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Abstract

This paper experimentally studies an essential institutional feature of matching markets: Randomization of allocation priorities. I compare single and multiple randomization in the student assignment problem with ties. The Gale-Shapley deferred acceptance algorithm is employed after indifferences in school priorities are resolved by either random procedure. The main result is that a significant fraction of individuals prefers multiple to single randomization, although both are equivalent in expectation. Multiple randomization is perceived to be fairer. One theoretical explanation is the failure to disregard compound lotteries. These results show that random procedures are not inherently neutral with respect to preferences and fairness perceptions.

JEL classification: C78, C91, D78, D81

Keywords: market design, school choice, mechanism design, experiment, deferred acceptance algorithm, randomization, tie-breaking.

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1 Introduction

Public resources can be allocated by mechanisms involving lotteries. This approach rests on the egalitarian assumption that a randomization is fair since everyone has an equal chance to obtain the public resource (Elster, 1991). There is a policy debate on such randomizations in the design of school choice mechanisms. Abdulkadiroğlu *et al.* (2005, 2009) redesigned the New York City (NYC) high school match based on a mechanism involving randomization. Here, randomization is applied if schools are indifferent between students. That is, schools do not admit students based on grades or other criteria, but use lotteries to determine priorities. Two types of randomization are discussed. Indifferences can either be resolved by a *single* lottery yielding one identical rank order of all students at all schools or by *multiple* lotteries resulting in a rank order at each school.

The policy debate reveals a tension between social welfare and social acceptability of both types of randomization. Abdulkadiroğlu *et al.* (2009) find that single randomization is favorable in terms of welfare based on simulations with NYC field data. More students receive their top choice under single randomization. Experimental results from Featherstone and Niederle (2014) also suggest that matching outcomes under single randomization are more efficient. In contrast, NYC policymakers argued that the "equitable approach" would be to employ multiple randomization (Pathak, 2011, p. 523). Single randomization was conjectured to be less socially acceptable. In addition, differences in truthful preference revelation between single and multiple randomization could offset the welfare advantages of single randomization.

Yet, there are no field data on this problem because only one procedure can be employed at a time. Up to now, we do not know whether and why individuals systematically prefer one type of randomization and whether this has an impact on their truth-telling behavior. I investigate the following research question: Do individuals prefer one random procedure to the other and does the procedure affect their behavior under the mechanism?

This paper examines behavior under the Gale and Shapley (1962) *deferred acceptance (DA) algorithm* with single and multiple randomization. I employ a controlled laboratory experiment: While the random procedure is varied, everything else is held constant. I investigate (i) whether individuals prefer either mechanism as well as their driving factors, and whether there is a causal impact of changing the randomization on (ii) truthful preference revelation and (iii) welfare.

I find that a significant fraction of participants prefers the multiple to the single randomization mechanism while the majority of participants is indifferent. The multiple randomization is perceived to be fairer, based on questionnaire data. Moreover, I find that differences in preferences about random procedures do not translate into differences in behavior under the two mechanisms. Consistent with the theory, truthful preference revelation and student welfare are not affected by the random procedure to resolve indifferences.

The main contribution of this paper is twofold. First, it tests the causal impact of changing the institution of randomization on truthful preference revelation and welfare. Second, it provides a behavioral theory and evidence consistent with preferences for multiple randomization. The paper shows that random procedures are not fair per se.

This paper relates to two lines of literature: decision-making involving random procedures and the design of school choice mechanisms. The first part of the research question addresses preferences for random procedures and their driving factors. Keren and Teigen (2010) and Eliaz and Rubinstein (2014) suggest that fairness judgements differ between distinct types of randomization. Bolton *et al.* (2005) find evidence that the acceptability of unequal outcomes is higher if they are generated by a random procedure. Results from Chlaß *et al.* (2016) indicate that individuals have procedural preferences, independent of the outcomes. Rubinstein (2002) finds evidence of false diversification in multiple lottery-decision problems. Dwenger *et al.* (2014) and Agranov and Ortoleva (2015) find evidence consistent with a preference for randomization. This paper contributes to this literature by providing evidence consistent with a preference for multiple randomization. Moreover, it relates these preferences to fairness judgements.

The second part of the research question relates to the experimental literature on school choice by addressing truthful preference revelation and the resulting welfare effects. School choice experiments employing the DA algorithm were initiated by Chen and Sönmez (2006), following the theoretical work of Abdulkadiroğlu and Sönmez (2003). A number of experimental papers investigate assignment mechanisms and their sensitivity to specific market design features. Pais and Pintér

(2008) provide evidence that less preference information provided is associated with more truth-telling behavior. Chen *et al.* (2015) find that truthful preference revelation also increases if the market size is enlarged. Concerning welfare results from the DA algorithm, Calsamiglia *et al.* (2010) find that constraining the number of schools in preference rankings can have a negative impact on efficiency. Departing from previous work, this paper is the first to focus on the random procedure resolving indifferences in allocation priorities prior to applying the DA algorithm. The novelty of the experimental design is that preference rankings under both random procedures are obtained and that subjects are asked to state which procedure they prefer to participate in. This paper contributes to the school choice literature by investigating the impact of the randomization design on truthful preference revelation and welfare.

The remainder of the paper is organised as follows. Section 2 introduces the DA algorithm and its properties under randomization, a behavioral theory consistent with a preference for multiple randomization, and the resulting predictions for the experiment. The experimental design and procedure is described in section 3. Section 4 provides the main results that a significant fraction of individuals prefer multiple to single randomization and that the amount of truth-telling does not differ between the two mechanisms. Section 5 concludes.

2 Theoretical framework

2.1 Deferred acceptance algorithm with randomization

Consider the problem of assigning indivisible goods to individuals without monetary transfers. I focus on a stylized school assignment problem following Abdulkadiroğlu *et al.* (2009).[1] A set of students is assigned to a set of schools according to student preferences and strict school priorities or indifferences. School priorities are exogenous. Each school has the capacity of one seat.

The student assignment variant of the Gale and Shapley (1962) student-proposing DA algorithm with prior randomization works as follows (adapted from Abdulkadiroğlu and Sönmez, 2003). At first, schools may have strict priorities or are indifferent between students. Students submit strict preference rankings over schools.

- If the school is indifferent between students, its priorities are determined randomly.
- Step 1: Students apply to the school at the top of their submitted ranking. One student is tentatively accepted at each school according to her priority. All others are rejected.
- Step k, $k \geq 2$: Rejected students apply to their next preferred school. Tentatively accepted and new applicants are jointly considered. The seat is tentatively assigned based on the priority and all unassigned students are rejected.
- Stop the algorithm if no student is rejected anymore.

When the algorithm terminates, the tentative matching turns into a final assignment. Every student is matched to her assigned school.

Strict student preferences and strict school priorities are a necessary condition of the DA algorithm (Gale and Shapley, 1962). In case of schools being indifferent between students, a strict priority ordering is obtained by randomization. I consider two random procedures to resolve indifferences at schools: Single and

[1] Allowing for schools with strict priorities and schools with indifferences at the same time constitutes a generalization of the canonical school assignment model in Abdulkadiroğlu and Sönmez (2003). If all schools are indifferent, this reduces to a version of the random priority mechanism (Pathak and Sethuraman, 2011).

multiple randomization. In the single randomization mechanism, one single lottery is employed to obtain one rank order of all students. This ranking is used to break ties at schools. Thereafter, the DA algorithm is applied. In the multiple randomization mechanism, school-specific lotteries are employed. Each school applies an independent lottery and thereby obtains its own priority order of students before running the DA algorithm.

2.2 Properties of the deferred acceptance algorithm with randomization

This section summarizes the theoretical properties of the student-proposing DA with random tie-breaking of priorities. In line with the literature, I consider the desirable properties stability, strategy-proofness, and Pareto efficiency for students.

Stability. The DA algorithm is stable and student-optimal under *strict* student preferences and school priorities (Gale and Shapley, 1962). That is, there is no unmatched blocking pair of a school and a student, such that the school prefers the student to an assigned student and the student prefers the school to her actually assigned school. The stability property holds independent of the random procedure to break ties.

Strategy-proofness. The student-proposing DA algorithm is strategy-proof for students (Dubins and Freedman, 1981; Roth, 1982). That is, truthful preference revelation is a dominant strategy for every student, independent of the preferences of the other students. Abdulkadiroğlu *et al.* (2009) show that strategy-proofness holds independent of the random procedure. They show that if the dominant strategy incentive compatibility holds for every student and for every priority and preference profile, then the DA is strategy proof for any arbitrary tie-breaking rule yielding a priority profile. The dominant strategy equilibrium of the preference revelation game is not changed by the tie-breaking.

Efficiency. The matching of the DA algorithm is efficient if there is no other matching outcome dominating it. In general, the DA algorithm is Pareto efficient when school and student welfare is considered. However, it is not Pareto efficient considering student welfare only (Roth, 1982). However, the DA algorithm under *strict* priority and preference profiles yields the *student-optimal stable* matching.

Every student prefers the DA matching outcome to any other stable outcome (Gale and Shapley, 1962).

Random tie-breaking can be shown to affect *ex post* efficiency negatively (Erdil and Ergin, 2008; Abdulkadiroğlu *et al.*, 2009; Kesten, 2010). The resulting matching outcome of the DA is not necessarily student-optimal anymore (Erdil and Ergin, 2008). Abdulkadiroğlu and Sönmez (2003) first discussed the additional efficiency loss due to exogenous randomization of originally indifferent priorities.

Efficiency can differ between tie-breaking rules. Single randomization can be shown to have at least equal welfare properties compared to multiple randomization. Abdulkadiroğlu *et al.* (2009) show that the set of stable matchings from single randomization is a subset of stable matchings from multiple randomization. The probability of obtaining a student-optimal matching outcome is smaller under multiple compared to single randomization. However, they admit that this property has no immediate effect on the outcome distribution from an *ex ante* perspective. Efficiency differences between matchings under single and multiple randomization depend on the setting.

2.3 Efficiency in the experiment

In the experimental environment, efficiency does not differ between matching distributions produced under single and multiple randomization. To see this, consider the assignment problem in the experiment with four students i_1, i_2, i_3, i_4, and four schools s_1, s_2, s_3, s_4, where each school is providing one seat. Assume truthful preference revelation. I construct the following student preference profiles over schools $P_i = \{P_{i_1}, ..., P_{i_4}\}$ such that the matching distributions do not depend on the randomization procedure:

$$\begin{array}{lllllll} s_1 & P_{i_1} & s_2 & P_{i_1} & s_3 & P_{i_1} & s_4 \\ s_1 & P_{i_2} & s_2 & P_{i_2} & s_3 & P_{i_2} & s_4 \\ s_1 & P_{i_3} & s_4 & P_{i_3} & s_3 & P_{i_3} & s_2 \\ s_1 & P_{i_4} & s_4 & P_{i_4} & s_3 & P_{i_4} & s_2 \end{array}.$$

The students i_1 and i_2, and the students i_3 and i_4 form pairs of preferences. Each pair has identical preferences.

Motivating example. The *original* school priorities over students $R_s = \{R_{s_1}, ..., R_{s_4}\}$ are strict $\succ_s$ for school 4 only. Schools s_1, s_2, and s_3 are originally indifferent $\sim_s$. Randomization induces strict priorities at the first three schools.

Assume the following strict priorities from **single randomization**:

$$\begin{array}{c} i_1 \succ_{s_1} i_2 \succ_{s_1} i_3 \succ_{s_1} i_4 \\ i_1 \succ_{s_2} i_2 \succ_{s_2} i_3 \succ_{s_2} i_4 \\ i_1 \succ_{s_3} i_2 \succ_{s_3} i_3 \succ_{s_3} i_4 \\ i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array}.$$

The resulting student-optimal stable matching under single randomization is indicated by the boxes:

$$\begin{array}{c} \boxed{s_1}\ P_{i_1}\ s_2\ P_{i_1}\ s_3\ P_{i_1}\ s_4 \\ s_1\ P_{i_2}\ \boxed{s_2}\ P_{i_2}\ s_3\ P_{i_2}\ s_4 \\ s_1\ P_{i_3}\ \boxed{s_4}\ P_{i_3}\ s_3\ P_{i_3}\ s_2 \\ s_1\ P_{i_4}\ s_4\ P_{i_4}\ \boxed{s_3}\ P_{i_4}\ s_2 \end{array}.$$

Assume three examples of **multiple randomizations**:

$$\begin{array}{c} i_1 \succ_{s_1} i_2 \succ_{s_1} i_3 \succ_{s_1} i_4 \\ i_4 \succ_{s_2} i_3 \succ_{s_2} i_1 \succ_{s_2} i_2 \\ i_1 \succ_{s_3} i_2 \succ_{s_3} i_4 \succ_{s_3} i_3 \\ i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array}, \quad \begin{array}{c} i_3 \succ_{s_1} i_4 \succ_{s_1} i_2 \succ_{s_1} i_1 \\ i_2 \succ_{s_2} i_1 \succ_{s_2} i_3 \succ_{s_2} i_4 \\ i_4 \succ_{s_3} i_3 \succ_{s_3} i_2 \succ_{s_3} i_1 \\ i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array}, \quad \begin{array}{c} i_2 \succ_{s_1} i_3 \succ_{s_1} i_1 \succ_{s_1} i_4 \\ i_4 \succ_{s_2} i_3 \succ_{s_2} i_1 \succ_{s_2} i_2 \\ i_1 \succ_{s_3} i_2 \succ_{s_3} i_4 \succ_{s_3} i_3 \\ i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array}.$$

The resulting matchings (box) from the DA under each multiple randomization are

$$\begin{array}{c} \boxed{s_1}\ P_{i_1}\ s_2\ P_{i_1}\ s_3\ P_{i_1}\ s_4 \\ s_1\ P_{i_2}\ \boxed{s_2}\ P_{i_2}\ s_3\ P_{i_2}\ s_4 \\ s_1\ P_{i_3}\ \boxed{s_4}\ P_{i_3}\ s_3\ P_{i_3}\ s_2 \\ s_1\ P_{i_4}\ s_4\ P_{i_4}\ \boxed{s_3}\ P_{i_4}\ s_2 \end{array}, \quad \begin{array}{c} s_1\ P_{i_1}\ s_2\ P_{i_1}\ \boxed{s_3}\ P_{i_1}\ s_4 \\ s_1\ P_{i_2}\ \boxed{s_2}\ P_{i_2}\ s_3\ P_{i_2}\ s_4 \\ \boxed{s_1}\ P_{i_3}\ s_4\ P_{i_3}\ s_3\ P_{i_3}\ s_2 \\ s_1\ P_{i_4}\ \boxed{s_4}\ P_{i_4}\ s_3\ P_{i_4}\ s_2 \end{array}, \quad \begin{array}{c} s_1\ P_{i_1}\ \boxed{s_2}\ P_{i_1}\ s_3\ P_{i_1}\ s_4 \\ \boxed{s_1}\ P_{i_2}\ s_2\ P_{i_2}\ s_3\ P_{i_2}\ s_4 \\ s_1\ P_{i_3}\ \boxed{s_4}\ P_{i_3}\ s_3\ P_{i_3}\ s_2 \\ s_1\ P_{i_4}\ s_4\ P_{i_4}\ \boxed{s_3}\ P_{i_4}\ s_2 \end{array}.$$

The matching distributions from the DA do not differ between single and multiple randomization. This result is generalized for the given P_i in Proposition 1.

Proposition 1. *Given the preference relations P_i in the experiment, one student gets into her top choice, two students get into their second preference, and one student gets into her third preference. This holds irrespective of the priority relations R_{s_1}, R_{s_2}, and R_{s_3}. The distribution of school seats does not depend on the randomization procedure.*

Sketch of proof. To see that Proposition 1 holds, assume truthful preference revelation of P_i, fix the priority relation R_{s_4}, and consider the priority relations R_{s_1}, R_{s_2}, and R_{s_3} produced by single and multiple randomization. Note that each pair of students has identical preference profiles. In this setting, the student-proposing DA produces the final matching in three steps.

Single randomization. The priority relations are identical: $R_{s_1} = R_{s_2} = R_{s_3}$. In step 1 of the DA, all students apply to their top choice s_1 and one student is accepted. In step 2, the three rejected students apply to their second preference, which is either s_2 or s_4. The student from the pair where the second student was accepted at s_1 in step 1 is accepted at her second preference (s_2 or s_4). I call the pair that is jointly applying to their second preference the *remaining pair*. For the remaining pair, the priority relation of either R_{s_2} or R_{s_4} determines who is accepted. The accepted student gets into her second preference and the rejected student gets into her third preference s_3 (after applying in step 3). Every student gets at least into her third preference. Importantly, it follows from the above argument that the fourth preference of each student and R_{s_3} are irrelevant for determining the final matching of the DA under truth-telling.

Multiple randomization. The priority relations R_{s_1}, R_{s_2}, and R_{s_3} may be identical, if the three lotteries yield identical outcomes (as in the single randomization). Consider the subset of priorities where $R_{s_1} \neq R_{s_2} \neq R_{s_3}$. Since R_{s_3} is irrelevant for determining the final matching, the relevant subset of priorities reduces to $R_{s_1} \neq R_{s_2}$. R_{s_2} determines who is accepted at her second preference. This holds for both $R_{s_1} \neq R_{s_2}$ and $R_{s_1} = R_{s_2}$. □

2.4 Preference for multiple randomization

A policymaker argued during the discussion about the NYC high school match:

> "*I believe that the equitable approach is for a child to have a new chance with each program. [...] If we use only one random number, and I had the bad luck to be the last student in the line this would be repeated 12 times and I would never get a chance*" (Pathak, 2011, p. 523).

The behavioral theory in this section accommodates choices consistent with this preference for multiple randomization.

Definitions. Let the set of students be $I = \{1, ..., N\}$ and the set of schools be $S = \{s_1, ..., s_M\}$, where each school has one seat capacity. Students have strict preferences $P_i = \{P_{i_1}, ..., P_{i_N}\}$. Schools have priorities $R_s = \{R_{s_1}, ..., R_{s_M}\}$ which are strict $\succ_s$ or indifference $\sim_s$. Let the single randomization be a simple lottery tie-breaking rule and multiple randomization be a compound lottery. In general, let $L = (p_1, ..., p_M)$ be the *simple lottery* over the possible material payoffs π_{i,s_m} from the DA algorithm. The payoff $\pi_{i,s_m} : S \to \mathbb{R}$ of student i is obtained with probability $p_m \in [0, 1]$, with $\sum_m p_m = 1$. Let $\mathcal{L} = \{L_{s_1}, ..., L_{s_M}; \alpha_1, ..., \alpha_M\}$ be the *compound lottery* yielding the simple lottery L_{s_m} with probability $\alpha_m \geq 0$ for $m = 1, ..., M$.

Figure 1 illustrates the compound lottery structure implied by the DA algorithm: The second lottery is only relevant if the first lottery is lost in expectation.

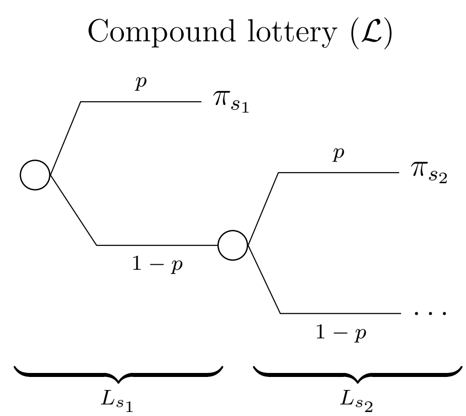

Figure 1: Tree representation of a two-stage compound lottery.

Expected utility theory. The reduction of the compound lottery under expected utility theory (EU) is considered as the benchmark. The following example illustrates the choice problem in the experiment mapping its environment of four students and four schools. Let $L = (q_1, ..., q_4)$ be the simple lottery over the payoffs π_{i,s_m} from the DA. Let $\mathcal{L} = \{L_{s_1}, ..., L_{s_4}; \alpha_1, ..., \alpha_4\}$ be the four-stage compound lottery of the four simple lotteries $L_{s_m} = (q_1^{s_m}, ..., q_4^{s_m})$, $m = 1, ..., 4$, over payoffs at the schools s_1, s_2, s_3, and s_4. The degenerate lottery L_{s_4} yields the payoff with certainty. Let the preference relation satisfy the standard axioms of EU. Assume truthful preference revelation. The consequentialist assumption states that individuals only care about the reduced lottery. Proposition 2 implies that the individuals are indifferent between the simple and the compound lottery (proof in Appendix B). This result holds irrespective of the functional form of the utility function U.

Proposition 2. *The expected utility of the simple lottery L is equivalent to the expected utility of the compound lottery $\mathcal{L}$, $EU(L) = EU(\mathcal{L})$.*

Standard EU predicts that single and multiple randomization are equivalent in expectation with respect to receiving utility from the payoff of the DA algorithm.

Behavioral theory. The following behavioral framework is based on the assumption that individuals do not consider the outcomes of the DA algorithm, but the ranking outcomes from the lotteries. This implies a cognitive failure to grasp the compound lottery structure. Since the distribution of matching outcomes does not depend on the randomization procedure, the shape of the utility function cannot explain the preference for multiple randomization under standard EU.

I propose that individuals do not evaluate the outcome distribution of payoffs from the DA algorithm, but rather that they evaluate individual lotteries in terms of their position in the priority ranking. They care about their position "in the line" (Pathak, 2011, p. 523). If this behavioral assumption is satisfied, then the concavity of the utility function is a sufficient condition to explain a preference for multiple randomization. In principle, risk or loss aversion suffice for concavity. Consistent with the quote, I show the result for loss aversion.

In order to accommodate choices that are consistent with a preference for multiple randomization, let the behavioral assumption be satisfied. Let f be defined over the set of ranking positions K:

A0. $f : (k_1, k_2) \to \mathbb{R}$, *with* $f(k_1, k_2) = k_1 + k_2$ *and* $f(k_1, k_2) > f(k_1', k_2)$ *if* $k_1 < k_1'$.

k_1 is the position in the ranking at school 1 and k_2 is the ranking position at school 2. Let the first position yield x and the second position yield y, with $x > y$, where the gain-loss utility function μ is defined over these outcomes from the ranking. Let the reference point $a \in (x, y)$.

Let the following assumptions from Kőszegi and Rabin (2007) about the gain-loss utility function μ be satisfied:

A1. $\mu(x)$ *is continuous* $\forall x$, *twice differentiable for* $x \neq 0$, *and* $\mu(0) = 0$.
A2. $\mu(x)$ *is strictly increasing.*
A3. *If* $y > x \geq 0$, *then* $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.
A4. $\forall x \neq 0$, $\mu''(x) = 0$.
A5. $\mu_-'(0)/\mu_+'(0) \equiv \lambda > 1$, $\mu_+'(0) \equiv lim_{x\to 0}\mu'(|x|)$ *and* $\mu_-'(0) \equiv lim_{x\to 0}\mu'(-|x|)$.

I call assumption $A0$ the *unconditional lottery evaluation*, as opposed to the reduction of compound lotteries. This assumption implies that the compound lottery implied by the DA algorithm is disregarded. Instead, the lottery at each school is evaluated unconditionally in terms of ranking positions. As a result, individuals have preferences over lotteries over a set of rankings.

Assumptions $A1$, $A2$, $A3$, and $A5$ are the Kahneman and Tversky (1979) prospect theory properties of the utility function as originally formulated in Bowman *et al.* (1999, p. 157).[2] $A3$ and $A5$ imply loss aversion (parameter λ).

Figure 2 presents the tree representation of single and multiple randomization under $A0$ in the two lottery case. It illustrates Proposition 3. The single randomization yields the same ranking distribution at both schools, $(x; x)$ or $(y; y)$, with one random draw.

[2]Assumption $A4$ ($A3'$ in Kőszegi and Rabin (2007)) excludes diminishing sensitivity to gains and losses. Linearity is the parsimonious and empirically plausible assumption in the lottery outcome space K of the school choice problem.

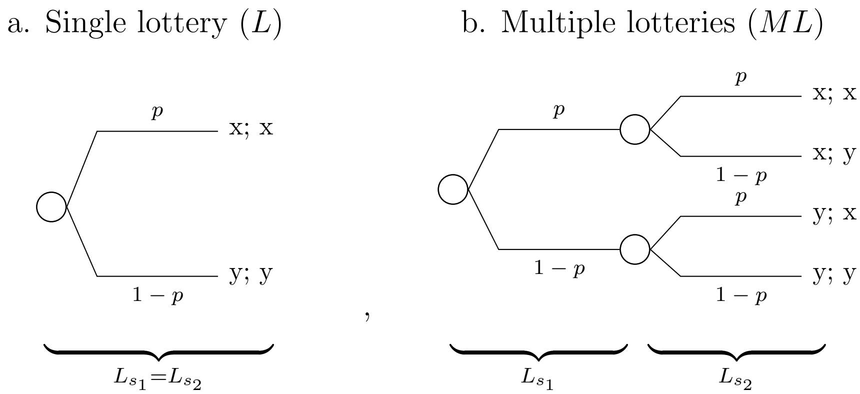

Figure 2: Tree representation of single and multiple randomization under $A0$.

Under single randomization, the probability of being "the last student in the line" (Pathak, 2011, p. 523) $(y; y)$ is $1 - p$. Under multiple randomization, the joint probability of running the misfortune of getting the second rank at each school $(y; y)$ is $(1-p)^2$. In contrast to single, the multiple randomization yields mixed ranking outcomes $(x; y)$ and $(y; x)$. Mixed ranking outcomes imply a "new chance" (Pathak, 2011, p. 523) distinct from the compound structure (see Figure 1).

Proposition 3. *Under loss aversion and assuming $A0$, the multiple randomization ML is strictly preferred to the single randomization L, $ML \succ L$.*

Proof. Consider the utility function U under single (L) and multiple (ML) randomization in the two lottery case:

$$
\begin{aligned}
& U(L) < U(ML) \\
& \text{By } A0: \\
& \Leftrightarrow p\mu(2x) + (1-p)\mu(2y) \\
& \quad < p^2\mu(2x) + p(1-p)\mu(x+y) + (1-p)p\mu(y+x) + (1-p)^2\mu(2y) \\
& \Leftrightarrow (p-p^2)\mu(2x) + [(1-p) - (1-p)^2]\mu(2y) - 2p(1-p)\mu(x+y) > 0 \\
& \Leftrightarrow (p-p^2)\mu(2x) + (p-p^2)\mu(2y) - 2p(1-p)\mu(x+y) > 0 \\
& \Leftrightarrow \mu(2y) + \mu(2x) < 2\mu(x+y). \qquad (1)
\end{aligned}
$$

By assumptions $A3$ and $A5$, equation (1) holds. □

2.5 Predictions

The experimental hypotheses are based on matching theory and the behavioral theory on preferences for procedures. First, consider the main hypothesis on preference for multiple randomization. The null hypothesis on preferences follows from equivalence of both random procedures in Proposition 2. Under standard EU assumptions, individuals are predicted to be indifferent between the simple lottery L and the compound lottery $\mathcal{L}$, $L \sim \mathcal{L}$.

By Proposition 3, individuals strictly prefer the multiple lotteries ML to the single lottery L under unconditional lottery evaluation and loss aversion, $L \prec ML$.

Hypothesis 1. (Preference for multiple randomization) *Individuals prefer the multiple randomization to the single randomization.*

Since both mechanisms are strategy-proof, truth-telling behavior is not predicted to differ. The dominant strategy equilibrium is not changed by the random procedure to break ties (Abdulkadiroğlu *et al.*, 2009). As a result, individuals are predicted to submit the same induced preference rankings under both mechanisms.

Hypothesis 2. (Truthful preference revelation) *Individuals truthfully reveal their preferences for schools.*

Truthful preference revelation implies that the proportions of truth-telling are identical under the single and the multiple randomization mechanism.

By Proposition 1, the single randomization and multiple randomization are predicted to yield equal matching distributions. Only student welfare is considered.

Hypothesis 3. (Student welfare) *The matching distribution from single randomization does not dominate the matching distribution from multiple randomization.*

Theoretical results on student welfare critically rely on truthful preference revelation. That is, Hypothesis 3 depends on the validity of Hypothesis 2.

3 Experimental design

Predictions are investigated in a laboratory experiment. Instructions are available in Appendix C.[3] The experiment is designed to test differences in preferences for random procedures and truth-telling behavior in assignment mechanisms with single (SR) and multiple (MR) randomization. Both assignment mechanisms are based on the DA algorithm. They differ only in one aspect: The randomization to obtain strict school priorities before applying the DA algorithm.

- In mechanism **SR**, one **single lottery** is employed to obtain one rank order of all students. This ranking is used to break ties at schools.
- In mechanism **MR**, **three lotteries** are employed. Each school breaks ties with an independent lottery and obtains its own priority order of students.

3.1 Environment

The stylized assignment problem in this experiment consists of a set of four students $I = \{i_1, i_2, i_3, i_4\}$ and a set of four schools $S = \{s_1, s_2, s_3, s_4\}$, where each school has one seat capacity. The designed environment comprises the following strict student preference profiles P_i and exogenously given school priorities R_s. Priorities may be strict $\succ_s$ or indifference $\sim_s$:

$$\begin{array}{lllllll} s_1 & P_{i_1} & s_2 & P_{i_1} & s_3 & P_{i_1} & s_4 \\ s_1 & P_{i_2} & s_2 & P_{i_2} & s_3 & P_{i_2} & s_4 \\ s_1 & P_{i_3} & s_4 & P_{i_3} & s_3 & P_{i_3} & s_2 \\ s_1 & P_{i_4} & s_4 & P_{i_4} & s_3 & P_{i_4} & s_2 \end{array} , \qquad \begin{array}{l} i_1 \sim_{s_1} i_2 \sim_{s_1} i_3 \sim_{s_1} i_4 \\ i_1 \sim_{s_2} i_2 \sim_{s_2} i_3 \sim_{s_2} i_4 \\ i_1 \sim_{s_3} i_2 \sim_{s_3} i_3 \sim_{s_3} i_4 \\ i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array} .$$

Induced preferences are correlated. For instance, students 1 and 2 prefer being matched to school 1 over school 2, school 2 over school 3, and school 3 over school 4. Each student is matched to one school only. Participants earn points during the experiment, which are transferred at a rate of 1/20 to euros. Payoffs are symmetric. If participants obtain their top school match, they are paid 14 euros. They receive 10 euros for their second-best, 6 euros for their third-best match,

[3] The school choice problem in the experiment is framed as a market with "applicants" applying for "positions" (see also Klijn *et al.*, 2013; Pais and Pintér, 2008). The advantage is that payments have a straightforward interpretation as salaries. See Appendix C for instructions.

and 2 euros for their match with the last school. Schools are not strategic players. Only school 4 has strict preferences *ex ante*. The first three schools are indifferent between students. In the experiment, mechanisms SR and MR differ in their random procedure to resolve these indifferences. Importantly, the lotteries to resolve indifferences in school priorities are not realized until the end of the experiment.

3.2 Setup

Figure 3 provides an overview of the experimental setup consisting of four parts. The experiment implements a one-shot game with complete information. In $SR - MR$, participants first submit their preference ranking for the assignment mechanism SR and then for MR. This order is reversed in $MR - SR$. In the beginning, individuals are randomly matched in groups of four and assigned to the $SR - MR$ or the $MR - SR$ conditions. Then, each participant is randomly assigned the role of a student within each group. Roles remain constant.

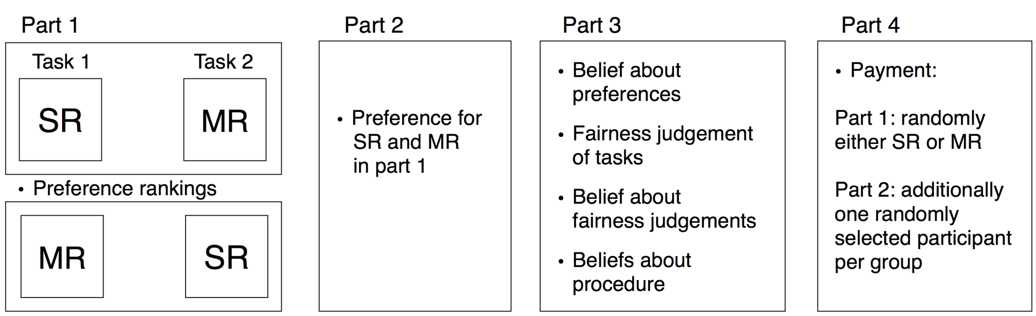

Figure 3: Experimental setup. Mechanisms: (SR) Single randomization and DA; (MR) multiple randomization and DA.

Part 1. Part one consists of two tasks: submitting preference rankings under single and multiple randomization. Participants start with task 1 by submitting their first preference ranking to apply to schools. Subjects know from the beginning that they face two different tasks where they submit preference rankings over schools. They also know that only one—either task one or task two—will be randomly determined to be payoff-relevant. However, they get to know the second task only after they have submitted their preference ranking in the first task. In task 2, participants submit the school ranking for the second mechanism.

Part 2. The preferences over the procedures in part 1 are elicited (without any announcement). Subjects may pay ten euro cents to choose between SR and MR. That is, they choose which of the procedures, from the first part of the experiment, should be relevant in determining their payoffs. If they do not pay to make an active choice, the computer randomly draws one of the two with equal probability.[4]

Part 3. Fairness judgements about either mechanism are elicited subsequently on a seven-point Likert scale. Fairness beliefs about the average fairness judgement of the other participants are elicited with a monetary incentive. Further, incentivized beliefs about the chance of getting the high payoff in one procedure, and incentivized beliefs about the average preferences of the other participants are elicited. All incentivations are based on point estimates with a payment of 0.5 euros for correct answers. The questionnaire contains a risk attitude measurement, as in Holt and Laury (2002), as well as questions on the demographic characteristics.[5]

Part 4. After the experiment, the uncertainty is resolved by playing the lotteries and running the DA algorithm for each matching group. Subjects are paid anonymously and privately. Everyone gets paid for part 1. In part 1, it is randomly determined for each subject individually whether she gets paid for SR or MR. Additionally, one randomly drawn participant per group is paid for part 2.

3.3 Procedure

Experimental sessions were conducted at BonnEconLab in December 2015 and January 2016. In total, $N = 192$ participants took part in the experiment. About 48% of them were female (in both conditions). Participants were recruited from the BonnEconLab subject pool (more than 6000 subjects) using hroot (Bock *et al.*, 2014). They were on average 23 years old. Mostly students from various fields of study took part: natural sciences (28%), economics or mathematics (19%), law (15%), medicine (7%), and humanities (16%). Sessions lasted between 58 and 76 minutes. Average earnings were 15.40 euros (min: 6.40; max: 35.50). The experiment was programmed using the software z-Tree (Fischbacher, 2007).

[4]A pilot study has been conducted. The results suggested that there could be a significant fraction of indifferences. Therefore, this option has been implemented in the main experiment.

[5]Risk aversion is found to impact behavior under the DA algorithm (Klijn *et al.*, 2013).

4 Results

4.1 Preferences for random procedures

This paper starts the analysis by comparing preferences for random procedures. Hypothesis 1 states that individuals prefer multiple to single randomization. This is consistent with the experimental evidence.

Result 1. (Preferences for random procedures) *On average, individuals prefer the multiple to the single randomization mechanism. A fraction of 23% of individuals prefers multiple randomization. The majority of individuals (65%) is not willing to pay ten euro cents to choose a procedure.*

Support. Figure 4 presents proportions of individuals holding strict preferences for single or multiple randomization, or indifferences. The proportion of individuals strictly preferring MR (23%) is significantly larger than the proportion of individuals preferring SR (12%) according to the binomial test ($N = 67$, $p = 0.007$, one-sided).

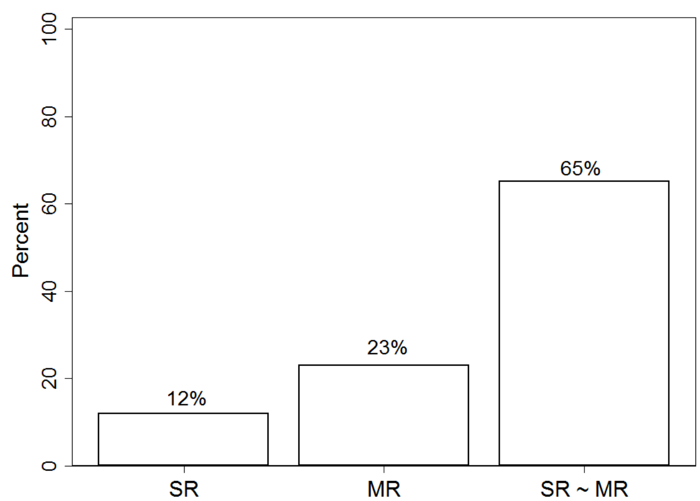

Figure 4: Proportion of individuals with preferences for single (SR) and multiple (MR) randomization, and indifferences ($SR \sim MR$) ($N = 192$).

The main result that individuals prefer multiple to single randomization does not depend on the submitted preference rankings. About half of the participants exhibiting strict preferences submitted the same preference rankings in task 1 and task 2. Individuals who submitted consistent rankings also prefer multiple (22%) to single (10%) randomization (binomial test, $N = 29$, $p = 0.031$, one-sided). The determinants of the preference for multiple randomization are investigated in subsection 4.4.

4.2 Truthful preference revelation

Differences in truthful preference revelation between SR and MR are analyzed on the basis of submitted preference rankings in task 1. The reason is that preference rankings in task 2 could depend on the experience from the submitted rankings and the procedure in task 1. Hypothesis 2 says that truth-telling rates are equal under both mechanisms. This paper finds evidence supporting the theory.

Result 2. (Top rank) *Truthful top-rank preference revelation does not differ significantly between the single* (69%) *and the multiple* (75%) *randomization mechanism.*

Support. Table 1 presents, in the first element of the diagonals, the proportions of truth-telling for the top choice under SR and MR. The proportion of truthful preference revelation of the top rank does not differ significantly between SR and MR according to the permutation test ($N = 48$, $p = 0.363$, two-sided).

Table 1: Distribution of submitted ranks (%).

	Submitted rank							
	SR				MR			
True rank	1	2	3	4	1	2	3	4
Rank 1	69	15	2	15	75	17	2	6
Rank 2	23	50	20	7	18	49	22	11
Rank 3	5	15	74	6	5	18	69	8
Rank 4	3	14	4	79	2	13	7	78

Proportions in percentages. Permutation tests are employed for individual rank comparison of the actual submitted ranks between SR and MR. No significant differences at $p < 0.05$, two-sided, $N = 48$.

The diagonal elements present percentages of truth-telling by submitted individual rank. Comparing average proportions of 16 individual combinations of true × submitted ranks between SR and MR in Table 1 indicates that there are no significant differences at conventional levels. In line with the literature, truthful preference revelation is analyzed based on top ranks and based on the whole ranking.

Result 3. (Entire ranking) *Truthful preference revelation of the entire rank-order list does not differ between single* (47%) *and multiple* (47%) *randomization.*

Support. Table 1 presents, in the diagonal elements, the proportions of truth-telling for each submitted rank. Overall, the average proportion of truthful preference revelation of the entire rank-order list is not significantly different between SR and MR (Mann-Whitney U, $N = 48$, $p = 0.949$, two-sided).[6]

On average, there are no significant differences in truthful preference revelation between SR and MR. This result holds if truth-telling for each individual student preference profile is compared between SR and MR (see Figure 6 in the Appendix).

The relation between the preference for a random procedure and truthful preference revelation is investigated in nonlinear probit estimations with standard errors clustered at matching group level. The regression of truthful top rank revelation on the preference for multiple randomization indicates that there is no significant relationship ($p = 0.483$). The same result holds for the regression of truthful preference revelation of the entire rank-order on the preference for multiple randomization ($p = 0.589$). These findings imply that the preference for multiple randomization is not related to truthful preference revelation.

[6] The average of each group is regarded as one data point, resulting in $N = 48$ observations.

4.3 Welfare

Matching outcomes of the SR and MR mechanism are evaluated *ex post* in terms of individuals receiving their preferred choices. I employ actual submitted preference rankings under SR and MR to simulate matching distributions.[7] Again, only submitted rankings from the first task of part one are considered. Hypothesis 3 predicts that single and multiple randomization yield identical matching distributions. This is consistent with the experimental evidence.

Result 4. (Welfare) *Matching distributions of SR and MR do not differ. Neither the proportion of top choices nor the average rank received differ.*

Support. Figure 5 shows cumulative distribution functions (CDF) of simulated matching outcomes using actual submitted preferences under SR and MR. The average proportion of individuals receiving their top choice does not differ significantly between SR (26%) and MR (33%) (permutation test, $N = 48$, $p = 0.152$, two-sided). The average rank received does not differ significantly between MR (2.29) and SR (2.27) (Mann-Whitney U, $N = 48$, $p = 1.000$, two-sided).

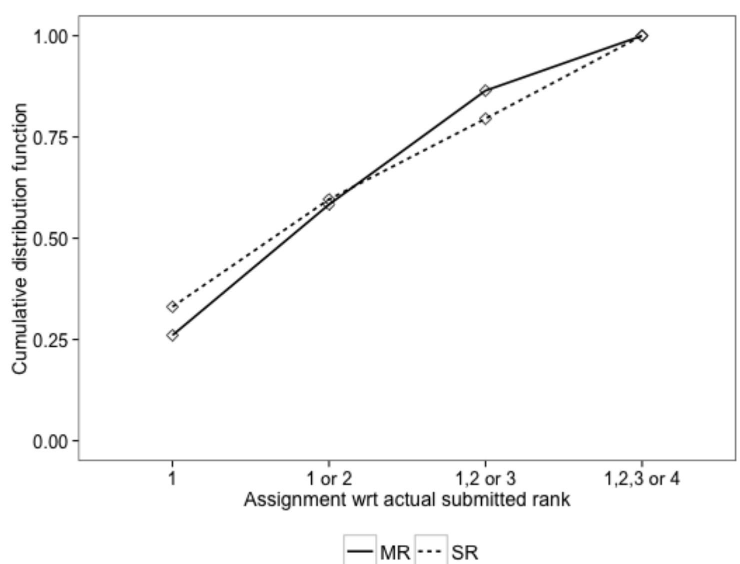

Figure 5: Cumulative matching distribution based on actual submitted preference rankings under single (SR) and multiple (MR) randomization.

[7]Simulations are based on 1000 single independent random draws and 3×1000 multiple random draws from a uniform distribution. Averages of the DA outcomes are calculated for each group.

4.4 Mechanism behind the preference for multiple randomization

A fraction of 23% of participants is found to prefer the multiple randomization mechanism. This section provides additional evidence from questionnaire data on the determinants of these preferences (see Appendix A). I focus on fairness judgements and beliefs as potential explanatory factors. In other words, this analysis sheds light on the question whether fairness perceptions or biased beliefs impact the preference for multiple randomization.

On average, participants judge the MR mechanism (3.0) to be fairer than the SR mechanism (2.7) on a Likert scale from 0 to 6 (Wilcoxon signed-rank, $N = 192$, $p = 0.003$, two-sided). This average difference is larger for subjects with preferences for multiple randomization (SR: 2.7, MR: 3.5; Wilcoxon signed-rank, $N = 44$, $p = 0.001$, two-sided). On average, subjects also expect the other participants to judge MR to be fairer than SR (Wilcoxon signed-rank, $N = 192$, $p < 0.001$, two-sided).

In terms of beliefs, the proportion of individuals expecting a higher chance to receive the highest payoff under the MR mechanism is larger than under the SR mechanism (binomial test, $N = 82$, $p = 0.060$, two-sided).

Result 5. (Fairness perception of multiple randomization) *The fairness judgement of multiple randomization is related to a preference for multiple randomization.*

Support. Table 2 provides the probit estimation results of the preference for multiple randomization. The fairness judgement (first row) of MR relative to SR is in all specifications significantly positively associated with the preference for MR.

A fairness judgement is defined as the difference between the fairness judgements of MR and of SR. It takes a positive value for the fairness judgement relation $MR > SR$. Belief MR is a binary variable taking the value one if the individual holds the belief (incentivized) that the chance of receiving the highest payoff is larger in MR. This chance is equal in SR and MR, since only one lottery at the top school is relevant for this highest payoff. This is not influenced by the number of additional lotteries at other schools.

Table 2: Probit Regression of Preference for Multiple Randomization.

	(1)	(2)	(3)	(4)	(5)
Fairness judgement	0.069**			0.068**	0.082***
	(0.034)			(0.034)	(0.030)
Belief MR		-0.112		-0.124	-0.126
		(0.117)		(0.116)	(0.092)
Risk aversion			-0.015	-0.007	-0.023
			(0.026)	(0.025)	(0.023)
Set of controls $^{+}$	No	No	No	No	Yes
Constant	0.326*	0.518**	0.636	0.590	4.443
	(0.166)	(0.202)	(0.423)	(0.494)	(9.120)
N	67	67	67	67	67
Pseudo R^2	0.05	0.01	0.00	0.07	0.33

Dependent variable: Binary preference taking value 0 if SR, and 1 if MR.
Average marginal effects reported. Robust standard errors in parentheses.
$^{+}$ Set of controls includes demographics, submitted ranks in task 1 and 2, player role, order effects, field of study, high-school math grade, beliefs about preferences/ fairness judgements/ procedure.
Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Fairness judgements (row one) are significantly positively associated with preferences for MR. This relation is robust to including other explanatory factors such as actual submitted rank orders and order effects. The relation of fairness judgements and preferences does not depend on education or math skills. The result is also robust to including indifferences: The full-sample probit regression of preference for MR on the fairness judgement of MR indicates that there is a significant relationship ($N = 192$, $p = 0.022$).

Biased beliefs (row two) that MR yields better outcomes are not associated with the preference for MR. These beliefs are not correlated with fairness judgements of MR either ($N = 192$, $r = 0.03$, $p = 0.639$). This finding suggests that the biased belief of receiving the highest payoff in MR is not associated with the fairness judgement of MR.

Risk aversion (row three) is not significantly related to the preferences for MR. That is, preferences for single or multiple lotteries do not depend on the risk attitudes of individuals.

5 Discussion and conclusion

This paper compares behavior under the DA mechanism with single and multiple randomization in terms of preferences for either random procedure, truthful preference revelation, and welfare. The first and main result clearly shows that a significant proportion of participants prefers multiple to single randomization. However, not all individuals prefer one random procedure to the other. Consistent with standard expected utility theory, the majority of participants is indifferent between both random procedures. This evidence suggests that there are preferences for multiple randomization in addition to the well documented preference for randomization (Dwenger *et al.*, 2014; Agranov and Ortoleva, 2015).

The second result indicates that the main mechanism behind a preference for multiple randomization is fairness, rather than biased beliefs. Multiple randomization is perceived to be fairer and fairness perceptions are related to preferences for the MR procedure. This result goes beyond previous literature on differences in fairness judgements of random procedures (Keren and Teigen, 2010; Eliaz and Rubinstein, 2014). There is no evidence that fairness judgements are associated with biased beliefs. This implies that individuals judge multiple randomization to be fairer, although they do not expect better chances in obtaining the highest outcome from this procedure. This result is limited to beliefs about school 1.

Preferences for random procedures and their fairness perception do not have an effect on behavior under the mechanism. The third result suggests that, consistent with matching theory, truthful preference revelation is not affected by the random procedure to break ties. Overall, however, truth-telling is far below the theoretical prediction of 100%. On average, 47% of participants submit their true preferences in this correlated environment. This result is in line with experimental findings on the DA algorithm employing different environments (Chen and Sönmez, 2006; Pais and Pintér, 2008; Calsamiglia *et al.*, 2010; Featherstone and Niederle, 2014).

The fourth result states that, consistent with the theory, there are no differences in matching outcomes between mechanisms. However, Abdulkadiroğlu *et al.* (2009) and Featherstone and Niederle (2014) find, based on field and experimental data, that single randomization yields more efficient results. They employ submitted preferences under one random procedure in their simulations. In contrast, this paper uses submitted preference rankings under both mechanisms. Moreover, Featherstone and Niederle (2014) report predictions and results based on an uncorrelated preference environment in contrast to the correlated environment employed in this experiment. The general theoretical result holds that single randomization yields outcomes that are at least as efficient as multiple randomization.

In terms of external validity, the real-life effect of a preference for multiple randomization may be underestimated by the evidence obtained in a stylized setting. The high-stakes school choice context could potentially increase concerns about the procedure. High stakes could be associated with stronger fairness perceptions. Further, increasing the number of indifferent schools could enhance fairness perceptions of multiple randomization. In turn, stronger fairness perceptions could lead to repugnance of unfair procedures. Anecdotal evidence from the NYC policy debate supports this conjecture (Pathak, 2011). NYC first implemented multiple randomization and changed it later to single randomization. Yet, this paper has shown that, even in an abstract lab environment, a significant proportion of individuals exhibits a preference for multiple randomization.

In sum, assignment mechanisms of public resources can be evaluated in terms of welfare and procedural fairness. The design of matching markets in practice often involves the choice between procedures to randomize allocation priorities. This design choice is made between single and multiple randomization. Single randomization is found to yield larger welfare in the field (Abdulkadiroğlu *et al.*, 2009). This paper has shown that fairness perceptions and preferences speak against single randomization. This implies that designers of such allocation procedures potentially face a trade-off between social welfare and procedural fairness perceptions.

In conclusion, this paper shows that randomization—although satisfying the "egalitarian principle" of equal chance (Elster, 1991)—is not inherently neutral with respect to procedural preferences and fairness perception. I provide evidence for a systematic distinction between a universal, single and a decentralized, multiple randomization. Random procedures as such neither guarantee equal fairness judgements nor indifference. This finding can have consequences for the social acceptability of market designs in practice. Fairness judgements and preferences point to a larger social acceptability of the multiple randomization mechanism.

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Appendix for online publication

A Tables and figures

Table 3: Fairness judgements.

Variable	Sample	N	Mean	(SE)	P-value signed-rank
SR	full	192	2.729	(0.108)	0.003
MR	full	192	3.010	(0.113)	
Belief SR	full	192	2.813	(0.085)	<0.001
Belief MR	full	192	3.068	(0.088)	
SR	reduced	44	2.727	(0.196)	0.001
MR	reduced	44	3.545	(0.224)	
Belief SR	reduced	44	2.841	(0.166)	0.002
Belief MR	reduced	44	3.432	(0.185)	

Fairness judgements are measured using a 7 point Likert scale $(0-6)$.
SR and MR present fairness judgements about the respective mechanism.
Belief SR and MR are incentivized beliefs about average judgement of others.
Reduced sample includes only participants with preference for MR.
P-values of two-sided Wilcoxon signed-rank test are reported.

Table 4: Belief distributions.

Variable	N	Distribution (%)			N	P-value
		SR	MR	$SR \sim MR$	(SR & MR)	binomial test
Preference	192	11.98	22.92	65.10	67	0.014
Belief preference	192	19.27	27.08	53.65	89	0.140
Belief procedure	192	16.67	26.04	57.29	82	0.060

Preference is the benchmark distribution of preferences reported in figure 4.
Belief preference is the incentivized belief about the average preferences of others.
P-values of two-sided binomial test ($SR = MR$) are reported (N of SR & MR).

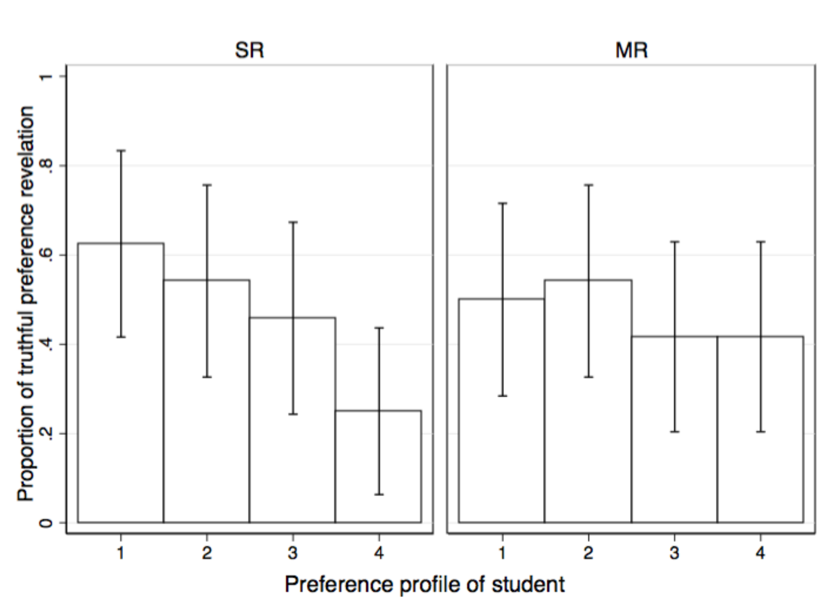


Figure 6: Proportions of truthful preference revelation by student preference profile under SR and MR (error bars present 95% confidence intervals).

B Proof of Proposition 2

Proposition 2. *The expected utility of the simple lottery L is equivalent to the expected utility of the compound lottery $\mathcal{L}$, $EU(L) = EU(\mathcal{L})$.*

Proof. The proof is shown based on the example used in the experiment. There are four students and four schools. Let $L = (q_1, ..., q_4)$ be the simple lottery and let $\mathcal{L} = \{L_{s_1}, ..., L_{s_4}; \alpha_1, ..., \alpha_4\}$ be the four-stage compound lottery of four simple lotteries. Lottery L_{s_4} is degenerate and yields the priority ranking with certainty.

First, note that there are 4! possible priority orderings over the number of students $N = 4$ of the set of students $I = \{1, ..., 4\}$ at each school $s \in S$. Lotteries are drawn from a uniform distribution. That is, each ordering is equally likely with probability $\frac{1}{4!}$ at school s. All $(4!)^3$ priority orderings are equally likely. Let the preference relation satisfy axioms of expected utility theory (EU).

Example. Consider the following environment from the experiment:

$$\begin{array}{lcl} s_1 \text{ P}_{i_1} \; s_2 \; P_{i_1} \; s_3 \; P_{i_1} \; s_4 & & i_1 \sim_{s_1} i_2 \sim_{s_1} i_3 \sim_{s_1} i_4 \\ s_1 \text{ P}_{i_2} \; s_2 \; P_{i_2} \; s_3 \; P_{i_2} \; s_4 & & i_1 \sim_{s_2} i_2 \sim_{s_2} i_3 \sim_{s_2} i_4 \\ s_1 \text{ P}_{i_3} \; s_4 \; P_{i_3} \; s_3 \; P_{i_3} \; s_2 & & i_1 \sim_{s_3} i_2 \sim_{s_3} i_3 \sim_{s_3} i_4 \\ s_1 \text{ P}_{i_4} \; s_4 \; P_{i_4} \; s_3 \; P_{i_4} \; s_2 & , & i_1 \succ_{s_4} i_2 \succ_{s_4} i_3 \succ_{s_4} i_4 \end{array} .$$

Simple lottery. Assuming truthful preference revelation of the preference profiles P_i, the expected utility of the simple lottery L is calculated as follows

$$\begin{aligned} EU(L) &= \sum_{n=1}^{4} q_n u(\pi_{i,s_m}) \\ i = 1, 2: \quad EU(L) &= \frac{1}{4}u(\pi_{i,s_1}) + \frac{1}{2}u(\pi_{i,s_2}) + \frac{1}{4}u(\pi_{i,s_3}), \\ i = 3: \quad EU(L) &= \frac{1}{4}u(\pi_{i,s_1}) + \frac{3}{4}u(\pi_{i,s_4}), \\ i = 4: \quad EU(L) &= \frac{1}{4}u(\pi_{i,s_1}) + \frac{3}{4}u(\pi_{i,s_3}), \end{aligned}$$

where q_n is the probability of getting the payoff; π_{i,s_m} is the payoff of student i receiving the seat at school s.

Reduction of the four-stage compound lottery. Since every simple lottery in the compound lottery is drawn from the same uniform probability distribution and using the Independence axiom of EU, the compound lottery can be reduced to the simple lottery:

$$EU(L) = q_1 u(\pi_{i,s_1}) + q_2 u(\pi_{i,s_2}) + q_3 u(\pi_{i,s_3}) + q_4 u(\pi_{i,s_4}) = EU(\mathcal{L})$$

Since the compound lottery ($\mathcal{L}$) can be reduced to the simple lottery (L), both are equal in expected utility: $EU(L) = EU(\mathcal{L})$. □

C Instructions

Notes: Part one consists of instructions on a handout. Part two is displayed on the computer screen during the experiment. Quiz questions are based on Klijn et al. (2013).

Part 1

Welcome to the experiment! You are going to take part in an economic study financed by the Max Planck Society. You will receive a show-up fee of 4 euros. Additionally, you will be able to earn a substantial amount of money. It is therefore crucial that you read these explanations carefully. Your payment depends on your decisions, on the decisions of the other participants, and on chance. The present instructions are identical for all participants.

From now on it is prohibited to use electronic devices. During the experiment there shall be absolutely no communication between participants. If you have any questions, please raise your hand. We will then come over to you. Any violation of these rules means you will be excluded from the experiment and from any payments.

During the experiment we will calculate in points. The total number of points you earn in the course of the experiment will be transferred into euro at the end, at a rate of

1 euro = 20 points.

The procedure and payment details are described below. Following the instructions, we ask you to answer a quiz. This shall help you to understand the decision situation.

Procedure

- Task 1
- Task 2
- Questionnaire
- Payment

The experiment consists of three parts: Task 1, task 2, and the questionnaire. **You will receive precise instructions for task 1 and task 2 on your computer screen** after the experiment has begun. At the beginning of the experiment, all participants are randomly assigned to groups of four. You will not get to know the identity of the other participants in your group. You stay in the same group during both tasks.

Your cash payment at the end of the experiment consists of the following parts. You receive 4 euros show-up fee. In the end, **one of both tasks (either 1 or 2) is randomly determined** and payd out to you. Additionally, you can earn money by answering the questionnaire.

Decision Situation

In tasks 1 and 2, we simulate a procedure that assigns positions to applicants. You and the other participants are applicants. Within each group, you are randomly assigned the role of an applicant. This role remains the same in both tasks.

Applicants use the procedure of a central clearinghouse to apply for positions. The computer determines who gets which position. In the following, payment table, priority lists, and the ranking are explained. Consequently, the procedure of allocating positions using this information is described.

There are four applicants (1,2,3, and 4) and four positions (W,X,Y, and Z). Every position accepts only one applicant. The following **payment table** determines your points during the experiment.

Payment table.

Points	Applicant 1	Applicant 2	Applicant 3	Applicant 4
280 points	Position W	Position W	Position W	Position W
200 points	Position X	Position X	Position Z	Position Z
120 points	Position Y	Position Y	Position Y	Position Y
40 points	Position Z	Position Z	Position X	Position X

In this payment table you can see how many points each applicant receives for each assigned position. This table is equivalent in both tasks. For instance, if applicant 1 is assigned position W in the allocation procedure, then he receives 280 points. If applicant 1 receives position X, then he receives 200 points; for position Y he receives 120 points and for position Z he receives 40 points. Positions have the following **priority lists** over the applicants in the experiment.

Priority lists.

Priority	Position W	Position X	Position Y	Position Z
Highest priority				Applicant 1
Second-highest priority				Applicant 2
Third-highest priority				Applicant 3
Lowest priority				Applicant 4

In this table, you can see that applicant 1 is on top of position Z's priority list. Position Z has highest priority for applicant 1, followed by applicant 2 with second-highest, applicant 3 with third-highest, and applicant 4 with lowest priority. Priority means, for example, that applicant 1 is favored by position Z compared to the other applicants. Please note that during the experiment only the priority list of position Z is given. **You receive more information about the priority lists of positions W, X, and Y on your computer screen during the experiment**.

Your Decision

In each task, you will make a decision about a ranking of positions. You may submit any ranking. All positions have to be listed. Rank 1 means the top rank, rank two the second-highest, rank 3 the third-highest, and rank 4 the lowest rank.

Every applicant submits a ranking of positions. Given these rankings and the priority lists, positions are assigned in the central allocation procedure. The final payment is made according to the payment table.

Description of the Allocation Procedure

1. Every applicant applies for his top rank position.

2. Every position compares applicants and temporarily accepts the applicant with highest priority on its priority list. All others are rejected.

3. If an applicant is rejected, then he applies for the position with the second-highest rank on his submitted ranking.

4. If a position gets new applications (previously rejected applicants at other positions), then it considers both new applicants and temporarily accepted applicant. The position compares new participants with the temporarily accepted participant and temporarily accepts the one with highest priority. All others are rejected.

5. This allocation procedure is repeated until no applicant is rejected anymore. Every applicant receives the position that accepts his application in the end.

An Example

Consider for illustration purposes the following example. There are three applicants (1, 2, and 3) and three positions (A, B, and C). Assume that the applicants submitted the following **rankings**:

Applicant 1:	rank 1 = B	rank 2 = C	rank 3 = A.
Applicant 2:	rank 1 = C	rank 2 = A	rank 3 = B.
Applicant 3:	rank 1 = B	rank 2 = C	rank 3 = A.

Important: These sample rankings are chosen arbitrarily and only serve illustrational purposes. They provide no guidance for your decision-making in the experiment!

Assume the following **priority lists**. In this example, all priority lists are given. In the experiment, only one priority list is given.

Example: Priority lists.

Priority	Position A	Position B	Position C
Highest priority	2	2	1
Second-highest priority	3	1	3
Lowest priority	1	3	2

You can see for example in the rankings that applicant 1 submitted position B at the top rank. Position C is on the second-highest rank and position A is on the lowest rank. Further, you can see from the priority lists that position A has highest priority for applicant 2 followed by applicants 3 and 1.

Questions about the Example

Please complete the following sentences.

1. In round one of the allocation procedure, every applicant applies for his top rank position. That is, applicant 1 applies for position ____, applicant 2 for position ____ and applicant 3 for position ____ . Based on these three applications, every position temporarily accepts one applicant and rejects all others according to their priority. Position B compares applicants 1 and 3 according to priority. Then, position B temporarily accepts applicant ____ and rejects applicant ____ . Position C temporarily accepts applicant ____ .

2. In round two, all previously rejected applicants apply for the position with the second highest rank. That is, applicant 3 applies for position ____ . Now, positions compare new applicants with the temporarily accepted ones from round one. Position C compares applicant 2 with 3. Applicant 3 has a higher priority then applicant 2. Then, position C temporarily accepts applicant ____ and rejects applicant ____ .

3. In round three, every applicant rejected in round two applies for the position with the next-highest rank in his ranking. That is, applicant ____ applies for position ____ . Since this position has not been assigned yet, all applicants are now assigned one position. The central allocation procedure is terminated.

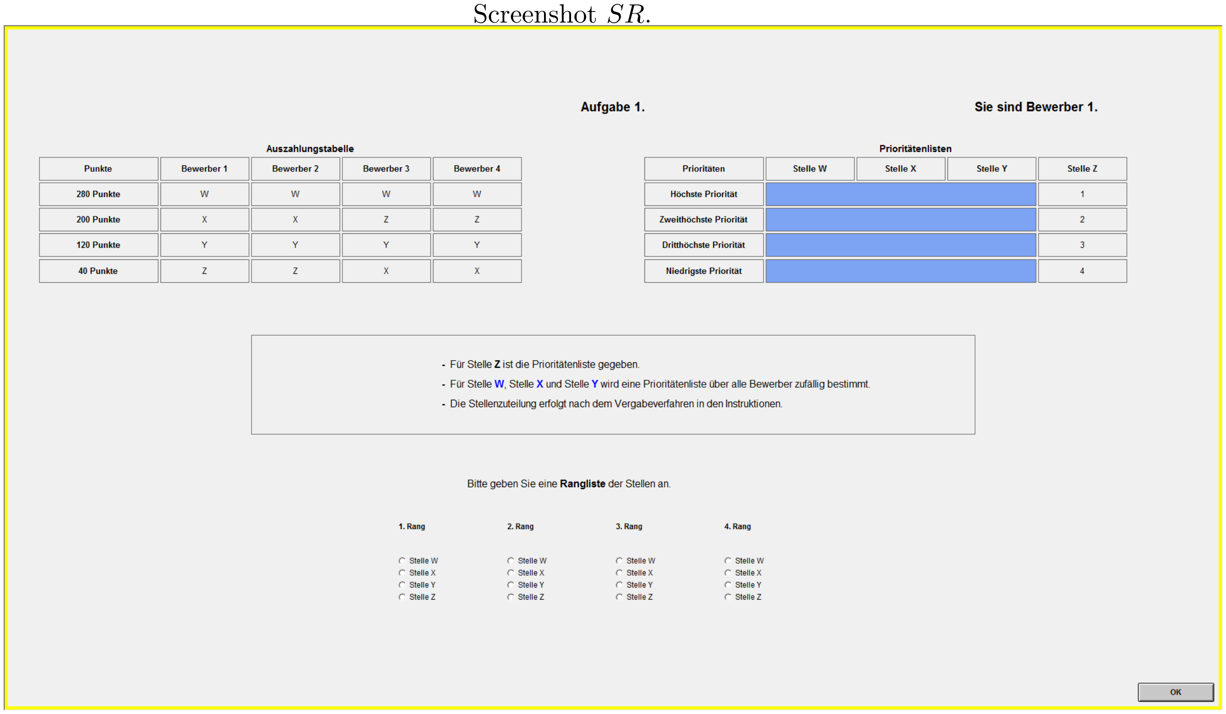
4. The final allocation of positions is as follows.
   Applicant ____ receives position A.
   Applicant ____ receives position B.
   Applicant ____ receives position C.

5. Are both tasks 1 and 2 selected to be payoff relevant in the end? Answer: ____ .

Do you have any questions? If this is the case, then please raise your hand. We will answer your questions individually. Thank you for participating in this experiment!

Part 2 [for order SR, MR]

Screenshot SR.

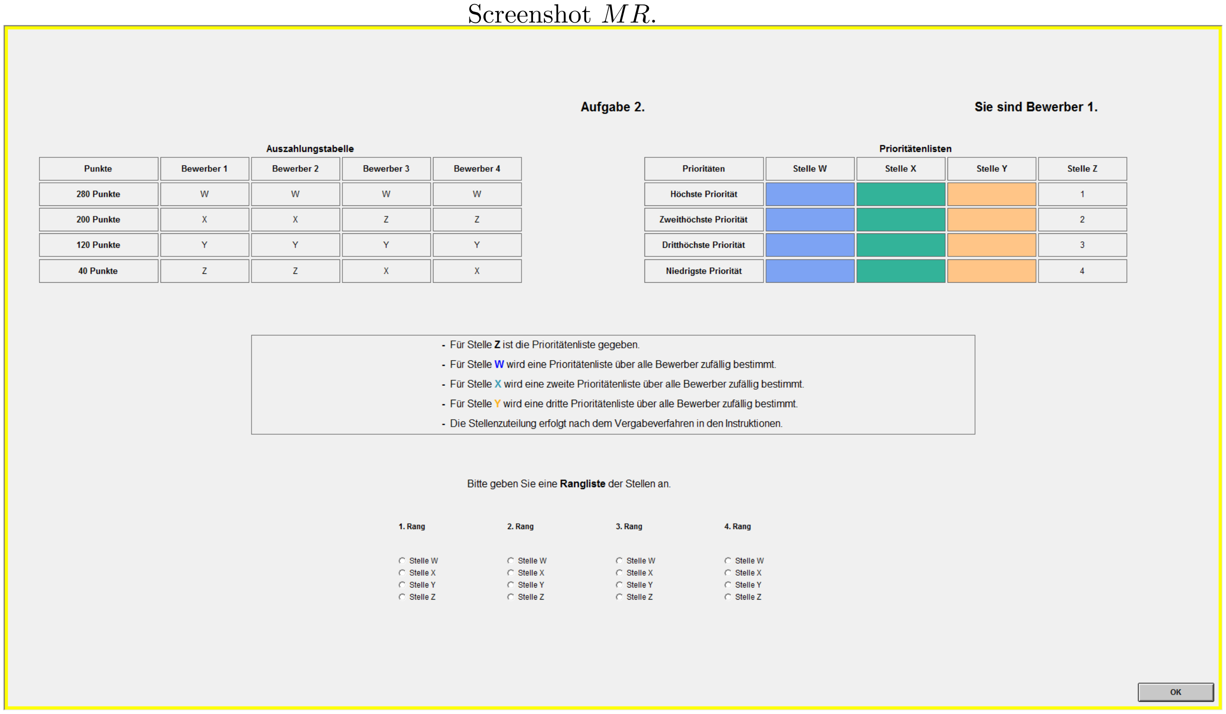


Task 1 [SR description]

- The priority list of the four applicants is given for position **Z**.
- One joint priority list of all applicants will be randomly drawn for position **W**, position **X**, and position **Y**.

  In the end, the priority list is randomly drawn by the computer.
  Any combination of applicants is equally likely.
  Then, assignment of positions takes place according to the procedure in the instructions.

Screenshot MR.



Task 2 [MR description]

- The priority list of the four applicants is given for position **Z**.
- One priority list will be randomly drawn for position **W**.
- One priority list will be randomly drawn for position **X**.
- One priority list will be randomly drawn for position **Y**.

  In the end, the priority lists are randomly drawn by the computer.
  Any combination of applicants is equally likely.
  Then, assignment of positions takes place according to the procedure in the instructions.

Question 1 *[Preferences for random procedures]*

Now you have the chance additionally to earn up to 280 points.

- You just made decisions about rankings in task 1 and task 2.
  One task has been randomly determined to be payoff-relevant for the previous part.

- Now you can earn additional money with your decisions from one of the tasks.
  You can decide for yourself which task should determine your additional payment.
  If you do not decide, then the computer randomly determines one task.

- If you choose one of the tasks, then you pay 2 points for your decision option.
  If you do not decide, you do not pay 2 points.

  - ○ I choose task 1.
  - ○ I choose task 2.
  - ○ I do not choose.