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ABSTRACT

Revisiting the Matching Function*

This paper shows analytically and numerically that there are two ways of generating an observationally equivalent comovement between matches, unemployment, and vacancies in dynamic labor market models: either by assuming a standard Cobb-Douglas contact function or by combining a degenerate contact function with idiosyncratic productivity shocks for new jobs. Despite this observational equivalence, we provide several reasons for why it is important to understand what happens inside the black box of job creation. We calibrate a combined model with both mechanisms to administrative German wage and labor market flow data. In contrast to the model without idiosyncratic shocks, the combined model is able to replicate the observed negative time trend in estimated matching functions. In addition, the full nonlinear combined model generates highly asymmetric business cycle responses to large aggregate shocks.

JEL Classification: E24, E32, J63, J64

Keywords: matching function, idiosyncratic productivity, job creation, vacancies, time trend, asymmetries

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1 Introduction

"The matching function is a modeling device that occupies the same place in the macroeconomist's tool kit as other aggregate function, such as the production function (. . .). Like the other aggregate functions its usefulness depends on its empirical viability and on how successful it is in capturing the key implications of heterogeneities and frictions in macro models."
(Petrongolo and Pissarides, 2001, 391-392)

There is widespread empirical evidence for a Cobb-Douglas constant returns matching function across countries, occupations or other disaggregation levels (see Blanchard and Diamond (1990) for an early work and Petrongolo and Pissarides (2001) for a survey). The coefficients from these matching function estimations are often used to parametrize Cobb-Douglas contact functions in theoretical models.[1] Thus, the job creation mechanism in search and matching models is usually exclusively driven by a theoretical contact function.[2] In reality, job creation consists of more than one margin. After workers and firms get in contact (e.g. in an interview), only a certain fraction of workers is selected. Not all workers are suitable for an employer and thus only those with the best characteristics (e.g. idiosyncratic shocks) are selected. This second mechanism is also well established in the literature.[3] However, most existing macro-labor business cycle papers use a degenerate selection mechanism (i.e. idiosyncratic shocks play no meaningful role).[4]

In this paper, we approach the matching function from a completely different angle. We focus on the potential role of idiosyncratic productivity for job creation. We start by choosing the opposite polar case to the existing literature, namely by assuming a degenerate contact function. Every worker gets in contact with some constant probability. Firms only select those workers with the largest idiosyncratic productivity.[5] In such a framework, due to the degenerate contact function, more vacancies do not lead to more matches in aggregate. However, both vacancies and the job-finding rate are procyclical. Imagine a positive aggregate productivity shock. This leads to a rise of the ex-ante expected profits in firms' vacancy free entry condition and thereby stimulates vacancy creation.

[1] In what follows, "contact function" refers to the theoretical function that establishes contacts between workers and firms. Due to idiosyncratic shocks, not all of the contacts may become matches. "Matching function" refers to the empirical connection between matches on the one hand and vacancies and unemployment on the other hand.

[2] See e.g. Hall (2005); Hagedorn and Manovskii (2008), and Shimer (2005). There are many papers that combine a contact function, a vacancy free entry condition and idiosyncratic productivity to model endogenous separations (e.g. Krause and Lubik, 2007; Thomas and Zanetti, 2009; Zanetti, 2011). However, in these papers idiosyncratic productivity shocks are used to model the behavior of separations (not hirings).

[3] For a seminal contribution with idiosyncratic productivity see Jovanovic (1979). Traditional search models (e.g. McCall, 1970; Mortensen, 1987) rely on exogenous wage distributions. If they are interpreted as the result of some underlying idiosyncratic productivity heterogeneity, they fall into the same category of models.

[4] See the upper panel in Figure 6 in the Appendix for an illustration.

[5] Brown et al. (forthcoming); Lechthaler et al. (2010) use similar mechanisms in context of more complex models.

In addition, larger aggregate productivity makes it profitable for firms to hire workers with less favorable characteristics (i.e. lower idiosyncratic productivity). This increases the job-finding rate.

We show analytically and numerically that this opposite polar case to the standard literature generates an observationally equivalent equilibrium comovement between matches, unemployment, and vacancies. To put it differently: Our first contribution is to show that dynamic labor market models with two standard modeling ingredients (vacancy free entry and idiosyncratic productivity) generate a simulated time series behavior that is in line with the results from matching function estimations (Cobb-Douglas, constant returns).[6]

We prove that the shape of the idiosyncratic productivity distribution at the cutoff point determines the precise nature of the comovement, i.e. the coefficients in an estimated matching function. We use high quality German administrative wage data to impose discipline on calibrating the density function. Our calibrated and simulated model (with degenerate contact function) generates an equilibrium comovement that is already fairly close to the correlations of the actual matches, unemployment, and vacancy time series in the German data.

In order to match the dynamics of the German labor market data, we replace the degenerate with a traditional contact function and set the weight on vacancies such that we can replicate the correlation structure from the data. Due to idiosyncratic productivity shocks, the required weight on vacancies in the contact function is much smaller than the coefficient from the matching function estimation. Thus, our paper reveals that the conventional practice to use matching function estimations in order to parametrize contact functions has caveats. More precisely: Assume that the weight on vacancies in the traditional contact function is parametrized with the values obtained from a matching function estimation. Then, in a model with idiosyncratic shocks, the model based comovement between matches, unemployment, and vacancies will not be in line with the data any more. In this scenario, a matching function estimation based on simulated data generates a substantially larger weight on vacancies than in the empirical data. This is very relevant from a welfare perspective. According to Hosios rule, an economy is constrained efficient when the bargaining power is equal to the elasticity of the contact function with respect to vacancies.

One important insight of our paper is that there are multiple ways of generating the same comovement between matches, unemployment, and vacancies in theoretical models. This observational equivalence applies to both the matching function and the Beveridge curve. But does it matter what happens inside the black box of job creation? We argue that it matters a lot. A model that combines both a traditional contact function and idiosyncratic productivity has very interesting implications. First, the combined model generates substantial labor market asymmetries to symmetric aggregate shocks. In the full nonlinear model,[7] large negative aggregate shocks generate stronger labor market reac-

[6]See the lower panel in Figure 6 in the Appendix for an illustration.

[7]Note that all our previous arguments were based on analytical steady state elasticities or numerical second order approximations, which are reasonable in the context of regular business cycle fluctuations. However, for very large shocks such as the Great Recession or the

tions than positive aggregate shocks of the same size. With large aggregate shocks, the curvature of the idiosyncratic productivity shock distributions matters, which we calibrate with wage data. The response of unemployment and the job-finding rate to a 5% negative productivity shock is nearly twice as large as to a positive shock of equal size. Second, the combined model provides a rationale for why matching function estimations often show a negative time trend (Petrongolo and Pissarides, 2001; Poeschel, 2012). When the selection margin is active, lower vacancy posting costs (e.g. due to new technology)[8] generate such a time trend even if the contact efficiency is constant. Intuitively, lower vacancy posting costs stimulate vacancy posting. This generates more matches via the contact margin. However, the selection margin is not directly affected and the number of extra jobs increases less than proportionally. An estimation of the matching function would therefore detect a decline in the matching efficiency. However, in a model with idiosyncratic shocks, this negative time trend would not be a sign for a worrisome deterioration of labor market conditions.

The rest of the paper proceeds as follows. Section 2 derives a simple model with contact function, free entry of vacancies and idiosyncratic productivity shocks for new jobs. Section 3 provides an analytical expression for the equilibrium comovement of the job-finding rate and the market tightness in this framework (starting with a degenerate contact function). Section 4 first calibrates the model with a degenerate contact function. Second, it combines a non-degenerate contact function and idiosyncratic productivity shocks to match the estimated matching function. Section 5 shows that the combined model has very interesting labor market dynamics and implications. Section 6 concludes.

2 A Simple Model

2.1 Model Environment

Our economy is populated with a continuum of workers who can either be employed or unemployed. Employed workers are separated with an exogenous probability ϕ. Unemployed workers search for a job. We assume that they get in contact with a firm with probability $p_t \leq 1$. The contact probability may either be driven by a standard contact function as in Mortensen and Pissarides (1994) and Pissarides (2000) or it may be degenerate, as standard in search models (e.g. McCall, 1970; Mortensen, 1987) or as assumed in selection models (Brown et al., forthcoming; Lechthaler et al., 2010).

When unemployed workers get in contact with a firm, they draw an idiosyncratic productivity realization ε_{it}, i.e. some workers are more productive than others. This nests the case of search and matching models where endogenous separations hit before production takes place (e.g. Krause and Lubik, 2007) or the stochastic job matching model (Pissarides, 2000, chapter 6). Firms will

Great Depression the full nonlinear model is more appropriate.

[8] There is anecdotal and quantitative support for our story that vacancy posting costs have actually declined (see e.g. Kuhn and Mansour, forthcoming).

only hire workers when the productivity realization, ε_{it}, is at least as large as the cutoff productivity $\tilde{\varepsilon}_t$, that makes a firm indifferent between hiring and not hiring. For illustration purposes, we start with a model where idiosyncratic productivity shocks are only drawn in the first period of employment. However, this assumption is without loss of generality. We show analytically in the Appendix that we obtain the same results for two additional polar cases. First, when we assume that the idiosyncratic shock, ε, is redrawn every period (both for new contacts and for existing matches) and *iid* across workers and time. Second, when we assume that ε is only drawn when a new contact is made, but it remains the same for the entire period of employment.

Firms have to post vacancies to obtain a share of the economy wide applicants (namely, the firm's vacancy divided by the overall number of vacancies, which is determined by a free-entry condition). With a traditional contact function, more vacancies lead to more contacts. By contrast, with a degenerate contact function, more vacancies do not lead to more contacts.

2.2 Contacts

Contacts are assumed to follow a Cobb-Douglas function with constant returns to scale (CRS)

$$c_t = \mu v_t^{\gamma} u_t^{1-\gamma}, \tag{1}$$

where c_t denotes the overall number of contacts, μ is the contact efficiency, and u_t and v_t are beginning of period unemployment and vacancies respectively. The contact probability for a worker is thus $p_t = \mu\theta_t^{\gamma}$ and the contact probability for a firm $q_t = \mu\theta_t^{\gamma-1} = p_t/\theta_t$, where $\theta_t = v_t/u_t$ denotes market tightness. With $\gamma = 0$ the contact function is degenerate in the sense that more vacancies do not lead to more contacts and jobs in the aggregate.

2.3 The Selection Decision

Once a contact between a searching worker and the firm has been established, firms decide whether to hire/select a particular worker or not. There is a random worker-firm pair specific idiosyncratic productivity shock, ε_{it}, which is *iid* across workers and time[9], with density function $f(\varepsilon_t)$ and the cumulative distribution $F(\varepsilon_t)$. ε_t is observed by the worker and the firm. Thus, the expected discounted profit of hiring an unemployed worker, $\pi_t^E(\varepsilon_t)$, is equal to the current aggregate productivity plus the idiosyncratic productivity shock, ε_t, minus the current wage (which may be a function of ε), $w_t(\varepsilon_t)$, plus the expected discounted future profits:

$$\pi_t^E(\varepsilon_t) = a_t + \varepsilon_t - w_t(\varepsilon_t) + \delta(1-\phi) E_t(\pi_{t+1}), \tag{2}$$

with

[9]Due to the *iid* assumption, we abstract from the worker-firm pair specific index i from here onward.

$$\pi_t = a_t - w_t + \delta(1-\phi) E_t(\pi_{t+1}), \tag{3}$$

where δ is the discount factor and ϕ is the exogenous separation probability. In the baseline scenario, incumbent worker-firm pairs are not subject to idiosyncratic productivity shocks, i.e. there is no ε_t and the wage for existing worker-firm pairs is not dependent on any idiosyncratic shock realization.

The firm selects an unemployed worker whenever there is an expected positive surplus:

$$\tilde{\varepsilon}_t = w_t(\varepsilon_t) - a_t - \delta(1-\phi) E_t(\pi_{t+1}). \tag{4}$$

Thus, the selection rate is given by:

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f(\varepsilon)\, d\varepsilon. \tag{5}$$

2.4 Vacancies

As in Pissarides (2000, chapter 1), we assume that each vacancy corresponds to one firm. For entering the market, firms have to pay a fixed vacancy posting cost κ. The value of a vacancy Ψ is

$$\Psi_t = -\kappa + q_t \eta_t E_t\left[\pi_t^E | \varepsilon_t \geq \tilde{\varepsilon}_t\right] + (1 - q_t\eta_t)\Psi_t, \tag{6}$$

where $q_t = c_t/v_t$ is the probability that a vacancy leads to a contact (i.e. overall contacts divided by overall vacancies). Thus:

$$\begin{aligned}\Psi_t = -\kappa + q_t\eta_t &\left(a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty}(\varepsilon_t - w(\varepsilon_t)) f(\varepsilon_t)\, d\varepsilon_t}{\eta_t} + \delta(1-\phi) E_t(\pi_{t+1})\right) \\ &+ (1-q_t\eta_t)\Psi_t,\end{aligned} \tag{7}$$

Firms will post vacancies up to the point where the value is driven to zero (free entry condition), i.e.

$$\frac{\kappa}{q_t\eta_t} = a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty}(\varepsilon_t - w(\varepsilon_t)) f(\varepsilon_t)\, d\varepsilon_t}{\eta_t} + \delta(1-\phi) E_t(\pi_{t+1}). \tag{8}$$

It is straightforward to see that the model nests the standard matching model where all workers are selected (i.e. with no role for idiosyncratic shocks), by setting $\eta_t = 1$ and $\varepsilon_t = 0$. In this case, the right hand side is $a_t - w_t + \delta(1-\phi) E_t(\pi_{t+1}) = a_t - w_t + \delta(1-\phi) E_t \frac{\kappa}{q_{t+1}}$.

Note that even in the case of a degenerate contact function, it is perfectly rational for firms to enter the market. Under a positive aggregate productivity shock, the expected returns of hiring a worker increase. Thus, more firms will enter the market to compete for these profits until the free-entry condition holds again. This makes vacancies procyclical in response to aggregate productivity shocks.

2.5 Wages

We assume that a larger idiosyncratic productivity shock leads to a proportionally larger wage:

$$w(\varepsilon_t) = \bar{w}_t + \alpha \varepsilon_t, \tag{9}$$

where α is the proportional component. $\bar{w}_t$ is the wage net of contemporaneous ε_t realization (i.e. the wage that holds in future periods if there are no future idiosyncratic shocks). $\bar{w}_t$ may be a function of current and future variables such as aggregate productivity, market tightness, future job-finding rates, or unemployment benefits, but not the current idiosyncratic productivity realization. Thus, our wage equation is very general and also nests standard Nash bargaining, i.e. a privately efficient wage formation.[10]

2.6 Employment

We assume an economy with a fixed labor force L, which is normalized to 1. The employment stock is thus equal to the employment rate, n. The employment dynamics in this economy is determined by

$$n_{t+1} = (1 - \phi - p_t \eta_t) n_t + p_t \eta_t. \tag{10}$$

The number of searching workers is equal to the number of unemployed workers at the beginning of period t, i.e.

$$u_t = 1 - n_t. \tag{11}$$

2.7 Labor Market Equilibrium

The labor market equilibrium consists of the contact function (1), the equations for firms' profits (3), the productivity cutoff point (4), the selection rate (5), the vacancy free entry condition (8), the wage equation (9), the employment dynamics equation (10) and the definition of unemployment (11).

3 Analytics

This section shows analytically that the model with degenerate contact function and idiosyncratic shocks generates an equilibrium comovement of matches, unemployment, and vacancies that is observationally equivalent to a model with a traditional contact function but degenerate selection. We prove for a degenerate contact function that the elasticity of the job-finding rate with respect to market tightness, which is equivalent to the weight on vacancies in an estimated matching function, is described by the first derivative of the expected idiosyncratic productivity shock. We illustrate the implications for different

[10] In the standard search and matching model (i.e. with degenerate selection), this corresponds to the well-known wage equation $w(\varepsilon_t) = \alpha(a_t - \varepsilon_t + \kappa\theta_t) + (1-\alpha) b$.

distributions and cutoff points. In a next step, we show how our results differ for a non-degenerate contact function. Finally, we discuss the robustness of our results. To obtain analytical results, all derivations in this section are based on a steady state version of our model, i.e. we assume that there is no aggregate uncertainty and we analyze the reaction of the job-finding rate and vacancies with respect to permanent changes in aggregate productivity.

These assumptions will be relaxed in Section 4.

3.1 Degenerate Contact Function

For illustration purposes, we start with a degenerate contact function ($\gamma = 0$). The equilibrium comovement between the job-finding rate and market tightness can be described by the three equations below plus one equation for the wage formation,[11] namely the hiring cutoff point $\tilde{\varepsilon}$, the job-finding rate $p\eta$, and the market tightness, defined as $\theta = v/u$:

$$\tilde{\varepsilon} = \frac{\bar{w} - a}{(1 - \delta (1 - \phi)) (1 - \alpha)}, \tag{12}$$

$$p\eta = p \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) \, d\varepsilon, \tag{13}$$

$$\theta = \frac{p\eta}{\kappa} \left(\frac{a - \bar{w}}{(1 - \delta (1 - \phi))} + \frac{(1 - \alpha) \int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) \, d\varepsilon}{\eta} \right), \tag{14}$$

or simplified

$$\theta = (1 - \alpha) \frac{p\eta}{\kappa} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) \, d\varepsilon}{\eta} - \tilde{\varepsilon} \right). \tag{15}$$

In standard empirical matching function estimations, the job-finding rate (jfr) is regressed on market tightness, where β_1 shows how strongly the job-finding rate and market tightness comove in percentage terms and ψ_t is an error term, namely:

$$\ln jfr_t = \ln p_t \eta_t = \beta_o + \beta_1 \ln \theta_t + \psi_t. \tag{16}$$

In a standard search and matching model with degenerate selection and a Cobb-Douglas CRS specification, β_1 is likewise the elasticity of matches with respect to vacancies.

The job-finding rate and market tightness are both dependent on aggregate productivity. By deriving the elasticity of the job-finding rate with respect to productivity and by deriving the elasticity of market tightness with respect to

[11] We suppress this equation for expositional convenience. We show below that the precise form of wage formation is irrelevant. Only one very general condition is required in this context.

productivity,[12] we obtain an analytical expression for the empirical elasticity of the job-finding rate with respect to market tightness,[13] namely:

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = \frac{-f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} a}{\eta}, \tag{17}$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}}. \tag{18}$$

Thus:

$$\frac{\partial \ln (p\eta)}{\partial \ln \theta} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) \, d\varepsilon}{\eta} - \tilde{\varepsilon} \right). \tag{19}$$

Interestingly, in equation (19) neither the wage nor the first derivative of the cutoff with respect to productivity, $\frac{\partial \tilde{\varepsilon}}{\partial a}$, shows up. We explicitly take into account the first derivative of the wage with respect to productivity in our derivations in the Appendix (i.e. we consider that different wage formation regimes or parameters lead to different wage reactions). However, as long as $\frac{\partial w}{\partial a} < 1$,[14] this term drops out in the analytical derivations. Thus, our results hold for a very broad set of wage formation mechanisms.

It is important to emphasize that the relationship between the job-finding rate and market tightness in equation (19) is not causal. With a degenerate contact function, more vacancies do not lead to more contacts and jobs in aggregate. However, the model with idiosyncratic shocks generates a positive comovement between the job-finding rate and market tightness in equilibrium. Thus, equation (19) shows that it is not necessary to assume a standard contact function to obtain a positive comovement of these two variables.

To put it differently: If a model with degenerate contact function, idiosyncratic shocks and free entry of vacancies is simulated and a matching function estimation is performed based on the simulated data, the estimation will generate a positive coefficient on vacancies, although the underlying contact function has a weight on vacancies of 0. We verify this numerically in Section 4.

What is the underlying economic mechanism and intuition? When aggregate productivity rises, firms have an incentive to hire workers with lower idiosyncratic productivity. Thus, the job-finding rate is procyclical. When productivity rises, this also increases the returns from posting a vacancy. Thus, firms compete for the larger pie of profits, more of them enter the market and thus increase the market tightness in the economy. These two mechanisms combined lead to the

[12] See Technical Appendix for details.

[13] Note that Merkl and van Rens (2012) show that the job-finding rate and its dynamics are isomorphic in a model with idiosyncratic training costs (under a Pareto distribution) and in the search and matching model. However, their model does not contain any vacancies and is thus silent on the shape of the matching function.

[14] If the wage comoves one to one with productivity (in absolute terms, e.g. $w_t = a_t + \varepsilon_t$), the job-finding rate and vacancies would have a zero elasticity with respect to productivity. This trivial case is excluded from our analysis.

positive equilibrium comovement between the job-finding rate and the market tightness that we observe in equation (19).

Interestingly, the elasticity of the job-finding rate with respect to market tightness derived in equation (19) has an economic interpretation. It corresponds to the first derivative of the conditional expectation of idiosyncratic productivity with respect to the cutoff point, i.e.:

$$\frac{\partial \ln (p\eta)}{\partial \ln \theta} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon} \right) = \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}. \tag{20}$$

Thus, up to a first order Taylor approximation, the comovement between the job-finding rate and the market tightness is determined by equation (20), and thus only depends on the distribution of idiosyncratic productivity and the position of the cutoff point. The quality of this approximation will be checked numerically in Section 4.

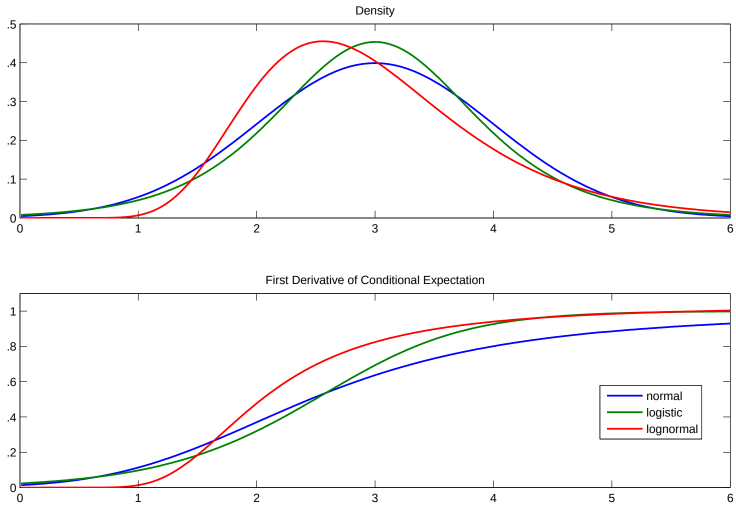
Figure 1 illustrates the prediction of our model for different idiosyncratic shock distributions. The upper panel plots the density functions of ε for normal, logistic and lognormal distributions. The lower panel plots equation (20) evaluated at the corresponding cutoff point (on the abscissa). As shown above, this value corresponds to the elasticity of the job-finding rate with respect to market tightness, i.e. the implied weight on vacancies in an estimated matching function. Two observations are worth pointing out. First, for these standard distributions the weight on vacancies is always between 0 and 1. Second, when the cutoff point is at the left hand side of the peak of the density function, the first derivative of the conditional expectation (i.e. the weight on vacancies) is smaller than 0.5, while it is larger than 0.5 on the right hand side. This will be important later on when we compute the idiosyncratic shock distribution based on the empirical wage distribution.

Why is the elasticity of matches with respect to vacancies larger than 0.5 on the right hand side of the peak of the density function and smaller than 0.5 on the left hand side? The reason is that market tightness is driven by the free entry condition of vacancies (see equation (15)). When aggregate productivity increases, workers with lower idiosyncratic productivity are hired, i.e. the hiring cutoff moves to the left. On the left hand side of the peak of the density function, a small mass of additional workers with low idiosyncratic productivity will be hired. Thus, vacancies move by a lot because the additional hiring activity does not lower the average idiosyncratic productivity by much. Large vacancy movements relative to the job-finding rate lead to a small estimated coefficient in equation (16).

3.2 Traditional Contact Function

Now, let us assume a traditional contact function with $0 < \gamma < 1$. In this case, the probability for a worker to make a contact ($p = c/u$) depends on aggregate productivity. In our real business cycle framework, we thus expect a procyclical movement of the contact rate ($\frac{\partial p}{\partial a} > 0$).

Figure 1: Predicted matching coefficients for standard distributions



Notes: Density function and first derivative of conditional expectation for different standard distributions (namely, normal, logistic, and lognormal). For comparability reasons, the variance is normalized to 1 and the mean is set to 3 (the lognormal distribution requires a positive mean).

To analyze the implications of this modification, we recalculate the elasticities of the job-finding rate and market tightness with respect to productivity:

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = -\frac{f(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\eta} + \frac{\partial \ln p}{\partial \ln a}, \tag{21}$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}} + \frac{\partial \ln p}{\partial \ln a}. \tag{22}$$

The elasticities of the job-finding rate and market tightness with respect to productivity are the elasticities with a fixed contact rate plus the elasticity of the contact rate with respect to productivity. Defining $\xi_{jfr/\theta} = \frac{\partial \ln jfr}{\partial \ln \theta}$, $\xi_{\eta/a} = -\frac{f(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\eta}$, $\xi_{\theta/a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}}$ and $\xi_{p/a} = \frac{\partial \ln p}{\partial \ln a}$, we can write the elasticity of the job-finding rate with respect to market tightness as:

$$\xi_{jfr/\theta} = \frac{\xi_{\eta/a} + \xi_{p/a}}{\xi_{\theta/a} + \xi_{p/a}}. \tag{23}$$

Taking first derivatives allows us to see how this elasticity changes with a procyclical contact rate:

$$\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{p/a}} = \frac{\xi_{\theta/a} - \xi_{\eta/a}}{\left(\xi_{\theta/a} + \xi_{p/a}\right)^2}. \tag{24}$$

In the previous section, we have shown that for a variety of standard assumptions and cutoff points, the elasticity of the selection rate with respect to market tightness is smaller than 1 ($\frac{\xi_{\eta/a}}{\xi_{\theta/a}} < 1$). Thus, the numerator of (24) is positive, and $\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{p/a}} > 0$, i.e. a stronger procyclicality of the contact rate increases the weight of vacancies in an estimated matching function. In different words: If both a traditional contact function and idiosyncratic shocks are important for match formation, both of them contribute to a positive weight on vacancies in an estimated matching function.

3.3 Robustness Checks

The model with a degenerate contact function is most similar to the selection model by Brown et al. (forthcoming). However, our results hold for a broad set of models that contain idiosyncratic productivity shocks. The Technical Appendix shows that we obtain the same analytical results in a search and matching model with endogenous separations (where iid shocks hit every period) and in a model where idiosyncratic shocks are drawn for the entire span of employment. In all these cases, the elasticity of the job-finding rate with respect to market tightness is given by equation (20).[15]

[15] For multiplicative idiosyncratic shocks, the results only change insofar as the equilibrium comovement is determined by the derivative of the logarithm of the conditional expectation of the shock with respect to the logarithm of the cutoff point.

4 Theory and Evidence

Our analytical results put us in a position to use wage data as a proxy for the idiosyncratic productivity shocks to calculate the model implied weight on vacancies. We first establish a reference point by estimating an empirical matching function. We then proceed to calibrate the steady state model with individual wage data. For a degenerate contact function we can thus directly calculate the elasticity of matches with respect to vacancies implied by the model. We also simulate the dynamic model and estimate a matching function from the simulated data. The simulation allows us to test for the quality of our steady state approximation and for the constant returns to scale assumption. Finally, we add a traditional contact function to the model and explore how it has to look like in order to obtain the same elasticity of matches with respect to vacancies as in our empirical estimations.

For all these exercises, we use administrative labor market data for Germany (see e.g. Dustmann et al., 2009; Schmieder et al., 2012). The German administrative database has several advantages over commonly used U.S. data. First, it provides actual labor market transitions on a daily basis. This means that we do not have to construct labor market flows from unemployment, employment and duration data and we do not face the problem of a time aggregation bias (see, e.g., Shimer, 2005, 2012; Nordmeier, 2014). Second, we can use several control variables that might influence the search and matching process. Third, we can observe wages for new matches. Importantly, these wages are from the same database that we construct our flow data from. Finally, we have real vacancies instead of a job advertising index for longer time series.[16]

4.1 Empirical Matching Function

We estimate a standard Cobb-Douglas CRS matching function for the German labor market. Thus, we regress the job-finding rate jfr on labor market tightness θ, a linear time trend t, and a shift dummy, d_{2005}, which accounts for the redefinition of unemployment in course of the so-called Hartz reforms:[17]

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \beta_2 t + \beta_3 d_{2005} + \psi_t, \tag{25}$$

where the job-finding rate at time t denotes all matches during month t over the beginning-of-month-t unemployment stock and market tightness refers to the beginning-of-month-t vacancy to unemployment ratio. The coefficient β_1 represents the matching elasticity with respect to vacancies and thus is the relevant reference point for our numerical exercises below.

We further include observable control variables to account for the effects of a changing unemployment pool and different search intensities on the aggregate

[16] See Appendix C for a detailed data description.

[17] In 2005, the official unemployment measure in Germany was extended to include recipients of former social assistance.

Table 1: Matching function estimations

jfr	(1)	(2)
constant	-2.3498***	-4.3403**
log θ	0.2458***	0.3463***
t	-0.0003***	-0.0054**
d_{2005}	-0.0798***	-0.0766
controls	no	yes
adj. R^2	0.5134	0.6162
DW statistic	1.3664	1.8407
CRS t-statistic	1.0074	1.6141

Note: OLS estimations (1993-2007). ***, ** and * indicate significance at the 1%, 5% and 10% levels (Newey-West standard errors). Control variables: *long* , *young*, *old*, *low-skilled*, *high-skilled*, *foreign*, *female*, *married*, *child*, *UB I*.

matching probability.[18] Table 1 displays the estimation results of the matching function specification with and without control variables. Both estimations show a fairly good fit in terms of the adjusted R^2 measure. However, the Durbin-Watson statistic indicates that it is important to control for the composition of the unemployment pool because this specification overcomes the positive autocorrelation in the error term.[19] The point estimate of β_1 in our preferred specification is 0.35 and the 95% confidence interval spans from 0.23 to 0.46. The matching elasticities of vacancies and unemployment are thus roughly one third and two thirds, respectively. These results are in line with the survey of matching function estimations by Petrongolo and Pissarides (2001). Moreover, the constant returns to scale assumption cannot be rejected. We also find a significantly negative time trend in matching efficiency, which is a common result in matching function estimations (see e.g. Petrongolo and Pissarides, 2001; Poeschel, 2012).

4.2 Model Implied Comovement

How closely does a model with a degenerate contact function but with free entry of vacancies and idiosyncratic productivity match the estimated matching function? To test for this, we use the wage distribution of the German administrative data for new matches to infer the shape of the actual distribution of idiosyncratic productivity.[20]

[18] It is well known that there is duration dependence of individual job-finding rates. Recent research by Hornstein (2012) and Barnichon and Figura (2011) suggests that this may be due to composition effects of the unemployment pool. Katz and Meyer (1990) find evidence for an influence of unemployment benefit receipt on workers' job acceptance behavior.

[19] We also performed an IV estimation to account for an endogeneity problem in specification (1), but the coefficients did not change notably. The results of the IV estimation are available on request.

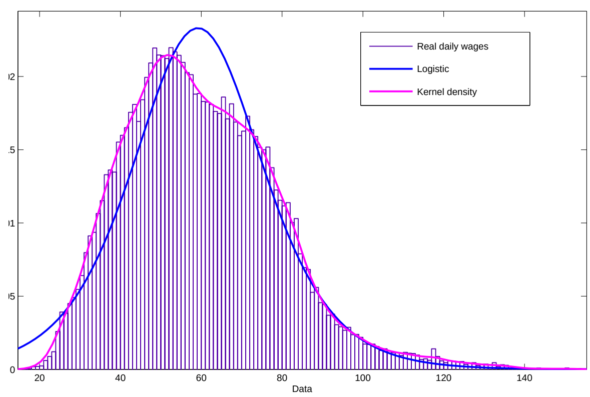
[20] See Appendix C for a description of the wage data.

We have assumed that wages are formed according to $w(\varepsilon_t) = \bar{w}_t + \alpha\varepsilon_t$, where $\bar{w}_t$ contains aggregate components (e.g. productivity and market tightness) and ε_t represents match-specific idiosyncratic productivity. Given that this wage formulation nests Nash bargaining, this is a standard assumption. We will use the proportionality between contemporaneous idiosyncratic productivity and the wage for our empirical analysis.

We focus on wages of a homogeneous reference group as we are not interested in wage differentials that can be explained by observable characteristics such as education, gender or unemployment history. We choose the following baseline group: male, German, not married, no children, age 25-55, medium skilled and short-term unemployed (before being hired). For comparability reasons, we restrict our attention to full-time employment.[21] As a robustness check we report results for various other group compositions in the Appendix.

Our proportionality assumption allows us to infer the shape of the distribution of the idiosyncratic productivity directly from the wage data. In line with our baseline model, we only use wages at the start of an employment spell.[22] The histogram of wages is displayed in Figure 2. When equation (9) holds, idiosyncratic productivity is just a scaled version of this distribution.[23]

Figure 2: Distribution of real daily wages



Notes: Wages are real daily gross wages of new job entrants with the following characteristics: male, German, full-time employed, not married, no children, age 25-55, mediums skilled (Hauptschule or Realschule plus vocational training), short-term unemployed.

We only observe part of the distribution, namely the realizations of productivity that result in a hire. We assume that the distribution is smooth at the

[21] Controlling for year fixed effects does not alter our results.

[22] This has the additional advantage that we do not have to control for tenure.

[23] Note that the scaling does not affect the shape of the distribution at the respective cutoff point and does not affect the calculation of equation (20).

hiring cutoff. This allows us to be otherwise agnostic about the part of the distribution that we do not observe.

According to our model, the relevant cutoff productivity would be determined by the lowest reported wage. In our data this wage is just below 17 Euro per calender day. In order to rule out that our results are driven by outliers we set the cutoff point at the 1st, 5th and 10th percentiles of the wage distribution. These correspond to daily gross wages of 26, 33 and 37 Euro respectively. It is quite standard in the literature to use the 10th percentile as a measure for the lowest wage. For example, the 50th to 10th percentile ratio is a conventionally used measure for wage dispersion (see e.g. Hornstein et al., 2011).

We fit the data non-parametrically using a kernel density estimation with a normal kernel (see Figure 2) and numerically calculate the derivative of the conditional expectation using equation (20). This gives us the numbers for the elasticity of matches with respect to vacancies. Based on the wage distribution, it is 0.12, 0.28, and 0.38 for the 1st, 5th, and 10th percentile, respectively (see Table 2). These model based numbers already come remarkably close to our empirical data estimate of 0.35. This is particularly interesting, given that these results are based on a degenerate contact function so far, where vacancies do not affect the aggregate number of contacts.

Table 2: Weight on vacancies, based on steady state approximation

	1st percentile	5th percentile	10th percentile
$\log \theta$	0.12	0.28	0.38

Note: Results are calculated numerically from the non-parametric fit of the distribution using equation (20).

Before we move to the dynamic analysis, it is worthwhile discussing some potential pitfalls of our analysis:

First, wage differentials may be driven by other factors than observables or idiosyncratic productivity, namely luck. This would change our wage equation to $w(\varepsilon_t) = \bar{w}_t + \alpha\varepsilon_t + \iota_t$, where ι_t is the luck component. But as long as there is no systematic correlation between ε_t and ι_t, the luck component simply adds noise to our analysis, but the results remain valid.

Second, collective bargaining is still the predominant wage formation mechanism in Germany. If collective bargaining prevents that idiosyncratic productivity differentials show up in the wage, our analysis is not valid. However, collective bargaining only defines a lower bound for the wage. If a worker with certain characteristics is particularly productive, firms can easily pay a higher wage. In addition, firms have a certain discretion into which pay scale they want to classify a particular worker (i.e. a worker with a lower idiosyncratic productivity can be assigned to a lower pay scale). Beyond this, collective bargaining has lost importance over the last decades. However, controlling for year fixed effects in our wage distribution does not alter our results. This lends support to our view that collective bargaining does not matter a lot in the context of our paper.

Third, we may have chosen our homogeneous reference group inappropriately. In particular, we may have defined it too broadly. Here, we face of course a trade off between the number of observations and a narrower group definition.[24] Therefore, we repeat the dynamic simulation for a set of different reference groups (in particular a finer differentiation along age and education). The results can be found in the Appendix and are fairly similar. We are therefore confident that our results are not driven by the choice of the reference group. In addition, the results in the Appendix show that our preferred reference group represents an intermediate case with respect to the range of estimates.

4.3 Dynamics

So far, the results have been based on our comparative static equation. In order to test for the validity of our results out of steady state, we now simulate the model with shocks to aggregate productivity. This also allows us to check whether non-constant returns to scale are present in the simulated data. Most importantly, we can use the dynamic simulation to quantitatively assess the interplay between a traditional contact function and idiosyncratic productivity.

For the dynamic simulation, we need to parametrize our model. In particular, we need to define a functional form for the idiosyncratic productivity distribution. We therefore fit several standard distributions to the data, i.e. we choose the parameters of the distributions that give the best fit of our data in terms of maximum likelihood. We choose the logistic distribution because it has the best fit. Figure 2 displays the distribution of real daily gross wages and the corresponding logistic distribution. The match is reasonably good especially near the cutoff.[25] We provide all other details on the parametrization in Appendix D. We simulate the model 1000 times with aggregate productivity governed by a first-order autocorrelation process. The simulation is based on a second-order Taylor approximation.[26] Each time we use 180 periods corresponding to the time span used for our empirical matching function estimation.

Again, we estimate a Cobb-Douglas CRS matching function:

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \psi_t. \qquad (26)$$

Table 3 compares the estimated coefficient β_1 to the comparative static results when we use equation (20). Again, we report results for different cutoff points depending on the chosen percentile for the lowest wage. The numbers from the simulation exercise and the comparative static exercise are literally the

[24] In addition, if we choose a too narrow subgroup, it may be difficult to make an inference for the aggregate matching function.

[25] We enforce a specific mass and shape for the part of the wages that we do not observe. This is without loss of generality as the non-observable part of the distribution does not affect our results as long as there is no sharp discontinuity in the vicinity of the cutoff. We are therefore confident that the logistic distribution provides a reasonable approximation for our purposes.

[26] This explicitly allows for some nonlinearities not covered by our analytical steady state results.

same. This shows that our comparative statics is a very good approximation for the dynamic exercise. Note that the discrepancy between the results in this section and the previous section only stem from the imperfect fit of the logistic distribution compared to the non-parametric fit.

Table 3: Matching function based on simulation (degenerate contact function)

	1st percentile	5th percentile	10th percentile
Simulation result			
constant	-2.66	-2.47	-2.32
log θ	0.14	0.22	0.28
Simulation result (unconstrained)			
constant	-2.65	-2.48	-2.32
log U	0.85	0.78	0.72
log V	0.14	0.22	0.28
Steady State prediction			
constant	-	-	-
log θ	0.14	0.22	0.28

Note: Steady State results are calculated numerically from the logistic fit of the distribution using equation (20). The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

We further analyze whether we have artificially imposed the CRS assumption in our comparative static exercise. We estimate the following unconstrained matching function:

$$\log m_t = \beta_0 + \beta_1 \log v_t + \beta_2 \log u_t + \psi_t, \tag{27}$$

where m_t denotes all matches in period t. Table 3 shows that the sum of estimated coefficients $(\beta_1 + \beta_2)$ is virtually 1. The estimated coefficients are also statistically significant at the 1% level in every single run of the simulation.[27]

Interestingly, when we estimate other functional forms such as CES, the Cobb-Douglas specification is confirmed. As a robustness check we have also performed IV estimations using the lagged value of market tightness as an instrument. This does not alter our results.[28]

4.4 Traditional Contact Function and Idiosyncratic Productivity

Finally, the dynamic simulation puts us in a position to combine idiosyncratic productivity with a traditional contact function. Hence, we assume that the contact probability is defined by $p_t(v_t, u_t) = \mu\theta_t^\gamma$ with $0 < \gamma < 1$, i.e. the job-finding rate is not only driven by the movement of the cutoff point for idiosyncratic productivity but also by a procyclical contact rate.

[27] We do not report t-statistics as means over t-values would not have a meaningful interpretation.

[28] Results are available from the authors on request.

We analyze how much of the empirical comovement between matches, unemployment, and vacancies is due to the contact function and how much is due to idiosyncratic productivity. For this purpose, we again use the logistic distribution for wages (see Table 3) and determine the contact elasticity γ so as to get an overall elasticity of matches with respect to vacancies of 0.35 as found in the empirical matching function. The results are shown in Table 4.

Table 4: Weight on vacancies, dynamic simulations with different contact function specifications.

	1st percentile	5th percentile	10th percentile
Matching function correlation with $\gamma = 0$			
$\log \theta$	0.14	0.22	0.28
Calibrated elasticity of the contact function (γ)			
$\log \theta$	0.24	0.17	0.09
Combined matching function correlation			
$\log \theta$	0.35	0.35	0.35

Note: The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

Our numerical results are in line with our theoretical results from Section 3.2. When a procyclical contact rate and idiosyncratic productivity are combined, this leads to a larger weight on vacancies in an estimated matching function. The first line in Table 4 shows the results for the model simulation with idiosyncratic productivity but with a degenerate matching function. The second line shows the elasticities of a traditional contact function that would correspond to the overall elasticity of matches with respect to vacancies if there was no idiosyncratic productivity. The third line shows the combination of the two mechanisms. Interestingly, the resulting estimated matching function has a weight on vacancies which is roughly equal to the sum of the weight on vacancies in the two cases.

Our calibration suggest that at least one third of the observed elasticity of matches with respect to vacancies is actually driven by idiosyncratic productivity. When we use the 10th percentile of the wage distribution, 80% of the weight on vacancies are driven by idiosyncratic shocks. Thus, our exercise shows that there is a potentially large bias in standard matching function estimations if idiosyncratic productivity plays a role. Hence, in many model applications, the contact functions may be misspecified, assigning too large a role for vacancies in the process of match formation.

Interestingly, when we compare the combined model in Table 4 to a standard search and matching model with degenerate selection but with a weight on vacancies in the contact function of 0.35, it turns out that the two models also produce the same Beveridge curve. Thus, observational equivalence does not only hold for the matching function but also for the Beveridge curve.

Given this observational equivalence, it is worthwhile emphasizing that the elasticity of the contact function with respect to vacancies is very important in search and matching models. Hosios (1990) shows that matching models with a CRS matching function are constrained efficient when firms' bargaining power in Nash bargaining is equal to the weight on vacancies in the contact function. We have shown that using the weight on vacancies from empirical estimations is inappropriate in the presence of idiosyncratic productivity shocks and leads to a misspecification. Against the background of Hosios rule, welfare implications of policy interventions may thus be judged incorrectly.

5 Why the Driving Forces of the Matching Function Matter

The theoretical section proves that idiosyncratic productivity shocks and a vacancy free entry condition alone generate an equilibrium comovement between matches, vacancies, and unemployment. We have used wage data to show that this may generate a large part of the observed comovement between these variables in the German data. Given that there are multiple ways of obtaining the observed comovement between matches, vacancies, and unemployment, does it matter whether the labor market is modeled with a contact function only (i.e. degenerate selection) or with a combined model (with both a standard contact function and idiosyncratic productivity shocks)? At the end of the previous section, we have briefly argued that this matters from a normative perspective (Hosios rule). This section shows that it also matters from a positive perspective. First, a model with idiosyncratic shocks and vacancy free entry generates highly asymmetric labor market reactions to business cycles. Second, we provide a novel explanation for the puzzling fact that many empirical matching function estimations document a decline of the matching efficiency over time.

5.1 Business Cycle Asymmetries

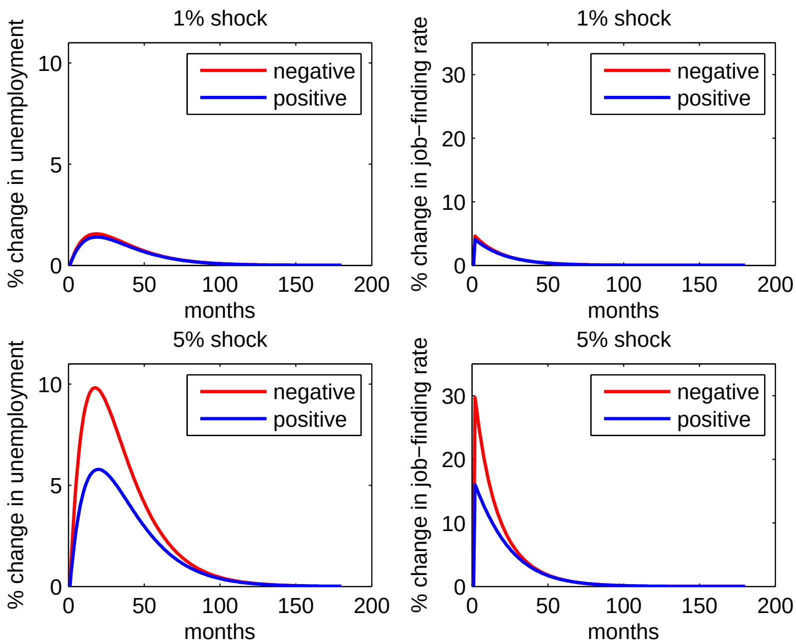
The analytical results in Section 3 are based on a steady state elasticity (i.e. a first order approximation). The numerical results in Section 4.3 use a second order approximation. Thus, in both cases higher order effects are not taken into account. Figure 3 displays the reactions of the combined model in response to a one percent (upper panel) and a five percent (lower panel) productivity shock (with autocorrelation coefficient 0.95), which are solved deterministically and fully nonlinearly.[29] In order to facilitate comparison of the quantitative responses, the responses are all in absolute terms. The upper panel shows that the nonlinear responses to a regularly sized productivity shock generates only minor asymmetries. Thus, our analytical results and the stochastic solution method appear to be appropriate approximations in a normal business cycle environment.

However, the lower panel of Figure 3 reveals large asymmetries of both the job-finding rate and unemployment to large business cycle shocks. The responses

[29]From now on all calculations for the combined model are based on the calibration for the 5th percentile of the wage distribution (see Table 4).

to a negative 5% productivity shock are almost twice as large as the responses to an equally sized positive productivity shock.

Figure 3: Response of unemployment and job-finding rate to productivity shock

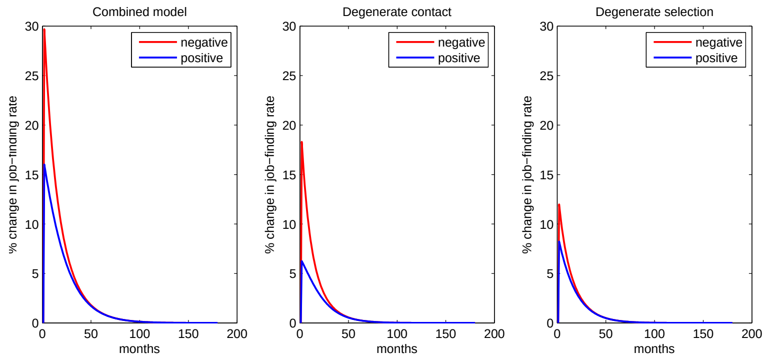


Notes: Responses are in percent deviation from steady state. For expositional convenience, the response of unemployment to a positive shock and the response of the job-finding rate to a negative shock are flipped upside down.

To isolate the driving forces of this asymmetry, we use our calibration for the 5th percentile of the wage distribution and impose a degenerate contact and degenerate selection mechanism respectively. More precisely, we assume that the contact rate (middle panel in Figure 4) and the selection rate (right panel) are constant over the business cycle. All parameter values of the model remain unchanged. Figure 4 shows that both the degenerate contact and the degenerate selection model generate some asymmetries. These asymmetries are particularly strong for the degenerate contact model. Or in different in words: the selection mechanism is the key driver for this asymmetry.

The asymmetry is straightforward to see in a model with a degenerate contact function, where the dynamics of the job-finding rate is exclusively driven by the selection rate:

Figure 4: Asymmetries with degenerate contact/selection



Notes: Responses are in percent deviation from steady state. For expositional convenience, the response of the job-finding rate to a negative shock is flipped upside down.

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f(\varepsilon)\, d\varepsilon. \tag{28}$$

Although the cutoff point $\tilde{\varepsilon}_t$ moves symmetrically over the business cycle, the selection condition generates large asymmetries. The reason is the shape of the underlying idiosyncratic productivity distribution, which we have calibrated to wage data (see Figure 2). In our calibration, a positive aggregate productivity shock moves the cutoff point to a thin part in the distribution. By contrast, a negative productivity shock pushes the cutoff point to a thick part of the distribution. This explains the strong asymmetric labor reaction to larger symmetric aggregate shocks.

Why does the contact function also generate asymmetries? This is straightforward to see with degenerate selection. In this case the dynamics of the job-finding rate is exclusively driven by the contact rate. In the steady state version, the contact rate is:

$$p = \left(\frac{a - w}{\kappa(1 - \delta(1 - \phi))} \right)^{\frac{\gamma}{1-\gamma}}. \tag{29}$$

Thus, there may be an asymmetry for the contact rate due to the exponent. Note that γ is the elasticity of the contact function with respect to vacancies. With $\gamma = 0.5$, this asymmetry would be absent (i.e. positive and negative productivity shocks would generate the same quantitative reactions, with opposite signs). In the mixed model, we have calibrated $\gamma = 0.17$. Thus, the exponent is 0.20. This generates a non-negligible additional asymmetry (see right panel in Figure 4).

Keep in mind that we have performed our entire analysis about asymmetries using the combined model and switching the contact and selection channel off respectively. If we did not take into account the selection mechanism and used the standard practice of parametrizing the contact function with parameters from our matching function estimation, we would have to set $\gamma = 0.35$. As a consequence, the exponent in equation (29) would increase from 0.20 to 0.54. The shown asymmetries would be reduced. Asymmetries in a standard search and matching model (e.g. Shimer 2005) in response to a 5% productivity shock are thus very small.[30]

To sum up, when we combine contact function and selection, the asymmetry is driven by two forces. First and most importantly, the curvature of the idiosyncratic productivity matters for the nonlinear dynamics of the selection rate. According to our calibration to wage data, this is a very powerful mechanism. Second, the behavior of the contact function becomes more nonlinear (due to the lower γ compared to a standard parametrization). To illustrate that the asymmetry in the mixed model is very meaningful, Figure 5 shows the impact reaction of the job-finding rate[31] in response to productivity shocks ranging from -8% to 8% in fully nonlinear simulations. The larger the shock, the larger is the discrepancy between the response to a negative and the response to a positive shock.

Thus, our paper establishes a mechanism for why large negative aggregate shocks generate very severe and asymmetric labor market reactions. This is certainly very interesting against the background of major recessions, such as the Great Recession. In Germany, the Great Depression and the two oil price crises might be of particular relevance here.[32] These events have at least doubled the unemployment rate in Germany and thus caused labor market effects that were a lot larger than those after strong business cycle booms.[33] In models with symmetric labor market reactions, the strong increase of unemployment in major recessions must be due to very severe aggregate shocks. By contrast, our combined model with traditional contact function and idiosyncratic productivity shocks suggests that part of the labor market responses could be due to the particularly strong propagation for large negative aggregate shocks.

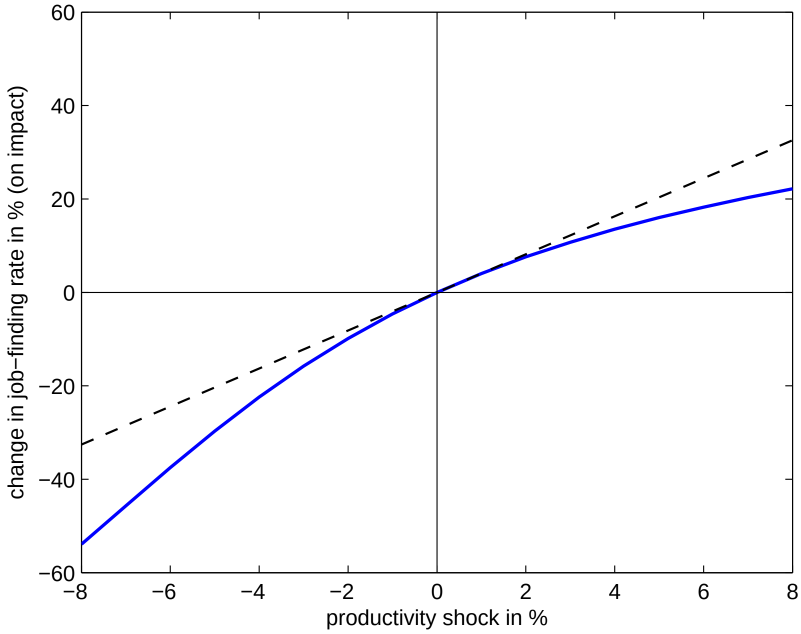
[30] First of all, as described, this is due to the larger elasticity of the contact function with respect to vacancies. Second, in Shimer's (2005) calibration, amplification to aggregate productivity shocks is small, which makes it even more difficult for asymmetries to show up (because the expected present value of a worker does not fluctuate a lot in response to aggregate productivity shocks). The combined model has stronger amplification. First, the selection mechanism generates extra amplification and extra asymmetry (as can be seen in Figure 4). Second, the contact function is also somewhat more volatile than in Shimer's calibration because the calibration of the idiosyncratic productivity shocks reduces the average size of the surplus.

[31] As the job-finding rate is a forward looking variable, the reaction is largest in the impact period.

[32] Note that the Great Recession in 2008/09 in Germany was different (i.e. no major job losses). However, this is due to exceptional factors (e.g. the internal flexibility of German firms, short-time work, the preceding wage moderation or the nature of the aggregate shock), which are outside our model and debated in the recent literature.

[33] The internet boom at the end of the 1990s caused for example a decline in unemployment from about 11.5% to 9.5%.

Figure 5: Impact response of job-finding rate to productivity shock



5.2 Vacancy Posting Costs and Time Trend in the Matching Function

There is a lot of anecdotal evidence that new technologies such as databases or the dissemination of the internet have made vacancy posting cheaper.[34] For example, the newspaper based job advertising index has collapsed because firms have started using internet based advertising (e.g. Barnichon, 2010). Obviously, firms have started substituting because there was a cheaper technology available. However, it is difficult to quantify this reduction of vacancy posting costs.

In a standard search and matching model with degenerate selection, a decline in vacancy posting costs would lead to a drop in unemployment but leave the estimated matching function unaffected.[35] In the presence of idiosyncratic shocks, the effects of a long run downward trend in vacancy posting costs are very different. To illustrate our point, we simulate a hypothetical situation in the combined model where vacancy posting costs drop by 50% over 420 periods (35 years). More specifically, we simulate the nonlinear trajectory of the economy to a new steady state, when vacancy posting costs decline linearly over time. In addition, the economy is subject to aggregate productivity shocks during the entire time span.[36] We take 180 periods (15 years) in the middle of this process and estimate a matching function based on the simulated data. This corresponds to the observation period in our empirical data and a 20% drop in vacancy posting costs during those 180 periods. We choose to estimate a subperiod of a longer time series because we consider the decline of vacancy posting costs as a long lasting process. In the estimations, we now also include a time trend, which turns out to be statistically significant and negative (see Table 5).

Table 5: Matching function with negative time trend

$\log jfr$	Coefficient	Significance (+,–)
constant	-2.13	
$\log \theta$	0.37	100% (+)
t	-0.0003	99.8% (–)

Note: The reported coefficients are means over 1000 simulations. The coefficients are OLS estimates. The third column indicates how often (in percent) coefficients are significantly above or below zero (Newey-West s.e.).

[34] It is important to distinguish ex ante hiring costs and ex post hiring costs in search and matching models. Given that vacancy posting costs are divided by the probability of filling a vacancy, these are costs prior to hiring a worker.

[35] To give an example: In a model with contact function only a drop in vacancy posting costs of 20% would lead to a reduction in steady state unemployment of 1.2 percentage points with our calibration. If both mechanisms are at work, the effect is more than halved.

[36] Productivity follows the same AR(1) process as before. As we are interested in the nonlinear solution, the shocks to productivity are deterministic (i.e. we pick one particular productivity path in each simulation).

The intuition for this negative time trend is most straightforward to understand in a model with a degenerate contact function, where the job-finding rate is solely driven by the optimal cutoff point of idiosyncratic productivity. When vacancy posting costs drop, the number of vacancies increases (see free entry condition). However, with a degenerate contact function this has no effect on matches and unemployment. Thus, the job-finding rate (matches divided by unemployment) remains constant, while the market tightness increases (vacancies divided by unemployment). In other words, it looks as if the estimated matching function has become less efficient (the job-finding rate remains constant although the market tightness has increased). If vacancy posting costs drop over a long time horizon, this leads to a negative time trend in the matching function.[37] If both a traditional contact function and the selection mechanism are at work, this effect is sustained as can be seen in Table 5.

The negative time trend can also be found in our estimation in Table 1. Interestingly, a negative time trend is a common feature of matching function estimations. According to Poeschel (2012), the sudies surveyed in Petrongolo and Pissarides (2001) that include a time trend "clearly suggest that there is a highly significant negative time trend, implying that labour market performance appears to deteriorate over time." For Germany, Fahr and Sunde (2004) have documented a negative time trend.[38]

Our paper shows that the decline in ex-ante hiring costs due to new technologies rationalizes why many matching function estimations may generate a negative time trend. In the absence of idiosyncratic productivity shocks, this negative time trend would be a sign for a worrisome instability of the contact function and a deteriorating labor market performance. We provide an explanation how a stable contact function and an estimated negative time trend can be reconciled. As long as there are no reliable proxies for the development of vacancy posting costs over time, it is impossible to control for this omitted variable bias, which is captured by the time trend. However, our paper provides an explanation that makes the observed time trend less worrisome.

[37] The coefficient on market tightness slightly deviates from 0.35. This is due to the fact that we simulate the nonlinear transition from one steady state to another. The selection rate is slightly lower in the final steady state, the weight on vacancies thus higher. Intuitively, due to lower vacancy posting costs, firms can afford to be pickier.

[38] Note that some empirical studies suggest that there was a recent increase of matching efficiency in Germany (e.g. Fahr and Sunde (2009); Klinger and Rothe (2012) or Hertweck and Sigrist (2013)). This is often attributed to the recent German labor market reforms. In contrast to the time trend, which is found in many different studies and for different observation periods, the labor market reforms seem to have implied a permanent upward shift of the matching efficiency.

6 Conclusion

We show that a wide class of models with degenerate contact function and idiosyncratic productivity (selection) generates a positive equilibrium relationship between matches on the one hand and unemployment and vacancies on the other hand. This comovement is Cobb-Douglas and constant returns. A combined model with traditional contact function and idiosyncratic productivity shocks (calibrated to wage data) has interesting implications such as asymmetric responses to large aggregate shocks.

Our paper provides important insights for future theoretical and empirical research. We have focused on a quantitative analysis with a careful calibration to high quality German data. One of our implications is that under the presence of idiosyncratic productivity shocks, the actual elasticity of the contact function with respect to vacancies is much lower than the number resulting from matching function estimations. Against the background of Hosios rule, the standard bargaining power in Nash bargaining must be much lower in order to establish constrained efficiency. It is certainly an interesting topic for future research to evaluate efficiency and the optimal use of policy instruments (e.g. fiscal policy) in the presence of idiosyncratic productivity shocks for new jobs.

In addition, our paper has shown that the decline in matching efficiencies in aggregate matching function estimations may be spurious in the presence of idiosyncratic productivity shocks and a time trend for vacancy posting costs. This also sounds a cautionary note on the conventional practice to use matching function estimations to quantify the effects of certain policy measures, such as unemployment benefit reforms.

Our paper also offers an interesting laboratory to analyze the quantitative effects of different policy interventions. We expect government spending to generate larger fiscal multipliers in severe recessions when the cutoff point is at a thicker part of the idiosyncratic shock distribution. This would complement theoretical results by Michaillat (2014) and empirical results by Auerbach and Gorodnichenko (2012). A detailed quantitative analysis is left for future research.

Overall, our paper suggests that it is important to have a better understanding what happens in the black box of matching. We have made a first step in this direction.

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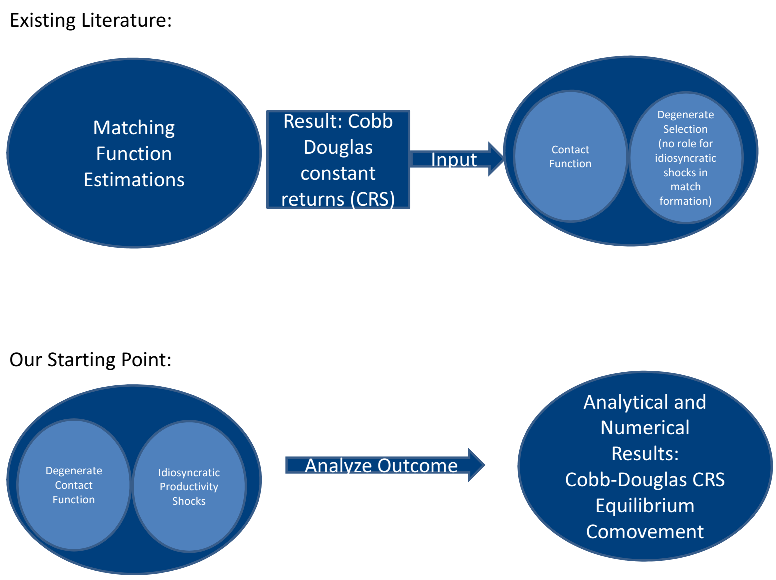
A Illustration

Figure 6



B Theory: Derivations

This Appendix proceeds in three steps. First, we show the intermediate steps for the results in Section 2. This corresponds to the case where the idiosyncratic shocks is only drawn during the first period of employment. Second, we show that the result also holds for a model with an *iid* shock in each period of employment, i.e. a model with endogenous separations (an assumption conventionally used in search and matching models with endogenous separations). Third, we show that the result does not change when workers draw an idiosyncratic shock realization at the beginning of their employment span and this realization does not change over time (an assumption conventionally used for the wage offer distribution in search models).

B.1 Baseline Results

In steady state, our model can be described by four equations: for the wage, the cutoff point, the selection rate, and the vacancy free entry condition.

$$w(\varepsilon_i) = \bar{w} + \alpha\varepsilon_i \tag{30}$$

with

$$\bar{w} = \omega(a, \eta, \theta, x), \tag{31}$$

$$\tilde{\varepsilon} = \frac{\bar{w} - a}{(1 - \delta(1 - \phi))} + \alpha\tilde{\varepsilon}, \tag{32}$$

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon)\, d\varepsilon, \tag{33}$$

$$\theta = \frac{p\eta}{\kappa}\left(\frac{a - \bar{w}}{1 - \delta(1 - \phi)} + \frac{(1 - \alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta}\right). \tag{34}$$

We account for very general wage formations, with $\bar{w}$ denoting the wage net of contemporaneous idiosyncratic productivity. We allow it to depend on all the endogenous variables. x could be a vector of exogenous shocks and parameters, such as unemployment compensation.

We can simplify the equations for the cutoff point and market tightness further:

$$\tilde{\varepsilon} = \frac{\bar{w} - a}{(1 - \delta(1 - \phi))(1 - \alpha)}, \tag{35}$$

$$\theta = (1 - \alpha)\frac{p\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon}\right). \tag{36}$$

Using the implicit function theorem, we can derive the derivatives of all the endogenous variables with respect to productivity.

The first derivative of the selection rate with respect to productivity is

$$\frac{\partial\eta}{\partial a} = -f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}. \tag{37}$$

Thus, the elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln(p\eta)}{\partial \ln a} = \frac{-f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}. \tag{38}$$

The first derivative of market tightness with respect to productivity is

$$\frac{\partial\theta}{\partial a} = -(1-\alpha)\frac{p}{\kappa}f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta}-\tilde{\varepsilon}\right)+ \tag{39}$$

$$(1-\alpha)\frac{p\eta}{\kappa}\left(\frac{-\tilde{\varepsilon}f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\eta+f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta^2}-\frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{40}$$

Simplified:

$$\frac{\partial\theta}{\partial a} = -(1-\alpha)\frac{p\eta}{\kappa}\left(\frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{41}$$

Thus, the elasticity of market tightness with respect to productivity is

$$\frac{\partial\ln\theta}{\partial\ln a} = \frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}}. \tag{42}$$

The first derivative of the cutoff point with respect to productivity is given by:[39]

$$\frac{\partial\tilde{\varepsilon}}{\partial a} = \frac{\omega_a'-1}{(1-\alpha)(1-\delta(1-\phi))+f(\tilde{\varepsilon})\omega_\eta'+(1-\alpha)\omega_\theta'}. \tag{43}$$

This term should be strictly smaller than zero for interesting cases. Imagine, for example, that the wage is given by $\bar{w}=\alpha(a+\kappa\theta)+(1-\alpha)b$. In this case $\partial\tilde{\varepsilon}/\partial a=(-1/((1-\delta(1-\phi))+\kappa\alpha))<0$.

We can now combine (38) and (42) to obtain the elasticity of the job-finding rate with respect to market tightness:

$$\begin{aligned}\frac{\partial\ln(p\eta)}{\partial\ln\theta} &= \frac{\frac{-f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}}{\frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}}}\\ &= \frac{f(\tilde{\varepsilon})}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta}-\tilde{\varepsilon}\right)\\ &= \frac{\partial\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{\partial\tilde{\varepsilon}}.\end{aligned}$$

[39] ω_y' denotes the partial derivative of ω, the wage function, with respect to the variable y.

B.2 Endogenous Separations

With endogenous separations, the cutoff point is

$$\tilde{\varepsilon} = \bar{w} - a + \alpha\tilde{\varepsilon} + \delta\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)\left(\bar{w} - a - \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)}\right) \tag{44}$$

$$+\delta^2\left(1-\phi\right)^2\left(\bar{w} - a - \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)}\right) + ... \tag{45}$$

$$\tilde{\varepsilon} = \frac{\left(\bar{w} - a - \delta\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)(1-\alpha)\frac{\int_{-\infty}^{\tilde{\varepsilon}} \varepsilon f(\varepsilon) d\varepsilon}{1-\phi(\tilde{\varepsilon})}\right)}{\left(1 - \delta\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)\right)(1-\alpha)}. \tag{46}$$

As usual, the selection rate is

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f\left(\varepsilon\right) d\varepsilon. \tag{47}$$

The elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln\left(p\eta\right)}{\partial \ln a} = \frac{-f\left(\tilde{\varepsilon}\right)\frac{\partial\tilde{\varepsilon}}{\partial a} a}{\eta}. \tag{48}$$

With endogenous separations, market tightness is:

$$\theta = \frac{p\eta}{\kappa}\left(\frac{a - \bar{w} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon) d\varepsilon}{(1-\phi(\tilde{\varepsilon}))}}{1 - \delta\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)}\right) \tag{49}$$

$$= (1-\alpha)\frac{p\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\left(1 - \phi\left(\tilde{\varepsilon}\right)\right)} - \tilde{\varepsilon}\right). \tag{50}$$

Taking into account that $\eta = 1 - \phi$ in this setting, we obtain:

$$\frac{\partial\theta}{\partial a} = -(1-\alpha)\frac{p}{\kappa}f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\eta} - \tilde{\varepsilon}\right) + \tag{51}$$

$$(1-\alpha)\frac{p\eta}{\kappa}\left(\frac{-\tilde{\varepsilon} f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\eta + f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\eta^2} - \frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{52}$$

After some algebra:

$$\frac{\partial\theta}{\partial a} = -\left(1-\alpha\right)\frac{p\eta}{\kappa}\left(\frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{53}$$

Thus, the elasticity of market tightness with respect to productivity is

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}}. \tag{54}$$

Combining equations (48) and (54), we obtain the heterogeneity based matching function:

$$\begin{aligned} \frac{\partial \ln (p\eta)}{\partial \ln \theta} &= \frac{\frac{-f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} a}{\eta}}{\frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}}} \\ &= \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon} \right) \\ &= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}. \end{aligned}$$

B.3 Same Idiosyncratic Shock for the Entire Employment Span

$$\tilde{\varepsilon} = \bar{w} - a + \alpha \tilde{\varepsilon} + \delta (1-\phi)(\bar{w} - a - (1-\alpha)\tilde{\varepsilon}) + \delta^2 (1-\phi)^2 (\bar{w} - a - (1-\alpha)\tilde{\varepsilon}) + ... \tag{55}$$

$$\tilde{\varepsilon} = \frac{\bar{w} - a}{1-\alpha}. \tag{56}$$

Selection Rate:

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) d\varepsilon. \tag{57}$$

The elasticity of the job-finding rate with respect to unemployment is

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = \frac{-f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} a}{\int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) d\varepsilon}. \tag{58}$$

Market tightness:

$$\theta = \frac{p\eta}{\kappa} \left(\frac{a - \bar{w} + \frac{(1-\alpha) \int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi)}}{(1 - \delta(1-\phi))} \right) \tag{59}$$

$$= \frac{(1-\alpha)}{1-\delta(1-\phi)} \frac{p\eta}{\kappa} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi)} - \tilde{\varepsilon} \right). \tag{60}$$

Using $\eta = 1 - \phi$, we get:

$$\frac{\partial\theta}{\partial a} = -\frac{(1-\alpha)}{1-\delta(1-\phi)}\frac{p}{\kappa}f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta}-\tilde{\varepsilon}\right)+ \tag{61}$$

$$\frac{(1-\alpha)}{1-\delta(1-\phi)}\frac{p\eta}{\kappa}\left(\frac{-\tilde{\varepsilon}f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\eta+f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta^2}-\frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{62}$$

After some algebra, we obtain:

$$\frac{\partial\theta}{\partial a} = -\frac{(1-\alpha)}{(1-\delta(1-\phi))}\frac{p\eta}{\kappa}\left(\frac{d\tilde{\varepsilon}}{da}\right). \tag{63}$$

Thus, the elasticity is

$$\frac{\partial\ln\theta}{\partial\ln a} = \frac{-\left(\frac{d\tilde{\varepsilon}}{da}\right)a}{\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}\right)}. \tag{64}$$

Combining equations (58) and (64), we obtain the heterogeneity based matching function:

$$\begin{aligned}\frac{\partial\ln(p\eta)}{\partial\ln\theta} &= \frac{\frac{-f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}}{\frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}}} \\ &= \frac{f(\tilde{\varepsilon})}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)\,d\varepsilon}{\eta}-\tilde{\varepsilon}\right) \\ &= \frac{\partial\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{\partial\tilde{\varepsilon}}.\end{aligned}$$

C Data Description

The German administrative database provides coherent definitions of the matching function variables. We use monthly data over the time period from 1993 to 2007. Matches and unemployment are obtained from the Sample of Integrated Labor Market Biographies (SIAB). The SIAB is a 2% random sample of all German residents who are registered by the Federal Employment Agency because of paying social security contributions or receiving unemployment benefits (see Dorner et al., 2010). Unemployment benefits may cover contribution-based benefits, means-tested benefits and income maintenance during training. We use an adjusted measure of unemployment benefit receipt according to Fitzenberger and Wilke (2010) to determine the unemployment stock. Matches are defined as transitions from unemployment to employment subject to social security. Even though marginal employment has become subject to social security since 1999, we do not consider this kind of employment as it is often ascribed to a stepping stone into regular jobs. The number of matches is calculated continuously, i.e. we take into account every daily transition. Hence, we do not neglect any job findings that are reversed within a month. See Nordmeier (2014) for more details on the time series.

Vacancies are taken from the official statistics and cover open positions that are reported to the Federal Employment Agency. The reported vacancies account for about 30-40% of overall vacancies in Germany. However, an adjustment of the reported vacancies by using the reporting rate of the IAB Job Vacancy Survey would not affect our estimation results because the reporting rate does not show a cyclical pattern in our observation period.

For our calibration exercise, we exploit the wage information included in the SIAB. Wages are shown as the employee's gross daily wage in Euros, which was calculated from the fixed-period earnings reported by the employer and the duration of the employment period in calendar days. Because we focus on new full-time jobs, we only consider wages above the marginal part-time income threshold. We use the consumer price index (CPI) from the National Accounts to obtain real daily wages.

Table C.1: Description of control variables

Variables	Extracted series	Definition
Unemployment duration	*long*	Share of long-term unemployed, i.e. unemployment duration $\geq$ 1 year
Age	*young* *old*	Share of unemployed with age $\leq$ 25 years Share of unemployed with age $\geq$ 55 years
Education	*low-skilled* *high-skilled*	Share of unemployed without vocational training (acc. to Fitzenberger et al., 2005) Share of unemployed with university degree (acc. to Fitzenberger et al., 2005)
Nationality	*foreign*	Share of unemployed with immigration background (see Wichert and Wilke, 2012)
Gender	*female*	Share of female unemployed
Family status	*married* *child*	Share of married unemployed Share of unemployed with at least one child
Benefit receipt	*UB I*	Share of contribution-based unemployment benefits recipients (unemployment benefits I)

Data source: SIAB.

D Parametrization of the Model

We parametrize the model on a monthly basis. For an overview of targets and parameters see Table D.2 and D.3. Aggregate productivity in steady state is normalized to 1. In all dynamic versions of the model productivity follows an AR(1) process with a correlation coefficient of 0.95 and a standard deviation for the shock of 0.44%. We have estimated these values from productivity data from the German National Accounts.[40] The discount factor is $0.99^{\frac{1}{3}}$. We set the value of non-work to 0.8. Unemployment benefits for short-term unemployed in Germany are 60 or 67% of the last net wage. Our value takes into account that there is a value of home production. Although this value is high compared to standard US calibrations, most workers are far from indifferent between working and not working.

In line with our data we set the separation rate to 1% and target an overall job-finding rate of 5% per month. Market tightness in the data is 0.09. Vacancy posting costs are chosen to hit this target. The value of market tightness in our data seems very low. This is partly due to the fact that we only consider reported vacancies. However, the level of market tightness, and hence kappa, is a matter of scaling only and does not affect any of our results. The relatively high values of vacancy posting costs are also due to the very low flow rates of the German labor market and to the fact that we abstain from hiring costs in this

[40] Productivity: Output per hours worked from the Federal Statistical Office (*Statistisches Bundesamt*), 1991Q1 to 2013Q1.

model. In all model versions with a Cobb-Douglas contact function we target an overall elasticity of matches with respect to vacancies of 0.35 in line with our estimated matching function. Taking the distribution of the idiosyncratic shock, market tightness and the job-finding rate as given, this determines the value of the contact efficiency parameter.

Wages are determined by Nash bargaining, which ensures private match efficiency. We set the bargaining power of workers to 0.5. Most of our findings are robust to the exact value of the bargaining power. It only matters for the calibration of the variance of idiosyncratic productivity, and hence for amplification.

We calibrate the distribution of idiosyncratic productivity from the distribution of individual entry wages. If there is some proportionality between wages and idiosyncratic productivity, the distribution of the latter is a scaled version of the former. We fit a logistic distribution to the wage data. A nice feature of the logistic distribution is that the derivative of the conditional expectation - the term that determines the elasticity of matches with respect to vacancies - is uniquely determined by the cumulative density to the right of the cutoff point (i.e. the selection rate). The 1st, 5th, and 10th percentile of our wages correspond to selection rates of 0.956, 0.921, and 0.885 in the fitted distribution. Note that these do not necessarily correspond to the real selection rate as we do not know the number of workers to the left of the distribution. However, for the dynamics of our model it is irrelevant whether we have a low selection rate with a high contact rate or vice versa. What matters is the shape of the idiosyncratic shock distribution at and to the right of the cutoff point which we calibrate with wage data. The contact rate is set to match the empirical job-finding rate of 5% per month is steady state.

We set the standard deviation of the idiosyncratic shock such that the cross-sectional wage dispersion in our model matches the one in the data. The cross-sectional wage dispersion in the data is measured by the coefficient of variation (standard deviation divided by the mean) in a given year for our homogeneous reference group. It is around 0.3 and constant across years. Accordingly, we target a standard deviation of wages in our model of 0.3. This implies an unconditional standard deviation of the idiosyncratic shock of 0.7.[41] The mean of the idiosyncratic productivity distribution is then determined endogenously to match the relevant selection rate. We interpret the mean of the entrant productivity as fixed ex-post hiring/training costs.

Note that the mean of idiosyncratic productivity seems unrealistically low in our parametrization. Two comments are in order. First, our baseline model is very simple. The mean of the idiosyncratic shock could equally be interpreted as a fixed training or hiring cost. In addition, the influence of unions may lead to larger average wages and there could be fixed costs of production. Including these features would lead to a larger average calibrated idiosyncratic productivity in the first period of employment. Second, our results for the elasticity of

[41] The standard deviation of wages is obtained by scaling with the bargaining power and by conditioning on the realized hires.

the matching function with respect to vacancies are independent of our parametrization strategy. The particular combination of mean and variance for a given selection rate does not matter for our key results. The variance only matters for amplification.

Table D.2: Comman parameters and targets

Parameter	Value	Source
Aggr. productivity	1	normalization
AR-coef. productivity	0.95	National Accounts data
SD productivity	0.0044	National Accounts data
Discount factor	$0.99^{\frac{1}{3}}$	set
Value of leisure	0.8	set
Bargaining power	0.5	set
Separation rate	0.01	SIAB data
Job-finding rate	0.05	SIAB data
Market tightness	0.09	SIAB and Vacancy data

Table D.3: Parameters (depending on percentile for lowest wage)

Parameter	1st	5th	10th	Source and/or target
SD of log dist.	0.7	0.7	0.7	SIAB data (wages)
Selection rate	0.956	0.921	0.885	SIAB data (wages)
Mean of log dist.	-11.48	-12.09	-12.47	selection rate
Vacancy posting cost	0.35	0.30	0.26	market tightness
		Combined model		
Contact elast. wrt vac.	0.24	0.17	0.09	match elast. wrt vac.
Contact efficiency	0.09	0.08	0.07	contact rate

E Different Wage Groups

The results in Section 4.3 are based on the distribution of entry wages of a homogeneous reference group. We repeat this exercise several times each time changing certain characteristics of the reference group. We consider the following group compositions: The reference group with...

- ... low-skilled (no vocational training) instead of medium-skilled.
- ... women instead of males.
- ... long-term instead of short-term unemployed.
- ... age further differentiated (ten year age spans).
- ... education further differentiated (no degree, vocational training degree, high school, high school and vocational training, technical college, university).

Table E.4 reports for each percentile the highest and the lowest estimates of all groups along with the baseline. Our conclusions from Section 4.3 are robust to the different group compositions. For the 5th percentile, for instance, we get a minimum of 0.13 for the coefficient on vacancies and a maximum of 0.26.

Table E.4: Weights on vacancies and unemployment: robustness

	1st percentile	5th percentile	10th percentile
		minimum	
constant	-2.87	-2.70	-2.49
log U	0.93	0.87	0.79
log V	0.06	0.13	0.21
		base	
constant	-2.65	-2.48	-2.32
log U	0.85	0.78	0.72
log V	0.14	0.22	0.28
		maximum	
constant	-2.55	-2.38	-2.22
log U	0.81	0.75	0.69
log V	0.19	0.26	0.32

Note: The reported coefficients are means over 1000 simulations. The matching function was estimated unconstrained.