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Fertility, Mortality and Environmental Policy

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ABSTRACT

Fertility, Mortality and Environmental Policy

This article examines pollution and environmental mortality in an economy where fertility is endogenous and output is produced from labor and capital by two sectors, dirty and clean. An emission tax curbs dirty production, which decreases pollution-induced mortality but also shifts resources to the clean sector. If the dirty sector is more capital intensive, then this shift increases labor demand and wages. This, in turn, raises the opportunity cost of rearing a child, thereby decreasing fertility and the population size. Correspondingly, if the clean sector is more capital intensive, then the emission tax decreases the wage and increases fertility. Although the proportion of the dirty sector in production falls, the expansion of population boosts total pollution, aggravating mortality.

JEL Classification: J13, Q56, Q58, O41

Keywords: environmental mortality, pollution tax, population growth, two-sector models

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1 Introduction

Can environmental policy boost population growth, thus making a successful wedding of high per capita consumption and clean environment ever harder for the generations to become? This paper shows that the the environmental policy, implemented by taxing the dirty sector, works in anti-natal direction only as long as the dirty sector is the more capital intensive one. Today, this is the case, mainly because productive capital exploits fossil fuels. But this may change! The novel clean-energy technologies, e.g. windmills and solar energy, utilize massive amounts of capital relative labor. Furthermore, many productive tasks of men may be replaced by clean robots and people may migrate to child rearing. Thus, the rise of the clean industry may generate an unintended natal side-effect. In this paper, we explore this possibility by a model of tree elements: the Stolper-Samuelson effect on resource allocation, endogenous fertility, and environmental mortality.

The Stolper-Samuelson effect (1941). Consider a two-sector economy that produces the final good according to two technologies (sectors), dirty and clean. A tax on the dirty sector curbs its production shifting capital to the clean one. If the dirty sector is capital intensive, then every shifted unit of capital demands more labor than before and wages rise. Correspondingly, if the clean sector is capital intensive, then a shifted unit of capital demands less labor and wages fall.

Endogenous fertility (Becker 1981). The demand for children depends on the opportunity cost of having a child. An emission tax shifts resources between the sectors generating changes in wages. If wages rise, then the opportunity cost of child increases and people move from child-rearing to the labor market so that fertility decreases. If, in contrast, wages fall, then fertility increases. Thus, emission taxes may have unintended demographic effects.

Environmental mortality (Lehmijoki 2013). There is more and more evidence on environmental degradation harming human health (Lehmijoki 2013). Following Lehmijoki and Rovenskaya (2010), we assume that mortality is an increasing function of total pollution which is a public good. For example, ambient air pollution harms all people equally. This provides the government every incentive to curb down dirty production by a pollution tax, but simultaneous changes in fertility may generate the opposite effect.

In the literature, De la Croix and Gosseries (2012) analyse the natalist bias of pollution control as follows. Polluting emissions can be decreased either by cutting down production per capita or by reducing the size of population. Emission taxes or quotas shift people from production to tax-free activities, such as reproduction. This deteriorates the environment even more, entailing the need to impose ever more stringent pollution rights per person. Consequently, future generations will face increasing population associated with decreasing production per capita.

Our model differs from de Croix and Gosseries (2012) in the following respects. First, they assume only one technology, but we two alternative technologies: dirty and clean. Consequently, emissions are proportional to total output in their model, but only to dirty output in our model. Second, they ignore physical capital and assume that spending on education accumulates human capital, while we assume that private saving accumulates aggregate capital (i.e. human and physical capital taken together). Thus, in our model, the allocation of capital and labor between the two sectors rather than aggregate capital plays an decisive role in pollution. Third, population growth decreases fertility through congestion in their model, but increases mortality through dirty output and pollution in our model.

The differences between the models lead to different policy recommendations for curbing pollution. De la Croix and Gosseries (2012) are pessimistic about the efficiency of taxation and quotas and suggest the use of population capping schemes in environmental policy. In contrast, we consider taxation as an efficient tool, but warn that its impact depends decisively on the intensity of the sectors: an emission tax alleviates pollution when the dirty sector, but aggravates that when the clean sector is more capital intensive.

The remainder of this article is organized as follows. Section 2 builds up a two-sector model of production and Section 3 focus on a family-optimization with endogenous fertility. Sections 4 and 5 construct the dynamics of the model and compare the traditional case where the dirty sector is more capital intensive with the modern case where the clean sector is more capital intensive. The results are summarized in Section 6.

2 The economy as a whole

Let N be population in the economy. The rate of population growth, $f - m$, is the difference between the fertility rate f and mortality rate m:

$$\frac{\dot{N}}{N} \doteq \frac{1}{N}\frac{dN}{dt} = f - m, \quad N(0) = N_0, \tag{1}$$

where t is time. We normalize the units so that it takes one unit of labor to rear one newborn. Then, total labor in child rearing is equal to the total number of newborns fN. Labor devoted to production, L, is equal to population N minus labor in child rearing, fN:

$$L = N - fN = (1 - f)N. \tag{2}$$

There is only one numeraire good that is consumed C and invested in capital. There are are two sectors that produce the good, clean and dirty. Clean output Y_c does not emit at all, but dirty output Y_d emits in one-to-one proportion. The government sets the tax x on emissions (the output of the dirty sector) Y_d and distributes the tax revenue xY_d to the families through a poll transfer s:

$$xY_d = sN. \tag{3}$$

Capital K and labor devoted to production, L, are transferable between the sectors:

$$L \geq L_c + L_d, \quad K \geq K_c + K_d, \tag{4}$$

where L_j and K_j are labor and capital in each sector $j \in \{c, d\}$, respectively. The production functions for sectors $j \in \{c, d\}$ are

$$\begin{aligned} &Y_j = F^j(K_j, L_j), \;\; F^j_K > 0, \;\; F^j_L > 0, \;\; F^j_{KK} < 0, \;\; F^j_{LL} < 0, \;\; F^j_{KL} > 0, \\ &F^j \text{ linearly homogeneous,} \end{aligned} \tag{5}$$

where the subscripts K and L denote the partial derivatives of the function F^j with respect to inputs K_j and L_j, correspondingly.

Let us denote the per capita terms

$$c \doteq C/N, \; k \doteq K/N \text{ and } k_i \doteq K_j/N \text{ and } l_i \doteq L_i j/N \text{ for } j \in \{c, d\}. \tag{6}$$

Then, dividing (2), (4) and (5) by population N yields

$$k \geq k_c + k_d, \quad 1 - f \geq l_c + l_d, \tag{7}$$

$$y_c \doteq Y_c/N = F^c(k_c, l_c), \;\; y_d \doteq Y_d/N = F^d(k_d, l_d). \tag{8}$$

3 The representative family

The representative family takes the mortality rate m, the tax x and the poll transfer s as given. Because it can save only in capital K, which is the only asset in the model, its budget is given by

$$\dot{K} \doteq \frac{dK}{dt} = Y_c + Y_d + sN - xY_d - C - \delta K, \quad K(0) = K_0, \quad \delta > 0, \tag{9}$$

where $Y_c + Y_d$ is factor revenue from production, sN poll transfers, xY_d, emission taxes, C consumption and δ the depreciation rate of capital. Noting (1), (5), (6), (8) and (13), the constraint (9) can be written in terms of capital per head, $k = K/N$, as follows:

$$\dot{k} = \frac{\dot{K}}{N} - \frac{K}{N}\frac{\dot{N}}{N} = F^c(k_c, l_c) + (1 - x)F^d(k_d, l_d) + s - c + (m - f - \delta)k,$$
$$k(0) = k_0. \tag{10}$$

Following Razin and Ben-Zion (1975) and Becker (1981), we assume that at each time t, a representative family derives temporary utility $u(t)$ from per capita consumption $c(t) \doteq C(t)/N(t)$ and fertility $f(t)$ so that these are substitutes:

$$u(t) \doteq c(t) + \psi\big(f(t)\big), \quad \psi' > 0, \quad \psi'' < 0. \tag{11}$$

Let the constant ρ be the family's rate of time preference given that the family could live forever. The adult's probability of dying in a short time dt is equal to $m\,dt$, where m is the adult's mortality rate. Thus, e^{-mt} is the probability that an individual in the family will survive beyond the period $[0, t]$, and $e^{-mt}u(t)$ is the family's expected temporary utility at time t. The family's expected utility for the planning period $t \in [0, \infty)$ is then [cf. (11)]

$$U \doteq \int_0^\infty \log\big[c(t) + \psi\big(f(t)\big)\big]e^{-(m+\rho)t}dt, \quad \psi' > 0, \quad \psi'' < 0. \tag{12}$$

The representative family maximizes its utility (12) by its per capita consumption c and the allocation of labor and capital, (l_c, l_d, f, k_c, k_d), subject to the accumulation of capital, (10), and the resource constraints (7), taking the mortality rate m, the tax x and the poll transfer s as given. This maximization can be presented in two stages. First, the family maximizes per capital output y by the allocation of labor and capital (l_c, l_d, f, k_c, k_d) subject to the resource constraints (7), given the fertility rate f. This leads to the function (cf. Appendix A)

$$\begin{aligned}
&y(k,f,x) \doteq \max_{(l_c,l_d,k_c,k_d)\text{ s.t. (7)}} [F^c(k_d,l_d) + (1-x)F^d(k_d,l_d)] \text{ with} \\
&\frac{\partial y}{\partial k}(x) > 0, \ \frac{\partial y}{\partial f}(x) < 0, \ \frac{\partial y}{\partial x}(k,f,x) = -y_d < 0, \ \frac{\partial y_d}{\partial x} < 0 \text{ and} \\
&\frac{\partial y_d}{\partial k}(x) > 0 \ \Leftrightarrow \ \frac{\partial y_d}{\partial k}(x) = \frac{l_c}{k_c}\frac{\partial y_d}{\partial f}(x) > 0 \ \Leftrightarrow \ \frac{k_d}{l_d} > \frac{k_c}{l_c},
\end{aligned} \tag{13}$$

Thus, production y_d and the fertility rate f are then positively correlated if and only if the dirty sector is more capital intensive:

$$\frac{\partial y_d}{\partial f}(x) > 0 \ \Leftrightarrow \ \frac{k_d}{l_d} > \frac{k_c}{l_c}.$$

If the dirty sector is capital intensive, then a transfer of one unit of capital from the clean sector to it decreases labor demand and wages. This lowers the opportunity cost of child rearing and promotes fertility. Correspondingly, with a capital intensive clean sector, that transfer increases labor demand and wages, raising the opportunity cost of child rearing and hampering fertility.

In the second stage, the family maximizes the real-valued Hamiltonian

$$\mathcal{H} \doteq \log\big[c + \psi(f)\big] + \phi[y(k,f,x) + s - c + (m - f - \delta)k], \tag{14}$$

by per capita consumption c and the fertility rate f, where the co-state variable $\phi(t)$ evolves according to

$$\begin{aligned}
&\dot{\phi} \doteq \frac{d\phi}{dt} = (\rho + m)\phi - \frac{\partial \mathcal{H}}{\partial k} = \left[\rho + \delta + f - \frac{\partial y}{\partial k}(x)\right]\phi, \\
&\lim_{t\to\infty} \phi(t)k(t)e^{-(m+\rho)t} = 0.
\end{aligned} \tag{15}$$

The first-order conditions for the maximization of (14) by c and f are

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{1}{c+\psi(f)} - \phi = 0, \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial f} = \frac{\psi'(f)}{c+\psi(f)} + \phi\left[\frac{\partial y}{\partial f}(x) - k\right] = \phi\left[\psi'(f) + \frac{\partial y}{\partial f}(x) - k\right] = 0. \tag{17}$$

Given these conditions, the fertility rate is a function of capital per person, k, and the tax rate x as follows [cf. (13)]:

$$f(k,x) \doteq (\psi')^{-1}\left(k - \frac{\partial y}{\partial f}(x)\right)_{,} \quad f_k \doteq \frac{\partial f}{\partial k} = \frac{1}{\psi''} < 0,$$

$$f_x \doteq \frac{\partial f}{\partial x} = -\frac{1}{\psi''}\frac{\partial^2 y}{\partial f \partial x} = \frac{1}{\psi''}\frac{\partial y_d}{\partial f} = f_k \frac{k_c}{l_c}\frac{\partial y_d}{\partial k} < 0 \quad \Leftrightarrow \quad \frac{k_d}{l_d} > \frac{k_c}{l_c}, \tag{18}$$

where $(\psi')^{-1}$ is the inverse of the function ψ'. From (16) and (18) it follows that consumption per capita is determined by

$$c = 1/\phi - \psi\big(f(k,x)\big). \tag{19}$$

Result (18) can be explained as follows. An increase in capital per head, k, increases the marginal product of labor and the wage. A tax on the dirty sector transfers resources from it to the clean sector. If the dirty sector is more capital intensive, then this increases labor demand and the wage. With a higher wage, people move from child rearing to production, which discourages fertility.

4 Dynamics

We assume that the mortality rate m is an increasing function of total pollution P which is a public good (e.g. air pollution, cf. Lehmijoki 2013):

$$m(P), \quad m' > 0. \tag{20}$$

Emissions Y_d generate pollution P according to [cf. (8), (13) and (18)]

$$\dot{P} \doteq \frac{dP}{dt} = Y_d - \omega P = y_d\big(k, f(k,x), x\big)N - \omega P, \quad 0 < \omega < 1, \quad P(0) = P_0, \tag{21}$$

where the constant ω is the proportional absorbtion of pollution by nature. Noting (18) and (20), the rate of population growth, (1), becomes

$$\frac{\dot{N}}{N} = f(k,x) - m(P), \quad N(0) = N_0. \tag{22}$$

Inserting the income function (13), the fertility function (18), per capita consumption (19), the mortality function (20) and the government budget constraint (3) (as $s = xy_d(k,f,x) = xy_d\big(k, f(k,x), x\big)$) back into (10), we obtain the evolution of capital per head as follows:

$$\dot{k} = y\big(k, f(k,x), x\big) + xy_d\big(k, f(k,x), x\big) - 1/\phi + \psi\big(f(k,x)\big) + (m - f - \delta)k,$$
$$k(0) = k_0. \tag{23}$$

The dynamics of the economy is dictated by four differential equations: the evolution of pollution (21), population (22) and capital per head, (23), and the behavior of the co-state variable (15). This system has pollution P, population N and capital per head, k, as predetermined variables and the co-state variable ϕ as a jump variable. In Appendix B, we show that with a small tax $x > 0$ (or a small subsidy $-x > 0$), this system has three stable roots and one unstable root. Thus, there is a saddle point solution and a unique steady-state equilibrium.

5 The effect of the emission tax x

We denote the steady-state value of a variable by superscript (*). In the steady state, the state variables – population N, pollution P and capital per head, k – must be constants. Given the system (15), (16), (18), (19), (21), (22) and (23), we obtain that also the co-state variable ϕ is constant and

$$\begin{aligned}
&\dot{N} = \dot{\phi} = 0 \quad \Leftrightarrow \quad m(P^*) = f(k^*, x^*) = f^* = \frac{\partial y}{\partial k}(x) - \rho - \delta,\\
&\dot{P} = 0 \quad \Leftrightarrow \quad y_d(k^*, f^*, x)N^* = \omega P^*,\\
&\dot{k} = 0 \quad \Leftrightarrow \quad 1/\phi^* - \psi(f^*) = c^* = y(k^*, f^*, x) + xy_d\big(k^*, f^*, x\big) - \delta k^*.
\end{aligned} \tag{24}$$

Differentiating equations (24) with respect to x and noting (12) and (13) yield (cf. Appendix C)

$$\frac{dm^*}{dx} = \frac{df^*}{dx} < 0 \;\Leftrightarrow\; \frac{dP^*}{dx} < 0 \;\Leftrightarrow\; \frac{k_d}{l_d} > \frac{k_c}{l_c}, \tag{25}$$

$$\frac{dN^*}{dx} < 0 \;\Leftrightarrow\; \frac{k_c}{l_c} < \frac{1}{2}\left[\left(\underbrace{\frac{\omega}{N^*}}_{+}\underbrace{\frac{1}{m'}}_{+}\frac{\partial y_d}{\partial k} + \underbrace{\frac{\partial y_d}{\partial x}}_{-}\right)\underbrace{\left(\frac{\partial y_d}{\partial k}\right)^{-2}}_{+} - \underbrace{\frac{1}{f_k}}_{-}\right]. \tag{26}$$

These results can be rephrased as follows:

Proposition 1 *If the dirty sector is more capital intensive (i.e. the clean sector more labor intensive), then an emission tax x decreases the fertility rate f^*, the mortality rate m^* and pollution P^*. Furthermore, if the clean sector is labor intensive enough (i.e. k_c/l_c small enough), then the emission tax x decreases total population (i.e. $\frac{dN^*}{dx} < 0$).*

The emission tax shifts resources from the dirty to the clean sector. If the dirty sector is more capital intensive, then the shift of resources increases the demand for labor, raising the wage and encouraging labor to migrate from child rearing to production. Thus, the fertility rate decreases, pollution diminishes and the mortality rate falls. If the clean sector is labor intensive enough (i.e. k_c/l_c small enough), then labor demand and the wages rise substantially, generating an extensive migration from child rearing to production. This decreases the size of population, further alleviating pollution.

Reversing the signs in (25) and (26) provides the following:

$$\frac{dm^*}{dx} = \frac{df^*}{dx} > 0 \;\Leftrightarrow\; \frac{dP^*}{dx} > 0 \;\Leftrightarrow\; \frac{k_d}{l_d} < \frac{k_c}{l_c},$$

$$\frac{dN^*}{dx} > 0 \;\Leftrightarrow\; \frac{k_c}{l_c} > \frac{1}{2}\left[\left(\underbrace{\frac{\omega}{N^*}}_{+}\underbrace{\frac{1}{m'}}_{+}\frac{\partial y_d}{\partial k} + \underbrace{\frac{\partial y_d}{\partial x}}_{-}\right)\underbrace{\left(\frac{\partial y_d}{\partial k}\right)^{-2}}_{+} - \underbrace{\frac{1}{f_k}}_{-}\right].$$

Proposition 2 *If the clean sector is more capital intensive, then an emission tax x increases the fertility rate f^*, the mortality rate m^* and pollution P^*. Furthermore, if the clean sector is capital intensive enough (i.e. k_c/l_c big enough), then the emission tax x increases total population (i.e. $\frac{dN^*}{dx} > 0$).*

As before, the emission tax shifts resources to the clean sector. If this sector is more capital intensive, then the demand for labor decreases, the wage falls and labor migrates from production to child rearing. Fertility increases, pollution aggravates and mortality rises. If the clean sector is capital intensive enough (i.e. k_c/l_c big enough), then labor demand and the wages fall drastically, generating an extensive flow from production to child rearing. This increases population, further aggravating pollution.

6 Discussion

In this article, we examine pollution-induced mortality when fertility is endogenous and output is produced from labor and capital by two sectors, dirty and clean. The sectors differ in capital intensity. In this setup, the emission tax raises the relative price of dirty technology. If, as usually assumed, the dirty sector is more capital intensive than the clean sector, then the emission tax curbs dirty production, hampers pollution and transfers resources from the capital-intensive dirty to the labor-intensive clean sector. This increases labor demand and wages, encouraging labor to migrate from child rearing to production. Consequently, the fertility rate decreases, population contracts, total pollution alleviates and the mortality rate falls. If, in contrast, the clean sector is more capital intensive, then the emission tax transfers resources to the capital-intensive clean sector, decreasing labor demand and wages, and attracting more people to child rearing. Thus, fertility increases, population expands, total pollution culminates, raising the mortality rate.

While a great deal of caution should be exercised when a highly stylized family-optimization model with two production sectors is used to explain the relationship of pollution, fertility, mortality and the accumulation of capital, the following judgement nevertheless seems to be justified. So far, the dirty sector has been more capital intensive than the clean sector. However, there are signs that the relative factor intensity may reverse in future. Many emerging clean technologies demand substantially capital, but only marginally labor. In that case, an emission tax that reallocates factors of production from the dirty to the clean sector makes people redundant in production. If those people change into reproduction, then the expansion of the clean sector may generate unintended natal side-effects that jeopardize the targets of environmental policy.

Appendix

A Results (13)

We obtain

$$
\begin{aligned}
y &= \max_{\substack{l_c,\, k_c,\, l_d,\, k_d \\ \text{s.t. (8)}}} [y_c + (1-x)y_d] = \max_{\substack{l_c,\, k_c,\, l_d,\, k_d \\ \text{s.t. (8)}}} [F^c(k_c, l_c) + (1-x)F^d(k_d, l_d)] \\
&= \max_{l_c, k_c, l_d, k_d} [F^c(k - k_d, n - l_d) + (1-x)F^d(k_d, l_d)].
\end{aligned}
$$

Because the production functions F^d and F^c are linearly homogeneous [cf. (5)], this implies

$$
\begin{aligned}
&F^d_L(1, \ell_d) = (1-x)F^c_L(1, \ell_c), \quad F^d_K(1, \ell_d) = (1-x)F^c_K(1, \ell_c), \\
&\ell_c \doteq l_c/k_c, \quad \ell_c \doteq l_c/k_c. \qquad (27)
\end{aligned}
$$

The zero homogeneity of the functions F^d_L and F^c_L yields $F^d_{KL} + F^d_{LL} d\ell_d = 0$, $F^c_{KL} + F^c_{LL} d\ell_d = 0$ and

$$
\ell_d = -\frac{F^d_{KL}}{F^d_{LL}}, \quad \ell_c = -\frac{F^c_{KL}}{F^c_{LL}}. \qquad (28)
$$

Differentiating the equations in (27) totally, we obtain

$$
F^d_{LL} d\ell_d = F^c_{LL} d\ell_c + F^c_L dp, \quad F^d_{KL} d\ell_d = F^c_{KL} d\ell_c + F^c_K dp.
$$

These can be written in the matrix form

$$
\begin{bmatrix} F^d_{LL} & -(1-x)F^c_{LL} \\ F^d_{KL} & -(1-x)F^c_{KL} \end{bmatrix} \begin{bmatrix} d\ell_d \\ d\ell_c \end{bmatrix} + \begin{bmatrix} F^c_L \\ F^c_K \end{bmatrix} dx = 0. \qquad (29)
$$

Noting (5) and (28), we obtain the Jacobian of this system as follows:

$$
\begin{aligned}
J &\doteq \begin{vmatrix} F^d_{LL} & -(1-x)F^c_{LL} \\ F^d_{KL} & -(1-x)F^c_{KL} \end{vmatrix} = p[F^d_{KL}F^c_{LL} - F^d_{LL}F^c_{KL}] \\
&= F^c_{LL}F^d_{LL}\left(\frac{F^d_{KL}}{F^d_{LL}} - \frac{F^c_{KL}}{F^c_{LL}}\right) = \underbrace{(1-x)}_{+}\underbrace{F^c_{LL}}_{-}\underbrace{F^d_{LL}}_{-}(-\ell_d + \ell_c) < 0 \\
&\Leftrightarrow \quad l_c/k_c = \ell_c < \ell_d = l_d/k_d. \qquad (30)
\end{aligned}
$$

Given this, (5) and (29), we obtain the functions

$$\ell_d(x), \quad \ell_c(x), \tag{31}$$

with derivatives

$$\frac{d\ell_d}{dx} = -\frac{1}{J}\begin{vmatrix} F_L^c & -F_{LL}^c \\ F_K^c & -F_{KL}^c \end{vmatrix} = -\frac{1}{J}(\underbrace{F_K^c}_{+}\underbrace{F_{LL}^c}_{-} - \underbrace{F_L^c}_{+}\underbrace{F_{KL}^c}_{+}) < 0 \;\Leftrightarrow$$

$$\frac{d\ell_c}{dx} = -\frac{1}{J}\begin{vmatrix} F_{LL}^d & F_L^c \\ F_{KL}^d & F_K^c \end{vmatrix} = -\frac{1}{J}(\underbrace{F_K^c}_{+}\underbrace{F_{LL}^d}_{-} - \underbrace{F_L^c}_{+}\underbrace{F_{KL}^d}_{+}) < 0$$

$$\Leftrightarrow \quad J < 0 \quad \Leftrightarrow \quad l_c/k_c < l_d/k_d.$$

From (8) it follows that

$$k_c = k - k_d, \quad 1 - f = l_d + l_c = \ell_d k_d + \ell_c k_c = (\ell_d - \ell_c)k_d + \ell_c k.$$

Solving for k_d and k_d yields

$$k_d = \frac{1 - f - \ell_c k}{\ell_d - \ell_c}, \quad k_c = k - \frac{1 - f - \ell_c k}{\ell_d - \ell_c} = \frac{\ell_d k - 1 + f}{\ell_d - \ell_c}.$$

Noting this, (5) and (31), we obtain

$$y_d(k, f, x) \doteq F^d(1, \ell_d)k_d = F^d\big(1, \ell_d(x)\big)\frac{1 - f - \ell_c(x)k}{\ell_d(x) - \ell_c(x)},$$

$$y_c(k, f, x) \doteq F^c(1, \ell_c)k_c = F^c\big(1, \ell_c(x)\big)\frac{\ell_d(x)k - 1 + f}{\ell_d(x) - \ell_c(x)},$$

for which [cf. (30)]

$$\frac{\partial y_d}{\partial f} = \frac{F^d\big(1, \ell_d(x)\big)}{\ell_c(x) - \ell_d(x)} < 0 \;\Leftrightarrow\; \frac{\partial y_d}{\partial k} = \frac{F^d\big(1, \ell_d(x)\big)\ell_c(x)}{\ell_c(x) - \ell_d(x)} = \frac{l_c}{k_c}\frac{\partial y_d}{\partial f} < 0 \;\Leftrightarrow$$

$$\frac{\partial y_c}{\partial f} = \frac{F^c\big(1, \ell_c(x)\big)}{\ell_d(x) - \ell_c(x)} > 0 \;\Leftrightarrow\; \frac{\partial y_c}{\partial k} = \frac{F^c\big(1, \ell_c(x)\big)\ell_d(x)}{\ell_d(x) - \ell_c(x)} = \frac{l_d}{k_d}\frac{\partial y_c}{\partial f} > 0 \;\Leftrightarrow$$

$$\frac{l_c}{k_c} = \ell_c < \ell_d = \frac{l_d}{k_d}; \quad \frac{\partial^2 y_d}{\partial k^2} = \frac{\partial^2 y_d}{\partial n^2} = \frac{\partial^2 y_d}{\partial k \partial n} = 0. \tag{32}$$

B Saddle-point solution

Given (17), the system (15), (21), (22) and (23) can be written as follows:

$$
\begin{aligned}
\frac{\dot{\phi}}{\phi} &= \rho + \delta + f(k,x) - \frac{\partial y}{\partial k}(x), \quad \dot{P} = y_d\big(k, f(k,x), x\big)N - \omega P, \\
\frac{\dot{N}}{N} &= f(k,x) - m(P), \\
\dot{k} &= y\big(k, f(k,x), x\big) + x y_d\big(k, f(k,x), x\big) - 1/\phi + \psi(f(k,x)) \\
&\qquad + [m - f(k,x) - \delta]k \ \text{ with} \\
\frac{\partial \dot{k}}{\partial f} &= \underbrace{\frac{\partial y}{\partial f} + \psi' - k}_{=0} + x\frac{\partial y_d}{\partial f} = x\frac{\partial y_d}{\partial f}.
\end{aligned}
$$

We linearize this system in the neighborhood of the steady state, where $\dot{\phi} = \dot{P} = \dot{N} = \dot{k} = 0$:

$$
\begin{bmatrix}
0 & 0 & 0 & f_k \\
0 & -\omega & y_d & \big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big)N \\
0 & -m' & 0 & f_k \\
\phi^{-2} & 0 & 0 & \frac{\partial y}{\partial k} + x\big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big) - \delta
\end{bmatrix}
\begin{bmatrix} d\phi \\ dP \\ dN \\ dk \end{bmatrix} = 0.
$$

The characteristic equation of this system is

$$
\begin{aligned}
&\begin{vmatrix}
-\lambda & 0 & 0 & f_k \\
0 & -\omega - \lambda & y_d & \big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big)N \\
0 & -m' & -\lambda & f_k \\
\phi^{-2} & 0 & 0 & \frac{\partial y}{\partial k} + x\big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big) - \delta - \lambda
\end{vmatrix} \\
&= -\lambda \begin{vmatrix}
-\omega - \lambda & y_d & \big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big)N \\
-m' & -\lambda & f_k \\
0 & 0 & \frac{\partial y}{\partial k} + x\big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big) - \delta - \lambda
\end{vmatrix} \\
&\quad - \frac{1}{\phi^2} \begin{vmatrix}
0 & 0 & f_k \\
-\omega - \lambda & y_d & \big(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\big)N \\
-m' & -\lambda & f_k
\end{vmatrix} \\
&= -\lambda \left[\frac{\partial y}{\partial k} + x\left(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k\right) - \delta - \lambda\right] \begin{vmatrix} -\omega - \lambda & y_d \\ -m' & -\lambda \end{vmatrix} \\
&\quad - \frac{f_k}{\phi^2} \begin{vmatrix} -\omega - \lambda & y_d \\ -m' & -\lambda \end{vmatrix}
\end{aligned}
$$

$$= \left\{ \lambda^2 - \left[\frac{\partial y}{\partial k} + x\left(\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial f} f_k \right) - \delta \right] \lambda - \frac{f_k}{\phi^2} \right\} (\lambda^2 + \omega\lambda + m' y_d) = 0.$$

This yields two polynomes. The first of these is $\lambda^2 + \omega\lambda + m' y_d = 0$, which has roots

$$\lambda_{1,2} = -\frac{\omega}{2} \pm \sqrt{\omega^2 - 4m' y_d}.$$

Because $4m' y_d > 0$ [cf. (20)], there are two stable roots λ_1 and λ_2.

In the neighborhood of no taxation $x = 0$, the second polynome is

$$\lambda^2 - \left(\frac{\partial y}{\partial k} - \delta \right) \lambda - \frac{f_k}{\phi^2} = 0.$$

This has one stable and one unstable root:

$$\begin{aligned}
\lambda_3 &= \frac{1}{2}\left(\frac{\partial y}{\partial k} - \delta \right) + \frac{1}{2}\sqrt{\left(\frac{\partial y}{\partial k} - \delta \right)^2 + \underbrace{\frac{4}{\phi^2}}_{+} \underbrace{f_k}_{-}} \\
&< \frac{1}{2}\left(\frac{\partial y}{\partial k} - \delta \right) + \frac{1}{2}\sqrt{\left(\frac{\partial y}{\partial k} - \delta \right)^2} = 0, \\
\lambda_4 &= \frac{1}{2}\left(\frac{\partial y}{\partial k} - \delta \right) - \frac{1}{2}\sqrt{\left(\frac{\partial y}{\partial k} - \delta \right)^2 + \underbrace{\frac{4}{\phi^2}}_{+} \underbrace{f_k}_{-}} \\
&> \frac{1}{2}\left(\frac{\partial y}{\partial k} - \delta \right) - \frac{1}{2}\sqrt{\left(\frac{\partial y}{\partial k} - \delta \right)^2} = 0.
\end{aligned}$$

C Proof of proposition 1

Differentiating equations (24) with respect to x and noting (12), (13) and (18) yield

$$\begin{aligned}
& f_k \frac{dk^*}{dx} + f_x = \frac{df^*}{dx} = \frac{\partial^2 y}{\partial k \partial x} = -\frac{\partial y_d}{\partial k} < 0 \;\; \Leftrightarrow \;\; \frac{dP^*}{dx} = \underbrace{\frac{1}{m'}}_{+} \frac{df^*}{dx} < 0 \\
& \Leftrightarrow \;\; \frac{k_d}{l_d} > \frac{k_c}{l_c}, \\
& \frac{dk^*}{dx} = -\frac{f_x}{f_k} - \frac{1}{f_k}\frac{\partial y_d}{\partial k} = -\frac{k_c}{l_c}\frac{\partial y_d}{\partial k} - \frac{1}{f_k}\frac{\partial y_d}{\partial k} = -\frac{\partial y_d}{\partial k}\left(\underbrace{\frac{k_c}{l_c}}_{+} + \underbrace{\frac{1}{f_k}}_{-} \right),
\end{aligned}$$

$$
\begin{aligned}
\frac{dN^*}{dx} &= \frac{1}{y_d}\left[\omega\frac{dP^*}{dx} - N^*\left(\frac{\partial y_d}{\partial k}\frac{dk^*}{dx} + \frac{\partial y_d}{\partial f}\frac{df^*}{dx} + \frac{\partial y_d}{\partial x}\right)\right] \\
&= \frac{N^*}{y_d}\left[\frac{\omega}{N^*}\frac{dP^*}{dx} - \frac{\partial y_d}{\partial k}\frac{dk^*}{dx} - \frac{\partial y_d}{\partial f}\frac{df^*}{dx} - \frac{\partial y_d}{\partial x}\right] \\
&= \frac{N^*}{y_d}\left[\frac{\omega}{N^*}\frac{1}{m'}\frac{df^*}{dx} + \left(\frac{\partial y_d}{\partial k}\right)^2\left(\frac{k_c}{l_c} + \frac{1}{f_k}\right) - \frac{k_c}{l_c}\frac{\partial y_d}{\partial k}\frac{df^*}{dx} - \frac{\partial y_d}{\partial x}\right] \\
&= \frac{N^*}{y_d}\left[-\frac{\omega}{N^*}\frac{1}{m'}\frac{\partial y_d}{\partial k} + \left(\frac{\partial y_d}{\partial k}\right)^2\left(\frac{k_c}{l_c} + \frac{1}{f_k}\right) + \frac{k_c}{l_c}\left(\frac{\partial y_d}{\partial k}\right)^2 - \frac{\partial y_d}{\partial x}\right] \\
&= \underbrace{\frac{N^*}{y_d}}_{+}\left[\left(\frac{\partial y_d}{\partial k}\right)^2\left(2\frac{k_c}{l_c} + \frac{1}{f_k}\right) - \frac{\omega}{N^*}\frac{1}{m'}\frac{\partial y_d}{\partial k} - \frac{\partial y_d}{\partial x}\right] < 0 \\
\Leftrightarrow &\quad \left(\frac{\partial y_d}{\partial k}\right)^2\left(2\frac{k_c}{l_c} + \frac{1}{f_k}\right) < \frac{\omega}{N^*}\frac{1}{m'}\frac{\partial y_d}{\partial k} + \frac{\partial y_d}{\partial x} \\
\Leftrightarrow &\quad \frac{k_c}{l_c} < \frac{1}{2}\left[\left(\underbrace{\frac{\omega}{N^*}}_{+}\underbrace{\frac{1}{m'}}_{+}\frac{\partial y_d}{\partial k} + \underbrace{\frac{\partial y_d}{\partial x}}_{-}\right)\underbrace{\left(\frac{\partial y_d}{\partial k}\right)^{-2}}_{+} - \underbrace{\frac{1}{f_k}}_{-}\right].
\end{aligned}
$$

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References:

Arrow, K., Bolin, B., Costanza, R., Dasgupta, P., Folke, K., Holling, C.S., Jansson, B.O., Levin, S., Mäler, K.G., Perrings, C., Pimentel, D., 1995. Economic Growth, Carrying Capacity, and the Environment. Ecological Economics 15, 91–95.

Becker, G.S., 1981. A Treatise on the Family. Harvard University Press, Cambridge, Massachusetts.

Cole, M.A., Elliot, R.J.R., 2005. FDI and the Capital Intensity of "Dirty" Sectors: A Missing Piece of the Pollution Havens Puzzle. Review of Development Economics 9, 530-548.

De la Croix, D., Grosseries, A. 2012. The Natalist Bias of Pollution Control. Journal of Environmental Economics and Management 63, 271–287.

Grossman, G.M., Krueger, A.B., 1995. Economic Growth and the Environment. Quarterly Journal of Economics 110, 9353–9377.

Lehmijoki, U 2013 Optimal Pollution, Optimal Population, and Sustainability, in: Green Growth, and Sustainable Development. Edited by J. Crespo Cuaresma, T. Palokangas and A. Tarasyev. Heidelberg: Springer Verlag 2013.

Lehmijoki, U., Palokangas, T., 2009. Population Growth Overshooting and Trade in Developing Countries. Journal of Population Economics 22, 43–56.

Lehmijoki, U., Rovenskaya, E., 2010. Environmental Mortality and Long-Run Growth, in: Cuaresma, Jesus Crespo, Palokangas, Tapio, Taraseyev, Alexander (eds.): Dynamic Systems, Economic Growth, and the Environment, 239–258. Springer, Heidelberg.

Razin, A., Ben-Zion, U., 1975. An Intergenerational Model of Population Growth. American Economic Review 65, 923–933.

Stolper, W.F., Samuelson, P.A., 1941. Protection and Real Wages. Review of Economic Studies 9(1), 58–73.