I Z A Institute
of Labor Economics
Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 10402

Missing Data, Imputation, and Endogeneity

Ian K. McDonough
Daniel L. Millimet

DECEMBER 2016

DISCUSSION PAPER SERIES

IZA DP No. 10402

Missing Data, Imputation, and Endogeneity

Ian K. McDonough
University of Nevada, Las Vegas

Daniel L. Millimet
Southern Methodist University and IZA

DECEMBER 2016

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Missing Data, Imputation, and Endogeneity*

Basmann (Basmann, R.L., 1957, A generalized classical method of linear estimation of coefficients in a structural equation. *Econometrica* 25, 77-83; Basmann, R.L., 1959, The computation of generalized classical estimates of coefficients in a structural equation. *Econometrica* 27, 72-81) introduced two-stage least squares (2SLS). In subsequent work, Basmann (Basmann, R.L., F.L. Brown, W.S. Dawes and G.K. Schoepfle, 1971, Exact finite sample density functions of GCL estimators of structural coefficients in a leading exactly identifiable case. *Journal of the American Statistical Association* 66, 122-126) investigated its finite sample performance. Here, we build on this tradition focusing on the issue of 2SLS estimation of a structural model when data on the endogenous covariate is missing for some observations. Many such imputation techniques have been proposed in the literature. However, there is little guidance available for choosing among existing techniques, particularly when the covariate being imputed is endogenous. Moreover, because the finite sample bias of 2SLS is not monotonically decreasing in the degree of measurement accuracy, the most accurate imputation method is not necessarily the method that minimizes the bias of 2SLS. Instead, we explore imputation methods designed to increase the first-stage strength of the instrument(s), even if such methods entail lower imputation accuracy. We do so via simulations as well as with an application related to the medium-run effects of birth weight.

JEL Classification: C36, C51, J13

Keywords: imputation, missing data, instrumental variables, birth weight, childhood development

Corresponding author:
Daniel L. Millimet
Department of Economics
Southern Methodist University
Box 0496
Dallas, TX 75275-0496
USA
E-mail: millimet@smu.edu

* Authors are grateful for helpful comments from two anonymous referees, Charles Courtemanche, and conference participants at the American Society of Health Economists Sixth Biennial Conference.

1 Introduction

Basmann (1957) introduces Two-Stage Least Squares (2SLS) as a means of estimating structural models that suffer from endogeneity when exclusion restrictions are available. In particular, the estimator allows one to take advantage of having more instrumental variables than endogenous regressors, in which case researchers are able to conduct tests of overidentifying restrictions (Sargan 1958; Basmann 1960; Hansen 1982). In subsequent work, Basmann et al. (1971) investigate the finite sample performance of the 2SLS estimator. Because of this research, and the future research it spurred (e.g., Stock et al. 2002; Flores-Lagunes 2007), the properties of 2SLS are well understood in many settings. However, one setting that has been inadequately addressed to date pertains to 2SLS estimation of a structural model when data on the endogenous covariate(s) are missing for some observations.[1]

Dealing with missing data is a frequent challenge confronted by empirical researchers. Ibrahim et al. (2005) note that medical researchers analyzing clinical trials often face the problem of missing data for various reasons, including survey nonresponse, loss of data, human error, and failing to meet protocol standards in follow up visits. Burton and Altman (2004), reviewing 100 articles across seven cancer journals, found that 81 of the 100 articles involve analyses with missing covariate data. Empirical researchers in economics face similar challenges. Abrevaya and Donald (2013), surveying four of the top empirical economics journals over a recent three-year period (2006-2008), find that nearly 40% of papers inspected had to confront missing data.[2]

Given the pervasive nature of missing data in empirical research, the literature on handling missing data is vast. Unfortunately, the literature tends to ignore the distinction between exogenous and endogenous covariates (i.e., whether the covariate is endogenous in the absence of missing data). As we discuss below, this distinction is likely to be salient as the 'optimal' method for dealing with missing data on an exogenous covariate may not be 'optimal' for an endogenous covariate. Specifically, the finite sample performance of various approaches for dealing with a missing covariate may differ when the resulting model is estimated via 2SLS as opposed to Ordinary Least Squares (OLS). This is the subject we investigate here.

Methods for dealing with (exogenous) missing covariates can be divided into two broad categories: *ad hoc* approaches and *imputation* approaches. The most widely used methods for dealing with missing covariate data are considered ad hoc by many researchers despite their popularity. These ad hoc approaches include so-called complete case analysis and variations on missing-indicator methods (Schafer and Graham 2002; Burton and Altman 2004; Dardanoni et al. 2011; Abrevaya and Donald 2013). Popular imputation approaches include regression (conditional mean) imputation and variants of nearest neighbor matching (Allison 2002; Rosenbaum 2002; Mittinty and Chacko 2005). Multiple imputation methods, with the advancement of computational power, have also become more widely used in empirical research (Rubin 1987).

Complete case analysis, as the name suggests, uses only observations without missing data. With this approach, efficiency losses can be substantial and bias may be introduced depending on the nature of the missingness (Pigott 2001; Schafer and Graham 2002; Horton and Kleinman 2007). The missing-indicator method, in the context of continuous variables, entails creation of a binary indicator of missingness and replacement of the missing values with some common value. The created indicator variable and covariate imputed with some common value (usually the

[1] In complementary work, Feng (2016) consider the problem of missing data on the instrument for some observations.

[2] The journals inspected in Abrevaya and Donald (2013) inlcude *American Economic Review*, *Journal of Human Resources*, *Journal of Labor Economics*, and *Quarterly Journal of Economics*. See Table 1 in Abrevaya and Donald (2013) for more details.

mean) are included, along with their interaction, in the estimating equation. With missing categorical variables, an indicator for a 'missing' category is added to the model. Although widely used and convenient, this method has been severely criticized (Jones 1996; Schafer and Graham 2002; Dardanoni et al. 2011).

Imputation approaches augment the original estimating equation with an imputation model in order to predict values of the missing data. Once the missing data are replaced with their predicted values, the original model is estimated using the full sample. Regression imputation obtains predicted values for the missing data by utilizing data on observations with complete data to obtain an estimated regression function with the covariate containing missing values as the dependent variable. The estimated regression function is then used to impute missing values with the predicted conditional mean. Nearest neighbor matching is done by replacing missing data with the values from observations with complete data deemed to be 'closest' according to some metric. Common univariate distance metrics include the Mahalanobis measure or the absolute difference in propensity scores, where the propensity score is the predicted probability that an observation has missing data (Mittinty and Chacko 2005; Gimenez-Nadal and Molina 2016). Matching methods are a variant of so-called hot deck imputation where the 'deck' in this case is just a single nearest neighbor (Andridge and Little 2010). Multiple imputation methods specify *multiple* (M, where $M > 1$) imputation models, rather than just a single imputation model. As such, M complete data sets are obtained by imputing the missing values M times. Common methods for imputing the M data sets are extensions of the regression and nearest neighbor matching methods described above. Using each of the imputed data sets, the analysis of interest is carried out M times with the M estimates being combined into a single result.

Despite this robust literature on missing data methods, there is a lack of guidance for applied researchers in dealing with missingness in endogenous covariates. As stated in Schafer and Graham (2002, p. 149), the goal of a statistical procedure is to make "valid and efficient inferences about a population of interest" irrespective of whether any data are missing. In our case, the statistical procedure is 2SLS and we wish to make inferences about some population parameter(s), θ. As such, any treatment of missing data should be evaluated in terms of the properties of the resulting estimate of θ, $\widehat{\theta}$. It is well known that the finite sample properties of 2SLS are complex even in the absence of missing data. Complete case analysis may introduce additional complexities due to nonrandom selection depending on the nature of the missingness. The missing-indicator approach introduces an additional endogenous covariate (due to the interaction term between the missingness indicator and the endogenous covariate), as well as measurement error in the already endogenous covariate due to the replacement of the missing data with an arbitrary value. Finally, any imputation procedure almost surely introduces measurement error in the endogenous covariate. Thus, understanding the implications of handling missing data in the specific context of 2SLS seems necessary. In the context of imputation, this point is made even more salient since the finite sample bias of 2SLS is not monotonically decreasing in the degree of measurement, or imputation, accuracy (Millimet 2015). Furthermore, the finite sample bias depends on the strength of the instruments which may be impacted by the imputation method. As such, and perhaps counter to intuition, the most accurate imputation method may not be the method that minimizes the finite sample bias of 2SLS.

In light of this, we investigate the finite sample performance of several approaches to dealing with missing covariate data when the covariate is endogenous even in the absence of any missingness. Specifically, we focus on imputation approaches and discuss the finite sample properties of OLS and 2SLS when one imputes an endogenous covariate prior to estimation. Then, we assess the finite sample performance of various imputation approaches in a Monte Carlo study. For comparison, we also examine the performance of the complete case and missing-indicator approaches. Finally, we illustrate the different approaches with an application to the causal effect of birth weight on the cognitive development of children in low-income households using data from the Early Childhood Longitudinal Study, Kindergarten Class of 2010-11 (ECLS-K:2011). In the sample, birth weight is missing for roughly 16% of children. Moreover, because birth weight is likely to be endogenous, we utilize instruments based on state-level regulations that affect participation in the Supplemental Nutrition Assistance Program (SNAP) similar to Meyerhoefer and Pylypchuk (2008). SNAP (formerly known as the Food Stamp Program) has been show to affect the health of low-income pregnant women and, hence, affect pregnancy outcomes (Baum 2012).

The Monte Carlo results suggest that imputation methods that incorporate the instruments along with other exogenous covariates generally produce the smallest finite sample bias of the 2SLS estimator. This is attributable, at least in part, to the improved instrument strength in the resulting first-stage estimation, as well as the improved imputation accuracy since the endogenous covariate is a function of the instruments (assuming they are valid). Among the ad hoc approaches, the complete case approach often does surprisingly well, while the missing-indicator approach does not. In terms of our application, however, we find surprisingly little substantive difference across the various estimators in terms of the point estimates, although the estimators that incorporate the instruments into the imputation model do lead to better instrument strength. Nonetheless, we do find some statistically and economically significant evidence that birth weight has an impact on math achievement at the beginning of kindergarten. This result is driven entirely by non-white male children.

The remainder of the paper is organized as follows. Section 2 sets up the structural model and discusses different methods for handling missing covariate data. Section 3 describes the Monte Carlo Study. Section 4 contains the application. Finally, Section 5 concludes.

2 Model

2.1 Setup

We consider the following structural model

$$y = x_1\beta_1 + \beta_2 x_2^* + \varepsilon \quad (1)$$

$$x_2^* = x_1\pi_1 + z\pi_2 + \eta \quad (2)$$

where y is a $N \times 1$ vector of an outcome variable, x_1 is a $N \times K$ matrix of exogenous covariates with the first element equal to one, x_2^* is a $N \times 1$ continuous endogenous covariate vector, β_1 is a $K \times 1$ parameter vector on the exogenous covariates, β_2 is a scalar parameter on x_2^* and is the object of interest, z is a $N \times L$ matrix of instrumental variables ($L \geq 1$), π_1 is a $K \times 1$ parameter vector, π_2 is a $L \times 1$ parameter vector, and ε and η are $N \times 1$ vectors of mean zero error terms.[3] The covariance matrix of the errors is given by

$$\Sigma = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ & \sigma_\eta^2 \end{bmatrix},$$

where $\sigma_{\varepsilon\eta} \neq 0$.

In the absence of missing data and utilizing the Frisch-Waugh-Lovell Theorem, the finite sample bias of the OLS estimator of β_2 from a simple regression of $\widetilde{y}$ on $\widetilde{x}_2^*$ is approximately

$$\mathrm{E}\left[\widehat{\beta}_2^{ols}\right] - \beta_2 \approx \frac{\sigma_{\varepsilon\eta}}{\sigma_{\widetilde{x}_2^*}^2}, \tag{3}$$

where $\widetilde{y}$ ($\widetilde{x}_2^*$) is a $N \times 1$ vector of residuals from an OLS regression of y (x_2^*) on x_1 and $\sigma_{\widetilde{x}_2^*}^2$ is the variance of $\widetilde{x}_2^*$ (Hahn and Hausman 2002; Bun and Windmeijer 2011).[4] Nagar (1959) and Bun and Windmeijer (2011) provide two different approximations of the finite sample bias of the 2SLS estimator of β_2 using $\widetilde{z}$ to instrument for $\widetilde{x}_2^*$, where $\widetilde{z}$ is a $N \times L$ matrix of OLS residuals obtained from regressing each column of z on x_1. The approximations are given by

$$\mathrm{E}\left[\widehat{\beta}_2^{2sls}\right] - \beta_2 \approx \frac{\sigma_{\varepsilon\eta}}{\sigma_\eta^2 \tau^2}(L-2) \tag{4}$$

$$\mathrm{E}\left[\widehat{\beta}_2^{2sls}\right] - \beta_2 \approx \frac{\sigma_{\varepsilon\eta}}{\sigma_\eta^2}\left[\frac{L}{\tau^2+L} - \frac{2\tau^4}{(\tau^2+L)^3}\right], \tag{5}$$

respectively, where τ^2 is the concentration parameter (Basmann 1963) given by

$$\tau^2 \equiv \frac{\pi_2' \widetilde{z}' \widetilde{z} \pi_2}{\sigma_\eta^2}.$$

The Nagar approximation requires $\tau^2 \to \infty$ as $N \to \infty$, while the Bun and Windmeijer approximation requires that $\max\{\tau^2, L\} \to \infty$ as $N \to \infty$.

2.2 Missing Data

Suppose that x_2^* is missing for $m = N - n$ observations ($n < N$). Let m_i be a binary variable, equal to one if x_2^* is missing for observation i and zero otherwise. The *missingness mechanism* refers to the process that determines whether x_2^* is missing for a given observation. The data are referred to as Missing Completely at Random (MCAR) if

$$\Pr(m_i = 1 | y_i, x_{1i}, x_{2i}^*, z_i) = \Pr(m_i = 1). \tag{6}$$

[3] We focus on the case of a continuous endogenous covariate for two reasons. First, imputing a discrete covariate requires greater consideration as to whether the imputation should preserve the discreteness and potential boundedness of the covariate. Second, if the boundedness is preserved, then 2SLS is problematic as any measurement error introduced due to the imputation will necessarily be non-classical due to its negative correlation with the true value of the bounded covariate (see, e.g., Black et al. 2000).

[4] Formally, $\widetilde{y} \equiv My$ and $\widetilde{x}_2^* \equiv Mx_2^*$, where $M \equiv [\mathrm{I}_N - x_1(x_1'x_1)^{-1}x_1]$ and I_N is a $N \times N$ identity matrix.

Under MCAR the probability of the data being missing is completely random. The data are referred to as Missing at Random (MAR) if

$$\Pr(m_i = 1 | y_i, x_{1i}, x_{2i}^*, z_i) = \Pr(m_i = 1 | y_i, x_{1i}, z_i). \tag{7}$$

Under MAR the probability of the data being missing depends only on observed data. Finally, the data are referred to as Not Missing at Random (NMAR) if

$$\Pr(m_i = 1 | y_i, x_{1i}, x_{2i}^*, z_i) \tag{8}$$

cannot be simplified. Under NMAR the probability of the data being missing depends on unobserved data.

2.3 Missing Data Methods

In this section, we briefly present some widely used methods for dealing with missing covariate data. We discuss imputation approaches first followed by ad hoc approaches.

2.3.1 Imputation Approaches

All imputation approaches entail replacing the missing data with values. Let x_2 denote a $N \times 1$ vector with the i^{th} element given as

$$x_{2i} = \begin{cases} x_{2i}^* \text{ if } m_i = 0 \\ \widehat{x}_{2i} \text{ if } m_i = 1 \end{cases}$$

where $\widehat{x}_{2i}$ is the imputed value for observation i. Different imputation approaches differ simply in how $\widehat{x}_{2i}$ is constructed and how many times the imputation is performed. The model used to construct $\widehat{x}_{2i}$ is referred to as the *imputation model*. We focus on two types of imputation models: regression-based models and matching-based models.

Regression Regression-based imputation approaches posit an imputation model of the generic form

$$(1 - m_i)x_{2i}^* = (1 - m_i)g(w_i, \xi_i) \tag{9}$$

where w_i is a vector of observed attributes of observation i, ξ_i is a scalar unobserved attribute of observation i, and $g(\cdot)$ is some unknown function. In a linear, parametric framework, (9) may be written as

$$(1 - m_i)x_{2i}^* = (1 - m_i)(w_i\delta + \xi_i). \tag{10}$$

Regression-based approaches typically estimate (10) via OLS and then define

$$\widehat{x}_{2i} \equiv w_i\widehat{\delta} \qquad \forall i \text{ such that } m_i = 1. \tag{11}$$

If the imputation model in (10) satisfies the usual assumptions of the classical linear regression model, then $\mathrm{E}[\widehat{x}_{2i}] = x_{2i}^*$, the imputation errors are heteroskedastic and orthogonal to w_i, and the imputation error variance is weakly decreasing in the number of observations with non-missing data.

Matching Matching-based imputation utilizes an alternative approach to predict the values for missing data. Here, the imputed values have the generic form

$$\widehat{x}_{2i} = \frac{1}{\sum_{l \in \{m_l=0\}} \omega_{il}} \sum_{l \in \{m_l=0\}} \omega_{il} x^*_{2l} \qquad \forall i \text{ such that } m_i = 1, \tag{12}$$

where ω_{il} is the weight given by observation i to observation l. Thus, missing values of x^*_2 are replaced with a weighted average of the non-missing data. Different matching algorithms may be used to construct the weights, ω_{il}. Let A_i represent the set of observations receiving strictly positive weight by observation i and let d_{il} denote a scalar measure of 'distance' between observations i and l, $i \neq l$. Every matching algorithm defines

$$A_i = \{l | m_l = 0, |d_{il}| \in C\},$$

where C is a neighborhood around zero. Single nearest neighbor (NN) matching sets

$$\omega_{il} = \begin{cases} 1 \text{ if } l \in A_i \\ 0 \text{ otherwise} \end{cases}$$

and $C = \min_{l \in \{m_l=0\}} |d_{il}|$. Thus, with single NN matching, (12) reduces to the value of x^*_2 from the 'closest' observation with non-missing data. Alternative matching algorithms include various multiple neighbor matching and kernel matching methods.

To operationalize any matching algorithm requires one to compute the distance between observations, d_{il}. Two common distance metrics are the Mahalanobis distance measure and the difference in propensity scores. The Mahalanobis distance is given by

$$d_{il} = (w_i - w_l)' \Sigma_w^{-1} (w_i - w_l),$$

where w_i is a vector of observed attributes and Σ_w is the covariance matrix of w. Distance based on the propensity score is given by

$$d_{il} = |p(w_i) - p(w_l)|, \tag{13}$$

where

$$p(w) = \Pr(m = 1|w) \tag{14}$$

is the propensity score. Specifically, $p(w)$ is the probability of missing data conditional on observed attributes. In practice, the propensity score may be estimated using a probit or logit model, or some other alternative.

Choice of w Implementing either regression- or matching-based imputation necessitates that the researcher choose the observed covariates w to be used in the imputation process. Unfortunately, there is, to our knowledge, little formal guidance provided to researchers regarding this variable selection. The implicit criteria used by most, if not all, researchers is to choose w based on convenience and/or to produce the most accurate estimates of the missing data. Maximizing accuracy subject to convenience implies choosing w (as well as the resulting imputation approach) in an attempt to minimize the variance of the imputation errors given the data at hand, $\mathcal{Z} \equiv x_1 \cup z$. In other words, $w \subseteq \mathcal{Z}$. Alternatively, one may utilize multiple imputation (MI) models and combine the estimates into a single estimate. In our context, defining $\beta \equiv [\beta_1' \ \beta_2]'$ as a $(K+1) \times 1$ vector and letting $p = 1, ..., P$ index the alternative imputation models, the final MI estimates are given by

$$\widehat{\beta} = \frac{1}{P} \sum_p \widehat{\beta}_{(p)} \tag{15}$$

$$\text{Var}(\widehat{\beta}) = \overline{\Sigma} + \left(1 + \frac{1}{P}\right) \frac{1}{(P-1)} \sum_p \left[\left(\widehat{\beta}_{(p)} - \widehat{\beta}\right) \left(\widehat{\beta}_{(p)} - \widehat{\beta}\right)' \right] \tag{16}$$

where $\widehat{\beta}_{(p)}$ represents the estimated parameter vector from imputation model p and $\overline{\Sigma}$ is the average over $\text{Var}(\widehat{\beta}_{(p)})$.

Regardless of whether a single ($P = 1$) or multiple ($P > 1$) imputation procedure is used, the 'optimal' choice of w is unclear. In the current context, it may seem that the choice of w is obvious, given the specification of the first-stage in (2). However, the 'optimality' of this choice (and the use of OLS) is not transparent and is, in fact, the subject of investigation here. Schafer and Graham (2002) argue that any imputation model must be judged in terms of the properties of the quantity of interest being estimated (in our case, β_2). Unfortunately, there is little guidance for researchers in how the choice of imputation model(s) impacts the resulting estimator, $\widehat{\beta}_2$, obtained via 2SLS conditional on the observed and imputed data. To offer some insight, we can extend the analysis of the finite sample bias of 2SLS to account for the imputed data on the endogenous covariate.

Recalling that x_2 is a $N \times 1$ vector containing the true values of x_2^* for the n observations with complete data and imputed values of x_2^*, $\widehat{x}_2$, for the remainder, we can,without loss of generality, express the relationship between x_2 and x_2^* as

$$x_2 = x_2^* + \upsilon, \tag{17}$$

where υ is the imputation error. Specifically, $\upsilon_i = 0$ if $m_i = 0$ and $\upsilon_i = \widehat{x}_{2i} - x_{2i}^*$ otherwise. If the imputation estimator is perfect, then υ is a $N \times 1$ vector of zeros. If the imputation estimator is unbiased, then $\text{E}[\upsilon] = 0$.

To continue, assume to start that υ satisfies the properties of classical measurement error; υ is mean zero, uncorrelated with ε, η, x_1, x_2^*, and z, and has a strictly positive variance, σ_υ^2. Substituting (17) into (1) and (2), the structural model becomes

$$y = x_1\beta_1 + \beta_2 x_2 + \widetilde{\varepsilon} \tag{18}$$

$$x_2 = x_1\pi_1 + z\pi_2 + \widetilde{\eta} \tag{19}$$

where $\widetilde{\varepsilon} \equiv (\varepsilon - \beta_2\upsilon)$ and $\widetilde{\eta} \equiv \eta + \upsilon$. The model can be written more compactly as

$$\widetilde{y} = \beta_2 \widetilde{x}_2 + \widetilde{\varepsilon} \tag{20}$$

$$\widetilde{x}_2 = \widetilde{z}\pi_2 + \widetilde{\eta} \tag{21}$$

where $\widetilde{x}_2$ is a $N \times 1$ vector of residuals from an OLS regression of x_2 on x_1 and all other notation is previously defined.

Letting $\sigma^2_{\widetilde{x}_2}$ denote the variance of $\widetilde{x}_2$, the finite sample bias of the OLS estimator of β_2 from (20) is approximately

$$\mathrm{E}\left[\widehat{\beta}_2^{ols}\right] - \beta_2 \approx \frac{\sigma_{\widetilde{\varepsilon}\widetilde{\eta}}}{\sigma^2_{\widetilde{x}_2}}, \tag{22}$$

while the Nagar and Bun and Windmeijer approximations of the finite sample bias of the 2SLS estimator are

$$\mathrm{E}\left[\widehat{\beta}_2^{2sls}\right] - \beta_2 \approx \frac{\sigma_{\widetilde{\varepsilon}\widetilde{\eta}}}{\sigma^2_{\eta}\tau^2}(L-2) \tag{23}$$

$$\mathrm{E}\left[\widehat{\beta}_2^{2sls}\right] - \beta_2 \approx \frac{\sigma_{\widetilde{\varepsilon}\widetilde{\eta}}}{\sigma^2_{\eta}+\sigma^2_{\upsilon}}\left[\frac{L}{\tau^2+L} - \frac{2\tau^4}{(\tau^2+L)^3}\right]. \tag{24}$$

Utilizing the following approximations

$$\begin{aligned}
\sigma^2_{\widetilde{x}_2} &\approx \sigma^2_{\widetilde{\eta}}\left(\frac{\tau^2}{N}+1\right)\\
\varphi &\equiv 1 - \frac{\sigma^2_{\upsilon}}{\sigma^2_{\widetilde{x}_2}} \approx 1 - \frac{\sigma^2_{\upsilon}}{\sigma^2_{\widetilde{\eta}}\left(\frac{\tau^2}{N}+1\right)}
\end{aligned}$$

where φ is the reliability ratio of x_2, the three bias approximations can be rewritten in in terms of the reliability ratio and the concentration parameter, given by

$$\mathrm{Bias}_{OLS} \approx \beta_2(\varphi-1) + \frac{\sigma_{\varepsilon\eta}}{\sigma^2_{\eta}\Gamma_0}\frac{1}{\frac{\tau^2}{N}+1} \tag{25}$$

$$\mathrm{Bias}_{Nagar} \approx \beta_2(\varphi-1)\left(\frac{\tau^2}{N}+1\right)\Gamma_1 + \frac{\sigma_{\varepsilon\eta}}{\sigma^2_{\eta}\Gamma_0}\Gamma_1 \tag{26}$$

$$\mathrm{Bias}_{BW} \approx \beta_2(\varphi-1)\left(\frac{\tau^2}{N}+1\right)\Gamma_2 + \frac{\sigma_{\varepsilon\eta}}{\sigma^2_{\eta}\Gamma_0}\Gamma_2 \tag{27}$$

where

$$\begin{aligned}
\Gamma_0 &\equiv 1 + \frac{(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}{1-(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}\\
\Gamma_1 &\equiv \frac{L-2}{\tau^2}\\
\Gamma_2 &\equiv \frac{L}{\tau^2+L} - \frac{2\tau^4}{(\tau^2+L)^3}.
\end{aligned}$$

With imputation, each bias expression in (25)-(27) contains two terms. The first term in each vanishes if $\varphi \to 1$. A sufficient condition for this is that the imputation procedure is perfectly accurate. The second term in each converges to the usual finite sample bias of OLS or 2SLS when an endogenous covariate is fully observed. However, the bias expressions reveal what is perhaps a surprising result. As shown in Millimet (2015), the biases are not monotonically decreasing in the reliability ratio. As such, the most accurate imputation procedure – defined as the procedure that minimizes σ^2_{υ} – does not necessarily minimize the (absolute value of the) finite sample bias of the OLS or 2SLS estimator. Moreover, conditional on the reliability ratio, the (absolute value of the) biases are monotonically decreasing in τ^2/L, which is the population analog of the first-stage F-statistic (Bound et al. 1995; Stock et al. 2002). Thus, while improved imputation accuracy *will not necessarily* decrease the 2SLS bias in absolute value holding the first-stage strength of the instrument(s) constant, improving the first-stage strength of the instrument(s) *will* decrease the 2SLS bias in absolute value holding the imputation accuracy constant.

In sum, when the data are missing on an endogenous covariate, maximizing imputation accuracy *does not* necessarily minimize the finite sample bias of 2SLS. The first-stage strength of the instrument(s), z, is also critical. Because the imputation model alters the dependent variable in the first-stage, shown in (19), the imputation procedure alters both the reliability ratio *and* the concentration parameter. As such, to minimize the finite sample bias of the 2SLS estimator, the imputation model should be chosen with *both* of these in mind.

To illustrate, Figure 1 plots the Nagar bias (in absolute value) for a hypothetical situation. The parameter values are given in Table 1.

Table 1. Hypothetical Parameter Values.

$L = 3$	$\sigma^2_\eta = 1$	$\sigma^2_{\tilde{x}^*_2} = \sigma^2_{\tilde{x}_2} - \sigma^2_\upsilon$
$N = 100$	$\sigma^2_\upsilon = \frac{(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}{1-(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}\sigma^2_\eta$	$\sigma^2_\varepsilon = \beta^2_2\sigma^2_{\tilde{x}^*_2}$
$\beta_2 = 1$	$\sigma^2_{\tilde{x}_2} = \sigma^2_\eta\left(\frac{\tau^2}{N}+1\right)$	$\sigma_{\varepsilon\eta} = \rho_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta$

L is set to three such that the expectation exists. The variance of ε is chosen such that the population R^2 in (20) is 0.5. The correlation coefficient between ε and η, $\rho_{\varepsilon\eta}$, reflects the degree of endogeneity of x^*_2 and is set to 0.5. The reliability ratio, φ, is varied from 0.2 to one. Finally, two different values of instrument strength are utilized: $\tau^2/L \in \{3, 5\}$.

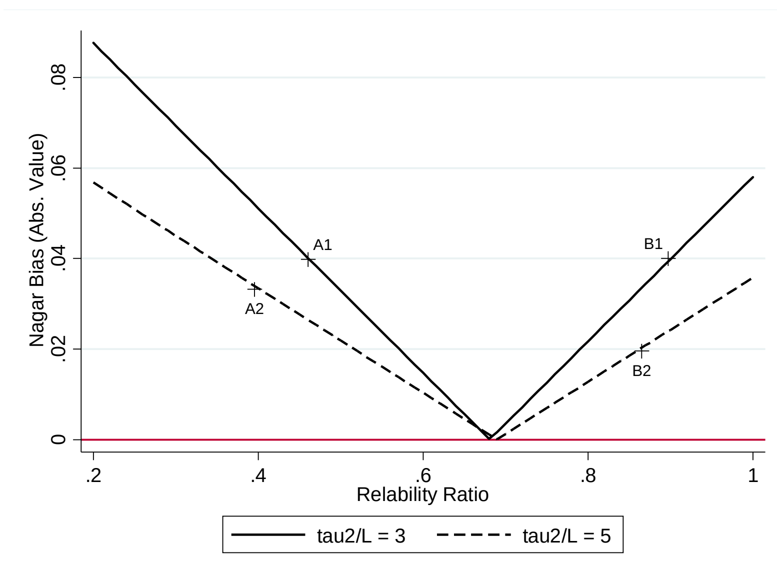

Figure 1. Hypothetical Illustration of Finite Sample Bias of 2SLS (Nagar Approximation).

Figure 1 highlights three key points. First, since any imputation procedure is likely to simultaneously alter both φ and τ^2/L, imputation will generally affect the finite sample performance of 2SLS. Second, as shown in Millimet (2015), the finite sample bias (in absolute value) is not minimized when $\varphi = 1$. Third, holding the reliability ratio constant, the finite sample bias (in absolute value) is strictly decreasing in τ^2/L. Together, these last two points have important implications for thinking about the properties of various imputation methods in the context of an endogenous covariate. For example, consider points A1 and A2 in Figure 1, as well as B1 and B2. Both sets of points illustrate situations where an imputation method that produces a smaller reliability ratio can yield a smaller finite sample bias (in absolute value). This is more likely to be the case if the improvement in the first-stage F-statistic is sufficiently great.

The analysis to this point, however, has assumed that the imputation errors in (17) satisfy the classical error-in-variables assumptions. With many imputation methods, this is not likely to be the case. Specifically, $\text{Cov}(\upsilon, \eta)$ is likely to be negative. This arises, for example, in the context of regression imputation because predicted values of the type shown in (11) tend to underpredict (in absolute value) the true value of x_2^*. To see this, consider the structural model as shown in (20) and (21). If $w = \widetilde{z}$ and without loss of generality we denote the first n observations as those with nonmissing data, then the imputation model becomes OLS applied to the following equation

$$\widetilde{x}_{2i}^* = \widetilde{z}_i \pi_2 + \widetilde{\eta}_i, \quad i = 1, ..., n \tag{28}$$

where $\widetilde{x}_2^*$ is a $n \times 1$ vector of residuals from an OLS regression of x_2^* on x_1. The imputed values are given by

$$\widehat{\widetilde{x}}_{2i} = \widetilde{z}_i \widehat{\pi}_2, \quad i = n+1, ..., N \tag{29}$$

where $\widehat{\pi}_2 = (\widetilde{z}^{o\prime}\widetilde{z}^o)^{-1}\widetilde{z}^{o\prime}\widetilde{x}_2^*$ and $\widetilde{z}^o$ is a $n \times L$ matrix of instruments for observations with non-missing data for x_2^*. The imputation errors are given by

$$\begin{aligned} \upsilon_i &= \widehat{\widetilde{x}}_{2i} - \widetilde{x}_{2i}^* \\ &= \widetilde{z}_i (\widetilde{z}^{o\prime}\widetilde{z}^o)^{-1}\widetilde{z}^{o\prime}\eta^o - \eta_i, \quad i = n+1, ..., N \end{aligned} \tag{30}$$

where η^o is a $n \times 1$ vector of errors for observations with non-missing data for x_2^*. With $\text{Cov}(\upsilon, \eta) < 0$, the reliability ratio may exceed unity and the bias expressions in (25)-(27) become

$$\text{Bias}_{OLS} \approx \beta_2(\varphi - 1) + \frac{\sigma_{\varepsilon\eta} + \sigma_{\varepsilon\upsilon} + \beta\sigma_{\upsilon\eta}}{\sigma_\eta^2 \Gamma_0} \frac{1}{\frac{\tau^2}{N} + 1} \tag{31}$$

$$\text{Bias}_{Nagar} \approx \beta_2(\varphi - 1)\left(\frac{\tau^2}{N} + 1\right)\Gamma_1 + \frac{\sigma_{\varepsilon\eta} + \sigma_{\varepsilon\upsilon} + \beta\sigma_{\upsilon\eta}}{\sigma_\eta^2 \Gamma_0}\Gamma_1 \tag{32}$$

$$\text{Bias}_{BW} \approx \beta_2(\varphi - 1)\left(\frac{\tau^2}{N} + 1\right)\Gamma_2 + \frac{\sigma_{\varepsilon\eta} + \sigma_{\varepsilon\upsilon} + \beta\sigma_{\upsilon\eta}}{\sigma_\eta^2 \Gamma_0}\Gamma_2 \tag{33}$$

where $\sigma_{\upsilon\eta}$ ($\sigma_{\varepsilon\upsilon}$) is the covariance between υ and η (ε) and $\sigma_{\varepsilon\upsilon}$ is likely to be non-zero as well since $\sigma_{\varepsilon\eta} \neq 0$.

While allowing for the fact that the imputation errors may be nonclassical complicates the bias expressions, it does not alter our general conclusions. To illustrate, Figure 2 plots the Nagar bias (in absolute value) for another hypothetical situation. The parameter values are given in Table 2.

Table 2. Hypothetical Parameter Values.

$L = 3$	$\sigma_\eta^2 = 1$	$\sigma^2_{\tilde{x}_2^*} = \sigma^2_{\tilde{x}_2} - \sigma^2_\upsilon - 2\sigma_{\upsilon\eta}$
$N = 100$	$\sigma_\upsilon^2 = \frac{(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}{1-(1-\varphi)\left(\frac{\tau^2}{N}+1\right)}\sigma_\eta^2 - 2\sigma_{\upsilon\eta}$	$\sigma^2_\varepsilon = \beta_2^2\sigma^2_{\tilde{x}_2^*}$
$\beta_2 = 1$	$\sigma^2_{\tilde{x}_2} = \left[\sigma^2_\eta + \sigma^2_\upsilon + 2\sigma_{\upsilon\eta}\right]\left(\frac{\tau^2}{N}+1\right)$	$\sigma_{\varepsilon\eta} = \rho_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta$
$\sigma_{\upsilon\eta} = -0.2$		
$\sigma_{\varepsilon\upsilon} = -0.1$		

As in Figure 1, points A and B illustrate a situation where an imputation procedure may produce a reliability ratio further from unity, but the bias (in absolute value) is smaller. This requires the improvement in the first-stage F-statistic to be sufficiently great.

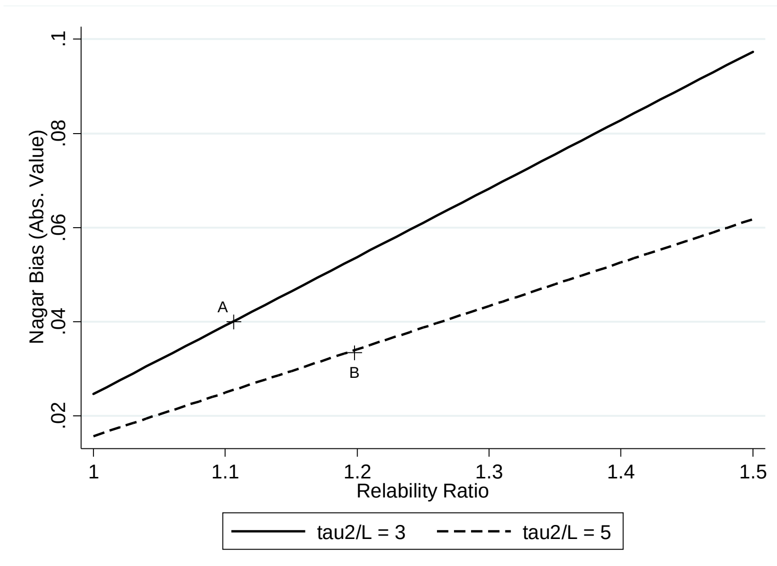

Figure 2. Hypothetical Illustration of Finite Sample Bias of 2SLS (Nagar Approximation).

Returning to the structural model in (18) and (19), we can now offer a few insights into the choice of w. First, letting $w = [x_1 \; z]$ will maximize the R^2 in the first-stage regardless of whether (19) is the true data-generating process for x_2^*. Moreover, with w defined as such, and utilizing regression-based imputation, the imputation errors will be orthogonal to x_1 and z. As such, if the instruments are valid in the absence of missing data, they will continue to be valid. However, maximizing the R^2 is not synonymous with maximizing the first-stage F-statistic. Second, letting $w = z$ may produce a higher first-stage F-statistic, although the imputed values may be less accurate if x_1 has predictive power. In addition, the imputation errors are no longer assured of being orthogonal to x_1. If the imputation errors are not orthogonal to x_1, then x_1 becomes endogenous in (18) and may affect the estimate of β_2 if z and x_1 are not orthogonal. Third, allowing for more flexibility by including higher order terms of z and/or x_1, as well as possible interactions between z and x_1, may improve accuracy as well as the strength of the first-stage relationship. Finally, bringing in data from outside the model to impute x_2^* may be desirable if the improvement in accuracy outweighs any reduction in the strength of the first-stage relationship.[5]

[5] Note, it may be possible to bring in 'outside' data if components of the first-stage error term in (2) could be observed and hence

It is the finite sample sensitivity of the 2SLS estimator to the choice of w, as well as the choice of regression- versus matching-based imputation and single versus multiple imputation, that we investigate below. However, before doing so, we present two ad hoc approaches for comparison.

2.3.2 Ad Hoc Approaches

Complete Case Analysis The most common method for dealing with missing data is the complete case (CC) approach (Schafer and Graham 2002). In the context of our structural model in (1) and (2), the complete case approach simply entails estimating the parameters via 2SLS applied to the $N-m$ observations with complete data. Aside from the efficiency loss due to the smaller sample size, the complete case approach will introduce additional bias if the sample is no longer random. Nonrandomness of the sample generally occurs unless the missingness mechanism satisfies MCAR.

Missing-Indicator Methods The other widely used method for dealing with missing data in empirical research is the missing-indicator method; also referred to as the dummy variable (DV) approach. Assuming x_2^* to be continuous, and utilizing the dummy variable m_i defined previously, the equation in (1) is replaced with an augmented model of the form

$$y_i = x_{1i}\beta_1 + \beta_2 x_{2i} + \alpha_1 m_i + \alpha_2 m_i x_{2i} + \zeta_i, \tag{34}$$

where

$$x_{2i} = \begin{cases} x_{2i}^* & \text{if } m_i = 0 \\ c & \text{if } m_i = 1 \end{cases} \tag{35}$$

and c is some scalar. A convenient choice for c, as it relates to interpretation, is the sample mean of x_2^* based on the observations without missing data. Note, however, since x_2^* (and, hence, x_2) is endogenous, the interaction term between m and x_2 is also endogenous. Additional instruments defined as $m \cdot z$ are potentially feasible depending on the process determining the missingness.

The benefits of the missing-indicator approach are the ease at which it can be implemented and the ability to leverage all data. This is evidenced by its pervasive use in empirical research. However, Jones (1996) and Dardanoni et al. (2011) show that this method generally yields biased and inconsistent estimates.

3 Monte Carlo Study

3.1 Design of the Data Generating Process

To assess the finite sample performance of 2SLS under different approaches to handle missing data, we utilize a Monte Carlo design similar to that in Abrevaya and Donald (2013). The general structure for the DGP, with one exogenous and one endogenous regressor, x_{1i} and x_{2i}^*, respectively, and instrumental variables, z_{li}, $l = 1, ..., L$, is as

included in the model as additional covariates. If these additional covariates also belong in (1), then the additional covariates may improve imputation accuracy but will not add additional exclusion restrictions.

follows:

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}^* + \varepsilon_i, \qquad i = 1, ..., N \\
x_{1i} &= \pi_{10} + \upsilon_{1i} \\
x_{2i}^* &= \pi_{20} + \pi_{21}\left(x_{1i} + \gamma_{21}\frac{x_{1i}^2}{2}\right) + \sum_{l=1}^{L} \pi_{22,l}\left(z_{li} + \gamma_{22,l}\frac{z_{li}^2}{2}\right) + \upsilon_{2i} \\
z_i &\sim N(\omega_0, \Sigma_z) \\
\varepsilon_i, \upsilon_{1i}, \upsilon_{2i} &\sim N(0, \Sigma),
\end{aligned}
$$

where $z_i = [z_{1i} \cdots z_{Li}]'$ is an $L \times 1$ vector of instrumental variables. In all simulations, $\{y_i, x_{1i}, z_i\}$ are observed for all observations. However, x_{2i}^* is missing for $m > 0$ observations. Moreover, in all simulations, we fix $(\beta_0, \beta_1, \beta_2, \pi_{20}) = (1, 1, 1, 1)$ and the covariance matrix of the errors is given by

$$
\Sigma = \begin{bmatrix} 1 & 0 & \rho \\ & 1 & 0 \\ & & 1 \end{bmatrix}.
$$

The number of instruments, L, is equal to three to follow our application as well as ensure that the first two moments of the estimator exist. The covariance matrix of z_i is given by

$$
\Sigma_z = \begin{bmatrix} 1/3 & 0 & 0 \\ & 1/3 & 0 \\ & & 1/3 \end{bmatrix}.
$$

Within this common framework, we consider numerous experiments. The experiments differ in terms of the degree of endogeneity, ρ, the data-generating process for the endogenous covariate, the correlation between the exogenous covariate and the instrumental variables, the strength of the instruments, and the nature of the missingness.

For the degree of endogeneity, we consider $\rho = \{0.1, 0.5\}$. For the determinants of the endogenous covariate, we alter the DGP along two dimensions. First, we vary the correlation between the exogenous and endogenous covariates by considering $\pi_{21} = \{0, 1\}$. Second, we consider both linear and nonlinear specifications for the endogenous covariate by setting $\gamma_{21} = \gamma_{22,l} = \{0, 1\}$. For the strength of the instrument, we consider values for $\pi_{22} = [\pi_{22,1} \cdots \pi_{22,L}]$ such that the elements are identical (i.e., $\pi_{22,1} = \cdots = \pi_{22,L}$) and the population analog of the first-stage F-statistic, τ^2/L, is one of $\{2, 5, 10\}$. Thus, $\tau^2/L = 2, 5$ correspond to the case of weak identification, whereas $\tau^2/L = 10$ is the typical rule-of-thumb benchmark for non-weak identification (Stock et al. 2002).[6] To obtain $\tau^2/L = \{2, 5, 10\}$, we set $\pi_{22} = \{\sqrt{2L/N}, \sqrt{5L/N}, \sqrt{10L/N}\}$, where N is the sample size.[7] If the exogenous covariate and instruments

[6] The focus on cases where the instruments are weak or very weak ($\tau^2/L \leq 10$) is motivated by two reasons. First, weak instruments are often encountered in applied research (and our application). Second, when instruments are strong, the choice of imputation model is less consequential as the 2SLS finite sample bias is relatively small and less dependent on imputation accuracy. While not presented, we conduct a few Monte Carlo experiments with $\tau^2/L = 20$. Results, available upon request, confirm our view.

[7] The first-stage regression is given by

$$x_{2i} = \pi_{20} + \pi_{21}x_{1i} + \pi_{22}z_i + \upsilon_{2i}$$

and the F-statistic used to test the null $\mathrm{H}_o : \pi_{22} = 0$ vs. $\mathrm{H}_1 : \pi_{22} \neq 0$ is given by

$$\widehat{\pi}_{22}'\Sigma^{-1}\widehat{\pi}_{22}/L,$$

are uncorrelated, then $\omega_0 = [1/3 \cdots 1/3]$; if they are correlated, then $\omega_0 = [x_{1i}/3 \cdots x_{1i}/3]$.

Finally, we consider four patterns of missingness. First, we create missingness in x_2^* by assuming a fraction, λ, of the sample has x_2^* missing completely at random (MCAR). In the second and third patterns, we create missingness in x_2^* for a fraction, λ, of the sample that is missing at random (MAR). In the second case, the probability of missingness depends on x_1 only. In the third case, the probability of missingness depends on x_1 and z. Formally, in the second and third cases, the probability of missingness for a given observation, p_i, is given by

$$p_i = \frac{e^{\Lambda_i}}{1 + e^{\Lambda_i}}, \tag{36}$$

where $\Lambda_i = x_{1i}$ in the second case and $\Lambda_i = x_{1i} + z_i$ in the third case.[8] In the second case, π_{10} is chosen such that $\mathrm{E}[p_i] = \lambda$ and $\omega_0 = 1$. In the third case, $\pi_{10} = 1$ and ω_0 is chosen such that it is equal across instruments and $\mathrm{E}[p_i] = \lambda$. In all simulations, we set $\lambda = 0.20$; x_2^* is missing for 20% of the sample in expectation. This simulation design yields a correlation coefficient between a binary indicator if x_{2i}^* is missing, m_i, and x_{1i} of approximately 0.35 in the second case; correlation coefficients of approximately 0.30 between m_i and x_{1i} and 0.17 between m_i and each element of z_i in the third case.

Altogether, we conduct 48 experiments for each of the four missingness mechanisms, for a total of 192 unique designs. In all cases, we set the sample size, N, to 500 and conduct 500 simulations.

3.2 Estimators

We compare the performance of 15 different estimators. The first two estimators, CC and DV, correspond to the ad hoc complete case and missing-indicator (dummy variable) approaches. The next five estimators are variants of single NN matching using the Mahalanobis distance measure and defined as follows:

- NN1: w includes x_1 and its quadratic, z and the quadratic of each element of z, and interactions between x_1 and each element of z
- NN2: w includes z and the quadratic of each element of z
- NN3: w includes x_1 and its quadratic
- MI1-NN: multiple imputation combining NN1 and NN2 using (15) and (16)

---

where Σ^{-1} is an $L \times L$ diagonal matrix of the form

$$\Sigma^{-1} = \begin{bmatrix} N/L & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & N/L \end{bmatrix}$$

since $\mathrm{Cov}(x_1, z) = 0$ and $\mathrm{Var}(z) = 1/L$ and$\mathrm{Var}(\upsilon_2) = 1$. Setting each element of π_{22} equal and solving as a function of F and N, yields

$$\widehat{\pi}_{22,l} = \sqrt{\frac{LF}{N}}.$$

[8]When $L = 3$, Λ_i is the sum of x_{1i} and the three instruments.

- MI2-NN: multiple imputation combining NN1, NN2, and NN3 using (15) and (16).

The final eight estimators are variants of regression-based imputation and defined as follows:

- Reg1: w includes x_1 and z
- Reg2: w includes x_1 and its quadratic, z and the quadratic of each element of z, and interactions between x_1 and each element of z
- Reg3: w includes z
- Reg4: w includes z and the quadratic of each element of z
- Reg5: w includes x_1
- Reg6: w includes x_1 and its quadratic
- MI1-Reg: multiple imputation combining Reg1, Reg2, Reg3, and Reg4 using (15) and (16)
- MI2-Reg: multiple imputation combining Reg1, Reg2, Reg3, Reg4, Reg5, and Reg6 using (15) and (16).

3.3 Simulation Results

The full simulation results are relegated to Tables A1-A16 in the Supplemental Appendix. In addition to the 15 estimators, we also present the results for the case of no missing data (i.e., 2SLS with x_2^* fully observed for the entire sample). We report the median bias and root mean squared error (RMSE) of the 2SLS estimates of β_2, as well as the median first-stage F-statistic for the test of instrument strength. Finally, we report the empirical standard deviations of the estimates and the mean robust standard errors for inference purposes.

The tables vary (i) the degree of endogeneity, $\rho = \{0.1, 0.5\}$, (ii) whether the true data-generation process for x_2^* is linear or nonlinear, $\gamma_{12} = \gamma_{22,l} = \{0, 1\}$, (iii) whether the true data-generation process for x_2^* depends on x_1, $\pi_{21} = \{0, 1\}$, and (iv) whether the exogenous covariate and instrumental variables are correlated, $\omega_0 = \{1/3, x_1/3\}$. Hence, there are $2 \times 2 \times 2 \times 2 = 16$ tables of results. Moreover, within each table, Panel A sets the expected value of the first-stage F-statistic to 2; Panel B (Panel C) sets it to 5 (10). Finally, the columns within each table represent the four different missingness mechanisms.

Given the number of experimental designs, we aggregate the performance of the estimators over numerous experiments using various metrics and report the results in Tables 3-8. Before discussing these results, we note a few over-arching findings that come from inspection of the detailed tables in the appendix. First, consistent with the analysis in Section 2, regression-based imputation approaches that include the instruments in the imputation procedure produce the strongest identification measured by the median first-stage F-statistic. Moreover, the imputation approaches (regression-based and matching) often produce the smallest median bias, sometimes even smaller than in the absence of missing data, due to the improvement in instrument strength. Second, imputation approaches that do not include the instruments in the imputation model – NN3, Reg5, and Reg6 – do not perform well and are not advisable. Third, despite the presence of a sometimes sizeable median bias, the CC approach generally performs well in terms of RMSE. Fourth, the DV approach is quite volatile. In some cases, its performance is virtually identical to the CC approach; in other cases, its performance is demonstrably worse. Fifth, the mean robust standard error is typically quite close to the empirical standard deviation for all estimators excluding the multiple imputation approaches. With multiple imputation, the mean standard errors tend to be conservative. Finally, the preferred estimators appear to belong to the set containing CC, NN1, NN2, MI1-NN, Reg1-4, and MI1-Reg.

We now turn to the results in Tables 3-8. To begin, we consider the performance of the different estimators aggregated over all experiments for each of the four missingness mechanisms. Panels A-D in Table 3 provide the median bias and RMSE of each estimator in each of the four cases. Under MCAR (Panel A), MAR with missingness depending on x_1 only (Panel B), and NMAR (Panel D), the estimators NN1, Reg1, and Reg2 yield median biases very close to zero. Thus, imputation approaches incorporating all exogenous variables in the model are preferred. In terms of RMSE, the estimators CC and MI1-Reg are preferred, although the performances of Reg1, Reg2, and MI1-NN are not much different. Under MAR with missingness depending on x_1 and z (Panel C), the performances of the estimators are notably worse. However, MI2-Reg achieves a median bias close to zero, while the four MI estimators produce the smallest RMSEs (with MI1-Reg producing the smallest RMSE).

Next, we consider the performance of the different estimators aggregated over all experiments for each of the three levels of instrument strength. Panels E-G in Table 3 provide the median bias and RMSE of each estimator. In all three cases, Reg1 and Reg2 yield median biases very close to zero and substantially better than the remaining estimators. In terms of RMSE, MI1-Reg is preferred, but CC is quite close. Thus, imputation approaches incorporating all exogenous variables in the model are preferred, and a regression approach tends to outperform more flexible methods based on (nonparametric) nearest neighbor matching. Moreover, while stronger instruments are clearly preferable, instrument strength does not affect recommendations concerning the preferred estimator.

In Table 4 we consider the performance of the different estimators aggregated over all experiments within different specifications of the data-generating process for the endogenous covariates, x_2^*, and correlation structures of the exogenous variables (x_1 and z). Panels A-D vary whether the true first-stage is linear or nonlinear and whether x_1 and z are correlated. Panels E and F vary whether the true first-stage depends on x_1 or not. In terms of median bias, we continue to find that Reg1 and Reg2 perform very well in every case. For RMSE, the estimators CC, MI2-NN, and MI1-Reg perform well across the various cases. Thus, imputation approaches incorporating all exogenous variables in the model continue to be preferred, along with the CC approach. It is also interesting to note that the performance of the DV estimator varies considerably across the different designs; its performance is particularly poor when x_1 and z are correlated (Panels C and D) and when the true first-stage depends on x_1 (Panel F). In other cases, the performances of DV and CC are quite similar.

To further evaluate the performance of the different estimators, we consider two alternative methods of aggregating performance across experiments. First, we rank the estimators from best (one) to worst (15) based on either median bias or RMSE within each of the 192 experimental designs. We then compute the median rank for each estimator across all designs of a particular type. The results are presented in Tables 5 and 6. Second, we compute Pitman's (1937) Nearness Measure, PN, over all experimental designs of a particular type. Formally, this measure is given by

$$PN = \Pr\left[\left|\widehat{\beta}_{2,A} - \beta_2\right| < \left|\widehat{\beta}_{2,B} - \beta_2\right|\right],$$

where $\widehat{\beta}_{2,j}$, $j = A, B$, represent two distinct estimators of the parameter β_2. Thus, $PN > (<) 0.5$ indicates superior performance of estimator A (B). The advantage of PN is that it summarizes the entire sampling distribution of an estimator. In practice, PN is estimated by its empirical counterpart: the fraction of simulated data sets where one estimator is closer (in absolute value) to the true parameter value than another estimator. The results are provided in Tables 7 and 8.[9]

The first four columns in Table 5 display the median rank of each estimator over all experimental designs within each of the four missingness mechanisms. Similar to Panels A-D in Table 3, we find that NN1, Reg1, and Reg2 performance best in terms of median bias, while CC and MI1-Reg perform best in terms of RMSE. Moreover, the first four columns of Table 5 indicate that CC, Reg1, and Reg2 dominate the remaining estimators as determined by the PN metric. Finally, Tables 5 and 7 point to a preference for CC under MCAR and NMAR and a preference for Reg1, Reg2, and MI1-Reg under both versions of MAR.

The final three columns in Table 5 display the median rank of each estimator across all experimental designs by instrument strength. The corresponding PN results are in the final three columns of Table 7. As in Table 3, the results indicate little variation in relative performance across different instrument strengths. Moreover, as in Table 3, the estimators NN1, Reg1, and Reg2 perform well in terms of median bias, while CC and MI1-Reg perform well in terms of RMSE. The PN metric continues to indicate very similar performances by CC, Reg1, and Reg2.

Tables 6 and 8 present the corresponding results aggregating across different data-generating processes for the endogenous covariates, x_2^*, and correlation structures of the exogenous variables (x_1 and z). The results continue to show that the estimators NN1, Reg1, and Reg2 perform well in terms of median bias, while CC and MI1-Reg perform well in terms of RMSE. The PN metric yields very similar performances by CC, Reg1, and Reg2. In addition, the PN metric indicates that MI1-Reg performs well when the true first-stage does not depend on x_1 (i.e., $\pi_{21} = 0$).

In sum, consistent with our expectations, we find that imputation methods that incorporate the instruments along with other exogenous covariates generally produce the smallest finite sample bias of the 2SLS estimator. This is attributable, at least in part, to the improved instrument strength in the resulting first-stage estimation. However, the CC estimator does very well in terms of RMSE across the range of experimental designs considered here, particularly under MCAR and NMAR. Multiple imputation, where the various regression imputation models incorporating the instruments via different specifications, also performs well in terms of RMSE. Specifically, multiple imputation seems to marginally outperform CC under MAR and when the endogenous covariate does not depend on the exogenous covariates in the structural model (i.e., $\pi_{21} = 0$). Nonetheless, the generally strong performance of the CC estimator in terms of RMSE is perhaps surprising. The DV approach and imputation methods that do not utilize the instrument in the imputation model are not recommended. We now illustrate these various estimators in practice.

[9] We compute the PN metric for all pairwise combinations of estimators. However, for brevity, Tables 7 and 8 present on a selection of the comparisons. Specifically, we do not report any comparisons involving NN2, NN3, Reg5, or Reg6 as these estimators do not perform well. Full results are available upon request.

4 Application

4.1 Motivation

Early childhood development is a major concern for policymakers worldwide as it is estimated that millions of children under the age of five are not meeting their developmental potential (Grantham-McGregor et al. 2007). Moreover, it is well documented that higher levels of cognitive development early in life are associated with better educational, health, and labor market outcomes later in life (Heckman et al. 2006; Conti and Heckman 2010; Bijwaard et al. 2015).

In light of this, several recent studies have examined the impact of infant health – proxied by birth weight – on cognitive development and, consequently, later life outcomes. Relative to infants with low birth weight, infants with higher birth weight tend to achieve greater levels of academic success, higher labor market earnings, and better health outcomes over the life cycle (Currie and Hyson 1999; Almond et al. 2005; Case et al. 2005; Black et al. 2007; Oreopoulos et al. 2008; Chatterji et al. 2014). However, the relationship is not necessarily monotonic as cognitive outcomes have also been found to be adversely impacted at the top end of the birth weight distribution. Richards et al. (2001) and Kirkegaard et al. (2006), for example, document a nonlinear relationship between birth weight and cognitive function with children at either end of the birth weight distribution displaying difficulties in math and reading. Cesur and Kelly (2010) find similar nonlinearities with cognitive outcomes. Further, Restrepo (2016) provides evidence that these nonlinearities may be related to maternal investment decisions. Specifically, maternal investment decisions are not homogenous across the distribution of socioeconomic status. Restrepo (2016) provides evidence that the consequences of low birth weight are exacerbated via reinforcing investment decisions by mothers with limited education, while the impacts of low birth weight are mitigated by compensatory investment decisions by well-educated mothers.

Here, we explore the role that infant health plays as it relates to *very early* childhood cognitive development, as opposed to longer-term outcomes, while confronting the challenges of missing data and endogeneity. In particular, we utilize data on children from low-income households, obtained from the ECLS-K:2011. In the ECLS-K:2011, birth weight is missing for a non-trivial fraction of the overall sample and is arguably endogenous even in the absence of missing data. The argument for birth weight being endogenous, in the current context, stems from the idea that unobserved maternal factors during pregnancy that impact birth weight may also be correlated with subsequent early childhood development. Since these latent factors are relegated to the error term, and at the same time correlated with birth weight, the zero conditional mean assumption fails to hold.

To confront this dual challenge of missing data and endogeneity, we first impute missing birth weight data using the imputation methods discussed previously. We then estimate various models of early childhood development via 2SLS instrumenting imputed birth weight with state-level SNAP rules. Meyerhoefer and Pylypchuk (2008) show that these state-level rules influence individual SNAP participation, and SNAP participation is associated with low-income expectant mothers gaining the requisite weight during pregnancy (Baum 2012). In turn, maternal weight gain during pregnancy is correlated with infant birth weight (Shapiro et al. 2000; Ludwig and Currie 2010).

4.2 Data

Collected by the US Department of Education, the ECLS-K:2011 follows a nationally representative sample of approximately 18,200 students across 970 different schools entering kindergarten in Fall 2010. Information is collected on a host of topics, including family background, teacher and school characteristics, and measures of student achievement. We focus on the Fall 2010 kindergarten wave of the survey where nearly 30% of children in the overall sample have missing values for birth weight.

Our outcome of interest is a standardized (mean zero, unit variance) item response theory (IRT) test score for mathematics. In all specifications, we control for a parsimonious set of covariates: birth weight, age, an index of socioeconomic status (SES) and its square, gender, four racial group dummies, an indicator for whether the child's mother was married at birth, three parental education group dummies, an indicator for whether or not the attended school is a public institution, state-level unemployment rate, state-level expenditure per pupil on pre-kindergarten programs, and state-level current expenditure per pupil on public primary and secondary school.[10] The set of controls is intentionally parsimonious as we do not wish to hold constant current attributes of the children that may act as mediators along the causal pathway between birth weight and current cognitive ability (Pearl 2014).

In all estimations, we exclude students with missing test scores and non-singleton births. We further restrict the sample to children living in low-income households, defined as those below 200% of the federal poverty line, and also drop children in the top 1% and bottom 1% of the age distribution. The final sample includes roughly 5,200 students, of which about 15.7% have missing values for birth weight.[11] Of those not missing birth weight, roughly 6.2% can be classified as *low* birth weight and 0.3% can be classified as *very low* birth weight.[12] Survey weights are used throughout the analysis.

To address the potential endogeneity of birth weight, we use data from the USDA SNAP Policy Database and exploit exogenous variation in state-level SNAP participation rules and outreach that were in place while the child was *in utero*. To capture the SNAP rules faced by the mother for the majority of her pregnancy, we use the state-level SNAP variables from the child's birth year if the child was *not* born in the first quarter of the year. Otherwise, we use the state-level SNAP variables from the year preceding the child's birth year. The three exclusion restrictions used include: state-level per capita outreach expenditures (in 2005 dollars), an indicator for whether SNAP applicants must be fingerprinted in all or part of the state, and an indicator for the state using simplified reporting measures. Each of these variables is potentially correlated with birth weight in low-income households, through SNAP participation, by making households more aware of program benefits and/or lowering certification/recertification costs associated with satisfying SNAP eligibility requirements. However, since the exclusion restrictions affect birth weight via SNAP participation, the instruments may be weak. Thus, the choice of imputation approach becomes even more salient.[13]

Summary statistics can be found in Table 9. Roughly 60% of the sample is non-white, with 35% being Hispanic, and less than 50% of the children were born to parents who were married at the time. Additionally, roughly 23%

[10] Components utilized by the National Center for Education Statistics in construction of the SES index include father and mother's education, father and mother's occupation, and household income.

[11] The number of observations is rounded to nearest ten per NCES restricted data guidelines. The restricted version of the data is utilized in order to have state of residence for the children.

[12] Conventional thresholds for low and very low birth weight are 2,500 grams ($\approx$ 88 ounces) and 1,500 grams ($\approx$ 53 ounces), respectively.

[13] Further weakening the instruments is the fact that the data only contain a child's current state of residence (during fall kindergarten), not the state of birth. However, given the historically low interstate mobility rates during the sample period, particularly among low-income households, this should not have a large impact on the quality of the instruments (Molloy et al. 2011).

of interviewed parents had less than a high school diploma, 38% had at most graduated from high school, and 29% had taken some college classes, but not attained a four year degree. The average age for children in the sample is approximately 5.5 years. Birth weight ranges from approximately 3 to 11 pounds, with the mean about seven pounds.

4.3 Results

The results are reported in Table 10. Focusing on our covariate of interest, birth weight, we report the results for the two ad hoc approaches (CC, DV) as well as the imputation approaches that include the instruments in the imputation procedure (NN1, NN2, MI1-NN, MI2-NN, Reg1, Reg2, Reg3, Reg4, MI1-Reg, and MI2-Reg).[14]

Looking at the full sample results in Panel A, three findings stand out. First, the instruments are quite weak; the first-stage F-statistics are less than five in all cases. However, the imputation procedures utilizing only the instruments in the imputation model (NN2, Reg3, and Reg4) yield much stronger instruments than the other approaches, particularly the ad hoc approaches. Second, despite the weak instruments, the effects of birth weight are reasonably precise; the standard errors are less than or equal to 0.03 in all cases. Third, the effect of birth weight is statistically significant at conventional levels according to all estimators; the magnitude of the impacts range from 0.034 to 0.039 for the ad hoc approaches, 0.042 to 0.051 for the NN estimators, and 0.035 to 0.039 for the Reg estimators. Additionally, the instruments pass the under identification test in all cases except for the DV and NN1 cases and pass the over identification test in all cases.

To assess heterogeneity in the effect, we partition the data into four subgroups: non-white boys, non-white girls, white boys, and white girls. The results are presented in Panels B-E. What becomes immediately clear, in terms of both the impact of birth weight on math achievement and the overall performance of the instruments, is that the results for the full sample are driven by non-white boys. The point estimates for non-white boys range from 0.024 to 0.031, with the estimates being statistically significant at conventional levels across all estimators. The point estimates for non-white girls, white boys, and white girls are closer to zero in magnitude and are statistically insignificant at conventional levels across all estimators. Additionally, though arguably still weak, the instruments are much stronger in the non-white boy subsample relative to the remaining subsamples. In particular, the first-stage F-statistics, across all estimators, are on average 1.17, 1.89, and 9.68 times stronger relative to those obtained using the non-white girl, white boy, and white girl subsamples, respectively. Further, the F-statistics now range between 6.97 (NN2) and 7.53 (Reg1) for the imputation estimators. The instruments also tend to do better in terms of passing the under/over identification tests for non-white children (both boys and girls), yet tend to not perform well when looking at just white children. This is especially true for the subsample of white girls, where the instruments fail the under/over identification tests across all estimators with the first-stage F-statistics ranging from only 0.35 (DV) to 1.03 (Reg2). Lastly, and similar to the results for the full sample, the imputation procedures utilizing only the instruments in the imputation model (NN2, Reg3, and Reg4) generally result in stronger instruments relative to the other approaches, with this result holding across all subsamples. This result continues to be particularly true relative to the ad hoc approaches.

In sum, using a recent cohort of children in low-income households, we find evidence of an economically and

[14] Estimation results for the other covariates are available upon request.

statistically significant impact of birth weight on early childhood development as measured by math test scores at the beginning of kindergarten for non-white male children. To put the results in context, a 10% increase in birth weight for the average non-white male child in the sample yields an approximate 0.35 standard deviation (SD) improvement in math test scores. For comparison, Figlio et al. (2014) find about a 0.05 SD improvement when averaging math and reading test scores and pooling students in grades 3-8. Chatterji et al. (2014) find about a 0.04 SD improvement in math scores using children of all ages below 18.

Our much larger effects could be due to our focus on test scores upon kindergarten entry, rather than later in primary and secondary school. For example, Del Bono and Ermisch (2009) find small effects of birth weight on the cognitive performance of three year olds and that the effects fade over time. The larger effects found here could also be attributable to heterogeneous effects, not only for low-income, non-white, male children, but also for the set of 'compliers' with our instruments (Imbens and Angrist 1994). Moreover, our sizeable effects hold only for math. While not reported, 2SLS estimates of the effect of birth weight on standardized reading test scores are statistically and economically insignifcant at conventional levels.[15] Finally, the magnitude of our results pertaining to math test scores do not appear to be driven by weak instruments biasing the estimates toward OLS estimates nor our use of birth weight in levels as opposed to logs. While not shown, the OLS estimates (in levels) are positive and statistically significant at conventional levels but are an order of magnitude smaller than the 2SLS estimates.[16] Furthermore, 2SLS estimates using the log of birth weight yields similar results to our level estimates.[17]

5 Conclusion

Basmann (1957) introduces 2SLS as a means of estimating structural models that suffer from endogeneity when exclusion restrictions are available. To state that this method is at the core of the applied econometrician's toolkit is an understatement. Subsequent work by Basmann and coauthors, as well as others building on his work, investigates the finite sample performance of the 2SLS estimator. However, one setting where the properties of 2SLS have not been adequately assessed pertains to 2SLS estimation of a structural model when data on the endogenous covariate(s) are missing for some observations. Not only does such an investigation pay homage to the enduring legacy of Basmann, but it also fills an important gap in the literature as missing data is a common occurrence in data analysis. Ad hoc approaches to this problem, often used by researchers, lack any formal justification and imputation procedures introduce measurement error in the endogenous covariate. Recently, Millimet (2015) shows that the finite sample bias of 2SLS is not monotonically decreasing with the measurement accuracy of an endogenous covariate. As such, the most accurate imputation method may not be the method that minimizes the finite sample bias of 2SLS.

In light of this, we investigate the finite sample performance of several approaches to dealing with missing covariate data when the covariate is endogenous even in the absence of any missingness. Our Monte Carlo results suggest that imputation methods that incorporate the instruments along with other exogenous covariates into the imputation model generally produce the smallest finite sample bias of the 2SLS estimator. This is attributable, at least in

[15] The OLS and 2SLS point estimates for reading are smaller than 0.012 across all estimators and typically smaller than 0.005. However, the instruments often fail the overidentification tests in the sub-samples, although not the full sample. Results are available upon request.

[16] For example, the CC estimate for non-white boys is 0.0037 (s.e. = 0.0010). The Reg1 – Reg4 point estimates range from 0.0037 to 0.0039, each with a standard error of 0.0010.

[17] For example, the CC estimate for non-white boys is 3.21 (s.e. = 1.48, $F = 3.85$). The Reg1 – Reg4 point estimates range from 3.17 (s.e. = 1.30, $F = 5.91$) to 3.34 (s.e. = 1.38, $F = 5.40$).

part, to the improved instrument strength in the resulting first-stage estimation. Among the ad hoc approaches, the complete case approach often does surprisingly well, while the missing-indicator approach does not. In terms of our application, however, we find surprisingly little substantive difference across the various estimators, although the estimators that incorporate the instruments into the imputation model do lead to better instrument strength.

References

[1] Abrevaya, J. and S.G. Donald (2013), "A GMM approach for dealing with missing data on regressors and instruments," Working Paper.

[2] Allison, P.D. (2002), *Missing Data*. Series: Quantitative Applications in the Social Sciences. California: Sage Publications.

[3] Almond, D., K. Chay, and D. Lee (2005), "The costs of low birth weight," *Quarterly Journal of Economics*, 120, 1031-1083.

[4] Andridge, R.R. and R.J.A. Little (2010), "A review of hot deck imputation for survey non-response," *International Statistical Review*, 78, 40-64.

[5] Baum, C.L. (2012), "The effects of food stamp receipt on weight gained by expectant mothers," *Journal of Population Economics*, 25, 1307-1340.

[6] Bassman, R.L. (1957), "A generalized classical method of linear estimation of coefficients in a structural equation," *Econometrica*, 25, 77-83.

[7] Bassman, R.L. (1959), "The computation of generalized classical estimates of coefficients in a structural equation," *Econometrica*, 27, 72-81.

[8] Basmann, R.L. (1960), "On finite sample distributions of generalized classical linear identifiability test statistics," *Journal of the American Statistical Association*, 55, 650-659.

[9] Basmann, R.L. (1963), "Remarks concerning the application of exact finite sample distribution functions of GCL estimators in econometric statistical inference," *Journal of the American Statistical Association*, 58, 943-976.

[10] Basmann, R.L., F.L. Brown, W.S. Dawes, and G.K. Schoepfle (1971), "Exact finite sample density functions of GCL estimators of structural coefficients in a leading exactly identifiable case," *Journal of the American Statistical Association*, 66, 122-26.

[11] Bijwaard, G.E., H. van Kippersluis, and J. Veenman (2015), "Education and health: The role of cognitive ability," *Journal of Health Economics*, 42, 29-43.

[12] Black, D.A., M.C. Berger, and F.A. Scott (2000), "Bounding Parameter Estimates with Nonclassical Measurement Error," *Journal of the American Statistical Association*, 95, 739-748.

[13] Black, S.E., P.J. Devereux, and K.G. Salvanes (2007), "From the cradle to the labor market? The effect of birth weight on adult outcomes," *Quarterly Journal of Economics*, 122, 409-439.

[14] Bound, J., D.A. Jaeger, and R.M. Baker (1995), "Problems with instrumental variables estimation when the correlation between the instruments and the endogenous explanatory variable is weak," *Journal of the American Statistical Association*, 90, 443-450.

[15] Bun, M.J.G. and F. Windmeijer (2011), "A comparison of bias approximations for the two stage least squares (2SLS) estimator," *Economics Letters*, 113, 76-79.

[16] Burton, A. and D.G. Altman (2004), "Missing covariate data within cancer prognostic studies: A review of current reporting and proposed guidelines," *British Journal of Cancer*, 91, 4-8.

[17] Case, A., A. Fertig, and C. Paxson (2005), "The lasting impact of childhood health and circumstance," *Journal of Health Economics*, 24, 365-389.

[18] Cesur, R. and I.R. Kelly (2010), "From cradle to classroom: High birth weight and cognitive outcomes," *Forum for Health Economics & Policy*, 13, 1-24.

[19] Chatterji, P., D. Kim, and K. Lahiri (2014), "Birth weight and academic achievement in childhood," *Health Economics*, 23, 1013-1035.

[20] Conti, G. and J.J. Heckman (2010), "Understanding the early origins of the education-health gradient: A framework that can also be applied to analyze gene-environment interactions," *Pespectives on Psychological Science*, 5, 585-605.

[21] Currie, J. and R. Hyson (1999), "Is the impact of health shocks cushioned by socioeconomic status? The case of low birth weight," *American Economic Review*, 89, 245-250.

[22] Dardanoni, V., S. Modica, and F. Peracchi (2011), "Regression with imputed covariates: A generalized missing indicator approach," *Journal of Econometrics*, 162, 362-368.

[23] Del Bono, E., and J. Ermisch (2009) "Birth weight and the dynamics of early cognitive and behavioural development," IZA Discussion Paper No. 4270.

[24] Feng, Q. (2016), "Instrumental Variables Estimation with Missing Instruments," U Texas, unpublished manuscript.

[25] Figlio, D., J. Guryan, K. Karbownik, and J. Roth (2014), "The effects of poor neonatal health on children's cognitive development," *American Economic Review*, 104, 3921-3955.

[26] Flores-Lagunes, A. (2007), "Finite sample evidence of IV estimation under weak instruments," *Journal of Applied Econometrics*, 22, 677-694.

[27] Gimenez-Nadal, J.I. and J.A. Molina (2016), "Commuting time and household responsibilities: Evidence using propensity score matching," *Journal of Regional Science*, 56, 332-359.

[28] Grantham-McGregor, S., Y.B. Cheung, S. Cueto, P. Glewwe, L. Richter, B. Strupp, and the International Child Development Steering Group (2007), "Developmental potential in the first 5 years for children in developing countries," *Lancet*, 369, 60-70.

[29] Hahn J. and J. Hausman (2002), "Note on bias in estimators for simultaneous equation models," *Economics Letters*, 75, 237-241.

[30] Heckman, J.J., J. Strixrud, and S. Urzua (2006), "The effects of cognitive and noncognitive abilities on labor market outcomes and social behavior," *Journal of Labor Economics*, 24, 411-481.

[31] Hansen, L.P. (1982), "Large sample properties of generalized method of moments estimators," *Econometrica*, 50, 1029-1054.

[32] Horton, N.J. and K.P. Kleinman (2007), "Much ado about nothing: A comparison of missing data methods and software to fit incomplete data regression models," *American Statistician*, 61, 79-90.

[33] Ibrahim, J.G., M-H. Chen, S.R. Lipsitz, and A.H. Herring (2005), "Missing-data methods for generalized linear models: A comparative review," *Journal of the American Statistical Association*, 100, 332-347.

[34] Imbens, G.W. and J.D. Angrist (1994), "Identification and estimation of local average treatment effects," *Econometrica*, 62, 467-475.

[35] Jones, M.P. (1996), "Indicator and stratification methods for missing explanatory variables in multiple linear regression," *Journal of the American Statistical Association*, 91, 222-230.

[36] Kirkegaard, I., C. Obel, M. Hedegaard, T.B. Henriksen (2006), "Gestational age and birth weight in relation to school performance of 10-year-old children: a follow-up study of children born after 32 completed weeks," *Pediatrics*, 118, 1600-1606.

[37] Ludwig, D.S. and J. Currie (2010), "The association between pregnancy weight gain and birth weight: a within-family comparison," *Lancet*, 376, 984-990.

[38] Meyerhoefer, C.D. and Y. Pylypchuk (2008), "Does participation in the food stamp program increase prevalence of obesity in health care spending?" *American Journal of Agricultural Economics*, 90, 287-305.

[39] Millimet, D.L. (2015), "Covariate measurement and endogeneity," *Economics Letters*, 136, 59-63.

[40] Mittinty, M.N. and E. Chacko (2005), "Imputation by propensity matching," In *American Statistical Association, Proceedings of the Survey Research Methods*, 4022-4028.

[41] Molloy, R.S., C.L. Smith, and A. Wozniak (2011), "Internal migration in the united states," *Journal of Economic Perspectives*, 25, 173-196.

[42] Nagar, A.L. (1959), "The bias and moment matrix of the general k-class estimators of the parameters in simultaneous equations," *Econometrica*, 27, 575-595.

[43] Oreopoulos, P., M. Stabile, R. Walld, and L.L. Roos (2008), "Short-, medium-, and long-term consequences of poor infant health: An analysis using siblings and twins," *Journal of Human Resources*, 43, 88-138.

[44] Pearl, J. (2014), "Interpretation and identification of causal mediation," *Psychological Methods*, 19, 459-481.

[45] Pigott, T.D. (2001), "A review of methods for missing data," *Educational Research and Evaluation*, 7, 353-383.

[46] Pitman, E.J.G. (1937), "The 'closest' estimates of statistical parameters," *Mathematical Proceedings of the Cambridge Philosophical Society*, 33, 212-222.

[47] Restrepo, B.J. (2016), "Parental investment responses to a low birth weight outcome: who compensates ans who reinforces?" *Journal of Population Economics*, 29, 969-989.

[48] Richards, M., R. Hardy, D. Kuh, M.E.J. Wadsworth (2001), "Birth weight and cognitive function in the British 1946 birth cohort: longitudinal population based study," *British Medical Journal*, 322, 199-203.

[49] Rosenbaum, P.R. (2002), *Observational Studies, 2nd Edition*. New York: Springer.

[50] Rubin, D.B. (1987), *Multiple Imputation for Nonresponse in Surveys*. New York: Wiley.

[51] Sargan, J.D. (1958), "The estimation of economic relationships using instrumental variables," *Econometrica*, 26, 393-415.

[52] Schafer, J.L. and J.W. Graham (2002), "Missing data: Our view of the state of the art," *Psychological Methods*, 7, 147-177.

[53] Shapiro, C., V.G. Sutija, and J. Bush (2000), "Effect of maternal weight gain on infant birth weight," *Journal of Perinatal Medicine*, 28, 428-431.

[54] Stock, J.H., J.H. Wright, and M. Yogo (2002), "A survey of weak instruments and weak identification in generalized method of moments," *Journal of Business and Economic Statistics*, 20, 518-529.

Table 3. Monte Carlo Results: Aggregated Across All Experiments by Missingness Mechanism or Instrument Strength.

	CC	DV	NN1	NN2	NN3	MI1-NN	MI2-NN	Reg1	Reg2	Reg3	Reg4	Reg5	Reg6	MI1-Reg	MI2-Reg
A. MCAR															
Median Bias	0.020	-0.194	-0.004	-0.131	0.191	-0.065	0.026	-0.009	-0.008	-0.109	-0.117	0.232	0.233	-0.064	0.036
RMSE	0.323	0.548	0.359	0.411	0.485	0.342	0.341	0.341	0.330	0.390	0.392	0.493	0.493	0.331	0.354
B. MAR (x_1 only)															
Median Bias	0.076	-0.003	-0.004	-0.055	0.206	-0.022	0.060	0.015	0.003	-0.035	-0.038	0.263	0.259	-0.013	0.081
RMSE	0.471	0.512	0.471	0.560	0.639	0.452	0.454	0.460	0.465	0.541	0.519	0.634	0.638	0.438	0.471
C. MAR (x_1, z)															
Median Bias	0.128	-0.219	-0.090	-0.233	0.150	-0.160	-0.049	-0.027	-0.038	-0.187	-0.192	0.234	0.280	-0.106	0.014
RMSE	0.733	1.001	0.819	0.865	0.956	0.715	0.661	0.804	0.742	0.868	0.856	1.163	0.965	0.649	0.700
D. NMAR															
Median Bias	0.032	-0.192	0.012	-0.130	0.211	-0.057	0.036	-0.005	0.001	-0.107	-0.114	0.243	0.245	-0.061	0.042
RMSE	0.317	0.542	0.393	0.422	0.530	0.359	0.364	0.345	0.334	0.389	0.395	0.504	0.502	0.336	0.360
E. τ^2 = 2															
Median Bias	0.103	-0.176	-0.047	-0.207	0.150	-0.118	-0.021	-0.006	-0.011	-0.151	-0.163	0.236	0.248	-0.082	0.029
RMSE	0.601	0.759	0.653	0.702	0.783	0.584	0.560	0.655	0.608	0.716	0.668	0.954	0.813	0.550	0.597
F. τ^2 = 5															
Median Bias	0.052	-0.144	-0.011	-0.122	0.195	-0.064	0.027	-0.007	-0.009	-0.100	-0.105	0.242	0.249	-0.057	0.045
RMSE	0.467	0.660	0.514	0.564	0.662	0.467	0.452	0.481	0.454	0.538	0.559	0.673	0.639	0.437	0.468
G. τ^2 = 10															
Median Bias	0.031	-0.122	0.007	-0.079	0.218	-0.036	0.051	-0.003	-0.006	-0.071	-0.072	0.246	0.252	-0.043	0.055
RMSE	0.380	0.617	0.438	0.496	0.573	0.402	0.390	0.401	0.405	0.458	0.475	0.571	0.553	0.364	0.387

Notes: Results from 2SLS estimation based N=500 with approximinately 20% missing data and 500 simulations per experimental design. RMSE = root mean squared error. Median bias and RMSE are computec over all experiments for a given missingness mechanism (Panels A-D) or instrument strength (Panels E-G) reported in Tables A1-A16. Lowest value highlighted in gray. 'No missing' refers to the situation where x_2 is fully observed. CC is the complete case estimator. DV is the ad hoc dummy variable approach. NN1 is the nearest neighbor matching estimator with matching based on x_1 and its quadratic, z and the quadratic of each element, and the interaction between x_1 and each element of z. NN2 (NN3) matches only on z and the quadratic of each element (x_1 and its quadratic). Reg1-Reg6 are various regression imputation estimators. Reg1 uses x_1 and z. Reg2 uses the variables in Reg1 plus quadratics of each variable and interactions between x_1 and each element of z. Reg3 uses z only. Reg4 uses the variables in Reg3 plus quadratics of each variable. Reg5 uses x_1 only. Reg6 uses the variables in Reg5 plus the quadratic for x_1. MI1-NN (MI2-NN) is multiple imputation using NN1-NN2 (NN1-NN3). MI1-Reg (MI2-Reg) is multiple imputation using Reg1-Reg4 (Reg1-Reg6). See text for further details.

Table 4. Monte Carlo Results: Aggregated Across All Experiments by DGP.

	CC	DV	NN1	NN2	NN3	MI1-NN	MI2-NN	Reg1	Reg2	Reg3	Reg4	Reg5	Reg6	MI1-Reg	MI2-Reg
A. Linear: $\rho(x_1,z) = 0$															
Median Bias	0.046	0.005	0.010	-0.019	0.221	0.001	0.081	0.001	-0.005	0.010	0.005	0.287	0.271	0.006	0.100
RMSE	0.366	0.389	0.433	0.544	0.595	0.426	0.428	0.381	0.386	0.440	0.528	0.622	0.568	0.388	0.432
B. Nonlinear: $\rho(x_1,z) = 0$															
Median Bias	0.050	-0.055	-0.017	-0.094	0.191	-0.055	0.029	-0.012	-0.008	-0.051	-0.069	0.207	0.239	-0.038	0.049
RMSE	0.542	0.621	0.573	0.613	0.706	0.514	0.482	0.600	0.530	0.669	0.586	0.873	0.699	0.474	0.511
C. Linear: $\rho(x_1,z) \neq 0$															
Median Bias	0.051	-0.534	-0.010	-0.184	0.188	-0.088	0.010	-0.003	-0.011	-0.206	-0.198	0.254	0.249	-0.096	0.024
RMSE	0.431	0.670	0.485	0.446	0.594	0.416	0.416	0.457	0.451	0.440	0.434	0.632	0.635	0.393	0.430
D. Nonlinear: $\rho(x_1,z) \neq 0$															
Median Bias	0.074	-0.286	-0.030	-0.244	0.175	-0.127	-0.016	-0.007	-0.009	-0.237	-0.216	0.229	0.241	-0.111	0.007
RMSE	0.593	0.933	0.652	0.734	0.796	0.586	0.551	0.617	0.595	0.718	0.708	0.839	0.788	0.552	0.578
E. $\pi_{21} = 0$															
Median Bias	0.057	0.000	-0.006	-0.018	0.197	-0.008	0.064	0.001	-0.009	0.002	-0.002	0.253	0.258	-0.001	0.086
RMSE	0.523	0.535	0.525	0.535	0.688	0.479	0.484	0.542	0.506	0.564	0.503	0.789	0.712	0.482	0.540
F. $\pi_{21} = 1$															
Median Bias	0.053	-0.552	-0.018	-0.267	0.191	-0.128	-0.013	-0.011	-0.008	-0.250	-0.256	0.232	0.243	-0.117	0.005
RMSE	0.457	0.801	0.559	0.647	0.668	0.502	0.461	0.503	0.487	0.598	0.635	0.709	0.641	0.430	0.438

Notes: Median bias and RMSE over all experiments for different DGPs reported in Tables A1-A16. Lowest value highlighted in gray. See Table 3 and text for further details.

Table 5. Monte Carlo Results: Median Ranks Across Experiments by Missingness Mechanism.

Estimator	MCAR	MAR (x_1 only)	MAR (x_1, z)	NMAR	$\tau^2 = 2$	$\tau^2 = 5$	$\tau^2 = 10$
A. BIAS							
CC	4	9	4.5	4.5	6.5	5.5	6
DV	12	9.5	12	12	11.5	12	12
NN1	4	4	6.5	4	5	4	5
NN2	10	6	11	10.5	12	9	7.5
NN3	13	13	8.5	13	10.5	13	13
MI1-NN	7	6.5	8.5	7	9	7	7
MI2-NN	9	10	8	9.5	6.5	9	10
Reg1	4	5	5	4	5	5	4
Reg2	4.5	4	4	4	5	4	4
Reg3	7	6	9	8	8	8	7
Reg4	8	6	9.5	7.5	8.5	7	7.5
Reg5	14	14	12.5	14	14	14	14
Reg6	15	14	12	14	14	14	14
MI1-Reg	7	6	8	7	7	7	6
MI2-Reg	9.5	11	6.5	10	6	10.5	10.5
B. RMSE							
CC	3.5	5	5	1	4	3	3
DV	11.5	8	8.5	11	9	10.5	11
NN1	9	9	10	9	9	9	9
NN2	9	9	10	10	11	9	9
NN3	14	15	13	15	13	14	15
MI1-NN	7	7	6	7.5	6	7	7
MI2-NN	8.5	9	3.5	9	5	8.5	11
Reg1	4	4.5	6	5	7	4	3
Reg2	5	5	6	4	5	4.5	4
Reg3	5	5	7.5	6	7	6	5.5
Reg4	7	5	7	6.5	6.5	6	7
Reg5	13	14	14	13	14	13	13
Reg6	14	13	13	14	14	14	14
MI1-Reg	4	3	2	4	3	4	4
MI2-Reg	9	9	7	9.5	8	8.5	8.5

Notes: Median rank over all experiments for a given missingness mechanism or instrument strength reported in Tables A1-A16. 1 = lowest bias/RMSE, 15 = highest. Lowest value highlighted in gray. See Table 3 and text for further details.

Table 6. Monte Carlo Results: Median Ranks Across Experiments by DGP.

Estimator	Linear $\rho(x_1,z) = 0$	Nonlinear $\rho(x_1,z) = 0$	Linear $\rho(x_1,z) \neq 0$	Nonlinear $\rho(x_1,z) \neq 0$	$\pi_{21} = 0$	$\pi_{21} = 1$
A. BIAS						
CC	8.5	6.5	6	5.5	8	5
DV	9	9.5	14	13.5	9	15
NN1	4.5	6	4	4	6	4
NN2	8	9.5	11	11.5	5.5	12
NN3	13	13	9.5	9	13	9
MI1-NN	5	7	7	7	5.5	7
MI2-NN	10	10	6	6.5	10	6
Reg1	5	5	5	4	6	4
Reg2	5	4	4	4	5	3
Reg3	6	7.5	10.5	9.5	5.5	11.5
Reg4	6.5	8	10	7	6	12
Reg5	14	14	14	13.5	14	12.5
Reg6	15	14	12.5	11.5	15	11.5
MI1-Reg	6	6	8	8	6	8
MI2-Reg	11	11	5	7	11	6
B. RMSE						
CC	1	2	6	3.5	4	3
DV	5	7	12	12	7	12
NN1	9.5	10	9	8.5	10	8
NN2	11	11	9	10.5	9	12
NN3	15	14.5	14	12.5	15	13
MI1-NN	8	8	7	7	7	7
MI2-NN	10	8	6.5	4.5	11	5
Reg1	3	5.5	4	6	5	4
Reg2	5	4	5	4	5	3.5
Reg3	5	6	5.5	9.5	4	9
Reg4	6.5	6.5	5	10	4	10
Reg5	13	13	13	13	13	13
Reg6	14	14	14	13	14	12
MI1-Reg	4	3	3	4	3	4
MI2-Reg	10	9	7.5	5.5	10	4

Notes: Median rank over all experiments for a given DGP reported in Tables A1-A16. 1 = lowest bias/RMSE, 15 = highest. Lowest value highlighted in gray. See Table 3 and text for further details.

Table 7. Monte Carlo Results: Pitman's Nearness Measures Aggregated Across All Experiments by Missingness Mechanism or Instrument Strength.

Estimator	MCAR	MAR (x_1 only)	MAR (x_1, z)	NMAR	$\tau^2 = 2$	$\tau^2 = 5$	$\tau^2 = 10$
CC vs. DV	0.668	0.565	0.649	0.664	0.615	0.637	0.657
CC vs. NN1	0.540	0.513	0.547	0.558	0.532	0.544	0.544
CC vs. NN2	0.625	0.552	0.593	0.634	0.590	0.601	0.613
CC vs. MI1-NN	0.554	0.507	0.559	0.568	0.534	0.548	0.559
CC vs. MI2-NN	0.552	0.533	0.540	0.571	0.526	0.549	0.571
CC vs. Reg1	0.499	0.486	0.486	0.527	0.497	0.501	0.501
CC vs. Reg2	0.500	0.482	0.487	0.520	0.494	0.502	0.496
CC vs. Reg3	0.594	0.535	0.568	0.609	0.561	0.580	0.589
CC vs. Reg4	0.600	0.530	0.584	0.612	0.568	0.584	0.593
CC vs. MI1-Reg	0.532	0.492	0.504	0.548	0.504	0.522	0.531
CC vs. MI2-Reg	0.542	0.532	0.529	0.560	0.523	0.542	0.557
NN1 vs. NN2	0.606	0.551	0.566	0.595	0.576	0.580	0.582
NN1 vs. Reg1	0.433	0.453	0.418	0.439	0.444	0.432	0.431
NN1 vs. Reg2	0.429	0.453	0.431	0.431	0.443	0.433	0.432
Reg1 vs. MI1-NN	0.585	0.546	0.583	0.577	0.563	0.576	0.580
Reg1 vs. MI2-NN	0.561	0.561	0.554	0.558	0.543	0.559	0.574
Reg1 vs. Reg2	0.498	0.500	0.513	0.476	0.498	0.499	0.493
Reg1 vs. MI1-Reg	0.555	0.515	0.533	0.541	0.524	0.540	0.545
Reg1 vs. MI2-Reg	0.551	0.565	0.551	0.554	0.542	0.553	0.570
Reg2 vs. MI1-NN	0.587	0.544	0.576	0.583	0.561	0.576	0.580
Reg2 vs. MI2-NN	0.566	0.552	0.551	0.569	0.542	0.559	0.578
Reg2 vs. MI1-Reg	0.546	0.512	0.517	0.557	0.518	0.534	0.547
Reg2 vs. MI2-Reg	0.558	0.556	0.538	0.563	0.539	0.553	0.570
MI1-Reg vs. MI2-Reg	0.483	0.549	0.524	0.490	0.504	0.508	0.522

Notes: Figures represent the empirical probablity that the first estimator listed in first column is closer in absolute value to the true parameter than the second estimator. See Table 3 and text for further details.

Table 8. Monte Carlo Results: Pitman's Nearness Measures Aggregated Across All Experiments by DGP.

Estimator	Linear $\rho(x_1,z) = 0$	Nonlinear $\rho(x_1,z) = 0$	Linear $\rho(x_1,z) \neq 0$	Nonlinear $\rho(x_1,z) \neq 0$	$\pi_{21} = 0$	$\pi_{21} = 1$
CC vs. DV	0.525	0.575	0.734	0.710	0.520	0.752
CC vs. NN1	0.545	0.546	0.527	0.541	0.530	0.549
CC vs. NN2	0.573	0.602	0.589	0.640	0.526	0.676
CC vs. MI1-NN	0.545	0.551	0.528	0.565	0.514	0.580
CC vs. MI2-NN	0.577	0.560	0.521	0.538	0.548	0.549
CC vs. Reg1	0.493	0.510	0.480	0.515	0.490	0.508
CC vs. Reg2	0.504	0.503	0.488	0.495	0.496	0.499
CC vs. Reg3	0.523	0.546	0.590	0.646	0.492	0.661
CC vs. Reg4	0.545	0.565	0.585	0.631	0.491	0.672
CC vs. MI1-Reg	0.508	0.514	0.504	0.550	0.489	0.549
CC vs. MI2-Reg	0.574	0.553	0.503	0.533	0.548	0.534
NN1 vs. NN2	0.544	0.572	0.588	0.614	0.492	0.666
NN1 vs. Reg1	0.425	0.439	0.420	0.459	0.437	0.435
NN1 vs. Reg2	0.439	0.431	0.441	0.434	0.445	0.427
Reg1 vs. MI1-NN	0.571	0.568	0.579	0.574	0.541	0.604
Reg1 vs. MI2-NN	0.589	0.563	0.551	0.531	0.562	0.555
Reg1 vs. Reg2	0.529	0.480	0.510	0.468	0.508	0.485
Reg1 vs. MI1-Reg	0.525	0.509	0.562	0.549	0.492	0.580
Reg1 vs. MI2-Reg	0.585	0.555	0.535	0.545	0.574	0.536
Reg2 vs. MI1-NN	0.552	0.576	0.570	0.591	0.533	0.611
Reg2 vs. MI2-NN	0.571	0.574	0.535	0.558	0.553	0.567
Reg2 vs. MI1-Reg	0.492	0.522	0.548	0.570	0.478	0.589
Reg2 vs. MI2-Reg	0.571	0.572	0.515	0.558	0.559	0.549
MI1-Reg vs. MI2-Reg	0.582	0.536	0.464	0.464	0.580	0.443

Notes: Figures represent the empirical probablity that the first estimator listed in first column is closer in absolute value to the true parameter than the second estimator. See Table 3 and text for further details.

Table 9. Summary Statistics: ECLS-K:11.

	N	Mean	SD	Min	Max
Math Achievement (z-score)	5200	-0.015	0.983	-2.097	4.381
Birthweight (oz.)	4360	114.854	21.384	48.000	175.000
Birthweight Missing (1 = Yes)	5200	0.157	0.364	0	1
Child Age (months)	5200	67.443	4.189	59.050	81.140
Female (1 = Yes)	5200	0.493	0.500	0	1
Black (1 = Yes)	5200	0.181	0.385	0	1
Hispanic (1 = Yes)	5200	0.354	0.478	0	1
Asian (1 = Yes)	5200	0.028	0.164	0	1
Other Race (1 = Yes)	5200	0.059	0.235	0	1
Socioeconomic Status (Index)	5200	-0.597	0.559	-2.330	1.950
Parents Married at Birth (1 = Yes)	5200	0.496	0.500	0	1
Parental Education (1 = Less than High School)	5200	0.225	0.418	0	1
Parental Education (1 = High School)	5200	0.381	0.486	0	1
Parental Education (1 = Some College, No Four Year Degree)	5200	0.286	0.452	0	1
Public School (1 = Yes)	5200	0.953	0.213	0	1
State Unemployment Rate	5200	8.960	1.848	4.167	13.058
State Pre-K Expenditures (all sources, per pupil)	5200	4.428	2.621	0	11.669
State Current Public School Expenditures (per pupil)	5200	10.194	2.646	6.212	19.076
SNAP State-Level Per Capita Outreach Expenditures	5200	0.042	0.125	0	1
SNAP Fingerprinting (1 = Yes)	5200	0.309	0.462	0	1
SNAP Simplified Reporting Procedures (1 = Yes)	5200	0.792	0.394	0	1

Notes: Sample restricted to singleton children in households in below 200% of the federal poverty line. Parental education refers to the "first parent" interviewed in the survey. For 94% (2%) of children, this is the biological mother (father). State-level SNAP variables are taken from a child's birth year if the child was not born in the first quarter, the preceding calendar year otherwise. Sample weights used. Number of observations rounded to nearest ten per NCES restricted data guidelines. See text for further details.

Table 10. Determinants of Math Achievement.

	CC	DV	NN1	NN2	MI1-NN	MI2-NN	Reg1	Reg2	Reg3	Reg4	MI1-Reg	MI2-Reg
A. Full Sample												
Birth Weight (oz.)	0.039 †	0.034 †	0.051 ‡	0.035 †	0.043	0.042 ‡	0.038 †	0.035 †	0.036 †	0.036 †	0.036 †	0.039 †
	(0.019)	(0.017)	(0.030)	(0.016)	(0.027)	(0.025)	(0.017)	(0.017)	(0.016)	(0.016)	(0.016)	(0.018)
First-Stage F	3.109	2.050	1.797	4.650			4.252	4.051	4.863	4.738		
Under Id. (p-value)	p = 0.059	p = 0.107	p = 0.223	p = 0.011			p = 0.017	p = 0.022	p = 0.008	p = 0.010		
Over Id. (p-value)	p = 0.226	p = 0.455	p = 0.373	p = 0.237			p = 0.264	p = 0.177	p = 0.282	p = 0.259		
N	4360	5200	5200	5200	5200	5200	5200	5200	5200	5200	5200	5200
B. Non-White Boys												
Birth Weight (oz.)	0.028 †	0.024 ‡	0.029 †	0.029 †	0.029 †	0.031 †	0.028 †	0.027 †	0.028 †	0.029 †	0.028 †	0.030 †
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.013)	(0.011)	(0.011)	(0.011)	(0.012)	(0.011)	(0.013)
First-Stage F	4.928	2.897	5.752	6.969			7.533	7.239	7.388	7.003		
Under Id. (p-value)	p = 0.009	p = 0.054	p = 0.004	p = 0.001			p = 0.001	p = 0.001	p = 0.001	p = 0.001		
Over Id. (p-value)	p = 0.272	p = 0.067	p = 0.316	p = 0.297			p = 0.260	p = 0.236	p = 0.288	p = 0.276		
N	1350	1690	1690	1690	1690	1690	1690	1690	1690	1690	1690	1690
C. Non-White Girls												
Birth Weight (oz.)	0.009	0.011	0.004	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.006	0.006
	(0.013)	(0.013)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.013)
First-Stage F	3.907	2.657	5.489	6.690			5.742	5.670	6.202	6.187		
Under Id. (p-value)	p = 0.020	p = 0.040	p = 0.003	p = 0.001			p = 0.004	p = 0.004	p = 0.002	p = 0.002		
Over Id. (p-value)	p = 0.130	p = 0.052	p = 0.258	p = 0.259			p = 0.268	p = 0.262	p = 0.279	p = 0.278		
N	1320	1640	1640	1640	1640	1640	1640	1640	1640	1640	1640	1640
D. White Boys												
Birth Weight (oz.)	0.008	0.010	0.017	0.011	0.014	0.015	0.012	0.012	0.012	0.012	0.012	0.012
	(0.015)	(0.015)	(0.016)	(0.014)	(0.016)	(0.016)	(0.015)	(0.016)	(0.014)	(0.014)	(0.015)	(0.015)
First-Stage F	2.975	1.666	3.351	3.794			3.761	3.093	3.888	3.902		
Under Id. (p-value)	p = 0.050	p = 0.090	p = 0.031	p = 0.016			p = 0.017	p = 0.039	p = 0.015	p = 0.014		
Over Id. (p-value)	p = 0.018	p = 0.152	p = 0.063	p = 0.042			p = 0.042	p = 0.040	p = 0.042	p = 0.042		
N	860	960	960	960	960	960	960	960	960	960	960	960
E. White Girls												
Birth Weight (oz.)	-0.001	-0.011	-0.014	0.004	-0.005	-0.012	0.001	-0.003	0.002	0.002	0.000	0.000
	(0.036)	(0.036)	(0.041)	(0.033)	(0.040)	(0.056)	(0.033)	(0.028)	(0.036)	(0.034)	(0.033)	(0.034)
First-Stage F	0.564	0.353	0.557	0.671			0.698	1.028	0.610	0.656		
Under Id. (p-value)	p = 0.758	p = 0.683	p = 0.752	p = 0.708			p = 0.700	p = 0.569	p = 0.742	p = 0.720		
Over Id. (p-value)	p = 0.126	p = 0.376	p = 0.173	p = 0.117			p = 0.119	p = 0.129	p = 0.119	p = 0.119		
N	830	920	920	920	920	920	920	920	920	920	920	920

Notes: ‡ $p<0.10$, † $p<0.05$, and * $p<0.01$. Data from the ECLS-K:11. Robust standard errors in parentheses. Achievement (z-score) measured in fall kindergarten. Other covariates included in the model include those listed in Table 9. Instruments for birthweight are real state-level per capita SNAP outreach, a binary indicator if the some or all of the state requires fingerprinting for SNAP, and a binary indicator of SNAP simplified reporting procedures. Instruments are taken from a child's birth year if the child was not born in the first quarter, the preceding calendar year otherwise. Number of observations rounded to nearest ten per NCES restricted data guidelines. Sample weights used. See text for further details.

Supplemental Appendix

"Missing Data, Imputation, and Endogeneity"

Ian K. McDonough
UNLV

Daniel L. Millimet
SMU & IZA

Table A1. Monte Carlo Results: Linear Specification, L = 3, ρ = 0.1, π_{21} = 0, ρ(x_1,z) = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2/L = 2																				
No Missing	0.010	0.438	2.483	0.438	0.453	0.027	0.463	2.681	0.463	0.460	0.028	0.451	2.674	0.451	0.452	0.020	0.458	2.506	0.457	0.473
CC	-0.009	0.549	2.139	0.548	0.544	0.052	0.501	2.245	0.501	0.512	0.015	0.573	2.162	0.573	0.571	0.022	0.495	2.443	0.494	0.505
DV	-0.011	0.551	1.335	0.551	0.543	0.051	0.508	1.397	0.508	0.514	0.033	0.555	1.338	0.554	0.561	0.017	0.504	1.527	0.502	0.507
NN1	-0.110	0.540	2.711	0.528	0.542	-0.087	0.545	2.954	0.537	0.563	-0.113	0.590	2.794	0.581	0.582	-0.063	0.557	3.019	0.554	0.575
NN2	-0.115	0.532	2.766	0.522	0.527	-0.078	0.537	3.038	0.532	0.557	-0.106	0.609	2.924	0.601	0.564	-0.102	0.607	3.241	0.605	0.596
NN3	0.053	0.661	1.894	0.658	0.676	0.113	0.670	1.981	0.660	0.685	0.144	0.790	1.798	0.783	0.794	0.105	0.683	2.040	0.669	0.714
MI1-NN	-0.111	0.511		0.500	0.586	-0.069	0.505		0.498	0.632	-0.091	0.530		0.520	0.657	-0.075	0.536		0.533	0.647
MI2-NN	-0.044	0.511		0.508	0.678	-0.003	0.513		0.512	0.715	-0.011	0.548		0.547	0.791	-0.006	0.531		0.531	0.739
Reg1	-0.073	0.598	3.480	0.593	0.530	-0.038	0.474	3.596	0.473	0.486	-0.050	0.591	3.559	0.589	0.537	-0.049	0.522	3.787	0.522	0.509
Reg2	-0.077	0.543	3.425	0.537	0.510	-0.055	0.498	3.713	0.494	0.486	-0.076	0.574	3.742	0.569	0.523	-0.055	0.510	3.785	0.510	0.512
Reg3	-0.070	0.563	3.464	0.557	0.516	-0.037	0.479	3.623	0.477	0.486	-0.052	0.626	3.637	0.624	0.544	-0.048	0.519	3.825	0.519	0.509
Reg4	-0.077	0.553	3.460	0.547	0.511	-0.046	0.466	3.602	0.464	0.485	-0.067	0.585	3.735	0.582	0.528	-0.055	0.512	3.794	0.512	0.509
Reg5	0.148	0.752	2.122	0.738	0.691	0.194	0.630	2.230	0.598	0.635	0.245	0.796	2.114	0.759	0.741	0.178	0.691	2.472	0.653	0.663
Reg6	0.154	0.757	2.121	0.743	0.698	0.199	0.626	2.234	0.594	0.634	0.273	0.848	2.019	0.803	0.777	0.181	0.696	2.465	0.657	0.666
MI1-Reg	-0.074	0.563		0.557	0.519	-0.044	0.474		0.472	0.492	-0.060	0.575		0.571	0.552	-0.052	0.515		0.515	0.511
MI2-Reg	0.003	0.617		0.617	0.607	0.034	0.513		0.511	0.568	0.052	0.635		0.634	0.670	0.033	0.564		0.561	0.591
B. τ^2/L = 5																				
No Missing	0.007	0.297	5.811	0.297	0.275	0.016	0.254	5.928	0.253	0.262	-0.002	0.273	5.544	0.273	0.267	0.007	0.270	5.387	0.270	0.272
CC	0.000	0.328	4.812	0.328	0.308	0.033	0.278	5.036	0.277	0.294	0.013	0.322	4.088	0.322	0.319	0.013	0.426	5.032	0.426	0.361
DV	-0.001	0.325	2.977	0.325	0.308	0.036	0.279	3.123	0.278	0.294	0.013	0.323	2.536	0.323	0.319	0.011	0.415	3.139	0.414	0.357
NN1	-0.024	0.344	5.654	0.344	0.333	-0.018	0.339	5.968	0.339	0.332	0.000	0.386	5.562	0.386	0.357	0.002	0.350	6.030	0.350	0.355
NN2	-0.034	0.340	5.932	0.340	0.326	-0.023	0.315	6.201	0.315	0.326	-0.024	0.362	5.775	0.362	0.340	-0.046	0.331	6.196	0.331	0.341
NN3	0.179	0.480	3.849	0.440	0.429	0.183	0.505	3.947	0.448	0.430	0.280	0.584	3.196	0.512	0.515	0.172	0.524	3.998	0.481	0.459
MI1-NN	-0.014	0.325		0.324	0.364	-0.023	0.308		0.308	0.363	-0.006	0.358		0.358	0.387	-0.004	0.328		0.328	0.375
MI2-NN	0.053	0.347		0.342	0.435	0.057	0.345		0.335	0.439	0.089	0.382		0.371	0.515	0.038	0.362		0.356	0.456
Reg1	-0.038	0.326	7.485	0.326	0.299	-0.018	0.289	7.924	0.289	0.290	-0.024	0.323	7.009	0.323	0.304	-0.038	0.346	7.902	0.345	0.333
Reg2	-0.034	0.319	7.581	0.318	0.297	-0.015	0.303	7.881	0.303	0.291	-0.042	0.357	7.086	0.356	0.310	-0.036	0.333	7.867	0.332	0.322
Reg3	-0.039	0.323	7.557	0.323	0.298	-0.017	0.290	7.854	0.290	0.289	-0.024	0.327	6.924	0.327	0.305	-0.038	0.340	7.910	0.339	0.328
Reg4	-0.036	0.322	7.561	0.321	0.297	-0.018	0.286	7.927	0.286	0.289	-0.030	0.340	7.130	0.340	0.307	-0.034	0.334	7.890	0.333	0.323
Reg5	0.206	0.474	4.780	0.416	0.389	0.227	0.439	5.027	0.361	0.373	0.282	0.527	4.039	0.432	0.418	0.201	0.490	4.975	0.434	0.433
Reg6	0.205	0.474	4.742	0.416	0.389	0.227	0.442	5.020	0.363	0.374	0.326	0.559	3.726	0.445	0.434	0.201	0.494	4.940	0.437	0.437
MI1-Reg	-0.039	0.322		0.322	0.298	-0.019	0.290		0.290	0.293	-0.030	0.332		0.332	0.313	-0.037	0.337		0.337	0.328
MI2-Reg	0.047	0.358		0.352	0.364	0.063	0.324		0.313	0.356	0.079	0.378		0.365	0.408	0.044	0.374		0.369	0.399
C. τ^2/L = 10																				
No Missing	-0.005	0.183	10.656	0.183	0.184	0.011	0.181	10.320	0.181	0.185	0.004	0.190	10.598	0.190	0.184	-0.012	0.180	9.995	0.180	0.189
CC	-0.014	0.207	8.511	0.207	0.206	0.024	0.205	8.643	0.204	0.208	0.008	0.230	7.717	0.230	0.219	0.004	0.206	9.291	0.205	0.210
DV	-0.014	0.207	5.335	0.207	0.206	0.023	0.205	5.358	0.204	0.208	0.012	0.230	4.812	0.230	0.219	0.001	0.206	5.786	0.206	0.210
NN1	-0.007	0.228	10.244	0.228	0.225	0.004	0.250	10.183	0.249	0.232	0.028	0.282	9.786	0.277	0.249	-0.010	0.227	11.156	0.227	0.237
NN2	-0.013	0.227	10.732	0.227	0.224	-0.007	0.224	10.744	0.224	0.225	0.013	0.259	10.202	0.258	0.239	-0.037	0.236	11.353	0.235	0.234
NN3	0.195	0.373	6.983	0.304	0.294	0.208	0.365	6.872	0.291	0.295	0.323	0.541	5.848	0.411	0.364	0.179	0.357	7.369	0.294	0.303
MI1-NN	-0.011	0.220		0.220	0.244	0.000	0.229		0.229	0.248	0.030	0.259		0.256	0.274	-0.021	0.225		0.225	0.251
MI2-NN	0.060	0.247		0.237	0.310	0.073	0.253		0.240	0.310	0.127	0.319		0.285	0.391	0.040	0.246		0.238	0.315
Reg1	-0.031	0.211	13.347	0.210	0.204	-0.013	0.213	13.466	0.213	0.207	-0.006	0.224	13.273	0.224	0.211	-0.034	0.214	14.625	0.213	0.217
Reg2	-0.033	0.212	13.381	0.211	0.204	-0.015	0.219	13.681	0.218	0.207	-0.017	0.229	13.504	0.228	0.213	-0.033	0.214	14.595	0.213	0.218
Reg3	-0.029	0.211	13.333	0.210	0.204	-0.010	0.213	13.515	0.213	0.206	-0.004	0.224	13.240	0.224	0.211	-0.034	0.213	14.638	0.213	0.217
Reg4	-0.029	0.211	13.443	0.210	0.204	-0.013	0.213	13.456	0.213	0.207	-0.004	0.224	13.118	0.224	0.212	-0.033	0.214	14.705	0.213	0.217
Reg5	0.216	0.352	8.573	0.268	0.261	0.235	0.366	8.590	0.271	0.266	0.321	0.447	7.479	0.305	0.293	0.205	0.355	9.306	0.271	0.278
Reg6	0.218	0.353	8.547	0.268	0.262	0.233	0.367	8.580	0.271	0.266	0.367	0.489	6.969	0.318	0.307	0.209	0.356	9.260	0.272	0.278
MI1-Reg	-0.029	0.211		0.210	0.204	-0.011	0.213		0.213	0.208	-0.007	0.223		0.223	0.215	-0.034	0.213		0.213	0.217
MI2-Reg	0.054	0.237		0.228	0.266	0.063	0.245		0.231	0.271	0.107	0.276		0.250	0.319	0.049	0.240		0.231	0.279

Notes: Results from 2SLS estimation based N=500 with approximinately 20% missing data and 500 simulations. RMSE = root mean squared error. SD = standard deviation. SE = standard error. 'No missing' refers to the situation where x2 is fully observed. 'CC' is the complete case estimator. DV is the ad hoc dummy variable approach. NN1 (NN2) is the nearest neighbor matching estimator, where matching is based on x1 and its quadratic (as well as z, its quadratic, and the interaction between z and x1). Reg1-Reg6 are various regression imputation estimators. Reg1 uses x1 and z. Reg2 adds quadratics and the linear interaction term to the model. Reg3 uses z only. Reg4 adds the quadratic for z. Reg5 uses x1 only. Reg6 adds the quadratic for x1. MI1-NN (MI2-NN) is multiple imputation using NN1-NN2 (NN1-NN3). MI1-Reg (MI2-Reg) is multiple imputation using Reg1-Reg4 (Reg1-Reg6). See text for further details.

Table A2. Monte Carlo Results: Linear Specification, L = 3, $\rho = 0.1$, $\pi_{21} = 1$, $\rho(x_1,z) = 0$.

Estimator	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2/L = 2$																				
No Missing	0.009	0.452	2.664	0.452	0.467	0.027	0.452	2.629	0.450	0.468	-0.010	0.418	2.789	0.418	0.424	0.025	0.448	2.566	0.448	0.441
CC	-0.011	0.468	2.191	0.468	0.505	0.061	0.475	2.178	0.470	0.510	0.032	0.487	2.270	0.487	0.509	0.006	0.509	2.491	0.508	0.503
DV	-0.208	0.528	1.516	0.473	0.469	-0.156	0.544	1.447	0.511	0.467	-0.118	0.545	1.506	0.530	0.489	-0.165	0.549	1.614	0.507	0.467
NN1	-0.091	0.534	2.884	0.529	0.552	-0.069	0.542	2.684	0.541	0.564	-0.061	0.647	2.406	0.646	0.639	-0.028	0.601	3.104	0.600	0.563
NN2	-0.210	0.673	2.257	0.654	0.620	-0.195	0.659	2.375	0.638	0.671	-0.368	1.204	1.221	1.135	1.097	-0.159	0.677	2.426	0.664	0.648
NN3	0.072	0.625	1.918	0.616	0.671	0.120	0.767	1.867	0.744	0.772	0.124	0.753	1.895	0.735	0.759	0.118	0.666	2.084	0.652	0.700
MI1-NN	-0.159	0.524		0.511	0.701	-0.124	0.502		0.493	0.768	-0.201	0.710		0.678	1.248	-0.092	0.568		0.562	0.720
MI2-NN	-0.084	0.508		0.506	0.754	-0.037	0.500		0.500	0.871	-0.087	0.582		0.575	1.195	-0.009	0.543		0.543	0.782
Reg1	-0.063	0.486	3.405	0.483	0.493	-0.045	0.470	3.399	0.470	0.488	-0.076	0.540	3.842	0.538	0.490	-0.024	0.537	3.877	0.536	0.502
Reg2	-0.061	0.513	3.356	0.511	0.502	-0.053	0.516	3.482	0.516	0.507	-0.112	0.569	3.931	0.565	0.496	-0.015	0.526	3.893	0.526	0.503
Reg3	-0.115	0.590	2.880	0.585	0.538	-0.096	0.502	2.917	0.500	0.521	0.027	0.785	1.923	0.782	0.783	-0.081	0.596	3.268	0.591	0.540
Reg4	-0.142	0.615	2.981	0.607	0.555	-0.135	0.518	2.892	0.514	0.535	-0.138	1.261	1.461	1.237	1.100	-0.123	0.612	3.375	0.603	0.537
Reg5	0.161	0.628	2.163	0.606	0.629	0.191	0.667	2.131	0.621	0.664	0.422	1.159	1.272	1.065	1.147	0.225	0.723	2.451	0.690	0.659
Reg6	0.162	0.694	2.194	0.668	0.666	0.208	0.644	2.191	0.592	0.641	0.252	0.762	2.090	0.709	0.695	0.217	0.709	2.484	0.673	0.658
MI1-Reg	-0.079	0.490		0.485	0.567	-0.062	0.458		0.457	0.562	-0.096	0.557		0.553	1.002	-0.066	0.537		0.534	0.557
MI2-Reg	0.002	0.507		0.507	0.628	0.017	0.500		0.496	0.627	0.047	0.593		0.588	1.090	0.034	0.576		0.575	0.624
B. $\tau^2/L = 5$																				
No Missing	0.000	0.279	5.415	0.279	0.270	0.024	0.271	5.517	0.269	0.269	0.010	0.277	5.572	0.277	0.266	0.009	0.266	5.388	0.265	0.272
CC	-0.012	0.289	4.508	0.289	0.301	0.054	0.309	4.682	0.307	0.305	0.006	0.343	4.331	0.343	0.326	0.010	0.318	4.996	0.317	0.308
DV	-0.124	0.359	2.738	0.328	0.311	-0.059	0.374	2.767	0.361	0.316	0.017	0.384	2.440	0.383	0.356	-0.107	0.378	2.865	0.355	0.317
NN1	-0.032	0.341	5.328	0.341	0.336	-0.004	0.403	5.481	0.403	0.357	0.049	0.478	4.657	0.468	0.467	-0.007	0.343	6.070	0.343	0.352
NN2	-0.099	0.369	4.431	0.360	0.357	-0.057	0.379	4.392	0.378	0.375	0.071	1.005	1.837	1.004	0.866	-0.077	0.386	4.502	0.383	0.388
NN3	0.138	0.472	3.664	0.438	0.445	0.197	0.471	3.721	0.413	0.427	0.256	0.619	3.346	0.554	0.535	0.174	0.474	4.206	0.435	0.447
MI1-NN	-0.066	0.327		0.323	0.404	-0.028	0.344		0.344	0.435	0.066	0.583		0.577	0.974	-0.041	0.336		0.336	0.428
MI2-NN	0.029	0.339		0.338	0.469	0.052	0.353		0.346	0.483	0.130	0.501		0.480	0.906	0.035	0.348		0.345	0.479
Reg1	-0.035	0.303	7.035	0.302	0.297	0.008	0.290	7.301	0.290	0.298	-0.039	0.334	7.396	0.333	0.310	-0.017	0.311	7.782	0.311	0.315
Reg2	-0.032	0.309	7.049	0.308	0.298	0.000	0.326	7.394	0.326	0.303	-0.050	0.379	7.482	0.378	0.322	-0.017	0.312	7.753	0.312	0.317
Reg3	-0.053	0.327	6.086	0.324	0.317	-0.017	0.329	6.146	0.329	0.324	0.181	0.605	3.705	0.564	0.524	-0.045	0.340	6.397	0.339	0.339
Reg4	-0.077	0.340	6.094	0.335	0.320	-0.034	0.336	5.990	0.336	0.328	0.292	1.080	2.216	1.037	0.896	-0.050	0.349	6.418	0.348	0.341
Reg5	0.196	0.440	4.432	0.385	0.384	0.230	0.463	4.406	0.378	0.393	0.558	0.948	2.547	0.715	0.689	0.226	0.462	4.902	0.392	0.407
Reg6	0.204	0.441	4.499	0.384	0.382	0.257	0.453	4.686	0.367	0.384	0.295	0.568	3.947	0.460	0.444	0.215	0.463	4.980	0.392	0.406
MI1-Reg	-0.044	0.309		0.307	0.324	-0.018	0.306		0.306	0.332	0.097	0.462		0.445	0.747	-0.037	0.317		0.317	0.344
MI2-Reg	0.038	0.333		0.330	0.378	0.077	0.338		0.325	0.386	0.214	0.511		0.451	0.761	0.048	0.347		0.339	0.398
C. $\tau^2/L = 10$																				
No Missing	-0.006	0.179	10.448	0.179	0.185	0.008	0.185	10.669	0.185	0.185	0.006	0.203	10.459	0.203	0.189	-0.009	0.188	10.318	0.188	0.188
CC	-0.013	0.208	8.390	0.207	0.208	0.025	0.217	8.716	0.217	0.208	-0.007	0.244	7.790	0.244	0.225	-0.006	0.206	9.506	0.206	0.209
DV	-0.071	0.257	4.770	0.243	0.218	-0.035	0.250	4.934	0.243	0.216	0.048	0.280	4.184	0.277	0.251	-0.084	0.241	5.177	0.226	0.217
NN1	-0.012	0.224	10.050	0.224	0.231	-0.010	0.236	10.559	0.236	0.231	0.078	0.332	8.492	0.316	0.285	-0.013	0.241	11.422	0.241	0.239
NN2	-0.059	0.261	8.102	0.260	0.252	-0.039	0.271	7.981	0.270	0.256	0.311	0.805	3.261	0.715	0.592	-0.059	0.272	8.495	0.270	0.260
NN3	0.175	0.356	6.740	0.285	0.298	0.207	0.376	6.747	0.295	0.297	0.315	0.506	5.780	0.377	0.374	0.210	0.388	7.380	0.319	0.309
MI1-NN	-0.025	0.226		0.225	0.279	-0.012	0.238		0.238	0.282	0.216	0.498		0.439	0.645	-0.028	0.243		0.242	0.283
MI2-NN	0.060	0.240		0.232	0.334	0.062	0.255		0.244	0.340	0.243	0.457		0.369	0.595	0.050	0.261		0.253	0.345
Reg1	-0.026	0.202	13.165	0.201	0.206	-0.010	0.208	13.783	0.208	0.206	-0.019	0.226	13.447	0.226	0.216	-0.029	0.217	14.892	0.216	0.216
Reg2	-0.027	0.204	13.140	0.203	0.207	-0.012	0.213	13.728	0.212	0.206	-0.023	0.238	13.414	0.237	0.218	-0.024	0.218	14.715	0.218	0.217
Reg3	-0.027	0.224	11.231	0.223	0.220	-0.028	0.217	11.261	0.216	0.220	0.182	0.423	7.421	0.362	0.314	-0.040	0.238	12.236	0.236	0.229
Reg4	-0.035	0.235	11.363	0.234	0.221	-0.036	0.224	11.095	0.222	0.223	0.389	0.711	4.613	0.547	0.481	-0.039	0.248	12.178	0.247	0.231
Reg5	0.214	0.345	8.243	0.259	0.265	0.226	0.359	8.361	0.267	0.269	0.574	0.730	5.096	0.416	0.418	0.209	0.359	9.305	0.278	0.277
Reg6	0.211	0.347	8.303	0.259	0.265	0.225	0.360	8.653	0.266	0.265	0.338	0.475	7.100	0.314	0.312	0.214	0.357	9.410	0.275	0.276
MI1-Reg	-0.027	0.210		0.209	0.223	-0.017	0.209		0.208	0.224	0.129	0.332		0.290	0.448	-0.035	0.224		0.223	0.233
MI2-Reg	0.055	0.232		0.223	0.278	0.070	0.236		0.225	0.282	0.242	0.405		0.304	0.472	0.044	0.247		0.239	0.287

Notes: See Table A1.

Table A3. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.1, π_{21} = 0, ρ(x_1,z) = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**
A. $\tau^2/L = 2$																				
No Missing	-0.011	0.333	4.062	0.333	0.320	0.025	0.319	4.152	0.317	0.324	0.091	0.964	0.802	0.958	1.213	0.016	0.324	3.940	0.324	0.326
CC	-0.021	0.381	3.259	0.381	0.370	0.039	0.368	3.529	0.366	0.365	0.092	0.897	0.818	0.889	1.102	0.034	0.384	3.733	0.384	0.369
DV	-0.026	0.377	2.026	0.377	0.369	0.038	0.367	2.206	0.366	0.365	0.097	0.893	0.509	0.886	1.076	0.035	0.386	2.323	0.386	0.369
NN1	-0.046	0.407	3.966	0.407	0.400	-0.028	0.408	4.280	0.407	0.405	-0.272	0.954	1.153	0.910	1.031	-0.018	0.426	4.565	0.426	0.412
NN2	-0.058	0.408	4.179	0.408	0.402	-0.026	0.402	4.511	0.402	0.400	-0.249	1.083	1.234	1.049	1.182	-0.046	0.452	4.626	0.451	0.424
NN3	0.127	0.527	2.683	0.494	0.512	0.181	0.534	2.926	0.498	0.494	-0.127	1.156	0.750	1.154	1.421	0.162	0.602	3.021	0.569	0.555
MI1-NN	-0.057	0.386		0.385	0.441	-0.032	0.383		0.383	0.443	-0.274	0.883		0.837	1.318	-0.032	0.419		0.418	0.457
MI2-NN	0.016	0.402		0.399	0.512	0.052	0.394		0.390	0.513	-0.219	0.833		0.806	1.539	0.034	0.434		0.431	0.547
Reg1	-0.042	0.369	5.118	0.368	0.357	-0.022	0.372	5.532	0.372	0.357	-0.140	0.942	1.306	0.935	1.044	-0.047	0.397	5.866	0.396	0.374
Reg2	-0.045	0.369	5.071	0.369	0.357	-0.023	0.393	5.469	0.393	0.358	-0.216	1.001	1.396	0.981	1.054	-0.037	0.395	5.783	0.395	0.373
Reg3	-0.039	0.369	5.152	0.368	0.357	-0.016	0.356	5.547	0.356	0.356	-0.156	0.870	1.313	0.862	0.975	-0.043	0.396	5.846	0.396	0.374
Reg4	-0.038	0.368	5.101	0.367	0.356	-0.020	0.358	5.463	0.358	0.357	-0.176	0.920	1.318	0.909	1.035	-0.042	0.395	5.805	0.395	0.374
Reg5	0.199	0.519	3.247	0.465	0.463	0.215	0.524	3.562	0.467	0.461	0.093	1.131	0.798	1.122	1.378	0.200	0.546	3.715	0.497	0.483
Reg6	0.200	0.522	3.247	0.468	0.466	0.213	0.522	3.570	0.465	0.461	0.103	1.227	0.787	1.216	1.501	0.198	0.547	3.710	0.497	0.484
MI1-Reg	-0.040	0.368		0.368	0.358	-0.021	0.350		0.349	0.363	-0.183	0.867		0.856	1.119	-0.042	0.396		0.395	0.375
MI2-Reg	0.045	0.405		0.400	0.426	0.053	0.390		0.384	0.428	-0.087	0.943		0.942	1.286	0.040	0.432		0.428	0.442
B. $\tau^2/L = 5$																				
No Missing	-0.002	0.195	9.015	0.195	0.199	0.017	0.196	9.152	0.195	0.199	0.093	1.140	0.835	1.136	1.366	0.000	0.187	9.156	0.187	0.198
CC	-0.004	0.218	7.482	0.218	0.225	0.037	0.218	7.701	0.217	0.224	0.083	0.951	0.761	0.949	1.150	-0.004	0.213	8.542	0.213	0.222
DV	-0.002	0.212	4.668	0.212	0.224	0.030	0.219	4.772	0.218	0.224	0.092	0.951	0.473	0.949	1.141	0.005	0.214	5.313	0.214	0.223
NN1	-0.004	0.261	8.877	0.260	0.251	0.010	0.255	9.021	0.254	0.249	-0.299	1.071	1.101	1.027	1.133	-0.003	0.243	10.400	0.243	0.254
NN2	-0.011	0.241	9.112	0.241	0.245	0.013	0.232	9.324	0.231	0.243	-0.277	1.072	1.159	1.033	1.080	-0.019	0.236	10.659	0.236	0.250
NN3	0.181	0.388	6.127	0.318	0.321	0.231	0.383	5.945	0.299	0.318	-0.155	1.413	0.765	1.403	1.890	0.191	0.382	6.836	0.318	0.326
MI1-NN	-0.004	0.243		0.243	0.267	0.015	0.233		0.233	0.269	-0.283	0.948		0.900	1.303	-0.010	0.230		0.230	0.271
MI2-NN	0.064	0.266		0.255	0.334	0.096	0.261		0.244	0.334	-0.236	0.931		0.895	1.800	0.053	0.257		0.247	0.337
Reg1	-0.013	0.220	11.770	0.219	0.222	0.005	0.215	12.055	0.215	0.222	-0.157	0.908	1.241	0.895	1.026	-0.022	0.224	13.354	0.223	0.231
Reg2	-0.018	0.224	11.737	0.224	0.223	-0.001	0.224	12.105	0.224	0.222	-0.214	0.901	1.363	0.877	0.958	-0.017	0.225	13.328	0.224	0.231
Reg3	-0.013	0.219	11.762	0.219	0.222	0.003	0.216	12.023	0.216	0.222	-0.170	0.891	1.238	0.876	0.995	-0.020	0.224	13.375	0.224	0.231
Reg4	-0.012	0.222	11.827	0.222	0.223	0.004	0.215	11.955	0.215	0.222	-0.198	0.914	1.281	0.894	1.011	-0.023	0.223	13.478	0.223	0.230
Reg5	0.229	0.365	7.401	0.277	0.284	0.264	0.379	7.697	0.274	0.287	0.065	1.143	0.749	1.141	1.423	0.219	0.371	8.441	0.285	0.296
Reg6	0.231	0.366	7.381	0.277	0.285	0.265	0.380	7.619	0.274	0.287	0.068	1.167	0.721	1.163	1.468	0.217	0.373	8.396	0.286	0.297
MI1-Reg	-0.014	0.221		0.221	0.223	0.002	0.216		0.216	0.224	-0.181	0.859		0.840	1.051	-0.020	0.224		0.223	0.231
MI2-Reg	0.065	0.250		0.238	0.284	0.093	0.252		0.234	0.288	-0.083	0.937		0.933	1.232	0.062	0.253		0.243	0.293
C. $\tau^2/L = 10$																				
No Missing	-0.004	0.132	17.967	0.132	0.138	0.005	0.138	18.327	0.138	0.138	0.099	1.729	0.855	1.729	1.912	-0.014	0.136	17.914	0.136	0.139
CC	-0.010	0.155	14.425	0.155	0.155	0.013	0.158	14.906	0.158	0.154	0.099	0.906	0.783	0.903	1.081	-0.010	0.154	16.505	0.154	0.155
DV	-0.008	0.154	8.947	0.153	0.155	0.012	0.159	9.300	0.159	0.154	0.092	0.932	0.488	0.930	1.096	-0.007	0.154	10.265	0.154	0.155
NN1	0.001	0.170	17.416	0.169	0.172	0.006	0.177	17.756	0.176	0.171	-0.294	0.931	1.115	0.878	1.002	0.014	0.174	19.595	0.173	0.177
NN2	-0.012	0.160	17.883	0.160	0.168	-0.004	0.171	18.244	0.171	0.169	-0.273	0.874	1.209	0.822	0.907	-0.009	0.168	20.245	0.168	0.173
NN3	0.203	0.318	11.396	0.219	0.226	0.215	0.326	11.583	0.223	0.224	-0.123	1.117	0.787	1.112	1.398	0.219	0.328	12.432	0.231	0.231
MI1-NN	-0.001	0.160		0.159	0.183	-0.001	0.169		0.168	0.185	-0.260	0.806		0.746	1.110	0.004	0.166		0.166	0.187
MI2-NN	0.071	0.189		0.170	0.254	0.075	0.199		0.179	0.255	-0.218	0.804		0.768	1.383	0.074	0.197		0.179	0.261
Reg1	-0.016	0.147	22.584	0.147	0.154	-0.010	0.154	23.209	0.154	0.154	-0.136	0.886	1.291	0.874	0.969	-0.019	0.159	25.895	0.159	0.161
Reg2	-0.012	0.148	22.322	0.148	0.154	-0.012	0.157	23.031	0.157	0.154	-0.204	0.937	1.454	0.912	0.974	-0.018	0.158	25.747	0.158	0.160
Reg3	-0.016	0.147	22.496	0.147	0.154	-0.009	0.154	23.200	0.154	0.154	-0.137	0.888	1.297	0.876	0.991	-0.018	0.159	26.029	0.158	0.161
Reg4	-0.014	0.147	22.471	0.147	0.154	-0.012	0.154	23.071	0.154	0.154	-0.154	0.890	1.400	0.872	1.020	-0.015	0.158	25.741	0.157	0.160
Reg5	0.221	0.304	14.143	0.192	0.199	0.232	0.314	14.572	0.199	0.200	0.120	1.124	0.779	1.119	1.376	0.215	0.311	16.193	0.203	0.209
Reg6	0.223	0.306	14.102	0.192	0.200	0.234	0.315	14.568	0.200	0.200	0.102	1.133	0.775	1.127	1.371	0.218	0.312	16.115	0.204	0.209
MI1-Reg	-0.014	0.147		0.147	0.154	-0.010	0.154		0.154	0.155	-0.171	0.833		0.815	1.068	-0.017	0.158		0.158	0.161
MI2-Reg	0.066	0.177		0.161	0.220	0.071	0.184		0.168	0.223	-0.086	0.901		0.898	1.222	0.064	0.186		0.172	0.228

Notes: See Table A1.

Table A4. Monte Carlo Results: Nonlinear Specification, L = 3, $\rho = 0.1$, $\pi_{21} = 1$, $\rho(x_1,z) = 0$.

Estimator	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2/L = 2$																				
No Missing	0.005	0.306	3.025	0.306	0.316	0.026	0.348	3.061	0.347	0.337	0.085	0.846	0.764	0.841	1.121	0.004	0.324	2.984	0.324	0.333
CC	-0.017	0.373	2.510	0.373	0.369	0.060	0.485	2.716	0.482	0.417	0.087	0.818	0.810	0.807	1.083	0.043	0.392	2.685	0.391	0.390
DV	-0.358	0.565	1.731	0.408	0.359	0.029	0.387	1.755	0.385	0.367	-0.941	1.056	1.528	0.487	0.486	-0.396	0.557	1.905	0.391	0.365
NN1	-0.065	0.463	3.092	0.461	0.434	-0.055	0.441	3.329	0.439	0.428	-0.370	0.938	1.249	0.837	0.864	-0.052	0.778	3.173	0.773	0.667
NN2	-0.299	0.685	2.223	0.637	0.585	-0.107	0.504	3.214	0.501	0.477	-0.865	1.092	3.307	0.605	0.487	-0.327	0.615	2.165	0.530	0.567
NN3	0.107	0.552	2.172	0.534	0.520	0.129	0.635	2.173	0.608	0.589	-0.061	0.940	0.772	0.938	1.184	0.147	0.821	2.243	0.793	0.719
MI1-NN	-0.162	0.482		0.460	0.672	-0.065	0.422		0.420	0.531	-0.615	0.861		0.546	1.024	-0.168	0.567		0.533	0.820
MI2-NN	-0.062	0.426		0.423	0.695	0.013	0.423		0.422	0.623	-0.434	0.698		0.522	1.296	-0.057	0.469		0.465	0.914
Reg1	-0.043	0.400	3.911	0.399	0.370	-0.030	0.458	4.284	0.458	0.412	-0.212	0.915	1.363	0.905	1.046	-0.058	0.418	4.248	0.417	0.404
Reg2	-0.033	0.388	3.629	0.387	0.365	-0.026	0.401	3.704	0.401	0.378	-0.141	0.806	1.104	0.794	1.041	-0.018	0.372	3.779	0.371	0.377
Reg3	-0.159	0.553	2.899	0.532	0.475	-0.048	0.378	3.948	0.377	0.376	-0.642	0.930	2.010	0.649	0.674	-0.135	0.549	3.111	0.533	0.500
Reg4	-0.209	0.518	3.021	0.481	0.440	-0.069	0.402	3.968	0.400	0.382	-0.867	0.993	4.403	0.423	0.394	-0.204	0.601	3.269	0.569	0.495
Reg5	0.189	0.560	2.470	0.522	0.499	0.225	0.559	2.661	0.507	0.509	-0.518	1.032	1.619	0.857	0.846	0.176	0.587	2.733	0.548	0.549
Reg6	0.184	0.528	2.403	0.493	0.469	0.185	0.799	2.382	0.762	0.752	0.064	0.895	0.758	0.892	1.088	0.183	0.533	2.532	0.491	0.499
MI1-Reg	-0.102	0.389		0.376	0.491	-0.030	0.387		0.387	0.412	-0.449	0.659		0.471	1.208	-0.105	0.413		0.403	0.525
MI2-Reg	-0.001	0.396		0.396	0.533	0.043	0.451		0.446	0.580	-0.378	0.636		0.503	1.272	-0.009	0.418		0.418	0.568
B. $\tau^2/L = 5$																				
No Missing	-0.017	0.196	6.561	0.195	0.198	0.008	0.200	6.790	0.199	0.198	0.042	0.723	0.726	0.720	0.928	0.008	0.204	6.119	0.204	0.203
CC	-0.019	0.232	5.527	0.232	0.224	0.032	0.228	5.927	0.227	0.220	0.090	0.837	0.750	0.831	1.097	0.015	0.225	5.692	0.225	0.228
DV	-0.195	0.362	2.952	0.281	0.254	0.036	0.233	3.601	0.232	0.223	-0.978	1.094	1.451	0.521	0.500	-0.241	0.411	2.968	0.319	0.262
NN1	-0.031	0.282	6.351	0.282	0.260	-0.002	0.250	6.914	0.249	0.246	-0.455	0.972	1.218	0.841	0.924	0.018	0.287	6.480	0.286	0.273
NN2	-0.164	0.397	3.884	0.380	0.347	-0.059	0.266	6.398	0.265	0.268	-0.917	1.021	3.128	0.411	0.411	-0.125	0.474	3.826	0.470	0.411
NN3	0.159	0.365	4.455	0.311	0.320	0.203	0.369	4.637	0.298	0.313	-0.108	0.988	0.720	0.983	1.205	0.211	0.413	4.523	0.344	0.338
MI1-NN	-0.086	0.303		0.296	0.391	-0.021	0.236		0.236	0.299	-0.683	0.880		0.518	0.986	-0.041	0.335		0.334	0.451
MI2-NN	0.004	0.276		0.275	0.424	0.063	0.253		0.242	0.350	-0.497	0.744		0.546	1.286	0.050	0.318		0.312	0.468
Reg1	-0.050	0.239	8.643	0.237	0.230	-0.013	0.229	9.312	0.229	0.233	-0.224	0.982	1.239	0.969	1.064	-0.026	0.254	9.022	0.254	0.244
Reg2	-0.043	0.233	7.822	0.232	0.223	0.001	0.225	7.909	0.225	0.220	-0.169	0.861	1.000	0.844	1.019	-0.009	0.245	7.824	0.245	0.234
Reg3	-0.097	0.281	5.834	0.270	0.268	-0.019	0.222	8.448	0.222	0.224	-0.702	0.954	1.841	0.609	0.657	-0.061	0.333	5.842	0.332	0.309
Reg4	-0.122	0.301	5.944	0.284	0.269	-0.015	0.226	8.501	0.225	0.225	-0.904	1.022	4.276	0.442	0.411	-0.086	0.368	5.918	0.364	0.314
Reg5	0.179	0.370	5.354	0.308	0.300	0.229	0.368	5.781	0.280	0.291	-0.561	1.033	1.441	0.859	0.901	0.231	0.407	5.472	0.329	0.322
Reg6	0.185	0.362	5.025	0.296	0.287	0.234	0.367	5.144	0.276	0.283	0.039	0.982	0.703	0.973	1.201	0.223	0.409	4.991	0.328	0.309
MI1-Reg	-0.069	0.243		0.236	0.279	-0.010	0.218		0.218	0.236	-0.500	0.705		0.502	1.174	-0.027	0.271		0.270	0.318
MI2-Reg	0.019	0.253		0.251	0.326	0.064	0.247		0.234	0.294	-0.410	0.672		0.537	1.270	0.059	0.288		0.280	0.359
C. $\tau^2/L = 10$																				
No Missing	0.017	0.140	12.163	0.139	0.138	-0.003	0.145	12.432	0.145	0.140	0.028	0.775	0.834	0.772	0.922	0.007	0.137	12.628	0.137	0.136
CC	0.013	0.156	9.778	0.155	0.155	0.000	0.162	10.705	0.162	0.158	0.058	0.780	0.823	0.776	0.980	-0.002	0.150	11.177	0.150	0.153
DV	-0.117	0.266	4.768	0.226	0.184	-0.003	0.171	6.234	0.171	0.160	-0.909	1.057	1.531	0.535	0.506	-0.131	0.250	5.210	0.211	0.180
NN1	0.021	0.187	11.587	0.185	0.180	0.011	0.196	12.274	0.194	0.178	-0.440	0.988	1.263	0.899	0.954	0.016	0.197	13.029	0.195	0.183
NN2	-0.070	0.261	6.715	0.260	0.248	-0.005	0.218	10.632	0.217	0.194	-0.902	1.042	3.024	0.446	0.435	-0.075	0.276	7.349	0.273	0.248
NN3	0.219	0.334	7.987	0.226	0.229	0.217	0.343	8.056	0.249	0.233	-0.147	0.906	0.794	0.893	1.129	0.216	0.344	8.974	0.242	0.234
MI1-NN	-0.024	0.197		0.197	0.283	0.010	0.194		0.193	0.217	-0.666	0.864		0.538	1.047	-0.022	0.209		0.209	0.281
MI2-NN	0.068	0.205		0.188	0.321	0.086	0.220		0.200	0.274	-0.503	0.732		0.531	1.264	0.061	0.217		0.204	0.323
Reg1	0.002	0.166	15.336	0.166	0.160	-0.013	0.182	16.451	0.182	0.169	-0.227	0.918	1.293	0.901	0.985	-0.010	0.176	17.600	0.176	0.166
Reg2	0.001	0.158	14.019	0.158	0.154	-0.008	0.170	14.197	0.170	0.157	-0.138	0.789	1.144	0.771	0.875	-0.005	0.167	16.084	0.167	0.158
Reg3	-0.024	0.209	10.267	0.209	0.196	-0.012	0.178	14.697	0.178	0.163	-0.749	1.004	1.863	0.652	0.713	-0.042	0.204	11.725	0.202	0.195
Reg4	-0.035	0.227	10.401	0.227	0.199	-0.014	0.183	14.411	0.183	0.164	-0.921	1.047	4.043	0.444	0.428	-0.053	0.221	11.813	0.218	0.195
Reg5	0.242	0.342	9.529	0.221	0.214	0.236	0.336	10.329	0.226	0.215	-0.607	1.051	1.579	0.822	0.871	0.244	0.337	10.838	0.227	0.220
Reg6	0.246	0.331	9.006	0.207	0.204	0.228	0.330	9.103	0.222	0.207	0.032	0.964	0.822	0.964	1.139	0.241	0.332	10.314	0.220	0.211
MI1-Reg	-0.007	0.176		0.176	0.202	-0.013	0.174		0.174	0.171	-0.503	0.691		0.463	1.123	-0.024	0.178		0.178	0.201
MI2-Reg	0.081	0.202		0.183	0.255	0.070	0.204		0.188	0.235	-0.438	0.675		0.503	1.212	0.075	0.202		0.188	0.257

Notes: See Table A1.

Table A5. Monte Carlo Results: Linear Specification, L = 3, ρ = 0.1, π_{21} = 0, $\rho(x_1,z) \neq 0$.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2/L = 2																				
No Missing	0.010	0.438	2.483	0.438	0.453	0.027	0.463	2.681	0.463	0.460	0.050	0.639	1.696	0.637	0.675	0.020	0.458	2.506	0.457	0.473
CC	-0.009	0.549	2.139	0.548	0.544	0.052	0.501	2.245	0.501	0.512	0.054	0.840	1.493	0.838	0.869	0.022	0.495	2.443	0.494	0.505
DV	-0.111	0.591	1.532	0.566	0.494	-0.068	0.519	1.580	0.502	0.478	0.023	0.720	1.005	0.720	0.721	-0.071	0.500	1.764	0.482	0.472
NN1	-0.110	0.541	2.718	0.530	0.544	-0.093	0.538	2.971	0.530	0.559	-0.179	0.880	1.977	0.870	0.811	-0.068	0.557	2.995	0.553	0.572
NN2	-0.119	0.559	2.830	0.547	0.560	-0.083	0.541	3.009	0.536	0.537	-0.191	0.682	2.116	0.664	0.707	-0.088	0.594	3.224	0.592	0.566
NN3	0.053	0.661	1.894	0.658	0.676	0.111	0.670	1.981	0.660	0.685	0.009	0.796	1.360	0.793	0.925	0.106	0.683	2.040	0.669	0.714
MI1-NN	-0.116	0.526		0.513	0.606	-0.078	0.511		0.504	0.606	-0.171	0.720		0.706	0.866	-0.072	0.539		0.536	0.630
MI2-NN	-0.035	0.524		0.521	0.692	-0.007	0.513		0.512	0.699	-0.089	0.666		0.662	0.988	-0.020	0.538		0.538	0.721
Reg1	-0.073	0.598	3.480	0.593	0.530	-0.038	0.474	3.596	0.473	0.486	-0.089	1.056	2.392	1.055	0.862	-0.049	0.522	3.787	0.522	0.509
Reg2	-0.077	0.543	3.425	0.537	0.510	-0.055	0.498	3.713	0.494	0.486	-0.163	0.850	2.382	0.839	0.754	-0.055	0.510	3.785	0.510	0.512
Reg3	-0.061	0.511	3.424	0.505	0.491	-0.032	0.467	3.607	0.465	0.471	-0.076	0.767	2.395	0.765	0.729	-0.046	0.506	3.780	0.506	0.497
Reg4	-0.060	0.512	3.403	0.506	0.492	-0.037	0.467	3.660	0.464	0.471	-0.103	0.720	2.414	0.713	0.782	-0.045	0.503	3.758	0.502	0.497
Reg5	0.148	0.741	2.119	0.727	0.685	0.195	0.629	2.229	0.597	0.635	0.159	1.356	1.494	1.339	1.216	0.181	0.691	2.469	0.653	0.663
Reg6	0.154	0.757	2.121	0.743	0.698	0.199	0.626	2.234	0.594	0.634	0.174	1.366	1.458	1.346	1.215	0.181	0.696	2.465	0.657	0.666
MI1-Reg	-0.073	0.533		0.527	0.514	-0.042	0.469		0.467	0.488	-0.100	0.709		0.705	0.857	-0.048	0.508		0.508	0.507
MI2-Reg	0.007	0.593		0.593	0.604	0.035	0.508		0.507	0.566	-0.010	0.867		0.867	1.042	0.035	0.559		0.556	0.589
B. τ^2/L = 5																				
No Missing	0.004	0.277	5.401	0.277	0.274	0.009	0.274	5.540	0.274	0.268	0.012	0.497	3.168	0.497	0.452	0.009	0.268	5.424	0.268	0.273
CC	-0.006	0.317	4.491	0.317	0.309	0.037	0.312	4.676	0.311	0.301	0.024	0.487	2.674	0.487	0.478	0.005	0.298	5.130	0.298	0.303
DV	-0.142	0.374	3.259	0.335	0.303	-0.098	0.343	3.285	0.316	0.297	-0.022	0.495	1.803	0.487	0.462	-0.128	0.347	3.714	0.308	0.295
NN1	-0.042	0.342	5.518	0.342	0.337	-0.031	0.337	5.919	0.336	0.328	-0.096	0.622	3.187	0.619	0.537	-0.008	0.346	6.155	0.346	0.347
NN2	-0.056	0.334	5.771	0.332	0.327	-0.023	0.334	5.950	0.334	0.323	-0.096	0.508	3.459	0.501	0.481	-0.038	0.335	6.399	0.335	0.331
NN3	0.135	0.479	3.682	0.445	0.453	0.172	0.455	3.826	0.408	0.419	0.119	0.652	2.184	0.633	0.668	0.183	0.523	4.100	0.476	0.451
MI1-NN	-0.053	0.326		0.325	0.360	-0.011	0.324		0.324	0.353	-0.086	0.541		0.536	0.563	-0.026	0.329		0.329	0.366
MI2-NN	0.015	0.342		0.339	0.443	0.050	0.341		0.336	0.423	0.014	0.500		0.500	0.676	0.042	0.362		0.357	0.447
Reg1	-0.039	0.307	6.986	0.306	0.302	-0.027	0.301	7.374	0.301	0.296	-0.044	0.482	4.201	0.478	0.451	-0.023	0.311	8.003	0.311	0.311
Reg2	-0.034	0.311	7.049	0.310	0.304	-0.024	0.310	7.552	0.309	0.297	-0.088	0.487	4.187	0.481	0.451	-0.027	0.313	7.893	0.312	0.313
Reg3	-0.029	0.296	6.998	0.296	0.297	-0.020	0.297	7.322	0.297	0.292	-0.047	0.466	4.238	0.462	0.449	-0.029	0.305	7.938	0.305	0.306
Reg4	-0.031	0.297	7.027	0.297	0.298	-0.027	0.299	7.378	0.299	0.292	-0.079	0.465	4.192	0.460	0.438	-0.027	0.305	7.810	0.305	0.307
Reg5	0.200	0.448	4.493	0.387	0.388	0.212	0.445	4.625	0.380	0.380	0.204	0.645	2.642	0.613	0.602	0.208	0.457	5.106	0.392	0.398
Reg6	0.200	0.449	4.473	0.388	0.389	0.210	0.445	4.646	0.380	0.380	0.224	0.650	2.597	0.611	0.612	0.207	0.460	5.084	0.394	0.400
MI1-Reg	-0.033	0.302		0.301	0.302	-0.023	0.300		0.299	0.298	-0.060	0.459		0.455	0.462	-0.024	0.307		0.307	0.311
MI2-Reg	0.045	0.335		0.329	0.364	0.051	0.331		0.325	0.360	0.027	0.504		0.504	0.547	0.059	0.341		0.335	0.374
C. τ^2/L = 10																				
No Missing	-0.002	0.184	10.224	0.184	0.187	0.002	0.191	10.631	0.191	0.186	0.000	0.275	5.670	0.275	0.270	-0.006	0.187	10.282	0.187	0.188
CC	-0.007	0.216	8.404	0.216	0.211	0.027	0.221	8.678	0.220	0.208	0.000	0.323	4.512	0.323	0.311	0.000	0.210	9.571	0.210	0.209
DV	-0.160	0.290	6.188	0.238	0.212	-0.114	0.263	6.195	0.225	0.209	-0.069	0.337	3.110	0.324	0.308	-0.155	0.275	7.003	0.220	0.207
NN1	-0.011	0.235	10.260	0.235	0.232	-0.005	0.239	10.621	0.239	0.230	-0.025	0.393	5.344	0.393	0.361	0.011	0.240	11.348	0.240	0.239
NN2	-0.024	0.225	10.450	0.224	0.225	0.006	0.234	10.716	0.234	0.226	-0.032	0.363	5.691	0.363	0.342	-0.016	0.233	11.773	0.232	0.230
NN3	0.190	0.369	6.814	0.296	0.302	0.208	0.372	6.927	0.292	0.295	0.191	0.507	3.607	0.458	0.467	0.211	0.393	7.478	0.318	0.309
MI1-NN	-0.028	0.222		0.222	0.248	0.006	0.229		0.229	0.246	-0.019	0.365		0.365	0.385	-0.008	0.229		0.229	0.253
MI2-NN	0.047	0.246		0.235	0.316	0.076	0.253		0.240	0.314	0.054	0.379		0.374	0.475	0.068	0.260		0.248	0.324
Reg1	-0.019	0.208	13.183	0.208	0.208	-0.010	0.211	13.632	0.211	0.207	-0.028	0.317	7.271	0.315	0.300	-0.016	0.217	14.915	0.216	0.216
Reg2	-0.019	0.210	13.197	0.210	0.209	-0.015	0.217	13.679	0.217	0.207	-0.052	0.332	7.186	0.329	0.306	-0.012	0.218	14.875	0.217	0.217
Reg3	-0.011	0.203	13.113	0.203	0.206	-0.006	0.210	13.508	0.210	0.205	-0.021	0.310	7.220	0.308	0.298	-0.018	0.215	14.811	0.214	0.214
Reg4	-0.014	0.203	13.141	0.203	0.207	-0.008	0.212	13.607	0.211	0.205	-0.043	0.314	7.250	0.312	0.298	-0.018	0.215	14.881	0.215	0.215
Reg5	0.222	0.357	8.425	0.267	0.267	0.226	0.362	8.623	0.269	0.265	0.229	0.473	4.432	0.411	0.395	0.225	0.363	9.497	0.275	0.276
Reg6	0.223	0.359	8.366	0.268	0.268	0.225	0.363	8.616	0.270	0.266	0.258	0.487	4.356	0.415	0.404	0.227	0.365	9.583	0.275	0.277
MI1-Reg	-0.016	0.206		0.205	0.209	-0.012	0.211		0.211	0.208	-0.030	0.314		0.312	0.308	-0.013	0.215		0.215	0.216
MI2-Reg	0.063	0.236		0.225	0.271	0.066	0.241		0.229	0.271	0.059	0.348		0.343	0.381	0.063	0.244		0.234	0.279

Notes: See Table A1.

Table A6. Monte Carlo Results: Linear Specification, $L = 3$, $\rho = 0.1$, $\pi_{21} = 1$, $\rho(x_1,z) \neq 0$.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**
A. $\tau^2/L = 2$																				
No Missing	0.010	0.438	2.483	0.438	0.453	0.027	0.463	2.681	0.463	0.460	0.050	0.639	1.696	0.637	0.675	0.020	0.458	2.506	0.457	0.473
CC	-0.009	0.549	2.139	0.548	0.544	0.052	0.501	2.245	0.501	0.512	0.054	0.840	1.493	0.838	0.869	0.022	0.495	2.443	0.494	0.505
DV	-0.887	0.918	11.984	0.212	0.184	-0.854	0.893	10.062	0.218	0.200	-0.823	0.881	6.054	0.282	0.260	-0.890	0.915	13.297	0.195	0.175
NN1	-0.117	0.541	2.747	0.530	0.551	-0.107	0.528	2.953	0.521	0.539	-0.192	0.899	1.733	0.887	0.903	-0.091	0.585	2.954	0.580	0.576
NN2	-0.475	0.540	6.510	0.297	0.309	-0.416	0.519	6.192	0.306	0.301	-0.374	0.622	2.885	0.506	0.522	-0.460	0.551	6.957	0.297	0.295
NN3	0.051	0.663	1.899	0.660	0.677	0.108	0.670	1.970	0.660	0.684	-0.006	0.886	1.368	0.885	0.989	0.105	0.680	2.045	0.666	0.712
MI1-NN	-0.295	0.469		0.377	0.601	-0.250	0.464		0.390	0.573	-0.261	0.664		0.614	0.899	-0.256	0.487		0.407	0.626
MI2-NN	-0.167	0.461		0.430	0.711	-0.126	0.465		0.447	0.696	-0.160	0.643		0.625	1.050	-0.144	0.476		0.457	0.735
Reg1	-0.073	0.598	3.480	0.593	0.530	-0.038	0.474	3.596	0.473	0.486	-0.089	1.056	2.392	1.055	0.862	-0.049	0.522	3.787	0.522	0.509
Reg2	-0.077	0.543	3.425	0.537	0.510	-0.055	0.498	3.713	0.494	0.486	-0.163	0.850	2.382	0.839	0.754	-0.055	0.510	3.785	0.510	0.512
Reg3	-0.456	0.535	9.097	0.264	0.274	-0.432	0.531	9.307	0.271	0.269	-0.468	0.638	5.907	0.385	0.367	-0.460	0.543	9.873	0.275	0.277
Reg4	-0.457	0.539	9.087	0.264	0.273	-0.419	0.526	8.929	0.280	0.276	-0.417	0.607	4.665	0.417	0.407	-0.467	0.547	9.918	0.277	0.276
Reg5	0.155	0.683	2.138	0.669	0.654	0.182	0.619	2.214	0.589	0.632	0.269	1.014	1.190	0.978	1.109	0.171	0.691	2.437	0.656	0.661
Reg6	0.154	0.757	2.121	0.743	0.698	0.199	0.626	2.234	0.594	0.634	0.174	1.366	1.458	1.346	1.215	0.181	0.696	2.465	0.657	0.666
MI1-Reg	-0.268	0.475		0.387	0.528	-0.225	0.436		0.356	0.500	-0.268	0.573		0.499	0.789	-0.262	0.450		0.377	0.528
MI2-Reg	-0.126	0.506		0.487	0.644	-0.082	0.441		0.429	0.610	-0.112	0.666		0.658	1.053	-0.100	0.474		0.466	0.640
B. $\tau^2/L = 5$																				
No Missing	0.004	0.277	5.401	0.277	0.274	0.009	0.274	5.540	0.274	0.268	0.012	0.497	3.168	0.497	0.452	0.009	0.268	5.424	0.268	0.273
CC	-0.006	0.317	4.491	0.317	0.309	0.037	0.312	4.676	0.311	0.301	0.024	0.487	2.674	0.487	0.478	0.005	0.298	5.130	0.298	0.303
DV	-0.820	0.853	15.415	0.189	0.162	-0.789	0.821	13.282	0.192	0.174	-0.768	0.824	7.595	0.258	0.229	-0.835	0.852	17.207	0.175	0.155
NN1	-0.053	0.352	5.484	0.351	0.342	-0.040	0.339	5.721	0.339	0.333	-0.048	0.772	2.771	0.771	0.663	-0.026	0.388	6.037	0.388	0.361
NN2	-0.366	0.411	10.715	0.213	0.217	-0.322	0.395	10.061	0.231	0.224	-0.283	0.460	4.414	0.375	0.377	-0.359	0.421	11.371	0.221	0.221
NN3	0.135	0.485	3.660	0.451	0.456	0.166	0.455	3.818	0.408	0.419	0.110	0.661	2.222	0.643	0.673	0.183	0.525	4.099	0.478	0.452
MI1-NN	-0.201	0.323		0.265	0.424	-0.167	0.321		0.272	0.401	-0.160	0.550		0.530	0.660	-0.196	0.337		0.285	0.446
MI2-NN	-0.087	0.308		0.301	0.508	-0.055	0.306		0.302	0.479	-0.065	0.496		0.494	0.749	-0.056	0.327		0.324	0.524
Reg1	-0.039	0.307	6.986	0.306	0.302	-0.027	0.301	7.374	0.301	0.296	-0.044	0.482	4.201	0.478	0.451	-0.023	0.311	8.003	0.311	0.311
Reg2	-0.034	0.311	7.049	0.310	0.304	-0.024	0.310	7.552	0.309	0.297	-0.088	0.487	4.187	0.481	0.451	-0.027	0.313	7.893	0.312	0.313
Reg3	-0.360	0.411	14.712	0.194	0.202	-0.342	0.410	15.091	0.203	0.201	-0.395	0.496	8.976	0.279	0.272	-0.370	0.422	16.312	0.207	0.207
Reg4	-0.362	0.415	14.761	0.194	0.201	-0.330	0.404	14.558	0.209	0.205	-0.342	0.477	7.442	0.328	0.306	-0.376	0.425	16.240	0.209	0.207
Reg5	0.198	0.453	4.483	0.394	0.389	0.200	0.446	4.572	0.383	0.382	0.317	0.866	2.118	0.782	0.778	0.206	0.455	5.118	0.392	0.397
Reg6	0.200	0.449	4.473	0.388	0.389	0.210	0.445	4.646	0.380	0.380	0.224	0.650	2.597	0.611	0.612	0.207	0.460	5.084	0.394	0.400
MI1-Reg	-0.201	0.309		0.242	0.349	-0.180	0.308		0.246	0.341	-0.204	0.423		0.361	0.466	-0.205	0.315		0.251	0.358
MI2-Reg	-0.063	0.293		0.289	0.430	-0.051	0.292		0.288	0.423	-0.043	0.439		0.436	0.656	-0.061	0.299		0.295	0.439
C. $\tau^2/L = 10$																				
No Missing	-0.002	0.184	10.224	0.184	0.187	0.002	0.191	10.631	0.191	0.186	0.000	0.275	5.670	0.275	0.270	-0.006	0.187	10.282	0.187	0.188
CC	-0.007	0.216	8.404	0.216	0.211	0.027	0.221	8.678	0.220	0.208	0.000	0.323	4.512	0.323	0.311	0.000	0.210	9.571	0.210	0.209
DV	-0.753	0.779	20.153	0.167	0.141	-0.711	0.741	17.759	0.167	0.150	-0.706	0.752	9.766	0.229	0.199	-0.760	0.781	22.531	0.155	0.136
NN1	-0.013	0.238	10.086	0.238	0.234	-0.006	0.241	10.400	0.241	0.233	0.028	0.516	4.667	0.513	0.427	-0.002	0.247	11.066	0.247	0.242
NN2	-0.286	0.322	16.715	0.166	0.169	-0.255	0.308	15.741	0.181	0.175	-0.203	0.346	6.716	0.291	0.288	-0.287	0.332	17.832	0.174	0.174
NN3	0.188	0.369	6.825	0.296	0.302	0.202	0.370	6.895	0.291	0.296	0.186	0.511	3.598	0.462	0.467	0.211	0.394	7.456	0.318	0.309
MI1-NN	-0.146	0.234		0.191	0.326	-0.123	0.235		0.201	0.311	-0.085	0.379		0.373	0.465	-0.144	0.242		0.201	0.337
MI2-NN	-0.037	0.214		0.213	0.391	-0.014	0.220		0.220	0.381	0.013	0.374		0.372	0.528	-0.022	0.228		0.228	0.402
Reg1	-0.019	0.208	13.183	0.208	0.208	-0.010	0.211	13.632	0.211	0.207	-0.028	0.317	7.271	0.315	0.300	-0.016	0.217	14.915	0.216	0.216
Reg2	-0.019	0.210	13.197	0.210	0.209	-0.015	0.217	13.679	0.217	0.207	-0.052	0.332	7.186	0.329	0.306	-0.012	0.218	14.875	0.217	0.217
Reg3	-0.283	0.325	22.909	0.149	0.156	-0.282	0.326	23.548	0.159	0.157	-0.325	0.397	13.394	0.214	0.212	-0.296	0.337	25.427	0.163	0.162
Reg4	-0.289	0.328	22.972	0.149	0.156	-0.273	0.319	22.746	0.163	0.159	-0.279	0.367	11.435	0.238	0.229	-0.303	0.339	25.417	0.164	0.162
Reg5	0.222	0.356	8.328	0.268	0.267	0.221	0.361	8.588	0.270	0.266	0.361	0.595	3.659	0.476	0.463	0.223	0.362	9.458	0.276	0.276
Reg6	0.223	0.359	8.366	0.268	0.268	0.225	0.363	8.616	0.270	0.266	0.258	0.487	4.356	0.415	0.404	0.227	0.365	9.583	0.275	0.277
MI1-Reg	-0.154	0.229		0.174	0.262	-0.140	0.232		0.182	0.258	-0.161	0.314		0.263	0.332	-0.158	0.240		0.185	0.268
MI2-Reg	-0.028	0.204		0.203	0.337	-0.017	0.209		0.208	0.335	-0.002	0.318		0.318	0.462	-0.028	0.214		0.213	0.344

Notes: See Table A1.

Table A7. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.1, π_{21} = 0, ρ(x_1,z) ≠ 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2/L = 2																				
No Missing	-0.018	0.316	4.218	0.316	0.327	0.061	0.753	1.136	0.751	0.838	0.089	0.911	0.754	0.902	1.130	0.033	0.338	4.178	0.337	0.322
CC	0.012	0.354	3.461	0.353	0.364	0.115	0.813	1.005	0.805	0.902	0.062	0.990	0.752	0.983	1.210	0.024	0.358	3.854	0.357	0.365
DV	-0.124	0.385	2.545	0.357	0.350	0.009	0.821	0.680	0.821	0.855	0.055	0.960	0.460	0.955	1.110	-0.082	0.414	2.834	0.392	0.342
NN1	-0.051	0.421	4.151	0.421	0.410	-0.180	0.773	1.498	0.757	0.843	-0.267	0.962	1.057	0.927	1.100	-0.031	0.396	4.948	0.396	0.403
NN2	-0.060	0.368	4.428	0.365	0.376	-0.188	0.819	1.382	0.796	0.987	-0.329	1.315	1.120	1.258	1.622	-0.042	0.396	4.961	0.395	0.384
NN3	0.089	0.536	3.024	0.512	0.521	-0.046	1.097	0.994	1.096	1.247	-0.169	1.047	0.774	1.041	1.323	0.174	0.583	3.302	0.548	0.552
MI1-NN	-0.053	0.371		0.369	0.433	-0.173	0.692		0.670	1.063	-0.277	0.965		0.911	1.643	-0.035	0.380		0.379	0.428
MI2-NN	0.009	0.390		0.389	0.518	-0.127	0.739		0.731	1.282	-0.245	0.854		0.817	1.761	0.056	0.415		0.412	0.527
Reg1	-0.055	0.359	5.505	0.358	0.356	-0.081	0.834	1.558	0.833	0.866	-0.128	0.963	1.187	0.959	1.085	-0.021	0.393	6.148	0.393	0.382
Reg2	-0.057	0.363	5.326	0.362	0.359	-0.119	1.018	1.687	1.016	0.976	-0.198	0.930	1.350	0.918	1.067	-0.015	0.387	6.069	0.386	0.375
Reg3	-0.049	0.343	5.435	0.341	0.348	-0.050	0.823	1.563	0.823	0.866	-0.113	0.932	1.129	0.928	1.063	-0.013	0.371	6.058	0.370	0.361
Reg4	-0.050	0.346	5.453	0.343	0.348	-0.090	0.805	1.616	0.804	0.837	-0.123	0.925	1.177	0.921	1.065	-0.005	0.370	6.153	0.370	0.360
Reg5	0.183	0.500	3.478	0.450	0.461	0.155	1.045	1.028	1.028	1.172	0.083	1.225	0.759	1.217	1.510	0.234	0.544	3.849	0.494	0.495
Reg6	0.185	0.499	3.443	0.449	0.461	0.153	1.026	1.021	1.007	1.137	0.083	1.183	0.740	1.175	1.478	0.238	0.548	3.816	0.497	0.496
MI1-Reg	-0.055	0.349		0.348	0.356	-0.101	0.832		0.830	0.944	-0.127	0.873		0.868	1.151	-0.013	0.378		0.378	0.373
MI2-Reg	0.022	0.383		0.380	0.422	-0.006	0.882		0.882	1.056	-0.058	0.962		0.962	1.315	0.071	0.421		0.416	0.447
B. τ^2/L = 5																				
No Missing	-0.005	0.196	9.417	0.196	0.198	0.046	0.661	1.743	0.658	0.726	0.093	0.903	0.775	0.895	1.139	-0.009	0.200	9.350	0.200	0.198
CC	-0.002	0.228	7.879	0.228	0.223	0.046	1.157	1.340	1.150	1.259	0.075	1.044	0.770	1.043	1.253	0.005	0.224	8.192	0.224	0.222
DV	-0.138	0.281	5.795	0.234	0.222	-0.065	0.723	0.907	0.719	0.772	0.057	1.048	0.473	1.048	1.204	-0.166	0.287	6.047	0.228	0.220
NN1	0.002	0.252	9.334	0.252	0.247	-0.123	0.713	1.989	0.706	0.759	-0.298	1.002	1.164	0.941	1.189	0.006	0.274	9.836	0.273	0.256
NN2	-0.019	0.244	9.655	0.243	0.237	-0.092	0.650	1.870	0.642	0.742	-0.298	1.019	1.165	0.969	1.097	-0.013	0.251	10.109	0.251	0.246
NN3	0.168	0.374	6.450	0.310	0.313	0.095	0.850	1.214	0.840	0.939	-0.096	1.042	0.736	1.037	1.304	0.208	0.399	6.503	0.327	0.326
MI1-NN	-0.006	0.239		0.239	0.264	-0.097	0.610		0.602	0.857	-0.271	0.869		0.805	1.351	0.007	0.253		0.253	0.273
MI2-NN	0.062	0.259		0.249	0.328	0.007	0.618		0.617	0.981	-0.227	0.800		0.759	1.513	0.073	0.279		0.267	0.338
Reg1	-0.021	0.228	12.416	0.228	0.220	-0.007	0.712	2.158	0.712	0.789	-0.157	1.151	1.195	1.131	1.230	-0.022	0.239	12.950	0.239	0.231
Reg2	-0.029	0.226	12.296	0.225	0.221	-0.074	0.695	2.488	0.689	0.716	-0.250	0.994	1.307	0.954	1.051	-0.016	0.239	12.656	0.238	0.230
Reg3	-0.027	0.224	12.248	0.224	0.218	0.019	0.652	2.141	0.652	0.740	-0.144	1.233	1.147	1.216	1.276	-0.017	0.236	12.912	0.236	0.229
Reg4	-0.027	0.223	12.106	0.223	0.218	-0.046	0.717	2.397	0.713	0.746	-0.179	1.149	1.202	1.126	1.199	-0.016	0.235	12.823	0.235	0.228
Reg5	0.223	0.379	7.869	0.295	0.283	0.251	0.933	1.366	0.901	1.093	0.023	1.480	0.763	1.480	1.776	0.230	0.386	8.111	0.300	0.297
Reg6	0.223	0.380	7.878	0.294	0.284	0.253	0.886	1.354	0.852	1.056	0.063	1.546	0.744	1.546	1.925	0.231	0.388	8.022	0.301	0.297
MI1-Reg	-0.026	0.224		0.224	0.221	-0.014	0.636		0.635	0.790	-0.182	1.063		1.037	1.281	-0.017	0.237		0.236	0.231
MI2-Reg	0.061	0.257		0.246	0.283	0.082	0.706		0.704	0.931	-0.106	1.180		1.170	1.552	0.065	0.268		0.257	0.294
C. τ^2/L = 10																				
No Missing	-0.004	0.136	18.264	0.136	0.137	0.024	0.476	2.533	0.476	0.469	0.108	0.933	0.862	0.924	1.196	0.000	0.136	17.488	0.136	0.139
CC	-0.008	0.152	14.658	0.152	0.154	0.056	0.554	1.827	0.553	0.620	0.126	0.840	0.871	0.830	1.105	0.002	0.145	15.750	0.145	0.155
DV	-0.158	0.239	10.671	0.169	0.157	-0.122	0.690	1.266	0.658	0.587	0.121	0.962	0.535	0.949	1.077	-0.151	0.229	11.661	0.160	0.156
NN1	0.007	0.166	17.468	0.166	0.169	-0.058	0.605	2.616	0.603	0.599	-0.245	0.930	1.248	0.889	1.030	0.020	0.182	19.182	0.181	0.178
NN2	-0.006	0.163	17.825	0.163	0.167	-0.071	0.595	2.753	0.593	0.570	-0.297	0.958	1.268	0.926	1.051	-0.004	0.173	19.694	0.172	0.173
NN3	0.206	0.322	11.468	0.224	0.225	0.158	0.809	1.610	0.785	0.854	-0.066	1.178	0.864	1.175	1.457	0.228	0.327	12.229	0.227	0.232
MI1-NN	-0.007	0.159		0.159	0.182	-0.072	0.564		0.561	0.648	-0.269	0.837		0.796	1.208	0.005	0.171		0.171	0.190
MI2-NN	0.066	0.190		0.172	0.254	0.007	0.582		0.582	0.822	-0.189	0.798		0.773	1.490	0.079	0.200		0.181	0.261
Reg1	-0.014	0.152	22.865	0.151	0.153	-0.003	0.612	2.870	0.609	0.605	-0.120	0.777	1.381	0.770	0.956	-0.016	0.163	24.930	0.163	0.163
Reg2	-0.020	0.152	22.952	0.152	0.153	-0.065	0.489	3.464	0.487	0.494	-0.178	0.923	1.429	0.907	1.130	-0.008	0.161	24.477	0.161	0.161
Reg3	-0.012	0.150	22.976	0.150	0.153	0.011	0.592	2.993	0.588	0.580	-0.110	0.809	1.278	0.806	0.968	-0.014	0.162	24.796	0.162	0.162
Reg4	-0.013	0.150	22.702	0.149	0.152	-0.049	0.580	3.502	0.580	0.520	-0.117	0.787	1.326	0.780	0.922	-0.010	0.160	24.543	0.159	0.160
Reg5	0.220	0.309	14.258	0.197	0.199	0.275	0.836	1.821	0.768	0.831	0.123	0.969	0.868	0.961	1.269	0.229	0.316	15.445	0.206	0.210
Reg6	0.225	0.310	14.232	0.197	0.199	0.279	0.841	1.808	0.773	0.826	0.132	0.953	0.863	0.945	1.235	0.229	0.317	15.455	0.206	0.211
MI1-Reg	-0.016	0.150		0.150	0.154	-0.022	0.506		0.506	0.578	-0.143	0.755		0.746	1.105	-0.012	0.161		0.161	0.163
MI2-Reg	0.066	0.180		0.164	0.221	0.058	0.573		0.560	0.714	-0.062	0.802		0.801	1.205	0.069	0.190		0.174	0.229

Notes: See Table A1.

Table A8. Monte Carlo Results: Nonlinear Specification, $L = 3$, $\rho = 0.1$, $\pi_{21} = 1$, $\rho(x_1,z) \neq 0$.

Estimator	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2/L = 2$																				
No Missing	0.021	0.317	3.001	0.317	0.327	0.052	0.629	1.042	0.626	0.746	0.057	0.679	0.755	0.678	0.944	0.020	0.314	2.877	0.314	0.328
CC	0.014	0.359	2.473	0.359	0.369	0.084	0.706	0.927	0.704	0.840	0.055	0.780	0.845	0.780	0.971	0.035	0.381	2.580	0.381	0.387
DV	-0.933	0.953	23.303	0.131	0.128	0.212	0.406	3.179	0.346	0.346	-1.782	1.806	5.409	0.299	0.304	-0.931	0.955	25.075	0.134	0.124
NN1	-0.060	0.423	3.034	0.420	0.415	-0.142	0.823	1.232	0.817	0.916	-0.469	1.044	1.369	0.928	0.961	-0.037	0.696	3.070	0.696	0.626
NN2	-0.551	0.583	8.870	0.188	0.195	-0.210	1.038	1.041	1.023	1.085	-0.700	1.005	1.691	0.695	0.783	-0.555	0.590	9.272	0.193	0.199
NN3	0.117	0.511	2.182	0.497	0.513	-0.025	0.910	0.906	0.910	1.025	-0.087	0.970	0.732	0.960	1.174	0.156	0.565	2.220	0.543	0.542
MI1-NN	-0.300	0.412		0.279	0.573	-0.146	0.789		0.776	1.231	-0.584	0.874		0.633	1.196	-0.280	0.494		0.400	0.740
MI2-NN	-0.161	0.363		0.325	0.624	-0.063	0.709		0.703	1.289	-0.457	0.755		0.607	1.373	-0.136	0.419		0.394	0.763
Reg1	-0.056	0.362	3.891	0.359	0.371	-0.073	0.769	1.437	0.764	0.837	-0.297	0.943	1.340	0.884	0.991	-0.056	0.438	4.013	0.437	0.407
Reg2	-0.043	0.386	3.713	0.385	0.378	-0.080	0.678	1.374	0.672	0.820	-0.155	0.922	1.087	0.903	1.134	-0.020	0.395	3.561	0.394	0.386
Reg3	-0.555	0.586	13.382	0.172	0.182	-0.261	1.022	1.296	0.989	1.138	-0.782	0.949	3.720	0.515	0.517	-0.552	0.592	14.223	0.184	0.186
Reg4	-0.565	0.591	13.413	0.173	0.181	-0.170	1.001	1.194	0.985	1.117	-0.669	1.029	2.048	0.777	0.836	-0.556	0.595	13.895	0.181	0.184
Reg5	0.178	0.490	2.494	0.453	0.476	0.193	0.935	0.945	0.921	1.093	-0.303	1.764	0.994	1.729	1.802	0.170	0.582	2.584	0.544	0.533
Reg6	0.173	0.498	2.403	0.466	0.483	0.111	1.111	0.952	1.094	1.223	0.012	0.866	0.773	0.866	1.063	0.189	0.560	2.414	0.524	0.517
MI1-Reg	-0.297	0.391		0.249	0.476	-0.115	0.691		0.674	1.264	-0.492	0.699		0.491	1.293	-0.279	0.402		0.274	0.493
MI2-Reg	-0.134	0.339		0.308	0.552	-0.003	0.669		0.668	1.374	-0.368	0.699		0.578	1.647	-0.131	0.374		0.352	0.582
B. $\tau^2/L = 5$																				
No Missing	-0.002	0.199	6.395	0.199	0.202	-0.006	0.621	1.352	0.621	0.688	0.061	0.727	0.792	0.726	0.914	0.003	0.197	6.104	0.197	0.206
CC	-0.006	0.224	5.108	0.224	0.227	0.017	0.720	1.222	0.720	0.795	0.068	0.709	0.861	0.703	0.866	0.003	0.216	5.109	0.216	0.231
DV	-0.910	0.916	27.769	0.127	0.118	0.313	0.489	2.916	0.379	0.367	-1.807	1.834	5.348	0.317	0.310	-0.911	0.919	28.932	0.124	0.116
NN1	-0.014	0.260	5.962	0.260	0.263	-0.108	0.635	1.544	0.615	0.701	-0.467	1.119	1.297	0.993	1.026	-0.007	0.280	5.968	0.280	0.281
NN2	-0.447	0.482	13.350	0.160	0.156	-0.128	0.975	1.103	0.967	1.016	-0.690	0.989	1.707	0.716	0.746	-0.467	0.482	13.460	0.153	0.159
NN3	0.184	0.371	4.225	0.317	0.326	0.009	0.838	0.978	0.838	0.993	-0.111	0.983	0.746	0.973	1.183	0.185	0.400	4.351	0.334	0.344
MI1-NN	-0.218	0.298		0.192	0.455	-0.107	0.694		0.680	1.048	-0.593	0.888		0.655	1.199	-0.232	0.297		0.197	0.473
MI2-NN	-0.093	0.231		0.213	0.485	-0.030	0.653		0.648	1.137	-0.427	0.753		0.608	1.388	-0.083	0.236		0.224	0.504
Reg1	-0.039	0.230	7.950	0.229	0.232	-0.084	0.733	1.927	0.725	0.747	-0.250	0.882	1.324	0.844	0.913	-0.032	0.243	8.072	0.243	0.248
Reg2	-0.028	0.219	7.300	0.218	0.224	-0.128	0.666	1.725	0.655	0.687	-0.142	0.752	1.098	0.738	0.895	-0.012	0.233	7.328	0.233	0.239
Reg3	-0.454	0.486	20.350	0.142	0.144	-0.119	1.092	1.177	1.087	1.235	-0.750	0.929	3.635	0.506	0.500	-0.468	0.488	21.066	0.140	0.148
Reg4	-0.463	0.491	20.266	0.144	0.143	-0.059	1.037	1.178	1.034	1.150	-0.620	0.968	2.207	0.736	0.710	-0.470	0.492	20.569	0.140	0.146
Reg5	0.193	0.361	5.086	0.291	0.300	0.151	0.811	1.202	0.803	0.933	-0.251	1.043	1.005	1.023	1.125	0.210	0.387	5.139	0.310	0.320
Reg6	0.214	0.353	4.737	0.279	0.291	0.118	0.787	1.080	0.782	0.900	0.052	0.901	0.756	0.901	1.131	0.227	0.401	4.884	0.315	0.318
MI1-Reg	-0.234	0.301		0.172	0.348	-0.070	0.692		0.684	1.247	-0.455	0.653		0.471	1.096	-0.243	0.297		0.177	0.367
MI2-Reg	-0.094	0.223		0.203	0.414	0.006	0.671		0.670	1.214	-0.320	0.632		0.536	1.254	-0.088	0.229		0.215	0.435
C. $\tau^2/L = 10$																				
No Missing	-0.003	0.139	11.660	0.138	0.139	0.033	0.672	2.027	0.672	0.742	0.013	0.815	0.792	0.814	0.992	-0.002	0.134	11.401	0.134	0.139
CC	-0.001	0.154	9.437	0.154	0.155	0.048	0.573	1.586	0.569	0.613	0.079	0.746	0.824	0.742	0.891	0.010	0.152	10.153	0.152	0.156
DV	-0.857	0.863	33.060	0.116	0.112	0.465	0.606	2.831	0.378	0.371	-1.836	1.865	5.201	0.301	0.315	-0.842	0.861	35.002	0.112	0.109
NN1	0.001	0.179	10.933	0.178	0.179	-0.025	0.835	2.140	0.835	0.755	-0.404	0.939	1.295	0.841	0.920	0.016	0.199	11.831	0.198	0.189
NN2	-0.362	0.388	19.822	0.127	0.127	0.057	1.261	1.425	1.261	1.147	-0.669	0.984	1.692	0.695	0.711	-0.378	0.391	20.477	0.129	0.129
NN3	0.208	0.306	7.509	0.215	0.225	0.166	0.935	1.409	0.923	0.932	-0.085	0.907	0.806	0.900	1.127	0.220	0.336	7.998	0.235	0.238
MI1-NN	-0.181	0.229		0.140	0.361	0.013	0.808		0.808	1.225	-0.550	0.818		0.598	1.116	-0.182	0.231		0.151	0.377
MI2-NN	-0.044	0.161		0.153	0.396	0.085	0.705		0.704	1.289	-0.405	0.699		0.568	1.290	-0.044	0.171		0.167	0.412
Reg1	-0.020	0.160	14.937	0.160	0.161	0.016	0.616	2.524	0.616	0.608	-0.247	0.821	1.317	0.786	0.889	-0.020	0.176	15.934	0.176	0.169
Reg2	-0.015	0.153	13.455	0.153	0.154	-0.024	0.735	2.534	0.735	0.725	-0.146	0.764	1.098	0.741	0.852	-0.007	0.163	14.371	0.163	0.162
Reg3	-0.378	0.396	29.817	0.112	0.116	0.181	1.061	1.367	1.051	0.960	-0.752	1.030	3.538	0.607	0.591	-0.385	0.398	31.596	0.115	0.119
Reg4	-0.382	0.400	29.484	0.113	0.115	0.176	0.875	1.475	0.863	0.836	-0.609	0.979	2.203	0.698	0.706	-0.383	0.400	30.224	0.115	0.117
Reg5	0.217	0.302	9.347	0.203	0.210	0.279	0.813	1.570	0.766	0.834	-0.292	1.027	0.983	0.997	1.176	0.222	0.325	9.973	0.223	0.221
Reg6	0.222	0.301	8.599	0.195	0.202	0.253	0.979	1.538	0.958	0.934	0.070	0.833	0.753	0.826	1.034	0.224	0.321	9.048	0.212	0.214
MI1-Reg	-0.199	0.235		0.126	0.277	0.105	0.631		0.627	1.033	-0.468	0.674		0.468	1.098	-0.196	0.235		0.133	0.285
MI2-Reg	-0.056	0.156		0.145	0.342	0.154	0.633		0.621	1.059	-0.322	0.616		0.510	1.250	-0.057	0.164		0.156	0.351

Notes: See Table A1.

Table A9. Monte Carlo Results: Linear Specification, L = 3, ρ = 0.5, π_{21} = 0, $\rho(x_1,z)$ = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2 = 2																				
No Missing	0.128	0.423	2.716	0.414	0.427	0.154	0.473	2.710	0.465	0.431	0.166	0.432	2.597	0.417	0.414	0.111	0.454	2.578	0.444	0.440
CC	0.132	0.455	2.281	0.445	0.471	0.164	0.451	2.375	0.429	0.444	0.191	0.509	2.204	0.489	0.495	0.117	0.518	2.442	0.509	0.499
DV	0.141	0.459	1.399	0.448	0.470	0.159	0.463	1.489	0.441	0.446	0.186	0.519	1.375	0.500	0.500	0.127	0.513	1.532	0.504	0.497
NN1	0.030	0.569	2.850	0.569	0.580	0.012	0.588	2.934	0.588	0.581	0.052	0.648	2.812	0.647	0.597	0.045	0.606	3.148	0.604	0.611
NN2	0.005	0.529	2.862	0.529	0.580	0.000	0.518	3.099	0.517	0.523	0.023	0.617	2.851	0.617	0.586	0.011	0.646	3.237	0.645	0.604
NN3	0.190	0.718	1.937	0.679	0.716	0.242	0.708	1.965	0.668	0.687	0.293	0.907	1.737	0.862	0.849	0.223	0.820	2.095	0.775	0.784
MI1-NN	0.025	0.515		0.515	0.642	0.020	0.512		0.512	0.619	0.032	0.582		0.581	0.686	0.043	0.602		0.601	0.663
MI2-NN	0.074	0.532		0.525	0.741	0.114	0.538		0.532	0.709	0.132	0.612		0.601	0.850	0.114	0.615		0.605	0.783
Reg1	0.032	0.501	3.572	0.499	0.499	0.056	0.497	3.733	0.494	0.479	0.070	0.532	3.758	0.530	0.520	0.057	0.617	3.887	0.616	0.564
Reg2	0.038	0.500	3.552	0.498	0.499	0.057	0.534	3.756	0.533	0.500	0.045	0.579	3.679	0.578	0.543	0.045	0.545	3.901	0.545	0.540
Reg3	0.030	0.501	3.558	0.499	0.499	0.062	0.490	3.720	0.488	0.478	0.067	0.527	3.727	0.525	0.515	0.055	0.588	3.863	0.587	0.555
Reg4	0.035	0.504	3.511	0.503	0.501	0.073	0.486	3.706	0.484	0.476	0.048	0.540	3.705	0.538	0.520	0.052	0.581	3.891	0.580	0.553
Reg5	0.291	0.691	2.277	0.623	0.625	0.322	0.689	2.374	0.616	0.598	0.388	0.783	2.172	0.691	0.685	0.324	0.830	2.449	0.773	0.722
Reg6	0.287	0.693	2.273	0.624	0.625	0.325	0.695	2.355	0.621	0.602	0.438	0.814	2.046	0.708	0.703	0.326	0.807	2.456	0.749	0.714
MI1-Reg	0.034	0.501		0.499	0.500	0.065	0.493		0.491	0.493	0.064	0.525		0.523	0.544	0.056	0.576		0.575	0.557
MI2-Reg	0.124	0.555		0.540	0.573	0.153	0.548		0.532	0.562	0.182	0.599		0.577	0.649	0.149	0.646		0.634	0.645
B. τ^2 = 5																				
No Missing	0.028	0.280	5.687	0.279	0.275	0.066	0.256	5.899	0.253	0.260	0.062	0.282	5.505	0.280	0.271	0.059	0.263	5.266	0.261	0.271
CC	0.038	0.316	4.618	0.315	0.304	0.090	0.293	4.763	0.288	0.292	0.087	0.341	4.122	0.335	0.324	0.062	0.294	4.966	0.290	0.297
DV	0.040	0.315	2.875	0.315	0.304	0.091	0.294	2.979	0.289	0.292	0.083	0.342	2.550	0.337	0.324	0.063	0.295	3.099	0.290	0.297
NN1	0.000	0.337	5.555	0.337	0.347	0.022	0.344	5.793	0.342	0.343	0.070	0.502	5.276	0.493	0.423	0.014	0.352	5.978	0.351	0.374
NN2	0.002	0.345	5.879	0.345	0.348	0.040	0.326	5.696	0.324	0.337	0.021	0.412	5.530	0.411	0.373	0.006	0.347	6.267	0.346	0.370
NN3	0.194	0.478	3.836	0.426	0.430	0.218	0.471	4.018	0.410	0.416	0.324	0.657	3.155	0.545	0.539	0.245	0.539	4.102	0.483	0.465
MI1-NN	0.004	0.329		0.329	0.376	0.033	0.321		0.319	0.371	0.055	0.417		0.411	0.452	0.018	0.334		0.333	0.402
MI2-NN	0.072	0.351		0.343	0.442	0.107	0.351		0.337	0.428	0.155	0.459		0.427	0.563	0.092	0.372		0.359	0.474
Reg1	0.009	0.315	7.316	0.315	0.318	0.024	0.303	7.585	0.302	0.309	0.025	0.371	6.925	0.369	0.337	0.018	0.301	7.813	0.301	0.335
Reg2	0.004	0.318	7.194	0.317	0.319	0.025	0.311	7.601	0.310	0.310	0.023	0.387	6.992	0.386	0.342	0.015	0.301	7.797	0.301	0.337
Reg3	0.008	0.316	7.350	0.316	0.318	0.026	0.303	7.623	0.302	0.309	0.026	0.373	7.013	0.371	0.338	0.019	0.301	7.774	0.301	0.335
Reg4	0.011	0.318	7.273	0.318	0.318	0.023	0.300	7.501	0.299	0.309	0.031	0.368	7.069	0.367	0.338	0.009	0.302	7.802	0.301	0.336
Reg5	0.243	0.464	4.599	0.394	0.388	0.286	0.473	4.720	0.378	0.377	0.360	0.606	4.086	0.482	0.438	0.274	0.470	4.988	0.383	0.411
Reg6	0.247	0.466	4.601	0.397	0.389	0.284	0.475	4.668	0.380	0.378	0.410	0.641	3.755	0.493	0.452	0.277	0.471	4.964	0.384	0.412
MI1-Reg	0.006	0.316		0.316	0.319	0.029	0.303		0.302	0.312	0.030	0.371		0.369	0.345	0.012	0.301		0.301	0.336
MI2-Reg	0.087	0.351		0.342	0.377	0.111	0.345		0.327	0.371	0.146	0.434		0.407	0.440	0.099	0.342		0.327	0.396
C. τ^2 = 10																				
No Missing	0.015	0.187	10.455	0.187	0.188	0.046	0.199	10.399	0.197	0.184	0.043	0.184	10.955	0.181	0.180	0.032	0.194	10.656	0.193	0.186
CC	0.038	0.214	8.700	0.213	0.208	0.069	0.213	8.511	0.209	0.206	0.063	0.220	8.094	0.216	0.212	0.036	0.216	9.525	0.215	0.205
DV	0.034	0.214	5.375	0.214	0.208	0.069	0.213	5.292	0.209	0.206	0.066	0.219	5.087	0.215	0.212	0.039	0.216	5.892	0.215	0.205
NN1	0.015	0.231	10.128	0.230	0.242	0.043	0.256	10.364	0.253	0.241	0.081	0.280	10.134	0.269	0.259	0.034	0.263	11.761	0.260	0.255
NN2	-0.003	0.230	10.405	0.230	0.239	0.030	0.240	10.482	0.239	0.239	0.046	0.253	10.417	0.247	0.251	0.006	0.253	11.818	0.253	0.251
NN3	0.200	0.367	6.896	0.292	0.299	0.260	0.406	6.730	0.311	0.303	0.379	0.544	5.859	0.373	0.356	0.248	0.419	7.683	0.331	0.318
MI1-NN	0.003	0.223		0.223	0.258	0.030	0.239		0.237	0.262	0.063	0.253		0.243	0.286	0.019	0.252		0.251	0.268
MI2-NN	0.074	0.249		0.235	0.319	0.107	0.272		0.250	0.330	0.185	0.321		0.267	0.397	0.103	0.289		0.269	0.337
Reg1	0.000	0.207	13.363	0.207	0.220	0.032	0.226	13.358	0.225	0.218	0.021	0.212	13.973	0.211	0.221	0.015	0.236	15.013	0.235	0.233
Reg2	-0.005	0.208	13.440	0.208	0.220	0.023	0.228	13.233	0.227	0.219	0.020	0.218	14.074	0.217	0.222	0.009	0.237	15.026	0.237	0.233
Reg3	0.000	0.207	13.389	0.207	0.220	0.031	0.225	13.389	0.224	0.218	0.020	0.211	13.893	0.210	0.221	0.011	0.236	15.055	0.236	0.233
Reg4	-0.002	0.208	13.372	0.208	0.220	0.034	0.225	13.380	0.224	0.219	0.025	0.211	13.961	0.210	0.222	0.009	0.237	15.002	0.236	0.233
Reg5	0.230	0.358	8.600	0.267	0.267	0.287	0.396	8.503	0.284	0.267	0.368	0.454	7.853	0.286	0.285	0.272	0.400	9.501	0.299	0.286
Reg6	0.233	0.360	8.558	0.268	0.267	0.292	0.398	8.443	0.285	0.268	0.412	0.501	7.342	0.301	0.296	0.271	0.401	9.412	0.300	0.287
MI1-Reg	-0.003	0.207		0.207	0.221	0.029	0.225		0.224	0.220	0.021	0.211		0.210	0.224	0.008	0.236		0.236	0.233
MI2-Reg	0.081	0.238		0.226	0.279	0.116	0.265		0.243	0.282	0.145	0.273		0.236	0.324	0.101	0.273		0.256	0.294

Notes: See Table A1.

Table A10. Monte Carlo Results: Linear Specification, L = 3, ρ = 0.5, π_{21} = 1, $\rho(x_1,z)$ = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**	**Median Bias**	**RMSE**	**Median F**	**SD**	**Mean SE**
A. τ^2 = 2																				
No Missing	0.144	0.473	2.868	0.454	0.433	0.168	0.458	2.597	0.440	0.458	0.111	0.423	2.561	0.417	0.437	0.150	0.484	2.602	0.471	0.470
CC	0.150	0.466	2.287	0.444	0.461	0.195	0.475	2.320	0.443	0.456	0.155	0.502	2.090	0.488	0.508	0.179	0.587	2.405	0.559	0.525
DV	-0.076	0.533	1.536	0.517	0.498	-0.015	0.534	1.531	0.528	0.488	-0.002	0.551	1.428	0.548	0.547	-0.057	0.549	1.653	0.531	0.493
NN1	-0.001	0.584	2.948	0.582	0.574	-0.007	0.522	2.879	0.521	0.555	0.038	0.717	2.157	0.714	0.747	0.057	0.613	2.956	0.609	0.626
NN2	-0.114	0.639	2.355	0.634	0.617	-0.071	0.566	2.297	0.561	0.593	-0.247	1.193	1.114	1.167	1.137	-0.082	0.676	2.528	0.669	0.674
NN3	0.220	0.708	1.997	0.660	0.703	0.239	0.754	1.980	0.717	0.781	0.227	1.124	1.676	1.085	1.057	0.301	0.886	1.977	0.830	0.816
MI1-NN	-0.040	0.536		0.536	0.716	0.008	0.468		0.468	0.688	-0.068	0.728		0.723	1.345	0.001	0.563		0.563	0.787
MI2-NN	0.069	0.526		0.520	0.792	0.086	0.505		0.501	0.808	0.054	0.643		0.642	1.388	0.113	0.594		0.586	0.885
Reg1	0.063	0.495	3.638	0.491	0.496	0.075	0.464	3.581	0.458	0.483	0.009	0.537	3.480	0.537	0.535	0.081	0.631	3.871	0.622	0.573
Reg2	0.065	0.486	3.599	0.482	0.496	0.058	0.492	3.673	0.489	0.491	-0.025	0.604	3.508	0.603	0.596	0.086	0.598	3.842	0.589	0.563
Reg3	0.003	0.720	3.170	0.720	0.599	0.040	0.582	3.213	0.582	0.576	0.195	0.884	1.731	0.856	0.883	0.008	0.606	3.213	0.606	0.637
Reg4	-0.042	0.648	3.239	0.646	0.599	0.030	0.586	3.163	0.585	0.572	-0.020	1.278	1.292	1.278	1.206	-0.016	0.630	3.356	0.629	0.625
Reg5	0.301	0.712	2.262	0.634	0.628	0.342	0.675	2.157	0.584	0.631	0.642	1.227	1.113	1.015	1.121	0.355	0.882	2.363	0.797	0.775
Reg6	0.324	0.695	2.277	0.613	0.622	0.334	0.669	2.280	0.576	0.609	0.361	0.789	1.906	0.711	0.724	0.349	0.875	2.392	0.785	0.750
MI1-Reg	0.031	0.497		0.496	0.596	0.066	0.482		0.480	0.584	0.067	0.603		0.601	1.133	0.050	0.565		0.563	0.649
MI2-Reg	0.118	0.546		0.532	0.648	0.160	0.515		0.496	0.635	0.208	0.651		0.619	1.174	0.146	0.643		0.623	0.730
B. τ^2 = 5																				
No Missing	0.061	0.272	5.817	0.269	0.261	0.087	0.290	5.813	0.287	0.269	0.068	0.279	5.572	0.276	0.265	0.052	0.248	5.727	0.245	0.266
CC	0.071	0.315	4.700	0.311	0.299	0.083	0.304	4.610	0.299	0.299	0.098	0.344	4.160	0.339	0.314	0.046	0.288	4.899	0.285	0.296
DV	-0.026	0.333	2.822	0.326	0.326	-0.014	0.386	2.743	0.380	0.329	0.111	0.392	2.443	0.388	0.364	-0.068	0.360	2.950	0.343	0.326
NN1	0.045	0.342	5.663	0.341	0.345	0.034	0.370	5.523	0.368	0.356	0.101	0.510	4.542	0.491	0.444	0.038	0.361	5.829	0.358	0.372
NN2	-0.031	0.379	4.504	0.378	0.379	-0.018	0.392	4.446	0.392	0.395	0.327	1.025	1.618	0.978	0.907	-0.023	0.391	4.888	0.390	0.399
NN3	0.238	0.480	3.938	0.416	0.426	0.309	0.515	3.714	0.440	0.445	0.340	0.637	3.206	0.522	0.512	0.245	0.489	4.117	0.421	0.456
MI1-NN	0.008	0.331		0.331	0.424	0.026	0.351		0.351	0.436	0.234	0.642		0.603	0.968	0.015	0.345		0.345	0.449
MI2-NN	0.103	0.344		0.334	0.478	0.121	0.369		0.355	0.498	0.268	0.579		0.513	0.876	0.110	0.358		0.346	0.501
Reg1	0.042	0.306	7.398	0.305	0.313	0.054	0.321	7.384	0.319	0.315	0.032	0.362	7.252	0.362	0.327	0.036	0.330	7.741	0.328	0.338
Reg2	0.035	0.306	7.382	0.305	0.314	0.047	0.329	7.264	0.328	0.316	-0.007	0.348	7.218	0.348	0.328	0.033	0.329	7.744	0.328	0.338
Reg3	0.019	0.319	6.422	0.319	0.331	0.043	0.350	6.157	0.350	0.343	0.280	0.708	3.606	0.629	0.575	0.006	0.330	6.404	0.330	0.352
Reg4	0.006	0.328	6.451	0.328	0.330	0.024	0.347	5.874	0.347	0.349	0.484	1.102	2.134	0.992	0.852	-0.007	0.342	6.496	0.342	0.353
Reg5	0.300	0.479	4.654	0.387	0.384	0.325	0.508	4.328	0.417	0.398	0.720	0.959	2.382	0.643	0.676	0.291	0.500	4.827	0.409	0.418
Reg6	0.297	0.481	4.625	0.388	0.385	0.336	0.503	4.569	0.407	0.388	0.412	0.631	3.864	0.500	0.441	0.283	0.506	4.867	0.414	0.417
MI1-Reg	0.022	0.305		0.304	0.337	0.047	0.323		0.323	0.350	0.194	0.502		0.457	0.751	0.019	0.321		0.321	0.361
MI2-Reg	0.116	0.345		0.329	0.392	0.142	0.366		0.349	0.404	0.322	0.564		0.463	0.755	0.113	0.364		0.348	0.417
C. τ^2 = 10																				
No Missing	0.029	0.197	10.245	0.197	0.186	0.042	0.201	10.701	0.200	0.187	0.030	0.186	10.752	0.185	0.184	0.035	0.184	10.521	0.182	0.183
CC	0.040	0.220	8.415	0.219	0.210	0.054	0.221	8.613	0.220	0.209	0.029	0.217	7.882	0.216	0.216	0.032	0.200	9.565	0.199	0.202
DV	-0.024	0.261	4.642	0.256	0.228	-0.013	0.260	4.882	0.256	0.225	0.093	0.269	4.202	0.259	0.247	-0.026	0.241	5.242	0.237	0.218
NN1	0.019	0.243	9.803	0.242	0.244	0.041	0.257	10.132	0.256	0.247	0.128	0.330	8.273	0.298	0.287	0.037	0.258	11.222	0.255	0.253
NN2	-0.010	0.271	7.749	0.271	0.271	-0.004	0.289	8.061	0.289	0.272	0.412	0.828	3.212	0.660	0.589	-0.023	0.272	9.194	0.271	0.268
NN3	0.231	0.390	6.751	0.302	0.302	0.245	0.402	7.072	0.320	0.304	0.341	0.562	5.913	0.413	0.366	0.250	0.418	7.773	0.323	0.313
MI1-NN	0.006	0.240		0.240	0.295	0.015	0.258		0.258	0.297	0.277	0.530		0.422	0.645	0.016	0.249		0.248	0.298
MI2-NN	0.087	0.262		0.246	0.350	0.098	0.279		0.265	0.352	0.305	0.508		0.378	0.586	0.101	0.277		0.259	0.357
Reg1	0.006	0.218	13.051	0.218	0.221	0.026	0.226	13.381	0.226	0.221	0.016	0.229	13.400	0.229	0.225	0.020	0.230	15.048	0.229	0.230
Reg2	0.006	0.219	12.943	0.219	0.221	0.025	0.232	13.282	0.231	0.222	0.011	0.240	13.250	0.240	0.227	0.017	0.231	14.908	0.230	0.231
Reg3	0.008	0.233	10.735	0.233	0.234	-0.007	0.250	11.226	0.250	0.235	0.224	0.408	7.443	0.326	0.301	0.010	0.235	12.334	0.234	0.241
Reg4	0.003	0.242	10.845	0.242	0.236	-0.003	0.268	11.010	0.268	0.240	0.459	0.784	4.639	0.584	0.461	0.001	0.241	12.347	0.241	0.242
Reg5	0.255	0.380	8.270	0.277	0.270	0.267	0.392	8.222	0.293	0.274	0.591	0.748	5.118	0.405	0.381	0.274	0.401	9.369	0.292	0.284
Reg6	0.253	0.379	8.275	0.276	0.268	0.284	0.391	8.499	0.286	0.270	0.386	0.504	7.014	0.321	0.303	0.274	0.404	9.393	0.294	0.284
MI1-Reg	0.003	0.223		0.222	0.237	0.002	0.236		0.236	0.241	0.182	0.369		0.310	0.449	0.017	0.229		0.229	0.244
MI2-Reg	0.087	0.255		0.238	0.290	0.097	0.267		0.252	0.296	0.285	0.439		0.318	0.465	0.103	0.268		0.248	0.301

Notes: See Table A1.

Table A11. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.5, π_{21} = 0, ρ(x_1,z) = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2 = 2																				
No Missing	0.062	0.355	4.151	0.354	0.339	0.104	0.432	4.403	0.429	0.342	0.520	1.048	0.738	0.931	1.128	0.054	0.315	4.181	0.313	0.326
CC	0.093	0.405	3.554	0.398	0.371	0.114	0.411	3.595	0.408	0.374	0.552	1.694	0.739	1.630	3.257	0.069	0.361	3.723	0.359	0.363
DV	0.088	0.407	2.199	0.400	0.371	0.121	0.406	2.247	0.403	0.372	0.535	1.793	0.455	1.728	2.773	0.072	0.357	2.305	0.355	0.363
NN1	0.057	0.473	4.122	0.472	0.429	0.040	0.433	4.539	0.432	0.417	-0.017	1.044	1.156	1.040	1.181	0.026	0.457	4.638	0.456	0.455
NN2	0.012	0.489	4.290	0.489	0.437	0.025	0.413	4.651	0.413	0.405	-0.012	0.957	1.097	0.956	1.103	-0.005	0.434	4.603	0.434	0.441
NN3	0.219	0.577	2.828	0.543	0.514	0.291	0.614	2.984	0.561	0.565	0.209	1.222	0.716	1.203	1.485	0.201	0.580	3.106	0.539	0.572
MI1-NN	0.028	0.466		0.465	0.472	0.039	0.401		0.401	0.454	-0.011	0.891		0.891	1.327	0.011	0.425		0.425	0.487
MI2-NN	0.112	0.478		0.472	0.530	0.133	0.440		0.430	0.557	0.085	0.828		0.826	1.577	0.083	0.446		0.439	0.566
Reg1	0.031	0.442	5.527	0.442	0.390	0.052	0.392	5.662	0.391	0.379	0.154	1.995	1.206	1.994	3.164	0.005	0.401	5.848	0.401	0.406
Reg2	0.022	0.446	5.456	0.446	0.394	0.050	0.390	5.547	0.390	0.379	0.089	1.056	1.360	1.050	1.120	0.012	0.399	5.945	0.399	0.405
Reg3	0.030	0.442	5.490	0.442	0.390	0.063	0.391	5.633	0.390	0.378	0.156	2.669	1.212	2.656	3.304	0.009	0.400	5.906	0.400	0.406
Reg4	0.027	0.456	5.378	0.455	0.395	0.061	0.393	5.684	0.392	0.381	0.143	1.037	1.267	1.025	1.186	0.011	0.395	5.959	0.395	0.404
Reg5	0.291	0.619	3.509	0.560	0.487	0.320	0.566	3.592	0.489	0.469	0.478	3.818	0.730	3.804	7.905	0.242	0.560	3.730	0.499	0.502
Reg6	0.291	0.612	3.486	0.552	0.485	0.317	0.569	3.590	0.492	0.473	0.479	1.500	0.713	1.424	1.897	0.245	0.560	3.704	0.498	0.502
MI1-Reg	0.027	0.445		0.445	0.393	0.062	0.389		0.388	0.383	0.145	1.035		1.023	2.681	0.012	0.399		0.398	0.406
MI2-Reg	0.113	0.491		0.481	0.458	0.160	0.436		0.421	0.448	0.276	1.203		1.180	4.257	0.089	0.440		0.431	0.468
B. τ^2 = 5																				
No Missing	0.034	0.207	9.427	0.207	0.196	0.066	0.194	9.620	0.186	0.191	0.509	0.951	0.787	0.782	1.029	0.036	0.193	9.580	0.192	0.193
CC	0.038	0.236	7.768	0.235	0.223	0.069	0.216	7.818	0.207	0.213	0.506	1.084	0.773	0.976	1.248	0.032	0.215	8.572	0.214	0.215
DV	0.045	0.236	4.849	0.235	0.223	0.068	0.216	4.834	0.207	0.213	0.506	1.047	0.476	0.930	1.156	0.031	0.215	5.359	0.214	0.215
NN1	0.048	0.268	9.102	0.265	0.259	0.063	0.266	9.296	0.257	0.254	-0.043	0.922	1.145	0.919	1.080	0.048	0.268	10.168	0.265	0.266
NN2	0.022	0.259	9.607	0.259	0.254	0.047	0.237	9.806	0.232	0.248	-0.033	1.059	1.154	1.059	1.238	0.022	0.259	10.669	0.259	0.261
NN3	0.222	0.414	6.144	0.328	0.322	0.289	0.435	6.505	0.319	0.317	0.230	1.434	0.706	1.403	1.942	0.252	0.422	6.768	0.331	0.335
MI1-NN	0.032	0.255		0.254	0.278	0.058	0.243		0.235	0.274	-0.030	0.838		0.837	1.381	0.043	0.257		0.255	0.281
MI2-NN	0.098	0.283		0.264	0.345	0.142	0.286		0.251	0.343	0.061	0.852		0.849	1.896	0.116	0.290		0.270	0.350
Reg1	0.024	0.231	12.276	0.231	0.233	0.043	0.229	12.284	0.224	0.228	0.108	1.105	1.245	1.095	1.229	0.018	0.241	13.383	0.241	0.245
Reg2	0.021	0.232	12.210	0.232	0.234	0.044	0.236	12.298	0.232	0.229	0.036	0.884	1.420	0.882	0.954	0.020	0.239	13.279	0.239	0.244
Reg3	0.021	0.231	12.217	0.231	0.233	0.040	0.229	12.289	0.225	0.228	0.101	1.149	1.310	1.142	1.242	0.015	0.241	13.412	0.240	0.245
Reg4	0.020	0.231	12.059	0.231	0.233	0.043	0.228	12.320	0.223	0.228	0.096	1.015	1.274	1.009	1.128	0.025	0.239	13.398	0.239	0.243
Reg5	0.282	0.397	7.718	0.295	0.284	0.303	0.418	7.863	0.282	0.278	0.425	1.448	0.764	1.371	1.701	0.271	0.406	8.531	0.305	0.302
Reg6	0.283	0.399	7.673	0.295	0.285	0.305	0.421	7.866	0.284	0.279	0.433	1.460	0.740	1.378	1.707	0.270	0.408	8.442	0.306	0.303
MI1-Reg	0.021	0.231		0.231	0.234	0.044	0.229		0.225	0.230	0.094	0.962		0.956	1.224	0.018	0.240		0.239	0.245
MI2-Reg	0.105	0.269		0.251	0.294	0.131	0.277		0.243	0.292	0.223	1.105		1.081	1.456	0.110	0.279		0.260	0.305
C. τ^2 = 10																				
No Missing	0.010	0.136	18.078	0.136	0.138	0.018	0.148	17.724	0.147	0.138	0.473	0.940	0.776	0.820	1.011	0.021	0.148	18.307	0.148	0.139
CC	0.019	0.153	14.612	0.153	0.154	0.020	0.165	14.623	0.165	0.154	0.512	0.972	0.833	0.848	1.039	0.027	0.157	16.662	0.156	0.152
DV	0.021	0.153	9.011	0.153	0.154	0.017	0.165	8.988	0.165	0.155	0.505	0.981	0.527	0.858	1.047	0.028	0.157	10.350	0.156	0.152
NN1	0.016	0.171	17.723	0.169	0.179	0.025	0.190	17.349	0.187	0.180	-0.053	0.918	1.258	0.917	1.085	0.034	0.199	19.728	0.197	0.189
NN2	0.021	0.167	17.949	0.167	0.177	0.027	0.182	18.062	0.181	0.178	-0.056	1.096	1.263	1.096	1.225	0.012	0.198	20.210	0.197	0.186
NN3	0.238	0.338	11.345	0.226	0.228	0.235	0.348	11.611	0.243	0.226	0.173	1.360	0.808	1.355	1.643	0.241	0.346	12.966	0.247	0.235
MI1-NN	0.024	0.164		0.163	0.191	0.025	0.181		0.179	0.193	-0.019	0.862		0.861	1.371	0.016	0.194		0.193	0.201
MI2-NN	0.099	0.200		0.176	0.264	0.100	0.217		0.191	0.261	0.046	0.851		0.851	1.693	0.093	0.226		0.204	0.264
Reg1	0.003	0.154	22.977	0.154	0.163	0.018	0.166	22.811	0.165	0.163	0.143	1.060	1.400	1.054	1.135	0.003	0.184	25.948	0.184	0.175
Reg2	0.007	0.154	23.112	0.154	0.164	0.006	0.172	22.547	0.172	0.163	0.068	1.089	1.429	1.088	1.140	0.005	0.181	25.952	0.181	0.173
Reg3	0.005	0.154	23.032	0.154	0.163	0.017	0.166	22.761	0.165	0.163	0.158	1.050	1.369	1.043	1.105	0.004	0.183	26.017	0.183	0.175
Reg4	0.002	0.154	23.045	0.154	0.163	0.014	0.166	22.503	0.166	0.163	0.112	1.294	1.434	1.287	1.636	0.008	0.182	25.863	0.182	0.173
Reg5	0.244	0.318	14.477	0.196	0.200	0.257	0.333	14.262	0.211	0.200	0.459	1.401	0.827	1.337	1.599	0.254	0.345	16.435	0.232	0.216
Reg6	0.246	0.319	14.473	0.196	0.200	0.257	0.335	14.250	0.212	0.201	0.461	1.409	0.789	1.341	1.594	0.256	0.346	16.417	0.232	0.217
MI1-Reg	0.004	0.154		0.154	0.164	0.013	0.166		0.166	0.165	0.141	1.012		1.007	1.452	0.004	0.182		0.182	0.174
MI2-Reg	0.085	0.187		0.167	0.227	0.094	0.202		0.180	0.228	0.253	1.112		1.092	1.604	0.091	0.217		0.198	0.238

Notes: See Table A1.

Table A12. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.5, π_{21} = 1, $\rho(x_1,z)$ = 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2 = 2																				
No Missing	0.070	0.314	3.089	0.308	0.317	0.115	0.301	3.029	0.288	0.308	0.320	0.740	0.733	0.665	0.868	0.072	0.341	2.923	0.336	0.336
CC	0.086	0.347	2.499	0.339	0.354	0.140	0.368	2.670	0.353	0.350	0.283	0.744	0.761	0.678	0.851	0.078	0.363	2.661	0.356	0.366
DV	-0.275	0.565	1.821	0.451	0.401	0.128	0.398	1.704	0.383	0.354	-0.908	1.078	1.490	0.602	0.557	-0.322	0.568	1.876	0.428	0.397
NN1	0.006	0.434	3.125	0.433	0.449	0.042	0.397	3.162	0.393	0.407	-0.352	1.076	1.325	1.016	1.018	0.006	0.491	3.111	0.490	0.496
NN2	-0.264	0.643	2.246	0.592	0.558	-0.019	0.449	3.119	0.449	0.436	-0.903	1.042	3.249	0.507	0.490	-0.264	0.589	2.096	0.530	0.535
NN3	0.218	0.557	2.180	0.510	0.534	0.256	0.520	2.253	0.459	0.494	0.057	1.128	0.720	1.127	1.389	0.234	0.610	2.181	0.562	0.587
MI1-NN	-0.118	0.439		0.424	0.677	0.019	0.383		0.382	0.494	-0.645	0.881		0.612	1.138	-0.084	0.435		0.420	0.686
MI2-NN	0.003	0.406		0.406	0.702	0.125	0.391		0.378	0.547	-0.386	0.759		0.644	1.508	0.028	0.427		0.427	0.720
Reg1	0.024	0.370	3.966	0.369	0.385	0.056	0.386	4.227	0.383	0.389	-0.042	0.931	1.252	0.929	1.024	0.020	0.436	4.212	0.435	0.429
Reg2	0.043	0.368	3.675	0.366	0.378	0.046	0.365	3.552	0.362	0.361	0.018	0.889	1.040	0.889	1.024	0.039	0.430	3.814	0.429	0.424
Reg3	-0.102	0.537	3.030	0.528	0.505	0.049	0.370	3.873	0.368	0.368	-0.643	1.130	1.847	0.899	0.848	-0.108	0.494	3.018	0.482	0.500
Reg4	-0.153	0.555	3.207	0.536	0.509	0.029	0.380	3.864	0.379	0.373	-0.885	1.069	4.425	0.526	0.480	-0.156	0.520	3.121	0.499	0.496
Reg5	0.273	0.557	2.523	0.492	0.503	0.315	0.544	2.704	0.451	0.468	-0.492	1.212	1.492	1.071	1.055	0.265	0.612	2.586	0.558	0.564
Reg6	0.290	0.546	2.407	0.473	0.479	0.298	0.529	2.401	0.443	0.451	0.251	1.012	0.712	0.978	1.202	0.266	0.621	2.522	0.556	0.547
MI1-Reg	-0.035	0.395		0.392	0.524	0.044	0.356		0.354	0.397	-0.411	0.658		0.516	1.276	-0.037	0.408		0.404	0.538
MI2-Reg	0.079	0.407		0.403	0.563	0.132	0.399		0.379	0.453	-0.322	0.651		0.565	1.385	0.064	0.441		0.438	0.592
B. τ^2 = 5																				
No Missing	0.025	0.190	6.643	0.190	0.199	0.052	0.202	6.599	0.200	0.198	0.357	0.794	0.787	0.717	0.879	0.029	0.208	6.540	0.208	0.200
CC	0.019	0.226	5.432	0.225	0.226	0.075	0.223	5.844	0.221	0.224	0.336	0.814	0.779	0.759	0.930	0.033	0.233	5.999	0.233	0.221
DV	-0.216	0.414	2.920	0.341	0.282	0.070	0.245	3.543	0.243	0.232	-0.833	1.041	1.504	0.624	0.560	-0.155	0.380	3.124	0.321	0.269
NN1	0.027	0.252	6.422	0.252	0.272	0.040	0.265	6.778	0.262	0.262	-0.275	1.042	1.349	0.985	1.021	0.011	0.303	6.711	0.303	0.286
NN2	-0.123	0.433	4.008	0.424	0.387	-0.004	0.269	6.333	0.269	0.278	-0.899	1.064	3.186	0.508	0.482	-0.109	0.456	3.879	0.447	0.403
NN3	0.213	0.397	4.414	0.322	0.330	0.247	0.398	4.582	0.308	0.329	0.114	1.113	0.758	1.108	1.357	0.212	0.432	4.699	0.369	0.349
MI1-NN	-0.032	0.298		0.296	0.430	0.023	0.247		0.247	0.313	-0.616	0.868		0.589	1.163	-0.047	0.330		0.328	0.444
MI2-NN	0.060	0.280		0.275	0.455	0.100	0.271		0.253	0.366	-0.379	0.719		0.602	1.493	0.047	0.320		0.316	0.468
Reg1	-0.008	0.242	8.412	0.242	0.245	0.033	0.244	9.305	0.243	0.249	-0.019	0.970	1.254	0.968	1.087	-0.015	0.268	9.377	0.268	0.259
Reg2	-0.006	0.224	7.651	0.224	0.237	0.023	0.234	7.817	0.234	0.235	0.077	0.922	1.157	0.918	1.079	-0.013	0.258	8.452	0.258	0.249
Reg3	-0.054	0.322	5.754	0.321	0.298	0.009	0.235	8.401	0.235	0.240	-0.599	1.021	1.891	0.817	0.811	-0.059	0.352	6.207	0.350	0.313
Reg4	-0.081	0.360	5.950	0.357	0.304	0.006	0.239	8.382	0.239	0.242	-0.874	1.048	4.225	0.519	0.475	-0.091	0.384	6.195	0.380	0.323
Reg5	0.239	0.397	5.287	0.312	0.308	0.275	0.403	5.888	0.296	0.300	-0.440	1.135	1.559	1.050	1.014	0.229	0.418	5.812	0.347	0.328
Reg6	0.234	0.379	4.947	0.286	0.293	0.258	0.399	5.087	0.293	0.293	0.320	1.110	0.775	1.064	1.339	0.229	0.406	5.428	0.333	0.312
MI1-Reg	-0.023	0.265		0.264	0.307	0.023	0.231		0.231	0.252	-0.347	0.653		0.535	1.323	-0.025	0.286		0.284	0.325
MI2-Reg	0.071	0.278		0.269	0.347	0.105	0.268		0.248	0.310	-0.242	0.633		0.573	1.437	0.061	0.302		0.296	0.365
C. τ^2 = 10																				
No Missing	0.016	0.151	12.314	0.150	0.138	0.033	0.142	13.147	0.140	0.135	0.373	0.707	0.836	0.625	0.774	0.020	0.145	12.431	0.145	0.141
CC	0.021	0.158	10.062	0.158	0.153	0.042	0.160	11.309	0.158	0.151	0.329	0.832	0.861	0.779	0.954	0.023	0.162	10.426	0.162	0.157
DV	-0.095	0.280	4.818	0.247	0.193	0.040	0.167	6.654	0.165	0.154	-0.800	1.008	1.513	0.610	0.547	-0.115	0.279	4.996	0.241	0.195
NN1	0.021	0.197	12.143	0.196	0.185	0.026	0.190	13.149	0.187	0.176	-0.278	1.019	1.336	0.978	0.972	0.030	0.209	12.608	0.207	0.201
NN2	-0.049	0.323	7.148	0.322	0.265	0.001	0.189	12.082	0.189	0.186	-0.895	1.056	3.284	0.527	0.481	-0.047	0.297	6.767	0.295	0.272
NN3	0.231	0.338	8.127	0.235	0.228	0.250	0.351	8.549	0.243	0.224	0.089	1.091	0.774	1.083	1.323	0.243	0.351	8.338	0.251	0.246
MI1-NN	-0.015	0.233		0.233	0.293	0.011	0.179		0.178	0.207	-0.584	0.848		0.599	1.137	-0.010	0.225		0.225	0.303
MI2-NN	0.079	0.230		0.216	0.330	0.095	0.212		0.188	0.272	-0.338	0.696		0.597	1.457	0.079	0.231		0.217	0.340
Reg1	-0.002	0.175	15.943	0.175	0.168	0.005	0.170	17.544	0.170	0.169	-0.079	1.022	1.408	1.022	1.089	0.004	0.187	16.476	0.187	0.183
Reg2	0.000	0.167	14.518	0.167	0.164	0.011	0.164	15.307	0.164	0.159	0.027	1.114	1.184	1.111	1.324	0.009	0.174	15.084	0.174	0.176
Reg3	-0.014	0.223	10.429	0.223	0.204	0.007	0.165	15.943	0.165	0.163	-0.642	0.974	2.087	0.727	0.746	-0.017	0.236	10.581	0.236	0.219
Reg4	-0.024	0.272	10.687	0.272	0.210	0.001	0.167	15.840	0.167	0.164	-0.886	1.026	4.672	0.496	0.473	-0.032	0.259	10.806	0.258	0.220
Reg5	0.245	0.337	9.840	0.229	0.212	0.260	0.335	11.065	0.212	0.204	-0.509	1.095	1.743	0.954	0.973	0.261	0.355	9.989	0.245	0.235
Reg6	0.244	0.331	9.241	0.217	0.203	0.253	0.331	9.901	0.211	0.197	0.315	0.935	0.822	0.890	1.145	0.256	0.341	9.646	0.227	0.225
MI1-Reg	-0.006	0.193		0.193	0.211	0.010	0.163		0.163	0.170	-0.381	0.665		0.551	1.440	-0.002	0.200		0.200	0.224
MI2-Reg	0.088	0.212		0.197	0.259	0.095	0.198		0.176	0.232	-0.288	0.632		0.562	1.490	0.083	0.223		0.207	0.274

Notes: See Table A1.

Table A13. Monte Carlo Results: Linear Specification, $L = 3$, $\rho = 0.5$, $\pi_{21} = 0$, $\rho(x_1,z) \neq 0$.

Estimator	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2 = 2$																				
No Missing	0.129	0.494	2.596	0.484	0.451	0.132	0.468	2.603	0.459	0.474	0.261	0.598	1.605	0.551	0.575	0.122	0.546	2.611	0.539	0.509
CC	0.138	0.527	2.160	0.521	0.538	0.183	0.552	2.324	0.540	0.539	0.273	0.722	1.441	0.672	0.695	0.147	0.442	2.437	0.417	0.453
DV	0.009	0.631	1.546	0.626	0.526	0.034	0.576	1.569	0.573	0.514	0.252	0.763	0.983	0.739	0.679	0.026	0.454	1.723	0.452	0.465
NN1	0.020	0.576	2.646	0.575	0.610	-0.011	0.529	2.894	0.529	0.558	0.000	0.810	1.912	0.810	0.888	0.039	0.576	3.131	0.573	0.602
NN2	0.010	0.533	2.818	0.533	0.561	0.000	0.534	2.920	0.534	0.549	-0.037	0.780	2.006	0.779	0.792	0.015	0.532	3.011	0.531	0.564
NN3	0.185	0.714	1.864	0.692	0.740	0.212	0.712	2.015	0.683	0.719	0.232	0.903	1.330	0.869	0.971	0.229	0.805	2.153	0.763	0.765
MI1-NN	0.026	0.519		0.518	0.651	-0.009	0.499		0.499	0.608	-0.035	0.685		0.684	0.973	0.034	0.518		0.516	0.645
MI2-NN	0.087	0.515		0.509	0.752	0.062	0.518		0.514	0.711	0.071	0.665		0.662	1.097	0.094	0.566		0.554	0.755
Reg1	0.048	0.612	3.349	0.612	0.584	0.036	0.546	3.622	0.545	0.540	0.071	0.724	2.300	0.719	0.727	0.068	0.504	3.868	0.501	0.522
Reg2	0.039	0.573	3.363	0.572	0.576	0.029	0.496	3.393	0.496	0.513	0.003	0.730	2.407	0.729	0.723	0.064	0.504	3.818	0.501	0.526
Reg3	0.049	0.644	3.383	0.644	0.576	0.042	0.523	3.556	0.522	0.520	0.090	0.721	2.324	0.713	0.701	0.068	0.490	3.876	0.488	0.510
Reg4	0.052	0.653	3.394	0.653	0.584	0.062	0.520	3.507	0.520	0.532	0.027	0.696	2.371	0.691	0.712	0.064	0.493	3.943	0.490	0.513
Reg5	0.298	0.818	2.161	0.774	0.745	0.287	0.744	2.303	0.684	0.689	0.343	0.990	1.453	0.913	0.979	0.320	0.708	2.462	0.629	0.657
Reg6	0.300	0.850	2.156	0.809	0.778	0.290	0.748	2.304	0.688	0.692	0.365	0.986	1.385	0.908	1.009	0.321	0.710	2.443	0.631	0.658
MI1-Reg	0.048	0.615		0.615	0.585	0.050	0.508		0.508	0.538	0.045	0.672		0.667	0.768	0.064	0.496		0.493	0.521
MI2-Reg	0.133	0.679		0.672	0.679	0.129	0.574		0.563	0.625	0.155	0.758		0.736	0.899	0.147	0.556		0.537	0.598
B. $\tau^2 = 5$																				
No Missing	0.055	0.263	5.432	0.259	0.267	0.061	0.272	5.564	0.270	0.266	0.096	0.420	3.162	0.415	0.409	0.053	0.269	5.412	0.267	0.269
CC	0.053	0.329	4.526	0.327	0.309	0.100	0.314	4.785	0.310	0.299	0.126	0.523	2.608	0.517	0.507	0.073	0.282	5.028	0.278	0.293
DV	-0.101	0.406	3.275	0.385	0.326	-0.030	0.354	3.310	0.341	0.317	0.101	0.499	1.771	0.499	0.482	-0.095	0.333	3.611	0.310	0.308
NN1	0.027	0.342	5.374	0.340	0.353	0.004	0.352	5.898	0.351	0.350	0.026	0.549	3.203	0.549	0.554	0.042	0.353	6.126	0.351	0.369
NN2	-0.003	0.334	5.652	0.334	0.343	0.008	0.341	5.891	0.341	0.339	-0.014	0.535	3.382	0.534	0.537	0.014	0.344	6.243	0.344	0.352
NN3	0.212	0.479	3.807	0.419	0.445	0.215	0.474	3.813	0.412	0.428	0.225	0.728	2.191	0.681	0.716	0.223	0.538	4.173	0.476	0.456
MI1-NN	0.018	0.325		0.324	0.378	0.002	0.335		0.335	0.373	0.000	0.513		0.512	0.598	0.028	0.334		0.333	0.392
MI2-NN	0.096	0.346		0.333	0.450	0.083	0.355		0.344	0.440	0.081	0.530		0.524	0.726	0.094	0.375		0.361	0.463
Reg1	0.028	0.312	7.068	0.311	0.319	0.020	0.308	7.522	0.308	0.315	0.031	0.486	4.229	0.486	0.495	0.029	0.312	7.963	0.312	0.331
Reg2	0.027	0.316	7.062	0.316	0.321	0.014	0.321	7.390	0.321	0.317	-0.008	0.553	4.151	0.553	0.530	0.026	0.313	7.920	0.312	0.332
Reg3	0.025	0.307	7.133	0.306	0.314	0.023	0.310	7.358	0.310	0.311	0.032	0.475	4.124	0.475	0.483	0.026	0.313	7.852	0.312	0.327
Reg4	0.021	0.309	7.142	0.308	0.315	0.022	0.314	7.411	0.314	0.312	0.017	0.475	4.165	0.475	0.478	0.027	0.313	7.788	0.313	0.328
Reg5	0.271	0.479	4.530	0.395	0.390	0.265	0.471	4.754	0.388	0.384	0.298	0.675	2.577	0.606	0.623	0.289	0.479	5.056	0.392	0.406
Reg6	0.276	0.482	4.520	0.397	0.391	0.271	0.472	4.739	0.389	0.384	0.324	0.691	2.515	0.614	0.628	0.288	0.482	5.030	0.394	0.407
MI1-Reg	0.029	0.310		0.309	0.319	0.027	0.311		0.311	0.318	0.020	0.474		0.474	0.520	0.032	0.312		0.311	0.331
MI2-Reg	0.110	0.352		0.337	0.379	0.106	0.349		0.335	0.376	0.117	0.525		0.514	0.596	0.117	0.353		0.338	0.391
C. $\tau^2 = 10$																				
No Missing	0.022	0.182	10.333	0.181	0.185	0.035	0.189	10.540	0.189	0.185	0.042	0.271	5.645	0.271	0.274	0.025	0.186	10.332	0.186	0.187
CC	0.026	0.215	8.427	0.214	0.209	0.059	0.217	8.784	0.216	0.207	0.070	0.331	4.537	0.330	0.319	0.040	0.199	9.539	0.198	0.205
DV	-0.127	0.295	6.229	0.256	0.227	-0.075	0.273	6.265	0.245	0.224	0.008	0.362	3.092	0.357	0.329	-0.129	0.268	6.867	0.225	0.219
NN1	0.031	0.239	10.135	0.237	0.243	0.012	0.238	10.756	0.237	0.241	0.043	0.384	5.400	0.383	0.372	0.038	0.245	11.227	0.243	0.255
NN2	0.002	0.232	10.473	0.231	0.238	0.008	0.238	10.805	0.238	0.237	0.010	0.394	5.568	0.394	0.384	0.009	0.245	11.672	0.245	0.246
NN3	0.224	0.384	6.741	0.295	0.303	0.223	0.375	7.023	0.287	0.297	0.253	0.566	3.600	0.500	0.484	0.233	0.401	7.526	0.317	0.314
MI1-NN	0.020	0.227		0.225	0.261	0.013	0.231		0.230	0.257	0.032	0.374		0.374	0.410	0.025	0.237		0.236	0.270
MI2-NN	0.094	0.256		0.237	0.325	0.081	0.256		0.239	0.321	0.106	0.401		0.387	0.506	0.096	0.270		0.253	0.335
Reg1	0.009	0.209	13.265	0.208	0.220	0.004	0.210	13.918	0.210	0.219	0.013	0.328	7.343	0.328	0.326	0.011	0.220	14.852	0.220	0.232
Reg2	0.006	0.211	13.185	0.210	0.221	0.003	0.216	13.844	0.216	0.220	-0.003	0.351	7.139	0.351	0.342	0.013	0.220	14.554	0.220	0.233
Reg3	0.006	0.209	13.412	0.209	0.218	0.007	0.215	13.724	0.215	0.218	0.027	0.330	7.244	0.330	0.325	0.011	0.223	14.665	0.223	0.230
Reg4	0.005	0.210	13.265	0.210	0.219	0.004	0.216	13.719	0.216	0.219	0.003	0.327	7.240	0.327	0.324	0.008	0.223	14.571	0.223	0.231
Reg5	0.260	0.375	8.278	0.269	0.268	0.250	0.372	8.694	0.268	0.266	0.275	0.504	4.487	0.424	0.408	0.262	0.378	9.403	0.278	0.284
Reg6	0.262	0.377	8.276	0.270	0.268	0.251	0.373	8.679	0.268	0.267	0.302	0.524	4.286	0.433	0.416	0.266	0.380	9.416	0.279	0.284
MI1-Reg	0.012	0.209		0.209	0.221	0.002	0.213		0.213	0.221	0.013	0.329		0.329	0.338	0.011	0.221		0.221	0.232
MI2-Reg	0.095	0.246		0.228	0.280	0.088	0.247		0.230	0.280	0.103	0.371		0.360	0.408	0.096	0.254		0.239	0.292

Notes: See Table A1.

Table A14. Monte Carlo Results: Linear Specification, L = 3, $\rho = 0.5$, $\pi_{21} = 1$, $\rho(x_1,z) \neq 0$.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2 = 2$																				
No Missing	0.146	0.490	2.502	0.478	0.450	0.143	0.433	2.590	0.421	0.434	0.230	0.771	1.678	0.744	0.769	0.124	0.553	2.537	0.546	0.514
CC	0.143	0.550	2.147	0.540	0.559	0.188	0.625	2.354	0.616	0.581	0.241	0.768	1.493	0.731	0.776	0.148	0.453	2.431	0.432	0.470
DV	-0.872	0.916	11.879	0.251	0.215	-0.843	0.885	10.012	0.254	0.232	-0.778	0.872	6.033	0.327	0.301	-0.872	0.904	13.491	0.211	0.199
NN1	0.013	0.553	2.756	0.553	0.598	-0.021	0.521	2.918	0.521	0.554	-0.043	0.837	1.664	0.837	0.918	0.034	0.587	3.065	0.585	0.616
NN2	-0.419	0.531	6.460	0.319	0.332	-0.396	0.525	6.362	0.341	0.338	-0.273	0.651	2.873	0.591	0.599	-0.410	0.550	7.127	0.327	0.336
NN3	0.159	0.760	1.884	0.743	0.771	0.225	0.739	2.002	0.707	0.752	0.207	0.922	1.340	0.885	1.000	0.219	0.823	2.106	0.782	0.788
MI1-NN	-0.197	0.449		0.400	0.674	-0.200	0.450		0.402	0.617	-0.154	0.622		0.608	0.957	-0.183	0.460		0.414	0.700
MI2-NN	-0.065	0.468		0.461	0.803	-0.051	0.458		0.454	0.763	-0.047	0.597		0.597	1.108	-0.043	0.498		0.496	0.823
Reg1	0.064	0.616	3.335	0.615	0.593	0.054	0.569	3.596	0.568	0.555	0.060	0.796	2.369	0.791	0.795	0.074	0.510	3.861	0.506	0.535
Reg2	0.063	0.603	3.363	0.602	0.604	0.039	0.964	3.668	0.964	0.917	-0.003	0.718	2.416	0.715	0.714	0.079	0.509	3.818	0.504	0.539
Reg3	-0.423	0.528	9.021	0.294	0.310	-0.411	0.533	9.447	0.306	0.306	-0.433	0.659	5.997	0.453	0.432	-0.435	0.541	10.022	0.302	0.315
Reg4	-0.425	0.532	9.154	0.293	0.309	-0.400	0.528	8.933	0.315	0.314	-0.368	0.631	4.649	0.491	0.472	-0.439	0.545	9.945	0.303	0.315
Reg5	0.329	0.904	2.132	0.864	0.789	0.315	0.772	2.293	0.714	0.717	0.474	1.104	1.155	0.984	1.097	0.344	0.710	2.391	0.633	0.672
Reg6	0.330	0.873	2.131	0.827	0.809	0.327	0.742	2.332	0.677	0.677	0.365	1.036	1.414	0.968	1.087	0.348	0.720	2.415	0.638	0.675
MI1-Reg	-0.172	0.464		0.417	0.620	-0.161	0.518		0.474	0.760	-0.181	0.538		0.506	0.796	-0.172	0.426		0.380	0.580
MI2-Reg	-0.006	0.550		0.548	0.769	-0.013	0.534		0.532	0.841	0.018	0.586		0.585	1.050	0.000	0.460		0.460	0.686
B. $\tau^2 = 5$																				
No Missing	0.055	0.263	5.432	0.259	0.267	0.061	0.272	5.564	0.270	0.266	0.096	0.420	3.162	0.415	0.409	0.053	0.269	5.412	0.267	0.269
CC	0.053	0.329	4.526	0.327	0.309	0.100	0.314	4.785	0.310	0.299	0.126	0.523	2.608	0.517	0.507	0.073	0.282	5.028	0.278	0.293
DV	-0.817	0.853	15.241	0.223	0.188	-0.783	0.817	13.182	0.224	0.200	-0.745	0.819	7.580	0.296	0.265	-0.814	0.845	17.306	0.189	0.175
NN1	0.023	0.343	5.315	0.342	0.355	-0.003	0.356	5.732	0.356	0.355	0.051	0.622	2.670	0.618	0.620	0.028	0.353	5.910	0.352	0.372
NN2	-0.332	0.407	10.475	0.235	0.242	-0.306	0.397	10.359	0.249	0.247	-0.217	0.474	4.347	0.417	0.421	-0.341	0.422	11.578	0.242	0.247
NN3	0.217	0.480	3.780	0.420	0.445	0.215	0.471	3.818	0.410	0.428	0.226	0.726	2.195	0.679	0.725	0.225	0.538	4.185	0.476	0.456
MI1-NN	-0.151	0.312		0.272	0.454	-0.151	0.326		0.289	0.433	-0.088	0.487		0.481	0.654	-0.151	0.320		0.279	0.472
MI2-NN	-0.032	0.300		0.299	0.522	-0.020	0.314		0.313	0.504	0.034	0.498		0.497	0.766	-0.005	0.325		0.325	0.540
Reg1	0.028	0.312	7.068	0.311	0.319	0.020	0.308	7.522	0.308	0.315	0.031	0.486	4.229	0.486	0.495	0.029	0.312	7.963	0.312	0.331
Reg2	0.027	0.316	7.062	0.316	0.321	0.014	0.321	7.390	0.321	0.317	-0.008	0.553	4.151	0.553	0.530	0.026	0.313	7.920	0.312	0.332
Reg3	-0.335	0.407	14.922	0.213	0.226	-0.332	0.412	15.133	0.224	0.226	-0.367	0.505	9.033	0.319	0.312	-0.352	0.422	16.316	0.225	0.233
Reg4	-0.337	0.410	14.766	0.213	0.226	-0.323	0.404	14.421	0.229	0.230	-0.312	0.479	7.319	0.349	0.341	-0.355	0.425	16.203	0.226	0.233
Reg5	0.277	0.475	4.536	0.393	0.389	0.266	0.472	4.717	0.391	0.386	0.452	0.829	2.066	0.676	0.709	0.288	0.474	5.038	0.390	0.405
Reg6	0.276	0.482	4.520	0.397	0.391	0.271	0.472	4.739	0.389	0.384	0.324	0.691	2.515	0.614	0.628	0.288	0.482	5.030	0.394	0.407
MI1-Reg	-0.154	0.303		0.254	0.376	-0.152	0.307		0.260	0.368	-0.163	0.426		0.388	0.536	-0.160	0.311		0.261	0.385
MI2-Reg	-0.008	0.298		0.298	0.453	-0.005	0.301		0.300	0.446	0.032	0.454		0.454	0.691	-0.009	0.302		0.302	0.463
C. $\tau^2 = 10$																				
No Missing	0.022	0.182	10.333	0.181	0.185	0.035	0.189	10.540	0.189	0.185	0.042	0.271	5.645	0.271	0.274	0.025	0.186	10.332	0.186	0.187
CC	0.026	0.215	8.427	0.214	0.209	0.059	0.217	8.784	0.216	0.207	0.070	0.331	4.537	0.330	0.319	0.040	0.199	9.539	0.198	0.205
DV	-0.747	0.779	19.971	0.195	0.163	-0.707	0.739	17.675	0.195	0.172	-0.689	0.750	9.791	0.262	0.230	-0.747	0.776	22.523	0.167	0.153
NN1	0.029	0.240	9.872	0.238	0.245	0.003	0.243	10.430	0.242	0.246	0.098	0.523	4.604	0.512	0.439	0.027	0.248	10.940	0.246	0.257
NN2	-0.262	0.320	16.335	0.180	0.186	-0.245	0.310	16.018	0.193	0.191	-0.167	0.355	6.772	0.316	0.315	-0.276	0.334	17.983	0.189	0.192
NN3	0.226	0.384	6.737	0.295	0.303	0.221	0.374	6.962	0.286	0.297	0.242	0.564	3.606	0.497	0.485	0.234	0.401	7.544	0.317	0.314
MI1-NN	-0.117	0.230		0.198	0.343	-0.116	0.236		0.208	0.326	-0.031	0.382		0.381	0.487	-0.121	0.242		0.208	0.355
MI2-NN	0.000	0.218		0.218	0.403	0.002	0.223		0.223	0.390	0.063	0.388		0.381	0.553	0.006	0.233		0.233	0.414
Reg1	0.009	0.209	13.265	0.208	0.220	0.004	0.210	13.918	0.210	0.219	0.013	0.328	7.343	0.328	0.326	0.011	0.220	14.852	0.220	0.232
Reg2	0.006	0.211	13.185	0.210	0.221	0.003	0.216	13.844	0.216	0.220	-0.003	0.351	7.139	0.351	0.342	0.013	0.220	14.554	0.220	0.233
Reg3	-0.270	0.323	23.118	0.163	0.174	-0.275	0.327	23.529	0.172	0.174	-0.312	0.400	13.363	0.238	0.240	-0.288	0.338	25.463	0.176	0.181
Reg4	-0.273	0.325	23.027	0.163	0.173	-0.259	0.320	22.575	0.176	0.177	-0.256	0.371	11.282	0.259	0.257	-0.287	0.340	25.295	0.177	0.181
Reg5	0.261	0.373	8.359	0.269	0.268	0.253	0.372	8.646	0.269	0.268	0.407	0.635	3.727	0.487	0.469	0.262	0.376	9.518	0.277	0.283
Reg6	0.262	0.377	8.276	0.270	0.268	0.251	0.373	8.679	0.268	0.267	0.302	0.524	4.286	0.433	0.416	0.266	0.380	9.416	0.279	0.284
MI1-Reg	-0.133	0.226		0.181	0.277	-0.135	0.231		0.188	0.273	-0.136	0.320		0.283	0.367	-0.142	0.241		0.194	0.285
MI2-Reg	0.000	0.208		0.208	0.350	-0.008	0.212		0.212	0.346	0.035	0.336		0.335	0.495	-0.005	0.220		0.220	0.359

Notes: See Table A1.

Table A15. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.5, π_{21} = 0, ρ(x_1,z) ≠ 0.

Estimator	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. $\tau^2 = 2$																				
No Missing	0.080	0.306	4.381	0.301	0.307	0.385	0.892	1.126	0.812	0.910	0.475	1.079	0.780	0.957	1.224	0.076	0.340	4.338	0.338	0.323
CC	0.091	0.358	3.489	0.351	0.350	0.437	0.827	1.024	0.711	0.803	0.468	0.939	0.814	0.816	1.008	0.106	0.374	3.935	0.367	0.346
DV	-0.043	0.395	2.505	0.384	0.365	0.330	0.773	0.653	0.718	0.804	0.470	1.005	0.491	0.892	1.037	-0.039	0.407	2.779	0.393	0.361
NN1	0.019	0.419	4.436	0.419	0.412	0.061	0.791	1.302	0.790	0.905	-0.027	0.910	1.156	0.909	1.060	0.055	0.456	4.525	0.453	0.439
NN2	0.003	0.365	4.613	0.365	0.390	0.066	1.024	1.458	1.023	1.103	-0.080	0.952	1.149	0.947	1.144	0.015	0.425	4.694	0.425	0.417
NN3	0.221	0.538	2.820	0.478	0.514	0.243	1.137	0.988	1.099	1.429	0.145	1.122	0.786	1.115	1.404	0.225	0.606	3.286	0.558	0.546
MI1-NN	0.004	0.372		0.372	0.441	0.075	0.819		0.818	1.164	-0.024	0.790		0.787	1.294	0.037	0.421		0.420	0.468
MI2-NN	0.075	0.393		0.383	0.519	0.162	0.792		0.782	1.455	0.005	0.769		0.769	1.508	0.103	0.455		0.445	0.544
Reg1	0.017	0.365	5.504	0.365	0.369	0.206	0.880	1.572	0.856	0.865	0.165	0.947	1.292	0.932	1.084	0.029	0.411	6.204	0.410	0.394
Reg2	0.020	0.368	5.495	0.367	0.370	0.133	0.839	1.666	0.826	0.835	0.061	0.968	1.392	0.966	1.068	0.032	0.406	6.112	0.406	0.394
Reg3	0.022	0.350	5.498	0.350	0.360	0.219	0.886	1.529	0.857	0.875	0.158	0.979	1.202	0.966	1.126	0.029	0.406	6.133	0.406	0.390
Reg4	0.021	0.349	5.499	0.348	0.361	0.167	0.845	1.604	0.824	0.857	0.131	0.855	1.235	0.847	1.049	0.030	0.404	6.111	0.404	0.389
Reg5	0.276	0.532	3.492	0.455	0.454	0.513	1.184	1.026	1.073	1.164	0.473	1.238	0.817	1.150	1.446	0.271	0.583	3.938	0.518	0.488
Reg6	0.276	0.537	3.486	0.460	0.455	0.513	1.178	1.011	1.069	1.169	0.482	1.312	0.793	1.221	1.530	0.281	0.586	3.940	0.520	0.490
MI1-Reg	0.022	0.356		0.355	0.368	0.183	0.846		0.824	0.884	0.138	0.883		0.874	1.156	0.023	0.406		0.405	0.394
MI2-Reg	0.103	0.402		0.388	0.430	0.300	0.948		0.902	1.019	0.252	0.990		0.961	1.327	0.106	0.453		0.442	0.458
B. $\tau^2 = 5$																				
No Missing	0.035	0.199	9.309	0.198	0.197	0.205	0.528	1.780	0.483	0.534	0.475	0.979	0.782	0.850	1.045	0.025	0.185	9.570	0.185	0.195
CC	0.031	0.216	7.562	0.215	0.220	0.310	0.674	1.291	0.610	0.663	0.471	0.865	0.755	0.735	1.023	0.026	0.205	8.551	0.205	0.217
DV	-0.114	0.289	5.510	0.252	0.240	0.171	0.709	0.860	0.698	0.683	0.443	0.927	0.459	0.788	0.964	-0.124	0.281	6.238	0.233	0.232
NN1	0.024	0.262	9.016	0.260	0.258	0.042	0.659	1.874	0.654	0.705	-0.040	0.933	1.153	0.930	1.038	0.024	0.249	10.289	0.248	0.268
NN2	0.019	0.256	9.408	0.255	0.252	0.056	0.714	1.889	0.711	0.759	-0.060	0.977	1.176	0.972	1.105	0.021	0.246	10.411	0.246	0.261
NN3	0.227	0.396	6.215	0.321	0.312	0.298	1.053	1.257	0.998	1.117	0.163	1.203	0.749	1.190	1.427	0.219	0.423	6.705	0.342	0.336
MI1-NN	0.026	0.250		0.249	0.277	0.049	0.613		0.608	0.834	-0.052	0.818		0.813	1.238	0.020	0.240		0.240	0.283
MI2-NN	0.091	0.279		0.263	0.333	0.173	0.664		0.643	1.056	0.035	0.788		0.788	1.472	0.076	0.278		0.262	0.354
Reg1	0.015	0.232	11.832	0.232	0.232	0.153	0.702	2.050	0.677	0.705	0.177	1.032	1.188	1.026	1.221	0.004	0.228	13.292	0.228	0.247
Reg2	0.014	0.233	11.610	0.233	0.232	0.068	0.587	2.459	0.579	0.623	0.092	0.958	1.295	0.956	1.039	0.013	0.225	13.288	0.225	0.245
Reg3	0.015	0.234	11.860	0.234	0.230	0.160	0.727	2.026	0.702	0.703	0.206	0.977	1.103	0.961	1.102	0.006	0.228	13.298	0.228	0.245
Reg4	0.012	0.234	11.836	0.234	0.230	0.100	0.605	2.340	0.593	0.634	0.135	1.042	1.163	1.031	1.124	0.007	0.225	13.195	0.225	0.244
Reg5	0.269	0.392	7.491	0.294	0.281	0.438	0.961	1.294	0.833	0.920	0.465	1.395	0.756	1.339	1.860	0.255	0.384	8.466	0.292	0.303
Reg6	0.275	0.394	7.449	0.295	0.282	0.439	0.960	1.296	0.831	0.929	0.472	1.361	0.746	1.297	1.949	0.259	0.385	8.452	0.292	0.303
MI1-Reg	0.016	0.232		0.232	0.232	0.123	0.633		0.616	0.694	0.176	0.909		0.900	1.261	0.003	0.226		0.226	0.246
MI2-Reg	0.096	0.268		0.252	0.291	0.239	0.727		0.680	0.819	0.276	1.032		1.009	1.595	0.086	0.260		0.247	0.305
C. $\tau^2 = 10$																				
No Missing	0.019	0.140	18.032	0.139	0.139	0.152	0.485	2.668	0.469	0.440	0.459	0.781	0.874	0.638	0.821	0.034	0.142	17.913	0.140	0.136
CC	0.028	0.156	14.484	0.155	0.155	0.191	0.575	1.921	0.541	0.545	0.454	0.880	0.846	0.753	0.936	0.048	0.153	16.393	0.150	0.150
DV	-0.132	0.232	10.779	0.177	0.171	0.012	0.622	1.292	0.618	0.564	0.462	1.018	0.529	0.912	0.957	-0.122	0.221	12.149	0.175	0.163
NN1	0.025	0.179	17.509	0.178	0.181	0.051	0.552	2.712	0.548	0.574	-0.011	1.001	1.238	1.001	1.152	0.050	0.195	19.494	0.189	0.188
NN2	0.008	0.173	17.775	0.173	0.176	0.065	0.605	2.775	0.601	0.586	-0.035	0.898	1.193	0.898	1.092	0.024	0.184	20.261	0.183	0.184
NN3	0.245	0.335	11.294	0.229	0.228	0.299	0.810	1.779	0.764	0.776	0.140	1.060	0.808	1.050	1.317	0.261	0.362	12.630	0.245	0.237
MI1-NN	0.010	0.170		0.169	0.193	0.071	0.531		0.527	0.658	-0.018	0.828		0.827	1.320	0.042	0.185		0.181	0.199
MI2-NN	0.089	0.201		0.180	0.265	0.169	0.547		0.530	0.789	0.061	0.781		0.780	1.491	0.114	0.226		0.196	0.269
Reg1	0.005	0.159	22.684	0.159	0.165	0.166	0.567	3.032	0.542	0.554	0.156	0.795	1.313	0.778	0.960	0.017	0.178	25.891	0.177	0.174
Reg2	0.005	0.160	22.337	0.160	0.165	0.040	0.502	3.499	0.500	0.492	0.073	0.922	1.443	0.914	1.038	0.018	0.173	25.344	0.172	0.172
Reg3	0.010	0.162	22.904	0.162	0.165	0.148	0.518	3.041	0.495	0.523	0.158	0.813	1.289	0.794	0.967	0.021	0.179	25.836	0.178	0.174
Reg4	0.008	0.163	22.495	0.163	0.165	0.053	0.514	3.512	0.509	0.501	0.149	0.873	1.358	0.853	1.001	0.022	0.176	25.625	0.174	0.171
Reg5	0.253	0.327	14.172	0.208	0.203	0.450	0.831	1.927	0.691	0.726	0.466	1.066	0.852	0.962	1.267	0.279	0.364	16.040	0.231	0.217
Reg6	0.253	0.329	14.149	0.209	0.203	0.449	0.843	1.908	0.702	0.736	0.449	1.144	0.845	1.043	1.367	0.277	0.365	16.039	0.231	0.217
MI1-Reg	0.007	0.160		0.160	0.166	0.106	0.495		0.483	0.544	0.153	0.768		0.751	1.097	0.020	0.176		0.175	0.174
MI2-Reg	0.087	0.194		0.175	0.230	0.230	0.585		0.538	0.663	0.259	0.847		0.805	1.238	0.105	0.221		0.192	0.241

Notes: See Table A1.

Table A16. Monte Carlo Results: Nonlinear Specification, L = 3, ρ = 0.5, π_{21} = 1, ρ(x_1,z) ≠ 0.

	MCAR					MAR (x_1 only)					MAR (x_1, z)					NMAR				
Estimator	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE	Median Bias	RMSE	Median F	SD	Mean SE
A. τ^2 = 2																				
No Missing	0.068	0.372	3.046	0.368	0.355	0.270	0.696	1.026	0.630	0.755	0.341	0.715	0.812	0.641	0.818	0.082	0.337	3.087	0.333	0.330
CC	0.100	0.386	2.657	0.383	0.399	0.349	0.781	0.964	0.701	0.842	0.349	0.786	0.944	0.699	0.819	0.107	0.379	2.737	0.375	0.374
DV	-0.939	0.952	23.694	0.153	0.143	0.265	0.463	3.145	0.369	0.375	-1.736	1.792	5.328	0.373	0.343	-0.935	0.952	25.759	0.148	0.136
NN1	-0.006	0.455	3.184	0.455	0.449	0.058	0.654	1.206	0.654	0.804	-0.221	1.211	1.306	1.173	1.220	0.031	0.443	3.111	0.441	0.471
NN2	-0.536	0.594	9.109	0.229	0.221	0.055	1.012	1.029	1.011	1.111	-0.578	1.044	1.748	0.855	0.934	-0.548	0.592	9.363	0.232	0.223
NN3	0.202	0.588	2.114	0.554	0.560	0.223	0.903	0.858	0.870	1.165	0.162	1.073	0.802	1.062	1.309	0.234	0.633	2.281	0.583	0.580
MI1-NN	-0.270	0.416		0.313	0.628	0.050	0.705		0.704	1.175	-0.404	0.903		0.783	1.488	-0.248	0.398		0.307	0.650
MI2-NN	-0.103	0.372		0.353	0.693	0.111	0.673		0.665	1.311	-0.194	0.774		0.732	1.685	-0.066	0.379		0.368	0.707
Reg1	0.003	0.504	4.155	0.504	0.454	0.155	0.744	1.490	0.736	0.855	-0.006	0.843	1.467	0.843	0.946	0.032	0.461	4.337	0.461	0.435
Reg2	0.024	0.366	3.687	0.366	0.383	0.110	0.695	1.327	0.685	0.795	0.065	0.839	1.053	0.835	0.889	0.055	0.397	3.808	0.396	0.407
Reg3	-0.548	0.600	14.143	0.211	0.207	-0.011	1.053	1.139	1.053	1.065	-0.690	1.005	3.642	0.661	0.607	-0.546	0.596	14.787	0.220	0.209
Reg4	-0.556	0.604	13.902	0.212	0.206	0.080	0.845	1.156	0.842	0.950	-0.499	1.246	2.200	1.123	0.988	-0.543	0.599	14.712	0.218	0.206
Reg5	0.239	0.673	2.632	0.633	0.558	0.431	1.013	0.962	0.930	1.122	0.005	1.113	1.091	1.113	1.209	0.284	0.649	2.760	0.599	0.569
Reg6	0.252	0.516	2.399	0.457	0.496	0.326	0.925	0.926	0.856	1.094	0.316	0.977	0.790	0.909	1.106	0.276	0.589	2.559	0.520	0.524
MI1-Reg	-0.270	0.396		0.279	0.539	0.102	0.655		0.650	1.160	-0.283	0.648		0.569	1.278	-0.232	0.394		0.288	0.530
MI2-Reg	-0.086	0.363		0.346	0.628	0.188	0.685		0.661	1.250	-0.134	0.595		0.576	1.430	-0.065	0.369		0.358	0.625
B. τ^2 = 5																				
No Missing	0.015	0.213	6.277	0.213	0.204	0.189	0.647	1.381	0.613	0.735	0.335	0.869	0.750	0.811	0.990	0.048	0.189	6.196	0.187	0.200
CC	0.019	0.244	5.148	0.244	0.229	0.236	0.623	1.179	0.593	0.675	0.327	0.820	0.868	0.768	0.933	0.053	0.211	5.495	0.209	0.227
DV	-0.904	0.912	27.531	0.136	0.132	0.369	0.554	2.944	0.400	0.396	-1.753	1.793	5.416	0.342	0.345	-0.896	0.918	29.648	0.144	0.125
NN1	0.008	0.277	6.151	0.277	0.276	0.032	0.660	1.556	0.658	0.773	-0.296	1.028	1.325	0.967	1.050	0.023	0.280	6.373	0.278	0.296
NN2	-0.453	0.483	13.772	0.174	0.173	0.067	1.082	1.045	1.080	1.163	-0.596	0.941	1.640	0.724	0.810	-0.447	0.476	13.864	0.172	0.177
NN3	0.220	0.473	4.205	0.413	0.366	0.194	0.796	1.083	0.776	0.918	0.113	1.595	0.791	1.579	1.916	0.250	0.448	4.477	0.361	0.369
MI1-NN	-0.219	0.307		0.209	0.464	0.080	0.731		0.728	1.217	-0.468	0.819		0.667	1.280	-0.196	0.289		0.205	0.489
MI2-NN	-0.065	0.264		0.254	0.518	0.121	0.661		0.653	1.222	-0.250	0.805		0.767	1.856	-0.038	0.239		0.234	0.532
Reg1	-0.010	0.247	8.072	0.246	0.247	0.086	0.666	1.861	0.661	0.723	-0.070	0.931	1.362	0.931	1.043	0.004	0.251	8.654	0.251	0.267
Reg2	-0.005	0.243	7.355	0.243	0.241	0.069	0.574	1.691	0.570	0.632	0.033	0.931	1.067	0.930	1.102	0.024	0.229	7.864	0.228	0.254
Reg3	-0.447	0.490	20.681	0.161	0.160	0.141	1.124	1.169	1.114	1.169	-0.726	0.926	3.579	0.512	0.560	-0.455	0.483	21.708	0.157	0.165
Reg4	-0.453	0.493	20.603	0.161	0.159	0.207	1.149	1.116	1.130	1.138	-0.568	0.944	2.244	0.740	0.775	-0.460	0.486	21.014	0.157	0.162
Reg5	0.237	0.385	5.093	0.311	0.304	0.370	0.907	1.169	0.820	0.958	-0.024	1.299	1.014	1.299	1.530	0.268	0.413	5.383	0.315	0.335
Reg6	0.240	0.389	4.773	0.306	0.300	0.327	0.823	1.119	0.751	0.903	0.227	0.952	0.764	0.922	1.137	0.279	0.414	5.016	0.309	0.326
MI1-Reg	-0.214	0.304		0.191	0.368	0.155	0.709		0.698	1.175	-0.328	0.631		0.533	1.313	-0.219	0.287		0.184	0.385
MI2-Reg	-0.067	0.238		0.224	0.431	0.234	0.691		0.660	1.168	-0.170	0.612		0.584	1.538	-0.061	0.225		0.218	0.454
C. τ^2 = 10																				
No Missing	0.006	0.134	11.595	0.134	0.138	0.149	0.455	2.008	0.436	0.419	0.297	0.742	0.723	0.678	0.926	0.013	0.141	11.543	0.141	0.140
CC	0.005	0.148	9.587	0.148	0.154	0.207	0.611	1.442	0.574	0.596	0.353	0.775	0.848	0.688	0.924	0.014	0.152	9.831	0.152	0.155
DV	-0.847	0.856	33.133	0.133	0.121	0.509	0.687	2.747	0.436	0.407	-1.772	1.837	5.153	0.361	0.353	-0.858	0.866	34.696	0.130	0.117
NN1	0.016	0.198	11.212	0.197	0.188	0.061	0.526	2.070	0.524	0.541	-0.279	0.913	1.285	0.863	0.953	0.028	0.198	11.562	0.196	0.203
NN2	-0.367	0.388	19.804	0.138	0.138	0.191	0.812	1.370	0.790	0.838	-0.576	0.934	1.594	0.758	0.832	-0.357	0.391	20.012	0.151	0.142
NN3	0.220	0.336	7.625	0.239	0.230	0.344	0.822	1.258	0.751	0.821	0.160	1.274	0.687	1.271	1.662	0.223	0.329	8.042	0.231	0.243
MI1-NN	-0.168	0.230		0.154	0.376	0.135	0.572		0.561	0.858	-0.428	0.748		0.618	1.221	-0.161	0.233		0.162	0.381
MI2-NN	-0.033	0.174		0.171	0.408	0.206	0.591		0.561	0.927	-0.243	0.718		0.672	1.654	-0.031	0.176		0.172	0.412
Reg1	-0.011	0.168	14.823	0.168	0.170	0.135	0.671	2.287	0.651	0.629	-0.026	0.888	1.323	0.887	1.044	0.000	0.179	15.264	0.179	0.183
Reg2	-0.003	0.160	13.422	0.160	0.164	0.066	0.520	2.400	0.516	0.535	0.073	0.814	1.012	0.811	0.987	-0.007	0.172	13.909	0.172	0.175
Reg3	-0.374	0.394	29.503	0.119	0.128	0.341	0.935	1.370	0.874	0.864	-0.709	0.973	4.009	0.593	0.590	-0.367	0.398	30.684	0.135	0.133
Reg4	-0.380	0.398	28.711	0.121	0.127	0.322	0.876	1.419	0.807	0.816	-0.541	0.967	2.390	0.784	0.758	-0.370	0.401	30.163	0.134	0.130
Reg5	0.228	0.323	9.429	0.218	0.212	0.444	0.888	1.455	0.756	0.826	0.012	1.117	1.016	1.115	1.327	0.245	0.327	9.715	0.224	0.228
Reg6	0.242	0.323	8.574	0.213	0.205	0.396	0.798	1.390	0.699	0.750	0.301	1.089	0.689	1.055	1.309	0.234	0.329	9.087	0.224	0.222
MI1-Reg	-0.187	0.233		0.134	0.287	0.247	0.616		0.575	0.896	-0.309	0.600		0.504	1.271	-0.187	0.240		0.148	0.292
MI2-Reg	-0.045	0.163		0.156	0.350	0.317	0.644		0.575	0.908	-0.163	0.589		0.560	1.483	-0.040	0.174		0.168	0.356

Notes: See Table A1.