DISCUSSION PAPER SERIES

IZA

IZA DP No. 7471

Relative Consumption, Working Time, and Trade Unions

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June 2013

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zur Zukunft der Arbeit
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Discussion Paper No. 7471
June 2013

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IZA Discussion Paper No. 7471
June 2013

ABSTRACT

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Status considerations with respect to consumption give rise to negative externalities because individuals do not take into account that their decisions affect the relative consumption position of others. Further, status concerns create incentives for excessive labour supply in competitive markets. We show that trade unions which are unable to internalise the externality can nevertheless mitigate the resulting distortion. The reason is that wages above the market clearing level are only feasible if people work less and, therefore, fewer hours than in a competitive market. Accordingly, the theoretical model establishes that trade unions can have a welfare-enhancing role in a world with relative consumption effects.

JEL Classification: D62, J22, J51

Keywords: externality, hours of work, relative consumption, trade union

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* We are grateful to an anonymous referee, Florian Baumann, Giacomo Corneo, Ronnie Schöb and participants of seminars in Beer-Sheva, Berlin, Dresden, and Tübingen, the CESifo area conference on Employment and Social Protection in Munich, and the annual meeting of the Verein für Socialpolitik in Frankfurt for helpful comments. The usual disclaimer applies.

1. Introduction

Trade unions are often viewed as an impediment to efficiency because they drive a wedge between marginal productivity and the marginal rate of substitution between leisure and consumption. In this paper, we show that such a view may not be justified if consumption exhibits status effects. Such effects will exist if higher consumption on the part of a reference group negatively affects an individual, for a given level of the individual's own consumption. Since this negative externality is not taken into account when choosing labour supply and, hence, consumption individually, status effects create incentives for working time to be excessive (see, for example, Frank 1985 and Schor 1991 for a detailed illustration). However, market power of workers can reduce this distortion. This classic second-best argument also applies in the present context: we show that a small, firm-specific trade union, which is not able to internalise the consumption externality, will set wages in such a manner that working time falls to below the level prevailing in a competitive market. Therefore, trade unions mitigate the negative impact of status considerations with respect to consumption.

The theoretical analysis is based on two well-supported empirical observations. First, preference interdependencies are pervasive and strong (Solnick and Hemenway 1998, 2005, Johansson-Stenman et al. 2002, Alpizar et al. 2005, Carlsson et al. 2007, Senik 2008, and Clark et al. 2008). Second, trade unions prefer reductions in working time and, historically, one of their central demands has been a cut in hours of work. Moreover, both weekly and annual working time decline with the strength of trade unions (Hubermann and Minns 2005, Alesina et al. 2005, Berger and Heylen 2011).[1] Finally, there is a negative association between union membership and (regular) hours of work (Aidt and Tzannatos 2002).

Accordingly, the present contribution is chiefly related to two strands of the literature which take these empirical observations as their point of departure. First, the effects of relative consumption on labour supply have primarily been discussed in relation to the impact and optimal structure of income taxation (see, for example, Duesenberry 1949, Boskin and Sheshinski 1978, Persson 1995, Ireland 1998, Corneo 2002, or Dodds 2012). In none of these contributions, however, do trade unions play a role. Second, models of collective bargaining rely on a wide variety of specifications relating hours and the number of employees to output. In models without overtime, either working hours are varied exogenously, while the trade union can set the wage

[1] The findings presented in Table 9 in Huberman and Minns (2005) are not contained in a later, substantially revised version (Huberman and Minns 2007). Moreover, the evidence suggesting a negative effect of trade unions on working time is not uncontroversial. Faggio and Nickell (2007) find that union density raises annual hours of work, but assert that this effect vanishes if the negative impact on earnings dispersion is taken into account. Causa (2009) observes a negative (positive) impact for males (females), while Burgoon and Baxandall (2004) report positive relationships between hours of work and union density. More recently, Oh et al. (2012) find either no correlation between working time and union density or a positive one for non-centralised collective bargaining regimes.

(cf. Calmfors 1985, Booth and Schiantarelli 1987, Andrews and Simmons 2001), or alternatively both wages and hours of work (Calmfors 1985, Booth and Ravallion 1993, Andrews and Simmons 2001, FitzRoy et al. 2002, Kramarz et al. 2008, Wehke 2009), with negotiations constituting a special case. Booth and Schiantarelli (1987) and Hart and Moutos (1991) also consider (sequential) negotiations with respect to hours, wages and employment. In none of these analyses are status effects incorporated.

A number of further studies link working time and trade union activity in the presence of leisure or consumption externalities. Hansen et al. (2012) enquire how coordination between various trade unions and the openness of the economy affect the difference between hours of work chosen individually and by unions. However, trade unions are, in contrast to the present contribution, assumed to be large and, thus, incorporate the leisure externality in their objective. In Oh et al. (2012), employers choose hours of work in a shirking model of efficiency wages. Their choice may differ from the working time preferred by employees because hours affect the gain from shirking. Further, relative consumption concerns affect the no-shirking constraint. Oh et al. (2012) demonstrate that a small trade union may indirectly raise working time because a wage increase above the competitive level makes the union better off but forces a firm to raise hours of work in order to prevent shirking. In partial contrast, Alesina et al. (2005) show that trade unions tend to reduce hours of work in the presence of shocks and argue that they thereby partially internalise the leisure externality, albeit unintentionally. Moreover, Frank (1985) claims that trade unions facilitate coordination among co-workers and may therefore mitigate the under-consumption of goods which have no status effects. Finally, Oswald (1979) considers a trade union characterised by a utility function that increases in the wage and employment levels of its own members and declines with the wage paid to members of other unions. However, working time per worker is fixed.

The remainder of the paper unfolds as follows. In Section 2, we specify the theoretical framework and derive the features characterising the optimal allocation and the competitive labour market outcome. We use a simple analytical framework based on a ratio comparisons model (Clark and Oswald 1998) in order to succinctly establish the main effects of trade unions. Furthermore, in Section 3 we assume that workers who are identical ex-ante are also treated identically ex-post. In addition, a utilitarian (monopoly) trade union, which is firm-specific and, therefore, cannot internalise the status externality, sets wages, while the firm chooses working time. Assuming identical payoffs implies that no worker is unemployed and enables us to illustrate the potentially efficiency-enhancing impact of trade unions most clearly. In Section 4, we relax the full employment restriction and follow the main strand of the literature by assuming that the trade union sets wages and hours of work, while the firm determines employment. Such set-up also mirrors observable features of collective bargaining contracts which generally include provisions about wages and working conditions, but much less often employment. For both settings analysed in Sections 3 and 4, we compare the resulting working time with the competitive and welfare-maximising levels. In Section 5, we modify the assumption that utility from status is determined by the ratio of own consumption to the reference level. Instead, status utility depends on the difference between the two consumption levels. The analysis clarifies that the exact specification of preferences is without impact. Finally, Section 6 briefly summarises. The proofs and most calculations underlying the exposition in Section 5 are relegated to an Appendix.

2. Model

Preferences

There are two types of individuals i, i = 1, 2, who differ only with respect to their productivity. In particular, we assume that type 2 is the high-productivity individual. The number of individuals of each type n_i is very large, so that the actions of a single individual do not affect aggregate outcomes. Each individual has a time endowment t, t > 0; and working time is denoted by h_i, so that leisure equals $t - h_i$. We assume that utility u_i is increasing in individual consumption c_i and leisure $t - h_i$. Furthermore, the utility function is not type-specific and increasing in (cardinal) status. Status is determined by the ratio of consumption levels $c_i/\bar{c}$ (Clark and Oswald 1998, but see Section 5), where $\bar{c}$ is the average level of consumption. Following, for example, Persson (1995) and Corneo (2002), we assume that utility is additive in its components, in order to avoid problems of non-uniqueness of the equilibrium and to clearly derive the effects of a trade union:

$$u_i = \ln c_i + \lambda \ln(t - h_i) + \rho \ln\left(\frac{c_i}{\bar{c}}\right) \tag{1}$$

The parameters λ, λ > 0 and ρ, ρ ≥ 0, indicate the weight of leisure and status concerns, relative to the value of consumption. Empirical studies mentioned in Section 1 indicate that for many goods a considerable part of the utility from consumption results from comparing own consumption levels with those of others (Solnick and Hemenway 1998, 2005, Johansson-Stenman et al. 2002, and Alpizar et al. 2005), implying that ρ is strictly positive. For leisure in contrast, the empirical evidence suggests that the comparison is not overly important. We, therefore, concentrate on the consumption externality in equation (1) for simplicity, but note in passing that our results will basically continue to apply also in the presence of leisure externalities as long as status concerns relating to leisure are relatively less pronounced than with respect to consumption (as, f. e., in Choudhary and Levine 2006).

Production

There are two types of firms. Their respective numbers are given and both types produce the same commodity. Each firm employs only labour of one type. Therefore, a firm's type is given by i. For simplicity, we set the number of firms of each type equal to one. Consequently, the output of type i individuals equals $\alpha_i f(n_i E_i h_i)$, where E_i, $E_i \leq 1$, describes the employment ratio. The production function f is increasing, strictly concave ($f' > 0 > f''$) and satisfies $f(0) = 0$ and $f'(n_i E_i h_i) \to \infty$ for $n_i E_i h_i \to 0$. We suppose that the firm's labour demand function is weakly concave or not too convex, an assumption that is commonly made in models of collective bargaining as a sufficiency condition for an interior solution (cf. Oswald 1982). Total output is given by $\alpha_1 f(n_1 E_1 h_1) + \alpha_2 f(n_2 E_2 h_2)$, where $0 < \alpha_1 < \alpha_2$. Alternatively, and without affecting subsequent findings, the representative firm could also employ both types of labour, so that its total production consisted of the sum of output of both types. Importantly, the additive structure of the production process and the given number of firms ensure that neither complementarities between the two types of labour nor profit level effects alter the labour supply consequences of status considerations in equilibrium.

Pareto Efficiency and Welfare Maximum

We assume that all individuals of type i are treated equally in a Pareto-efficient allocation. While this assumption obviously restricts the set of allocations, it implies that no individual is unemployed (so that $E_i = 1$) and that average consumption equals $\bar{c} = \frac{n_1 c_1 + n_2 c_2}{n_1 + n_2}$.[2] The set of Pareto-efficient allocations can then be characterised by maximising, for example, the utility of all individuals of type 1, subject to a given utility level $\bar{u}_2$ of individuals of type 2 and aggregate output coinciding with aggregate consumption $n_1 c_1 + n_2 c_2$.

$$\Gamma(c_1, c_2, h_1, h_2, \mu_1, \mu_2) = n_1 u_1(c_1, h_1, c_2) + \mu_1 n_2 [u_2(c_2, h_2, c_1) - \bar{u}_2] \\ + \mu_2 [\alpha_1 f(n_1 h_1) + \alpha_2 f(n_2 h_2) - n_1 c_1 - n_2 c_2] \qquad (2)$$

The first-order conditions for a maximum of the objective Γ are, inter alia, given by:

[2] An individualistic specification of the reference level of consumption $\bar{c}_i = \frac{\beta n_i c_i + (1-\beta) n_j c_j}{n_i + n_j}$, $0 \leq \beta \leq 1$, $i, j = 1, 2$, $i \neq j$, will generate the same predictions as derived below. A proof is available upon request.

$$\frac{\partial\Gamma}{\partial c_1} = n_1 \frac{1+\rho}{c_1} - [n_1 + \mu_1 n_2] \frac{\rho n_1}{\bar{c}[n_1 + n_2]} - \mu_2 n_1 = 0 \tag{3a}$$

$$\frac{\partial\Gamma}{\partial c_2} = \mu_1 n_2 \frac{1+\rho}{c_2} - [n_1 + \mu_1 n_2] \frac{\rho n_2}{\bar{c}[n_1 + n_2]} - \mu_2 n_2 = 0 \tag{3b}$$

$$\frac{\partial\Gamma}{\partial h_1} = -\frac{n_1\lambda}{t - h_1} + \mu_2 \alpha_1 f'(n_1 h_1) n_1 = 0 \tag{3c}$$

$$\frac{\partial\Gamma}{\partial h_2} = -\mu_1 \frac{n_2\lambda}{t - h_2} + \mu_2 \alpha_2 f'(n_2 h_2) n_2 = 0 \tag{3d}$$

Substituting for μ_2 in (3a) and (3b) in accordance with (3c) and (3d), and using $c_2 = \mu_1 c_1$, which results from the combination of (3a) and (3b), for i = 1, 2 we obtain:

$$\alpha_i f'(n_i h_i) = \frac{c_i \lambda}{t - h_i} \tag{4}$$

While the right-hand side of equation (4) describes the marginal rate of substitution between leisure and consumption, the left-hand side equals individual marginal productivity. As preferences are identical and the consumption externality is completely internalised, condition (4) is independent of ρ. Equation (2) will constitute a utilitarian welfare function if $\mu_1 = 1$ and $\bar{u}_2 = 0$ are imposed. For $\mu_1 = 1$, optimal consumption quantities are the same for both types and denoted by c*. Equation (4) then implies that more productive individuals of type 2 work longer hours, $h_2^* > h_1^*$, and obtain less utility in the welfare maximum.[3]

It could be argued that this property of the utilitarian welfare maximum is undesirable, namely that $u_1(c_1^*, h_1^*, c_2^*) > u_2(c_2^*, h_2^*, c_1^*)$ holds. A Nash-welfare function $[u_1 - u_1^M][u_2 - u_2^M]$, for example, with the utility levels u_i^M obtained in the market equilibrium as fallback levels, will not exhibit this feature, because it generates $c_1^* < c_2^*$ and $h_1^* < h_2^*$ as outcomes. Since the findings derived below are basically the same for the Nash-welfare function as they are for the utilitarian objective, we base our subsequent analysis on the utilitarian specification.[4]

[3] This ranking of optimal hours of work may not prevail if the production function is specified differently.

[4] While the proofs of Propositions 2 and 3 are basically the same for the Nash-specification, the proof of Proposition 1 is substantially more elaborate. We are grateful for Giacomo Corneo for suggesting this extension.

Labour Supply and Labour Market Equilibrium

The price of the good produced by all firms and consumed by all individuals is normalised to unity. Furthermore, the hourly wage for a type i worker is denoted by w_i. Accordingly, profit maximisation implies that marginal productivity equals the wage. In addition, competition ensures full employment ($E_i = 1$). Since average consumption $\bar{c}$ is given from an individual's perspective, each individual chooses labour supply h_i such as to maximise:

$$u_i(h_i) = [1+\rho]\ln(w_i h_i) + \lambda \ln(t - h_i) - \rho \ln \bar{c} \tag{5}$$

Labour supply in a competitive market, which equals working hours in equilibrium in the present setting, is defined by:

$$\frac{1+\rho}{h_i^M} - \frac{\lambda}{t - h_i^M} = 0 \tag{6}$$

Labour supply is independent of the wage. Consequently, individual choices will result in identical decisions $h^M = h_1^M = h_2^M$ (see also Corneo 2002) and, given different productivity levels, in different wage and consumption levels $\frac{w_2^M}{w_1^M} = \frac{c_2^M}{c_1^M} = \frac{\alpha_2}{\alpha_1}$. In addition, the reaction function implied by equation (6) is independent of the other individuals' choices regarding working time (cf. Persson 1995).

In a number of contributions it has been argued (cf. Frank 1985, Schor 1991) or shown formally (see, e.g., Cahuc and Postel-Vinay 2005, Persson 1995, Alvarez-Cuadrado 2007, Pérez-Asenjo 2011, Dodds 2012) that status concerns induce people to supply too much labour.[5] One measure for labour supply being excessive could be the undistorted amount, namely labour supply in the absence of status concerns, i.e. for $\rho = 0$. Using this standard, the model analysed here also predicts that working time is too high, because market supply h^M is increasing in ρ and, hence, greater than the undistorted level of labour supply.

However, labour supply in the absence of a consumption externality constitutes just one benchmark. As an alternative, we consider the utilitarian welfare optimum. The subsequent Proposition summarises our findings:

[5] Pérez-Asenjo (2011) and Goerke and Pannenberg (2013) provide empirical evidence for a positive relationship between relative income and hours of work for the United States and Germany, respectively.

Proposition 1 (Labour Supply in a Competitive Setting)

a) In a competitive market, labour supply of both types of individuals is $h^M = t\frac{1+\rho}{1+\rho+\lambda}$. The wage ratio equals the ratio of marginal productivities, $\frac{w_2^M}{w_1^M} = \frac{\alpha_2}{\alpha_1}$.

b) Competitive labour supply h^M exceeds the undistorted level $\frac{t}{1+\lambda}$ for $\rho > 0$.

c) For $\mu_1 = 1$ and $\bar{u}_2 = 0$, that is, for a utilitarian welfare function, labour supply h_1^* of a type 1 individual falls short of the competitive market outcome h^M, and the optimal labour supply h_2^* of a type 2 individual will be less than h^M if $\rho > \rho_{2,crit}$, where $\rho_{2,crit} := \frac{c_2^M(h_2^*)}{c^*} - 1$ and $c_2^M(h_2^*)$ is the consumption of a type 2 individual working h_2^* hours in a competitive market.

Proof: see Appendix 8.1

The intuition for Proposition 1 is the following. Status concerns enhance the gain from consumption and, as a result, individual labour supply is excessive, relative to the undistorted level. Furthermore, the Pareto-efficient level of working time declines in the level of consumption, as can be seen from equation (4). Accordingly, if consumption in the market equilibrium is less than in a Pareto-efficient allocation, there is a further reason for working time to be excessive in a competitive labour market. The particular Pareto-efficient allocation resulting from the maximisation of a utilitarian welfare function fulfils this condition with respect to consumption unambiguously for an individual of type 1, because this individual consumes less than individuals of type 2, and because output would be less in the market economy if working time equalled the optimal amount of type 1 individuals. However, if productivities and, hence, wages differ, individuals of type 2 may consume more in a market economy than their optimal level. This would, in the absence of status concerns, provide an incentive to reduce labour supply in the market outcome below the optimal level. If the status externality is sufficiently strong, however, a type 2 individual will also work more in the market equilibrium than is optimal.

The above argument has ignored profits. To clarify that taking profit income into consideration would not qualitatively affect results, assume that consumption on the part of individuals of type i in a competitive market were to equal labour income $w_i h_i$ plus profit income π_i. In this case, consumption by type 1 individuals in a competitive market would not have to fall short of the optimal level c* if optimal labour supply h_1^* and market supply coincided. However, the positive impact on labour supply resulting from status concerns would persist. If the status externality were strong enough, that is if $\rho > \frac{c_i^M(h_i^*, \pi_i)}{c^*} - 1$ were to hold, where $c_i^M(h_i^*, \pi_i)$ describes the consumption level of type i in a market economy in which individuals also received profit income π_i, individuals of type i would supply a greater amount of labour than would be optimal. Since consumption for both types cannot be higher in the market economy than the level resulting from the maximisation of a utilitarian welfare function, unless individuals of at least one type were to work more, the above condition will hold for at least one type who would, thus, work more than is optimal. However, since the distribution of profit income is not specified, it is not necessarily a type 1 individual whose labour supply is excessive. In consequence, the inclusion of profit income does not affect the essence of Proposition 1c).[6]

If there is just one type of individual and all of them are treated equally, the Pareto-efficient allocation will be defined uniquely. Further, note that consumption in a competitive market cannot exceed the Pareto-efficient consumption level at the same level of labour supply, while status concerns represent incentives to raise working time above the efficient level. In consequence, the present model also generates the prediction that labour supply will unambiguously be excessive in a competitive market, relative to the (unique) Pareto-efficient outcome, if there is only one type of individual.[7] Finally, Proposition 1 clarifies that the number of individuals n_i of each type does not have an impact on outcomes, because all individuals of a type are treated equally and behave in the same manner. To save on notation, we subsequently set $n_1 = n_2 = 1$, so that E_i equals the employment level of type i individuals (and no longer the employment ratio).

[6] Note that considering profits does not alter Proposition 1b) because undistorted labour supply would also decline with profit income.
[7] See, for example, Persson (1995), Cahuc and Postel-Vinay (2005), and Alvarez-Cuadrado (2007).

3. A Constrained Trade Union

Suppose that all workers of type i belong to a firm-specific, utilitarian trade union which sets wages and employment, while the firm determines working time. In line with the usual monopoly union set-up, we assume that the trade union can prevent its members from underbidding colleagues and can effectively control labour supply (Oswald 1979, 1982). In consequence, the market outcome is no longer determined by the intersection of supply and demand but the tangency of the trade union's indifference curve with the labour demand schedule. Since, conceptually, there are many firms, each trade union is small in the sense that it does not consider the effects of its own actions on relative consumption. We subsequently focus on one (representative) union and assume that workers who do not find a job obtain utility $\bar{u}$, $\bar{u} < u_i(w_i, h_i)$, which results for example, from home production. Therefore, the objective of a trade union which represents individuals of type i is given by:

$$\Omega_i(w_i, E_i) = E_i u_i(w_i, h_i) + [1 - E_i]\bar{u} \tag{7}$$

Importantly, in this section we assume that although the trade union can (formally) set E_i, it is implicitly restricted to a choice of full employment ($E_i = 1$). Such a restriction allows the effects of trade unions on working time to be derived most clearly and can be justified on the basis of various arguments. First, each worker may be able to veto the union's choices if he/she is worse off than a colleague ex-post, implying that all workers need to have the same utility. This will rule out unemployment in the absence of a redistributive mechanism. Second, the leadership of a trade union which 'caused' unemployment by setting $E_i < 1$ would lose the support of its members. Third, the costs of unemployment, that is, the difference between u_i and $\bar{u}$, may be so high, that full employment is optimal. We call this trade union which effectively cannot determine employment a constrained union, and use the superscript C to represent it. For $E_i = 1$, the firm's first-order condition $w_i = \alpha_i f'(h_i)$ implies that $\frac{dh_i}{dw_i} = \frac{1}{\alpha_i f''(h_i)} < 0$ holds. Maximising $\Omega_i^C(w_i) = u_i(w_i, h_i(w_i))$ with respect to the wage w_i, and taking average consumption as given, yields[8]

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[8] Note that $\frac{\partial^2 \Omega_i^C}{\partial (w_i)^2} < 0$ will unambiguously be warranted if labour demand $E_i h_i = h_i$ (for $E_i = 1$) is not too convex. Obviously, the first-order condition (8) will also hold if the trade union sets hours (instead of wages). In this case, the wage adjusts in equilibrium to guarantee that all workers are employed at the prescribed working time. Accordingly, the precise specification of the constrained union's objective is without relevance.

$$\frac{\partial \Omega_i^C}{\partial w_i} = \frac{1}{\alpha_i f''(h_i(w_i))}\left[\frac{1+\rho}{h_i(w_i)} - \frac{\lambda}{t-h_i(w_i)}\right] + \frac{1+\rho}{w_i} = 0$$

$$\Rightarrow \quad 1+\rho+\varepsilon_i^C(h_i^C)\left[h_i^C \frac{\lambda}{t-h_i^C} - 1 - \rho\right] = 0, \qquad (8)$$

where the union's (implicit) choice of working time is denoted by h_i^C. The labour demand elasticity $\varepsilon_i^C(h_i^C) := -\frac{w_i^C}{\alpha_i f''(h_i^C) h_i^C}$ is greater than unity at the optimal wage. We can summarise the characterisation of the constrained trade union's behaviour in

Proposition 2 (Constrained Trade Union):

Suppose a constrained trade union.

a) Working time of both types of individuals is the same $h^C = h_1^C = h_2^C$, while the wage ratio equals the ratio of marginal productivities, $\frac{w_2^C}{w_1^C} = \frac{\alpha_2}{\alpha_1}$.

b) Working time is less than in a competitive market ($h^C < h^M$).

c) For $\varepsilon_i^C(h_i^*) := -\frac{f'(h_i^*)}{f''(h_i^*) h_i^*}$ and $\rho_{i,crit}^C := \frac{\varepsilon_i^C(h_i^*)}{\varepsilon_i^C(h_i^*)-1} \frac{c_i^M(h_i^*)}{c^*} - 1$, where $c_i^M(h_i^*)$ denotes the consumption level at the wage $w_i(h_i^*)$, working time h^C will be less than the optimal level h_i^* of a type i individual if $\varepsilon_i^C(h_i^*) \leq 1$ or if $\varepsilon_i^C(h_i^*) > 1$ and the strength of the relative consumption effect falls short of a critical level $\rho_{i,crit}^C$.

d) Individuals of both types will be better off than in a competitive market.

Proof: see Appendix 8.2

To provide an intuition for Proposition 2, observe that part a) implies that consumption levels differ and the marginal utility from consumption is greater for the less productive individual. This is the case because a trade union balances the gain from higher wages and more leisure with the reduction in consumption resulting from a decrease in working time. The gain and the loss are proportional to a worker's productivity indicator α_i because of its multiplicative impact on (marginal) output. Therefore, the relative gain from higher wages and lower working time is the same for both types of individuals. In consequence, each trade union chooses the same level of working time, irrespective of an individual's productivity.

Wage setting will reduce working time to below the competitive levels (cf. part b)) because a firm will respond to a wage increase above the level resulting in a competitive market by a reduction in labour demand, that is, the amount of hours. Since individual labour supply, in contrast to actual working time, will not affect a trade union's payoff if wages exceed the market-clearing level, the nature, though not the strength of this finding is independent of the existence of a consumption externality.

Turning to part c) of the Proposition, working time implicitly chosen by the trade union will be less than the optimal level h_i^* for a type i individual if the importance of status concerns is relatively small ($\rho < \rho_{i,crit}^C$). A weak consumption externality makes the inequality $h^C < h_i^*$ more likely for two reasons. First, the weaker the negative consumption externality is, as measured by ρ, the less the difference between competitive hours h^M and the optimal amount h_i^*. Since the market outcome constitutes the trade union's benchmark, a weak externality makes it more likely that hours of work are reduced below h_i^*. Second, a less pronounced externality raises the wage set by a trade union for $\varepsilon_i^C(h_i^C) > 1$ because the reduction in utility from consumption resulting from a higher wage is mitigated. In consequence, hours of work decline (as the negative derivative of equation (8) with respect to ρ clarifies).

Each trade union on its own will, of course, increase the wellbeing of its members. However, a general wage increase can raise average consumption $\bar{c}$ and will then make individuals worse off, ceteris paribus. When establishing the aggregate impact of union wage setting on an individual's wellbeing, this negative externality obviously has to be taken into account. In doing so, it may be noted that consumption is proportional to marginal productivity in a competitive market and also in the unionised world (see Appendix 8.2). Furthermore, hours of work are the same for both types of individuals in a competitive market (cf. equation (6)), on the one hand, and in a unionised world on the other. Therefore, the consumption externality is constant. Effectively, independent wage setting by firm-specific trade unions neutralises the negative consumption externalities which each individual union's behaviour causes.

In the above analysis it has been assumed that the trade union can set wages (and employment to $E_i = 1$). If, in contrast, the trade union negotiated the wage level with the firm (in a Nashbargain), the resulting wage would be lower than the union's preferred wage and higher than the competitive wage. The extent to which a trade union reduces working time would then depend on its bargaining power. The weaker a trade union is, the less likely that collective bargaining reduces working time to below the level h_i^*.[9] Furthermore, if unions for different types of individuals are characterised by different opportunities to influence wages, those results in Proposition 2 which rely on the implicit assumption that unions of both types of individuals behave equally, need no longer hold.

In consequence, we can summarise the results of this section as follows: a firm-specific trade union which (1) does not take into account the negative consumption externality of higher working hours, (2) has to ensure full employment, and (3) can set either wages or hours of work will reduce working time to below the level chosen individually in a competitive market. Therefore, trade unions can reduce the distortion arising from negative consumption externalities. Working time in a unionised world is more likely to remain above the level which characterises the maximum of a utilitarian welfare function, the more significant the negative consumption externality is. Furthermore, less bargaining power on the part of the trade union makes it more likely that collective bargaining actually mitigates the distortion.

4. An Unconstrained Trade Union

In this section, we relax the assumption that all workers are treated equally and always work for a positive number of hours. To generate unemployment, we assume that the trade union can set wages and hours of work, while the firm chooses the number of employees. We denote this setting as one with an unconstrained union and indicate outcomes with a superscript U. Otherwise, the assumptions of the previous section are retained. Alternatively and without there being an impact on results, the trade union could decide on wages and employment, or hours and employment, while the firm determines the remaining variable. Furthermore, our results also apply to a trade union that maximizes aggregate utility of employed workers $E_i u_i$, which can be seen from setting $\bar{u} = 0$ in the subsequent equations.

[9] FitzRoy et al. (2002) indicate that hours of work rise with the firm's bargaining power towards the competitive level in a setting without consumption externalities, in which the firm and the trade union bargain over wages and hours. Kramarz et al. (2008) obtain a similar prediction. Andrews and Simmons (2001) derive a sufficient condition for hours to fall with union power in a framework with negotiations regarding wages and hours, while Hart and Moutos (1991) show that hours will be unaffected by the level of union power if wages, hours, and employment are bargained over and the production function is of the Cobb-Douglas type.

If the union sets wages and hours, labour demand E_i will be decreasing in wages and hours, $\frac{\partial E_i}{\partial w_i} = \frac{1}{h_i \alpha_i f''(h_i E_i)} < 0$, $\frac{\partial E_i}{\partial h_i} = -\frac{E_i}{h_i} < 0$. The trade union's objective then is:

$$\Omega_i^U(w_i, h_i, E_i(w_i, h_i)) = E_i(w_i, h_i) u_i(w_i, h_i) + [1 - E_i(w_i, h_i)]\bar{u} \tag{9}$$

Average consumption $\bar{c}$ is likely to be affected by unemployment. However, since it is given from a single trade union's perspective, we do not have to specify $\bar{c}$ in detail. Accordingly, the first-order conditions for the maximum of Ω_i^U are:

$$\frac{\partial \Omega_i^U}{\partial h_i} = -\frac{u_i(h_i, w_i) - \bar{u}}{h_i} + \frac{1+\rho}{h_i} - \frac{\lambda}{t - h_i} = 0 \tag{10}$$

$$\frac{\partial \Omega_i^U}{\partial w_i} = \frac{u_i(h_i, w_i) - \bar{u}}{h_i \alpha_i f''(h_i E_i(h_i, w_i))} + E_i(h_i, w_i)\frac{1+\rho}{w_i} = 0 \tag{11}$$

Inspection of these equations clarifies that choices made by unconstrained trade unions will not result in identical amounts of working time for both types of individuals.[10] Moreover, while workers may be better off ex-ante, not all of them will benefit ex-post, because some workers will be unemployed. We summarise the features of the union's optimal choices in

Proposition 3 (Unconstrained Trade Union):

Suppose an unconstrained trade union.

a) Hours of work are lower than in the competitive market ($h_i^U < h^M$).

b) If there is unemployment, working time will be higher than the level (implicitly) chosen by a constrained trade union ($h_i^U > h^C$).

c) For $\varepsilon_i^U(h_i^*) := -\frac{f'(E_i h_i^*)}{f''(E_i h_i^*) E_i h_i^*}$ and $\rho_{i,crit}^U := \frac{\varepsilon_i^U(h_i^*)}{\varepsilon_i^U(h_i^*) - 1} \frac{c_i^M(h_i^*)}{c^*} - 1$, where $c_i^M(h_i^*)$ denotes the consumption level at the wage $w_i(h_i^*)$, working time h^U will be less than the optimal level h_i^* of a type i individual if $\varepsilon_i^U(h_i^*) \le 1$ or if

[10] If $h_1^U = h_2^U$, (10) can only hold if utility levels and, hence, wages are the same. This implies that $E_2 > E_1$ as labour demand rises with α_i at a given wage, which is incompatible with (11) for f ''' ≤ 0.

$\varepsilon_i^U(h_i^*) > 1$ and the strength of the relative consumption effect falls short of a critical level $\rho_{i,crit}^U$.

Proof: See Appendix 8.3

The intuition for parts a) and c) of Proposition 3 is the same as for the respective parts of Proposition 2. However, it should be emphasised that trade unions which set wages and hours can reduce hours to below the competitive level in the absence of consumption externalities as well (Calmfors 1985, Wehke 2009, Andrews and Simmons 2001, FitzRoy et al. 2002). This monopoly effect of trade unions cannot have positive welfare effects unless there are other distortions, as is the case here in the presence of consumption externalities. To clarify part b) of Proposition 3, suppose the opposite, namely that constrained hours exceed unconstrained working time. If the trade union could determine wages and hours directly and preferred less working time than in a setting in which, effectively, it can only set wages, the reduction in working time would have to make the union better off. However, the trade union could have lowered hours of work in the constrained world as well, namely by raising the wage. Since higher wages increase utility, ceteris paribus, unconstrained hours of work being lower than constrained hours implies that the union has not exhausted all gains from setting wages and hours. In consequence, constrained hours cannot exceed unconstrained working time.

5. Alternative Specification of Preferences

The entire above analysis has been based on the assumption that status utility is determined by the ratio $c_i/\bar{c}$ of an individual's own consumption, c_i, to the reference level of consumption, $\bar{c}$. Alternatively, status is often viewed as a function of the difference between own consumption and the reference level, $c_i - \bar{c}$. Such an additive comparisons model has also been used to show that labour supply in a competitive market can exceed the welfare-maximising level (Ljungvist and Uhlig 2000, Choudhary and Levine 2006, and Pérez-Asenjo 2011). Moreover, the ratio and additive comparisons approach explain actual choices equally well (Mujcic and Frijters 2013). Therefore, we subsequently show that an alternative specification of preferences $\tilde{u}_i$, based on an additive comparisons approach, does not fundamentally affect the findings summarised in Propositions 1 to 3. Let $\tilde{u}_i$ be given by:[11]

[11] We are very grateful to an anonymous referee for bringing to our attention this possibility of strengthening the findings, which in previous versions of the paper were solely based on the ratio comparisons model.

$$\tilde{u}_i = \ln c_i + \lambda \ln(t - h_i) + \rho \ln(c_i + K - \bar{c}) \tag{1'}$$

In equation (1'), the parameter K, K > $\bar{c}$ + 1, ensures that the status effect is well-defined. Since individuals of each type behave identically, average consumption equals $\bar{c} = \dfrac{n_1 c_1 + n_2 c_2}{n_1 + n_2}$. Subsequently, we denote all endogenous variables of interest by a tilde in order to clearly differentiate outcomes for $\tilde{u}_i$ from those resulting in the setting based on ratio comparisons (u_i).

The set of Pareto-efficient allocations can be characterised by maximising a modified equation (2) with respect to consumption levels c_i, working time h_i, and the Lagrange-multipliers μ_1 and μ_2, where $\tilde{u}_i$ replaces u_i. The first-order conditions for a maximum of this objective $\tilde{\Gamma}$ are, inter alia, given by equations (3c) and (3d) and by:

$$\frac{\partial \tilde{\Gamma}}{\partial c_1} = \frac{n_1}{c_1} + \frac{\rho n_1 n_2}{K[n_1 + n_2] + n_2[c_1 - c_2]} - \frac{\mu_1 \rho n_1 n_2}{K[n_1 + n_2] + n_1[c_2 - c_1]} - \mu_2 n_1 = 0 \tag{3a'}$$

$$\frac{\partial \tilde{\Gamma}}{\partial c_2} = \frac{\mu_1 n_2}{c_2} + \frac{\mu_1 \rho n_1 n_2}{K[n_1 + n_2] + n_1[c_2 - c_1]} - \frac{\rho n_1 n_2}{K[n_1 + n_2] + n_2[c_1 - c_2]} - \mu_2 n_2 = 0 \tag{3b'}$$

Cancelling n_1 and n_2, simplifying and combining both first-order conditions, yields:

$$\frac{1}{c_1} - \frac{\mu_1}{c_2} + \frac{\rho[n_2 + n_1]}{K[n_1 + n_2] + n_2[c_1 - c_2]} - \frac{\mu_1 \rho[n_2 + n_1]}{K[n_1 + n_2] + n_1[c_2 - c_1]} = 0 \tag{12}$$

The set of Pareto-efficient allocations will not generally be characterized by $c_1\mu_1 = c_2$, as it is true for the ratio comparisons model. This is the case because differential weights for both types of individuals in the objective $\tilde{\Gamma}$ imply that symmetric changes in the reference level of consumption alter the value of $\tilde{\Gamma}$. Setting μ_1 = 1 (and $\bar{u}_2$ = 0) clarifies that equation (12) is solved by $c_1 = c_2$ and that the utilitarian allocation characterized by $\tilde{c}^* = \tilde{c}_1 = \tilde{c}_2$. Furthermore, substituting for μ_2 in (3a') and (3b') in accordance with (3c) and (3d) for $\tilde{c}_1 = \tilde{c}_2$, we obtain equation (4). In consequence, the more productive individuals of type 2 work longer hours and obtain less utility ($\tilde{h}_1^* < \tilde{h}_2^*$). Therefore, the characterisation of the welfare maximum is unaffected by modelling status via the ratio or the difference of consumption levels.

Turning to labour supply, the maximisation of (1') for a given reference consumption, $\bar{c}$, and taking into account $c_i = w_i h_i$, where $\tilde{w}_i^M$ denotes the wage paid to workers of type i, yields:

$$\frac{\partial \tilde{u}_i}{\partial h_i} = 0 \qquad => \qquad \frac{1}{\tilde{h}_i^M} - \frac{\lambda}{t - \tilde{h}_i^M} + \frac{\rho \tilde{w}_i^M}{\tilde{w}_i^M \tilde{h}_i^M + K - \bar{c}} = 0 \tag{13}$$

Since $\frac{\partial^2 \tilde{u}_i}{\partial (h_i)^2} < 0$, labour supply $\tilde{h}_i^M$ is increasing with the wage $\tilde{w}_i^M$, the strength of consumption comparisons ρ and the reference level of consumption $\bar{c}$. Therefore, labour supply will be higher than the undistorted level ($\tilde{h}_i^M > \tilde{h}_i^*$; cf. Proposition 1b).

Because productivity of type 2 workers exceeds that of type 1 workers, labour demand will be higher at the same wage. If there are weakly fewer type 2 individuals ($n_1 \geq n_2$), higher labour demand translates into a greater demand for hours and type 2 workers will, hence, work more than type 1 individuals in the market equilibrium. They will also obtain a higher wage and have a higher level of consumption. Furthermore, the wage ratio will equal the ratio of marginal productivities. Finally, a critical value of the parameter ρ can be derived which ensures that labour supply of a type i individual exceeds the optimal amount. If there is only one type of individual, this restriction will always be fulfilled (see Appendix 8.4). In consequence, the main content of Proposition 1 relating to a ratio comparisons model, namely that working time in the competitive equilibrium exceeds the undistorted level and that it will be higher than the optimal amount if the consumption externality is sufficiently strong, also obtains for an additive comparisons framework. This implies that the starting point of our analysis is independent of the exact specification of preferences: Labour supply is excessive in a competitive setting in the presence of consumption externalities.

The decisive difference between the ratio comparisons specification investigated in Sections 2 to 4 and the additive comparisons model is that all individual choices depend on the reference level of consumption, $\bar{c}$, in the later set-up only. This reference level is affected by wages, working hours and the number of employees. In consequence, we can show in Appendix 8.4 that the findings summarised in Propositions 2 to 3 continue to hold for a given reference level of consumption.[12] Furthermore, most parts will also be valid for an additive comparisons set-up if the reference level in the unionised settings is lower than in the non-unionised world and, furthermore, is weakly less for the unconstrained trade union than in a framework with a constrained union. This is the case because a lower reference level $\bar{c}$ tends to raise a trade union's wage demands and increases utility, ceteris paribus. Therefore, a decline in $\bar{c}$ provides

[12] The only difference is Proposition 2, part a) that working time of both types is the same in the presence of a constrained trade union and that the wage ratio equals the ratio of marginal productivities.

further incentives to lower hours of work and, hence, strengthens the findings derived for the ratio comparisons approach in which the reference level is without impact.

6. Summary

If people exhibit status concerns with respect to consumption, they will have an incentive to increase consumption and, hence, labour supply beyond the level which is optimal in the absence of the consumption externality. Therefore, status considerations distort individual decisions in a competitive market towards too much labour supply and production. Taking such an outcome as our starting point, we show that trade unions which are firm-specific and, hence, cannot internalise the consumption externality reduce working time to below the level that results in a competitive market. This prediction can be obtained irrespective of whether status depends on the ratio or difference between own consumption and the respective reference level. Therefore, trade unions can help to reduce the distortion arising from status concerns. The reason for this potentially beneficial impact of trade unions is that wages above the market clearing level reduce labour demand and, therefore, effectively curtail production and consumption opportunities. This outcome is a classic second-best feature.

The interpretation of trade union activity produced here bears some similarities, but also several differences, to the voice interpretation of unions (Freeman and Medoff 1984), which emphasises their productivity-enhancing impact. The monopoly impact and the voice effect of trade unions, which helps to raise productivity because workers are less inclined to leave the firm if dissatisfied with working conditions, are two sides of the same coin. In the present model, union monopoly power has beneficial effects on its own. Therefore, the potentially positive role of trade unions in a world with relative consumption does not require these collective organisations to do more than raise wages (and reduce employment and hours).

7. References

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8. Appendix:

8.1 Proof of Proposition 1

Parts a) and b) are obvious from the discussion prior the Proposition. To derive the last part, multiply the denominator of condition (6) by $w_i^M = \alpha_i f'(n_i h_i^M)$. Then, condition (4) can be rewritten for $\mu_1 = 1$ and the resulting expression $\frac{1}{c*} - \frac{\lambda}{[t - h_i^*]\alpha_i f'(n_i h_i^*)} = 0$ can be subtracted from the modified version of condition (6). This yields an equation denoted by Z_i:

$$Z_i(w_i^M, h_i) := \frac{1+\rho}{h^M w_i^M} - \frac{\lambda}{t - h^M} \frac{1}{\alpha_i f'(n_i h^M)} - \frac{1}{c*} + \frac{\lambda}{t - h_i^*} \frac{1}{\alpha_i f'(n_i h_i^*)} \qquad \text{(P.1.1)}$$

Suppose now that labour supply in the market equilibrium is given by $h^M = h_i^*$. Simplifying Z_i accordingly and substituting $c_i^M(h_i^*)$ for $h_i^* w_i^M(h_i^*)$ in (P.1.1), we obtain:

$$Z_i(w_i^M(h_i*), h_i^*) = \frac{1+\rho}{c_i^M(h_i^*)} - \frac{1}{c*} \qquad \text{(P.1.2)}$$

If labour supply of an individual of type 1 in the market equilibrium does not exceed supply in the welfare maximum and individuals of type 2 supply less labour than is optimal, total market output will be less than the welfare maximising level. Accordingly, since $c_1^M < c_2^M$ for any arbitrary labour supply level, given $w_2^M > w_1^M$, consumption of a type 1 individual will have to be less than in the welfare maximum if $h^M = h_1^*$ holds ($c_1^M < c*$). In consequence, $Z_1 > 0$ holds for any $h^M \leq h_1^M = h_2^M = h_1^*$, and individual 1 has an incentive to increase labour supply beyond h_1^*. For a type 2 individual, $\rho > \rho_{2,crit}$, where $\rho_{2,crit} := \frac{c_2^M(h_2^*)}{c*} - 1$, ensures that Z_2 is positive and $h_2^M > h_2^*$. This proves part c).

8.2 Proof of Proposition 2

Setting $n_i = 1$ and substituting $\alpha_i f'(h_i)$ for w_i in the last term of the first line of (8), and cancelling α_i leaves labour supply h_i as the only type-specific variable. Given the strict concavity of Ω_i^C in w_i, the value of h_i which maximises Ω_i^C is the same for both types of individuals. If hours of work coincide ($h^C = h_1^C = h_2^C$), so too will the derivatives of the production function. Therefore, the wage ratio $\frac{w_2^C}{w_1^C}$ equals $\frac{\alpha_2 f'(h^C)}{\alpha_1 f'(h^C)} = \frac{\alpha_2}{\alpha_1}$. This establishes part a).

The trade union increases the wage beyond the competitive level. Since employment is determined by labour demand in a monopoly union setting, given the wage, and the trade union can prevent underbidding by individual workers, all of them will be treated identically. Higher labour costs and the employment of all workers implies that hours of work will decline below h^M. As working time is independent of the wage in the competitive setting, this proves part b) of the Proposition.

If $\mu_1 = 1$ holds, consumption levels are the same, i. e. $c_1^* = c_2^* = c^*$. Multiplying the term in square brackets in (8) by $\alpha_i f'(h)$, evaluating the expression at h_i^*, and substituting $c_i^M(h_i^*)$ for $\alpha_i f'(h_i^*)h_i^*$ and $\frac{\alpha_i f'(h_i^*)}{c^*}$ for $\frac{\lambda}{t - h_i^*}$ in accordance with equation (4), the sign of the first-order condition (8), evaluated at the optimal level of hours h_i^*, can be determined.

$$\left.\frac{\partial \Omega_i^C}{\partial w_i}\right|_{h=h_i^*} = [1+\rho][1-\varepsilon_i^C(h_i^*)] + \frac{\varepsilon_i^C(h_i^*) c_i^M(h_i^*)}{c^*} \qquad \text{(P.2.1)}$$

In (P.2.1), the labour demand elasticity ε_i^C is evaluated at the optimal working time h_i^*, $\varepsilon_i^C(h_i^*) := -\frac{\partial h_i}{\partial w_i}\frac{w_i}{h_i^*} = \frac{-w_i}{\alpha_i f''(h_i^*)h_i^*} = \frac{-f'(h_i^*)}{\alpha_i f''(h_i^*)h_i^*}$. (P.2.1) will be positive for $\varepsilon_i^C(h_i^*) < 1$ and also for $\varepsilon_i^C(h_i^*) > 1$ if $\rho < \rho_{i,crit}^C := \frac{\varepsilon_i^C(h_i^*)}{\varepsilon_i^C(h_i^*) - 1}\frac{c_i^M(h_i^*)}{c^*} - 1$. In both cases the union has an incentive to raise the wage beyond a level which induces h_i^* hours, so that hours fall to below h_i^*. This proves part c) of Proposition 2.

Total utility of a worker consists of the components the trade union can influence and the consumption externality, which is taken as given. The utility reduction due to the status externality in a competitive market is given by:

$$\rho \ln\left(\frac{c_i}{\bar{c}}\right) = \rho \ln\left(\frac{\alpha_i f'(h^M)h^M}{0.5\alpha_1 f'(h^M)h^M + 0.5\alpha_2 f'(h^M)h^M}\right) = \rho \ln\left(\frac{2\alpha_i}{\alpha_1 + \alpha_2}\right).$$

Since this externality will be constant if wages reflect marginal productivities and working time is the same for both types, wages above the competitive level will make workers better off if $\hat{u}_i := \ln c_i + \lambda \ln(t - h_i)$ increases with the wage up to the level set by the trade union. The derivatives of $\hat{u}_i$ are:

$$\frac{\partial \hat{u}_i}{\partial w_i} = \frac{1}{\alpha_i f''(h_i)}\left[\frac{1}{h_i} - \frac{\lambda}{t - h_i}\right] + \frac{1}{w_i} \tag{P.2.2}$$

$$\frac{\partial^2 \hat{u}_i}{\partial (w_i)^2} = -\frac{f'''(h_i)}{(\alpha_i)^2 (f''(h_i))^3}\left[\frac{1}{h_i} - \frac{\lambda}{t - h_i}\right] - \frac{1}{(\alpha_i f''(h_i))^2}\left\{\frac{1}{(h_i)^2} + \frac{\lambda}{(t - h_i)^2}\right\} - \frac{1}{(w_i)^2} \tag{P.2.3}$$

The derivative (P.2.2) is unambiguously positive at the competitive working time h^M and hours h^C resulting from wage setting by a constrained trade union, for $\varepsilon_i^C > 1$. The derivative (P.2.2) is also positive at $h^\# := \frac{t}{1+\lambda} < h^M$. Therefore, $\hat{u}_i$ is increasing in the wage at the competitive wage, at a wage resulting in a working time $h^\# = \frac{t}{1+\lambda}$, which implies that the term in square brackets in (P.2.2) is zero, and at the wage which is set by the constrained trade union, implying that $h = h^C$. We now need to show that (P.2.2) cannot be negative for any $h \in [h^C, h^M]$ because $\hat{u}_i$ having a positive slope for any wage that results in hours of work between h^C and h^M establishes the positive impact of trade unions on workers' wellbeing. Observe that the expression in square brackets in (P.2.2) and (P.2.3) is decreasing in h_i and, therefore, increasing in the wage. If the expression in square brackets is negative, the function $\hat{u}_i$ is strictly increasing in the wage. Therefore, for all $h \in [h^\#, h^M]$, (P.2.2) has a positive sign. For $h^C > h^\#$, the above considerations provide the proof for part d). However, a priori, the sign of $h^\# - h^C$ is indeterminate. Let us assume, therefore, that $h^C < h^\#$, note that the sign of (P.2.3) will be negative for $h < h^\#$ and recall that (P.2.2) is positive at h^C and $h^\#$. In consequence, $\hat{u}_i$ is strictly concave for $h^C < h^\#$ and increasing in the wage at h^C and $h^\#$. Thus, the derivative $\frac{\partial \hat{u}_i}{\partial w_i}$ will be positive at all relevant wages.

8.3 Proof of Proposition 3

Evaluating (10) at the competitive working time h^M (cf. equation (6)), we obtain:

$$\left.\frac{\partial\Omega_i^U}{\partial h_i}\right|_{h_i=h^M} = -\frac{u_i(h^M,w_i)-\bar{u}}{h^M}+\frac{1+\rho}{h^M}-\frac{\lambda}{t-h^M} = -\frac{u_i(h^M,w_i)-\bar{u}}{h^M}<0 \quad \text{(P.3.1)}$$

Given the first-order condition (11), the numerator of (P.3.1) is positive and the trade union has an incentive to reduce working time to below h^M.

To prove part b), note that there are nine potential combinations of hours and wages, relative to the setting with a constrained union. Wages w_i^U, as implicitly defined by equations (10) and (11), can be below or above the constrained outcome or they can coincide ($w_i^C < w_i^U$, $w_i^C = w_i^U$, $w_i^C > w_i^U$). The same can be true for hours ($h^C < h_i^U$, $h^C = h_i^U$, $h^C > h_i^U$). If wages and hours are weakly less than in the constrained setting ($w_i^U \le w_i^C$; $h_i^U \le h^C$), the firm will employ more people than if the trade union were constrained and there will be no unemployment. Accordingly, either hours or wages must strictly exceed constrained levels. However, we can also show that situations in which wages are higher than in the constrained setting ($w_i^U > w_i^C$), while hours are weakly less ($h_i^U \le h^C$), provide incentives for the unconstrained union to lower wages. Thus, these two possible combinations cannot constitute the optimal unconstrained outcome either. To establish this claim, we substitute out the utility difference in (11), making use of (10) and setting $n_i = 1$. This yields:

$$\frac{\partial\Omega_i^U}{\partial w_i} = \frac{1}{\alpha_i f''(h_i E_i(h_i,w_i))}\left[\frac{1+\rho}{h_i}-\frac{\lambda}{t-h_i}\right]+E_i(h_i,w_i)\frac{1+\rho}{w_i}=0 \quad \text{(P.3.2)}$$

Next, we deduct the first-order condition (8) of the constrained union from (P.3.2) and evaluate the resulting expression at the constrained wage w_i^C, assuming that the third derivative of the production function is zero, which implies that $f''(h^C) = f''(h_i^U E_i)$.

$$\Delta\Omega := \frac{\partial\Omega_i^U}{\partial w_i}-\frac{\partial\Omega_i^C}{\partial w_i} = \frac{1}{\alpha_i f''}\left[\frac{1+\rho}{h_i^U}-\frac{\lambda}{t-h_i^U}-\frac{1+\rho}{h^C}-\frac{\lambda}{t-h^C}\right]+\frac{1+\rho}{w_i^C}\left[E_i-1\right] \quad \text{(P.3.3)}$$

Suppose next that h_i^U is weakly less than h^C ($h_i^U \leq h^C$). This implies that the first term in (P.3.3) is non-negative, as $\frac{1+\rho}{h} - \frac{\lambda}{t-h} > 0$. Since this expression is multiplied by $\frac{1}{\alpha_i f''} < 0$ < 0, the first term in (P.3.3) is non-positive. In addition, the last term in (P.3.3) is negative, given unemployment in the unconstrained setting. Accordingly, evaluating the derivative of the unconstrained trade union's objective with respect to the wage at the constrained wage and presuming $h_i^U \leq h^C$, we find this derivative to be negative for $f''' = 0$. If $f''' < 0$ holds, $0 > f''(hE_i) > f''(h)$ for $E_i < 1$ will result. This implies that $\frac{1}{f''(h_i^U E_i)} < \frac{1}{f''(h^C)} < 0$ for $h_i^U \leq h^C$ and, hence, that the first term in (P.3.3) is more likely to be negative than for $f''(h_i^U E_i) = f''(h^C)$. The above considerations imply that the unconstrained trade union will lower the wage – given the restriction $h_i^U \leq h^C$ – below the level set by the constrained union. In consequence, a combination of hours and wages characterised by $w_i^U > w_i^C$ and $h_i^U \leq h^C$ cannot maximise the unconstrained union's objective. Therefore, $h_i^U > h^C$ represents the only combination of hours which is not incompatible with the first-order conditions (10) and (11), profit-maximising firm behaviour and the existence of unemployment. This proves part b).

The proof of part c) of Proposition 3 is similar to that of part c) of Proposition 2. We denote the employment level in equation (11) by E_i^U, insert condition (11) into the derivative in (10) and evaluate the resulting expression at the optimal working time h_i^*. We then substitute $\frac{\alpha_i f'(h_i^*)}{c*}$ and $\frac{c_i^M(h_i^*)}{h_i^*}$ for $\frac{\lambda}{t-h_i^*}$ and $\alpha_i f'(h_i^*)$ respectively, to obtain $\rho_{i,crit}^U$.

8.4 Derivations for Additive Comparisons Model

All endogenous variables of interest in the setting with an additive specification of status concerns are denoted by a tilde. In order to compare working time in a utilitarian welfare maximum, resulting in a consumption level $\tilde{c}*$ for both types of individuals, and the market equilibrium, we follow the proof of Proposition 1c) (cf. Appendix 8.1). We first multiply the denominator of (13) by $\tilde{w}_i^M = \alpha_i f'(n_i \tilde{h}_i^M)$ and then subtract a slightly rewritten characterisation of optimal working time (cf. equation (4)).

$$\tilde{Z}_i(\tilde{w}_i^M, h_i) = \frac{1}{\tilde{h}_i^M \tilde{w}_i^M} - \frac{\lambda}{[t - \tilde{h}_i^M]\alpha_i f'(n_i \tilde{h}_i^M)} + \frac{\rho}{\tilde{w}_i^M \tilde{h}_i^M + K - \bar{c}}$$

$$- \frac{1}{\tilde{c}^*} + \frac{\lambda}{[t - \tilde{h}_i^*]\alpha_i f'(n_i \tilde{h}_i^*)} \tag{P.4.1}$$

Suppose now that working time in market equilibrium equals the optimal amount, $\tilde{h}_i^M = \tilde{h}_i^*$. Evaluating (P.4.1) at $\tilde{h}_i^*$, and using $\tilde{c}_i^M(\tilde{h}_i^*) = \tilde{w}_i^M(\tilde{h}_i^*)\, \tilde{h}_i^*$, we obtain:

$$\tilde{Z}_i(w_i^M(\tilde{h}_i^*), \tilde{h}_i^*) = \frac{1}{\tilde{c}_i^M(\tilde{h}_i^*)} - \frac{1}{\tilde{c}^*} + \frac{\rho}{\tilde{c}_i^M(\tilde{h}_i^*) + K - \bar{c}} \tag{P.4.2}$$

This expression will be positive, implying that there are incentives to increase labour supply beyond the level $\tilde{h}_i^M = \tilde{h}_i^*$, if the parameter ρ exceeds a critical level $\tilde{\rho}_{i,crit}$:

$$\tilde{\rho}_{i,crit} := \left[\frac{\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{c}^*} - 1\right]\left[1 + \frac{K - \bar{c}}{\tilde{c}_i^M(\tilde{h}_i^*)}\right] \tag{P.4.3}$$

If there is only one type of individual, consumption $\tilde{c}_i^M(\tilde{h}_i^*)$ at the optimal level of labour supply $\tilde{h}_i^*$ cannot exceed the welfare maximising level $\tilde{c}^*$ and $\tilde{\rho}_{i,crit} < \rho$ applies. Thus, parts b) and c) of Proposition 1 basically also apply for an additive comparisons approach.

Turning, next, to unionised labour markets, we again assume $n_1 = n_2 = 1$. The first-order condition for a constrained trade union is given by:

$$\frac{\partial \tilde{\Omega}_i^C}{\partial w_i} = \frac{\partial h_i}{\partial w_i}\left\{\frac{1}{h_i(w_i)} - \frac{\lambda}{t - h_i(w_i)} + \frac{\rho w_i}{w_i h_i(w_i) + K - \bar{c}}\right\} + \frac{1}{w_i} + \frac{\rho h_i(w_i)}{w_i h_i(w_i) + K - \bar{c}} = 0$$

$$\Rightarrow \left[1 + \frac{\rho \tilde{h}_i^C w_i}{\tilde{w}_i^C \tilde{h}_i^C + K - \bar{c}^C}\right]\left[1 - \tilde{\varepsilon}_i^C(\tilde{h}_i^C)\right] + \tilde{\varepsilon}_i^C(\tilde{h}_i^C)\frac{\tilde{h}_i^C \lambda}{t - \tilde{h}_i^C} = 0 \tag{8'}$$

In equation (8'), the union's (implicit) choice of working time is denoted by $\tilde{h}_i^C$, the reference level of consumption by $\bar{c}^C$, and the labour demand elasticity $\tilde{\varepsilon}_i^C(\tilde{h}_i^C)$ is greater than unity at the union's optimal choice of wages $\tilde{w}_i^C$.

If type 2 individuals work the same number of hours than type 1 individuals, their wages will be higher because of the productivity differential. However, the term in square brackets in (8') increases in wages, implying that the derivative of the trade union's objective for type 2 individuals evaluated at the optimal working time $\tilde{h}_1^C$ of type 1 individuals is negative because the labour demand elasticity $\tilde{\varepsilon}_i^C(\tilde{h}_i^C) = -\dfrac{f'(\tilde{h}_i^C)}{f''(\tilde{h}_i^C)\tilde{h}_i^C} > 0$ is independent of the productivity parameter α_i. Accordingly, wages of type 2 individuals have to fall to below the level which induces $\tilde{h}_2 = \tilde{h}_1^C$. Hence, individuals of type 2 work more hours than type 1 individuals ($\tilde{h}_1^C < \tilde{h}_2^C$). This prediction differs from the finding for the ratio comparisons model, in which working time is independent of productivity differentials.

To compare the market outcome to the situation resulting in the presence of constrained trade unions, suppose that unions set the wage equal to the competitive level ($\tilde{w}_i^C = \tilde{w}_i^M$). Therefore, labour demand would be the same ($\tilde{h}_i^C = \tilde{h}_i^M$). But if hours of work maximize utility at the market wage, the term in curly brackets in the first line of (8') will be zero at the reference level of consumption prevailing in a competitive labour market. Thus, trade unions have an incentive to raise wages beyond $\tilde{w}_i^M$. This implies that hours of work fall to below the competitive level.

Note further that the wage defined by equation (8') is decreasing with $\bar{c}^C$. In consequence, part b) of Proposition 2 also holds for an additive comparisons model and a reference level of consumption which is not higher than the in a competitive setting.

Evaluating the trade union's first-order condition at $\tilde{h}_i^*$ yields:

$$\left.\frac{\partial \tilde{\Omega}_i^C}{\partial w_i}\right|_{h_i = \tilde{h}_i^*} = \left[1 + \frac{\rho \tilde{h}_i^* w_i(\tilde{h}_i^*)}{w_i(\tilde{h}_i^*)\tilde{h}_i^* + K - \bar{c}^C}\right]\left[1 - \tilde{\varepsilon}_i^C(\tilde{h}_i^*)\right] + \tilde{\varepsilon}_i^C(\tilde{h}_i^*)\frac{\tilde{h}_i^* \lambda}{t - \tilde{h}_i^*} \qquad \text{(P.4.4)}$$

Substituting $\frac{\alpha_i f'(\tilde{h}_i^*)}{\tilde{c}_i^*}$ for $\frac{\lambda}{t-\tilde{h}_i^*}$ in accordance with equation (4) and $\tilde{c}_i^M(\tilde{h}_i^*)$ for $\alpha_i f'(\tilde{h}_i^*)\tilde{h}_i^*$ $= w_i(\tilde{h}_i^*)\tilde{h}_i^*$, (P.4.4) collapses to:

$$\left.\frac{\partial\tilde{\Omega}_i^C}{\partial w_i}\right|_{h_i=\tilde{h}_i^*} = \left[1+\frac{\rho\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{c}_i^M(\tilde{h}_i^*)+K-\bar{c}^C}\right]\left[1-\tilde{\varepsilon}_i^C(\tilde{h}_i^*)\right]+\tilde{\varepsilon}_i^C(\tilde{h}_i^*)\frac{\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{c}_i^*} \qquad \text{(P.4.5)}$$

(P.4.5) will obviously be positive for $\tilde{\varepsilon}_i^C(\tilde{h}_i^*) < 1$ and have the same sign for $\tilde{\varepsilon}_i^C(\tilde{h}_i^*) > 1$ if $\rho < \tilde{\rho}_{i,crit}^C$, where $\tilde{\rho}_{i,crit}^C := \left[\frac{\tilde{\varepsilon}_i^C(\tilde{h}_i^*)}{\tilde{\varepsilon}_i^C(\tilde{h}_i^*)-1}\frac{\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{c}^*}-1\right]\left[1+\frac{K-\bar{c}^C}{\tilde{c}_i^M(\tilde{h}_i^*)}\right]$. This establishes the qualitative equivalent to Proposition 2, part c).

Proposition 2, part d), finally, asserts that each worker will be better off in comparison to the market outcome. In an additive comparisons model, trade union wage setting increases utility for a given reference level of consumption above the level resulting in a competitive environment. If that were not the case, the trade union would simply retain the competitive wage-hours combination. Because utility decreases with the reference level of consumption, part d) of Proposition 2 will also hold for an additive comparisons model if the reference level of consumption does not rise above the level resulting in a competitive labour market.

The first-order conditions of an unconstrained trade union are given by:

$$\frac{\partial\tilde{\Omega}_i^U}{\partial h_i} = -\frac{u_i(h_i,w_i)-\bar{u}}{h_i}+\frac{1}{h_i}-\frac{\lambda}{t-h_i}+\frac{\rho w_i}{w_i h_i+K-\bar{c}^U}=0 \qquad (10')$$

$$\frac{\partial\tilde{\Omega}_i^U}{\partial w_i} = \frac{u_i(h_i,w_i)-\bar{u}}{h_i\alpha_i f''(h_i E_i(h_i,w_i))}+E_i(h_i,w_i)\left[\frac{1}{w_i}+\frac{\rho h_i}{w_i h_i+K-\bar{c}^U}\right]=0 \qquad (11')$$

Evaluating (10') at the competitive working time $\tilde{h}_i^M$ (cf. equation (13)), we obtain an analogue to (P.3.1). Thus the derivative (10') is negative for $\tilde{h}_i = \tilde{h}_i^M$ and the trade union has an incentive to reduce working time to a level below $\tilde{h}_i^M$ for a given reference consumption. Since, moreover, the derivative in (10') rises with the reference level of consumption $\bar{c}^U$, these considerations establish the validity of Proposition 3, part a) for an additive comparisons model and $\bar{c}^U \leq \bar{c}$.

To prove the analogue of part b) of Proposition 3, we follow the procedure applied in Appendix 8.3. Hence, we show that in situations in which wages are higher than in the constrained setting ($w_i^U > w_i^C$), while hours are weakly less ($h_i^U \leq h^C$), the unconstrained union prefers lower wages. We can substitute out the utility difference in (11'), making use of (10'). This yields:

$$\frac{\partial \tilde{\Omega}_i^U}{\partial w_i} = \frac{\partial E_i}{\partial w_i} h_i \left[\frac{1}{h_i} - \frac{\lambda}{t-h_i} + \frac{\rho w_i}{w_i h_i + K - \bar{c}^U} \right] + \frac{E_i(h_i, w_i)}{w_i} + \frac{E_i(h_i, w_i)\rho h_i}{w_i h_i + K - \bar{c}^U} = 0 \quad \text{(P.4.6)}$$

Next, we deduct (8') from (P.4.6), using $\frac{\partial E_i}{\partial w_i} h_i = \alpha_i f''$, and assuming f ''' = 0.

$$\Delta\tilde{\Omega} := \frac{\partial \tilde{\Omega}_i^U}{\partial w_i} - \frac{\partial \tilde{\Omega}_i^C}{\partial w_i} = \tilde{E}_i \left[\frac{1}{\tilde{w}_i^U} + \frac{\rho \tilde{h}_i^U}{\tilde{w}_i^U \tilde{h}_i^U + K - \bar{c}^U} \right] - \left[\frac{1}{\tilde{w}_i^C} + \frac{\rho \tilde{h}_i^C}{\tilde{w}_i^C \tilde{h}_i^C + K - \bar{c}^C} \right]$$

$$+ \frac{1}{\alpha_i f''} \left[\frac{1}{\tilde{h}_i^U} - \frac{1}{\tilde{h}_i^C} - \frac{\lambda}{t - \tilde{h}_i^U} + \frac{\lambda}{t - \tilde{h}_i^C} + \frac{\rho \tilde{w}_i^U}{\tilde{w}_i^U \tilde{h}_i^U + K - \bar{c}^U} - \frac{\rho \tilde{w}_i^C}{\tilde{w}_i^C \tilde{h}_i^C + K - \bar{c}^C} \right] \quad \text{(P.4.7)}$$

We evaluate the resulting expression at $\tilde{w}_i^C$ and suppose additionally that $\tilde{h}_i^U \leq \tilde{h}_i^C$ holds.

$$\Delta\tilde{\Omega}\Big|_{\tilde{w}_i^C = \tilde{w}_i^U, \tilde{h}_i^C \geq \tilde{h}_i^U} = \underbrace{\frac{\tilde{E}_i - 1}{\tilde{w}_i^C}}_{<0} + \underbrace{\frac{\tilde{E}_i \rho h_i^U}{\tilde{w}_i^C \tilde{h}_i^U + K - \bar{c}^U} - \frac{\rho \tilde{h}_i^C}{\tilde{w}_i^C \tilde{h}_i^C + K - \bar{c}^C}}_{\leq 0 \text{ for } \bar{c}^U \leq \bar{c}^C}$$

$$+ \underbrace{\frac{\frac{1}{\tilde{h}_i^U} - \frac{1}{\tilde{h}_i^C} - \frac{\lambda}{t - \tilde{h}_i^U} + \frac{\lambda}{t - \tilde{h}_i^C}}{\alpha_i f''}}_{<0} + \underbrace{\frac{\rho \tilde{w}_i^C}{\alpha_i f''}}_{<0} \left[\frac{\tilde{w}_i^C (\tilde{h}_i^C - \tilde{h}_i^U) + \bar{c}^U - \bar{c}^C}{(\tilde{w}_i^C \tilde{h}_i^U + K - \bar{c}^U)(\tilde{w}_i^C \tilde{h}_i^C + K - \bar{c}^C)} \right] \quad \text{(P.4.8)}$$

Therefore, the first-order condition of the unconstrained trade union with respect to the wage will be negative if evaluated the constrained union's optimal wage and if additionally the reference level of consumption is given ($\bar{c}^U = \bar{c}^C$) and working time is less than the level chosen by the constrained union. Hence, the unconstrained trade union has an incentive to reduce the wage to below a level $\tilde{w}_i^C$, given $\tilde{h}_i^U \leq \tilde{h}_i^C$. Since lower wages and hours than in the constrained setting are not compatible with unemployment, the only feasible equilibrium is one in which hours of work in the unconstrained setting are higher than in the framework in which the trade union is constrained to choose ensure full employment. Therefore, Proposition 3b) also holds for an additive comparisons setting.

The proof of part c) of Proposition 3, applied to a setting based on additive comparisons, follows Proposition 2. We denote employment in equation (11') by $\tilde{E}_i$, insert condition (11') into the derivative in (10') and evaluate the resulting expression at the optimal working time $\tilde{h}_i^*$. We then substitute $\frac{\alpha_i f'(\tilde{h}_i^*)}{\tilde{c}^*}$ in accordance with (4) for $\frac{\lambda}{t-\tilde{h}_i^*}$ and $n_i = 1$, and $\frac{\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{h}_i^*}$ for $w_i^* = \alpha_i f'(\tilde{h}_i^*)$, to obtain $\tilde{\rho}_{i,crit}^U$, where $\tilde{\varepsilon}_i^U(\tilde{h}_i^*) := -\frac{\partial \tilde{E}_i}{\partial w_i}\frac{w_i^U}{\tilde{E}_i} = -\frac{\tilde{w}_i^U}{\alpha_i f''(\tilde{E}_i \tilde{h}_i^*)\tilde{E}_i \tilde{h}_i^*}$.

$$\tilde{\rho}_{i,crit}^U(\tilde{h}_i^*) := \frac{\tilde{w}_i^U \tilde{h}_i^* + K - \bar{c}^U}{\tilde{w}_i^U \tilde{h}_i^*}\left[\frac{\varepsilon_i^U(\tilde{h}_i^*)}{\varepsilon_i^U(\tilde{h}_i^*)-1}\frac{\tilde{c}_i^M(\tilde{h}_i^*)}{\tilde{c}^*}-1\right] \tag{P.4.9}$$