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Abstract

This paper develops a model of risky investment in education under disappointment aversion, modelled as loss aversion around one's endogenous expectation. The model shows that disappointment aversion reduces the optimal investment in education for lower ability people and increases it for higher ability people, thereby magnifying the investment gap between them generated by the riskiness of education. Policies aimed at influencing students' expectations can reduce early dropout.

JEL classification: D03; D81; I21.

Keywords: education; risk; disappointment aversion; endogeneous reference points.

1. Introduction

Leaving education too soon has negative consequences on future earnings and non-pecuniary dimensions such as crime, drug use, health and teenage pregnancy (Oreopoulos & Salvanes 2011). There is no single explanation for early dropout. Previous literature has explored the role played by inaccurate perception of returns to schooling (Jensen 2010), impatience (Cadena & Keys 2015) and other "behavioural barriers" (Lavecchia et al. 2014).

This paper considers another possible explanation for early drop out: potential disappointment at doing worse than expected. Evidence shows that people's decision to stay in school is affected by their expectations (Goux et al. 2014). It is reasonable to think that people who have lower expectations, such as less academically talented people, may leave school too soon because they are afraid not to reach their expected outcome. In this paper, we develop a model of risky investment in education under disappointment aversion, where disappointment aversion is defined as loss aversion around the agent's expected outcome, and the expected outcome depends on the investment level chosen by the agent and on her ability. The model is used to explore the impact of disappointment aversion on investment in education. Understanding the effects of

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people's reference points on their education choices is crucial in order to better design education policies.[1]

2. The model

Set-up There are two periods: the "present" or "youth", and the "future" or "adulthood". The individual devotes a fraction $\lambda \in [0,1]$ of the first period to investing in human capital (acquiring education). The remaining fraction of the first period, $1-\lambda$, is spent working for a wage $w_0 > 0$. The cost of investing in education is thus given by foregone earnings. All consumption occurs in the second period, hence the individual's earnings in youth are carried forward to adulthood in the form of savings, $s(\lambda) \equiv (1-\lambda) w_0$. For simplicity, we ignore interest on savings, hence one unit of savings carried forward increases adult consumption by one unit.

In the second period, the individual consumes; the investment in education is zero. Preferences over consumption, v, are strictly increasing, strictly concave, twice differentiable and the third derivative equals zero. The individual will obtain one of two potential wages $\{w_L, w_H\}$, where $w_H > w_L \geq w_0$. The probability of wage w_j, denoted $\pi^j(\lambda,\alpha) \equiv \Pr(w_j|\lambda,\alpha)$, depends on the individual's investment, λ, and on her academic ability, α, where $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. It is assumed that an increase in either education or ability increases the probability of a high wage but at a decreasing rate, and that education and ability are complements.

Assumption 1. $\pi^H(\lambda,\alpha)$ is a continuous function with $\pi^H_\lambda(\lambda,\alpha) > 0$, $\pi^H_{\lambda\lambda}(\lambda,\alpha) < 0$, $\pi^H_\alpha(\lambda,\alpha) > 0$, $\pi^H_{\alpha\alpha}(\lambda,\alpha) \leqslant 0$ and $\pi^H_{\lambda\alpha}(\lambda,\alpha) > 0$ for all α and λ.

The timing is as follows. In the first period, the individual learns her ability and chooses a level of investment in education λ. In the second period, her wage realization is revealed and she then chooses consumption.

Disappointment Aversion In disappointment aversion models (see Bell (1985), Loomes & Sugden (1986), Koszegi & Rabin (2006), Koszegi & Rabin (2007)), the agent is sensitive to deviations from what she expected to receive. In particular, she is loss averse around her expectation, which implies that losses relative to her expectation are more painful than equal-sized gains are pleasurable. We model disappointment aversion as loss aversion around the agent's expected utility, which depends on the investment level actually chosen by the agent. Hence, as in Koszegi & Rabin (2007), the expectation-based reference point is choice-acclimating: it adjusts to the agent's choice.

Let $\overline{v}$ denote the expected utility of an individual with ability α who has made the investment λ. Then,

$$\overline{v} \equiv \sum_{j=L,H} \pi^j(\lambda,\alpha)\, v\left(w_j + (1-\lambda) w_0\right). \tag{1}$$

[1] Lecouteux & Moulin (2015) show that, when students' reference points are given by their origins (i.e. whether they are from a poorer or wealthier background), the optimal policy implies lower tuition fees for poorer students.

The individual's reference point is given by (1). Disappointment aversion implies that the individual suffers a utility loss when failing to reach her expectation that is larger than the utility gain from exceeding her expectation. This can be formalised by assuming that the individual's "experienced" utility in the low wage and in the high wage states are $v^L + \psi^L \left(v^L - \overline{v}\right)$ and $v^H + \psi^H \left(v^H - \overline{v}\right)$ respectively, where $\psi^L \geq \psi^H \geq 0$, and v^L and v^H shorthand for the indirect utility at the low- and the high wage. Taking the expectation over the wage realization yields the individual's *ex-ante* utility inclusive of disappointment aversion,

$$V(\lambda, \alpha) = \sum_{j=L,H} \pi^j(\lambda, \alpha)\, v^j - \psi\sigma(\lambda, \alpha)\, \Delta v, \tag{2}$$

where $\psi \equiv \psi^L - \psi^H$, $\Delta v \equiv v^H - v^L$ and $\sigma(\lambda, \alpha) \equiv \pi^L(\lambda, \alpha)\, \pi^H(\lambda, \alpha)$.[2] Note that, from the properties of $\pi^H(\lambda, \alpha)$ and $v(\cdot)$, it follows that the ex ante utility function is strictly concave.

The second term, that captures disappointment aversion, has three components: (i) the strength of the aversion towards disappointment ψ; (ii) the utility gap between the good and the bad outcome Δv; (iii) the product of the probabilities $\sigma(\lambda, \alpha)$. For any α, $\sigma(\lambda, \alpha)$ is an increasing function of λ up to the point where $\pi^H(\lambda, \alpha) = 1/2$ and decreasing in λ thereafter. Note that with two potential wages, the wage variance is proportional to $\sigma(\lambda, \alpha)$. Hence, consistent with empirical evidence (Chen 2008)[3], the model features an inverted U-shaped relation between λ and wage risk at the individual level, although for low ability agents wage risk monotonically increases in investment in so far as for them $\pi^H(\lambda, \alpha) < 1/2$ for any λ.

3. Optimal investment

In youth the individual chooses $\lambda^*(\alpha)$ to maximize (2), leading to the first order condition

$$V_\lambda = \left(\pi^H_\lambda(\lambda^*(\alpha), \alpha) - \psi\sigma_\lambda(\lambda^*(\alpha), \alpha)\right)\Delta v - w_0\left\{E\left[v'\right] - \psi\sigma(\lambda^*(\alpha), \alpha)\,\Delta v'\right\} = 0. \tag{3}$$

The unique solution, denoted $\lambda^*(\alpha)$, is, by the theorem of the maximum, a continuous function of α. Due to the complementarity between investment and ability in the success probability, it is easy to see that $\lambda^*(\alpha)$ and $\pi^*(\alpha) \equiv \pi^H(\lambda^*(\alpha), \alpha)$ are strictly increasing in α.

It is assumed that the ability distribution is such that for some low types, getting a low wage is more likely than getting a high wage, and, vice versa, for some high types, getting a high wage is more likely than getting a low wage.

Assumption 2. $\pi^*(\underline{\alpha}) < 1/2 < \pi^*(\overline{\alpha})$.

[2] A natural restriction on ψ is that it does not exceed unity. This upper bound ensures that an agent with a sufficiently low success probability will always benefit from an increase in the success probability, i.e. will never prefer her success probability to be reduced even further.

[3] Chen (2008) shows that decision to enter college in particular is associated with an increase in the level of earning risk at the individual level.

Let $\widehat{\alpha}$ denote the ability type who chooses a level of education that induces equal probability of success and failure, i.e. $\pi^*(\widehat{\alpha}) = 1/2$. Assumption 2 implies that $\underline{\alpha} < \widehat{\alpha} < \overline{\alpha}$. Note that $\widehat{\alpha}$ is unique due to the monotonicity of the success probability in ability.

The rate of return on education is defined as the expected increase in adulthood earnings from a marginal increase in investment in education relative to the loss in youth earnings.

$$\rho^*(\alpha) \equiv \frac{\pi_\lambda^H(\lambda^*(\alpha), \alpha)\Delta w}{w_0}. \tag{4}$$

3.1. Benchmark case

Consider first the case with no disappointment aversion. The individual is choosing between two alternative investments: investment in human capital (education) and investment in physical capital (savings). Following Levhari & Weiss (1974), the former is risky while the latter is safe,[4] which causes the following deviation: $\rho^*(\alpha) > 1$ if a marginal increase in education increases wage risk and $\rho^*(\alpha) < 1$ otherwise. In particular, in our model the latter implies that $\rho^*(\alpha) > 1$ for lower ability types and $\rho^*(\alpha) < 1$ for higher ability types, as wage risk increases with investment up to the point where the success probability is 1/2 and decreases thereafter. Thus, lower ability types invest in education less than it would be socially optimal to, whereas higher ability types more. This result is stated by Proposition 1.

Proposition 1. *In the absence of disappointment aversion, $\rho^*(\alpha) > 1$ for any $\alpha < \widehat{\alpha}$ and $\rho^*(\alpha) \leq 1$ otherwise.*

3.2. Disappointment aversion

The introduction of disappointment aversion will differentially affect the privately optimal investments in education by low and high ability individuals. Low ability individuals respond by reducing their investments, while high ability individuals increase theirs. Proposition 2 demonstrates this result for the case of limited risk aversion. By limited risk aversion we mean that that the difference in marginal utility between the low and the high outcome, $\Delta v'$, is arbitrarily small.

Proposition 2. *Suppose that there is positive but limited risk aversion. Then there exists a unique type $\widetilde{\alpha} \in (\widehat{\alpha}, \overline{\alpha})$ such that disappointment aversion reduces thex investment in education for ability types below $\widetilde{\alpha}$ and increases it for ability types above $\widetilde{\alpha}$.*

The intuition is that less talented agents are discouraged from increasing their investment, as doing so would raise their expectations and hence the potential disappointment associated with a negative outcome. Conversely, more talented agents are encouraged to increase their investment

[4] Human capital is more risky than physical capital, as it cannot be bought, sold or separated from its owner, whereas physical capital allows for more diversification.

in order to secure a favourable outcome and avoid potential disappointment. Both higher and lower ability types respond to disappointment averse preferences by adjusting their investment towards certainty. It is important to point out that limited risk aversion is a sufficient, but far from necessary condition, as the example below will show.[5] Indeed, the example shows that, with an empirically relevant level of risk aversion (relative risk aversion lies between 0.5 and 1 over the relevant range of consumption), the deviation between the rate of return on education and unity in Proposition 1 is modest. Disappointment aversion, however, strongly reinforces the "Levhari-Weiss effect" in Proposition 1, significantly increasing the investment gap.

3.3. Numerical example

Let $w_0 = 0.5$, $w_L = 1$ and $w_H = 1.5$. Let the success probability be isoelastic in investment and multiplicative in ability – a common specification in the literature – $\pi^H(\lambda, \alpha) = \alpha\beta_0\lambda^{\beta_1}$, with $\beta_0 = 1.5$ and $\beta_1 = 0.5$. The ability space is $A = [0.5, 0.75]$. To be consistent with the assumption $v''' = 0$, assume a simple quadratic utility, $v(c) = 1.5c - 0.25c^2$, where c is consumption.[6] Figure 1 highlights the benchmark case using solid lines. The top panel shows how investment increases with ability. Ability types below $\widehat{\alpha} = 0.62$ have a success probability below 1/2. The bottom panel shows that the implicit rate of return on education, $\rho^*(\alpha)$, while strictly decreasing in ability, deviates from the return on savings (unity) by a modest degree for all ability types. Consider then the impact of disappointment aversion (here set at $\psi = 0.2$), as indicated by the hatched lines. Ability types below $\widetilde{\alpha} = 0.64$ reduce their investment relative to the benchmark case, while higher types increase their investment. The differential impact of disappointment by ability is reflected in the significantly more sloped gradient in the implicit rate of return on education, with lower ability types having $\rho^*(\alpha)$ well above unity and higher types having $\rho^*(\alpha)$ well below unity.

4. Discussion

While both lab and field evidence show that reference points are determined by expectations (Marzilli Ericson & Fuster (2011); Bartling et al. (2015)), as assumed in this paper, it is anyways interesting to note how our result would change under an alternative reference point considered in previous literature, the best outcome. If we assume the agent's reference point to be the highest wage, the discouragement effect described above no longer occurs: the optimal investment in education increases for any ability level. The intuition is that people of any ability level want to reduce the probability of a low wage (particularly higher ability people). Further research is

[5] Note that $\tilde{\alpha} > \widehat{\alpha}$ because, while for type $\widehat{\alpha}$ a marginal investment in education has no impact on wage risk, it increases the gap in utility between the high and the low states. In the limiting case where there is no risk aversion, $\tilde{\alpha}$ and $\widehat{\alpha}$ coincide.

[6] Being quadratic, $v(c)$ does not satisfy $v' > 0$ globally. However, it does within the relevant range.

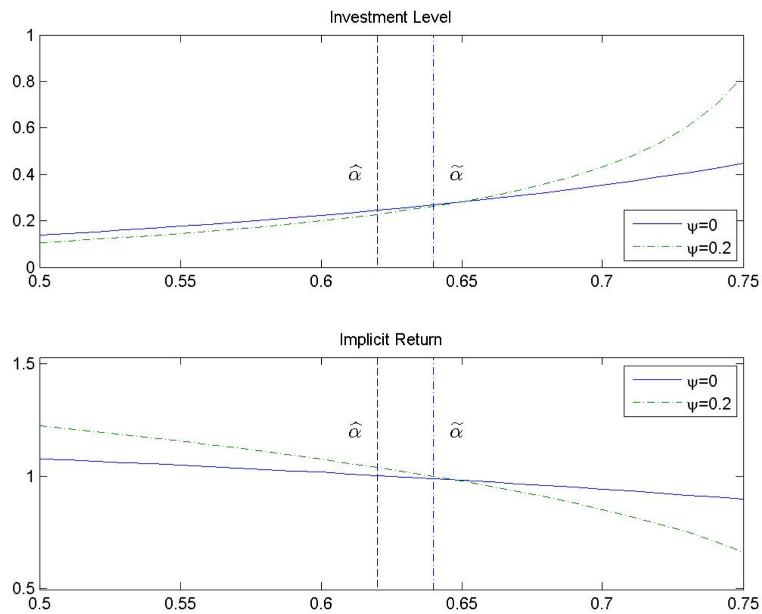

Figure 1: Investment and implicit return in the benchmark case ($\psi = 0$) and under disappointment ($\psi = 0.2$).

needed on how people's reference points are formed in the context of education decisions, and on how education policies can influence such reference points.

5. Conclusion

This paper shows that disappointment aversion reduces the optimal investment in education for lower ability agents and increases it for higher ability agents, thereby magnifying the educational investment gap between them.

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Appendix

A. Proofs

Proof of Proposition 1

When $\psi = 0$, the individual chooses $\lambda^*(\alpha)$ to maximize (1), leading to the first order condition

$$\pi_\lambda^H(\lambda^*(\alpha),\alpha)\Delta v = w_0 E\left[v'\right]. \tag{A1}$$

Using the assumption that $v''' = 0$ it is easy to show that

$$\Delta v = \frac{\left[v^{H\prime} + v^{L\prime}\right]}{2}\Delta w. \tag{A2}$$

Combining the above with the rate of return on educational investment in (4) yields

$$\rho^* = \frac{\pi_\lambda^H(\lambda^*(\alpha),\alpha)}{w_0}\frac{\Delta v}{\left[v^{H\prime} + v^{L\prime}\right]/2} = \frac{E\left[v'\right]}{\left[v^{H\prime} + v^{L\prime}\right]/2}, \tag{A3}$$

where the first equality uses (A2) and the second equality uses (A1). It follows that $\rho^* > 1$ if and only if $E\left[v'\right] > \left[v^{H\prime} + v^{L\prime}\right]/2$. Given that $v^{H\prime} < v^{L\prime}$, this will be the case for any agent for whom $\pi^H(\lambda^*(\alpha),\alpha) < 1/2$. Conversely, $\rho^* < 1$ for any agent for whom $\pi^H(\lambda^*(\alpha),\alpha) > 1/2$.

Proof of Proposition 2

By comparative statics, $\partial\lambda^*(\alpha)/\partial\psi$ has the same sign as the cross-partial derivative

$$V_{\lambda\psi} = -\sigma_\lambda\left(\lambda^*(\alpha),\alpha\right)\Delta v + w_0\sigma\left(\lambda^*(\alpha),\alpha\right)\Delta v'. \tag{A4}$$

Note first that $\Delta v' < 0$, whereby the second term is non-positive. Moreover, noting that

$$\sigma_\lambda(\lambda,\alpha) = \pi_\lambda^H(\lambda,\alpha)\left[1 - 2\pi^H(\lambda,\alpha)\right], \tag{A5}$$

it follows that $\sigma_\lambda\left(\lambda^*(\alpha),\alpha\right) > 0$ for any ability type $\alpha \leq \widehat{\alpha}$. Hence for all such low ability types, disappointment aversion strictly reduces the privately optimal investment. For any ability type $\alpha > \widehat{\alpha}$, $\sigma_\lambda\left(\lambda^*(\alpha),\alpha\right) < 0$, whereby $V_{\lambda\psi}$ may be positive.

Consider the case of limited risk aversion, that is where v' is close to constant. In the limiting case where there is no risk aversion, v' is constant and hence $\Delta v' = 0$. From (A4) we then see that, in the limit, $V_{\lambda\psi} = 0$ for type $\widehat{\alpha}$. Hence in the limiting case $\widetilde{\alpha} = \widehat{\alpha}$. Outside the limit, if $\Delta v'$ is negligible, it follows that $V_{\lambda\psi\alpha} = -\left(\sigma_{\lambda\lambda}\lambda' + \sigma_{\lambda\alpha}\right)\Delta v > 0$ for all types $\alpha > \widehat{\alpha}$, where we used that $\sigma_{\lambda\alpha} = \pi_{\alpha\lambda}^H\left(1 - 2\pi^H\right) - 2\pi_\lambda^H\pi_\alpha^H < 0$. This ensures that, for positive but limited risk aversion, $\widetilde{\alpha} > \widehat{\alpha}$ exists and is unique, and is close to $\widehat{\alpha}$.

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