF model is given by

$$\ln(y_{it}) = x_{it}\beta + \alpha_i - u_{it} + v_{it}. \tag{25}$$

The set of covariates includes the the average applied tariff on manufacturing goods, an indicator for being a party to the Kyoto Protocol, log per capita gross domestic product (GDP), lagged log per capita CO_2 emissions, and a quartic time trend. The sample is an unbalanced panel of 1906 observations across 152 countries. The variance of the error term is modeled as

$$\sigma^2_{u_{it}} = \exp(z_{it}\gamma), \tag{26}$$

where z_{it} includes an intercept, log per capita GDP and its quadratic, lagged log per capita CO_2 emissions and its quadratic, an indicator for being a party to the Kyoto Protocol, and three pairwise interactions between log per capita GDP, lagged log per capita CO_2 emissions, and the binary measure of the Kyoto Protocol. The NLLS model is identical.

Finally, the author is concerned about the possible endogeneity of their primary regressor of interest, manufacturing tariffs.[21] As such, this variable is instrumented for using two instruments: years since the conclusion of the Uruguay Round interacted with an indicator for whether country i is a World Trade Organization (WTO) member and the average tariff rate in country i's contiguous neighbors. The sample size is 1608 in the models allowing for endogeneity.

Again, we address endogeneity via a control function approach with bootstrap standard errors. Specifically, (25) is augmented with the first-stage residual, becoming

$$\ln(y_{it}) = x_{it}\beta + \alpha_i + \widehat{\eta}_{it} - u_{it} + v_{it}, \tag{27}$$

where $\widehat{\eta}_{it}$ is the estimated residual from the first-stage model given by

$$x_{1it} = w_{it}\pi + \alpha_i + \eta_{it}. \tag{28}$$

In (25), x_{1it} represents the element of x corresponding to the tariff covariate and w_{it} is a vector including the excluded instruments and remaining (exogenous) elements of x. Note, in the model treating manufacturing tariffs as endogenous, z_{it} in (26) includes all the covariates from the model assuming exogeneity plus the two instruments, their quadratics, and all pairwise interactions between the covariates.

Coefficient estimates are presented in Table 5. The OLS and IV estimates are identical to columns 1 and 2 in Table 4 in Kono (2017). Pagan and Hall (1983) tests of heteroskedasticity always reject the null of homoskedasticity ($p < 0.01$ in all cases) in the original models. Similarly, we easily reject the null that the error term in the original models are not skewed ($p < 0.01$ in all cases). While not shown, the variance of the one-sided error component is found to be at least marginally related to the covariates included in (26). That said, the SF estimates are mostly in agreement with Kono (2017). The sole exception relates to the impact of being a member of the Kyoto Protocol. Whereas Kono (2017) finds that members have about 3% lower per capita CO_2 emissions on average, the SF models indicate an 8% reduction. In contrast, while the NLLS estimates under endogeneity are similar to the IV estimates in Kono (2017), the NLLS estimates under exogeneity differ (although the signs of the coefficients are in agreement). As in the previous applications, this appears to arise from the more tenuous identification in the NLLS model as suggested by the larger standard errors.

Conclusion

Nonclassical measurement error in the dependent variable can be problematic in a linear regression framework, especially if it is heteroskedastic. This holds even if the measurement error is "small." This article draws researchers' attention to potential solutions. While it is not a panacea, the usage of SF and NLLS models should be a part of the researcher's tool box

Table 5. Determinants of Per Capita Carbon Dioxide Emissions.

Variable	OLS	IV	Heteroskedastic Stochastic Frontier		NLLS	
			Exogenous	Endogenous	Exogenous	Endogenous
Manufacturing	-0.0009	-0.0014	-0.0007	0.0007	-0.0005	-0.0038
Tariff	(0.0004)	(0.0035)	(0.0004)	(0.0044)	(0.0004)	(0.0035)
ln(Per Capita	0.1351	0.1477	0.1534	0.1586	0.3271	0.1506
GDP)	(0.0345)	(0.0361)	(0.0221)	(0.0335)	(0.2140)	(0.0359)
Kyoto Protocol	-0.0317	-0.0297	-0.0871	-0.0712	-0.1486	-0.0247
	(0.0110)	(0.0158)	(0.0183)	(0.0219)	(0.1059)	(0.0152)
Time	0.0012	0.0016	0.0000	-0.0007	0.0022	0.0002
	(0.0015)	(0.0032)	(0.0017)	(0.0042)	(0.0013)	(0.0036)
Time^2	-0.0001	-0.0002	-0.0001	-0.0001	-0.0002	-0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ln(Lagged	0.7405	0.7419	0.7227	0.7389	0.4109	0.7083
Per Capita CO_2)	(0.0403)	(0.0462)	(0.0465)	(0.0505)	(0.3821)	(0.0371)
Control Function				-0.0014		0.0027
				(0.0044)		(0.0035)

Notes: OLS = Ordinary Least Squares. IV = Instrumental Variable. NLLS = Nonlinear Least Squares. All regressions also include controls for cohort. All regressions except for those estimated via NLLS also include controls for district. Clustered standard errors in parentheses for the OLS, IV, and exogenous stochastic frontier and NLLS models. Standard errors obtained from 500 bootstrap repetitions for the endogenous stochastic frontier and NLLS models. Number of observations = 1,906 in models under exogeneity and 1,608 in models under endogeneity. Heteroskedasticity refers to the variance of the one-sided error and depends on in the models under exogeneity; in the models under endogeneity.

when there is concern of asymmetric, heteroskedastic measurement error. Moreover, recent developments in the SF literature allow for semiparametric specifications of the model, as well as an assessment of the determinants of the measurement error. Future research ought to explore alternative methods of addressing skewed, systematic measurement error. In particular, research is needed to address this problem in situations where the outcome is not continuous, but rather a limited dependent variable such as a binary, multinomial, or count random variable, as these frequently appear in empirical research.

Finally, concerning recommendations for best practice, our results here point to, at a minimum, practitioners checking their residuals for both heteroskedasticity and skewness. If there is evidence of both and there is suspicion that measurement error in the dependent variable exists, then it is likely that some of the initial estimates are biased.

Notes

[1]Semi- and nonparametric stochastic frontier approaches also exist (Parmeter and Kumbhakar 2014). We do not consider such models here.

[2]We thank an anonymous referee for raising this issue.

[3]See Santos Silva and Tenreyro (2006) for a similar argument in the context of log-linearizing a regression model.

[4]To see this, consider a quite general setup for skewed measurement error, known as the skewed generalized t distribution, which has density

$$f(\varepsilon; \mu, \sigma, \lambda, p, q) = \frac{p}{2vq^{1/p}B\left(p^{-1}, q\right)\left(\frac{|\nu|^p}{q(v\sigma)^p(\lambda sign(\nu)+1)^p} + 1\right)^{p^{-1}+q}}, \qquad (29)$$

where $\nu = \varepsilon - \mu + m$, $m = \frac{2v\sigma\lambda q^{p^{-1}}B(2p^{-1}, q-p^{-1})}{B(p^{-1}, q)}$,

$$v = \frac{q^{-p^{-1}}}{\sqrt{(3\lambda^2+1)\frac{B(3p^{-1}, q-2p^{-1})}{B(p^{-1}, q)} - 4\lambda^2\frac{B(2p^{-1}, q-p^{-1})^2}{B(p^{-1}, q)^2}}},$$

and $B(\cdot, \cdot)$ is the beta function (Theodossiou 1998). Here μ captures the location, σ the scale, and λ the skewness, with p and q controlling kurtosis together. The skewed generalized t distribution contains many popular skewed densities as special cases, including the skewed t ($p = 2$), skewed Laplace ($p = 1$, $q \to \infty$), and skewed normal ($p = 2$, $q \to \infty$) distributions.

One common simplification of this density is to set $m = 0$ and $v = 1$. In this case, a random variable with skewed generalized t has a mean of $\mu + \frac{2\sigma\lambda q^{p^{-1}}B(2p^{-1}, q-p^{-1})}{B(p^{-1}, q)}$ provided $pq > 1$. Again, even if $\mu = 0$, so long as $\lambda \neq 0$ the skewed generalized t random variate will no longer be mean zero. Even more problematic, the mean is no longer constant if

heteroskedasticity is present.

[5] Other distributions for u are common in the SF literature, such as the exponential and truncated normal. The half-normal and exponential have modes at zero, whereas the mode of truncated normal depends on the estimated mean. This has implications for the nature of the measurement error. For brevity, we focus on the half-normal, implying that the modal value of the measurement error is zero. In some applications, it may make sense to relax this restriction with the truncated normal distribution.

[6] Distributional assumptions (such as half-normal or truncated-normal) can be imposed on u_i^*, but unlike the setting just described, they are not necessary for consistent estimation. In the stochastic frontier literature u_i^* is assumed to be one-sided. However, this is not a necessary requirement.

[7] The downside of this approach is that distributional assumptions are required to properly disentangle σ_v^2 and σ_u^{2*} which undermines the use of NLLS in the first place.

[8] See Wang and Schmidt (2002) for a similar MC study motivated by traditional efficiency analysis.

[9] Given the inclusion of γ_0 in the NLLS model, μ^* is normalized to one.

[10] We also conduct Design 3 when continuing to under-specify the heteroskedastic variance. These results are available upon request.

[11] We are grateful to the authors for making the data available at http://kishoregawande.net/.

[12] When inequality is treated as endogenous, it is excluded from $\sigma_{u_i}^2$ in (16). If the variance of u depends on inequality, then its expectation does as well and any instrument for inequality will not be independent of u. It appears that the variance of u, however, does not depend on inequality and therefore this choice matters little in practice.

[13] NLLS with a linear skedastic function is omitted as the models had difficulty

converging.

[14]While the point estimates are identical, the IV standard errors differ. Nepal et al. (2011) use heteroskedasticity-robust standard errors in their IV regressions, but not in the OLS regressions. We use non-robust throughout for the SF models; heteroskedasticity-robust standard errors for the NLLS models.

[15] The imprecise NLLS estimates result from the more tenuous identification in the NLLS model relative to the SF model that also incorporates distributional assumptions. However, examining the individual bootstrap estimates reveals that the large standard errors are driven by replications in which the NLLS model is poorly identified and, as a result, produce unreasonable estimates. If we use the estimated total killings reported in Table 3, eliminate bootstrap replications which produce the top quartile of estimated death tolls in the exogenous or endogenous NLLS models, and obtain standard errors using the remaining bootstrap estimates, then the NLLS standard errors are much more in line with those in the heteroskedastic SF models (ranging from about equal to about 30% larger, with a few still about twice as large). This suggests that the NLLS is a potentially viable model, but researchers need to be wary of weakly identified models.

[16]The data are available at

`https://www.aeaweb.org/articles?id=10.1257/app.3.2.119`.

[17]The data are available at `https:`

`//dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/SJN8ZE`.

[18]See `http://wdi.worldbank.org/table/3.8`. Accessed on 21 June 2019.

[19]See `https://unstats.un.org/unsd/energy/`. Accessed on 21 June 2019.

[20]See `http://wdi.worldbank.org/table/3.8`. Accessed on 21 June 2019.

[21]The author also is concerned about the endogeneity of lagged per capita CO_2 emissions due to the inclusion of country fixed effects. We do not pursue this here.

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Supplemental Appendix

(A) β_0

(B) β_1
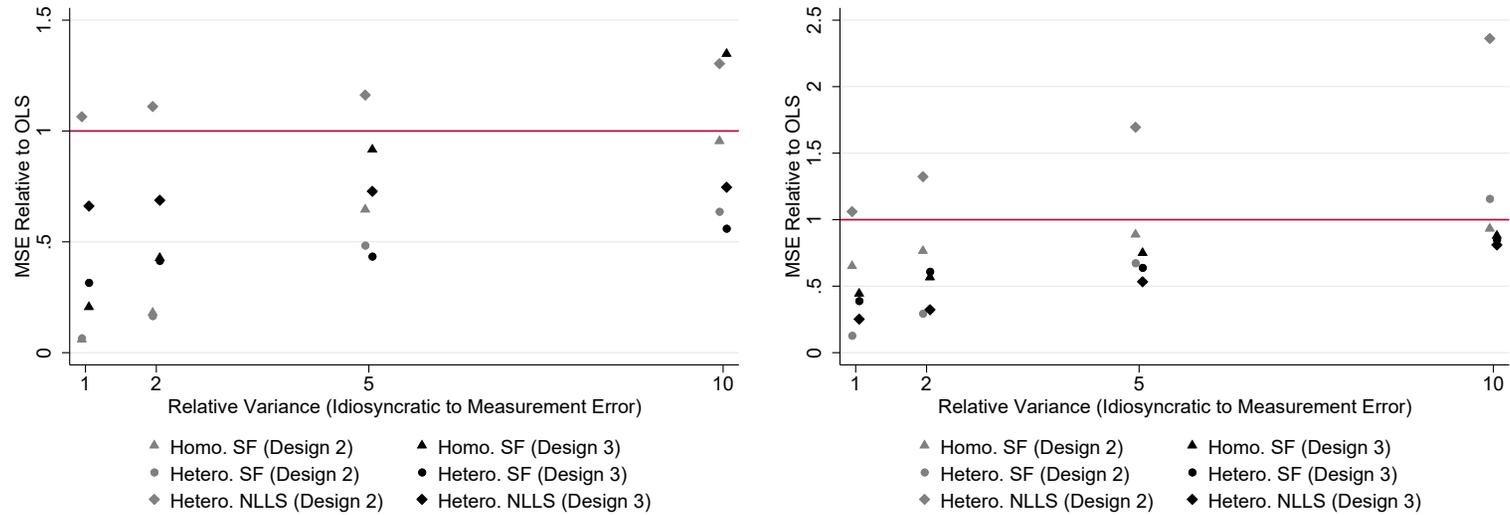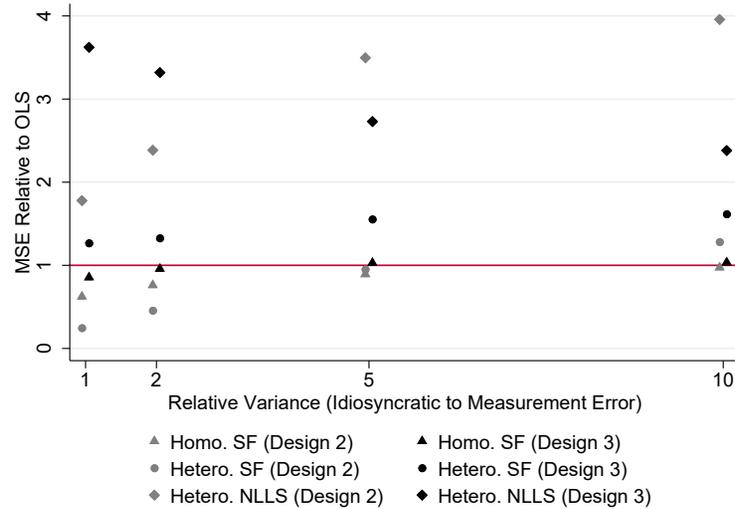
(C) β_2

Figure A1. Monte Carlo Results (One-Sided Measurement Error): Median Squared Error Relative to Ordinary Least Squares.
Notes: Results based on 1,000 simulations with N=100. See text for further details.

(A) β₀

(B) β₁
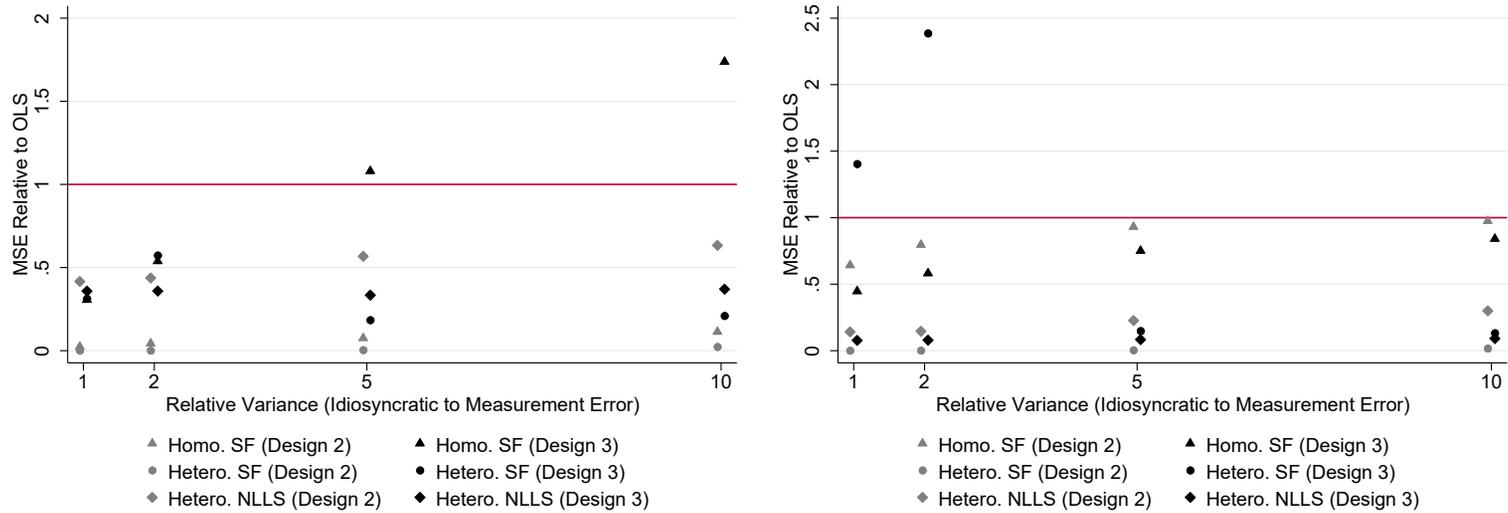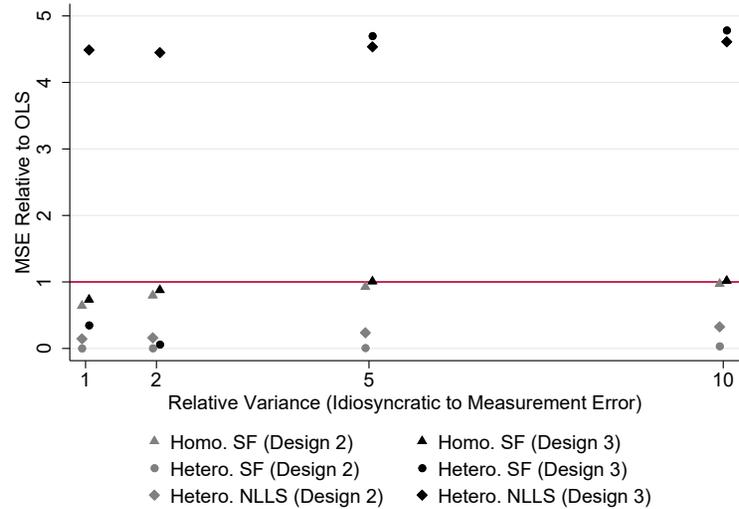
(C) β₂

Figure A2. Monte Carlo Results (One-Sided Measurement Error): Median Squared Error Relative to Ordinary Least Squares.
Notes: Results based on 1,000 simulations with N=10,000. See text for further details.

(A) $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$

(B) $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$

(C) $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$

(D) $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$
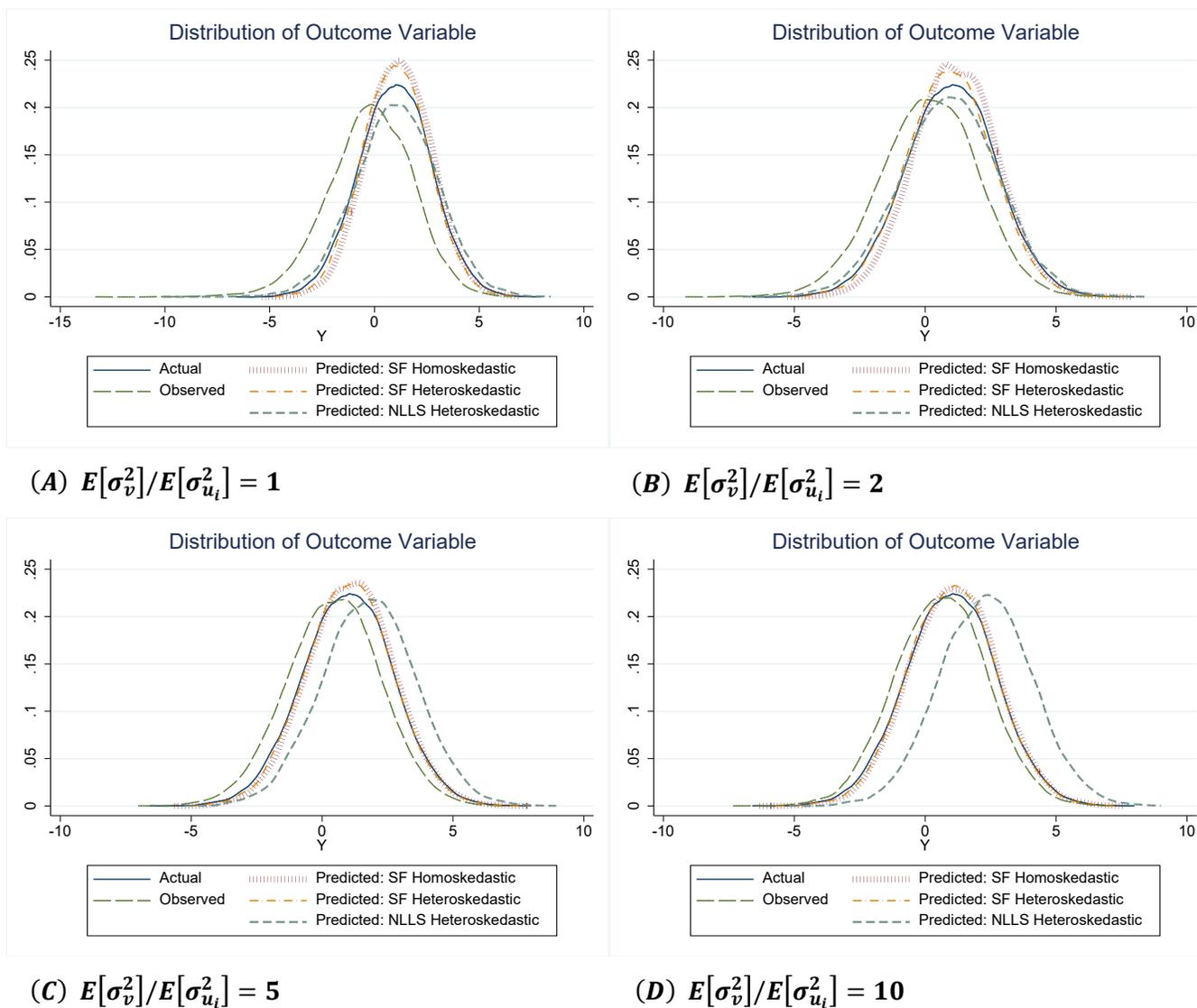
Figure A3. Monte Carlo Results (One-Sided Measurement Error): Distribution of Observed and Predicted Dependent Variable.
Notes: Results based on a single repetition of DGP2 with N=10,000. Kernel densities obtained using an Epanechnikov kernel and Silverman's (1986) rule-of-thumb bandwidth. See text for further details.

(A) β_0

(B) β_1
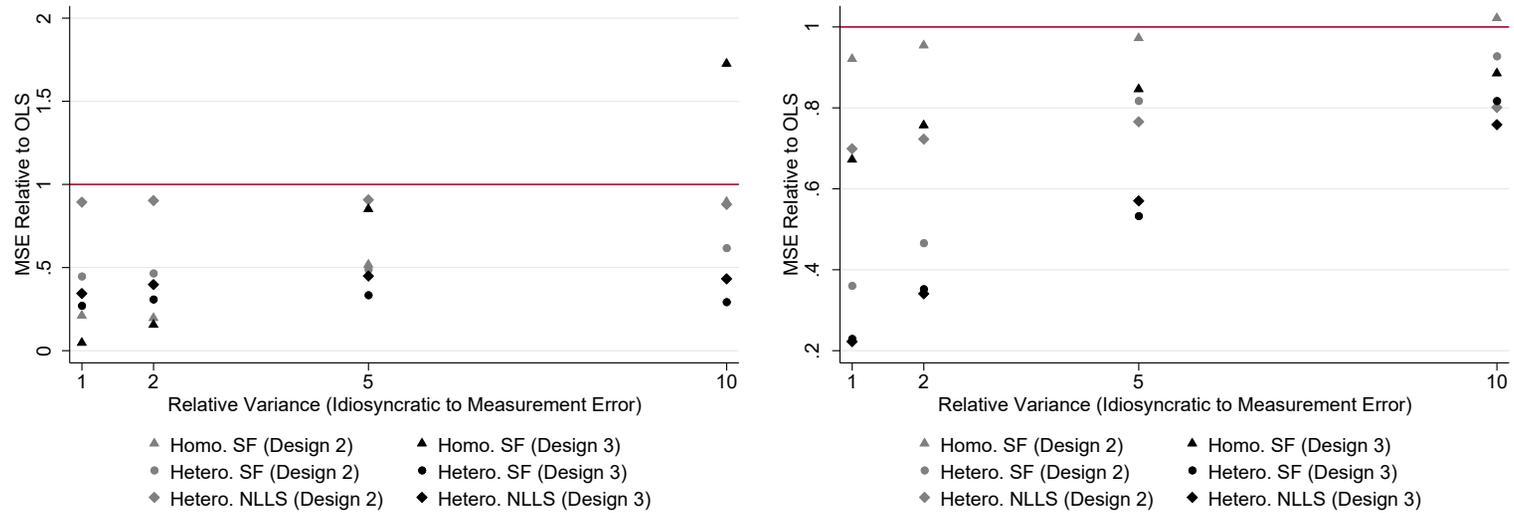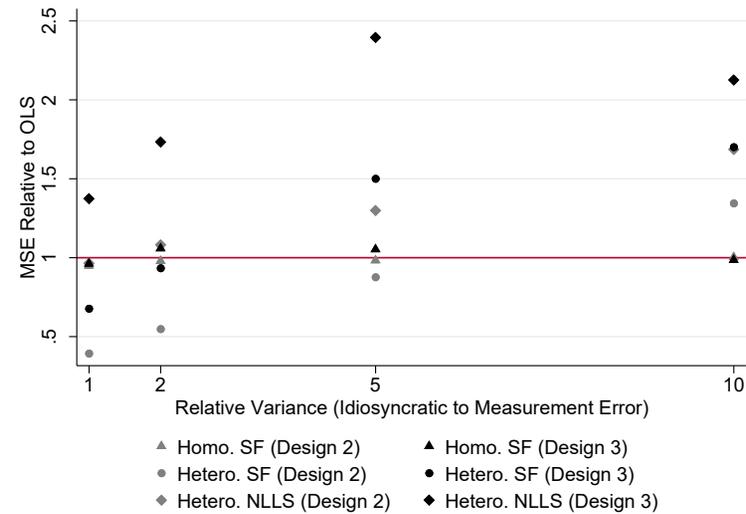
(C) β_2

Figure A4. Monte Carlo Results (Skewed Measurement Error): Median Squared Error Relative to Ordinary Least Squares.
Notes: Results based on 1,000 simulations with N=100. See text for further details.

(A) β₀

(B) β₁
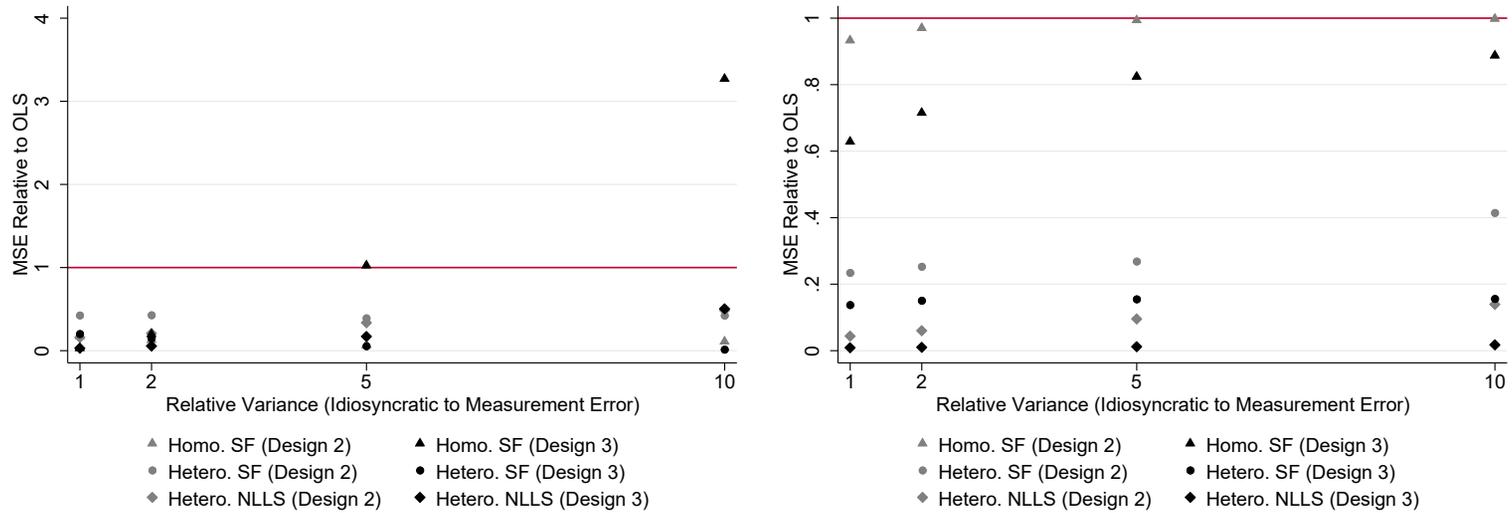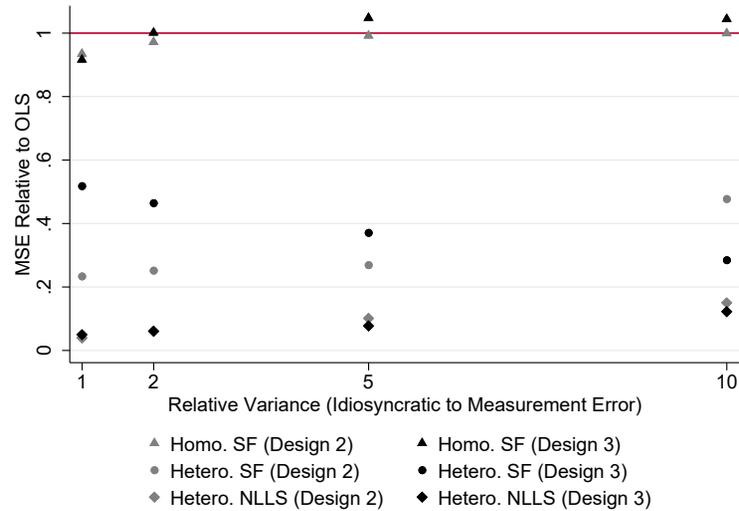
(C) β₂

Figure A5. Monte Carlo Results (Skewed Measurement Error): Median Squared Error Relative to Ordinary Least Squares.
Notes: Results based on 1,000 simulations with N=10,000. See text for further details.

Table A1. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$.

| Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-1.327	-0.104	-0.282	35.610	1.327	0.363	0.492	37.206	1.783	0.261	0.424	5377.428	0.000	0.921	0.719	0.117
β_1	-0.002	0.000	-0.015	0.885	0.113	0.113	0.177	1.164	0.020	0.020	0.050	5.702	0.949	0.934	0.898	0.879
β_2	-0.003	-0.004	-0.019	0.816	0.119	0.119	0.177	1.167	0.022	0.022	0.051	5.786	0.950	0.929	0.902	0.884
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-1.209	0.073	-0.151	47.615	1.209	0.376	0.398	48.917	1.485	0.241	0.270	9024.858	0.000	0.834	0.749	0.169
β_1	-0.422	-0.335	-0.092	0.082	0.422	0.336	0.195	1.638	0.204	0.135	0.069	8.905	0.188	0.357	0.832	0.594
β_2	0.299	0.238	0.050	1.404	0.302	0.245	0.174	1.900	0.114	0.081	0.049	13.327	0.485	0.601	0.876	0.618
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-1.098	0.416	-0.478	1.832	1.098	0.528	0.606	3.091	1.246	0.378	0.480	245.908	0.002	0.525	0.406	0.432
β_1	-0.483	-0.320	-0.210	0.292	0.484	0.324	0.390	0.515	0.279	0.130	0.245	1.461	0.172	0.438	0.539	0.894
β_2	0.119	0.114	0.131	0.354	0.162	0.149	0.178	0.365	0.040	0.035	0.049	0.435	0.874	0.827	0.839	0.776
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-1.324	-0.001	-0.002	-0.285	1.324	0.029	0.029	1.830	1.754	0.001	0.001	6.403	0.000	0.950	0.950	0.068
β_1	0.000	0.000	0.000	0.028	0.011	0.011	0.017	0.041	0.000	0.000	0.000	0.008	0.944	0.953	0.952	0.962
β_2	0.000	-0.001	-0.001	0.029	0.012	0.011	0.018	0.043	0.000	0.000	0.000	0.008	0.944	0.941	0.949	0.957
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-1.207	0.186	-0.017	1.352	1.207	0.186	0.042	2.056	1.458	0.036	0.017	27.239	0.000	0.000	0.935	0.738
β_1	-0.423	-0.339	-0.005	0.068	0.423	0.339	0.020	0.234	0.179	0.115	0.003	0.138	0.000	0.000	0.942	0.793
β_2	0.302	0.242	0.005	-0.039	0.302	0.242	0.019	0.166	0.092	0.059	0.002	0.067	0.000	0.000	0.936	0.804
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-1.030	0.600	-0.619	-0.541	1.030	0.600	0.619	0.589	1.067	0.373	0.430	0.415	0.000	0.000	0.000	0.261
β_1	-0.441	-0.294	-0.246	0.072	0.441	0.294	0.420	0.156	0.195	0.087	0.238	0.041	0.000	0.000	0.001	0.610
β_2	0.118	0.101	0.136	0.251	0.118	0.101	0.136	0.251	0.014	0.010	0.030	0.067	0.003	0.000	0.293	0.000

Notes: Results based on 1000 repetitions. OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Coverage rates based on 95% confidence interval. In Design 1, the one-sided measurement error is homoskedastic. In Designs 2 and 3, the one-sided measurement error is heteroskedastic. In Design 2, the heteroskedasticity function is linear in the covariates and the SF2 and NLLS models use the correct functional form for the heteroskedasticity. In Design 3, the heteroskedasticity function is nonlinear in the covariates and the SF2 and NLLS models use a linear functional form for the heteroskedasticity. See text for further details.

Table A2. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.938	-0.116	-0.241	30.704	0.938	0.403	0.459	31.821	0.895	0.261	0.314	4074.036	0.000	0.913	0.638	0.129
β_1	-0.002	-0.001	-0.011	0.756	0.098	0.099	0.159	0.990	0.015	0.016	0.040	4.114	0.959	0.943	0.897	0.890
β_2	0.001	0.000	-0.006	0.744	0.102	0.104	0.157	0.994	0.016	0.017	0.040	4.186	0.954	0.938	0.909	0.884
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.859	0.027	-0.155	41.005	0.859	0.411	0.404	41.902	0.754	0.245	0.239	6495.392	0.000	0.841	0.647	0.173
β_1	-0.300	-0.263	-0.104	0.292	0.301	0.265	0.192	1.399	0.108	0.087	0.061	6.685	0.336	0.454	0.785	0.679
β_2	0.216	0.190	0.069	1.100	0.221	0.198	0.165	1.529	0.065	0.054	0.042	8.514	0.610	0.661	0.872	0.681
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.777	0.334	-0.353	1.260	0.777	0.503	0.471	2.173	0.628	0.334	0.283	90.887	0.004	0.586	0.437	0.426
β_1	-0.341	-0.259	-0.220	0.206	0.343	0.264	0.335	0.389	0.144	0.089	0.182	0.697	0.277	0.478	0.555	0.904
β_2	0.083	0.085	0.090	0.254	0.127	0.126	0.149	0.274	0.025	0.025	0.035	0.218	0.897	0.870	0.869	0.843
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.937	-0.003	-0.004	0.106	0.937	0.035	0.035	1.559	0.878	0.002	0.002	5.783	0.000	0.954	0.954	0.076
β_1	0.000	0.000	0.000	0.027	0.010	0.010	0.018	0.038	0.000	0.000	0.001	0.006	0.956	0.952	0.941	0.962
β_2	0.000	0.000	-0.002	0.029	0.010	0.010	0.018	0.041	0.000	0.000	0.001	0.007	0.951	0.945	0.947	0.959
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.854	0.174	-0.003	1.650	0.854	0.174	0.032	2.146	0.729	0.032	0.002	39.082	0.000	0.011	0.952	0.754
β_1	-0.300	-0.267	0.001	0.080	0.300	0.267	0.013	0.203	0.090	0.072	0.000	0.125	0.000	0.000	0.955	0.795
β_2	0.213	0.190	0.001	-0.049	0.213	0.190	0.014	0.144	0.046	0.036	0.000	0.061	0.000	0.000	0.954	0.803
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.730	0.562	-0.510	-0.377	0.730	0.562	0.510	0.412	0.535	0.327	0.274	0.204	0.000	0.000	0.000	0.278
β_1	-0.312	-0.237	-0.371	0.055	0.312	0.237	0.418	0.111	0.098	0.057	0.197	0.021	0.000	0.000	0.003	0.641
β_2	0.083	0.079	0.047	0.179	0.083	0.079	0.049	0.179	0.007	0.006	0.007	0.034	0.006	0.001	0.638	0.001

Notes: See Table A1 and text for further details.

Table A3. Monte Carlo Results (One-Sided Measurement Error): $E\left[\sigma_v^2\right]/E\left[\sigma_{u_i}^2\right] = 5$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.595	-0.067	-0.125	26.717	0.595	0.415	0.382	27.397	0.367	0.220	0.193	3199.117	0.000	0.885	0.555	0.159
β_1	-0.003	-0.002	-0.003	0.684	0.089	0.090	0.143	0.892	0.013	0.013	0.033	3.375	0.955	0.939	0.897	0.879
β_2	0.003	0.002	-0.005	0.611	0.093	0.096	0.140	0.862	0.014	0.014	0.032	3.265	0.948	0.938	0.907	0.890
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.544	0.017	-0.103	32.335	0.544	0.425	0.367	32.918	0.309	0.225	0.174	4436.361	0.001	0.832	0.557	0.194
β_1	-0.190	-0.179	-0.103	0.499	0.195	0.186	0.176	1.136	0.050	0.046	0.048	4.889	0.623	0.639	0.798	0.736
β_2	0.137	0.128	0.082	0.751	0.152	0.146	0.156	1.124	0.033	0.031	0.038	4.909	0.761	0.771	0.857	0.756
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.495	0.231	-0.159	1.205	0.495	0.482	0.333	1.785	0.261	0.291	0.146	93.919	0.015	0.681	0.540	0.477
β_1	-0.212	-0.181	-0.100	0.176	0.217	0.189	0.212	0.316	0.062	0.048	0.075	0.522	0.569	0.649	0.733	0.929
β_2	0.058	0.060	0.075	0.198	0.105	0.106	0.130	0.227	0.017	0.018	0.027	0.176	0.909	0.892	0.894	0.896
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.592	-0.012	-0.021	0.405	0.592	0.064	0.071	1.318	0.351	0.007	0.011	6.121	0.000	0.952	0.934	0.083
β_1	0.000	0.000	0.000	0.028	0.008	0.008	0.021	0.037	0.000	0.000	0.001	0.007	0.949	0.951	0.939	0.961
β_2	0.000	0.000	-0.001	0.028	0.009	0.009	0.022	0.038	0.000	0.000	0.001	0.007	0.944	0.940	0.935	0.960
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.540	0.141	-0.005	0.980	0.540	0.142	0.043	1.360	0.292	0.024	0.004	17.734	0.000	0.316	0.939	0.682
β_1	-0.190	-0.183	-0.001	0.039	0.190	0.183	0.015	0.139	0.036	0.034	0.001	0.057	0.000	0.000	0.954	0.755
β_2	0.135	0.130	0.003	-0.014	0.135	0.130	0.015	0.095	0.018	0.017	0.000	0.023	0.000	0.000	0.952	0.743
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.461	0.503	-0.211	-0.230	0.461	0.503	0.211	0.263	0.214	0.264	0.048	0.088	0.000	0.000	0.000	0.287
β_1	-0.198	-0.171	0.026	0.037	0.198	0.171	0.101	0.073	0.039	0.030	0.018	0.010	0.000	0.000	0.055	0.698
β_2	0.052	0.053	0.103	0.114	0.052	0.053	0.104	0.114	0.003	0.003	0.012	0.014	0.019	0.015	0.106	0.003

Notes: See Table A1 and text for further details.

Table A4. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.422	0.018	-0.041	24.594	0.422	0.387	0.333	25.066	0.189	0.188	0.152	2753.755	0.024	0.866	0.560	0.211
β_1	0.000	0.001	0.009	0.618	0.085	0.086	0.130	0.828	0.011	0.012	0.028	2.837	0.953	0.944	0.916	0.876
β_2	0.004	0.003	0.009	0.585	0.089	0.091	0.133	0.820	0.012	0.013	0.029	3.008	0.950	0.939	0.913	0.885
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.384	0.068	-0.045	28.864	0.384	0.393	0.316	29.266	0.159	0.196	0.134	3503.779	0.045	0.834	0.562	0.259
β_1	-0.134	-0.129	-0.091	0.521	0.146	0.143	0.164	1.012	0.030	0.029	0.041	3.889	0.753	0.748	0.823	0.792
β_2	0.100	0.096	0.081	0.690	0.124	0.123	0.148	0.998	0.023	0.023	0.035	3.971	0.830	0.824	0.858	0.795
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.353	0.200	-0.054	0.922	0.353	0.448	0.286	1.344	0.137	0.259	0.117	50.474	0.115	0.733	0.604	0.555
β_1	-0.153	-0.138	-0.068	0.146	0.162	0.150	0.175	0.267	0.036	0.032	0.050	0.337	0.705	0.740	0.801	0.939
β_2	0.040	0.042	0.060	0.155	0.095	0.097	0.125	0.195	0.014	0.015	0.026	0.126	0.925	0.910	0.889	0.924
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.419	-0.044	-0.085	0.564	0.419	0.127	0.153	1.195	0.176	0.031	0.041	4.974	0.000	0.923	0.743	0.083
β_1	0.000	0.000	0.002	0.025	0.008	0.008	0.024	0.035	0.000	0.000	0.001	0.006	0.945	0.946	0.926	0.957
β_2	0.000	0.000	-0.002	0.026	0.009	0.009	0.024	0.036	0.000	0.000	0.001	0.006	0.946	0.945	0.927	0.958
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.382	0.083	-0.089	0.822	0.382	0.134	0.123	1.163	0.146	0.025	0.031	15.910	0.000	0.676	0.695	0.600
β_1	-0.134	-0.132	-0.047	0.013	0.134	0.132	0.057	0.117	0.018	0.018	0.008	0.043	0.000	0.000	0.694	0.665
β_2	0.095	0.094	0.035	0.004	0.095	0.094	0.044	0.082	0.009	0.009	0.004	0.020	0.000	0.000	0.696	0.660
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.326	0.448	-0.150	-0.160	0.326	0.448	0.150	0.190	0.107	0.211	0.024	0.049	0.000	0.001	0.031	0.312
β_1	-0.139	-0.128	0.022	0.027	0.139	0.128	0.066	0.055	0.020	0.017	0.008	0.006	0.000	0.000	0.290	0.767
β_2	0.037	0.038	0.076	0.081	0.037	0.038	0.077	0.081	0.002	0.002	0.006	0.007	0.074	0.058	0.086	0.005

Notes: See Table A1 and text for further details.

Table A5. Monte Carlo Results: Total Outcomes.

Design	Bias				Mean Absolute Percentage Error			
	Observed	SF1	SF2	NLLS	Observed	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-132.43	-10.13	-27.88	3560.78	137.28	36.32	49.36	3813.06
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-93.63	-11.43	-23.98	3069.75	96.89	40.68	46.31	3277.86
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-59.31	-6.41	-12.34	2671.26	61.23	42.42	38.74	2826.62
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-41.92	2.11	-3.84	2459.25	43.37	39.98	34.48	2595.19
II. One-Sided Heteroskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-120.87	7.29	-15.46	4760.20	125.10	37.89	40.01	4958.92
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-85.46	3.09	-15.21	4100.62	88.53	41.61	40.51	4237.35
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-54.11	2.01	-10.28	3233.49	55.96	43.62	37.42	3350.74
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-38.27	6.98	-4.55	2885.97	39.58	40.74	32.47	3014.80
III. One-Sided Heteroskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-109.76	41.77	-47.67	183.10	113.89	53.92	62.12	318.56
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-77.38	33.75	-34.86	125.87	80.59	51.58	48.18	229.25
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-49.19	23.45	-15.47	120.33	51.28	50.02	33.99	186.74
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-34.93	20.42	-4.96	92.11	36.59	46.61	29.07	134.46
I. One-Sided Homoskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-13240.66	-13.08	-17.19	-2852.40	132.44	2.81	2.82	183.03
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-9365.27	-27.53	-35.91	1065.03	93.68	3.46	3.48	155.95
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-5922.19	-121.57	-206.41	4054.10	59.25	6.34	7.01	132.01
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-4188.02	-441.64	-847.72	5646.30	41.90	12.63	15.31	119.72
II. One-Sided Heteroskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-12071.91	1864.77	-171.44	13525.38	120.81	18.66	4.12	205.97
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-8538.05	1746.84	-27.79	16500.19	85.42	17.48	3.15	214.42
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-5399.33	1405.70	-54.84	9801.80	54.02	14.18	4.19	135.71
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-3818.28	826.39	-887.41	8219.59	38.18	13.40	12.28	116.32
III. One-Sided Heteroskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=1$	-10301.51	6003.24	-6189.49	-5408.91	103.10	60.07	61.93	58.89
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=2$	-7296.29	5623.96	-5103.19	-3770.71	73.01	56.27	51.07	41.17
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=5$	-4607.49	5036.01	-2111.80	-2294.14	46.09	50.37	21.12	26.28
$E[\sigma_v^2]/E[\sigma_{u_i}^2]=10$	-3261.51	4482.90	-1504.50	-1597.75	32.64	44.86	15.06	19.03

Notes: OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Values represent the bias and mean absolute percentage error of the sum of the observed and predicted dependent variables relative to the sum of the actual dependent variable, where the sum is computed over the observations in the simulated data. See Table A1 and text for further details.

Table A6. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-1.719	-1.121	-1.116	-1.233	1.719	1.123	1.118	1.853	2.969	1.493	1.444	13.847	0.000	0.748	0.277	0.125
β_1	-0.003	-0.002	-0.012	0.056	0.095	0.097	0.150	0.147	0.014	0.015	0.036	0.057	0.952	0.938	0.917	0.972
β_2	0.003	0.004	-0.009	0.092	0.087	0.088	0.149	0.159	0.012	0.012	0.034	0.125	0.963	0.949	0.929	0.965
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.556	-0.835	-1.011	0.610	1.556	0.840	1.013	3.013	2.442	0.963	1.189	104.460	0.000	0.875	0.242	0.262
β_1	-0.618	-0.590	-0.391	-0.300	0.618	0.590	0.393	0.608	0.399	0.365	0.203	0.757	0.002	0.002	0.444	0.360
β_2	0.441	0.424	0.282	0.390	0.441	0.424	0.291	0.515	0.207	0.193	0.115	0.510	0.035	0.053	0.574	0.336
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.380	-0.071	-0.717	2.246	1.380	0.429	0.727	3.163	1.956	0.344	0.640	177.730	0.003	0.814	0.301	0.652
β_1	-0.657	-0.535	-0.304	0.090	0.657	0.535	0.325	0.510	0.472	0.314	0.153	1.089	0.019	0.073	0.578	0.771
β_2	0.191	0.191	0.165	-0.224	0.216	0.208	0.201	0.553	0.065	0.059	0.061	2.872	0.676	0.620	0.725	0.855
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-1.716	-1.034	-1.050	-0.575	1.716	1.034	1.050	2.095	2.944	1.075	1.117	5.750	0.000	0.000	0.000	0.071
β_1	0.000	0.000	-0.003	0.032	0.010	0.010	0.022	0.041	0.000	0.000	0.001	0.008	0.949	0.946	0.938	0.959
β_2	0.000	0.000	-0.002	0.030	0.009	0.009	0.023	0.041	0.000	0.000	0.001	0.007	0.949	0.949	0.935	0.966
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.547	-0.654	-1.010	0.648	1.547	0.654	1.010	1.063	2.392	0.429	1.022	3.314	0.000	0.000	0.000	0.832
β_1	-0.611	-0.591	-0.297	0.035	0.611	0.591	0.297	0.161	0.374	0.349	0.088	0.049	0.000	0.000	0.000	0.827
β_2	0.437	0.422	0.211	-0.026	0.437	0.422	0.211	0.115	0.191	0.178	0.045	0.026	0.000	0.000	0.000	0.826
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.289	0.269	-0.584	0.218	1.289	0.269	0.584	0.294	1.673	0.102	0.347	0.144	0.000	0.020	0.000	0.227
β_1	-0.637	-0.502	-0.235	0.007	0.637	0.502	0.235	0.075	0.407	0.253	0.059	0.010	0.000	0.000	0.007	0.506
β_2	0.170	0.164	0.126	-0.002	0.170	0.164	0.126	0.045	0.030	0.027	0.017	0.003	0.000	0.000	0.003	0.644

Notes: Results based on 1000 repetitions. OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Coverage rates based on 95% confidence interval. In Design 1, the skewed measurement error is homoskedastic. In Designs 2 and 3, the skewed measurement error is heteroskedastic. In Design 2, the heteroskedasticity function is linear in the covariates and the SF2 and NLLS models use the correct functional form for the heteroskedasticity. In Design 3, the heteroskedasticity function is nonlinear in the covariates and the SF2 and NLLS models use a linear functional form for the heteroskedasticity. See text for further details.

Table A7. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-1.157	-0.669	-0.629	-0.530	1.157	0.688	0.647	1.490	1.351	0.648	0.568	21.045	0.000	0.996	0.603	0.155
β_1	-0.002	-0.002	-0.007	0.073	0.087	0.088	0.145	0.153	0.012	0.012	0.032	0.077	0.964	0.955	0.907	0.968
β_2	0.003	0.003	-0.001	0.096	0.083	0.084	0.149	0.160	0.011	0.011	0.034	0.119	0.958	0.934	0.902	0.962
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.035	-0.504	-0.621	0.438	1.035	0.556	0.644	2.087	1.088	0.468	0.520	63.113	0.000	0.988	0.438	0.239
β_1	-0.438	-0.426	-0.320	-0.234	0.438	0.426	0.327	0.443	0.205	0.195	0.150	0.419	0.026	0.036	0.498	0.382
β_2	0.312	0.305	0.228	0.352	0.312	0.305	0.247	0.392	0.108	0.104	0.084	0.297	0.175	0.183	0.600	0.404
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.914	0.063	-0.477	2.338	0.914	0.451	0.509	2.972	0.867	0.320	0.327	161.489	0.010	0.764	0.393	0.657
β_1	-0.464	-0.403	-0.289	0.083	0.464	0.403	0.308	0.462	0.240	0.182	0.146	1.075	0.065	0.139	0.554	0.773
β_2	0.136	0.140	0.116	-0.228	0.162	0.160	0.165	0.533	0.038	0.037	0.042	2.918	0.768	0.740	0.780	0.886
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-1.154	-0.699	-0.711	-0.016	1.154	0.699	0.711	1.742	1.333	0.510	0.528	5.327	0.000	0.022	0.006	0.081
β_1	0.000	0.000	-0.002	0.032	0.009	0.009	0.026	0.042	0.000	0.000	0.001	0.008	0.944	0.941	0.913	0.954
β_2	0.000	0.000	-0.001	0.028	0.009	0.009	0.026	0.040	0.000	0.000	0.001	0.007	0.950	0.950	0.918	0.959
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.035	-0.373	-0.675	0.873	1.035	0.373	0.675	1.151	1.072	0.143	0.457	10.797	0.000	0.000	0.000	0.869
β_1	-0.432	-0.426	-0.218	0.048	0.432	0.426	0.218	0.148	0.187	0.182	0.048	0.057	0.000	0.000	0.000	0.847
β_2	0.309	0.304	0.155	-0.036	0.309	0.304	0.155	0.107	0.095	0.093	0.024	0.031	0.000	0.000	0.000	0.852
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.853	0.414	-0.338	0.213	0.853	0.414	0.338	0.245	0.733	0.194	0.120	0.096	0.000	0.000	0.001	0.239
β_1	-0.451	-0.379	-0.174	0.005	0.451	0.379	0.174	0.056	0.204	0.145	0.034	0.005	0.000	0.000	0.028	0.586
β_2	0.121	0.121	0.086	0.000	0.121	0.121	0.086	0.036	0.015	0.015	0.008	0.002	0.000	0.000	0.010	0.731

Notes: See Table A6 and text for further details.

Table A8. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-0.657	-0.237	-0.245	-0.249	0.657	0.417	0.390	0.852	0.443	0.233	0.207	5.341	0.000	0.948	0.575	0.206
β_1	-0.003	-0.003	-0.007	0.066	0.082	0.083	0.130	0.133	0.010	0.011	0.026	0.061	0.953	0.944	0.919	0.968
β_2	0.003	0.003	0.004	0.076	0.081	0.082	0.134	0.144	0.011	0.011	0.029	0.065	0.951	0.938	0.910	0.968
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.577	-0.158	-0.252	0.306	0.577	0.389	0.374	1.202	0.345	0.195	0.178	23.351	0.004	0.940	0.514	0.267
β_1	-0.278	-0.274	-0.243	-0.142	0.278	0.275	0.257	0.296	0.088	0.087	0.091	0.232	0.258	0.264	0.545	0.578
β_2	0.204	0.202	0.181	0.262	0.205	0.204	0.206	0.287	0.052	0.051	0.061	0.163	0.554	0.541	0.680	0.626
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.502	0.173	-0.201	1.927	0.503	0.479	0.295	2.280	0.270	0.294	0.114	172.351	0.037	0.732	0.550	0.704
β_1	-0.293	-0.269	-0.222	0.099	0.294	0.271	0.252	0.398	0.102	0.087	0.100	1.406	0.272	0.348	0.600	0.818
β_2	0.087	0.089	0.075	-0.176	0.119	0.120	0.139	0.412	0.022	0.022	0.030	1.932	0.860	0.835	0.853	0.922
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-0.657	-0.396	-0.357	0.456	0.657	0.396	0.357	1.440	0.431	0.190	0.149	6.005	0.000	0.728	0.373	0.090
β_1	0.000	0.000	0.000	0.029	0.009	0.009	0.028	0.039	0.000	0.000	0.001	0.007	0.950	0.950	0.890	0.948
β_2	0.000	0.000	0.000	0.029	0.008	0.008	0.027	0.039	0.000	0.000	0.001	0.008	0.950	0.948	0.900	0.962
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.581	-0.191	-0.363	1.179	0.581	0.193	0.363	1.351	0.338	0.063	0.134	17.760	0.000	0.960	0.000	0.879
β_1	-0.274	-0.273	-0.144	0.061	0.274	0.273	0.144	0.141	0.075	0.075	0.022	0.064	0.000	0.000	0.000	0.853
β_2	0.195	0.195	0.103	-0.041	0.195	0.195	0.103	0.100	0.038	0.038	0.011	0.032	0.000	0.000	0.000	0.855
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.466	0.506	-0.116	0.205	0.466	0.506	0.117	0.212	0.220	0.271	0.017	0.064	0.000	0.000	0.153	0.189
β_1	-0.285	-0.258	-0.115	0.002	0.285	0.258	0.115	0.039	0.082	0.067	0.017	0.002	0.000	0.000	0.040	0.731
β_2	0.076	0.078	0.049	0.001	0.076	0.078	0.050	0.027	0.006	0.006	0.003	0.001	0.007	0.000	0.210	0.835

Notes: See Table A6 and text for further details.

Table A9. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-0.401	-0.008	-0.052	0.132	0.401	0.359	0.317	0.786	0.171	0.160	0.129	16.752	0.022	0.896	0.541	0.259
β_1	0.000	0.000	0.005	0.067	0.082	0.083	0.120	0.136	0.010	0.011	0.024	0.065	0.957	0.940	0.925	0.970
β_2	0.002	0.002	0.012	0.082	0.079	0.080	0.123	0.146	0.010	0.010	0.024	0.101	0.955	0.944	0.925	0.961
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.350	0.040	-0.039	0.444	0.350	0.356	0.296	0.965	0.134	0.161	0.119	27.017	0.085	0.883	0.612	0.398
β_1	-0.196	-0.194	-0.170	-0.100	0.198	0.197	0.199	0.234	0.049	0.049	0.058	0.187	0.534	0.534	0.701	0.733
β_2	0.141	0.140	0.145	0.214	0.148	0.148	0.180	0.236	0.030	0.030	0.047	0.134	0.739	0.739	0.767	0.763
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.295	0.236	-0.055	0.971	0.297	0.452	0.193	1.167	0.101	0.275	0.060	37.066	0.210	0.737	0.739	0.817
β_1	-0.211	-0.200	-0.184	0.040	0.214	0.203	0.218	0.297	0.059	0.053	0.074	0.498	0.503	0.541	0.631	0.843
β_2	0.060	0.061	0.054	-0.093	0.101	0.101	0.128	0.296	0.016	0.016	0.025	0.681	0.909	0.888	0.858	0.938
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-0.406	-0.195	-0.203	0.605	0.406	0.216	0.218	1.205	0.165	0.070	0.068	5.154	0.000	0.984	0.618	0.116
β_1	-0.001	-0.001	0.001	0.025	0.008	0.008	0.024	0.036	0.000	0.000	0.001	0.006	0.949	0.949	0.903	0.957
β_2	0.000	0.000	0.001	0.031	0.008	0.008	0.023	0.039	0.000	0.000	0.001	0.007	0.950	0.950	0.906	0.963
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.352	-0.097	-0.220	1.182	0.352	0.163	0.220	1.301	0.124	0.042	0.050	18.572	0.000	0.952	0.053	0.867
β_1	-0.194	-0.194	-0.172	0.051	0.194	0.194	0.172	0.127	0.038	0.038	0.036	0.053	0.000	0.000	0.000	0.834
β_2	0.138	0.138	0.123	-0.030	0.138	0.138	0.123	0.089	0.019	0.019	0.018	0.026	0.000	0.000	0.015	0.825
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.272	0.520	-0.027	0.207	0.272	0.520	0.051	0.208	0.075	0.284	0.005	0.056	0.000	0.000	0.694	0.117
β_1	-0.202	-0.190	-0.118	0.002	0.202	0.190	0.118	0.032	0.041	0.036	0.027	0.002	0.000	0.000	0.113	0.807
β_2	0.054	0.056	0.031	0.000	0.054	0.056	0.035	0.023	0.003	0.003	0.002	0.001	0.009	0.004	0.583	0.882

Notes: See Table A6 and text for further details.