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ABSTRACT

Bounds with Imperfect Instruments: Leveraging the Implicit Assumption of Intransitivity in Correlations

Instrumental variables (IV) is an indispensable tool for establishing causal relationships between variables. Recent work has focused on improving bounds for cases when an ideal instrument does not exist. We leverage a principle, "Intransitivity in Correlations," related to an under-utilized property from the statistics literature. From this principle, it is straightforward to obtain new bounds. We argue that these new theoretical bounds become increasingly useful as instruments become increasingly weak or invalid.

JEL Classification: C26

Keywords: instrumental variables, bounding, partial identification, transitivity in correlations

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1. Introduction

The use of instrumental variables (IV) has answered many important questions and greatly improved the credibility of research that attempts to establish causal links in the social sciences and applied statistics literatures. Some important empirical questions remain unanswered on account of endogenous regressors and the absence of a truly exogenous IV. Nevo and Rosen (2012) (NR, henceforth) address estimation with what they term "Imperfect Instrumental Variables" (IIV): instruments that are correlated with an endogenous variable but that are not entirely exogenous. They show that it is possible, under the assumption that the instrument is no more endogenous than the endogenous variable itself, to improve upon one of two previously known bounds. This is done through the use of an additional IV, which is simply a weighted sum of the endogenous regressor and the main instrumental variable.

Here, without making any additional assumptions, we present new bounds that have the potential to improve upon the bound that NR does not address. We do so by leveraging results from the statistics literature that provide additional information about the relationship between the assumed sets of parameter values in each of the cases NR examine. The rest of the paper proceeds as follows: Section 2 summarizes the NR result, Section 3 explains our method and the new bounds that we arrive at and Section 4 concludes and motivates avenues for future research.

2. Bounds from Nevo and Rosen

Assume the following data generating process:

$$y_i = x_i\beta + u_i$$

Then an IIV exists when, for some z, $E[xu] \neq 0$, $E[zu] \neq 0$ and $E[xz] \neq 0$. It is widely known that assumptions about the direction of correlations between x, z, and u describe the direction of the bias for both β_{OLS} and the standard β_{IV}^{Z}, thus providing bounds. NR show that a new set of bounds can be obtained when Assumption 1 holds:

Assumption 1. *The instrument and the endogenous variable are correlated with the unobservables in the same direction:* $\rho_{zu}\rho_{xu} > 0$.

NR propose a new instrument:

$$V(\lambda) = \sigma_x z - \lambda\sigma_z x$$

They show that for $\lambda = \lambda^* = \rho_{zu}/\rho_{xu}$, $V(\lambda)$ is both relevant and valid. While $V(\lambda^*)$ is not feasible, as it is a function of unknown parameters ρ_{zu} and ρ_{xu}, NR show that $V(\lambda = 1)$ improves upon bounds for the true value of β. A value of $\lambda = 1$ corresponds to the most conservative case of Assumption 2:

Assumption 2. *The instrument is no more endogenous than the endogenous variable:* $\rho_{zu}^2 \leq \rho_{xu}^2$.

Using the above assumptions, NR arrive at the following bounds.

Table 1: Bounds for an IIV Under Assumptions 1 and 2

	$\rho_{xz} \leq 0$	$\rho_{xz} \geq 0$
$\rho_{xu}, \rho_{zu} < 0$:	Case 1: $\beta_{OLS} < \beta_{IV}^{V(1)} < \beta < \beta_{IV}^{Z}$	Case 3: $\beta > \max\{\beta_{IV}^{V(1)}, \beta_{IV}^{Z}\} > \beta_{OLS}$
$\rho_{xu}, \rho_{zu} > 0$:	Case 2: $\beta_{IV}^{Z} < \beta < \beta_{IV}^{V(1)} < \beta_{OLS}$	Case 4: $\beta < \min\{\beta_{IV}^{V(1)}, \beta_{IV}^{Z}\} < \beta_{OLS}$

3. New Bounds Leveraging an Implicit Assumption

We first discuss "Intransitivity in Correlations," which follows from the concept of "Transitivity in Correlations" described in the statistics literature. We then apply this new principle to obtain new bounds for the two sided bounds cases shown in Table 1 (Cases 1 and 2).

3.1. Transitivity in Correlation

In general, if two variables A and C are each positively correlated with a third variable B, it is possible that A and C could be correlated positively, negatively, or not at all with each other. However, Langford et al. (2001) show that, in some circumstances, correlations can be transitive. They prove that a sufficient condition for positive correlation between A and C, when $\rho_{AB}\rho_{BC} > 0$, can be stated as follows: $\rho_{AB}^2 + \rho_{BC}^2 > 1 \implies \rho_{AC} > 0$. Later work by Lipovetsky (2002) shows the above condition holds regardless of whether ρ_{AB} and ρ_{BC} are both positive or both negative. He also shows that a sufficient condition for negative correlation between A and C, when $\rho_{AB}\rho_{BC} < 0$, can be stated as follows: $\rho_{AB}^2 + \rho_{BC}^2 > 1 \implies \rho_{AC} < 0$. We combine these two results to define Theorem 1, similarly to Lipovetsky and Conklin (2004):

Theorem 1. *A sufficient condition for the product of three related correlations to be positive ($\rho_{AB}\rho_{BC}\rho_{AC} > 0$) is for the sum of the squares of at least one pair of correlations to be greater than one:*
$\rho_{AB}^2 + \rho_{BC}^2 > 1 \mid \rho_{AB}^2 + \rho_{AC}^2 > 1 \mid \rho_{AC}^2 + \rho_{BC}^2 > 1 \implies \rho_{AB}\rho_{BC}\rho_{AC} > 0$

Without loss of generality, consider when $\rho_{AB}^2 + \rho_{BC}^2 > 1$. From Langford et al. (2001), we see that if $\rho_{AB}\rho_{BC} > 0$, then the sum of the square of these two terms implies that $\rho_{AC} > 0$. Thus all three terms are positive and the product of the three terms is then positive. From Lipovetsky (2002), we see that if $\rho_{AB}\rho_{BC} < 0$, then the sum of the square of these two terms implies that $\rho_{AC} < 0$. Thus the product of the first two terms is negative, the third term is negative, and the product of all three terms is then positive.

Since the sum of any two of the squared correlations being greater than one guarantees a positive product of the three correlations ($\rho_{AB}\rho_{BC}\rho_{AC} > 0$), then, by transposition, if the product of the three correlations is negative ($\rho_{AB}\rho_{BC}\rho_{AC} < 0$), it must be the case that none of the pairs of the correlations sum to more than one. We define the following corollary, which describes the Principle of Intransitivity in Correlations:

Corollary 1. *The necessary conditions for the product of three related correlations to be negative ($\rho_{AB}\rho_{BC}\rho_{AC} < 0$) is for the sum of the squares of each of the pairwise correlations to be less than one, i.e.,*

$$\begin{aligned}\rho_{AB}^2 + \rho_{BC}^2 &\leq 1 \\ \rho_{AB}^2 + \rho_{AC}^2 &\leq 1 \\ \rho_{AC}^2 + \rho_{BC}^2 &\leq 1\end{aligned}$$

Note that for Cases 1 and 2 in Table 1, the product of the three ρ terms is negative. Thus the above corollary gives a general definition of the necessary conditions for two-sided bounds. This approach is not applicable to Cases 3 and 4, as in these cases $\rho_{AB}\rho_{BC}\rho_{AC} > 0$. Below we use this result to obtain a new set of two-sided bounds.

To our knowledge, we are the first to apply the above result to create a new set of bounds in the context of instrumental variables. Lipovetsky and Conklin (2004) also describes a case where transitivity not holding yields an important implication for linear regression. Specifically, they describe how the inclusion of multi-collinear regressors affects the R^2 of a model. A paper by Mauro (1990) from the Psychology literature, concerning omitted variables bias, presents an equivalent condition to that used by Langford et al. (2001) and then uses it in the context of sensitivity analysis. However, no explicit bounding formula for an estimator is provided.

3.2. Tightening the Bounds

Returning to the NR bounds, it is clear that, for Cases 1 and 2, $\rho_{xu}\rho_{zu}\rho_{xz} < 0$. Thus, we can apply Corollary 1 to obtain the following inequality:

$$\rho_{xu}^2 + \rho_{xz}^2 \leq 1 \tag{1}$$

We solve Equation 1 for ρ_{xu}^2, substitute for the definition of ρ_{xu} and take the square root of each side:

$$\begin{aligned}\rho_{xu}^2 &\leq 1 - \rho_{xz}^2 \\ \rho_{xu} = \frac{\sigma_{xu}}{\sigma_x\sigma_u} &\in \pm\sqrt{1 - \rho_{xz}^2}\end{aligned}$$

We then multiply each side by $\frac{\sigma_u}{\sigma_x}$ and substitute the definition of bias to obtain the following.[1]

$$(\beta_{OLS} - \beta) \in \pm\frac{\sigma_u}{\sigma_x}\sqrt{1 - \rho_{xz}^2}$$

[1]For the simple linear model $y_i = x_i\beta + u_i$, the well known probability limit of the bias in β_{OLS} is $\frac{\sigma_{xu}}{\sigma_x^2}$.

Finally, shifting the set by $-\beta_{OLS}$ and then transforming it by a negative we have two new bounds, which we will denote by β_L and β_U:

$$\begin{aligned} -\beta &\in \pm\frac{\sigma_u}{\sigma_x}\sqrt{1-\rho_{xz}^2}-\beta_{OLS} \\ \beta &\in \beta_{OLS}\pm\frac{\sigma_u}{\sigma_x}\sqrt{1-\rho_{xz}^2} \qquad (2) \\ \beta &\in (\beta_L,\beta_U) \qquad (3) \end{aligned}$$

We now provide some intuition about the usefulness of these new bounds. In Case 1 (see Table 1), β_L is clearly dominated by β_{OLS} (the pre-existing lower bound), as well as by $\beta_{IV}^{V(1)}$ (NR's new tighter lower bound). However, β_U may dominate the pre-existing upper bound of β_{IV}^Z. If so, this provides evidence that the instrument being used is not valid. Furthermore, while the bias in β_{IV}^Z, $\frac{\rho_{zu}}{\rho_{xz}}\frac{\sigma_u}{\sigma_x}$, approaches infinity as ρ_{xz} approaches 0, the bias in β_U is always finite.[2] In short, weak instruments provide the worst case scenario for both β_U and β_{IV}^Z, but the worst case scenario for β_{IV}^Z is more problematic.

The new β_U bound also improves relative to β_{IV}^Z as the instrument becomes more strongly correlated with the unobserved term, as β_{IV}^Z is a function of ρ_{zu} while β_U is not.[3] Similar logic applies to β_L in Case 2. The potentially tighter bounds in these two cases are shown in Table 3.2.

Table 2: New Bounds for Two Cases

	$\rho_{xz} \leq 0$
$\rho_{xu}, \rho_{zu} < 0$:	Case 1: $\beta_{OLS} < \beta_{IV}^{V(1)} < \beta < min\{\beta_U, \beta_{IV}^Z\}$
$\rho_{xu}, \rho_{zu} > 0$:	Case 2: $max\{\beta_L, \beta_{IV}^Z\} < \beta < \beta_{IV}^{V(1)} < \beta_{OLS}$

4. Conclusion and Future Work

In this paper, we contribute to the literature on bounds and imperfect instrumental variables. Our first contribution is to present the Principle of Intransitivity in Correlations, which follows from an under-used tool in the statistics literature. Application of this principle to Nevo and Rosen (2012) brings to light a previously unknown necessary condition for two cases of their bounds. Using this necessary condition, we show that a new set of bounds can be straightforwardly derived without any additional assumptions.

Our new bounds are more likely to dominate pre-existing bounds as stronger violations of the traditional assumptions of instrument validity and instrument

[2] β_U approaches a maximum bias of $\beta_{OLS} + \frac{\sigma_u}{\sigma_x}$.

[3] Simulations, using an estimate of σ_u from $\beta_{IV}^{V(1)}$, support these conclusions, with the bias of β_{IV}^Z becoming large relative to that of β_U when ρ_{xz} is small even when ρ_{zu} is not large. The result of Bound et al. (1995) regarding bias from weak instruments provides the intuition for this results.

relevance occur. These bounds improve the ability of a researcher to make inferences about the true value of a parameter when only very poor instruments are available. However, these bounds depend upon the value of an unknown parameter, σ_u. In this context, neither β_{OLS}, β_{IV}^{Z} nor $\beta_{IV}^{V(1)}$ are consistent estimators of β and thus do not provide consistent estimates of the residuals nor σ_u. This leaves a gap between theory and application, similar to the gap between Generalized Least Squares and Feasible Generalized Least Squares. Future work exploring estimation or bounding of σ_u in this framework is needed to allow for application of these new bounds. Application of these bounds would increase the usefulness of IV, an important tool for empirical research in many social science and applied statistics applications.

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