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IZA DP No. 12005

Artificial Intelligence, Jobs, Inequality and Productivity: Does Aggregate Demand Matter?

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ABSTRACT

Artificial Intelligence, Jobs, Inequality and Productivity: Does Aggregate Demand Matter?

Rapid technological progress in artificial intelligence (AI) has been predicted to lead to mass unemployment, rising inequality, and higher productivity growth through automation. In this paper we critically re-assess these predictions by (i) surveying the recent literature and (ii) incorporating AI-facilitated automation into a product variety-model, frequently used in endogenous growth theory, but modified to allow for demand-side constraints. This is a novel approach, given that endogenous growth models, and including most recent work on AI in economic growth, are largely supply-driven. Our contribution is motivated by two reasons. One is that there are still only very few theoretical models of economic growth that incorporate AI, and moreover an absence of growth models with AI that takes into consideration growth constraints due to insufficient aggregate demand. A second is that the predictions of AI causing massive job losses and faster growth in productivity and GDP are at odds with reality so far: if anything, unemployment in many advanced economies is historically low. However, wage growth and productivity is stagnating and inequality is rising. Our paper provides a theoretical explanation of this in the context of rapid progress in AI.

JEL Classification: O47, O33, J24, E21, E25

Keywords: technology, artificial intelligence, productivity, labour demand, innovation, growth theory

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1 Introduction

'Will Humans Go the Way of Horses?' is the title of a paper by Brynjolfsson and McAfee (2015) asking whether recent innovations in technology, particularly in Artificial Intelligence (AI) will make human labour obsolete. This reflects a growing fear and obsession with potential mass technological unemployment. For instance a headline in the popular media claimed that 'AI will Put 10 million Jobs at High Risk More Than Were Eliminated by the Great Recession' [1]. Moreover fears about technological unemployment and the 'Future of Work' is taken seriously by by governments and global development organizations. Amongst others, in 2017 the International Labour Organization (ILO) established the 'Global Commission on the Future of Work' to ensure 'social justice in the 21st century'[2] . The World Trade Report 2017 of the WTO dealt with 'Trade, Technology and Jobs', while the World Development Report 2019 is focused on the topic of 'The Changing Nature of Work'. It addresses what it describes as 'anxiety about the sweeping impact of technology on employment' (World Bank, 2018, p.1).

As many have justifiably pointed out, these are not new anxieties. Concerns that technology will adversely impact jobs and 'social justice' have been around. As a result, economic theory has built up a significant scholarship dealing with the relationship between technology, growth and development, employment, inequality and productivity. In this, it is well appreciated that technology can possibly be a cause of unemployment and inequality. In theories of skill-biased technological change (SBTC) technology and capital raise the demand for high-skilled workers, and hence their wage premium, causing increasing wage inequality (Autor et al., 2003). Technology can also substitute for labour, leading to higher unemployment and polarization of jobs (Autor and Dorn, 2013).

While past anxieties and doomsday scenarios have so far not been realized, the current wave of anxiety about the impact of AI on jobs is often claimed to be different than before. 'This time is different' is a refrain that is often heard. The main reason it is different this time is said to be due to the fast, exponential nature of technological change driven by AI. As Friedman (2016, p.38) who describes the current age as one of 'accelerations' laments, 'one of the hardest things for the human mind to grasp is the power of exponential growth'. Chiacchio et al. (2018) cite the McKinsey Global Institute that has calculated the disruption presently caused by new exponential technologies is ten times faster and 300 times the scale of that caused by technologies during the Industrial Revolution in the 18th century, thus having '3000 times the impact' (p.3) compared to the past.

As a result, a number of reports of possible mass unemployment to be caused by AI-facilitated automation, have received huge media coverage. The reports in question include Frey and Osborne (2013, 2017) and Bowles (2017) who estimated that up to 47 per cent and 54 per cent respectively of USA and EU jobs could be automated in 10 to 20 years and Frey et al. (2016) who concluded that up to 66 per cent of all jobs in developing countries are susceptible to automation.

It is not just jobs that may be disrupted. Labour productivity and income distribution are also likely to be affected. Brynjolfsson et al. (2017) argue that the impact of new technologies like AI is subject to an 'implementation lag'. As the implementation of AI progresses, it is expected that 'economic growth will accelerate sharply as an ever-increasing pace of improvements cascade through the economy' (Nordhaus, 2015, p.2). To the extent that technology would complement

[1] see : https://www.cbinsights.com/research/jobs-automation-artificial-intelligence-risk

[2] See http://www.ilo.org/global/topics/future-of-work

certain types of labour and replace others, some workers would see their productivity and wages increase, while others may experience the opposite. The net outcome may be faster GDP and productivity growth, but also sharp increases in income and wealth inequality (Korinek and Stiglitz, 2017).

In this paper we critically re-assess these predictions of the impacts of AI on jobs, inequality and productivity, and on overall economic growth. We do so for two reasons. One is that there are still very few theoretical growth models that incorporates AI, and virtually none that considers demand side constraints. A second is that the predictions of AI causing massive job losses and faster growth in productivity and GDP are at odds with reality: if anything, unemployment in many advanced economies are at historical lows. However, wage growth and productivity is stagnating and inequality rising. Our model is attempt to provide a theoretical explanation of this in the context of rapid progress in AI.

As far as the current state of formal theoretical modelling of AI in growth is concerned, a small recent fluttering of papers include work by Acemoglu and Restrepo (2017a); Aghion et al. (2017); Agrawal et al. (2018); Bessen (2018); Nordhaus (2015) and Sachs (2018). These innovative papers each tend to focus on a salient aspect of AI in relation to economic growth, productivity and/or inequality; for instance Acemoglu and Restrepo (2017a) elaborates a key feature of AI, namely its ability to substitute for labour in production; Aghion et al. (2017) identifies constraints on growth due to Baumol's cost-disease type of effects; Sachs (2018) recognises that as AI progresses, it may shift income from workers to the owners of 'business capital'; and Bessen (2018) make the important point of stressing the potential importance of the prince and income elasticity of demand in assessing the impacts of AI (see also our discussion of this aspect in section 2.1 below).

The contribution of our paper is to incorporate AI-facilitated automation into a standard product variety-model, frequently used in endogenous growth, and moreover to modify the model to allow demand-side constraints to affect outcomes. This is a novel approach, given that endogenous growth models, and including most of the papers mentioned dealing with AI in economic growth in the previous paragraph, tend to be supply-driven.

The rest of the paper is structured as follows. In section 2 we survey the relevant literature on the impact of technological innovation on jobs, inequality and productivity, highlighting the importance of taking the effect of aggregate demand into account. In section 3 we introduce an endogenous growth model based on (Gries, 2018) that is potentially demand constrained. In section 4 we use this model to provide a theoretical evaluation the impact of progress in AI on jobs, inequality, productivity and growth. Given assumptions on the elasticity of substitution between AI and human labour services, the results predicted by the model is consistent with the recent experience of advanced countries in terms of sluggish labour productivity and wage growth, and increasing inequality. Section 5 concludes.

2 Relevant Literature

In this section we survey the extant literature on the relationship between AI and automation, and jobs, inequality and productivity growth.

2.1 Impact on Jobs

There has been a number of much quoted initial reports which predicted that a significant percentage of the human labour force may be replaced by automation of jobs and tasks. In a much quoted report, Frey and Osborne (2013, 2017) predicted that up to 47 per cent of USA jobs could be automated in 10 to 20 years. Using a similar methodology, Bowles (2017) estimated that in case of the EU this could be even higher, at up to 54 per cent of jobs automated in 10 to 20 years.

Acemoglu and Restrepo (2017b) calculated that one additional robot per 1,000 workers reduces the employment: population ratio in the USA by 0,37 per cent and wages by 0,25 to 0,5 percent on average. Using a similar approach Chiacchio et al. (2018) who models the impact of robotics on employment in six EU countries, finding that one additional robot per 1,000 workers 'reduces the employment rate by 0,16 to 0,20 percentage points'.

Scary as these predictions may be, the bulk of subsequent theoretical and empirical work have suggested that the impact of AI-automated job losses may be greatly overestimated. Recent theoretical refinements, for instance by Bessen (2017, 2018) show that theoretically, there is the possibility that employment will actually increase as a result of automation, depending on the elasticity of demand for the product in question. If the demand is elastic (>1) then there is the possibility of an increase in employment.

Consistent with these theoretical expectations are newer empirical studies that shows that (i) the methods used to calculate potential job losses in initial reports are sensitive to assumptions used; (ii) while some jobs and sectors may be at risk from automation, the impact is heterogeneous and moreover many new jobs and tasks may be created in other sectors; (iii) automation may affect tasks, rather than jobs; (iv) the tempo of innovation in AI is slowing down, and (v) the diffusion of AI may be much slower than thought previously.

Because of these reasons, we think it unlikely that the pessimistic scenarios of large jobs losses due to new technological innovations will materialize. Rather, the challenge for theory is to explain the co-existing of progress in AI with stagnating wages and productivity - which is what we attempt to do in sections 3 and 4. For now however, let us consider each the empirical results that we just mentioned, in more detail.

First, methodological issues in predicting future jobs losses as a result of automation. The initial and much-quoted report of Frey and Osborne (2013) has been subject to criticism of their methodology. Arntz et al. (2016, 2017) refines the Frey and Osborne (2013) method for predicting possible job losses due to automation in the USA for 21 OECD countries. They find a much lower likelihood of job losses in the OECD: only 9 per cent. And for the USA Atkinson and Wu (2017) presents data to show that the changes in jobs lost and gained has actuallly declined in recent years, and that since 2000 the levels of job churn in the USA has been only 42 percent of the levels the country experienced between 1850 and 2000. They conclude that the much feared disruption of labour markets by technology is 'false alarmism'.

Second, job displacement rather than job replacement by AI may be more likely. Thus, many new jobs or tasks are likely to be created by AI, including jobs that may not at present exist. Empirical evidence seems to bear out that this is already happening. For example, Dauth et al. (2017) finds in the case of Germany **no** net jobs losses as a result of automation. Berriman and Hawksworth (2017) similar reckon that in the UK there will be jobs at risk from automation (they estimate around 30 percent) but conclude that overall the net impact of automation on jobs will be neutral as a result of new jobs being created elsewhere in the economy. These new jobs elsewhere in the economy are more likely to be created due to a rise in the demand for products, which may be the result of new technological innovation establishing new products that consumers want, or making products much cheaper so that the demand for them rises sufficiently to spur production, and hence indirectly, labour demand.

Thirdly, automation may affect tasks more directly rather than jobs themselves. Autor (2015) has argued that claims of mass unemployment due to automation or robotics are exaggerated because automation tends rather to change the nature and content of jobs, such as the tasks that a job consist of, rather than eliminate a job altogether.

Fourth, automation is not likely to lead to mass unemployment due to the fact that that the pace of innovation may be declining. In fields directly relevant for automation, such as in artificial intelligence (AI) there are fears that ICT progress is slowing down, as a result for instance of declining computer processing power (declines in Moores Law) and the declining marginal cost-benefit ratio of obtaining large dataset used for developing AI through deep learning (House of Lords, 2018). Gordon (2018) finds that automation is having a 'evolutionary' rather than a 'revolutionary' impact on jobs in the USA, replacing workers 'slowly' and 'only in a minority of sectors' (p.1).

Fifth, the diffusion of AI technology is much slower than is thought (and may even be slowing down), thereby limiting the impact of automation on jobs (OECD, 2015).

Thus, the employment impact of automation is not likely to be as negative as predicted (and may even be positive). Ultimately it may be an empirical issue for different countries, depending on the extent and sizes of elasticities between AI and labour, on demand elasticities and the extent to which AI can result in product innovations that fosters growth in demand. Stagnating demand may explain why we see progress in AI, but neither huge job losses nor significant wage and productivity growth.

2.2 Impact on Inequality

The impact of AI on income inequality may be negative due to the differential impact it has on different jobs and different workers. Chiacchio et al. (2018) for instance found from a study of six EU countries that younger workers, men, and those with mid-level education are more likely to be displaced by automation. More generally, Korinek and Stiglitz (2017) identify two major channels through which AI-automation will worsen income distribution: one is through the growing 'innovation rents' from AI for instance the benefits to AI may only accrue to a small number of companies; and a second channel is through AI changing the relative demand for labour, and thus change relative wages.

However, as in the case of the impact of AI on jobs, AI may also, perhaps counter-intuitively, improve the distribution of income. One positive impact may be due to what is known as 'Moravec's Paradox'. This explains that 'high-level reasoning requires very little computation while low-level sensor-motor skills require enormous computational resources' (Van de Gevel and Noussair, 2013, p.17). Thus, it will be hardest for new technology to replace the tasks and jobs that workers in the lower-skill level occupations perform, such as security staff, cleaners, gardeners, receptionists, chefs, and the like. A second positive impact of AI on income distribution may be, as Acemoglu and Restrepo (2017a) illustrated, that AI may lead reduce the wage gap due to 'high-skill automation'. In their model of automation there are both high-skill labour and low-skilled labour with the possibility that if only high-skilled automation occur that the consequence may be *neither productivity nor inequality increases.* The intuition behind their result is that automation of high-skilled labour would reduce high-skilled wages and raise the price of capital (at least over the short-term, as more firms replace workers with machines) and hence reduce productivity gains.

In conclusion, given the nature of current AI technologies and their applications it is not certain that at least over the short to medium term, that increases in inequality due to AI automation will be significant. It is rather more likely, as Naudé and Nagler (2018) found for the case of Germany, that reductions in social security, erosion of the bargaining power of labour unions, and labour market deregulation will contribute more towards wage inequality. Stagnating aggregate demand, perhaps due to a declining share of labour in aggregate income (see e.g. Sachs (2018)) or a declining working population, will reduce innovation, growth and put further pressure on public resources that can be used in redistribution.

2.3 Impact on Productivity

In the USA and most of Western Europe there has been a long-run decline in labour productivity growth over the past seventy years. For instance in the USA average labour productivity growth per decade since the 1950 was 2,3 percent (in the 1950s), 2,5 percent (1960s), 1,03 percent (1970s), 1,8 percent (1980s), 2,5 percent (1990s), 1,8 percent (2000s) and since 2010 it was on average 1,8 percent per annum. In European countries such as the UK, Germany and France it was even more sluggish, at respectively 0,43 percent, 0,44 and 0.54 percent on average per year since 2010. Lewis (2018) depicts labour productivity growth in the UK since 1761, showing that the UK's ten-year average labour productivity growth since 2007 was the lowest since the late 18th century.

From the labour productivity growth rates cited in the first paragraph it can be seen that the USA experienced a decline in average annual labour productivity until the 1990s, when it achieved an almost historically high labour productivity growth rate of 2,5 percent on average for the decade, after which it has declined to currently historically low rates. The 1990s upsurge in labour productivity in the USA has been ascribed by some to be a (short-lived) result of the impact of the 1980s ICT revolution in the country (Byrne et al., 2013; Gordon, 2018). That the slowdown in labour productivity growth has taken place over the period when technological innovation has been robust in the USA and many European countries in terms of patents registered and R&D expenditure (Gordon, 2018), has been labelled as the 'productivity paradox'.

The 'productivity paradox' is implicated in the 'secular stagnation' that many advanced economies such as the USA, European countries and Japan experienced since roughly the 1980s (Teulings and Baldwin, 2014). The term 'secular stagnation' has been traced back to Hansen (1939) and is defined by Eichengreen (2015, p.1) as a 'downward tendency of the real interest rate, reflecting an excess of desired saving over desired investment, resulting in a persistent output gap and/or slow rate of economic growth'. The slowing down in economic growth in the USA is discussed by Gordon (2018) who emphasizes a particular 'deceleration' in USA economic growth after 2006, noting that between 1920 and 1970 average annual growth in the USA was 3,7 percent, which then slightly declined between 1970 and 2006 to 3,1 percent but then slowed down much significantly in the decade after 2006 to just 1,35 percent per year. Economic growth also declined in European countries, for instance between 2006 and 2016 average annual GDP growth in Germany and the UK was respectively 1,5 and 1,3 percent, down from 1,9 percent and 2,6 percent over the period 1970 to 2006. A significant proportion - almost half - of this slowdown in economic growth is ascribed to the slowdown in labour productivity growth (Gordon, 2018).

For purposes of the present paper, we are particularly interested in the causes of the 'productivity paradox', and how the emergence of new technologies such as AI may in the near future affect labour productivity growth. In the economics literature there is no consensus on these matters, however.

One one side there are those, such as Gordon (2018), Cowen (2016) and Bloom et al. (2017) who believes that the ICT revolution as reached 'maturity' and that despite much innovative activity we are in an age of 'diminished impact of ongoing innovation' (Gordon, 2018, p.20). According to Gordon (2018) the ICT revolution has reached maturity so that innovations have less and less impact on productivity. Other authors, such as Cowen (2016, p.43) shares in this pessimistic diagnosis of the current state of innovation stating that 'most new technologies today generate only marginal improvements in well-being'. For Jones (2009) the difficulty for researchers and innovators to move beyond the current maturity ICT technologies are reflected in what he calls the 'burden of knowledge'.

Gordon (2018) argues that slower growth in educational attainment, lower investment and slower population growth and labour market participation are more important causes of the slowdown in productivity and economic growth stating that 'productivity growth depends not only on innovation but also on the rate of increase in physical and human capital' (Ibid, p.2).

As for how new technologies such as AI will affect near future labour productivity growth, Gordon (2018) is not very optimistic, concluding that '... robots and artificial intelligence are likely to evolve gradually rather than suddenly causing a sharp jump in productivity growth' (Gordon, 2018, p.23).

Not all economists are in agreement with (Gordon, 2018). Some have argued that the impact of new technologies are not only *mis*-measured but *un*-measured, given that many ICT services add to consumer surplus without being measured, and that TFP (total factor productivity) growth is not a a good indicator of innovation anymore (Mokyr, 2014, 2018). Others such as Aghion et al. (2017) have invoked Baumol's cost disease to argue that the sectors of the economy where productivity growth has been fastest have seen their overall share in the economy decline, so that the average productivity growth is dragged down. A further contrary view is that it is not that the impact of technological innovation on labour productivity is being mismeasured (Syverson, 2017) or that the innovation engine has run out of steam, but rather that the impact of technological progress on labour productivity is subject to an 'implementation lag' and that hence the full effects of recent technologies on productivity is still to be experienced (Brynjolfsson et al., 2017).

Mokyr (2018) concurs with Brynjolfsson et al. (2017) that the impacts of new technological innovations will take some time to be felt, arguing that 'the twenty-first century productivity slow-down described by Gordon is temporary, until new General Purpose Technologies such artificial intelligence (AI) and genetic editing have fully been incorporated into production lines' (Ibid, p. 13). One reason why it can take time for the incorporation of new technologies into production lines is that the 'diffusion machine' has been less effective to spread new technologies to all firms and countries (OECD, 2015). Evidence[3] suggest that this is the result of inefficient

[3] For instance for Italy, Spain and Portugal it has been estimated that inefficient management practices contributed between a quarter and a third of the difference in labour productivity growth with a country

management practices (Schivardi and Schmitz, 2018) and skills mismatches in labour markets (OECD, 2015).

Once new technologies are more widely adopted the impacts on labour productivity are predicted to be significant. For instance Krishnan et al. (2018) predict that as AI diffuse through the USA labour productivity growth would average 2 per cent per year for a decade or so, which would be around four times higher than in the recent past. However, this is dependent on the diffusion actually taking place. Tuuli and Batten (2015, p.3) argue in this regard that the spectre of secular stagnation will remain and that the principal danger is not lack of progress in AI technologies, but rather 'the inability to exploit future opportunities by failing to invest in new technologies today, as well as in new skills required in the future', and that with the potential inherent in AI, that 'the opportunity costs of not investing in new technologies and skills becomes even larger'.

In conclusion, it can be concluded from this section that AI in itself will not affect productivity significantly unless it is much more thoroughly diffused and complemented by appropriately skilled workers. Understanding the factors that determine the speed of diffusion necessitates an understanding of how demand side constraints can reduce incentives to spend on adoption of new technologies or invest in human skill upgrading.

2.4 The Role of Demand

Subsections 2.1 to 2.3 surveyed the extant literature on the relationship between AI-automation and jobs, inequality and productivity. Summarising this literature, we may conclude that AI has the potential to reduce jobs (substitute for labour), to raise inequality (as the share of GDP going to labour decline) and speed-up productivity growth (and GDP growth). However, none of this will be automatic, and existing empirical evidence corroborates that at least until present, AI did not cause massive job losses, or productivity increases, and neither is it likely to carry the blame for rises in inequality. One of the factors that mediates and/or qualifies the impact of AI on these macro-variables, we argued, is aggregate demand.

Aggregate demand has mostly been neglected in predictions and analysis of the impacts of AI so far. The literature surveyed earlier in this section tends to take an overtly supply-side approach. While the supply side is of great importance, because innovations in AI technologies improve the capacity of the economy and determines potential economic growth, the demand side can restrict the actual growth process. If AI-automation generates economic growth from which labour does not earn income, then consumption demand will not grow sufficiently to absorb the additional production capacity.

Furthermore, even if wage and price adjustments take place, a lack of aggregate demand will continue to restrict the growth process if: (i) AI and human labour are highly substitutable, so that labour's income shares decline; and (ii) other economic agents, who gain income shares from AI - like technology providers or financial wealth holders - do not spend their new income to absorb the growing potential production. Both conditions imply that growth could potentially be higher if it were not restricted by a lack of demand.

While a lack of demand is *excluded* in standard growth theory due to the assumption of Say's

such as Germany during the 1990s (Schivardi and Schmitz, 2018). And in the OECD countries, around 25 percent of workers are mismatched to their jobs in terms of skills, i.e they are either over-or-under qualified for their job (OECD, 2015).

Law and the use of the Euler Equation, we will show in our novel theoretical model in the next section that the demand side can indeed be a long-term constraint and that this can explain the empirical facts discussed earlier this section.

3 Theoretical Model

In this section we set out a theoretical, endogenous growth model where aggregate demand can act as constraint on growth, and hence affect the impact of AI on jobs, inequality, and productivity. We start out by providing an intuitive explanation of our model.

3.1 Intuitive Explanation

Our model elaborates on and extends the growth models introduced by Romer (1987, 1990). Apart from the nested production of human capital, it is roughly similar to textbook types of the product variety model as e.g. described by Aghion (2009).

We assume that final goods are produced with the human service good H and a number of innovative intermediate goods x. Final goods firms sell their output to a competitive final goods market. Entrepreneurs bring innovative ideas and new technologies to the market in the form of intermediate goods. They are not always successful, because irrespective of their innovation and effort, there may be no market demand for their product. The human service is produced by labour and a technology which can substitute labour in a quasi-labour-augmenting way. This technology is in fact the AI service, which has the potential to substitute for a very broad range of labour, both for high and low skilled labour.

In the initial steady state we start with pure product innovation, and at first do not allow for progress in AI (which is largely a process innovation). In the initial steady state all factors of production share symmetrically in the gains from product innovations. We show that a decoupling of investments from saving and the introducing the concept of *no expectation-error equilibrium* , where as a result of stochastic shocks affecting the market effective or real sales of final goods may be lower than potential sales, we can model the impact of demand constraints. Under such conditions we show that effective GDP may be below potential GDP, and effective productivity growth may be less than potential productivity growth.

If we allow for progress in AI then we allow for AI to substitute for labour in the production or generation of the human service good. If the elasticity of substitution in this production process is high, labour is replaced and wages will fall. We show that under normal conditions AI progress will still allow for a net (but relatively low) growth in labour productivity and wages. Wage growth however, will be slow and the income share of labour will decline. Financial wealth owners, who finance the innovative intermediate firms can realize a large productivity gain and am increase of their factor rewards. As a result inequality increases. With a slow increase in labour income and consumption, and thus a slow growth in aggregate demand, total GDP growth will also be affected and will be slower. If progress in AI technologies do not allow labour to share in income growth the slowdown in aggregate demand growth will restrict total economic growth as well. The results predicted by the model seem to be consistent with recent observations in some advanced economies, such as sluggish labour productivity and wage growth and increasing inequality.

3.2 Final-goods-producing firms

The first set of economic agents to introduce are the firms that produce final consumption goods. They do so using human services, AI-services, and intermediate inputs. Their potential production output may fall short of effective or actual production output, due to frictions and shocks in final good markets. They will spend resources to sell as much as possible, and pay labour, the providers of AI and the entrepreneurs that provide the intermediate goods, for these inputs.

Potential and effective final output: Let firm $i \in \mathcal{F}$ be a representative firm that produces final goods using human service inputs H_i and $N_i(t)$ of differentiated intermediate inputs $x_{ji}(t)$ offered by $N(t)$ intermediate input producing firms. $Q_i^p(t)$ represents total production by firm i of final goods:

$$Q_i^p(t) = H_i^{1-\alpha} \sum_{j=1}^{N_i} x_{ji}^{\alpha}(t) = N_i(t) H_i^{1-\alpha} x_j^{\alpha}(t) \tag{1}$$

with $x_j(t)$ being a representative variation of the intermediate goods $x_{ji}(t)$. Total production is also the potential output of final goods Q_i^p in the final goods markets.

Firms are aware that, due to what can be categorised broadly as market frictions, not all of their final production will be sold to final consumers. Thus, they will spend $\theta_i(t)$ on promotional activities in order to ensure more of their products are sold. Their eventual output, which can differ from their potential output, is termed the *effective output* (or *effective sales*) of final goods, and can be written as:

$$Q_i(t) = H_i^{1-\alpha} N_i(t) x_i^{\alpha}(t) (1 - \theta_i(t)) \tag{2}$$

Market frictions and effective sales: Market frictions can be denoted by $\delta_i(t)$ and the share of total potential production sold to final consumers as $\Phi_i(t) < 1$. We assume that this share decreases as market frictions δ_i becomes more prevalent, and that the share increases with more promotional expenditures θ_i:

$$\Phi_i = \Phi_i(\delta_i, \theta_i), \text{ with } \frac{\partial \Phi_i}{\partial \delta_i} < 0, \frac{\partial \Phi_i}{\partial \theta_i} > 0.$$

Effective sales[4] can be specified as:

[4] At this point it would be sufficient to suggest that the expectations function $\Phi(\theta_i, \delta_i)$ is monotonic increasing in θ_i and decreasing in $E[\delta_i]$. Taking expectations for (3) we obtain $E[\Phi] = 1 - E\left[(\delta_i)^2\right] + E[\delta_i]\theta_i$, and applying $E\left[(\delta_i)^2\right] = E[\delta_i]E[\delta_i] + Var(\delta_i)$ we arrive at (4). For specification (3), $\frac{\partial E[\Phi_i]}{\partial E[\delta_i]} = -2E[\delta_i] + \theta_i < 0$, as long as $Var(\delta_i) \leq 2E[\delta_i] + E[\delta_i]E[\delta_i]$ at the sales maximizing θ_i (see 5). With the specific random distribution (34) we ensure this condition holds.

$$\Phi_i(t) = 1 - (\delta_i(t))^2 + (\delta_i(t))\,\theta_i(t) \quad (3)$$

$$E[\Phi(t)] = 1 - E[\delta_i(t)]\,E[\delta_i(t)] - Var(\delta_i(t)) + E[\delta_i(t)]\,\theta_i(t) \quad (4)$$

Here $E[\Phi_i(\delta_i, \theta_i)]$ is the expected share of potential final output that is sold and $(1-\theta_i)$ is the share that is supplied. Firms will increase their spending θ_i on the promoting their products to a level where they expect that all goods will indeed be sold. The expected effective sales equilibrium is $(1-\theta_i) - E[\Phi_i(\delta_i, \theta_i)] = 0$, and the promotional spend to maximizing sales is:

$$\theta_i(t) = \frac{E[\delta_i(t)]\,E[\delta_i(t)] + Var(\delta_i(t))}{(1 + E[\delta_i(t)])} \quad (5)$$

For the moment the promotional spend θ_i is a function of subjectively expected market frictions; later we will link these expectations, and hence the promotional spend, to the state the aggregate economy, specifically aggregate demand.

Demand for labour and wages: A central contribution of our model is that we allow for a production process in which labour can be automated. This takes place when labour is substituted for by AI in the production of the human service input H_i. Formally we model the human service input H_i as being produced according to a Constant Elasticity of Substitution (CES) production function. This function contains labour and AI inputs, the latter represented by $A_{Li}(t)$. The human service input production function can be written as:

$$H_i = \left(\beta L_i^{-\rho} + (1-\beta)\,A_{Li}^{-\rho}\right)^{-\frac{1}{\rho}}, -1 < \rho < 0 : \text{high substitution} \quad (6)$$

with ρ indicating the degree of substitution. The relation of ρ and the elasticity of substitution is $\sigma = \frac{1}{1+\rho}$. If $\sigma > 1$ $(-1 < \rho < 0)$ the elasticity of substitution is high and AI can easily substitute for labour; if $\sigma < 1$ $(0 < \rho < \infty)$ the elasticity of substitution is low and AI can substitute for some labour, but not as easily. In this case technology and labour tend to be more complementary.

Including H_i in the final good production process we derive the factor demand and factor rewards for the profit maximizing firm:

$$\max_{L_i, x_i} : Q_i(t) = N_i(t)\,H^{1-\alpha} x_i^{\alpha}(t)\,(1-\theta_i(t)) - w_L L_i(t) - w_A(t) A_L - N_i(t)\,p_x(t)\,x_i(t) \quad (7)$$

The above means that real wages $w_L(t)$ are determined by the marginal productivity of labour. From the first-order-conditions we can derive wages as:[5]

[5] F.O.C.: $\frac{dQ}{dL_i} = N_i\left(-\frac{1-\alpha}{\rho}\right) x_i^{\alpha}\left(\beta L_i^{-\rho} + (1-\beta)\,A_{Li}^{-\rho}\right)^{-\frac{1-\alpha}{\rho}-1} \beta(-\rho)\,L_i^{-\rho-1} - w = 0$

$\Leftrightarrow w_L = (1-\alpha)\,N_i x_i^{\alpha} H^{1-\alpha}\left(\beta + (1-\beta)\,L_i^{\rho} A_{Li}^{-\rho}\right)^{-1} L_i^{\rho} \beta L_i^{-\rho-1}$

$$w_L(t) = (1-\alpha)\,\beta Q_i^p(t)\,(1-\theta_i(t))\left(\beta + (1-\beta)\left(\frac{L_i}{A_L}\right)^{-\rho}\right)^{-1} L_i^{-1}. \tag{8}$$

Income of the provider of the AI: The factor reward, or income, of the economic agent that provides the AI can be derived from the above in a symmetrical manner as in (8) and can thus be specified as:[6]

$$w_A(t) = (1-\alpha)(1-\beta)\,Q_i^p(t)\,(1-\theta_i(t))\left(\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho} + (1-\beta)\right]\right)^{-1} A_L^{-1} \tag{9}$$

Demand for intermediate goods: Firms' demand for intermediate goods is similarly determined. We assume that the prices of the intermediate inputs are set by the entrepreneurs that offer these as unique, monopoly-like intermediate goods (see section 3.3 below). Using the first-order-conditions the demand for each intermediate input is[7], namely

$$x_i(t) = \left(\frac{\alpha(1-\theta_i(t))}{p_x(t)}\right)^{\frac{1}{1-\alpha}} H_i. \tag{10}$$

3.3 Intermediate-goods-producing firms

The second set of economic agents in our model are entrepreneurs who provide intermediate inputs to the final good producing firms. These entrepreneurs each provide a unique intermediate input, through being innovative, and as a result enjoy being a monopolist. However, not all new intermediate innovative products are successful: this depends as we show in this subsection, *inter alia* on market conditions, specifically the total effective demand in the economy.

Market entry of new monopolistic firms: The intermediate good-supplying entrepreneur establishes a monopoly firm because it sells an unique product which is the outcome of entrepreneurial (product) innovation. The costs (denominated in units of final output) to produce one unit of x is c_x and the profits this result in is $\pi_x = (p_x - c_x)\,x$.

Using the demand function (10) and plugging in $p_x = \alpha(1-\theta_i)\,H^{1-\alpha}x^{-(1-\alpha)}$ we obtain:

$$\pi_x(t) = \alpha(1-\theta_i(t))\,H^{1-\alpha}x(t)^{-(1-\alpha)}\,x(t) - c_x x(t) \tag{11}$$

From the first-order-conditions, and using (10) and (13) we can determine[8] the optimal price p_x and optimal production of $x(t)$:

[6] F.O.C.: $\frac{dQ}{dA_L} = N_i\left(-\frac{1-\alpha}{\rho}\right)\left(\beta L_i^{-\rho} + (1-\beta)\,A_L^{-\rho}\right)^{-\frac{1-\alpha}{\rho}-1} x_i^{\alpha}(1-\theta_i)(1-\beta)(-\rho)\,A_L^{-\rho-1} - w_A = 0$
$\Leftrightarrow w_A = (1-\alpha)(1-\theta_i)\,N_i x^{\alpha} H^{1-\alpha}\left(\left[\beta L_i^{-\rho}A_L^{\rho} + (1-\beta)\right]\right)^{-1} A_L^{\rho}(1-\beta)\,A_L^{-\rho-1}$

[7] F.O.C is $\frac{d\Pi_i}{dx_i} = \alpha(1-\theta_i)\,x_i^{\alpha-1}H_i^{1-\alpha} - p_x = 0$, thus $p_x = \alpha(1-\theta_i)\,H_i^{1-\alpha}x_i^{-(1-\alpha)} \Leftrightarrow x_i^{1-\alpha} = \alpha\frac{(1-\theta_i)}{p_x}H_i^{1-\alpha}$.

[8] F.O.C. is $\frac{\partial \pi_x}{\partial x} = \alpha^2(1-\theta_i)\,H^{1-\alpha}x^{\alpha-1} - c_x = 0$, thus $c_x = \alpha^2(1-\theta_i)\,H^{1-\alpha}x^{\alpha-1} \Leftrightarrow x^{1-\alpha} = (c_x)^{-1}\alpha^2(1-\theta_i)\,L^{1-\alpha}$.

$$p_x = \frac{c_x}{\alpha}. \tag{12}$$

$$x\left(t\right) = \alpha^{\frac{2}{1-\alpha}}\left(1-\theta_i\left(t\right)\right)^{\frac{1}{1-\alpha}}\left(c_x\right)^{-\frac{1}{1-\alpha}}H, \tag{13}$$

With (13) and (12) we derive the expression for maximum profits $\pi_x\left(t\right)$ as:

$$\pi_x\left(t\right) = \left(\frac{1}{\alpha}-1\right)\left(c_x\right)^{\frac{-\alpha}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\left(1-\theta_i\left(t\right)\right)^{\frac{1}{1-\alpha}}H \tag{14}$$

The present value of this infinite profit flow, discounted at the steady-state interest rate r is:

$$V_x(t) = \frac{1}{r}\pi_x\left(t\right) = \int_t^{\infty}\pi_x\left(t\right)e^{-r(v,t)(v-t)}dv. \tag{15}$$

Here $\frac{1}{r}\pi_x$ is the present value of profits per innovation and $\frac{1}{r}\pi_x\dot{N}$ are the total profits of the start-up firm of introducing $\dot{N}(t)$ new goods. In addition to costs of innovation, the new firm also has to cover the costs of market entry (e.g. commercialization costs) for the new intermediate good, which is ν_x. Thus, total entry costs of the start-up with innovation rate $\dot{N}$ is $\dot{N}\nu_x = I_x$. With competitive market entry the net rents of a new firm turns to zero, and the net present value of the new firm just about covers its total start-up costs:

$$\frac{1}{r}\pi_x(t)\dot{N}(t) - I_x(t) = 0. \tag{16}$$

With $\dot{N}\nu_x = I_x$ the steady-state interest rate is:

$$r = \frac{\pi_x(t)}{\nu_x(t)} \tag{17}$$

Market entry of new technologies and goods: Because we are more interested in this paper in the impacts of technology (AI) and not in its creation or innovation, we assume that initial innovation in the intermediate goods market is exogenously given with $\dot{A}\left(t\right) = \frac{dA(t)}{dt}$ the exogenous number of innovative intermediate products invented at t. These innovative intermediate products are however not automatically successful in the market. We model the process of market entry as an *aggregate matching* process.[9]

New, innovative intermediate goods $\dot{A}\left(t\right)$ are offered on the market and entrepreneurs try to find buyers amongst the final goods producing firms. The number of new intermediate products successfully entering the market $\dot{N}$ is a function of two elements: (i) the given number of new, innovative intermediate products $\dot{A}\left(t\right)$ potentially ready for market entry, and (ii) the number of opportunities for market entry that entrepreneurs (start-ups) discover. These opportunities

[9] For a micro-foundation of this process see Gries and Naudé (2011) .

are determined by the capacity of the market. Absorption capacity for intermediate goods is a function of total effective demand of intermediate goods in the economy $X^{eD}(t)$.

In an aggregate matching function we can combine these two elements and describe the resulting process of market entry as $\dot{N} = f\left(\dot{A}, X^{eD}\right)$. For simplicity, we assume a matching technology with constant economies to scale so that the number of new products in the market will be given by:

$$\dot{N}(t) = \left(X^{eD}(t)\right)^{\gamma} (\dot{A}(t))^{1-\gamma} \tag{18}$$

where γ is the contribution of market opportunities. Although the assumption of a macro-matching process is basic it represents the main idea behind the mechanism. Given (18) the growth of new products in the economy is a *semi*-endogenous process because the number of new products $\dot{A}$ is fixed, but the number of new technologies implemented to establish intermediate products $\dot{N}$ is endogenous.

3.4 Aggregate production and income distribution

Having specified final goods and intermediate goods production in our stylized economy in the previous two subsections, we can now add these, including the financing of the costs of production and innovation to obtain an expression of and decomposition of aggregate production (GDP). We can also specify the distribution of income to the various agents in the economy.

Calculating and Decomposing GDP: From the subsections above, it is clear that effective output of the total economy has to be divided amongst intermediate goods x, labor L and the technology service provider A_L. The budget constraint for effective output is therefore:

$$Q(t) = N(t)H^{1-\alpha}x^{\alpha}(t)\left(1-\theta(t)\right) = N(t)p_x(t)x(t) + w_L(t)L + w_A(t)A_L \tag{19}$$

Note that effective output is not the same as GDP or aggregate income. As x is produced by using c_x units of final goods, net final output, and thus *income* is

$$Y(t) = Q(t) - N(t)x(t)c_x. \tag{20}$$

Further, from (19) and (20) we obtain $Q - Nxc_x = Np_x x - Nxc_x + w_L L_i$. With the definition of profits in the intermediate goods sector (11), the income constraint becomes:

$$Y(t) = N(t)\pi_x(t) + w_L(t)L + w_A(t)A_L \tag{21}$$

According to (21) total income in the economy is composed of profits, labour and technology income. Given equation (17) we obtain $Y = rN\nu_x + w_L + w_A A_L$.

As $N\nu_x$ is the value of all debt ever issued ($N\nu_x = F$), and all new products are financed by newly issued debt, $\dot{N}(t)\nu_x = \dot{F}(t)$ profits are channelled to financial investors who finance the process

$$N(t)\,\pi_x(t) = r(t)\,F(t)\,. \tag{22}$$

Value added generated by innovative intermediate firms generates an income to the owners of financial assets $r(t)\,F(t)$. The growth process is thus essentially a process of financial wealth accumulation through the financing of new products (innovation), which we can label a 'Silicon-Valley' model of growth.

Finally, using (22) we arrive at the familiar income decomposition of GDP:

$$Y(t) = r(t)\,F(t) + w_L(t)\,L + w_A(t)A_L. \tag{23}$$

In addition to income of financial wealth owners, value added generated by the human service input is distributed to labour ($w_L(t)\,L$) and the providers of the AI-technology ($w_A(t)A_L$).

Income distribution: Because we are interested in the distributional aspects of asymmetrical technical change (AI), we derive the income shares of the three input and resource providing agents in our model, namely labour ($w_L L$), AI-service providers ($w_A A_L$) and financial investors (rF).

The income share of labour is derived by using (8), (13) and (20) and is described by:

$$\frac{w_L(t)\,L}{Y(t)} = \frac{(1-\alpha)\,\beta}{[1-\iota]}\left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1} < 1 \tag{24}$$

with $\iota = \alpha^2 < \alpha$, denoting the share of the intermediate good of total production Nxc_x/Q.[10]

The income share of providers of the AI-service is

$$\frac{w_A(t)\,A_L}{Y(t)} = \frac{(1-\alpha)\,(1-\beta)}{[1-\iota]}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho} + (1-\beta)\right]^{-1}. \tag{25}$$

For details of the calculation see appendix 6.1.

Income share of financial investors can be determined using (20), (22) and (11), so that we arrive at

$$\frac{N(t)\,\pi_x(t)}{Y(t)} = \frac{\alpha-\iota}{1-\iota}. \tag{26}$$

Details of the calculation are contained in appendix 6.1.

[10] For details see appendix 6.1.

3.5 Specifying the aggregate demand side

In section 2 of this paper we argued that what happens to aggregate demand as AI-technology progresses will be an important mediator of the eventual impact and progression of AI in the economy. Therefore, in order to understand and analyse the role of aggregate demand we need to specify in our model the consumption and savings behaviour of the agents in our model. Using a *representative* intertemporal choice approach based on a *representative* household's 'Euler Equation'[11] as is the standard case in endogenous growth models, is however not adequate for the task. Instead, we follow the Keynesian tradition by assuming that some households only earn wage income wL, and another group of households earn only financial income from assets rF. Each group will have its own consumption preferences and patterns.

Consumption expenditure and labour income: According to (24) the share of labour income is $\frac{wL}{Y} = \frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}$.

We assume that total wage income will be fully consumed, and that total wage income is the only source of consumption expenditure. Given these simplifying assumptions total effective consumption in the economy is given by:

$$C^e(t) = w(t)L = cY^e(t), \tag{27}$$

$$\text{with} \quad c = \frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1} \tag{28}$$

the economy's marginal (and average) rate of consumption.

Investment expenditure and innovation: Innovation requires investment. We assume that such investments are identical for each innovation $\nu = \nu_x$. Thus total start-up investments $I^e(t)$ are given by:

$$I^e(t) = \nu\dot{N}(t). \tag{29}$$

Effective demand and Keynesian income-expenditure equilibrium: Effective income Y^e can be used for effective consumption C^e and investment I^e. Thus, effective demand is given by $Y^{eD} \equiv C^e + I^e$. While the consumption rate is determined by (27) and a constant fraction of total effective income, investments are driven only by the market entry of new goods (innovation) $\dot{N}$. With the consumption rate (28) given, the Keynesian income-expenditure mechanism can be applied to determine effective total demand, Y^{eD}.

In income-expenditure equilibrium we have:

$$Y^e(t) \overset{!}{=} Y^{eD}(t) \equiv C^e(t) + I^e(t), \tag{30}$$

[11] $\frac{\dot{C}}{C} = fracr - \rho\eta_U$ with ρ denoting the time representative agents preference rate and η_U the intertemporal elasticity of substitution.

From this we can derive the well-known Keynesian income-expenditure multiplier. Defining

$$\mu = \frac{\nu}{1-c} \tag{31}$$

as the standard multiplier gives:

$$Y^{eD}(t) = \frac{1}{1-c} I^{e}(t) = \mu \dot{N}(t). \tag{32}$$

3.6 Solving the model

Solving the model that we have outlined in the preceding sections requires finding values for the variables which will result in a stationary equilibrium for the economy. Such a stationary equilibrium will exist if firm behaviour is stationary, which will be the case if all the firm's expected values are equal to effective values, so that there is no need for change. We label this the 'no expectation-error equilibrium' (n-e-e**e**) to indicate that expectations are in accordance with effective values (i.e. there is no 'error' or deviance).

Definition 1 formally defines this equilibrium concept, first from the perspective of each firm, and second from the perspective of the aggregate economy:

Definition 1
For each firm "no expectation-error equilibrium" (n-e-ee) is defined as the condition for which (i) firm i's expected output sold in the market $E[Q_i(t)]$ is indeed firm i's effective output absorbed by the market $Q_i^e(t)$, and (ii) this holds for all firms

$$\begin{aligned} (i) \quad &: \quad E[Q_i(t)] = Q_i^e(t), \\ (ii) \quad &: \quad E[Q(t)] = Q^e(t). \end{aligned} \tag{33}$$

From each firm i's perspective the only stochastic variable is market frictions δ_i. In order to counter these stochastic market frictions and promote its sales firms will determine the expected level $E[\delta_i]$, and variance $Var(\delta_i)$ of these frictions and spend a fraction θ_i of real output to maximize the expected sales ratio $E[\Phi_i(\delta_i(t), \theta i(t))]$ (see 5).

We need to say more about market frictions. For the random distribution of market frictions facing each firm we assume an exponential distribution with

$$\begin{aligned} \Pr(\delta_i(t) > \varepsilon) \quad &= \quad e^{-\frac{\varepsilon}{\upsilon_i(t)}} \qquad \varepsilon \geq 0. \\ E[\delta_i(t)] \quad &= \quad \upsilon_i(t), \quad Var(\delta_i(t)) = \upsilon_i^2(t) \end{aligned} \tag{34}$$

Thus, the mean of the random distribution of market frictions δ_i for each firm is υ_i and the expected value of the required promotional spend is (according to 4 and 5) θ_i. Thus, in n-e-ee (defined by 33) the effective sales ratio of each firm is given by:

$$\lambda_i(t) = 1 - \theta_i(t) = E\left[\Phi_i\left(\delta_i(t), \theta_i(t)\right)\right] = 1 - \frac{E\left[\delta_i(t)\right] E\left[\delta_i(t)\right] + Var(\delta_i(t))}{\left(1 + E\left[\delta_i(t)\right]\right)}. \tag{35}$$

This indicates that when in equilibrium (n-e-ee 33) all planning at firm level is consistent with mean market conditions. Firm i's expected sales are indeed the effective sales so that

$$E\left[Q_i(t)\right] = Q_i^e(t) = H_i^{1-\alpha} N_i(t) x_i^{\alpha}(t) \lambda_i(t)$$

There is thus no need for a revision of production plans. We can now offer the solution to the model.

Solution: First, assume that firm i is representative of all firms, and $\delta_i(t)$ are i.i.d for $i \in I$. This implies $\lambda(t) = \lambda_i(t) = E\left[\Phi_i\left(\delta_i(t), \theta_i(t)\right)\right]$ and results in aggregate effective output being:

$$E\left[Q(t)\right] = Q^e(t) = N(t) H^{1-\alpha} x^{\alpha}(t) \lambda(t). \tag{36}$$

Second, because the market conditions that determine the properties of the exponential random distribution of frictions δ_i (34), and δ_i are i.i.d for $i \in I$, we obtain from (36) for the aggregate economy a relation between the ratio of effective sales λ and the parameter υ of the random distribution of frictions as:

$$\lambda(t) = \frac{1 + \upsilon(t) - 2\upsilon^2(t)}{1 + \upsilon(t)}. \tag{37}$$

In order to determine the ratio of effective sales λ we can use the equilibrium condition of the aggregate final goods market. From the Keynesian income-expenditure mechanism (32) we have established that effective aggregate demand is $\frac{\nu}{1-c}\dot{N}$. In n-e-ee (33 ii), expected sales in the market are equal to effective sales and thus, in aggregate goods market equilibrium, effective demand Y^{eD} is equal to effective supply of final goods $Y^{eS} = Q^e - Nxc_x$,

$$\begin{aligned}
\overbrace{\frac{\nu}{1-c}\dot{N}(t)}^{\text{effective demand } Y^{eD}} &= Y^e(t) = \overbrace{\lambda(t) N(t) H^{1-\alpha}\left(x(t)\right)^{\alpha} - N(t) x(t) c_x}^{\text{effective supply } Y^{eS}}, \qquad (38) \\
\text{with } x_j(t) &= \alpha^{\frac{2}{1-\alpha}} \left(\lambda(t)\right)^{\frac{1}{1-\alpha}} H \left(c_x\right)^{-\frac{1}{1-\alpha}} \quad \text{and } g_N = \frac{\dot{N}(t)}{N(t)}.
\end{aligned}$$

Before we can eventually solve the system (see proposition 1), we have to determine the endogenous growth rate of new products successfully entering and remaining in the market $\dot{N}/N$. Equation (18) describes the aggregate matching process for successful new intermediate products (product innovations) entering the market. Thus, the growth rate of implemented product innovations is[12]

[12] From (18) and (13) we obtain $X^{eD} = Nx = N\alpha^{\frac{2}{1-\alpha}} \left(\lambda\right)^{\frac{1}{1-\alpha}} \left(c_x\right)^{-\frac{1}{1-\alpha}} L.$ and $\dot{N} =$

$$g_N = \frac{\dot{N}(t)}{N(t)} = \left(\alpha^{\frac{2}{1-\alpha}} (\lambda(t))^{\frac{1}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} H\right)^{\gamma} (g_A)^{1-\gamma} \tag{39}$$

Thus in n-e-ee we have three equations, (37), (38), and (39) to solve for the three variables, the effective sales ratio λ, parameter υ of the random distribution of market frictions, and the semi-endogenous growth rate g_N. The result is stated in the following proposition:

Proposition 1 *No expectation-error equilibrium (definition 1 equation 33) and equations (37), (38), and (39) determine*

(i) the equilibrium ratio of effective sales and effective production:

$$\tilde{\lambda} = \mu^{\frac{1-\alpha}{1-\gamma}} \frac{g_A{}^{(1-\alpha)}}{H^{1-\alpha}(1-\alpha^2)^{\frac{1-\alpha}{1-\gamma}}}, \qquad \frac{d\tilde{\lambda}}{d\mu} > 0, \tag{40}$$

(ii) the equilibrium value of the mean and variance of the distribution of aggregate market frictions δ :

$$E[\delta] = \tilde{\upsilon} = \left[\left(1 + 8\left(1-\tilde{\lambda}\right)^{-1}\right)^{\frac{1}{2}} + 1\right] \frac{1}{4}\left(1-\tilde{\lambda}\right), \quad Var(\delta) = \tilde{\upsilon}^2, \tag{41}$$

(iii) the growth rate of effective income (GDP) :

$$g_Y = \frac{\dot{Y}^e(t)}{Y^e(t)} = g_N = \left(\alpha^{\frac{2}{1-\alpha}} \tilde{\lambda}^{\frac{1}{1-\alpha}} c_x^{-\frac{1}{1-\alpha}} H\right)^{\gamma} (g_A)^{1-\gamma}, \tag{42}$$

and (iv) the effective income at each point in time and hence the growth path:

$$\tilde{Y}^e(t) = Y^{eD}(t) = N(t)\frac{\nu_x}{1-c}\left(\alpha^{\frac{2}{1-\alpha}} \tilde{\lambda}^{\frac{1}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} H\right)^{\gamma} (g_A)^{1-\gamma} \tag{43}$$

For a proof see appendix 6.2.

In addition to the standard steady state values stated in proposition 1 we are also interested in the wages $w_L(t)$ and the earnings of the providers of AI-services A_L, $w_A(t)$. We first determine the wage rate by plugging (13) in (8)

$$w_L(t) = N(t)(1-\alpha)\left[\alpha^2 c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}} \tilde{\lambda}^{\frac{1}{1-\alpha}} \left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1+\rho}{\rho}} \beta \tag{44}$$

$\left(\alpha^{\frac{2}{1-\alpha}} (\lambda)^{\frac{1}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} L\right)^{\gamma} N^{\gamma}(\dot{A}))^{1-\gamma}$. Rearranging gives $\frac{\dot{N}(t)}{N(t)} = \left(\alpha^{\frac{2}{1-\alpha}} (\lambda)^{\frac{1}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} L\right)^{\gamma} (\frac{\dot{A}(t)}{A(t)})^{1-\gamma}$ for $N(t) = A(t)$.

For providing the AI service, providers of A_L are rewarded according to their marginal productivity, and by plugging (13) in (9) we obtain

$$w_A(t) = N(t)(1-\alpha)\left(\alpha^2 c_x^{-1}\right)^{\frac{\alpha}{1-\alpha}} \tilde{\lambda}^{\frac{1}{1-\alpha}} \left[\beta\left(\frac{A_L}{L_i}\right)^{\rho} + (1-\beta)\right]^{-\frac{1+\rho}{\rho}} (1-\beta), \tag{45}$$

4 Modelling the Impact of AI

In section three of this paper we proposed a theoretical model with which we can identify effects of AI that cannot be considered in the standard neoclassical model and/or (semi-)endogenous growth approach. In this section we will use the model to study the impact of AI on jobs, inequality and productivity, in order to enrich the discussion of the current state of the literature that we provided in section 2.

The essence of our model, as far as AI is concerned, is to have included in the production of the human service input a technological parameter A_L that describes the AI-facilitated automation technology. In section 3 we established a steady state equilibrium for a given level of A_L. Proposition 1 states the result.

In order to model the impact of AI, we can for instance consider a once-off change in AI, and perform a comparative static analysis, i.e. analyse the impact of $d\tilde{\lambda}/dA_L$. However, AI-technologies are more likely to diffuse into the economy gradually over time, as we stressed in the literature review. Thus we rather model the impact of AI as a sequence of steady-state changes in a continuous process, describing this as $\frac{\dot{A}_L}{A_L} \Longrightarrow \frac{\dot{\lambda}}{\lambda}$.

The impacts on jobs, productivity (and wages), inequality and the growth rate, are set out in the subsections that follow. First, we note the interaction between AI and labour in providing human service inputs, and emphasise the role of demand constraints.

4.1 Impact on human service production, H

The impact of AI most directly is on the human service input H. Given the labour supply and a given technology A_L will determine a given level of human service in this economy. However, now we assume that at least for a while AI will exogenously improve $\dot{A}_L > 0$, so that the owners of this technology can supply continuously more (or a better service) to the market. As a result of this additional technology growth the supply of the human service will continuously increase as long as process innovations continue[13]

$$\frac{\dot{H}}{H} = \frac{(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho} + (1-\beta)} \frac{\dot{A}_L}{A_L} > 0 \tag{46}$$

If this process continues for a long time $\frac{L}{A_L}$ will change noticeably and $\lim_{t\to\infty} \frac{\dot{H}}{H} = \frac{\dot{A}_L}{A_L}$ for $-1 < \rho < 0$.

[13] For calculations see the appendix 6.4.

4.2 The demand constraint

In our model, labour income is the only income that consumes, so if labour income does not profit from productivity gains, consumption may stagnate and restrict growth from the demand side. In order to evaluate this potential mechanism we can calculate the impact of continuous AI progress $\dot{A}_L$ on the effective sales ratio and obtain[14]

$$\frac{\dot{\lambda}}{\lambda} = \frac{(1-\alpha)(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho} + (1-\beta)} \left[\overset{(-)}{\rho} \frac{(1-\gamma)}{\alpha + \frac{(1+\alpha)(1-\beta)}{\beta}\left(\frac{L}{A_L}\right)^{\rho}} - 1 \right] \frac{\dot{A}_L}{A_L} < 0. \tag{47}$$

Thus, the effect of AI-progress and automation undoubtedly tightens the demand constraint as long as the elasticity of substitution between AI technologies and labour is high, $-1 < \rho < 0$. If the process continues for a while $\frac{L}{A_L}$ will change noticeably, and $\lim_{t\to\infty} \frac{\dot{\lambda}}{\lambda} = -(1-\alpha)\frac{\dot{A}_L}{A_L} < 0$ for $-1 \le \rho < 0$. Changes in the effective sales ratio have further implications for other variables like wages or GDP growth, which we will discuss below.

4.3 Impact on wages and labour productivity

Wages are equal to marginal labour productivity. Thus the effects of AI-progress on wage and labour productivity growth is:

$$\frac{\dot{w}_L(t)}{w_L(t)} = \overbrace{\frac{\dot{N}(t)}{N(t)}}^{(i)} + \overbrace{\frac{1}{1-\alpha}\frac{\dot{\lambda}(t)}{\lambda(t)}}^{(ii)} + \overbrace{\frac{\dot{H}(t)}{H(t)} + \rho\left(\frac{L}{A_L}\right)^{\rho}\left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\frac{\dot{A}_L(t)}{A_L(t)}}^{(iii)} \tag{48}$$

This equation shows that the wage rate is affected by product innovation $\frac{\dot{N}(t)}{N(t)}$ in intermediate good production. A growing number of intermediate goods has a positive impact on wages. In the final good Cobb-Douglas production function (1) the number of intermediate goods improves total factor productivity and labour can share in this [see (i) in 48]. However, if the elasticity of substitution is high ($-1 < \rho < 0$) the effective sales ratio $\lambda(t)$ declines, and negatively affects wage growth. This reflects demand constraints because aggregate demand is not adequately growing to absorb additional supply [see (ii) in 48].

Another influence on the wage rate is due to an increase in the human service input $\frac{\dot{H}(t)}{H(t)}$ which may raise productivity and wages. Growth in human service input is driven by IT and AI growth, and is described by (46). However, if the elasticity of substitution is high ($-1 < \rho < 0$) wage income growth is reduced and labour cannot participate in the positive effects of . Thus, IT and AI technologies affect labour productivity growth through various positive and negative channels.

The total effect on marginal labour productivity and wage growth is:

[14] For a proof and the respective conditions see appendix 6.4.

$$\frac{\dot{w}_L(t)}{w_L(t)} = \frac{\dot{N}(t)}{N(t)} + \eta_{w,A_L}\frac{\dot{A}_L}{A_L} \neq 0, \text{ and} \tag{49}$$

$$\eta_{w,A_L} = \frac{\overset{(-)}{\rho}(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho} + (1-\beta)}\left[\frac{1}{1-\gamma}\frac{1}{\alpha + \frac{1+\alpha}{\beta}(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}} + 1\right] < 0 \tag{50}$$

with $\eta_{w,A_L} < 0$ [15] being the wage elasticity of IT and AI automation technology. $\eta_{w,A_L} < 0$ is fully due to the assumption of a high elasticity of substitution between AI and labour ($-1 < \rho < 0$).

Finally, equation (49) indicates that while product innovations (in intermediate production) raises labour productivity and wage growth, as far as AI-innovation is concerned, the net effect on labour productivity and wage growth is *negative*.

4.4 Impact on inequality and jobs

Income share of labour: If we take the labour share of income as a measure of income distribution and inequality, we identify a continuously worsening of the labour share of income as long AI innovation continues and there is high substitutability of labour.

$$\frac{d\frac{w_L(t)L}{Y(t)}}{dt}\frac{1}{\frac{w_L(t)L}{Y(t)}} = \overset{(-)}{\rho}\frac{(1-\beta)}{\beta\left(\frac{A_L}{L}\right)^{\rho} + (1-\beta)}\frac{\dot{A}_L}{A_L} < 0 \quad \text{for } -1 < \rho < 0 \tag{51}$$

For a proof of (51) see appendix 6.4. If the process continues, and $\frac{L}{A_L}$ will change noticeably $\lim_{t\to\infty}\frac{d\frac{w_L(t)L}{Y(t)}}{dt}\frac{1}{\frac{w_L(t)L}{Y(t)}} = \rho$, and the income share of labour approaches

$$\lim_{t\to\infty}\frac{w_L(t)L}{Y(t)} = 0 \quad \text{for } -1 \le \rho < 0.$$

What does this process, described by wage development (49) and the development of the income share of labour income (51) mean for jobs?

In the regime of high substitutability AI will substitute labour and cause unemployment. Higher unemployment will drive wages down. If labour tried to establish a wage growth at a level of GDP growth (53) or a level that keeps the income share of labour constant, this would imply growing unemployment - an effect consistent with predictions of rising technological unemployment.

[15] For a proof and the respective conditions see appendix 6.4.

If the process continues for a while $\frac{L}{A_L}$ will change noticeably, and $\lim_{t\to\infty}\frac{\dot{w}_L(t)}{w_L(t)} = \frac{\dot{N}(t)}{N(t)} + \overset{(-)}{\rho}\frac{\dot{A}_L}{A_L}$ for $-1 \le \rho < 0$.

Income share of technology providers: The income share of the technology providers increases when there is high substitutability of labour, as can be seen from:

$$\frac{d\frac{w_A(t)A_L(t)}{Y(t)}}{dt}\frac{1}{\frac{w_A(t)A_L(t)}{Y(t)}} = -\overset{(-)}{\rho}\frac{\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}}{\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}+(1-\beta)}\frac{\dot{A}_L(t)}{A_L(t)} > 0. \tag{52}$$

In the long term, if the process continues, and $\frac{L}{A_L}$ will change noticeably $\lim_{t\to\infty}\frac{d\frac{w_A A_L}{Y(t)}}{dt}\frac{1}{\frac{w_A A_L}{Y(t)}} = 0$, and the income share of technology providers will approach

$$\lim_{t\to\infty}\frac{w_A(t)A_L(t)}{Y(t)} = \frac{(1-\alpha)}{1-\iota} \quad \text{for } -1 \leq \rho < 0$$

4.5 Impact on GDP growth

Considering the drivers of GDP we obtain from 42

$$\frac{\dot{g}_Y}{g_Y} = \frac{\gamma}{1-\alpha}\overset{(i)}{\frac{\dot{\lambda}}{\lambda}} + \overset{(ii)}{\frac{\dot{H}}{H}} \tag{53}$$

This shows a clearly positive impact of AI-automation on GDP growth.

So far, in the original steady state GDP growth was determined only by the rate of new innovative intermediate products entering the market. With the additional AI technology innovation a second engine of growth is introduced. Due to a continuing quasi-factor-augmenting technology growth $\frac{\dot{A}_L}{A_L} > 0$ the GDP growth rate will increase.

However, the extent of this increase is demand constrained. In an extreme case GDP growth may even be negative. The change in GDP growth rate in (53) is due to the standard supply-side effect which leads to growth in the human service inputs and raises production capacity. However, due to a decline in the labour share of income the consumption rate will decline, so that the effective sales ratio declines. The net effect in GDP is ambiguous. It will be positive when $(-1 < \rho < 0)$ is sufficiently large (see 54), in other words if the substitutability between labour and AI is not too high $(\sigma = \frac{1}{1+\rho})$[16]

$$\begin{aligned}\frac{\dot{g}_Y}{g_Y} &= \left[\frac{\rho(1-\beta)^2}{(1-\gamma)^2}\frac{\gamma}{\frac{(1-\alpha)\beta}{[1-\iota]}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1}+1\right]\frac{(1-\gamma)(1-\beta)^{-1}}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\frac{\dot{A}_L}{A_L} > 0 \\ \text{with} &: \overset{(-)}{\rho} > -\left[\frac{\beta}{(1+\alpha)}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]\frac{(1-\gamma)^2}{\gamma(1-\beta)^2}.\end{aligned} \tag{54}$$

[16] For a proof see appendix 6.4.

If the substitutability is very high and ρ is close to -1 demand constraints may dominate the supply effects and GDP may even shrink.

5 Concluding Remarks

Rapid technological progress in artificial intelligence (AI) has been predicted to lead to mass unemployment, rising inequality, and higher productivity growth through automation. In this paper we critically re-assessed these predictions by (i) surveying the recent literature and (ii) incorporating AI-facilitated automation into a product variety-model, frequently used in endogenous growth theory, but modified to allow for demand-side constraints. This is a novel approach, given that endogenous growth models, including most recent work on AI in economic growth, are largely supply-driven.

Our contribution was motivated by two reasons. One is that there are still only very few theoretical models of economic growth that incorporate AI, and moreover an absence of growth models with AI that takes into consideration growth constraints due to insufficient aggregated demand. A second is that the predictions of AI causing massive job losses and faster growth in productivity and GDP are at odds with reality so far: if anything, unemployment in many advanced economies is at historical lows. However, wage growth and productivity is stagnating and inequality is rising.

Our paper provided a theoretical explanation of this in the context of rapid progress in AI, showing that if labour income does not profit from the economic gains generated by progress in AI, consumption may stagnate and restrict growth. We incorporated AI into the model as a technology service that can substitute (or complement) labour and that diffuse into the economy gradually over time. As such the substitutability between labour and AI is a vital parameter. At high elasticities of substitution we illustrated that this will lead to a decline in employment, a decline in wages and the labour share of income, and greater inequality with a larger share of income accruing to the providers of the AI. Because the latter do not consume, the effect is that the consumption rate declines, which in turn mean less sales for final good producing firms.

The outcomes include sluggish GDP growth (even shrinking GDP), a declining rate of product innovation, and slower productivity growth. With slow diffusion of AI (and slowing innovation in AI), we will not see an immediate rise in unemployment, but rather slower growth in GDP and productivity as the economy does not benefit much from the supply-side driven capacity expansion potential that this technology can deliver; wages can however decline in line with slower GDP and productivity growth in order to maintain employment levels.

In short, our model is consistent with the recent empirical literature that suggests that due to slow diffusion and a slowdown in innovation rates that doomsday scenarios in terms of technological unemployment and inequality is unlikely to materialise soon, and it is also consistent with the reality that whilst AI-technologies are offering many potential applications, its lack of diffusion has so far not yet boosted either GDP or productivity growth, with the result that most advanced economies have been experiencing slow growth and stagnating wages, even in the face of stable employment. Humans will not go the way of horses any time soon. However to ensure that this does not happen eventually, the suggestions from this paper are that more research is needed to understand the sizes and determinants of the substitutability between AI and labour, and to understand the diffusion and applications of AI as an input into production.

6 Appendix

6.1 Income share of labour, technology owners and financial wealth

Income share of labour

$$
\begin{aligned}
w_L(t)\,L &= N(t)\,(1-\alpha)\,x^{\alpha}(t)\,(1-\theta_i(t))\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1-\alpha+\rho}{\rho}}\beta L^{1-\alpha} \\
&= N(t)\,(1-\alpha)\,x^{\alpha}(t)\,(1-\theta_i(t))\,H^{1-\alpha}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\beta \\
\frac{w_L(t)\,L}{Y(t)} &= \frac{(1-\alpha)\,Q(t)\,\beta\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}}{Q(t)-N(t)x(t)c_x} \\
&= \frac{(1-\alpha)\,\beta\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}}{1-N(t)x(t)c_x\frac{1}{Q(t)}}
\end{aligned}
$$

and applying (13) gives $\iota=\frac{N(t)x(t)c_x}{Q(t)}=\frac{Nc_x}{NH^{1-\alpha}\alpha^{\frac{-(1-\alpha)2}{1-\alpha}}(1-\theta_i)^{\frac{-(1-\alpha)}{1-\alpha}}(c_x)^{-\frac{-(1-\alpha)}{1-\alpha}}H^{-(1-\alpha)}(1-\theta_i)}=\frac{c_x}{\alpha^{-2}c_x}=\alpha^2$.
Thus,

$$
\frac{w_L(t)\,L}{Y(t)}=\frac{(1-\alpha)\,\beta}{(1-\iota)}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}=\frac{\beta}{(1+\alpha)}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}
$$

If A_L is time dependent $A_L(t)$ and would continuously increase, and we find that in the long term

$$
\lim_{t\to\infty}\frac{w_L(t)\,L}{Y(t)}=\frac{(1-\alpha)\,\beta}{(1-\iota)}\left[\beta+(1-\beta)*\infty\right]^{-1}=0.
$$

Income share of technology owners:

$$
\begin{aligned}
w_A A_L &= N_i\,(1-\alpha)\,x^{\alpha}\,(1-\theta_i)\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-\frac{1-\alpha+\rho}{\rho}}(1-\beta)\,A_L^{1-\alpha} \\
&= N(t)\,(1-\alpha)\,x^{\alpha}\,(1-\theta_i)\,H^{1-\alpha}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-1}(1-\beta) \\
\frac{w_A A_L}{Y(t)} &= \frac{(1-\alpha)\,Q(t)\,(1-\beta)\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-1}}{Q(t)-N(t)x(t)c_x} \\
&= \frac{(1-\alpha)\,(1-\beta)}{1-\iota}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-1}=\frac{(1-\beta)}{(1+\alpha)}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-1}
\end{aligned}
$$

If A_L is time dependent $A_L(t)$ and would continuously increase, we will find that over the long term

$$
\lim_{t\to\infty}\frac{w_A(t)A_L(t)}{Y(t)}=\frac{(1-\alpha)\,(1-\beta)}{1-\iota}\left[\beta*0+(1-\beta)\right]^{-1}=\frac{(1-\alpha)}{1-\iota}.
$$

Income share of financial wealth owners: From (3), (13) an (20) we know

$$
\begin{aligned}
\frac{N(t)\,\pi_x(t)}{Y(t)} &= \frac{N\left(\frac{1}{\alpha}-1\right)(c_x)\left[\alpha^{\frac{2}{1-\alpha}}(\lambda)^{\frac{1}{1-\alpha}}(c_x)^{-\frac{1}{1-\alpha}}H\right]}{NH^{1-\alpha}x^{\alpha}\lambda - Nxc_x} \\
&= \frac{\left(\frac{1}{\alpha}-1\right)Nc_x x}{NH^{1-\alpha}x^{\alpha}\lambda - Nxc_x} \\
&= \frac{N\left(\frac{1}{\alpha}-1\right)c_x x}{\left(1-\frac{Nxc_x}{Q}\right)NH^{1-\alpha}x^{\alpha}\lambda} \\
&= \frac{(1-\alpha)\frac{1}{\alpha}\alpha^2}{(1-\iota)} = \frac{(\alpha-\iota)}{(1-\iota)} = \frac{\alpha}{(1+\alpha)}
\end{aligned}
$$

6.2 Proof of proposition 1

Determine λ : Equation (38) and (39) determine the two variables, λ and g_N:

$$
\begin{aligned}
g_N &= \left(\alpha^{\frac{2}{1-\alpha}}(\lambda)^{\frac{1}{1-\alpha}}(c_x)^{-\frac{1}{1-\alpha}}H\right)^{\gamma}(g_A)^{1-\gamma} \\
\mu g_N &= \left(\lambda H^{1-\alpha}x^{\alpha} - xc_x\right), \\
\text{with (13)} \quad x &= \alpha^{\frac{2}{1-\alpha}}(\lambda)^{\frac{1}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}
\end{aligned}
$$

and plugging in for x and g_N results in:

$$
\begin{aligned}
\mu g_N &= \lambda\lambda^{\frac{\alpha}{1-\alpha}}H^{1-\alpha}\left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)^{\alpha} - \lambda^{\frac{1}{1-\alpha}}\left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)c_x \\
\mu\left(\alpha^{\frac{2}{1-\alpha}}(c_x)^{-\frac{1}{1-\alpha}}H\right)^{\gamma}(g_A)^{1-\gamma} &= \left[H^{1-\alpha}\left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)^{\alpha} - \left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)c_x\right]\lambda^{\frac{1}{1-\alpha}-\frac{\gamma}{1-\alpha}} \\
\lambda^{\frac{1-\gamma}{1-\alpha}} &= \mu\frac{\left(\alpha^{\frac{2}{1-\alpha}}(c_x)^{-\frac{1}{1-\alpha}}H\right)^{\gamma}(g_A)^{1-\gamma}}{\left[c_x\left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)(\alpha^{-2}-1)\right]} \\
\lambda &= \mu^{\frac{1-\alpha}{1-\gamma}}\frac{\left(\alpha^{\frac{2}{1-\alpha}}(c_x)^{-\frac{1}{1-\alpha}}H\right)^{\frac{\gamma}{1-\gamma}(1-\alpha)}(g_A)^{(1-\alpha)}}{\left[c_x\left(\alpha^{\frac{2}{1-\alpha}}H(c_x)^{-\frac{1}{1-\alpha}}\right)(\alpha^{-2}-1)\right]^{\frac{1-\alpha}{1-\gamma}}} \\
&= \mu^{\frac{1-\alpha}{1-\gamma}}\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}\left[(\alpha^{-2}-1)\right]^{\frac{1-\alpha}{1-\gamma}}}
\end{aligned}
$$

$$
\tilde{\lambda} = \mu^{\frac{1-\alpha}{1-\gamma}}\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}(\alpha^{-2}-1)^{\frac{1-\alpha}{1-\gamma}}}
$$

Determine $\tilde{\upsilon}$: With the effective sales ratio $\tilde{\lambda}$ we can determine the equilibrium $\tilde{\upsilon}$ using (37) and the assumption that δ_i are i.i.d for $i \in I$

$$\begin{aligned}
\tilde{\lambda} &= \frac{1+\upsilon-2\upsilon^2}{1+\upsilon} \\
\lambda(1+\upsilon) &= 1+\upsilon-2\upsilon^2 \\
0 &= -(\lambda+\lambda\upsilon)+1+\upsilon-2\upsilon^2 \\
\left(\upsilon-\frac{1}{4}(1-\lambda)\right)^2 &= \left(\frac{1}{4}(1-\lambda)\right)^2+\frac{1}{2}(1-\lambda) \\
\upsilon &= \pm\left[1+8(1-\lambda)^{-1}\right]^{\frac{1}{2}}\frac{1}{4}(1-\lambda)+\frac{1}{4}(1-\lambda)
\end{aligned}$$

$$\tilde{\upsilon} = \left[\left(1+8\left(1-\tilde{\lambda}\right)^{-1}\right)^{\frac{1}{2}}+1\right]\frac{1}{4}\left(1-\tilde{\lambda}\right)$$

GDP growth rate: Using (20) and taking the time derivative in equilibrium, with $\lambda = \tilde{\lambda}$ and $x = \tilde{x}$ we obtain $\dot{Y}(t) = \dot{N}(t)\left(\lambda H^{1-\alpha}\tilde{x}^\alpha - \tilde{x}c_x\right)$ and thus $g_Y = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{N}(t)}{N(t)}$.

6.3 Dynamic consistency:

- Consistent motion of the demand and supply side in n-e-ee:

From (38) we know

$$\begin{aligned}
\overbrace{\frac{\nu}{1-c}\dot{N}(t)}^{\text{effective demand}} &= Y^e(t) = \overbrace{N(t)\left(\lambda(t)H^{1-\alpha}x^\alpha - xc_x\right)}^{\text{effective supply}}, \\
\text{with } x(t) &= \alpha^{\frac{2}{1-\alpha}}\left(\lambda(t)\right)^{\frac{1}{1-\alpha}}H\left(c_x\right)^{-\frac{1}{1-\alpha}} \to x_j.
\end{aligned}$$

and in equilibrium $\tilde{x}_j = \alpha^{\frac{2}{1-\alpha}}\left(\tilde{\lambda}\right)^{\frac{1}{1-\alpha}}H\left(c_x\right)^{-\frac{1}{1-\alpha}}$ and $\frac{\nu}{1-c}\dot{N}(t) = N(t)\left(H^{1-\alpha}\tilde{x}^\alpha\tilde{\lambda} - \tilde{x}c_x\right)$. A change over time is described by

$$\begin{aligned}
\frac{1}{1-c}\nu\ddot{N}(t) &= \dot{N}(t)\left(H^{1-\alpha}\tilde{x}^\alpha\tilde{\lambda} - \tilde{x}c_x\right) \\
\frac{1}{1-c}\nu\frac{\ddot{N}(t)}{\dot{N}(t)} &= \left(H^{1-\alpha}\tilde{x}^\alpha\tilde{\lambda} - \tilde{x}c_x\right).
\end{aligned}$$

Second, the growth rate of innovation related investments is $\frac{\dot{N}}{N} = g_N$ for exponential growth $(N(t) = e^{g_N t})$ and

$$\frac{1}{1-c}\nu g_N = \left(H^{1-\alpha}\tilde{x}^\alpha\tilde{\lambda} - \tilde{x}c_x\right).$$

q.e.d.

- Consistent start values of Financial and Technology stocks:

As the last step we can show consistency by deriving the savings, and show that these savings indeed can finance the process from the start. Financial wealth income is $rF(t)$. We have assumed that only labour income will be consumed. Income of financial asset holders only serve for savings and these savings are financing investments into newly introduced (innovative) goods.

$$rF(t) + w_A(t) A_L = S(t) = \dot{F}(t) = \dot{N}(t) \nu_x$$

Defining $w_A(t) = N(t)(1-\alpha)\left(\alpha^2 c_x^{-1}\right)^{\frac{\alpha}{1-\alpha}}(1-\theta_i)^{\frac{1}{1-\alpha}}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho} + (1-\beta)\right]^{-\frac{1+\rho}{\rho}}(1-\beta) = N(t) z$ we can write

$$\begin{aligned}
rF(t) + N(t) z A_L &= \dot{N}(t) \nu_x \\
rF(t) + N(t) z A_L &= N(t) \frac{\dot{N}(t)}{N(t)} \nu_x \\
rF(0) e^{(g_N)t} + N(0) e^{g_N t} z A_L &= N(0) e^{g_N t} g_N \nu_x \\
r\frac{F(0)}{N(0)} + z A_L &= g_N \nu_x \\
\frac{F(0)}{N(0)} &= \frac{g_N \nu_x - z A_L}{r}
\end{aligned}$$

For this debt and technology growth mechanism we need to show that savings in deposits and financing investments are consistent in their stock and flow mechanism, and we can derive a relation for the start period $F(0)/N(0)$ that leads to this consistent growth process [17]

$$\begin{aligned}
r\frac{F(t)}{N(t)} &= \nu \frac{\dot{N}(t)}{N(t)} \iff \\
rF(0) e^{(g_N)t} &= \nu N(0) e^{g_N t} g_N \iff \\
rF(0) &= \nu N(0) g_N
\end{aligned}$$

$$\frac{F(0)}{N(0)} = \frac{\nu}{r} g_N$$

q.e.d.

6.4 Modelling the impact of AI

Effects on human service H: From the production of H and a given labour L and technology supply A_L we obtain

$$\begin{aligned}
H &= \left(\beta\left(\frac{A_L}{L}\right)^{\rho} + (1-\beta)\right)^{-\frac{1}{\rho}} A_L \\
H &= \left(\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-\frac{1}{\rho}} L
\end{aligned}$$

[17] $r\frac{F(t)}{N(t)} = \nu \frac{\dot{N}(t)}{N(t)} \iff rF(0) e^{(g_N)t} = \nu N(0) e^{g_N t} g_N \iff rF(0) = \nu N(0) g_N$

$$
\begin{aligned}
\dot{H} &= -\frac{1}{\rho}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-\frac{1}{\rho}-1}(-\rho)L\,(1-\beta)\left(\frac{L}{A_L}\right)^{\rho-1}\frac{L}{A_L}\frac{\dot{A}_L}{A_L} \\
&= \left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-\frac{1}{\rho}-1}L\,(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\frac{\dot{A}_L}{A_L} \\
\frac{\dot{H}}{H} &= \left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-\frac{1}{\rho}-1}L\,(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\frac{\dot{A}_L}{A_L}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{\frac{1}{\rho}}L^{-1} \\
&= \left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-1}(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\frac{\dot{A}_L}{A_L}
\end{aligned}
$$

Continuous innovation and effects of asymmetric technologies:

$$
\frac{\dot{H}}{H}=\frac{(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\frac{\dot{A}_L}{A_L},\quad \lim_{t\to\infty}\frac{\dot{H}}{H}=\frac{\dot{A}_L}{A_L}
$$

Effects on effective sales ratio: $\frac{\dot{\lambda}}{\lambda}$

$$
\mu=\frac{\nu}{1-c},\quad c=\frac{(1-\alpha)\,\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}
$$

$$
\begin{aligned}
\tilde{\lambda} &= \mu^{\frac{1-\alpha}{1-\gamma}}\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}\left(\alpha^{-2}-1\right)^{\frac{1-\alpha}{1-\gamma}}} \\
F_1 &= \mu^{\frac{1-\alpha}{1-\gamma}},\qquad F_2=\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}\left(\alpha^{-2}-1\right)^{\frac{1-\alpha}{1-\gamma}}}
\end{aligned}
$$

$$
\begin{aligned}
(1)\frac{dF_1}{dA_L} &= \frac{1-\alpha}{1-\gamma}\mu^{\frac{1-\alpha}{1-\gamma}-1} \\
&\quad (-)\frac{\nu}{\left(1-\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\right)^2}(-)\frac{(1-\alpha)\beta}{[1-\iota]} \\
&\quad (-)\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1-1}(1-\beta)\rho\left(\frac{L}{A_L}\right)^{\rho-1}(-)\frac{L}{A_L}\frac{\dot{A}_L}{A_L} \\
&= \frac{1-\alpha}{1-\gamma}\mu^{\frac{1-\alpha}{1-\gamma}-1} \\
&\quad \frac{\nu}{\left(1-\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\right)^2}\frac{(1-\alpha)\beta}{[1-\iota]} \\
&\quad \left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1-1}(1-\beta)\rho\left(\frac{L}{A_L}\right)^{\rho}\frac{\dot{A}_L}{A_L} \\
&= \frac{1-\alpha}{1-\gamma}\mu^{\frac{1-\alpha}{1-\gamma}}\frac{\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-2}(1-\beta)\rho\left(\frac{L}{A_L}\right)^{\rho}}{1-\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}}\frac{\dot{A}_L}{A_L} \\
&= \frac{1-\alpha}{1-\gamma}\mu^{\frac{1-\alpha}{1-\gamma}}\frac{\frac{(1-\alpha)\beta}{[1-\iota]}\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-2}(1-\beta)\rho\left(\frac{L}{A_L}\right)^{-\rho}}{1-\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}\left(\frac{L}{A_L}\right)^{-\rho}}\frac{\dot{A}_L}{A_L} \\
&= \overset{(-)}{\rho}\frac{1-\alpha}{1-\gamma}\mu^{\frac{1-\alpha}{1-\gamma}}\frac{\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-2}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}\left(\frac{L}{A_L}\right)^{\rho}-\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}}\frac{\dot{A}_L}{A_L}<0
\end{aligned}
$$

$$
\begin{aligned}
(2)\frac{dF_2}{dA_L} &= -(1-\alpha)\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}\left(\alpha^{-2}-1\right)^{\frac{1-\alpha}{1-\gamma}}H}\frac{\partial H}{\partial A_L} \\
&= -(1-\alpha)\frac{g_A{}^{(1-\alpha)}}{\alpha^2H^{1-\alpha}\left(\alpha^{-2}-1\right)^{\frac{1-\alpha}{1-\gamma}}}\frac{1}{\left(\frac{\beta}{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}+1\right)}\frac{\dot{A}_L}{A_L}<0
\end{aligned}
$$

bring $\frac{dF_1}{dA_L}$ and $\frac{dF_2}{dA_L}$ together:

$$
\begin{aligned}
\frac{\dot{\lambda}}{\lambda} &= \frac{1-\alpha}{1-\gamma}\left[\overset{(-)}{\rho}\,\frac{\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-2}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}\left(\frac{L}{A_L}\right)^{\rho}-\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}}-\frac{(1-\gamma)(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\right]\frac{\dot{A}_L}{A_L} \\
&= \frac{1-\alpha}{1-\gamma}\left[\overset{(-)}{\rho}\,\frac{\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-2}(1-\beta)}{\left[\frac{(1+\alpha)}{\beta}\left(\frac{L}{A_L}\right)^{\rho}\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]-1\right]\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}}-\frac{(1-\gamma)(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\right] \\
&= \frac{(1-\alpha)(1-\beta)}{1-\gamma}\left[\overset{(-)}{\rho}\,\frac{\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-1}}{\alpha+\frac{(1+\alpha)}{\beta}(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}-\frac{(1-\gamma)}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\right]\frac{\dot{A}_L}{A_L}<0
\end{aligned}
$$

and in the long term:

$$
\begin{aligned}
\lim_{t\to\infty}\frac{\dot{\lambda}}{\lambda} &= \frac{(1-\alpha)(1-\beta)}{1-\gamma}\left[\overset{(-)}{\rho}\,\frac{1}{\left[\alpha+\frac{(1+\alpha)}{\beta}(1-\beta)\infty\right][(\beta*0+(1-\beta))]}-\frac{(1-\gamma)}{\beta*0+(1-\beta)}\right]\frac{\dot{A}_L}{A_L}<0 \\
\lim_{t\to\infty}\frac{\dot{\lambda}}{\lambda} &= \frac{(1-\alpha)(1-\beta)}{1-\gamma}\left[\overset{(-)}{\rho}*0-\frac{(1-\gamma)}{(1-\beta)}\right]\frac{\dot{A}_L}{A_L}=-(1-\alpha)\frac{\dot{A}_L}{A_L}<0
\end{aligned}
$$

Effects on wages: $\frac{\dot{w}_L(t)}{w_L(t)}$ continuous changes and effects of asymmetric technologies:

$$
\begin{aligned}
w_L(t) &= \left[N(t)(1-\alpha)\left[\alpha^2c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}}\lambda(t)^{\frac{1}{1-\alpha}}H\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\beta\right] \\
&= \left[N(t)(1-\alpha)\left[\alpha^2c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}}\lambda(t)^{\frac{1}{1-\alpha}}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1+\rho}{\rho}}\beta\right] \\
\dot{w}_L(t) &= \frac{\dot{N}(t)}{N(t)}\left[N(t)(1-\alpha)\left[\alpha^2c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}}\lambda(t)^{\frac{1}{1-\alpha}}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1+\rho}{\rho}}\beta\right] \\
&\quad+\frac{1}{1-\alpha}\frac{\dot{\lambda}(t)}{\lambda(t)}\left[N(t)(1-\alpha)\left[\alpha^2c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}}\lambda(t)^{\frac{1}{1-\alpha}}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1+\rho}{\rho}}\beta\right] \\
&\quad+(1+\rho)(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\frac{\dot{A}_L}{A_L}\left[\begin{array}{c}N(t)(1-\alpha)\left[\alpha^2c_x^{-1}\right]^{\frac{\alpha}{1-\alpha}}\lambda(t)^{\frac{1}{1-\alpha}} \\ \left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-\frac{1+\rho}{\rho}}\beta\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\dot{w}_L(t)}{w_L(t)} &= \frac{\dot{N}(t)}{N(t)} + \frac{1}{1-\alpha}\frac{\dot{\lambda}(t)}{\lambda(t)} + (1+\rho)(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\left[\beta + (1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}\frac{\dot{A}_L}{A_L} \\
\frac{\dot{w}_L(t)}{w_L(t)} &= \frac{\dot{N}(t)}{N(t)} + \frac{1}{1-\alpha}\frac{\dot{\lambda}(t)}{\lambda(t)} + (1+\rho)(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\left(\beta\left(\frac{L}{A_L}\right)^{-\rho} + (1-\beta)\right)^{-1}\left(\frac{L}{A_L}\right)^{-\rho}\frac{\dot{A}_L}{A_L}
\end{aligned}
$$

$$
\frac{\dot{w}_L(t)}{w_L(t)} = \underbrace{\frac{\dot{N}(t)}{N(t)}}_{(1)} + \underbrace{\left[\begin{array}{c} \frac{\overset{(-)}{\rho}}{1-\gamma}\frac{\left[\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)\right]^{-2}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}\left(\frac{L}{A_L}\right)^{\rho}-\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}} \\ -\frac{1}{\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)} + (1+\rho)(1-\beta)\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)^{-1} \end{array}\right]}_{(2)}\frac{\dot{A}_L}{A_L}
$$

$$
(2):\ \frac{\overset{(-)}{\rho}(1-\beta)}{\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)}\left[\overbrace{\frac{1}{1-\gamma}\frac{1}{\left[\alpha+\frac{1+\alpha}{\beta}(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]}}^{<0}+1\right]
$$

$$
\begin{aligned}
\lim_{t\to\infty}\frac{\dot{w}_L(t)}{w_L(t)} &= \frac{\dot{N}(t)}{N(t)} + \frac{\overset{(-)}{\rho}(1-\beta)}{\beta*0+(1-\beta)}\left[\overbrace{\frac{1}{1-\gamma}\frac{1}{\left[\alpha+\frac{1+\alpha}{\beta}(1-\beta)\infty\right]}}^{<0}+1\right]\frac{\dot{A}_L}{A_L} \\
\lim_{t\to\infty}\frac{\dot{w}_L(t)}{w_L(t)} &= \frac{\dot{N}(t)}{N(t)} + \overset{(-)}{\rho}\frac{\dot{A}_L}{A_L}
\end{aligned}
$$

Effects on income share of labour: $\frac{wL}{Q}$:

$$
\frac{w_L(t)L}{Y(t)} = \frac{(1-\alpha)\beta}{(1-\iota)}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1} = \frac{\beta}{(1+\alpha)}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}
$$

$$
\begin{aligned}
\frac{w_L(t)L}{Y(t)} &= \frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1} < 1 \\
\frac{d\frac{w_L(t)L}{Y(t)}}{dt} &= -\frac{(1-\alpha)\beta}{[1-\iota]}\left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1-1}(1-\beta)\rho\left(\frac{L}{A_L}\right)^{\rho-1}(-)\frac{L}{A_L}\frac{\dot{A}_L}{A_L} \\
\frac{d\frac{w_L(t)L}{Y(t)}}{dt}\frac{1}{\frac{w_L(t)L}{Y(t)}} &= \left[\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right]^{-1}(1-\beta)\overset{(-)}{\rho}\left(\frac{L}{A_L}\right)^{\rho}\frac{\dot{A}_L}{A_L} < 0 \quad \text{for } -1<\rho<0
\end{aligned}
$$

$$
= \overset{(-)}{\rho}\frac{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}{\left[\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}+(1-\beta)\right]\left(\frac{L}{A_L}\right)^{\rho}}\frac{\dot{A}_L}{A_L}
$$

$$\lim_{t\to\infty}\frac{d\frac{w_L(t)L}{Y(t)}}{dt}\frac{1}{\frac{w_L(t)L}{Y(t)}} = \overset{(-)}{\rho}\frac{(1-\beta)}{\left[\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}+(1-\beta)\right]}\frac{\dot{A}_L}{A_L} = \overset{(-)}{\rho}\frac{(1-\beta)}{[\beta*0+(1-\beta)]} = \overset{(-)}{\rho}.$$

Effect on income share of the technology providers

$$\frac{w_A A_L}{Y(t)} = \frac{(1-\alpha)(1-\beta)}{1-\iota}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-1}$$

$$\frac{d\frac{w_A A_L}{Y(t)}}{dt} = (-)\frac{(1-\alpha)(1-\beta)}{1-\iota}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-2}\beta\rho\left(\frac{A_L}{L_i}\right)^{\rho-1}\frac{A_L}{L_i}\frac{\dot{A}_L}{A_L} = -\overset{(-)}{\rho}\beta\frac{(1-\alpha)(1-\beta)}{1-\iota}\left[\beta\left(\frac{A_L}{L_i}\right)^{\rho}+(1-\beta)\right]^{-2}\left(\frac{A_L}{L_i}\right)^{\rho}\frac{\dot{A}_L}{A_L}$$

$$\frac{d\frac{w_A(t)A_L(t)}{Y(t)}}{dt}\frac{1}{\frac{w_A(t)A_L(t)}{Y(t)}} = -\overset{(-)}{\rho}\frac{\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}}{\left[\beta\left(\frac{A_L(t)}{L_i}\right)^{\rho}+(1-\beta)\right]}\frac{\dot{A}_L(t)}{A_L(t)} > 0$$

$$\lim_{t\to\infty}\frac{d\frac{w_A(t)A_L(t)}{Y(t)}}{dt}\frac{1}{\frac{w_A(t)A_L(t)}{Y(t)}} = -\overset{(-)}{\rho}\frac{\beta*0}{[\beta*0+(1-\beta)]}\frac{\dot{A}_L(t)}{A_L(t)} = 0.$$

Effects on the income share of financial wealth holders: $\frac{N(t)\pi_x(t)}{Y(t)}$ According to (26) is

$$\frac{N(t)\pi_x(t)}{Y(t)} = \frac{(\alpha-\iota)}{(1-\iota)} = \frac{\alpha}{(1+\alpha)},$$

thus, the income share of financial wealth holders will not change over time.

Change in the growth rate: $\frac{\dot{g}_N}{g_N}$

$$g_N = \left(\alpha^{\frac{2}{1-\alpha}}\tilde{\lambda}^{\frac{1}{1-\alpha}}c_x^{-\frac{1}{1-\alpha}}H\right)^{\gamma}(g_A)^{1-\gamma}$$

$$\begin{aligned} g_N &= \alpha^{\frac{2\gamma}{1-\alpha}}\lambda^{\frac{\gamma}{1-\alpha}}c_x^{-\frac{\gamma}{1-\alpha}}H(g_A)^{1-\gamma} \\ \dot{g}_N &= \frac{\gamma}{1-\alpha}\frac{\dot{\lambda}}{\lambda}\lambda^{\frac{\gamma}{1-\alpha}}\alpha^{\frac{2\gamma}{1-\alpha}}c_x^{-\frac{\gamma}{1-\alpha}}H(g_A)^{1-\gamma} \\ &\quad +\frac{\dot{H}}{H}\alpha^{\frac{2\gamma}{1-\alpha}}\lambda^{\frac{\gamma}{1-\alpha}}c_x^{-\frac{\gamma}{1-\alpha}}H(g_A)^{1-\gamma} \\ \frac{\dot{g}_N}{g_N} &= \frac{\gamma}{1-\alpha}\frac{\dot{\lambda}}{\lambda}+\frac{\dot{H}}{H} \end{aligned}$$

$$
= \frac{\gamma}{1-\alpha}\left[\overset{(-)}{\rho}\frac{1-\alpha}{1-\gamma}\frac{\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)^{-2}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}\left(\frac{L}{A_L}\right)^{\rho}-\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}}-\frac{(1-\alpha)}{\left(\frac{\beta}{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}+1\right)}\right]\frac{\dot{A}_L}{A_L}
$$

$$
+\frac{1}{\left(\frac{\beta}{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}+1\right)}\frac{\dot{A}_L}{A_L}
$$

$$
=\left[\overset{(-)}{\rho}\frac{\gamma}{1-\gamma}\frac{\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)^{-2}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}\left(\frac{L}{A_L}\right)^{\rho}-\left[\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right]^{-1}}-\frac{\gamma}{\left(\frac{\beta}{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}+1\right)}+\frac{1}{\left(\frac{\beta}{(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}}+1\right)}\right]\frac{\dot{A}_L}{A_L}
$$

$$
= \left[\overset{(-)}{\rho}\frac{\gamma}{1-\gamma}\frac{\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-2}\left(\frac{L}{A_L}\right)^{\rho}(1-\beta)}{\frac{[1-\iota]}{(1-\alpha)\beta}-\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)^{-1}}+\frac{1-\gamma}{\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)(1-\beta)^{-1}\left(\frac{L}{A_L}\right)^{-\rho}}\right]\frac{\dot{A}_L}{A_L}
$$

$$
= \left[\overset{(-)}{\rho}\frac{\gamma}{1-\gamma}\frac{\left(\frac{L}{A_L}\right)^{\rho}(1-\beta)}{\left[\frac{[1-\iota]}{(1-\alpha)\beta}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)}+\frac{1-\gamma}{\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)(1-\beta)^{-1}\left(\cdots\right.}\right.
$$

$$
= \left[\overset{(-)}{\rho}\frac{1}{(1-\gamma)^2}\frac{\gamma}{\left[\frac{[1-\iota]}{(1-\alpha)\beta}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]}+1\right]\frac{(1-\gamma)}{(1-\beta)^{-1}\left(\beta\left(\frac{L}{A_L}\right)^{-\rho}+(1-\beta)\right)}\frac{\dot{A}_L}{A_L}<0
$$

$$
\overset{(-)}{\rho}\frac{1}{(1-\gamma)^2}\frac{\gamma}{\left[\frac{[1-\iota]}{(1-\alpha)\beta}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]}+1 < 0
$$

$$
\overset{(-)}{\rho}\frac{1}{(1-\gamma)^2}\frac{\gamma}{\left[\frac{[1-\iota]}{(1-\alpha)\beta}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]} < -1
$$

$$
\overset{(-)}{\rho} < -\left[\frac{[1-\iota]}{(1-\alpha)\beta}\left(\beta+(1-\beta)\left(\frac{L}{A_L}\right)^{\rho}\right)-1\right]\frac{(1-\gamma)^2}{\gamma}
$$

References

Acemoglu, D. and Restrepo, P. (2017a). Low-Skill and High-Skill Automation. *Working Paper no. 17-12, MIT Department of Economics.*

Acemoglu, D. and Restrepo, P. (2017b). Robots and Jobs: Evidence from US Labor Markets. *NBER Working Paper no. 23285. National Bureau for Economic Research.*

Aghion, P. (2009). Some Thoughts on Industrial Policy and Growth. *OFCE Working Paper no 2009-09, April.*

Aghion, P., Jones, B., and Jones, C. (2017). Artificial Intelligence and Economic Growth. *NBER Working Paper 23928, National Burea for Economic Research.*

Agrawal, A., McHale, J., and Oettl, A. (2018). Finding Needles in Haystacks: Artificial Intelligence and Recombinant Growth. *NBER Working Paper no. 24541. National Bureau for Economic Research.*

Arntz, M., Gregory, I., and Zierahn, U. (2017). Revisiting theRisk of Automation. *Economic Letters*, 159:157–160.

Arntz, M., Gregory, T., and Zierahn, U. (2016). The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis. *OECD Social, Employment and Migration Working Paper no. 189. Paris: OECD.*

Atkinson, R. and Wu, J. (2017). False Alarmism: Technological Disruption and the U.S. Labor Market, 1850-2015. *Information Technology and Innovation Foundation*, May:1–28.

Autor, D. (2015). Why are There Still so Many Jobs? The History and Future of Workplace Automation. *Journal of Economic Perspectives*, 29(3):3–30.

Autor, D. and Dorn, D. (2013). The growth of Low Skill Service Jobs and the Polarization of the US Labor Market. *American Economic Review*, (103):1553–1597.

Autor, D., Levy, F., and Murnane, R. (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *Quarterly Journal of Economics*, 118(4):12791333.

Berriman, R. and Hawksworth, J. (2017). Will Robots Steal our Jobs? The Potential Impact of Automation on the UK and other Major Economies. *PwC UK Economic Outlook, March.*

Bessen, J. (2017). Automation and Jobs: When Technology Boost Employment. *Law and Economics Research Paper no. 17-09. Boston University School of Law.*

Bessen, J. (2018). AI and Jobs: The Role of Demand. *NBER Working Paper no. 24235. National Bureau for Economic Research.*

Bloom, N., Jones, C., van Reenen, J., and Webb, M. (2017). Are Ideas Getting Harder to Find? *NBER Working Paper no. 23782, National Burea for Economic Research.*

Bowles, J. (2017). The Computerization of European Jobs. *Bruegel.org, 24th July.*

Brynjolfsson, E. and McAfee, A. (2015). Will Humans Go the Way of Horses? *Foreign Affairs*, 94:8–14.

Brynjolfsson, E., Rock, D., and Syverson, C. (2017). Artificial Intelligence and the Modern Productivity Paradox: A Clash of Expectations and Statistics. *NBER Working Paper no. 24001. National Bureau for Economic Research.*

Byrne, D., Oliner, S., and Sichel, D. (2013). Is the Information Technology Revolution Over? *International Productivity Monitor*.

Chiacchio, F., Petropoulos, G., and Pichler, D. (2018). The Impact of Industrial Robots on EU Employment and Wages: A Local Labor Market Approach. *Working Paper no. 02, Bruegel, Brussels.*

Cowen, T. (2016). Is Innovation Over? The Case Against Pessimism. *Foreign Affairs*, March/April:42–46.

Dauth, W., S.Findeisen, J.Suedekum, and Woessner, N. (2017). German Robots The Impact of Industrial Robots on Workers. *CEPR Discussion Paper No. 12306*.

Eichengreen, B. (2015). Secular Stagnation: The Long View. *NBER Working Paper 20836, National Bureau for Economic Research.*

Frey, C. and Osborne, M. (2013). The Future of Employment: How Susceptible are Jobs to Computerization? *Oxford Martin Programme on the Impacts of Future Technology, University of Oxford.*

Frey, C. and Osborne, M. (2017). The Future of Employment: How Susceptible are Jobs to Computerization? *Technological Forecasting and Social Change*, 114:254–280.

Frey, C., Osborne, M., and Holmes, C. (2016). *Technology at Work v2.0: The Future is Not What it Used to Be*.

Friedman, T. (2016). Thank You for Being Late: An Optimists Guide to Thriving in the Age of Accelerations. *New York: Picador.*

Gordon, R. (2018). Why Has Economic Growth Slowed When Innovation Appears to be Accelerating? *NBER Working Paper 24554. National Bureau for Economic Research.*

Gries, T. (2018). A New Theory of Demand Restricted Growth: A Semi-Endogenous Growth Model. *Mimeo: Paderborn University.*

Gries, T. and Naudé, W. (2011). Entrepreneurship and human development: A capability approach. *Journal of Public Economics*, (5):216–224.

Hansen, A. (1939). Economic Progress and Declining Population Growth. *American Economic Review*, 29(1):1–15.

House of Lords (2018). Ai in the UK: Ready, Willing and Able? *Select Committee on Artificial Intelligence, HL Paper 100.*

Jones, B. (2009). The Burden of Knowledge and the Death of Renaissance Man: Is Innovation Getting Harder? *Review of Economic Studies*, 76(1):283–317.

Korinek, A. and Stiglitz, J. (2017). Artificial Intelligence and its Implications for Income Distribution and Unemployment. *NBER Working Paper no. 24174. National Bureau for Economic Research.*

Krishnan, M., Mischke, J., and Remes, J. (2018). Is the Solow Paradox Back? *McKinsey Quarterly*, June.

Lewis, J. (2018). Bitesize: The Past Decade's Productivity Growth in Historical Context. *Bank Underground Blog*, 25 April.

Mokyr, J. (2014). Secular Stagnation? Not in Your Life. *In Teulings, C. and Baldwin, R. eds. Secular Stagnation: Facts, Causes and Cures. CEPR. Available at: https://voxeu.org/content/secular-stagnation-facts-causes-and-cures (accessed 23 July 2018).*

Mokyr, J. (2018). The Past and the Future of Innovation: Some Lessons from Economic History. *Explorations in Economic History*, 69:13–26.

Naudé, W. and Nagler, P. (2018). Technological Innovation, Entrepreneurship and Productivity in Germany, 1871-2015. *SPRU Working Paper Series (SWPS), 2018-02. Science Policy Research Unit, University of Sussex.*

Nordhaus, W. (2015). Are We Approaching an Economic Singularity? Information Technology and the Future of Economic Growth. *Cowles Foundation Discussion Paper no. 2021. Yale University.*

OECD (2015). The Future of Productivity. *OECD. Paris.*

Romer, P. (1987). Growth Based on Increasing Returns Due to Specialization. *American Economic Review*, 77(2):56–62.

Romer, P. (1990). Endogenous Technical Change. *World Development*, (5):S71–S102.

Sachs, J. (2018). R&d, Structural Transformation, and the Distribution of Income. *NBER Workshop on the Economics of Artificial Intelligence, 6 Jan.*

Schivardi, F. and Schmitz, T. (2018). The IT Revolution and Southern Europe's Two Lost Decades. *Working Paper no. 624, Innocenzo Gasparani Institute for Economic Research.*

Syverson, C. (2017). Challenges to Mismeasurement Explanations for the US Productivity Slowdown. *Journal of Economic Perspectives*, 31(2):165–186.

Teulings, C. and Baldwin, R. (2014). Secular Stagnation: Facts, Causes and Cures. *London: Centre for Economic Policy Research.*

Tuuli, S. and Batten, S. (2015). Back to the Future: Why we're Optimists in the Secular Stagnation Debate. *Bank Underground Blog, Bank of England.*

Van de Gevel, A. and Noussair, C. (2013). The Nexus between Artificial Intelligence and Economics. *Springer Briefs in Economics.*

World Bank (2018). The Changing Nature of Work. *World Development Report 2019. Washington DC : The World Bank.*